International Liquidity Management:
Sterilization Policy In Illiquid Financial Markets

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Abstract

During the booms that precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. The main policy tool for this task is sterilization – essentially a swap of international reserves for public bonds. However, there is an extensive debate on the effectiveness of this policy, with many arguing that it may be counterproductive once the (over-) reaction of the private sector is considered. But what forces account for the private sector’s reaction remain largely unexplained. In this paper we provide a model to discuss these issues. We emphasize the international liquidity management aspect of sterilization over the traditional monetary one, a re-focus that seems warranted when the main concern is external crisis prevention. We first demonstrate that policies to smooth expansions in anticipation of downturns can be Pareto improving in economies where domestic financial markets are underdeveloped. We then discuss the implementation and effectiveness of this policy via sterilization. The greatest risk of policy arises in situations where policy is most needed – that is, when financial markets are illiquid. Our mechanism is akin to the "implicit bailout" problem, although the central bank acts non-selectively and only intervenes through open markets in our model. Illiquidity replaces corruption and ineptitude. In addition to an appreciation of the currency and the emergence of a quasi-fiscal deficit, the private sector’s reaction to sterilization may lead to an expansion rather than the desired contraction in aggregate demand or nontradeables investment and to a bias toward short term capital inflows. The main insights extend to international liquidity management issues more generally.

JEL Classification Numbers: E590, F310, F340, G380

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1 Introduction

During the booms that invariably precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. They primarily rely on tight monetary policy. In particular, they attempt to sterilize capital inflows through an open market sale of domestic bonds or increased reserve requirements.\footnote{E.g., Calvo et al. (1993) p. 146 write: “Sterilized intervention has been the most popular response to the present episode of capital inflows in Latin America.” And so does the World Bank (1997), p. 181: “Sterilization was the most widely and intensively used policy response to the arrival of capital inflows among the countries in our sample.” The sample included 22 emerging economies.}

These sterilized interventions can be extremely large. During the early 1990s in Chile, for example, the exchange intervention meant that over three quarters of its large capital inflow — amounting to around seven percent of its GDP per year — went to international reserves accumulation at the central bank. The sterilization of this intervention increased the ratio of international reserves to monetary base from 3.5 around 1990 to over 6.0 by 1993. This pattern was repeated in many emerging economies during the early 1990’s, when capital flows to the developing world surged. In fact most of the economies involved in the crises of the second half of the 1990’s had heavily sterilized inflows. Illustratively, many of these countries’ central banks exhausted their stock of treasury securities in the process, and had to resort to alternative sterilization mechanisms (see e.g. Glick (1998)).

While sterilization is a widely used tool, both policy makers and academics have warned that it comes along with a number of difficulties and risks. Building on the Mundell-Fleming logic, many argue that sterilization is, at best, ineffective. When capital markets are integrated and there is a simultaneous attempt to stabilize the exchange rate, the central bank has no control over the money supply because the private sector can undo an open market sale of bonds for money (Mundell (1962)). The policy literature, noting the increase in capital inflows that accompany sterilization, argues that since it is these flows that fuel what is perceived as an excessive expansion in aggregate demand and exchange rate overvaluation, sterilization is counterproductive (e.g. Calvo et al. 1993, Williamson 1995, Corbo and Hernandez 1996, Massad 1997).\footnote{In his statement on behalf of the Latin American Governors of the Fund at the joint World Bank - IMF annual meeting of the Board of Governors held in Hong Kong (1997), Massad writes: “The high rate of return on capital in a booming economy attracts large inflows of external resources. These inflows are further encouraged by the appreciation of the domestic currency, which is characteristic of economies experiencing rapid productivity growth. Capital flows stimulate domestic demand and could push up domestic interest rates if the monetary authority safeguards domestic equilibrium. This, in turn, could provide a further incentive for capital inflows. The probable outcome will be continued appreciation of the local currency, the resulting risk of widening the current account deficit, and the greater danger that these capital flows will be reversed, should some negative external shock occur.” (page 4, our emphasis).}

Building on Sargent and Wallace’s (1981) unpleasant monetary arithmetics, Calvo (1991) formally shows that, by raising domestic
interest rates, the government increases its debt-service burden and creates a quasi-fiscal deficit that may jeopardize the very stabilization attempt that is supposedly being protected by the sterilization.

