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# THE GENERAL EQUILIBRIUM THEORY OF EFFECTIVE PROTECTION AND RESOURCE ALLOCATION

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THE GENERAL EQUILIBRIUM THEORY OF EFFECTIVE PROTECTION AND RESOURCE ALLOCATION[1]

The theory of effective protection appears to have been developed in recent years in an attempt to seek a concept of protection which, in the presence of traded inputs, would be able to perform analytically the role that nominal tariffs played in the "older," traditional theory which was premised on a model which excluded traded inputs.

Thus, in the traditional model, with two traded goods (1 and 2) produced with standard restrictions on the production functions[2] by two primary factors (k and L) in given endowment, and the small-country assumption, a tariff on a good (1) would lead to: i) a relative rise in the output of the protected good (1); ii) a relative rise in the value added therein; and iii) a rise in the use of each factor ($K_1$ and $L_1$) therein.[3] For two traded goods and n ($n > 2$) primary factors, a tariff on one good will continue to imply increase in its output and value-added, though not necessarily in each of the primary factors used therein. For n ($n > 2$) traded goods and m ($m \geq n$) primary factors, a tariff on one good will still increase its output and value-added but, when more than one tariff is imposed--implying more than one price change--even this cannot be asserted for the good with the highest tariff.

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[1]Thanks are due to the National Science Foundation for supporting the research reported in this paper. The present paper has benefited from our earlier work on the same subject (1971a)(1971b), which is not replaces: although the results of all three papers are "compatible," the formulation and results in the present paper are more insightful, in our view. We have also had the benefit of correspondence and/or mutual discussions over the last year with Chulsoon Khang and, in particular, Michael Bruno, whose paper (1973) in this Symposium complements ours admirably. John Chipman's careful comments have also led to many improvements.

[2]These should be linear homogeneous, and factor-intensities should differ in equilibrium.

[3]Proposition (i) follows from the concavity of the transformation function; Proposition (ii) follows from the identity of value-added with gross output; and Proposition (iii) follows from the Stolper-Samuelson theorem.

In this traditional analysis of "nominal" tariffs, the tariff leads to a change in the price of output and hence to change in output quantity: the change in value-added follows because value-added coincides with (gross) output and, in the two-primary-factors case, the uni-directional change in each primary factor used also follows because of the Stolper-Samuelson theorem. The basic proposition, however, consists in relating the change in the quantity of output to the change in the price of output, thanks to the nominal tariff structure. Since output change is the same as value-added change in these analyses, this may be taken as the primary proposition of the traditional theory concerning the effect of a tariff structure on resource allocation.

The task of the theory of effective protection may then be conceived essentially as one of relating, in a model allowing imported inputs, the tariff structure to change in value-added (where, with imported inputs, output does not coincide with value-added). Is it possible to devise a "price" of value-added, which can be used as an index to rank different activities such that, in exact analogy with the nominal tariff theory, the change in the "quantity" of value-added can be correctly predicted? If such an index can be devised, then we would be able to treat it as the total analog of the nominal tariff in the traditional model: for example, in the two primary-factors, two traded goods (and now, imported inputs) model, such an index would then be able to predict the shift of value-added between the two activities.

But one more dimension of the problem, which does not exist with nominal tariff theory, would be: can such an index be measured from observed or observable data without having to solve the general equilibrium (production) system for the two situations between which the resource-allocational shift is being predicted? For, if it cannot be, the index is "useless" because, to compute it, one would have to solve the full system and would thus already know the shift in value-added brought about by the tariff structure.[4]

In this paper, we use a general equilibrium, value-theoretic model with any number of primary factors, traded intermediates and goods, and discuss in terms thereof the question of the existence of a "price" of value-added that can serve as the "effective protection" index, predicting the shift in the value-added among the different activities. It is shown that such a price, and hence such an ERP index, cannot be constructed in general; this being the contention of the well-known Ramaswami-Srinivasan paper (1971). But that, two alternative sets of sufficiency conditions can be established, when such price of value-added and hence an ERP index exists. The first set of conditions consists in restricting the class of production functions to separable production functions--an approach implicit in the work of Corden (1969) who used what he called "two-stage" production functions. The other set of conditions consists in restricting the tariff structure to a range where, along with gross substitutability, it suffices to lead to a workable ERP index--an approach implicit in the work of Jones (1971) and Khang (1973), who work with a model where the tariff changes analysed are implicitly in the range which we spell out as sufficient for the ERP theory.

Note finally that we concentrate here on the problem of predicting resource allocation, as distinct from the problem of predicting (gross) output

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[4]In the analysis that follows, we will therefore find that the range of possibilities over which the ERP index works analytically is larger than the range over which it can be measured "usefully" in the sense defined in the text.

change. The latter question has also been the subject of inquiry. We therefore specify the workability of the ERP index for this purpose as well: however, the sufficiency conditions for predicting resource allocation (i.e. value-added) shifts are not valid for predicting output changes, as indeed we should expect.

## I: Sufficiency Conditions for ERP Theory

### The Model

Consider an economy producing n tradable goods for final use, using d ($d \geq n$) domestic primary inputs and m imported inputs. Let the production function for the $i^{th}$ good be $F^i(D^i, M^i)$ where $D^i = (D^i_1, \ldots D^i_d)'$ is the column vector of domestic inputs and $M^i = (M^i_1, \ldots M^i_m)'$ is the column vector of imported inputs used in its production. We shall assume, for simplicity, that all inputs enter into the production of each commodity and that each production function exhibits constant returns to scale and is concave. Let each domestic input be supplied inelastically to the extent of its availability.

