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ABSTRACT. We prove that the quantum double of the quasi-Hopf algebra $A_q(\mathfrak{g})$ of dimension $n^{\dim \mathfrak{g}}$ attached in [EG] to a simple complex Lie algebra $\mathfrak{g}$ and a primitive root of unity q of order n^2 is equivalent to Lusztig's small quantum group $\mathfrak{u}_q(\mathfrak{g})$ (under some conditions on n). We also give a conceptual construction of $A_q(\mathfrak{g})$ using the notion of de-equivariantization of tensor categories.

1. Introduction

It is well known from the work of Drinfeld [D] that the quantum group $U_q(\mathfrak{g})$ attached to a simple complex Lie algebra $\mathfrak{g}$ can be produced by the quantum double construction. Namely, the quantum double of the quantized Borel subalgebra $U_q(\mathfrak{b})$ is the product of $U_q(\mathfrak{g})$ with an extra copy of the Cartan subgroup $U_q(\mathfrak{h})$, which one can quotient out and get the pure $U_q(\mathfrak{g})$. This principle applies not only to quantum groups with generic q, but also to Lusztig's small quantum groups at roots of unity, $\mathfrak{u}_q(\mathfrak{g})$ ([L1, L2]). However, $\mathfrak{u}_q(\mathfrak{g})$ itself (without an additional Cartan) is not, in general, a quantum double of anything: indeed, its dimension is $d = m^{\dim \mathfrak{g}}$ (where m is the order of q), which is not always a square.

However, in the case when $m = n^2$ (so that the dimension d is a square), we have introduced in [EG], Section 4, a quasi-Hopf algebra $A_q = A_q(\mathfrak{g})$ of dimension $d^{1/2}$, constructed out of a Borel subalgebra $\mathfrak{b}$ of $\mathfrak{g}$. So one might suspect that the quantum double of $A_q(\mathfrak{g})$ is twist equivalent to $\mathfrak{u}_q(\mathfrak{g})$. This indeed turns out to be the case (under some conditions on n), and is the main result of this note. In other words, our main result is that the Drinfeld center $\mathcal{Z}(\mathrm{Rep}(A_q(\mathfrak{g})))$ of the category of representations of $A_q(\mathfrak{g})$ is $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g}))$.

We prove our main result by showing that the category $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b}))$ of representations of the quantum Borel subalgebra $\mathfrak{u}_q(\mathfrak{b})$ is the equivariantization of the category $\mathrm{Rep}(A_q(\mathfrak{g}))$ with respect to an action of a certain finite abelian group. Thus, $\mathrm{Rep}(A_q(\mathfrak{g}))$ can be conceptually defined as a de-equivariantization of $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g}))$. So, one may say that the main outcome of this paper is a demystification of the quasi-Hopf algebra $A_q(\mathfrak{g})$ constructed "by hand" in [EG].

The structure of the paper is as follows. In Section 2 we recall the theory of equivariantization and de-equivariantization of tensor categories. In Section 3 we recall the construction of the quasi-Hopf algebra $A_q(\mathfrak{g})$ from the paper [EG]. In Section 4 we state the main results. Finally, Section 5 contains proofs.

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2. Equivariantization and de-equivariantization

The theory of equivariantization and de-equivariantization of tensor categories was developed in [B, M] in the setting of fusion categories; it is now a standard technique in the theory of fusion categories, and has also been used in the setting of the Langlands program [F]. A detailed description of this theory is given in [DGNO] (see also [ENO], Sections 2.6 and 2.11). This theory extends without major changes to the case of finite tensor categories (as defined in [EO]), i.e, even if the semisimplicity assumption is dropped. Let us review the main definitions and results of this theory.

2.1. **Group actions.** Let $\mathcal{C}$ be a finite tensor category (all categories and algebras in this paper are over $\mathbb{C}$). Consider the category $\underline{\mathrm{Aut}}(\mathcal{C})$, whose objects are tensor auto-equivalences of $\mathcal{C}$ and whose morphisms are isomorphisms of tensor functors. The category $\underline{\mathrm{Aut}}(\mathcal{C})$ has an obvious structure of a monoidal category, in which the tensor product is the composition of tensor functors.

Let G be a group, and let $\underline{G}$ denote the category whose objects are elements of G, the only morphisms are the identities and the tensor product is given by multiplication in G.

Definition 2.1. An *action* of a group G on a finite tensor category $\mathcal{C}$ is a monoidal functor $\underline{G} \to \underline{\mathrm{Aut}}(\mathcal{C})$.