Uniformly, this debate has viewed the effects of sterilization as arising from changes in the composition of the government’s liabilities (money versus bonds). However, as noted above, in a typical sterilized intervention the central bank also accumulates substantial international reserves as assets and some mix of domestic currency and bonds as liabilities, while the private sector’s balance sheet changes in the opposite direction. We argue in this paper that the impact that sterilization has on the asset side of the government’s balance sheet and on its counterpart in the private sector is central — and perhaps the chief factor — in understanding its consequences. That is, we emphasize the international liquidity management aspect of sterilization over its monetary implications.

We build this view on two salient features of emerging economies vis-à-vis developed ones: First, it is crisis-prevention rather than day-to-day fine-tuning that typically shapes their macroeconomic policy. And second, external crises are invariably associated with a country’s shortage of international liquidity. The sovereign debt literature echoes both of these points (see Eaton and Gersovitz (1981), Bulow and Rogoff (1989)). A country’s debt capacity is tied to its aggregate international collateral (or liquidity), and therefore external crises occur when this external constraint binds. This paper studies the use of sterilization as a tool for international liquidity management. Since sterilization is effectively an intervention in domestic assets markets, an adequate treatment of this liquidity management issue requires us to be explicit not only about the presence of an aggregate external constraint, but also about the structure of domestic assets and their liquidity.

For a concrete example of the environment and policy issues addressed in this paper, consider the following: Suppose, that all domestic production in the economy requires only imported goods. In order to put up a building in downtown Bangkok, a Thai developer must import all the raw materials for the building. Suppose that this building is not acceptable collateral to a foreigner, so that loans to this developer, against the collateral of the building, will only be forthcoming from other domestics. Lacking any internationally liquid assets to exchange for the raw materials, it would appear that the real estate developer is in a dilemma. However, suppose that foreigners do accept, as internationally liquid assets, claims on export sector receivables. Then, construction can proceed as long as the real estate developer can find a domestic with export sector revenue who will accept the building

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3See Caballero and Krishnamurthy (1999) for a model of crises based on collateral shortages. For us, an internationally liquid asset is one that can be sold (or borrowed against) at a moment’s notice to an international investor without suffering a steep discount. A shortage of international liquidity means that the quantity of these assets, net of any pre-existing external debt, is insufficient to meet all external financial needs.
estate builders take out loans at this low cost of capital to “arbitrage” the government’s short-term liquidity commitment. We show that this mismatch leads to increased external borrowing and a shift in the composition of borrowing toward short maturities.⁵,⁶,⁷

Unlike the international finance literature, we emphasize the international liquidity management aspect of sterilization over its monetary aspect. Changes in international reserves and public debt are central to our analysis. While not addressing sterilization, both Woodford (1990) and Holmstrom and Tirole (1998) present models in which policy has real effects through changes in public debt. Government policies are non-Ricardian because public debt provides the private sector with liquid assets that they are unable to create for themselves. Holmstrom and Tirole (1998) show that when there are aggregate shocks the private sector may have a shortage of liquid assets. Public bonds make up this shortfall and government policy has real effects. In our model, on the other hand, issuing public bonds does not increase private liquidity. In fact we shall assume that if the government issue a public bond, the private sector anticipates a tax liability which reduces domestic private liquidity one for one. As a result, policy along the lines of Woodford (1990) or Holmstrom and Tirole (1998) will be Ricardian in our model. At a more abstract level, nonetheless, our policies are related to theirs in that government policy acts through changing the liquidity of the (in our case, international) assets ultimately held by the private sector.