Production is assumed to place under perfect competition, given the domestic price vectors. $P^0 = (P^0_1, \ldots, P^0_n)'$ and $P^M = (P^M_1, \ldots, P^M_m)'$ respectively of the outputs and imported inputs. For any given $P^0$ and $P^M$, the equilibrium outputs and inputs are assumed to be unique. For simplicity, we shall be concerned only with equilibria in which every commodity is produced.

We need some further notations. Let:

$$F^i_D = \left(\frac{\partial F^i}{\partial D^i_1}, \ldots, \frac{\partial F^i}{\partial D^i_d}\right)' \qquad i = 1,2,\ldots n. \qquad (1)$$

$$F^i_M = \left(\frac{\partial F^i}{\partial M^i_1}, \ldots, \frac{\partial F^i}{\partial M^i_m}\right)' \qquad i = 1,2,\ldots n. \qquad (2)$$

$$F^i_{DD} = \left(\left(\frac{\partial^2 F^i}{\partial D^i_j \partial M^i_k}\right)\right) \qquad i = 1,2,\ldots n;\ j,\ k = 1,2,\ldots d. \tag{3}$$

$$F^i_{DM} = \left(\left(\frac{\partial^2 F^i}{\partial D^i_j \partial M^i_k}\right)\right) \qquad i = 1,2,\ldots n;\ j = 1,2,\ldots d;\ k = 1,2,\ldots m. \tag{4}$$

$$F^i_{MD} = (F^i_{DM})' \tag{5}$$

$$F^i_{MM} = \left(\left(\frac{\partial^2 F^i}{\partial M^i_j \partial M^i_k}\right)\right) \qquad i = 1,2,\ldots n;\ j,\ k = 1,2,\ldots m. \tag{6}$$

$$V^i = P^0_i F^i - (P^M)' M^i = \text{domestic value added in industry } i. \tag{7}$$

The competitive equilibrium conditions are:

$$P^0_i F^i_D = P^0_n F^n_D \qquad i = 1,2,\ldots n-1. \tag{8}$$

$$P^0_i F^i_M = P^M \qquad i = 1,2,\ldots n. \tag{9}$$

$$\sum_{i=1}^{n} D^i_j = \bar{D}_j \qquad j = 1,2,\ldots d. \tag{10}$$

Equations (8) state that the marginal value product of each domestic input in each of the first (n-1) industries equals the marginal value product of the same input in the $n^{th}$ industry. Equations (9) state that the marginal value product of each imported input in any industry equals its price. Equations (10) state that the total amount used in all the n industries together of each domestic product equals the exogenously specified availability.

There are here n(d+m) endogenous variables, namely, $D^i_j$, $M^i_k$ where $i = 1,2,\ldots n$; $j = 1,2,\ldots d$; and $k = 1,2,\ldots m$. There are (n+d+m-1) exogenous variables, namely, $P_i^0$, $\overline{D}_j$, $P_k^M$ where i = 1,2,...n-1; j = 1,2,...d; k = 1,2,...m.[5] There are in all n(d+m) equations, consisting of d(n-1) in system (8), mn in system (9) and d in system (10). Thus the number of equations equals the number of endogenous variables. We have assumed throughout the analysis that the solution is unique and $D_j^i$, $M_k^i$ are positive for all i, j, k.

<u>The Analysis</u>:

We can look upon a change in tariff structure as a change in the domestic proce vectors $P^0$ and $P^M$. Let $\hat{P}^0$ and $\hat{P}^M$ denote a small change in $P^0$ and $P^M$ brought about by a small change in tariff structure. Let us denote by $\hat{V}^i$, $\hat{D}^i$, $\hat{M}^i$, the changes in $V^i$, $D^i$ and $M^i$ respectively. Differentiating (7) totally we get:

$$\hat{V}^i = \hat{P}_i^0 F^i - (\hat{P}^M)' M^i + P_i^0\{(F_D^i)'\hat{D}^i + (F_M^i)'\hat{M}^i\} - (P^M)'\hat{M}^i$$

$$= \hat{P}_i^0 F^i - (\hat{P}^M)' M^i + P_i^0 (F_D^i)'\hat{D}^i \text{ using (9).}$$

$$\frac{\hat{V}^i}{V^i} = \frac{\hat{P}_i^0 F^i - (\hat{P}^M)' M^i}{P_i^0 F^i - (P^M)' M^i} + \frac{P_i^0 (F_D^i)'\hat{D}^i}{P_i^0 - (P^M)' M^i} = \frac{(\hat{P}_i^0/P_i^0) - \sum_{k=1}^{k=m} \theta_{ik}^M (\hat{P}_k^M/P_k^M)}{1 - \sum_{k=1}^{k=m} \theta_{ik}^M} + \frac{\sum_{j=1}^{j=d} \theta_{ij}^D (\hat{D}_j^i/D_j^i)}{\sum_{j=1}^{j=d} \theta_{ij}^D} \tag{11}$$

$$\text{where } \theta_{ik}^M = \frac{P_k^M M_k^i}{P_i^0 F^i} = \frac{M_k^i \frac{\partial F^i}{\partial M_k^i}}{F^i} = \text{competitive share of } k^{th} \text{ imported input in } i^{th} \text{ output.} \tag{12}$$

