working paper
department
of economics

ENDOGENOUS CITY-SUBURB GOVERNMENTAL RIVALRY
THROUGH HOUSEHOLD LOCATION

by
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Number 190
March, 1976

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Through Household Location

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M.I.T.

I. Introduction

The subject matter of this paper is described closely in the title. We construct a model of endogenous local government decision making for both a central city and a suburban jurisdiction in a single metropolitan area. In this model the goals of public policy for each can be advanced by competing with the other to attract households - some households - to reside within the jurisdiction. But changes in location behavior by households in turn call for changes in governmental action to realize their goals. Thus, both government behavior and household location behavior are reciprocally endogenous. The key elements in the model are the specification of policy goals for the two jurisdictions, the submodel by which household location influences government action, and the submodel by which government action influences household location.

Despite notions about governmental goals, the approach is thoroughly individualistic: public goals are defined in terms of constituent welfare, and each local government is assumed to be a perfectly responsive representative mechanism. This approach is meant to perform two functions: to characterize behaviorally some real world complexities affecting the relations between local government and its constituents and among local governments; to contribute to the normative issue of the optimal degree of governmental fragmentation at a metropolitan area level. The normative issue is part of an extended discussion stemming from the classic Tiebout
model in which radical local fragmentation assertedly serves to make the public sector partake of the properties of a purely competitive market system in private goods.

The thrust of the model developed here is that the local public sector is a distinctive system. Its "publicness" is not diluted in the context of competitive relationships among governmental decision makers; rather it impresses a special configuration on the competitive process. The consequences do not lend an easy belief that fragmentation helps to integrate public and private sectors so as more closely to approximate an efficient use of resources. Quite the contrary, fragmentation of public decision making creates a set of "artificial" incentives for households which leads to systematic discrepancies from optimal resource use. This is established by first developing a model of a single metropolitan government, one in which household location patterns arise out of pure spatial distribution considerations. The consequences of such a model are then contrasted with the model of jurisdictional fragmentation, which is the main business of the paper.

The foregoing suggests that the paper is primarily normative. In fact, it is positive. The chief goal is to throw light on a number of complex relationships between the public and private sectors believed to be operative in metropolitan areas, and which have had little systematic exposure. We have deliberately attempted to create a more complicated, richer web of interrelationships than is usual either in local public finance or urban economics. The model in its totality is not tractable for elegant, analytical solutions. It is more amenable to simulation

\[1/\] See Roybout [27] and, for example, [5], [6], [9], [13], [19], [20], [21], [22].
for characterization of overall system outcomes. But several large submodels are detachable and capable of being analytically manipulated to reveal their properties. And certain broad qualitative judgments can be inferred as well for the larger system.

The paper will for the most part elaborate the model informally, explaining the nature of the relationships and their basic function in the overall system. Some qualitative conclusions will, similarly, be informally drawn from the model, results that bear on the normative issue noted above.

II. Preliminary Assumptions

The metropolitan area is assumed to be divided into two political jurisdictions, a central city and a suburb. The central city is totally surrounded by the suburb and its whole area is developed for urban use. The suburb also has a fixed area, but only part of it is developed for urban use in the initial period. Changes in the urban area depend endogenously on the amount of land that households are willing to buy for urban use.

Each jurisdiction is politically administered by a general form of local government, which provides the variety of so-called local public goods — streets, sewers, police and fire protection, the administration of law, public primary and secondary education, welfare services, health services, recreation, etc. In this model they are assumed to be offered in fixed proportions — with one class of exceptions to be noted below — and so can be treated as components of a single composite public good.
This public good is not a Samuelsonian "pure" public good. It suffers from a generalized form of "crowding". Thus, we can distinguish between the provision of the "stock" from which services will be rendered - which is an intermediate good - and the services themselves - which are the final outputs. The level, or quality, of services rendered to each household is equal for all households - except as related to the class of exceptions mentioned above and to be discussed below - and depends on both the size of the stock provided and the number of households who have to share services from it. In general the quality is a positive function of stock size and a negative function of population size.

In addition to this distinction between intermediate good and output we make as well a distinction between the input (resource) cost of the intermediate good and the amount of the intermediate good furnished. Thus, unlike most treatments, we do not measure public service output by input costs; they are separated by both the input cost of intermediate output and the intermediate cost of final output.

Both jurisdictions produce the same composite public services, but not necessarily at the same level. Expenditures to provide these services are raised by a set of two different kinds of taxes in the suburban jurisdiction and three different kinds in the central city. Common to both are a proportional tax on household income and an ad valorem tax on land. In addition to these the central city government uses a set of user charges. The reason for this asymmetry will be discussed below.

Two other public policy instruments are examined in this model: a business zoning and a residential zoning regulation. These are assumed
to be possessed by the suburban government but not by the central city government. The reason for this asymmetry will similarly be discussed below.

Finally, the model concentrates on the local public sector and the location decisions of households, but it also deals - albeit in a deliberately oversimplified way - with the location decisions of businesses. It is assumed that the suburban government designates a maximum permissible business acreage to be allowed in the suburbs - its business zoning, $Z^B$.

In the relevant range of situations dealt with, this represents less land than the metropolitan area business firms would like to occupy there. The aggregate zoning restriction is therefore binding. If the suburban government wishes more business activity in the suburb, it thus has only to increase the allowable maximum acreage. Business location is thus effectively controlled by suburban government use of its business zoning instrument.

III. The Location Decision of the Individual Household: Abstract Stage

We assume there are $N$ households in the urban area. Throughout this exercise we assume this number unchanged, although the model can easily be developed for changing population size. The purpose of the constant population treatment is to throw maximum light on the determinants of the share of the total which goes to each jurisdiction. The number in the central city is $N_1$, in the suburbs $N_2$. Expressed as percentage shares they are $N_1$ and $N_2$, respectively. The total amount of business activity is $B$, assumed to be distributed spatially in business acres of constant density. Variable density of business concentration is given
by percentage of a jurisdiction's acres which are business-occupied, not by differences in business activity per acre. This simple treatment is meant for convenience. Little of substance is involved, so long as the business location decision is the incomplete one already described.

B is divided into $B_1$ and $B_2$, the amount of business activity in city and suburb, respectively. The corresponding relative shares of the assumed constant total $B$ are $b_1$ and $b_2$.

We assume that all households have the same utility function, but differ only with respect to income level. This is tantamount to assuming away differences in demographic structure - size, age, ethnic group, education, job location, special consumption tastes, etc. The reason for abstracting from these known influences on location is to highlight the effect that income alone has and thus to gain some sense of the degree to which observable spatial patterns are understandable in terms of income alone. Further household differences can be subsequently superimposed onto the abstract income treatment to elaborate real world complexities, and thus examine spatial distributions stepwise.

The utility function for household $j$ is:

$$u_j = u[H, L, X, C, D].$$

$H$ is the amount of housing capital - or structure - consumed. $L$ is the amount of land used as a residence site consumed. $H$ and $L$ together reflect the amount of housing services consumed. These two components of the housing package are separated because some of the public policy instruments affect the two differently - indeed, are intended to do so. $X$ is the amount of all other private goods consumed - with their relative
quantities remaining unchanged (along with their relative prices); the constancy of relative shares enables us to treat X as a composite commodity. G is the quality level of public services consumed.

D is a different kind of "commodity" than any of these others. It is the crowdedness of the "social space". This variable is meant to portray more than simply the average density of land use - i.e., number of structures (or people) per acre. Individuals carry on market, social and other activities in public places - they use streets, sidewalks, shops, playgrounds, various public and private facilities. The density of others' intersecting uses of these same spaces contributes to the crowdedness, the press and tempo of the lifestyle of a household. We assume that the tastes of this population make this a negative commodity - on balance people prefer uncrowdedness. Moreover, it is a superior negative good: richer households dislike crowdedness more than poorer households.

Of the five commodities listed, four are location-specific and spatially linked. The two housing components are consumed in one place, and they determine the jurisdiction in which the household resides and thus the local public services that are obtainable only by being a resident. They also determine the home neighborhood and larger community whose character has great influence on the crowdedness of a household's life space. Only X can be consumed anywhere - and in many places within each time period - throughout the urban area.