In section 2 we lay out our basic model. We show that when domestic financial markets are underdeveloped an externality arises whereby the private sector draws down its international liquidity too fast (over time) relative to the constrained efficient outcome. This sets the stage for the policy discussion.

In section 3 we introduce a government/central bank and describe its rights and commitments. We demonstrate conditions under which sterilization policy is effective and leads to Pareto improvements and those under which it is completely undone by the private sector. When domestic secondary markets are illiquid, the government action can backfire, leading to a net loss of international liquidity and a Pareto loss.

In section 4 we show that our perspective naturally accommodates two additional sources

⁵With some relabeling, this mechanism can also be illustrated via a fixed exchange rate commitment. Suppose that the government has reserves to sustain a fixed exchange rate over the next year, but has issued bonds that expire later. Then capital inflows to purchase the bonds cannot take advantage of the fixed exchange rate unless the bonds can be sold in liquid secondary market in the next year. If this is not so, other shorter term assets will be created to take advantage of the government commitment. The domestic private sector finds that there is good demand for such assets and takes out loans (i.e. sells the asset) and increases real investment.

⁶The shift in composition of capital inflows during periods of intensive sterilization has been documented by, e.g., Montiel and Reinhart (1999) and Calvo, Leiderman, and Reinhart (1993).

⁷Dooley (1999) has also emphasized some of the insurance aspects of interventions.
anticipate a crisis, and still will not do enough about it.\textsuperscript{8}

2.1 The Economic Environment, Assets, and Balance Sheets

2.1.1 Basic setup

\textit{Time.} The world lasts three periods. Time is indexed as \( t = 0, 1, 2 \). Date 0 is the fully flexible period when agents design the productive structure, ownership structure, and portfolio allocations. Date 1 is the crisis period, when agents must shift resources from the future to cope with shocks in the present. Date 2 represents the unconstrained future, when the economy is (relatively) rich in resources.

\textit{Agents and heterogeneity.} There are two types of agents: (i) a continuum of unit measure of domestic entrepreneurs-consumers (henceforth, domestics) with linear preferences over date 2 consumption of a single good, and (ii) foreign financiers (henceforth, foreigners) with large endowments at all dates and linear preferences with no discounting, thus the international gross interest rate is one.\textsuperscript{9}

\textit{Endowment, Production, Investment, and Liquidity.} We assume that domestic agents are endowed at date 0 with \( w \) units of an internationally liquid asset — e.g., the present value of export sector receivables — and access to a production technology.

On net, domestic agents must import materials from the rest of the world in order to produce. They do this by pledging their international liquidity to foreigners and taking on foreign debt of \( d_{0,f} \). Production has a time-to-build aspect. Investments are made at dates 0 and 1, and output is realized at date 2. Let \( k \) denote the total amount of capital devoted to production at the beginning of date 1, inherited from date 0. Then creating capital of \( k \) requires a date 0 investment of \( c(k) \) units of imported goods. \( c(k) \) is assumed to be strictly increasing, convex and positive.

We capture the normal churn of the economy, with its implied domestic heterogeneity, with a simple Bernoulli process. At date 1, half of the firms are spared of further investment and go on to produce \( Rk \) units of goods at date 2. The rest experiences a productivity fall, \( \Delta \equiv R - r > 0 \), which can be offset by reinvesting a fraction \( \theta \leq 1 \) of \( k \), in units of the imported good, in order to realize output at date 2 of:

\[ \bar{R}(\theta)k = (r + \theta\Delta)k \leq Rk. \]

This time-to-build structure underlines a critical link between financing and production during a crisis. Firms in any economy have ongoing capital needs (working capital, etc.).

\textsuperscript{8}See Caballero and Krishnamurthy (1999) for a similar model with aggregate shocks.