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[5] We could have used the $n^{th}$ commodity as numeraire and set $P_n^0 = 1$. However, there is no reason why a tariff cannot be imposed on this commodity. As such we have not set $P_n^0 = 1$ by definition. Of course if a tariff structure changes all prices in the same proportion, i.e. $\hat{P}_i^0/P_i^0 = \hat{P}_k^M/P_k^M$ i = 1,2,...n; k = 1,2,...m, equilibrium outputs and

$$\theta^D_{ik} = \frac{D^i_k \frac{\partial F^i}{\partial D^i_k}}{F^i} = \text{competitive share of } k^{th} \text{ domestic primary input in output.} \tag{13}$$

It is seen from (11) that the proportionate change in value added in the $i^{th}$ industry, $\frac{\hat{V}^i}{V^i}$ , is the sum of two terms. The first term is the weighted average of the proportionate change in the exogenously given prices relevant to the $i^{th}$ industry, the proportionate change in price of output having a positive weight of unity and the proportionate change in price of each input having a negative weight equal to its competitive share in output. This term can therefore be interpreted as a proportionate change in the "net" price (as it were) of industry i or as a proportionate change in the "price" ($P^i_v$) of value added.

The second term, on the other hand, is a weighted average of the proportionate changes in domestic primary inputs used in industry i, each input having a weight equal to its competitive share in output. Thus, the second term can be interpreted as a proportionate change in "quantity" ($Q^i_v$) of value added by industry i.

Using these symbols, we can thus write:

$$\frac{\hat{V}^i}{V^i} = \frac{\hat{P}^i_v}{P^i_v} + \frac{\hat{Q}^i_v}{Q^i_v} \tag{14}$$

where
$$\frac{\hat{P}^i_v}{P^i_v} = \frac{(\hat{P}^0_i/P^0_i) - \sum\limits_{k=1}^{k=m} \theta^M_{ik}(\hat{P}^M_k/P^M_k)}{1 - \sum\limits_{k=1}^{k=m} \theta^M_{ik}} \tag{15}$$

$$\frac{\hat{Q}_v^i}{Q_v^i} = \frac{\sum_{j=1}^{j=d} \theta_{ij}^D (\hat{D}_j^i / D_j^i)}{\sum_{j=1}^{j=d} \theta_{ij}^D} \tag{16}$$

On examining (15), we see that $\frac{\hat{P}_v^i}{P_v^i}$ is <u>not</u> (in general) the "proportionate change in value-added per unit of output," which represents the original ERP definition of Corden (1966), Johnson (1965) and others. Rather, it is the ERP definition which is recommended by Corden (1969) for the case of substitution between imported inputs and domestic factors, and which is used by Jones (17 1), Ray (1973) and others.

Now, recall that, in the traditional model and the theory of nominal protection founded thereon, if the tariff on imports of a commodity is increased while tariffs on all other imports remain unchanged, the domestic output of that commodity will go up. Further, if the commodity experiencing a tariff increase is one of the only two commodities which the economy can produce (and produces) with two primary factors, then the Stolper-Samuelson theorem assures us that both primary factors will be attracted to its production. Analogously, one would expect that if the composition of a tariff structure results in equal ERP's for all industries but the $i^{th}$ and the $i^{th}$ has a <u>higher</u> ERP, the value added (the counterpart of output in the traditional model) by industry i would go up. Further, if there are only two industries and two primary factors in the economy, the industry receiving higher effective protection could be expected to attract both domestic factors in case all inputs are gross substitutes.[6]

We can therefore pose the analytical problem of ERP theory as one of

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[6]Some uncritical enthusiasts of ERP, however, have argued that a result that does not hold even for nominal tariffs in the traditional model, could hold for ERP's. Thus, they have held that the ranking of (more than two) industries according to their $\hat{V}^i/V^i$ after the imposition of a tariff structure would be the same as their ranking according to their ERP's.

defining sufficiency conditions for which these results would hold. To this problem, we now turn.

Suppose that a tariff structure results in $\frac{\hat{P}^1_v}{P^1_v} > (<) \frac{\hat{P}^2_v}{P^2_v} = \ldots = \frac{\hat{P}^n_v}{P^n_v}$. We wish to be able to say then that $\frac{\hat{V}^1}{V^1} > (<) 0$. Thus we wish to infer the sign of $\frac{\hat{V}^1}{V^1}$ from the <u>sign</u> of $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$ <u>alone</u> when all other $\frac{\hat{P}^i_v}{P^i_v}$'s equal $\frac{\hat{P}^n_v}{P^n_v}$. An inspection of (14) reveals that this is possible if and only if the sign of $\frac{\hat{Q}^1_v}{Q^1_v}$ is the same as that of $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$. From (15), note further that $\frac{\hat{P}^i_v}{P^i_v}$ depends upon $\frac{\hat{P}^0_i}{P^0_i}$, the changes in imported input prices $\left(\frac{\hat{P}^M_k}{P^M_k}\right)$ as well as the $\theta^M_{ik}$. From (16), we also see that $\frac{\hat{Q}^i_v}{Q^i_v}$ depends on $\theta^D_{ij}$ and $\frac{\hat{D}^i_j}{D^i_j}$.