The household is subject to a budget constraint in making its choice of an optimal budgetary allocation among these commodities. It is as follows:

2/ In the real world, individual tastes for crowdedness of the social space appear to differ markedly, being a positive good for some. The present treatment is based on the presumed average attitude in postwar urban America.
\[
Y_j^i + (P_{L_i} - P_{L_j}) L_{ij} = \tau_i Y_j^i + (tm)_j + P_{L_i} (1 + \tau_{L_i}) L_i + P_H + P_X.
\]

\(Y_j^i\) is the income level of household \(j\) residing in jurisdiction \(i\).

Actually, we assume that jurisdictional residence does not itself affect income level. (This is different from the fact that we later establish that there comes to be a systematic income stratification between the two jurisdictions - so that by self-selection the average income characteristics of the two jurisdictions come to differ.) \(\tau_i\) is the proportional income tax rate in jurisdiction \(i\). \((tm)_j\) together are the expected total annual travel costs from \(j\)'s chosen location. \(t\) is the travel cost per mile and \(m\) the expected annual travel miles. \(P_{L_i}\) is the price of land in \(i\), \(\tau_{L_i}\) is the land tax rate in \(i\), and \(L_i\) is the amount of land in \(i\) consumed. \(P_H\) is the price of housing capital per quantity unit and \(H\) the amount of housing capital consumed. \(P_X\) is the price of the composite private good and \(X\) is the amount of that good consumed. Finally, \(P_{L_{ij}}\) is the price of land in \(i\) at the time when household \(j\) first became a resident of \(i\) and \(L_{ij}\) is the amount of land bought by \(j\) at that time.

The budget constraint says that total purchasing power at the command of household \(j\) residing in jurisdiction \(i\) is exhausted in outlays due to income tax liability \((\tau_i Y_j^i)\), travel expense \((tm)_j\), land use costs \((P_{L_i} - \tau_{L_i}) L_i\), use of housing capital \((P_H)\), and consumption of the composite private good \((P_X)\). The income tax rate is a pure rate per dollar of income \((\tau_i)\). For travel expense, the expected annual travel miles \(m\) depends primarily on work and shopping trips. These depend on the distribution of business activity between central city and suburb \((B_1, B_2)\).
and the location of household j's residence - measured in terms of its distance from the nearest point of the central city (k): i.e.,

(3) \[ m = m(k, B_1, B_2) \]

Travel cost per mile, t, is assumed constant mostly for convenience.^[3/]

This treatment of location stems from the assumptions that the city has considerable more business activity than the suburb and that, unlike in the suburb, business activity is uniformly distributed. Thus, any location within the city (k = 0) has equal accessibility (expected travel costs) and total expected travel miles depends on distance from this city concentration.

Land use costs include the annual rental cost of land (an imputed rental, since every household is assumed to own the land it occupies - price per unit \( P_{L_1} \) times amount \( L_1 \)) and the annual liability of the land tax (tax rate \( t_{L_1} \) times price \( P_{L_1} \cdot L_1 \)).

The unusual aspect of this budget constraint is that total available purchasing power in each period is assumed to consist of that period's income earned \(^1\) and that period's share of imputed accumulated capital gains on land. Every household is assumed to own the land which renders it annual services. Capital gains on the land are a product of dynamic adjustments. The model has both a comparative static and a dynamic facet.

[^3/]: This is tantamount to including only out-of-pocket costs and assuming a single travel mode. This is not so much at variance as it seems with the oft-presumed importance of travel time, with its supposed large utility differences for different income groups. In a system with different travel modes, richer households are likely to choose higher money cost, quicker modes where long trips are involved, poorer households choosing lower money cost but slower modes. Our single, constant "cost" mode represents a fair compromise to this trade-off.
This stems from a distinction between a short and long-run adjustment process. This will be discussed below. These capital gains are an important link between constituents' welfare and the use of public policy to influence the long-run location of business and households. It will be seen that locational changes by others have an effect on each household's welfare as a consumer of the commodities listed above (by affecting their size and prices); such changes also affect their welfare as owners of the land that makes up the jurisdiction. Such changes in the asset values of their initial holdings of land increase or decrease their present period purchasing power over the commodities in their utility function. Since these gains are exempt from the income tax, they directly increase or decrease utility—enhancing consumption.

The capital gains are measured as the difference between present land values in the jurisdiction and those that prevailed when the household first became a resident of the jurisdiction, on the amount of land which the household bought at that beginning period.

In the budget constraint, each of the commodities consumed has a price except for crowdedness of life space (D). In fact, this last has an implicit price as well. We shall discuss it below.

Given these commodity preferences and price-income constraints, household \( j \) selects that bundle which maximizes its utility. In selecting a bundle, some elements can be chosen in fine gradations, while others involve an all-or-nothing choice. The composite private good, the amount of land and housing capital can be varied very finely. On the other hand, once a particular piece of land has been selected, a jurisdiction has been also. The household then passively has to accept both the one-and-only-one
level of crowdedness existing there (by assumption) and the one-and-only-one level of public services provided — both being outside the household's ability to influence them.

Thus, the choice can be looked on as proceeding from the following two stage process: household \( j \) considers the utility-maximizing combination of goods attainable from a city location (given the budget constraint) and that attainable from a suburban location, and then selects that bundle which gives the higher utility level. So the choice in effect simultaneously selects an optimal location and an optimal amount of the different commodities (considering the all-or-nothing constraint on crowdedness and public service level).

IV. The Location Decision of the Individual Household: Metropolitan Government

A major thrust of this paper is to contrast the consequences of metropolitan government and jurisdictional fragmentation. This is begun by examining how the aforementioned household location decision proceeds under the two situations. For this task we must clothe the highly abstract procedure above in particularities of the city-suburb choice.

Under metropolitan government, the level of public services provided and the effective tax rates are the same for all households: i.e.,

\[
G_1 = G_2, \quad \tau_1 = \tau_2, \quad t_{L_1} = t_{L_2} \quad (\text{subscripts always use 1 for the city, 2 for the suburb}).
\]

Moreover, there is no use of zoning to discriminate the use of residential land. Having assumed that business is disproportionately located in the city, the chief basis for selecting city vs. suburban location is simply the choice between the resulting lower crowdedness of the suburb vs. the greater accessibility of the city. This depends on the price of lower crowdedness relative to its desirability.
We have not yet spoken of a price for lower crowdedness. Such a price is implicit in the working of the land market. We shall discuss the land market in detail below; but one relevant aspect of it needs to be introduced at this time. Accessibility to desirable destinations is a positive good so long as overcoming distance incurs real costs. Since business destinations are heavily concentrated in the city portion of the area and there is no costless way to increase the supply of land with high accessibility (by increasing accessibility on less accessible land), a gradation of accessibility will come to constitute a dimension of the "quality" of land. Therefore, no one would be willing to pay the same price for an inferior (less accessible) piece of land as for a superior (more accessible). As a result, a competitive process will bid up the price of city land above that of the suburb - and lots in the suburb that differ in accessibility (by k) will have prices that inversely reflect these differences. In short, the land market will capitalize quality differences stemming from accessibility differences.\(^4\)

Differences in accessibility within the suburb - but not within the city (by our earlier assumption concerning k in the city) - will lead to different prices within the suburb. Previously we have spoken of a single price for land in suburb and city, \(P_{l_2}^{L_2}\) and \(P_{l_1}^{L_1}\). While the city does have one price, the suburb has a different one at each accessibility level k. Use of an aggregate refers to the mean for the suburb.

If uncrowdedness is not a valued characteristic of a living area, then city and suburban land will be occupied, but at different prices.