\textsuperscript{9}The distinction between foreigners and domestics needs not be as stark as posited here. Domestic savers, for example, may be grouped with one or the other depending on the nature of the shocks faced by the economy. Our insights can be easily extended to more complex environments along this direction.
another domestic in order to take on debt. On the other hand, we assume that \( rk \) cannot be pledged to a foreign investor. Foreigners only lend to a firm against its internationally liquid assets of \( w \). This means that the maximum amount of external debt the country can take on is \( w \).\(^{11}\) While much of this asymmetry can have a microeconomic origin, there are sovereign aspects that reinforce it. We return to this issue in the next section.

**Assumption 3 (Liquidity Bias)**

*Foreigners lend to domestic firms only against the backing of \( w \). Domestics lend against both \( w \) and \( rk \).*

*Financial structure and Balance Sheets.* Firms can raise finance at date 0 and date 1 from either domestic or foreign investors. We assume that all finance must be default free and fully secured debt – either by domestic liquidity in the case of domestics, or international liquidity in the case of foreign investors.

Date 0 decisions result in firms arriving at date 1 with installed capital of \( k \) and foreign debt of \( d_{0,f} \). At date 1, a firm that receives a shock is distressed (S), while a firm that escapes the shock is intact (I). The balance sheet of a domestic firm has assets of \( rk \) units of domestic liquidity and \( w \) units of international liquidity, and foreign debt of \( d_{0,f} \). The simplest way to think of the asymmetric treatment of collateral by foreigners and domestics, is to think of foreigners studying a balance sheet of the firm as perceiving only \( w \) as assets. On the other hand, a domestic sees both this quantity as well as \( rk \) as assets. Thus the debt constraint with respect to foreigners at date 0 is,

\[
d_{0,f} \leq w.
\]

At date 1, if a firm takes on additional debt with foreigners, the date 1 debt constraint is:

\[
d_{0,f} + d_{1,f} \leq w.
\]

### 2.1.2 Discussion of main assumptions

Let us pause at this juncture and discuss our main assumptions, starting from the two borrowing constraints.

We have assumed a friction that prevents a domestic entrepreneur from borrowing fully up to his output from another domestic. Investment at date 0 produces \( rk \) goods at date 2, and depending on the realization of shocks and reinvestment, an additional \( \Delta k \) of output.

\(^{11}\)The stark distinction between domestics and foreigners in their valuation of assets is only made for simplicity. In reality, many residents behave like our foreigners at time of distress (e.g., households may be behind capital flights), and many foreigners behave like our domestics (e.g., institutional specialists well informed and connected with the domestic establishment).
that only a fraction of exports or net exports is international collateral. On the other hand, the sovereign debt literature typically just imposes international collateral as an aggregate constraint. We take a microeconomic perspective by assuming that \( w \) is held by individual agents in the economy who can trade it among themselves and with foreigners.

Sensible as it may be, here we have simply posited assumption 3 regarding liquidity bias. In the next section we introduce a government and sovereign risk, thereby providing an alternative — and more explicitly modeled — grounding for the assumption.

The last assumption worth commenting on is the non-observability of the idiosyncratic production shock at date 1. Domestic agents are ex-ante identical at date 0. If the production shock was verifiable, domestic agents would write contracts amongst themselves to insure that they would be ex-post identical as well. There would be no heterogeneity at date 1 amongst domestics and frictions in the domestic market would have no economic effect. Assuming non-observability is necessary to study the impact of these frictions.

## 2.2 The Microeconomic Problem

Domestics have two sets of decisions. At date 1, given the date 0 choices of other firms (through prices) and the realization of the idiosyncratic shock, a domestic firm must decide how much to borrow (lend) and reinvest. At date 0, a firm must decide how much to invest and how much international liquidity to retain. We solve this problem by backward induction, starting from date 1.