One possible approach to establishing our sufficiency conditions then is to look for restrictions on the production function strong enough to ensure that $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$ and $\frac{\hat{Q}^1_v}{Q^1_v}$ have the same sign (in the case $\frac{\hat{P}^i_v}{P^i_v} = \frac{\hat{P}^n_v}{P^n_v}$ for 1 = 2,...n-1) regardless of (1) the alternative patterns of $\frac{\hat{P}^0_i}{P^0_i}$ and $\frac{\hat{P}^M_k}{P^M_k}$ that can result in a given sign for $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$ and $\frac{\hat{P}^i_v}{P^i_v} = \frac{\hat{P}^n_v}{P^n_v}$ i = 1,2,...n-1 and (2) the values of $\theta^D_{ij}$ and $\theta^M_{ik}$. The second approach is to look for restrictions on $\frac{\hat{P}^i_0}{P^i_0}$ and $\frac{\hat{P}^M_k}{P^M_k}$ such that $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$ and $\frac{\hat{Q}^1_v}{Q^1_v}$ have the same sign regardless of $\theta^D_{1j}$ and $\theta^M_{1k}$. Since we are not placing any special restrictions on the production functions in this approach, we have to look for such restrictions on the price changes that $\frac{\hat{D}^1_j}{D^1_j}$ has the same sign as $\left\{\frac{\hat{P}^1_v}{P^1_v} - \frac{\hat{P}^n_v}{P^n_v}\right\}$ for all j, thus ensuring that $\frac{\hat{Q}^1_v}{Q^1_v}$ is also of that sign.

I: <u>Sufficient Restrictions on Production Functions</u>:

It turns out that the first approach leads to the following restriction on the production functions $F^i$: that there exist functions $\phi^i(D^i)$ which depend only on $D^i$ such that $F^i$ could be written as:

$$F^i \equiv G^i[\phi^i, M^i]. \tag{17}$$

Given linear homogeneity of $F^i$ and its concavity, we can assume without loss of generality that $\phi^i$ is homogeneous of degree one and concave. In other words, each production function is "separable" in the sense that the domestic primary inputs used in each industry can be aggregated into an index $\phi^i$.

Now, given (17), we can write:

$$F^i_D = G^i_D \phi^i_D \quad \text{where} \quad \phi^i_D = \left(\frac{\partial \phi^i}{\partial D^i_1}, \ldots, \frac{\partial \phi^i}{\partial D^i_d}\right)' \tag{18}$$

$$F^i_M = G^i_M \tag{19}$$

where

$$G^i_D = \frac{\partial G^i}{\partial \phi^i} \tag{20}$$

$$G^i_M = \left(\frac{\partial G^i}{\partial M^i_1}, \ldots \frac{\partial G^i}{\partial M^i_m}\right)' \tag{21}$$

Suppose now we define $P^i_v = P^0_i G^i_D$. (22)

Then we can rewrite (8), (9) and (10) as:

$$P^i_v \phi^i_D = P^n_v \phi^n_D \qquad i = 1,2,\ldots n-1. \tag{8'}$$

$$P_i^0 G_M^i = P^M \qquad i = 1,2,\ldots n \qquad (9)'$$

$$\sum_{i=1}^{n} D_j^i = \bar{D}_j \qquad j = 1,2,\ldots d \qquad (10)'$$

It can be readily seen from (8)' and (10)' that the domestic input allocations $D_j^i$ depend only on $P_v^i$ ($i = 1,\ldots n$) and the total availability of each input. Given the linear homogeneity and concavity of $\phi^i$ we are back to the traditional model, if we interpret $\phi^i$ as the net output of industry i with $P_v^i$ as its net unit price. Hence, if the $P_v^i$ rises relative to $P_v^n$ while all other $P_v^i$'s remain the same relative to $P_v^n$, then the net output of i, i.e. $\phi^i$, will go up. Further, in the special case of a two-industry, two-primary factor world, this rise in net output (= value added) will come about by industry i attracting each domestic input from the other industry.[7] The gross output price $P_0^i$ and imported input price vector $P^M$ will influence domestic factor allocation only through their influence on $P_v^i$.

It is now easy to show that $P_v^i$, as defined by (22), satisfies (15) and that if we define $Q_v^i = \phi^i$, then (16) is also satisfied, thus linking up our results directly with the problem of ERP theory which we had formulated. For,

$$\begin{aligned} V^i &= P_i^0 F^i - (P_M)' M^i \\ &= P_i^0 G^i - (P_i^0 G_M^0)' M^i \\ &= P_i^0 [G^i - (G_M^0)' M^i] \\ &= P_i^0 G_D^i \phi^i \quad \text{since } G^i \text{ is linear homogeneous} \\ &= P_v^i \phi^i \quad \text{using (22).} \end{aligned}$$

Hence $$\frac{\hat{V}^i}{V^i} = \frac{\hat{P}_v^i}{P_v^i} + \frac{\hat{\phi}^i}{\phi^i} \qquad (23)$$

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[7] This also holds, in the case of more than two primary factors in a two-industry world provided all factors are gross substitutes, i.e. each primary factor will be attracted and net output will go up. The result, in neither

But $\quad \hat{\phi}^i = \sum \phi^i_j \hat{D}^i_j \quad$ where $\quad \phi^i_j = \dfrac{\partial \phi^i}{\partial D^i_j}$

and $\quad \phi^i = \sum \phi^i_j D^i_j \quad$ since $\phi^i$ is homogeneous of degree one.

Thus $$\frac{\hat{\phi}^i}{\phi^i} = \frac{\sum \phi^i_j \hat{D}^i_j}{\sum \phi^i_0 D^i_j} = \frac{\sum \phi^i_j D^i_j (\hat{D}^i_j / D^i_j)}{\sum \phi^i_j D^i_j}$$

But $$\frac{\phi^i_j D^i_j}{\sum \phi^i_j D^i_j} = \frac{G^i_D \phi^i_j D^i_j}{\sum G^i_D \phi^i_j D^i_j} = \frac{(\partial F^i / \partial D^i_j) D^i_j}{\sum (\partial F^i / \partial D^i_j) D^i_j} \left\{\text{using (19)}\right\} = \frac{\theta^D_{ij}}{\sum \theta^D_{ij}} \left\{\text{using (13)}\right\}.$$

Hence $$\frac{\hat{\phi}^i}{\phi^i} = \frac{\sum \theta^D_{ij} (\hat{D}^i_j / D^i_j)}{\sum \theta^D_{ij}} = \frac{\hat{Q}^i_v}{Q^i_v} \; . \tag{24}$$

This implies, given (11), that $P^i_v$ as defined by (22) satisfies (15). Hence, clearly the ERP index ($P^i_v$) will work so as to predict correctly incremental value-added in the protected industry.