\(^4\) This formulation has a classic presentation in Alonso [1]. Other important treatments are Wingo [28], Muth [17], Herbert and Stevens [10], Kain [11].
The mean difference is $P^o_{L_1} - P^o_{L_2}$, and the population divides with $N^o_1$ in the city and $N^o_2$ in the suburb. This represents a "neutral", base level price differential. Now suppose uncrowdedness becomes valued. Still at the earlier price differential, suburban residence now looks more attractive, so migration from city to suburb will occur, raising $P_{L_2}$ and lowering $P_{L_1}$ until a new set of price differentials is achieved that balances the attractiveness of higher $D$ with the new lower price differential for the least eager - the marginal - migrant. The new, lower average differential is $P^*_{L_1} - P^*_{L_2}$, and at this level $N^o_2$ has increased to $N^*_2$, $N^o_1$ has declined to $N^*_1$. The decline in the price differential between any lot in the suburb - and thus the average lot - and a lot in the city is a measure of the price of lower crowdedness (i.e., the difference between $D_1$ and $D_2$):

$$P_D = (P^o_{L_1} - P^o_{L_2}) - (P^*_{L_1} - P^*_{L_2})$$

$P_D$ depends, of course, on the size of $N^o_1$, $N^o_2$ and the distribution of preferences for $D$ among the whole population (i.e., it depends on the value of the particular differential $D_1 - D_2$ on the part of the marginal mover, both determined simultaneously by the competitive process).

The simple location decision under metropolitan government can therefore be looked on as involving the balance between the attractiveness of less crowded living and the price one has to pay for it. Those house-

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5/ The identity of the $N^o_2$ is not random. The trade-off involved is travel costs for land costs. Households buying larger lots are likely to benefit more from the trade than those buying smaller lots and so will be the households migrating under the differential price. Thus, each household will be only marginally different between city and suburb: everyone would lose if the assignment between city and suburb were reversed.
holds who place a high valuation on noncrowdedness relative to the larger amount of travel cost that becomes uncompensated through capitalization in land prices are likely to move to the suburb. There is some small loss to higher relative to lower income households in the lower land-price benefit relative to travel cost, but we assume that noncrowdedness is a commodity with high income elasticity\(^6\) - i.e., utility valuation rising rapidly with income level - so on balance richer households are highly likely to be the ones who outbid others for noncrowdedness.

This self-selected distribution of the population between city and suburb is in an import sense a "natural" one: it represents the voluntary balance of preference and technical trade-offs between land-use density and accessibility - which represent naturally competitive facets in the deployment of resources over space. Of course the outcome is not unique. It can be changed by a different distribution of business activity over the area, since this determines the pattern of travel costs and differential crowdedness. This in turn can be influenced by a change in business zoning. We shall consider this more specifically in the fragmented government case to follow.

V. The Location Decision of the Individual Household: Fragmented Government

City and suburb are now different political jurisdictions. Since public action is deemed to respond to the wants of the constituent population, and since the two jurisdictions can have populations that differ in both tax base and tastes (i.e., relative commodity preference differences

\(^6\) Consistent with many empirical studies. See, for example, Muth [17], Mills [14] and [15], Niedercorn [18].
for households with the same utility function but different income levels), both service levels and effective tax rates can differ. So such differences can now affect household location decisions since they enter both into the utility function (eq. (1)) and budget constraint (eq. (2)). Moreover, these very location decisions will in turn affect the governmental decisions about public service levels and effective tax rates, since they change tax base and tastes in the two jurisdictions.

A. The Demand for Public Services

In order to trace the effect of population composition on public choice, we must develop a concept of the demand for public services. We assume a responsive government, one that registers the desires of its electorate by majority rule. Moreover, the single dimensional expenditure side—i.e., the level of public services, G—and the relatively simple tax side of public action seem appropriate to permit the assumption that voter preferences among service level and tax alternatives are single-peaked. This assumption is not essential, but it is convenient to enable us to express the demand relationship transparently.

Under a set of single-peaked preferences, majority rule outcomes for a population of some magnitude are in principle completely determinate,

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7/ Single-peakedness is a property of the set of voter preferences such that everyone's preferences can be registered on the same single spatial array of alternatives in terms of a most preferred position and a set of spatial relations such that relative distances in the same direction from this preferred position are isomorphic with order of preference, and distance itself becomes a measure of preference intensity. Thus, while all individuals may have different preferences toward a set of alternatives, they all nonetheless share a common perspective about the structure of the differences among these alternatives. See Black [4] and Arrow [2].

8/ To avoid strategic misrepresentation of preferences in the course of voting.
and they form a transitive set of social preference orderings. In explicit or implicit elimination - contest voting procedures, the winning vote alternative will be that represented by the median most preferred position (on the compatible spatial array mentioned above). Given the regularity of preferences with income level in our present model, this can be approximated by the most preferred preference position of the median voter - in turn, of the median income recipient in each jurisdiction.

The demand for public service level by any household is discovered by examining the optimal budget allocation for the household under utility maximization. The hypothetical situation is to suppose that the household is faced with a given price for all of the commodities that enter its utility function, both private and public, and a specified level of D (different amounts cannot be chosen or even directly voted for), and asked how much of each it would buy - if it were free to choose any quantity it liked of every one of them, private and public alike. This is meaningful for the level of public services because this was defined as the quality of services as experienced by each household: i.e., it represents an "own consumption" level. The hypothetical question is answered by deriving the familiar first-order conditions for utility maximization, which call for marginal rates of substitution (ratios of marginal utilities) equal to price ratios for each pair of commodities. For variations in the prices of own and other commodities, in the level of D, and in the household's income level, it traces out the conventional commodity demand function. This demand function, interpreted as the amount of public ser-

\[9/\] See Black, loc. cit., and Arrow, loc. cit., for discussion of inferences from single-peaked preference voting situations.
ices a particular household would most favor under each particular configuration of income and prices, is as follows:

\[ G_j^i = G(Y_j^i, P_{X_i}, P_{H_i}, P_{L_i}, P_{D_i}, D) \]

We assume \( G \) is a normal good, so that if its price falls or income rises, more of it is wanted. What remains to be defined is the price of public services. It is the amount of total tax liability per unit of public services provided. Since there are two different taxes, this is as follows:

\[ P_j^i = (\gamma_j^i Y_j^i + t_{L_i} P_{L_i}) ; G_i \]

Notice that this is a price personal to household \( j \), because the two taxes are weighted by the specific tax bases applicable to household \( j \) - its income level and its purchase of land services (remember that every household owns the land it occupies and is thus directly liable to the land tax).

Since \( L_i \) is in principle determinate, given the other variables in the demand function, we can substitute (6) into (5) for the median income recipient in jurisdiction \( i \):

\[ G_i = \hat{G}_i = G(\hat{Y}_i, \tau_i, t_{L_i}, P_{X_i}, P_{H_i}, P_{L_i}, P_{D_i}, D) \]

In general \( G_i \) will rise if \( \hat{Y}_i \) rises, will fall if any of the rest rise (\( D \) is defined as a negative commodity).

The two tax rates are not unconstrained. In this model it is assumed that the government must balance its budget in every period. Total revenue from given tax rates \( \tau_i \) and \( t_{L_i} \) are:
\begin{align}
(8a) & \quad T_1 = \tau_1 \bar{Y}_1 N_1 + \tau_{L1} P_{L1} A_1 \\
(8b) & \quad T_2 = \tau_2 \bar{Y}_2 N_2 + \int_0^K t_{L2} P_{L2}(k) a(k) \, dk
\end{align}

$T_1$ is total revenue in the city

$T_2$ is total revenue in the suburb

$\bar{Y}_1, \bar{Y}_2$ are mean household income levels in city and suburb, respectively

$A_1$ is the total number of privately owned acres in the city

$K$ is the distance from the city border to the suburb border

$a(k)$ is the number of privately owned acres at distance $k$ from the city border.