**Date 1 problem.** Consider the problem of a distressed firm in raising funds to alleviate its production shock. A choice of \( \theta k \) will result in output at date 2 of \( \hat{R}(\theta) k \) goods. In order to save a fraction \( \theta \) of the distressed unit, the firm must raise finance and reinvest \( \theta k \) import goods. It can do this in two ways. First, the firm may have some international liquidity at date 1 that it can use to borrow directly from foreigners. That is the firm can always raise directly,

\[
d_{1,f} \leq w - d_{0,f}.
\]

The latter quantity is always positive, and represents the minimum that a firm can raise at date 1. The rest must come from intact firms, which also have access to foreign investors since their capacity to borrow abroad at date 1 is also \( w - d_{0,f} \).

A distressed firm can use its domestic liquidity to access the international liquidity of the intact firms. In equilibrium, the latter discount the domestic liquidity at a rate of \( L_1 \geq 1 \) in providing international liquidity. \( L_1 \) is the date 1 interest rate. It is not an interest rate that is driven by expectations of default or currency depreciation. Rather, it is driven by

respectively. The same statistics for firms that do not borrow in foreign currency (75 percent of firms) are: 46 percent, 139 employees, 2.36, 61 percent and 19 percent. See Dollar and Hallward-Dreimeier (1998).
2.3 Equilibrium and Crises

Equilibrium. Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

\[
D_{1,d} = \frac{1}{2} d_{1,d} \\
X_{1,d} = \frac{1}{2} x_{1,d}.
\]

Therefore, market clearing,

\[
D_{1,d} = X_{1,d},
\]

determines the gross interest rate, $L_1$.

An equilibrium of this economy consists of date 0 and date 1 decisions, $(k, d_{0,f})$ and $(\theta, d_{1,f}, d_{1,d}, x_{1,d})$, respectively, and prices $L_{1}$.\footnote{Where we have used the fact that at equilibrium prices, $\frac{x_{1,d}}{L_{1}} = d_{1,f}$.} Decisions are solutions to the firms’ problems (P1), (P2), and (P3) given prices. At these prices, the market clearing condition (1) holds.

Let us now study equilibrium in more detail. Starting from date 1, consider financing and investment choices of the distressed firm given $(k, d_{0,f})$. First, if $\Delta \geq 1$, then the distressed firm would choose to save as many of its production units as it can. It may borrow up to its international debt capacity,

\[
d_{1,f} = w - d_{0,f}.
\]

If the amount raised from international investors, $w - d_{0,f}$, is less than the funds needed for restructuring, $k$, the firm will have to access the domestic debt market to make up the shortfall. It will choose to do this as long as $\Delta \geq L_1$, or the return on restructuring exceeds the interest rate. If the firm borrows fully up to its domestic debt capacity, it will issue debt totalling,

\[
d_{1,d} = rk,
\]

and raise funds with which to pay for imported goods of \( \frac{rk}{L_{1}} \). As long as the sum of \( \frac{rk}{L_{1}} \) and the right hand side of (2) is more than the borrowing need, the firm is unconstrained in its reinvestment at date 1 and all production units will be saved. In this case, the firm will borrow less than its domestic debt capacity (and perhaps less than the international debt capacity).

Intact firms can tender at most their excess international debt capacity of $w - d_{0,f}$ in return for purchasing domestic debt. They will choose to do this as long as the domestic interest rate exceeds the international rate of one, $L_1 \geq 1$. 

\[\text{16} \]
where both liquidity constraints are binding. At the aggregate level, the economy is liquidity constrained with respect to foreigners; at the individual level, firms are liquidity constrained with respect to other domestics since they are selling all of their domestic liquidity in aggregation; real investment is constrained; domestic spreads are positive; and the interest rate of $L_1$ is above the international interest rate. This is the most interesting configuration for the prevention-policy questions we intend to address in the main section of the paper.

**Technical Assumption 1 (Conditions for Crisis)**

*Assume that:*

$$c'^{-1}\left(\frac{\Delta + R}{1 + \Delta}\right) + c\left(c'^{-1}\left(\frac{\Delta + R}{1 + \Delta}\right)\right) < w < c'^{-1}\left(\frac{1 + R}{2\Delta}\right) + c\left(c'^{-1}\left(\frac{1 + R}{2\Delta}\right)\right).$$

The assumption guarantees that in equilibrium $\Delta > L_1 > 1$ and $\theta < 1$. This ensures focus on a case where both (3) and (4) bind.