It is worth noting that, in the case of separable production functions, we can indeed meaningfully talk of $P^i_v$ as price per unit of value added and $\phi^i$ as quantity (in physical units) of value added in each industry. The reason is the following. Suppose we are given the prices $w_1, \ldots w_d$, of the domestic primary inputs. The minimal cost of producing one unit of the value added product of industry i is obtained by minimizing $c^i = \sum_1^d w_j D^i_j$ subject to $\phi^i(D^i) = 1$. The minimal value of $c^i$ is $\tilde{\lambda}(w)$ where $(\tilde{D}^i, \tilde{\lambda})$ are solutions of $\phi^i_D(\tilde{D}^i) = \tilde{\lambda} w$ and $\phi^i(\tilde{D}^i) = 1$. Now this minimal unit cost $\tilde{\lambda}(w)$ is exactly equal to the price $P^i_v$ of value added as defined by (22) if $w$ is set equal to the value of $P^0_i F^i_0$ when $D^i$, $M^i$ satisfy the equilibrium conditions (9)-(10). This is easily seen by appropriate substitutions and utilising the separability of $F^i$ and linear homogeneity of $\phi^i$.

II: Sufficient Restrictions on Tariff Change:

Let us now turn to the second approach, assuming that $F^i$ are not separable.* Let us differentiate the system (8)-(10) totally. We get:

$$P_i^0[F_{DD}^i\hat{D}^i + F_{DM}^i\hat{M}^i] - P_n^0[F_{DD}^n\hat{D}^n + F_{DM}^n\hat{M}^n] = -\hat{P}_i^0F_D^i + \hat{P}_n^0F_D^n \tag{25}$$

$$P_i^0[F_{MD}^i\hat{D}^i + F_{MM}^i\hat{M}^i] = \hat{P}^M - \hat{P}_i^0F_M^i \tag{26}$$

$$\sum_{i=1}^{d}\hat{D}^i = 0 \tag{27}$$

Eliminating $\hat{D}^n$ and $\hat{M}^i$ $(i = 1,2,\ldots n)$, we get:

$$(P_i^0A^i + P_n^0A^n)\hat{D}^i + P_n^0A^n\sum_{j\neq i}^{i-1}\hat{D}^i = s^i + t^i \tag{28}$$

where $A^i = F_{DD}^i - F_{DM}^i(F_{MM}^i)^{-1}F_{MD}^i$ (non-separability ensures that $F_{MM}^i$ has an inverse, barring pathologies)

$$s^i = (-\hat{P}_i^0F_D^i + \hat{P}_n^0F_D^n)$$

$$t^i = -F_{DM}^i(F_{MM}^i)^{-1}(\hat{P}^M - \hat{P}_i^0F_M^i) + F_{DM}^n(F_{MM}^n)^{-1}(\hat{P}^M - \hat{P}_n^0F_M^n).$$

Let $\Delta$ be the square matrix of order $(n-1)d$ whose $(ij)$ element

$$\left.\begin{aligned}\Delta_{ij} &= P_i^0A^i + P_n^0A^n && \text{if } i = j\\ &= P_n^0A^n && \text{if } i \neq j\end{aligned}\right\}\quad i = 1,2,\ldots n-1 \tag{29}$$

Then:

$$\Delta(\hat{D}_1,\ldots\hat{D}_{n-1})' = (s^1 + t^1,\ldots,s^{n-1} + t^{n-1})' = s + t. \tag{30}$$

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*This automatically rules out the case of imported inputs being used in fixed proportions to output, of course.

Change in Gross Outputs:

Let us evaluate the sign of the change in gross output of an industry, say the first industry, as well as the change in value added by it.

$$\begin{aligned}
\hat{F}^1 &= (F_D^1)'\hat{D}^1 + (F_M^1)'\hat{M}^1 \\
&= (F_D^1)'\hat{D}^1 + (F_M^1)(F_{MM}^1)^{-1}\left\{\frac{\hat{P}_M}{P_1^0} - \frac{\hat{P}_1}{P_1^0} F_M^1 - F_{MD}^1 \hat{D}_1\right\} \\
&= \left\{(F_D^1)' - (F_M^1)(F_{MM}^1)^{-1}F_{MD}^1\right\} \hat{D}_1 + (F_M^1)'(F_{MM}^1)^{-1}\left\{\frac{\hat{P}_M}{P_1^0} - \frac{\hat{P}_1}{P_1^0} F_M^1\right\} \qquad (31)
\end{aligned}$$

$$\begin{aligned}
\hat{V}^1 &= P_1^0\hat{F}^1 - (P^M)'\hat{M}^1 + \hat{P}_1^0 F^1 - (\hat{P}^M)'M^1 \\
&= P_1^0(F_D^1)'\hat{D}^1 + \hat{P}_1^0 F^1 - (\hat{P}^M)'M^1. \qquad (32)
\end{aligned}$$