The formulations differ for city and suburb because, by assuming that accessibility is equal for all lots in the city, all have the same price; while lots in the suburb differ in accessibility (as a function of $k$) and so differ in price correspondingly.\footnote{As we noted at the beginning, not all of the suburb's area is developed for urban use. Nonurban use does not depend on urban accessibility. Therefore, we assume that from $k = k^*$, the boundary separating urban from nonurban use, to $k = K$, the jurisdictional boundary, land price is not a function of $k$ and can be treated as a constant, $P_F$. A more explicit statement of (8b) is therefore:}

Total expenditures - which must balance total revenues - depend in the general case on two considerations: (1) the quality level of public services provided, $G_i$; (2) the number of households. Public services are neither pure private goods nor pure public goods: i.e., it is in general inappropriate to assume that total costs of providing each unit

\[ (8b') \quad T_2 = \tau_2 \bar{Y}_2 N_2 + \int_0^{k*} t_{L2} P_{L2}(k) a(k) \, dk + \int_0^{k*} t_{L2} P_{L2}(A_2 - \int_0^{k*} a(k) \, dk) \]

$A_2$ is the total private area of the suburb, so the last term is the revenues from the nonurban portion and the middle term is the revenues from the urban portion.
level of services to the total population is either simply the cost of providing that unit to each member of the population times the number of members (pure private) or the same regardless of the number of members (pure public). Consumption of public services is "shared consumption". Provision of the capacity to provide services to one member constitutes a capacity to provide services to others as well, since each member's "consumption" does not exhaust a particular set of resources used exclusively for that member's consumption. So total costs are less than per member cost times number of members. But various forms of "crowding" or inter-user interference make the quality of service fall to each as the same capacity is pressed to provide services to a larger and larger population. So total expenditures necessary to provide a given quality level rise as the population size rises.\footnote{11}

In addition to the influence of population size, the quality level of services imparts an unambiguous impact. Higher quality levels require higher total resource costs. So in the general case the total expenditure function (measure of real resource cost) can be shown as:

\begin{align}
(9) \quad E_1 &= E(N_1, G_i) \\
&= \frac{\partial E_1}{\partial N_1} > 0, \quad \frac{\partial E_1}{\partial G_i} > 0 \quad (b) \quad \frac{\partial^2 E_1}{\partial N_1^2} > 0 \quad (c) \quad \frac{\partial E_1}{\partial N_1} < 0 \text{ for } \frac{N_1}{N_i} < 0 \\
&= 0 \text{ for } N_1 = N_i \\
&> 0 \text{ for } N_i > N_1 \leq N
\end{align}

Properties (a) call for rising populations to increase E if some G_i is to

\footnote{11} For a discussion of some of the properties of pure public, pure private and quasi-public (mixed) goods, see Samuelson [24], Musgrave and Musgrave [16], Buchanan [7].
be maintained, and quality of services to increase E for any population size. Property (b) establishes the "crowding" of quasi-public goods as involving (very) small cost-augmenting impacts for small populations but rising disproportionally as population increases. Property (c) indicates that there are economies and diseconomies of population scale in providing the quasi-public goods. In the application being presented, we suppose that scale economies are exhausted at a quite low jurisdiction size (i.e., \( N_1 \) is small).

Central city and suburb are both subject to this expenditure function. We assume the budget balancing procedure to be as follows: at the beginning of the period a level \( G_1 \) is decided upon, for a given constituent population, \( N_1 \). This determines an expenditure total to be met out of tax revenues. A prior policy decision - to be explained below - has set the relative amounts to be raised by income and land taxes. Given the jurisdictional population and its income distribution, the mean income level is determined and thus the income tax rate necessary to raise the planned amount assigned. Similarly, the population and its income distribution, together with the calculated income tax rate, simultaneously determine the set of optimal demands for land and the land tax rate consistent with this set and with the total revenue goal assigned. Since this depends on the detailed working of the land market and we have not yet spelled that out, we shall not present here the explicit mathematics of this solution process.

B. Jurisdictional Differences in the Supply of Public Services

What we described above in equation (9) was "the general case" of the cost function for public services. In the present model there are
special features that distinguish the situation of local goods provision in a metropolitan area. Built onto the basic land-use asymmetry between city and suburb as described in the metropolitan government case above (namely, the specialized difference in land-use density and distribution of business activity) are a derived set of differences stemming from the existence of jurisdictional fragmentation. They fall into three categories: (1) differences in tax base, (2) differences in welfare load, (3) differences in interjurisdictional externalities. We shall argue that in the context of that initial land-use asymmetry all three create a systematic difference in the same direction between the price of public services in the two jurisdictions.

1. Differences in Tax Base

To facilitate the discussion about tax bases, we note that the proportional income tax base identically, and the land tax base indirectly, are positive functions of household income level. The higher the income, the higher will be the household's demand for land (as a normal, superior good). The amount of land demanded by the household and, when aggregated, the price of land subjected to increased demand, will both rise. Both will increase the household's base subject to ad valorem land tax. Thus, we can speak of the size of the tax base in terms of income level alone, as though only the proportional income tax were in effect.

Suppose there were two jurisdictions, 1 and 2, with equal populations and equal desired public service level. Then total planned expenditures would be the same for both. Suppose also that a proportional income tax were the sole tax source. Assume now that jurisdiction 2 had a total tax base twice as large as jurisdiction 1 because of an average income level
twice as large. Then the tax rate required in jurisdiction 2 would be half as large as that required in jurisdiction 1. Nonetheless, every household in the former would pay the same total amount of tax as every household in the latter. So there is no difference in treatment in this original allocation of population. But suppose that households were free to move from one jurisdiction to the other. Any household moving from jurisdiction 1 to jurisdiction 2 would lose a higher tax rate liability for one half as large—and thus on this particular household tax base a total amount of tax liability one half as great as previously. This incentive to move does not depend on the income level of the migrant being greater, equal to or less than that of the destination mean: every household beginning in jurisdiction 1 has a similar incentive.

Taken by itself, this systematic migration incentive would increase population in jurisdiction 2 and decrease it in 1. This would increase public service costs in the former relative to the latter. It would also tend to lower the mean tax base in the former, while its effect on that of the latter depends on which households migrated. On balance there would be a progressive diminution in the absolute value of the differential tax rate effect, unless the order of migration actually widened the difference in mean tax base enough to offset the worsening crowding of public service provision in jurisdiction 2. The initial tax incentive might or might not be eventually wiped out (at small enough sizes of jurisdiction 1 the inefficiencies of small scale provision of public services would raise their costs as well), but in any event a migration flow would have been elicited from 1 to 2 to take advantage of the tax rate differential.
In our model, the tax rate differential makes for a differential in the price of public services in the two jurisdictions and, all other things equal, creates an incentive for households to migrate from the low tax base to high tax base jurisdiction. This differential is purely a product of the character of public sector transactions in contrast to private sector transactions. Pricing is not based on an exact quid pro quo for benefits received by each household, since these are due to services provided with some degree of "publicness". Pricing stems from a communal fund, the tax base, which is - if at all - only remotely related to individual benefits. Thus the jurisdiction's total tax base represents a positive externality for every household in the jurisdiction. Its net benefits from public action is greater the greater is the size of the jurisdictional tax base. It is externalities operating with different magnitude in the two jurisdictions that generate differences in the price of public services and thus differential attractiveness for location.

2. Differences in Welfare Load

Up to now we have spoken about quality level of public services as a flow of benefits going equally to all households in the jurisdiction (although not necessarily with equal utility valuation). Now we introduce an important distinction within the population. Some households are eligible to receive the benefits of various welfare programs. In the real world this would include relief, food stamps, social work services, special health care services, compensatory education and vocational training, etc. In our model world of one composite public commodity this is approximated by giving those households eligible for "welfare
benefits"—generally poorer or afflicted households—a supplement to the normal bundle of public services. Thus, while nonwelfare recipients receive $G_i$, a welfare recipient receives $G_i + \lambda$. For convenience, and because in the real world the welfare supplement is likely to be quite similar in different parts of the same metropolitan area (however dissimilar in different metropolitan areas), we treat this supplement as strictly equal in city and suburb: $\lambda_1 = \lambda_2 = \lambda$.

Suppose now we have two hypothetical jurisdictions again, with equal populations and equal total expenditures. Jurisdiction 1, however, has twice as large a percentage of its population on welfare as does jurisdiction 2. What level of $G_i$ will the same total expenditures finance in the two jurisdictions? Clearly, since each welfare recipient receives $G_i + \lambda$ while nonrecipients receive only $G_i$, $G_2$ will exceed $G_1$; welfare recipients are more expensive and decrease the fund available to produce service quality. Abstracting from the relative size of tax bases and thus of tax rates (which would presumably go in the same direction as what follows anyway), any household in 1 that decided to move to 2 would gain a higher quality level. This holds for welfare recipients and nonwelfare recipients. Even welfare recipients have an incentive to run away from a jurisdiction with many other welfare recipients.