If $\mathcal{C}$ is equipped with a braided structure we say that an action $\underline{G} \to \underline{\mathrm{Aut}}(\mathcal{C})$ respects the braided structure if the image of $\underline{G}$ lies in $\underline{\mathrm{Aut}}^{br}(\mathcal{C})$, where $\underline{\mathrm{Aut}}^{br}(\mathcal{C})$ is the full subcategory of $\underline{\mathrm{Aut}}(\mathcal{C})$ consisting of braided equivalences.

2.2. **Equivariantization.** Let a finite group G act on a finite tensor category $\mathcal{C}$. For any $g \in G$ let $F_g \in \underline{\mathrm{Aut}}(\mathcal{C})$ be the corresponding functor and for any $g, h \in G$ let $\gamma_{g,h}$ be the isomorphism $F_g \circ F_h \simeq F_{gh}$ that defines the tensor structure on the functor $\underline{G} \to \underline{\mathrm{Aut}}(\mathcal{C})$. A *$G$-equivariant object* of $\mathcal{C}$ is an object $X \in \mathcal{C}$ together with isomorphisms $u_g : F_g(X) \simeq X$ such that the diagram

$$\begin{array}{ccc} F_g(F_h(X)) & \xrightarrow{F_g(u_h)} & F_g(X) \\ {\scriptstyle \gamma_{g,h}(X)}\downarrow & & \downarrow{\scriptstyle u_g} \\ F_{gh}(X) & \xrightarrow{u_{gh}} & X \end{array}$$

commutes for all $g, h \in G$. One defines morphisms of equivariant objects to be morphisms in $\mathcal{C}$ commuting with u_g, $g \in G$. The category of G-equivariant objects of $\mathcal{C}$ will be denoted by $\mathcal{C}^G$. It is called the **equivariantization** of $\mathcal{C}$.

Note that $\mathrm{Vec}^G = \mathrm{Rep}(G)$, so there is a natural inclusion $\iota : \mathrm{Rep}(G) \to \mathcal{C}^G$.

One of the main results about equivariantization is the following theorem (see [ENO], Proposition 2.10 for the semisimple case; in the non-semisimple situation, the proof is parallel).

Theorem 2.2. *Let G be a finite group acting on a finite tensor category $\mathcal{C}$. Then Rep(G) is a Tannakian subcategory of the Drinfeld center $\mathcal{Z}(\mathcal{C}^G)$ (i.e., the braiding of $\mathcal{Z}(\mathcal{C}^G)$ restricts to the usual symmetric braiding of Rep(G)), and the composition*

$$Rep(G) \to \mathcal{Z}(\mathcal{C}^G) \to \mathcal{C}^G$$

(where the last arrow is the forgetful functor) is the natural inclusion ι.

If $\mathcal{C}$ is a braided category, and the G-action preserves the braided structure, then $\mathcal{C}^G$ is also braided. Thus $\mathcal{C}^G$ is a full subcategory of $\mathcal{Z}(\mathcal{C}^G)$, and the inclusion ι factors through $\mathcal{C}^G$. Thus in this case Rep(G) is a Tannakian subcategory of $\mathcal{C}^G$.

2.3. **De-equivariantization.** Let $\mathcal{D}$ be a finite tensor category such that the Drinfeld center $\mathcal{Z}(\mathcal{D})$ contains a Tannakian subcategory Rep(G), and the composition Rep$(G) \to \mathcal{Z}(\mathcal{D}) \to \mathcal{D}$ is an inclusion. Let $A := \mathrm{Fun}(G)$ be the algebra of functions $G \to \mathbb{C}$. The group G acts on A by left translations, so A can be considered as an algebra in the tensor category Rep(G), and thus as an algebra in the braided tensor category $\mathcal{Z}(\mathcal{D})$. As such, the algebra A is braided commutative. Therefore, the category of A-modules in $\mathcal{D}$ is a tensor category, which is called the **de-equivariantization** of $\mathcal{D}$ and denoted by $\mathcal{D}_G$.

Let us now separately consider de-equivariantization of braided categories. Namely, let $\mathcal{D}$ be a finite braided tensor category, and Rep$(G) \subset \mathcal{D}$ a Tannakian subcategory. In this case Rep(G) is also a Tannakian subcategory of the Drinfeld center $\mathcal{Z}(\mathcal{D})$ (as $\mathcal{D} \subset \mathcal{Z}(\mathcal{D})$), so we can define the de-equivariantization $\mathcal{D}_G$. It is easy to see that $\mathcal{D}_G$ inherits the braided structure from $\mathcal{D}$, so it is a braided tensor category.