Suppose that effective protection is now conferred only on (against) industry 1 by the following change in the tariff structure:[8]

$$\frac{\hat{P}_1^0}{P_1^0} > (<) \frac{\hat{P}_i^0}{P_i^0} = \frac{\hat{P}_k^M}{P_k^M}, \quad i = 2,\ldots n; \; k = 1,2,\ldots m$$

i.e. the relative prices (in terms of good 1) of goods 2,...n and all imported inputs fall in the same proportion. This would mean that:

$$s^1 + t^1 = P_1^0\left(\frac{\hat{P}_n^0}{P_n^0} - \frac{\hat{P}_1^0}{P_1^0}\right)\left\{(F_D^1) - F_{DM}^1(F_{MM}^1)^{-1}F_M^1\right\} \qquad (33)$$

$$s^i + t^i = 0 \qquad i = 2,\ldots n-1. \qquad (34)$$

Solving for $\hat{D}^1$ from (30) and substituting in (31) we get:

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[8]It can be seen from (15) that this structure results in $(ERP)^1 > (<) (ERP)^i$, $i = 2,3,\ldots n$.

$$\hat{F}^1 = P_1^0\left(\frac{\hat{P}_n^0}{P_n^0} - \frac{\hat{P}_1^0}{P_1^0}\right)\left[\frac{1}{P_1^0}(F_M^1)'(F_{MM}^1)^{-1}F_M^1 + \frac{1}{|\Delta|}\right]$$

$$\cdot\begin{vmatrix} 0_1 & (F_D^1)' - (F_M^1)'(F_{MM}^1)^{-1}F_{MD}^1 & 0_2 \\ F_D^1 - F_{DM}^1(F_{MM}^1)^{-1}F_M^1 & & \\ & & \Delta \\ 0_3 & & \end{vmatrix} \qquad (35)$$

where: $|\Delta|$ = determinant of $\Delta$, and $0_1$, $0_2$, $0_3$ are null matrices of order 1x1, 1x(n-2)d and (n-2)dx1 respectively.

It is clear that the first term in the square bracket in (35) is negative since $(F_{MM}^1)^{-1}$ is a negative definite matrix. The second term is also negative since $|\Delta|$ is of the same sign as $(-1)^{(n-1)d}$ ($\Delta$ is a negative definite matrix of order (n-1)d) and

$$\begin{vmatrix} 0_1 & (F_D^1)' - (F_M^1)'(F_{MM}^1)^{-1}F_{MD}^1 & 0_2 \\ F_D^1 - F_{DM}^1(F_{MM}^1)^{-1}F_M^1 & & \\ & & \Delta \\ 0_3 & & \end{vmatrix}$$ is of the same sign as

$(-1)^{(n-1)d+1}$, being the determinant of a negative definite matrix of order (n-1)d+1. Thus $\hat{F}^1$ is of sign opposite to that of $\left\{\hat{P}_n^0/P_n^0 - \hat{P}_1^0/P_1^0\right\}$. Hence, if the first industry is conferred positive effective protection, i.e. $\left\{\hat{P}_1^0/P_1^0 > \hat{P}_n^0/P_n^0\right\}$, its gross output $F^1$ goes up and if effective protection is given against industry 1, i.e. $\left\{\hat{P}_1^0/P_1^0 < \hat{P}_n^0/P_n^0\right\}$ then its gross output goes down.

Change in Value Added:

This result on gross outputs paradoxically does not extend to value added. This is because, unfortunately, $(F_D^1)'\hat{D}^1$ is not of definite sign and hence it is not possible to assert, even with the earlier-imposed restrictions on tariff changes, that $V^1$ is positive (negative) according as effective protection is given to (against) the first industry.

However, in a two-industry economy, with inputs being gross substitutes, we can obtain the result we are after. This is seen as follows.

Given n = 2, $\Delta$ reduces to $P_1^0A^1 + P_2^0A^2$. Given that all inputs are gross substitutes, the off-diagonal elements of $F_{DD}^i$, $F_{MM}^i$ and all elements of $F_{DM}^i$ ( = $(F_{MD}^i)'$ ) are non-negative. This, together with the concavity of $F^i$, ensures that (a) $(F_{MM}^i)^{-1}$ consists of nonpositive elements and (b) the off-diagonal elements of $A^i$ are non-negative. Since $\Delta = P_1^0A^1 + P_2^0A^2$ is thus a negative definite matrix with non-negative off-diagonal elements, $\Delta^{-1}$ consists of nonpositive elements. Now $\hat{D}_1 = \Delta^{-1}\{s^1 + t^1\} =$ $P_1^0\{\hat{P}_2^0/P_2^0 - \hat{P}_1^0/P_1^0\}\Delta^{-1}\{F_D^1 - F_{DM}^1(F_{MM}^1)^{-1}F_M^1\}$ when the tariff change is restricted to $\{\hat{P}_2^0/P_2^0 = \hat{P}_k^M/P_k^M\}$, k = 1,2,...m. Since $\{F_D^1 - F_{DM}^1(F_{MM}^1)^{-1}F_M^1\} > 0$, it follows that $\hat{D}_1$ is of sign opposite to that of $\{\hat{P}_2^0/P_2^0 - \hat{P}_1^0/P_1^0\}$.

Hence, if the tariff structure results in effective protection being conferred on industry 1 (i.e. $\hat{P}_1^0/P_1^0 > \hat{P}_2^0/P_2^0 = \hat{P}_k^M/P_k^M$ , k = 1,2,...m), this industry attracts all domestic resources and its gross output and value added go up. If $\hat{P}_2^0/P_2^0 = \hat{P}_k^M/P_k^M > \hat{P}_1^0/P_1^0$ , then it is industry 2 which gets effective protection and it will attract each domestic resource resulting in its gross output and value added going up.