The mechanism here, intrinsic to public sector transactions, is structurally similar to that for unequal tax bases. Other inhabitants of the jurisdiction draw unequally out of a communal resource pool for providing public services. To participate in a pool with expensive other participants is a negative externality to each. The percentage of participants who are expensive withdrawers is the measure of this
negative externality. Different percentages imply different sized externalities. Since it affects the ratio of quality level to tax levies (via the total expenditure pool), it affects the price of public services in the two jurisdictions.

We are not supposing that only the poorest are eligible for welfare, since the real world list of services being reflected includes some that go to persons who are stochastically distressed and thus not subject to determinate prior identification in terms of income. Nonetheless, the probability that a family will be eligible for welfare in a given period is an inverse function of income. Thus, a poorer jurisdiction will have a higher expected proportion of its population as welfare recipients than a richer one. But such a difference will mean that the price of public services is greater in the poorer than in the richer jurisdiction.

Thus, tax base and welfare load considerations both establish that a poorer jurisdiction will experience a higher price of public services, all other things equal, than a richer. In our analysis of the single metropolitan area government, the trade-off between travel costs and density suggested that the suburb will become specialized as a residence for wealthier households relative to the rest of the urban area. That initial discrepancy in mean constituent income, arising from "natural" spatial considerations, will become the platform for generating a differential in public service price - thereby creating systematic incentives for additional households to move to the suburbs. The superimposed differential arises entirely out of externalities which are distinctive features of public sector, as opposed to private sector, processes.
3. **Differences in Interjurisdictional Externalities**

Another vital characteristic of the real-world metropolitan area is that it exists as the counterpart of a very high degree of economic and social interaction: such interaction is the raison d'être as well as the consequence of the spatial concentration known as an urban area. Much of the interaction is connected with the carrying out of business activity. The heavy concentration of such activity in the central city means that it is the desirable destination for a large proportion of all trips in the area. When a household moves to the suburb it does not thereby end all of its presence in the central city. It is likely to continue a significant presence there as for a job or for specialized shopping or recreation or cultural activities.

Such presence inadvertently makes use of public services provided in the city. Streets and roads, water, sewers, police and fire protection, special services like libraries, museums, parks, beaches, etc. Consider the impact of this on roads, for example. Suppose the city government wanted to provide quality level $G_1$. Since this is an output - not input - measure it implies for roads that a certain trip from A to B should be able to be made in a specified amount of time, with specified degree of convenience and safety. To determine the input cost of providing this output level the planners would have to know the expected traffic (since congestion affects all these output dimensions). If only the city residents were expected to use the road system, a certain input cost would be required. But suburbanites would typically want to use the system as well, and it is costly to try to exclude them from the availability of the roads.

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12/ See my earlier treatment, e.g., in Rothenberg [23].
With suburban use, the same capacity would result in lower $G$ for the city residents. So more, wider and stronger roads would have to be provided - at higher total cost - to achieve the same $G$, in the presence of suburban use. Thus, the interactive physical presence of suburbanites in the central city raises the real cost of providing public services to the city residents.

The same cost-enhancing effect is present in other types of public services as well. Thus, the physical presence of suburbanites in the central city is a negative externality for the provision of public services in the city. It is an interjurisdictional externality. Such externalities are not restricted to one direction: the presence of city residents in the suburb should have some of the same effects.

There is strong reason to believe that the externalities imposed by suburbanites on the city are greater than those imposed by city residents on the suburb. First, the disproportionate concentration of business and governmental and nonprofit institutional facilities in the city implies much more per capita suburbanite presence in the city than the reverse. Second, each instance of suburbanite presence in the city is likely to have a larger cost-enhancing (or quality depressing) impact than the reverse presence of city residents in the suburbs. That is because the marginal crowding effect on public services is greater the greater is the average crowding. This is inferred from properties (b) and (c) of the public expenditure function (9). So the net interjurisdictional externalities imposed on the suburb are likely to be negative, those imposed on the city positive.
The net increase in cost of public services due to net externalities may be less than suggested by the expenditure function and the differential size of the spillover because there are two potential financial offsets. One is governmental profits on business taxation, the other is the proceeds of special user charges. Since the overconcentration of business in the city is one of the chief causes of the net presence of the suburbanite there, it would seem reasonable to tax that business at rates which yield revenues in excess of the business-induced cost increases. This is hampered by the fact that it is extremely difficult to calculate the business-induced portion of cost increases - i.e., to assign differing liabilities to individual businesses on the basis of their impact on costs. Perhaps even more constraining, however, is that businesses are deemed to be both generally desirable in the urban area and mobile, so that the competitive attractions of alternative areas prevent obviously exploitative tax treatment for fear of losing these valuable mainstays of the urban economy.

Actual treatment of businesses is difficult to evaluate in terms of financial profits or losses relative to interjurisdictional externalities. We have accordingly chosen to exclude this factor from the model, assuming that business taxes are financially neutral: they just equal the marginal cost of providing public services to the business community.

The second financial offset is user charges. These are prices set to compel suburbanites to pay for their use of public services. Some public services are close enough to private goods in their production and distribution that individual instances of consumption and their associated production costs can be reasonably identified, and control of access to use restricted at minimal cost. User charges pinpointed to suburbanite
use, and approximating marginal costs, are appropriate here. Museums, libraries, swimming pools and other recreation facilities provide services that fall into this category. But clearly not all public services do: most are public services because they violate at least one of these characteristics substantially. Inability to pinpoint and control suburbanite use or to isolate its marginal cost make user charges inefficient. They would often involve a significant real cost to administer and be incapable of accurate calculation and impinge on city residents as well as suburbanites. Imperfect user charges do not obtain proper financial offsets and impose additional costs on city residents - in extra administrative costs and in the misallocation of resources that results from inappropriate marginal taxation.

Thus, while user charges can be resorted to by the city government to help offset negative net interjurisdictional externalities, they are not a perfect policy instrument. On balance, their use will leave a residual net loss to the city government.

In sum, the asymmetrical presence of interjurisdictional negative externalities in the metropolitan area will increase the real per capita cost of providing any level of G in the central city above that in the suburb, and thus the price of public services facing each resident. This is superimposed on any differential stemming from the previous two political sources. The differential involved here is not fixed but depends on such variables as the spatial distribution of business and other nonresidential activities, travel costs, and the characteristics of user charges. Of these, in the present model the distribution of business between city and suburb is subject to a policy instrument: business zoning.
C. Household Location Choice

We can now summarize the effects of the above differentials for explicit inclusion in the model. The effect of tax base can be directly shown by relating the formation of tax rates to the required expenditure totals and the two tax bases. The effect of welfare load can be shown by introducing the welfare recipient premium, \( \lambda \), and relating the probability of becoming a welfare recipient to household income level. The inter-jurisdictional externalities can be treated as a net effect in two respects: i.e., for the city but not for the suburban government, and as the residual left after imperfect user charges have been employed optimally (i.e., to minimize the net burden on the city).\(^{13/}\) It can be shown as a complicating of the city's - but not the suburb's - public expenditure function. This is expressed in equations (9a') and (9b'), which also incorporate a variable relating to welfare load (to be explained below):

\[
(9a') \quad E_1 = E_1(N_1, N_2, G_1, B_1, B_2, \phi_1, \lambda)
\]

where

- (1) \( \frac{\partial E_1}{\partial N_2} > 0 \),
- (2) \( \frac{\partial^2 E_1}{\partial N_2^2} < 0 \),
- (3) \( \frac{\partial^2 E_1}{\partial N_2 \partial N_1} > 0 \),
- (4) \( \frac{\partial^2 E_1}{\partial N_2 \partial B_1} > 0 \),
- (5) \( \frac{\partial^2 E_1}{\partial N_2 \partial B_2} < 0 \)

\[
(9b') \quad E_2 = E_2(N_2, G_2, \phi_2, \lambda)
\]

In (9a'), the \( E_1 \) function incorporates the probability that a suburbanite will have a physical presence in the central city. So expenditures rise with total number of suburbanites (via expected total suburbanite presence). This individual probability depends on the distribution of

\(^{13/}\) Optimal use is achieved where it is expanded to the point where the marginal increase in financial offset just equals the marginal real cost to the city (in administrative cost, resident incidence and resource allocation).
desirable nonresidential trip destinations - so the more they are located in the city the greater is the individual probability and thus the greater is the marginal impact of each suburbanite \( \frac{\partial^2 E}{\partial N_2 B_1} > 0 \); the reverse holds for suburban-located business \( \frac{\partial^2 E}{\partial N_2 B_2} < 0 \). Trips are made for socializing too, so an increase in absolute size of the suburban population diverts some of such trips away from the city \( \frac{\partial^2 E_1}{\partial N_2^2} < 0 \); similarly a larger city population attracts more such trips, as well as implying a more congested city, thus increasing the marginal crowding impact of each trip on city public services \( \frac{\partial^2 E_1}{\partial N_2 \partial N_1} > 0 \).