**Proposition 1 (Crisis Region)**

*Under assumption 1, date 0 decisions of $k$ and $d_{0,t}$ are such that both the international constraint and the domestic constraint are binding. The international liquidity premium and the domestic illiquidity index are positive, and some projects are downsized. $L_1 > 1, s_d > 0, \theta < 1.*

Graphically, the solution to (P3) is represented in figure 2. The inner curve represents the budget set. This is the set of points of $(w-d_{0,f}, k)$ such that the date 0 budget constraint
substitute the expression for $L_1$, (5), into the objective of (P4), arriving at an expression that is free of prices. The program for a central planner is,

\[(P5) \quad \max_{K,D_{0,f}} (R+r)K + 2\Delta(W - D_{0,f})\]

s.t. \[w \geq d_{0,f}\]
\[c(K) = D_{0,f}\]

The solutions to (P5) are the constrained efficient decisions of the economy. The only difference between the programs (P4) and (P5) is in the objective. Subtracting the objective in (P5) from that of (P4) we arrive at,

\[s_d \left( \frac{r}{L_1}K - (W - D_{0,f}) \right).\]

At a given equilibrium, this term must be zero. But it is apparent that individuals and the central planner value a marginal unit of international liquidity and domestic liquidity quite differently. Moreover, this misvaluation is directly proportional to $s_d$, the domestic illiquidity index.

The first order condition of (P5) gives,

\[c'(K) = \frac{R+r}{2\Delta},\]

while that of (P4) yields,

\[c'(k) = \frac{R+r\Delta}{\Delta + L_1}.\]

Graphically, we can represent the solutions to (P4) and (P5) in figure 3.

![Figure 3: (P4) and (P5)](image)
some of the international liquidity held by the private sector (which needs to borrow abroad to buy the public bonds).

Widespread as it may be, sterilization is perceived as a "risky" strategy, hampered by the possible overreaction of the private sector. But the mechanisms behind this "risk" are not well understood, let alone modeled. In this section we offer a methodic analysis of sterilization in an economy with underdeveloped financial markets for private and public instruments. We shall begin by noting conditions under which the private sector completely undoes the central bank's action. In cases where this does not occur, we demonstrate conditions under which the policy can be Pareto improving, and those under which it results in a Pareto loss. Before doing so, however, we must introduce the (consolidated) government, its instruments and constraints, and the implications of its action for asset liquidity.

3.1 Preliminaries

3.1.1 Public Bonds

We consolidate the central bank and the treasury. The minimum number of ingredients we need in order to address our policy question is one public financial instrument and a tax to finance any quasi-fiscal deficit that the sterilization policy may generate.\(^{19}\) We start with this minimum and enrich the set of public financial instruments to include money in section 4, when we discuss exchange rate systems.

We formally describe public bonds in the next assumption, and justify it at the end of the preliminaries-section, when we discuss the key assumptions of this section.

**Assumption 4 (Public Bonds)**

*At date 0, the government issues public bonds with face value \(B\):*

- (Long Maturity) These bonds mature at date 2, but can be bought and sold in secondary markets at date 1.

- (Iliquidity) A sale at date 1 of one unit of a date 2 government bond suffers a real cost of \(0 \leq \alpha < 1\). Selling \(X\) units of bonds only recovers \(\frac{X(1-\alpha)}{L_1}\) units of international liquidity.