The Ramaswami-Srinivasan and Jones-Khang Analyses:

We are now in a position to "explain" and reconcile the results reached on ERP theory by Ramaswami and Srinivasan (1971), Jones (1971) and Khang (1973); in the process we will be able to pinpoint the differences resulting from the fact that the J-K model is a special case of the more general R-S model, the significance of the nature of tariff changes considered by these authors and the reason why factor endowments critically matter in the R-S exercise and not in the J-K analyses.[9]

All these authors discuss effective protection in the context of a two-industry model with two domestic inputs and one imported input. However, while the R-S model allows the use of the imported input by both industries, the J-K model restricts its use only to the first industry. The tariff structure considered by R-S involves subsidisation of the imported input, leaving the output prices unchanged, while J-K change one output price (that of industry 1) and the price of its imported input.

To relate their results with ours, let us tabulate $\Delta$, $\hat{P}^0_i$, $\hat{P}^M_k$, and $(s^1 + t^1)$ for the R-S, J-K models and evaluate $\hat{D}_1 = \Delta^{-1}[s^1 + t^1]$. (See Table 1.)

In both models, the off-diagonal elements of $\Delta$ are non-negative if all inputs are gross-substitutes. Hence the elements of $\Delta^{-1}$ and $(F^i_{MM})^{-1}$ are nonpositive. However, the sign of $(s^1 + t^1)$ is not, in general, determinate in either model. Thus, in the R-S model, since $F^i_{DM}$, $F^i_{MM}$ depend both on prices ($P^0_1$, $P^0_2$ and $P^M_1$) and factor allocations, the same change in prices (and hence, the same pattern of effective protection) could result (as in the R-S example) in either positive or negative $\hat{D}_1$ depending on the factor endowment which helps determine the factor allocations. In the J-K model, on the other hand, it is clear that:

$$\hat{D}_1 \gtreqless 0 \text{ if either: (a) } \hat{P}^0_1 = 0 \text{ and } \hat{P}^M_1 \lesseqgtr 0; \text{ or}$$

$$\text{(b) } \hat{P}^M_1/P^M_1 = \hat{P}^0_1/P^0_1 \text{ and } \hat{P}^0_1 \gtreqless 0; \text{ or}$$

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[9] The Jones (1971) Appendix I, which attempts at reconciling the R-S analysis with Jones' own results, is not really to the point in ignoring the critical differences which exist between the two models and hence also in the types of tariff change which can lead to breakdown of ERP theory.

Table 1: Critical Values in the R-S and J-K Models

| Model | $\Delta$ | $\hat{P}_1^0$ | $\hat{P}_2^0$ | $\hat{P}_1^M$ | $(s^1 + t^1)$ |
|---|---|---|---|---|---|
| R-S | $P_1^0\left\{F_{DD}^1 - F_{DM}^1(F_{MM}^1)^{-1}F_{MD}^1\right\} + P_0^2\left\{F_{DD}^2 - F_{DM}^2(F_{MM}^2)^{-1}F_{MD}^2\right\}$ | 0 | 0 | $\gtreqless 0$ | $\left\{F_{DM}^2(F_{MM}^2)^{-1} - F_{DM}^1(F_{MM}^1)^{-1}\right\}\hat{P}_1^M$ |
| J-K | $P_1^0\left\{F_{DD}^1 - F_{DM}^1(F_{MM}^1)^{-1}F_{MD}^1\right\} + P_0^2F_{DD}^2$ | $\gtreqless 0$ | 0 | $\gtreqless 0$ | $-\hat{P}_1^0F_D^1 - F_{DM}^1(F_{MM}^1)^{-1}(\hat{P}_1^M - \hat{P}_1^0F_M^1)$ |

$$\text{(c)} \quad \hat{P}_1^M/P_1^M < (>)\ \hat{P}_1^0/P_1^0 \text{ and } \hat{P}_1^0 \geqq (\leqq)\ 0.$$

In all these three cases, the industry gaining effective protection gains domestic resources and increases its value added. However, <u>outside</u> of the range of tariff changes described in (a)-(c), one could observe domestic factor movements and of value added in a direction <u>opposite</u> to that indicated by the pattern of effective protection.[10]

To sum up, one can define a measure of effective protection which performs, in the non-traditional model with imported inputs, a role <u>completely</u> analogous to that of nominal tariffs in the traditional model without imported inputs, only in the case of separable production functions. In the case of non-separable production functions, the analogy between effective protection and nominal protection breaks down except in cases where effective protection is conferred on an industry through particular forms of tariff change.

## II: "Useful" Measurability of ERP Index

We now address ourselves to the question whether, even in the cases where sufficiency conditions obtain, for the ERP index to predict value-added shifts correctly, the ERP index can be measured "usefully," i.e. <u>without</u> having access to the kind of information which would enable us to solve <u>directly</u> for the resource allocational effects of the tariff structure. It turns out that this range of possibilities is even narrower.