The variable \( \phi_i \) is the percentage of the population who are welfare recipients. Increase in this variable increases \( E_i \). \( \lambda \) is the per household premium for a welfare recipient.

The overall effect on prices of public services is given as follows:

\[ (10) \quad T_i = E_i \]

This is the budget balancing requirement.

\[ (11) \quad \tau_1 \bar{Y}_1 N_1 = \Omega_i T_i \quad \text{or} \quad \Omega_i = \frac{\tau_1 \bar{Y}_1 N_1}{T_i} \]

where \( \Omega_i \) is the public policy instrument deciding on the percentage of total revenues to be raised by the income tax

\[ (12) \quad E = \bar{W}_E + W_E \]

where \( \bar{W}_E \) is the total expenditures excluding welfare payments (costs) \( W_E \) is the total welfare payments

\[ ^{14/} \quad \text{This should more properly be interpreted as the probability that a household in } i \text{ will be welfare recipients.} \]
where, as before, $\lambda$ is the per household premium for a welfare recipient

\[
\phi_i = \phi(\bar{Y}_i, \sigma_{Y_i})
\]

where $\sigma_{Y_i}$ is the standard deviation of the household income distribution in $i$

\[
\frac{\partial Q_i}{\partial \bar{Y}_i} < 0 , \quad \frac{\partial Q_i}{\partial \sigma_{Y_i}} > 0
\]

\[
\tau_i = \frac{\Omega_i E_i}{\bar{Y}_i N_i}
\]

\[
\tau_{L_i} = \frac{E_i - \Omega_i E_i}{P_i A_i}
\]

where $P_{L_i}$ is the mean value of land per acre in $i$

$A_i$ is the total number of private acres in $i$

Equations (10) and (11) simply formalize the budget balancing requirement and the definition of the policy instrument $\Omega$ that determines the relative mix of revenues from the two tax sources. Equations (12) and (13) show the determination of the costs of the welfare load ((12) is purely definitional), and equation (14) indicates the probabilistic basis of the welfare load. The higher the mean income the lower the probability; the higher the income dispersion the higher probability, since for a given mean income a higher dispersion means more people at very low absolute levels - and absolute deprivation is one criterion for welfare eligibility.
Equations (15) (which is mathematically equivalent to equations (10) and (11) together) and (16) show how the size of the tax base explicitly enters to affect the effective tax rates: both forms of tax base have a negative effect on their corresponding tax rates.

We can now return to the choice situation of the household under fragmented government. Begin at the locational situation established under metropolitan government. \( N_1^* \) households reside in the city, \( N_2^* \) in the suburb. Mean land prices are \( P_{L1}^* \) and \( P_{L2}^* \); \( D = D^* \) and \( G = G^* \); and the price of public services is \( P_G^* \). Now every household has the opportunity to face the prices of public services and the service levels resulting from the separate jurisdictional tax bases. From the preceding section and equations (5)-(16), the initial population distribution establishes a lower public service price in the suburb than in the city. Every household has at this point a greater incentive to migrate to the suburb than previously. Those who can gain overall by such a move will do so.

To see which households are likely to move we must examine the nature of the gains now available through political fragmentation. All three sources of advantage operate through a lowered set of tax rates in the suburb relative to that in the city. The absolute size of the money gains is therefore a positive function of household income. Thus, while fragmentation enhances the attraction of the suburb for all, degree of enhancement is a positive function of income. Since we wish to consider the use of a public policy instrument in the suburb that generally occurs in the real world only after fragmentation and some subsequent urban development, we shall modify the comparative static "short-run" flavor of the model to construct a fictional dynamics. We suppose that there is a
queue of city households in descending order of potential gross gains from migration to the suburb. From the preceding, this queue will isomorphically display decreasing household income levels.

At the start of the fragmentation period the population is divided between the last (marginal) migrant and the first (marginal) nonmigrant in the queue. The latter is nearly indifferent between moving and staying, and subsequent members of the queue would lose increasingly the lower the income level (because of the trade-off between crowdedness and travel costs). The political gains from fragmentation will create net gains from migration for households down from the head of the queue. Households will then actually migrate in the same temporal order as the size of these net gains. The net gains will decrease for each subsequent marginal nonmover as households ahead of it migrate, because: (1) the differential crowdedness between the two jurisdictions decreases with higher density in the suburb and lower density in the city: i.e.,

\[ D_1 = D(N_1, B_1, U_{A_1}); \]

where \( U_{A_1} \) is the number of private acres in urban use

(2) public services become more crowded in the suburb, less crowded in the city (by equation (9)); (3) the difference between average tax base in city and suburb may decline as poorer households move to the suburb (but this depends on the specifics of the income distribution); (4) land prices rise in the suburb and fall in the city, thus raising the price of noncrowdedness \( D \); (5) the marginal interjurisdictional externality impact falls with the population shift, thereby decreasing the per capita effect on tax rates; (6) whatever the tax rate differential, the lower income level results in a smaller absolute advantage.
For some household income level, implying as well a certain total of previous migrants, net benefits will be zero, and for lower levels negative net benefits. This level establishes the new stable marginal nonmover - and with it the equilibrium population distribution.

Consider the same sequential migration now from the point of view of early migrants (or even those who "hypothetically" became suburbanites under even metropolitan government). Their net gains from residing in the suburb instead of the city are likely to fall as more and more previous city dwellers migrate to the suburb because: (1) the D differential falls with population shift; (2) the per household tax base in the suburb declines, thereby raising the tax rate, as unambiguously lower income households move in (regardless of what happens to the per household tax base differential); (3) with lower income households moving in, the expected percentage of the population on welfare increases, thereby raising the tax rate further.

One force tends to increase gains to the early suburbanites: the population shift raises land prices in the suburb (as it lowers them in the city). This generates capital gains for them. The net effect of these opposing forces depends on the specifics of the relationships described here only abstractly, details including the characteristics of the subsequent movers.

Early suburbanites can influence the characteristics of movers so as to increase the probability that capital gains will exceed benefit losses for at least moderate in-migration. By discouraging lower income households from migrating they can minimize the losses due to tax base and welfare load changes. Moreover, if they can do this while maintaining
low density of urban land use despite a larger population, they can minimize the increasing crowdedness of the suburban life space.

Both goals can be accomplished simultaneously by instituting minimum residential lot zoning in the suburb. Requiring large lot sizes means that any given number of migrants will not seriously increase urban density and will be forced to channel much of the increased total demand for suburban land to the conversion of rural land to urban uses (i.e., increase $U_{A2}$), leading to similar land price rises as if more of the demand were for higher land-use density. Moreover, since the size of lot demanded by a household is positively related to its income, requiring a large lot to be purchased imposes a potentially large utility loss on the poor household by distorting its budgetary allocation considerably - and the loss is greater the lower the income level. Finally, such zoning can be used to control the overall numbers migrating as well as the composition by varying its required minimum and thus the whole schedule of losses to potential migrants.