We will need the following result (see [ENO], Section 2.6 and Proposition 2.10 for the semisimple case; in the non-semisimple situation, the proof is parallel).

Theorem 2.3. *(i) The procedures of equivariantization and de-equivariantization are inverse to each other.*

(ii) Let $\mathcal{C}$ be a finite tensor category with an action of a finite group G. Let $\mathcal{E}'$ be the Müger centralizer of $\mathcal{E} = Rep(G)$ in $\mathcal{Z}(\mathcal{C}^G)$ (i.e., the category of objects $X \in \mathcal{Z}(\mathcal{C}^G)$ such that the squared braiding is the identity on $X \otimes Y$ for all $Y \in \mathcal{E}$). Then the category $\mathcal{E}'_G$ is naturally equivalent to $\mathcal{Z}(\mathcal{C})$ as a braided category.

3. The quasi-Hopf algebra $A_q = A_q(\mathfrak{g})$

In this section we recall the construction of the finite dimensional basic quasi-Hopf algebras $A_q = A_q(\mathfrak{g})$, given in [EG], Section 4.

Let $\mathfrak{g}$ be a finite dimensional simple Lie algebra of rank r, and let $\mathfrak{b}$ be a Borel subalgebra of $\mathfrak{g}$.

Let $n \geq 2$ be an odd integer, not divisible by 3 if $\mathfrak{g} = G_2$, and let q be a primitive root of 1 of order n^2. We will also assume, throughout the rest of the paper, that n is relatively prime to the determinant $\det(a_{ij})$ of the Cartan matrix of $\mathfrak{g}$.

Let $\mathfrak{u}_q(\mathfrak{b})$ be the Frobenius-Lusztig kernel associated to $\mathfrak{b}$ ([L1, L2]); it is a finite dimensional Hopf algebra generated by grouplike elements g_i and skew-primitive elements e_i, $i = 1, \ldots, r$, such that

$$g_i^{n^2} = 1,\ g_i g_j = g_j g_i,\ g_i e_j g_i^{-1} = q^{\delta_{ij}} e_j,$$

e_i satisfy the quantum Serre relations, and

$$\Delta(e_i) = e_i \otimes K_i + 1 \otimes e_i,\ K_i := \prod_j g_j^{a_{ij}}.$$

The algebra $\mathfrak{u}_q(\mathfrak{b})$ has a projection onto $\mathbb{C}[(\mathbb{Z}/n^2\mathbb{Z})^r]$, $g_i \mapsto g_i$ and $e_i \mapsto 0$. Let $B \subset \mathfrak{u}_q(\mathfrak{b})$ be the subalgebra generated by $\{e_i\}$. Then by Radford's theorem [R], the multiplication map $\mathbb{C}[(\mathbb{Z}/n^2\mathbb{Z})^r] \otimes B \to \mathfrak{u}_q(\mathfrak{b})$ is an isomorphism of vector spaces. Therefore, $A_q := \mathbb{C}[(\mathbb{Z}/n\mathbb{Z})^r]B \subset \mathfrak{u}_q(\mathfrak{b})$ is a subalgebra. It is generated by g_i^n and e_i, $1 \leq i \leq r$.

Let $\{1_z | z = (z_1, \dots, z_r) \in (\mathbb{Z}/n^2\mathbb{Z})^r\}$ be the set of primitive idempotents of $\mathbb{C}[(\mathbb{Z}/n^2\mathbb{Z})^r]$ (i.e., $1_z g_i = q^{z_i} 1_z$).

Following [G], for $z, y \in \mathbb{Z}/n^2\mathbb{Z}$ let $c(z, y) = q^{-z(y-y')}$, where y' denotes the remainder of division of y by n.

Let

$$\mathbb{J} := \sum_{z,y \in (\mathbb{Z}/n^2\mathbb{Z})^r} \prod_{i,j=1}^{r} c(z_i, y_j)^{a_{ij}} 1_z \otimes 1_y.$$

It is clear that it is invertible and $(\varepsilon \otimes \mathrm{id})(\mathbb{J}) = (\mathrm{id} \otimes \varepsilon)(\mathbb{J}) = 1$. Define a new coproduct

$$\Delta_{\mathbb{J}}(z) = \mathbb{J}\Delta(z)\mathbb{J}^{-1}.$$

Lemma 3.1. *The elements $\Delta_{\mathbb{J}}(e_i)$ belong to $A_q \otimes A_q$.*