Remember that our analysis has been in terms of "differentials." To be of any <u>policy</u> use at all, one should be able to assess the impact of <u>non-infinitesimal</u> changes in the tariff structure. Indeed, in the traditional

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[10] Note that, in the cases where there are <u>only two</u> primary factors, the "workability" of the ERP index ($P_v^i$), in <u>both</u> the cases of sufficiency

conditions distinguished in this paper, is associated with the increment in value-added following ERP-production being accompanied by the increase in employment in this industry of both the primary factors. The Stolper-Samuelson theorem's validity in each instance is thus critical to this outcome, as noted by Bhagwati-Srinivasan (1971a). Note also that the R-S counter-example is characterised by the primary-factor-ratios in the two activities going in contrary directions, thus invalidating the Stolper-Samuelson argument; also read Khang (1973) from this point of view.

model, we have the comparitive static result that any increase in the tariff on imports of one commodity, ceteris paribus, will result in an increased production of that commodity in the new equilibrium. Formally of course, in the non-traditional model also, given that production functions are separable, we can say that any change in tariff structure which results in an increase in "price" of value added in one industry, ceteris paribus, will result in an increase in the "quantity" of value added of that industry in the new equilibrium.

However, one cannot in general compute the pattern of "prices" of value added from the knowledge of the tariff structure alone--one needs information on the production functions. This is in contrast to the traditional modelwhere one can predict that, ceteris paribus, the equilibrium output of a commodity will go up consequent on an increase in the tariff on this commodity without drawing upon any knowledge of its production function.

This fact is evident from our definition of ERP, in (15). In order to obtain the "price" of value added after a non-infinitesimal change in tariff structure, essentially we have to integrate (15). In the absence of imported inputs (15) reduces to $\hat{P}_i^0/P_i^0$ and hence the integral is the proportionate change in output price alone and can be computed directly from the tariff change. However, once imported inputs are admitted, $\theta_{ik}^M$ or the share of each imported input in output enters the expression and in general $\theta_{ik}^M$ depends on the prices, a functional dependence that can be derived from the production function. Without a knowledge of this dependence, one cannot, in general, carry out the required integration. For instance, this dependence can take the simple form that $\theta_{ik}^M$ are constant--a situation that arises in the case where the production function is Cobb-Douglas in the imported inputs and the index of domestic factors, such that $F^i = [\phi^i(D^i)]^{\alpha_0^i} (M_1^i)^{\alpha_1^i} \ldots (M_m)^{\alpha_m^i}$ with $\sum_{j=0}^{j=m} \alpha_j^i = 1$ and $\phi^i(D^i)$ is homogeneous of degree one in domestic inputs and concave--we can perform the integration with the information on $\theta_{ik}^M$ obtained from the initial equilibrium and with the knowledge of the proposed changes in tariff structure. Another instance is the case where each imported input is used in fixed proportions with output in each production function. In this case, also, the relevant information is contained in the initial equilibrium input/output ratios and the proposed changes in tariff structure.

In other cases, such as the general CES production function (which is, of course, separable) one can try to get by with "approximations" by assuming that the change in tariff structure is sufficiently small that either imported input coefficients or their shares in output remain approximately equal to their initial equilibrium values: but one really cannot get "correct" ERP indices measured usefully for the kinds of "real-life," "large" tariff structures which ERP-enthusiasts have been discussing in most recent contributions.

## CONCLUSIONS

Our analysis thus leads us to conclude, somewhat nihilistically, that:

(i) A measure of ERP which will unfailingly predict the domestic resource shift consequent on a change in the tariff structure does not always exist;

(ii) The range of sufficient conditions over which an ERP index will so work is significantly narrower than that over which the nominal tariff theory will so work in the traditional trade-theoretic model without imported inputs; and

(iii) The range of sufficient conditions over which such a working ERP index can be measured "usefully"--i.e. without solving the general equilibrium production system for both the situations over which the resource shift is sought to be predicted--is yet narrower.

These nihilistic conclusions are reinforced by four further observations:

(i) As we would expect, even when an ERP index works in predicting resource-allocation (i.e. value-added shifts), it does not necessarily work in predicting output shifts: and the latter are of greater interest in trade negotiations where ERP's may be thought of as replacing nominal tariffs in the future.

(ii) Recent studies, by Cohen (1969) and Guisinger and Schydlowsky (1970), of the relationship between the (calculated) nominal tariffs and ERP's in a number of empirical studies have shown that a remarkably high correlation exists between them: thus raising the question whether it is useful to spend vast resources on calculating ERP's when nominal tariffs seem to be adequate proxies for them anyway.

(iii) In a multi-commodity world where tariffs are levied on more than one commodity or input, we could not even tell, when the different processes were ranked by their ERP's in a chain, that the highest-ERP process would have gained resources and the lowest-ERP process would have lost them, in relation to the pre-trade situation.[11] As with nominal tariffs, the scope of purely "qualitative economics" is negligible in this real-world case, so that once again the vast empirical effort required in making up the ERP numbers seems grossly disproportionate to what can be done to predict actual resource-allocational impacts of the tariff structure without resort to the full general-equilibrium solution.

(iv) It also needs to be emphasised that attempts at arguing that the constancy of the (imported-factor) $a_{ij}$'s is a reasonable restriction because raw materials do not substitute with domestic factors and are in a fairly fixed proportion to output are based on a false equation of the imported factors with intermediates and raw materials. Most economies <u>import</u> capital goods and these <u>do</u> substitute with (domestic) labour quite generally. And, indeed, it is not at all uncommon for there to be substitution between intermediates and primary factors,[12] though admittedly this is less important in practice than the substitution among the primary factors, capital and labour.

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[11] Only when one price changes that resource-pull can be inferred unambiguously, with standard restrictions on production functions and on the relative numbers of the factors and products. For further discussion of this problem, see Bhagwati and Srinivasan (1971b).

[12] For examples, see Ramaswami and Srinivasan (1971).

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