So early suburbanites have a rational incentive for their jurisdictional government to impose residential minimum lot zoning. This government, assumed responsive to their welfare, is therefore assumed to comply. Thus, we must include the existence of minimum lot zoning in the suburb as part of the setting for describing the locational choice of the individual household.15/

We may now integrate all these elements into the choosing situation. The individual household will choose so as to maximize its utility.

15/ Compare this treatment with others in the literature, e.g., Bailey [3], Davis [8], Stull [26], Siegan [25].
It does this in two stages: (1) hypothetically select the best budgetary allocation for the city and suburb separately; (2) select from these two that one which gives higher utility.

Form the indirect utility function \( V^j \) for the household (where achievable utility level is given as a function of the income level and prices of all commodities (as well as the externally determined levels of \( G^1 \) and \( D^1 \)):

\[
\begin{align*}
(18a) \quad & \max U_1^j : V_1^j = V_1^j \left[ Y^j + (P_{L_1} - P_{L_1}) L_1^j + \tau_1, P_{L_1}, \tau_1, P_H, P_X, N_1, B_1, G_1, D_1 \right] \\
(18b) \quad & \max U_2^j : V_2^j = V_2^j \left[ Y^j + (P_{L_2} - P_{L_2}) L_2^j + \tau_2, P_{L_2}, \tau_2, (tm), P_H, P_X, L_2^j, N_2, B_2, G_2, D_2 \right]
\end{align*}
\]

where \( L_2^j \) is the minimum lot zoning requirement in the suburb.

Notice that travel costs are included in \( V_2^j \) but not \( V_1^j \) because travel costs are zero in the city.

These budget optimizing allocations will result in an optimal simultaneous choice of \( L, H \) and \( X \) in both jurisdictions (\( G \) and \( D \) being set externally). In the suburb this optimum is a second best, because the household may be forced to buy more land than it would like in order to satisfy the zoning requirement \( L_2 > L_2^j \). A distortion here means that \( H \) and \( X \) must in general diverge also from what an unconstrained choice would have warranted. So we characterize the respective choices as follows:

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16/ We assume that a household obtains utility only from public services it consumes in its resident jurisdiction. Those used - impinged on - in the other jurisdiction are deemed to be inadvertent and nonvalued elements of the job or shopping, etc., purpose of the transjurisdictional visits. This could be relaxed but at a cost of some complexity.
\[(19a) \quad \hat{U}_1^j = U(G_1, D_1, L_1, H_1, X)\]
\[(19b) \quad \hat{U}_2^j = U(G_2, D_2, L_2^*, H_2^*, X^*)\]

where \(^*\) signifies choice constrained by the minimum lot zoning requirement as well as the budget constraint.

The net gain from migration to the suburb, \(\pi^j\), is:
\[(20) \quad \pi^j = v_2^j - v_1^j\]

D. Equilibrium Population Distribution

We are now able to indicate how individual household location choice leads to an equilibrium distribution of population between city and suburb. Begin with some \((G_1, G_2, P_{G_1}, P_{G_2}, L_1^Z, P_H, P_X, B_1, B_2, t)\) for the given population of \(N\) households. Then all households \(j\) with \(\pi^j > 0\), when account is taken of their impact on land prices, are assumed to migrate to the suburb, all with \(\pi^j < 0\) assumed to stay. Consider now a sequence of movers and the last household in that sequence, that for which net gains are lowest. Designate this household the marginal mover in the sequence. We express its gains as follows:
\[(21) \quad \pi_j(N_2, N_1, B_2, B_1) = \pi^j(N_2, N_1, B_2, B_1) \text{ for } j \in N_2 \text{ and } \pi^j \leq \pi^w \]
\[w \neq j, \text{ all } w \in N_2.\]

We define the equilibrium population distribution between suburb and city as that resulting from an equilibrium sequence, which is a sequence of movers to the suburb for which the net gains of the marginal mover is zero:
(22) \( \hat{N}_2, \hat{N}_1 \) is an equilibrium distribution if \( \pi_j(\hat{N}_2, \hat{N}_1, B_2, B_1) = 0 \)

The analysis can be illustrated by Figure 1.

Figure 1 Determination of Equilibrium Population Distribution under Metropolitan Government and Fragmented Government

Curve \( \pi^D \) shows at each level \( N_2^i \) the net gains to the marginal mover in the migration sequence equal to \( N_2^i \) households under metropolitan government (i.e., when only land-use density gains are involved); curve \( \pi^{D+J} \) shows corresponding marginal mover gains under fragmented government (i.e., when density and jurisdictional gains are involved). Since the array of movers is monotonically decreasing with respect to household income level, this latter scale is shown on the top. The marginal mover at \( N_2 = N \) has the lowest income in the population, \( Y_L \); at \( N_2 = 0 \) it has
the highest, $Y_H$. $\pi^D$ is flatter than $\pi^{D+J}$ in the positive and negative sectors: $\pi^{D+J}$ has bigger potential gains, since jurisdictional benefits can be very substantial for rich households and adds to land-use density gains; it also has bigger potential losses, since in addition to adverse land price movements with large suburban populations, the presence of minimum lot zoning makes possible large budgetary distortions for poor households. The equilibrium suburban population is $N^*_2$ under metropolitan government, but rises to $N^*_2$ under fragmented government.

The result of jurisdictional fragmentation is: (1) a larger suburban population; (2) a lower income for the marginal mover; (3) a price of public services lower than before in the suburb and higher than before in the city ($\hat{P}_{G_2} < P^*_G < \hat{P}_{G_1}$); (4) a higher level of public services than before in the suburb, a lower level than before in the city ($\hat{G}_{G_2} > G^*_G > \hat{G}_{G_1}$); (5) migration exclusion in the suburb through minimum lot zoning. So fragmentation has improved the welfare of the richer part of the population - the migrants - while worsening it for the poorer part. This welfare shift in favor of the richer comes at the expense of the poorer because it stems heavily from the removal of the wealthier households' tax base from the erstwhile common pool and of their responsibility for tending welfare clients. In effect, fragmentation weakens the ability to use the local level of government to engage in progressive real income redistribution. 17/

The above consequences are predominantly distributional. There is an important resource allocational consequence as well. The larger suburban population results in: (1) a large total of interjurisdictional ex-

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17/ For a broader perspective on the relationship between jurisdictional fragmentation and the governmental income redistribution function, see Rothenberg [20].
ternalities; (2) a larger total transportation mileage. Externalities
drive a wedge between relevant benefits and costs of providing public
services. This complicates the problem of balancing resource use between
public and private uses and across jurisdictions. This is especially
obvious if we modified our model slightly to allow suburbanites to value
the public services they use in the city.

The larger total travel mileage necessary to enable the same popula-
tion to transact the same total amount of business involves a waste of
resources. This is because, while they obtain lower density living and
the extra travel cost is presumably the payment for this, the population
does not face the true marginal social cost of the lower density living
but a subsidized, possibly much lower cost - the offsets through juris-
dictional benefits. Thus they are "artificially" induced to buy too much
required travel - some of the increase therefore represents a waste
of resources.

Both allocational effects decrease the efficiency with which resources
are used. So fragmentation has an income stratifying, regressive dis-
tributional impact and a resource inefficiency impact.

VI. The Working of the Land Market

Under the metropolitan government we argued that the land market
would operate to permit a balancing of density with travel costs by changing
the relative prices of land parcels in accordance with tastes and ac-
cessibility. Under fragmented government we argued that jurisdictional
benefits are generated just by dint of residing in - and thus owning real
property in - the suburb. Since these benefits are specific to particular
real property, property which cannot be literally duplicated, shouldn't
the land market capitalize these benefits in these particular pieces of land, so that, while original owners of the land (farmers and early urban settlers) may benefit, the second and subsequent waves of suburban migrants should find prospective gains to be in fact snatched away before they are realized? If so, the analysis of the last section is wrong, and the impetus to suburbanization is much weaker.

This would occur if favored suburban land were in fact fixed in supply: jurisdictional benefits might then be perfectly capitalized in higher land values. But suburban land is not fixed in supply - there is a positive, price-responsive supply mechanism which prevents land prices from rising enough to wipe out these gains for migrants.