Lemma 3.2. *The associator $\Phi := d\mathbb{J}$ obtained by twisting the trivial associator by $\mathbb{J}$ is given by the formula*

$$\Phi = \sum_{\beta,\gamma,\delta \in (\mathbb{Z}/n\mathbb{Z})^r} \left(\prod_{i,j=1}^{r} q^{a_{ij}\beta_i((\gamma_j+\delta_j)'-\gamma_j-\delta_j)} \right) \mathbf{1}_\beta \otimes \mathbf{1}_\gamma \otimes \mathbf{1}_\delta,$$

where $\mathbf{1}_\beta$ are the primitive idempotents of $\mathbb{C}[(\mathbb{Z}/n\mathbb{Z})^r]$, $\mathbf{1}_\beta g_i^n = q^{n\beta_i}\mathbf{1}_\beta$, and we regard the components of β, γ, δ as elements of $\mathbb{Z}$.[1] *Thus Φ belongs to $A_q \otimes A_q \otimes A_q$.*

Theorem 3.3. *The algebra A_q is a quasi-Hopf subalgebra of $\mathfrak{u}_q(\mathfrak{b})^{\mathbb{J}}$, which has coproduct $\Delta_{\mathbb{J}}$ and associator Φ. It is of dimension $n^{\dim \mathfrak{g}}$.*

Remark 3.4. The quasi-Hopf algebra A_q is not twist equivalent to a Hopf algebra. Indeed, the associator Φ is non-trivial since the $3-$cocycle corresponding to Φ restricts to a non-trivial $3-$cocycle on the cyclic group $\mathbb{Z}/n\mathbb{Z}$ consisting of all tuples whose coordinates equal 0, except for the ith coordinate. Since A_q projects onto $(\mathbb{C}[(\mathbb{Z}/n\mathbb{Z})^r], \Phi)$ with non-trivial Φ, A_q is not twist equivalent to a Hopf algebra.

4. Main results

Let $T := (\mathbb{Z}/n^2\mathbb{Z})^r$. We have the following well known result.

Theorem 4.1. *The quantum double $D(\mathfrak{u}_q(\mathfrak{b}))$ of $\mathfrak{u}_q(\mathfrak{b})$ is twist equivalent, as a quasitriangular Hopf algebra, to $\mathfrak{u}_q(\mathfrak{g}) \otimes \mathbb{C}[T]$. Therefore,*

$$\mathcal{Z}(Rep(\mathfrak{u}_q(\mathfrak{b}))) = Rep(\mathfrak{u}_q(\mathfrak{g})) \boxtimes Vec_T$$

as a braided tensor category, where the braiding on $Rep(\mathfrak{u}_q(\mathfrak{g}))$ is the standard one, and Vec_T is the category of T-graded vector spaces with the braiding coming from the quadratic form on T defined by the Cartan matrix of $\mathfrak{g}$. [2]

Proof. It is well known ([D], [CP]) that $D(\mathfrak{u}_q(\mathfrak{b}))$ is isomorphic as a Hopf algebra to $H := \mathfrak{u}_q(\mathfrak{g}) \otimes \mathbb{C}[T]$, with standard generators $e_i, f_i, K_i \in \mathfrak{u}_q(\mathfrak{g})$ and $K_i' \in \mathbb{C}[T]$, and comultiplication

$$\Delta_*(e_i) = e_i \otimes K_i K_i' + 1 \otimes e_i, \ \Delta_*(f_i) = f_i \otimes K_i'^{-1} + K_i^{-1} \otimes f_i$$

(in fact, this is not hard to check by a direct computation). Note that the group algebra $\mathbb{C}[T \times T]$ is contained in H as a Hopf subalgebra (with the two copies of T

[1] $\mathbf{1}_\beta$ should not be confused with 1_z that appeared above.

[2] Actually, the quadratic form gives the inverse braiding, but this is not important for our considerations.

generated by K_i and K_i', respectively). Consider the bicharacter of $T \times T$ given by the formula

$$\langle (a,b),(c,d)\rangle = <a,d>,$$

where $<,>: T \times T \to \mathbb{C}^*$ is the pairing given by the Cartan matrix. Consider the twist $J \in \mathbb{C}[T \times T]^{\otimes 2}$ corresponding to this bicharacter. It is easy to compute directly that twisting by J transforms the above comultiplication Δ_* to the usual "tensor product" compultiplication of H:

$$\Delta(e_i) = e_i \otimes K_i + 1 \otimes e_i,\ \Delta(f_i) = f_i \otimes 1 + K_i^{-1} \otimes f_i,$$

and the same holds for the universal R-matrix (this computation uses that K_i' are central elements). This implies the theorem. □

Let $\Gamma \cong (\mathbb{Z}/n\mathbb{Z})^r$ be the $n-$torsion subgroup of T.