Space will not permit more than the briefest explanation about the land market. There are in fact two land markets, related but different: the city market and the suburban market. The city, being spatially totally encompassed by the suburb, has a fixed land area. Price elasticity of supply is zero here. The price of city land (by assumption homogeneous in everything, including accessibility) is set at that level at which the total demand for city land equals the fixed supply. The demand operates through price. The lower the price the more land each resident wants to use - and the more households want to use it; and vice versa. So total demand shows the conventional inverse relationship to price. If at some price total demand exceeds the fixed supply, price rises until the decreasing amount demanded exactly equals available supply. In this market increased jurisdictional attractiveness does result in substantial capitalization.

The suburban market is different. For one thing, lots are differentiated by accessibility. For another - and more importantly - not all of the
suburb's land is used for urban purposes. Some is under rural use. All land in the suburb is under competition by urban and nonurban users. We assume that the rural opportunity cost — its reservation price — is constant. Then an increase in urban demand for suburban land is transmitted throughout all developed suburban land in terms of accessibility to the external margin of urban development, and the marginal lot used for urban purposes, previously priced equal to the nonurban reservation price, now experiences an increase in price above this level. Urban users can now outbid rural users for land marginally subject to non-urban use, at a new price equal to the new price of the previous marginal urban lot less the value of differential accessibility between them.

So a conversion will occur: additional land will be supplied for urban use, and always at the same price — the reservation nonurban price plus the value of the slight decrease in accessibility relative to the previous margin. This constant supply price is likely to be considerably less than the value of suburban land to at least some new migrants. So the competition of new land supplied at the external margin for urban uses prevents all the land in the interior from rising enough to wipe out all prospective migratory gains — since its force is felt by every other lot adjusted only by differential accessibility.

Prospective migratory gains do, however, raise all suburban land prices some, since the newly converted, but less accessible land, sells at the same price as did the previous marginal land which is more accessible. So price rises everywhere in the interior according to relative accessibility. This set of rises does decrease the prospective gains from migration (without, by itself, wiping them out). At the same time, migration is
decreasing both density and jurisdictional gains. So the land market provides a third mechanism for equalizing net advantages in city and suburb: all three, but not any one of them, re-establishes an equilibrium that is disturbed by some new asymmetrical attraction between the two jurisdictions.18/

VII. Long-Run Public Policy Adjustment

In analyzing the previous adjustment process we began by assuming that earlier policy decisions had set the mix of taxes in both jurisdictions and the residential and business zoning regulations in the suburb. The consequent policy adjustment of $G$, $\tau$ and $t_L$ can be considered a short-run adjustment. We now consider what determines the policy decisions on tax mix and zoning. Adjustment of these constitutes the long-run process.

As noted above, adjustment of business zoning directly influences the distribution of business between city and suburb and through that, expected travel costs from the suburb, the level of land-use density in city and suburb, the size of interjurisdictional externalities, etc. All these will have some effect on the extent of suburbanization and thus, on the size of capital gains for pre-existing suburban residents. Thus, an adjustment of business zoning is called for in exactly the same way that the short-run service level and tax adjustments were called for - namely, by a majority vote based on utility impacts for constituents. If the median voter experiences utility gain from the capital gains - density, etc. - consequences, the adjustment will be undertaken.

18/ Under this treatment, adjacent parcels of land on the two sides of the suburb-city boundary will show a price discontinuity, since not only accessibility (in which they differ very little) but jurisdictional characteristics as well (in which they may differ greatly) influence price. This differs from the classic "homogeneous" land market theories, as for example, Alonso [1] and Muth [17]. It is more akin to zoning or segregation models, e.g., Stull, loc. cit., and Kain and Quigley [12].
Adjustment of residential zoning has the same analytic structure: its ability to influence the size, income level composition and land-use density characteristics of any new migration will have a composite of effects on the components of suburban voter utility functions. Adjustments will be adopted that positively affect the median voter.

Adjustment of the tax mix is more subtle. Its basis lies in the different kinds of influence a tax on income and a tax on land have on location decisions. While income is mobile, and can be completely removed from a jurisdiction's total tax base, land is immobile. It cannot be removed from the jurisdiction. It can lose value, but only in accordance with the working of the land market. It is ordinarily assumed that a rise in a land tax has a smaller disincentive effect on location than a comparable rise in an income tax. In the present model, the consequences are quite complicated, since land tax capitalization occurs, as does land conversion between urban and nonurban uses in the suburb, but not in the city.

The conversion process along with the full land price adjustment mechanism is too complicated to include in an already overlong paper; but the tax capitalization process can be briefly noted to help clarify the effect of a change in tax mix on utility.

\[
P_{L_1}^* = P_{L_1}^* (g_i, t_{L_1})
\]

\[
\frac{\partial P_{L_1}^*}{\partial t_{L_1}} < 0 < \frac{\partial P_{L_1}^*}{\partial g_i}
\]

where \( g_i \) is the value of the services of land excluding the land tax

This function shows that a land tax is shifted back on the owner - the tax is capitalized and subtracted from the selling price. Thus, if
a larger share of revenues is assigned by either jurisdiction to the land tax in the hope of attracting more residents so as to produce capital gains for existing residents, the upward impact on land prices of this greater migration is offset to some extent by the backward shifting of the land tax. The net effect depends on specifics of the relationships.

(VIII) Jurisdictional Rivalry

Given our assumptions of constant population and business in the metropolitan area, every use by a jurisdiction of its long-run policy instruments to gain higher utility for its existing residents by influencing further migration comes at the expense of the other jurisdiction (although not necessarily in zero-sum fashion). Moreover, the particular long-run adjustment that it will be profitable for it to make depends in the general case on the particular levels at which the other jurisdiction's policy instruments stand.

So we may think of a sequential long-run process which begins when one jurisdiction sets its long-run policies. This setting determines where it is worthwhile for the second to set its instruments. But the setting of the second's policies now makes it worthwhile for the first to adjust its instruments. This change in the first's instrument settings changes the second jurisdiction's optimal policies - and so on.

The process is a familiar one of duopolistic strategic interaction. Whether or not the process converges to a joint equilibrium where both long- and short-run adjustments are mutually compatible depends on the particular conjectural variation functions of the two. Analysis of conditions for stability is beyond the scope of the present paper. Two observations can be made about qualitative aspects of the interaction in conclusion, however.
First, the two jurisdictions are not symmetrical in their rivalrous relationship. Because of its spatially enclosed (and fully developed) character, and its lower average household income level, encouraging additional migration to the city, starting at any particular situation, is very likely to be beneficial for pre-existing city residents, since it bids up city land prices (giving capital gains) and lowers the price of public services. Crowding is worsened but is not a matter for deep concern by the relatively poor population. Thus, the city in general wants to become bigger.

For the suburb, additional migration brings a smaller average capital gain because of additional land conversion (the total potential capital gains have to be shared among a larger number of lots). Moreover, although the price of public services rises here too, increasing crowdedness occurs as well, and here it is a matter for concern, because the pre-existing residents are richer and care more to retain uncrowded conditions. Indeed, the suburb has a specialized function as an area of low crowding. So while the suburb wants more population in some circumstances, this desire is not unlimited: it has to balance its specialized low-density role with the possibility of more (but marginally falling) jurisdictional advantages and land capital gains.

In this strange game of tug of war, then, what one side gains is not always equal to what the other side loses. It is as though as the rope is pulled triumphantly closer, part of it dissolves and part of it grows.

Second, to indicate that the strategic game might in some cases have no harmonious resolution - that it might be unstable - does not thereby
disqualify it as a paradigm of the real world. The time scale on which we count the model's so-called short-run responses, not to speak of the long-run responses, is a long one. Over the decades or so necessary for a set of strategic iterations to occur, real world observations would also presumably show a number of broadly oscillatory movements. It is somewhat presumptuous to judge these movements after a much shorter experience with the modern metropolitan area as reflective of a grand general equilibrium. Our experience of city and suburb has, after all, been one of constant change, not of serene balance.
References