Our first main result is the following.

Theorem 4.2. *The group Γ acts on the category $\mathcal{C} = Rep(A_q)$, and the equivariantization $\mathcal{C}^\Gamma$ is tensor equivalent to $Rep(\mathfrak{u}_q(\mathfrak{b}))$.*

The proof of Theorem 4.2 will be given in the next section.

By Theorem 2.3(i), Theorem 4.2 implies that the category $\mathrm{Rep}(A_q)$ can be conceptually defined as the de-equivariantization of $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b}))$.

Our second main result is the following.

Theorem 4.3. *The Drinfeld center $\mathcal{Z}(Rep(A_q))$ of $Rep(A_q)$ is braided equivalent to $Rep(\mathfrak{u}_q(\mathfrak{g}))$. Equivalently, the quantum double $D(A_q)$ of the quasi-Hopf algebra A_q is twist equivalent (as a quasitriangular quasi-Hopf algebra) to the small quantum group $\mathfrak{u}_q(\mathfrak{g})$.*

Proof. Since $\mathcal{Z}(\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b}))) = \mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g})) \boxtimes \mathrm{Vec}_T$ as a braided category, and $\mathrm{Rep}\Gamma \subset \mathrm{Vec}_T$ is a Tannakian subcategory, we have that $\mathrm{Rep}(\Gamma) \subset \mathcal{Z}(\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b})))$ is a Tannakian subcategory. Moreover, $\mathrm{Rep}\Gamma \subset \mathrm{Vec}_T$ is a Lagrangian subcategory (i.e, it coincides with its Müger centralizer in Vec_T), so the Müger centralizer $\mathcal{D}$ of $\mathrm{Rep}\Gamma$ in $\mathcal{Z}(\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b})))$ is equal to $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g})) \boxtimes \mathrm{Rep}(\Gamma)$. This implies that the de-equivariantization $\mathcal{D}_\Gamma$ is $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g}))$. On the other hand, by Theorem 4.2, $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b})) = \mathrm{Rep}(A_q)^\Gamma$, so by Theorem 2.3(ii) we conclude that $\mathcal{Z}(\mathrm{Rep}(A_q)) = \mathrm{Rep}(\mathfrak{u}_q(\mathfrak{g}))$, as desired. □

5. Proof of Theorem 4.2

Let us first define an action of Γ on $\mathcal{C} = \mathrm{Rep}(A_q)$.

For $j = 0, \dots, n-1$, $i = 1, \dots, r$, let $F_{ij} : \mathrm{Rep}(A_q) \to \mathrm{Rep}(A_q)$ be the functor defined as follows. For an object (V, π_V) in $\mathrm{Rep}(A_q)$, $F_{ij}(V) = V$ as a vector space, and $\pi_{F_{ij}(V)}(a) = \pi_V(g_i^j a g_i^{-j})$, $a \in A_q$.

The isomorphism $\gamma_{ij_1,ij_2} : F_{ij_1}(F_{ij_2}(V)) \to F_{i,(j_1+j_2)'}(V)$ is given by the action of

$$(g_i^n)^{\frac{(j_1+j_2)'-j_1-j_2}{n}} \in A_q,$$

and $\gamma_{i_1j_1,i_2j_2} = 1$ for $i_1 \neq i_2$.

Let us now consider the equivariantization $\mathcal{C}^\Gamma$. By definition, an object of $\mathcal{C}^\Gamma$ is a representation V of A_q together with a collection of linear isomorphisms $p_{i,j} : V \to V$, $j = 0, \dots, n-1$, $i = 1, \dots, r$, such that

$$p_{i,j}(av) = g_i^j a g_i^{-j} p_{i,j}(v),\ a \in A_q,\ v \in V,$$

and

$$p_{i,j_1}p_{i,j_2} = p_{i,(j_1+j_2)'}(g_i^n)^{\frac{-(j_1+j_2)'+j_1+j_2}{n}}.$$

It is now straightforward to verify that this is the same as a representation of $\mathfrak{u}_q(\mathfrak{b})$, because $\mathfrak{u}_q(\mathfrak{b})$ is generated by A_q and the $p_{i,j} := g_i^j$ with exactly the same relations. Moreover, the tensor product of representations is the same as for $\mathfrak{u}_q(\mathfrak{b})^{\mathbb{J}}$. Thus $\mathcal{C}^\Gamma$ is naturally equivalent to $\mathrm{Rep}(\mathfrak{u}_q(\mathfrak{b}))$, as claimed.

This completes the proof of Theorem 4.2.

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