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\(^{19}\)Importantly, taxes are always paid in units of liquidity (either domestic or international). That is, suppose that a firm has \(rk\) units of domestic liquidity at date 1, and that the government levies a tax of \(T\) on the firm. Then, after the transaction the firm will be left with \(rk - T\) units of domestic liquidity, and the government will be left with \(T\) units of domestic liquidity. Thus, if the firm has a tax liability of \(T\) to the government, this simply reduces its liquidity with respect to other firms. That is, unlike Woodford (1990) and Holmstrom and Tirole (1998), our government cannot create liquidity. The simplest way to think of taxes is that they alter the balance sheet of a firm by introducing an additional liability. This then affects the firm's ability to raise finance from other agents.
0 sterilization transaction would be fully undone at date 1. Since the government reverses its own action very quickly, policy must fail. Essentially the one-period bonds are viewed as perfect substitutes for the reserves they replace. It is worth pointing out that we do not require the extreme assumption made above, but just that a fraction of the new government liabilities matures after some potential external crisis.

Long maturity of public bonds is not a sufficient condition for imperfect substitutability, because there is still a secondary market open and the government will inject its reserves into this market at date 1. There are two types of domestic assets, corporate assets and government bonds. Since foreigners cannot repatriate payments at date 2, any assets held until date 2 are effectively defaulted on to foreigners. As a result, foreigners must sell all assets at date 1, and limit their date 0 holdings to reflect this fact. In principle, if the government commits to inject all of the foreign reserves gained during sterilization back into the market at date 1, then a foreign investor would be willing to buy a domestic bond since he knows he can sell these at date 1 for the government’s reserves.\(^{20}\) Requiring that the public reserves are released only on presentation of an invoice for imported goods implies that the foreign investor cannot, directly, repatriate the return from selling the domestic bond at date 1. Taken together, the two assumptions on sovereign risk imply that domestic assets are imperfect substitutes for the foreign assets removed from the private sector during sterilizations, and this allows some space for the policy to be non-Ricardian. Finally, the sovereign risk assumption also provides a deeper foundation for our earlier assumption on liquidity bias.

Our sovereign risk assumption is similar to that made in the widely accepted and empirically supported sovereign debt literature (see e.g. Obstfeld and Rogoff (1998) ch. 6). The main difference is that, since decentralization is not their main concern, the sovereign debt literature simply posits a maximum repayment for a country. In our setup domestic financial markets are modeled explicitly and hence we have to be careful about how private actions may affect the maximum repayment and perhaps undo government actions. Both aspects of our assumption, suspension of convertibility and selective default/rescheduling have been observed in recent years (e.g. Malaysia for the former, and Ecuador, Korea and Mexico for the latter).

Finally, we will be interested in studying the role of illiquidity in the secondary public bond market. Our modeling of illiquidity – transactions costs incurred for early liquidation – is not unusual. As we will see below, illiquidity can exacerbate the impact of the government’s maturity mismatch on the private sectors’ asset liquidity. On one hand, illiquidity is

\(^{20}\)Imagine the following scenario: the government issues 1 bond for $1 at date 0. The foreigner buys this bond at date 0 for $1. He sells this at date 1, knowing that the government will inject the $1 back into the market, and this will provide the liquidity for his exit from the country.
3.3 Sterilization with Liquid Public Debt Markets

Let us start our study of the effectiveness of sterilization in a context where private asset markets are underdeveloped but secondary public debt markets are domestically liquid ($\alpha = 0$).

Lemma 2 (Date 0 Interest Rates)

*If the private sector holds government bonds in equilibrium, the date 0 interest rate must satisfy,*

$$L_0 \geq L_1$$

This is easy to verify. In order for domestics to hold government bonds, they must be compensated for losing their international liquidity. Taking on an extra unit of debt costs $\Delta + L_1$. However purchasing one bond yields an extra $L_0^{\Delta + L_1}$. This gives us the inequality in the lemma.

Sterilization is naturally associated with a capital inflow, as either foreigners or domestic investors will attempt to buy the high yield government bonds. Foreigners have short horizons, however. Thus, if foreigners see a scenario where they can purchase these bonds at date 0 and sell them back to the domestic private sector at date 1, they will step in to purchase the bonds. They will do this only to the extent that the private sector has some international liquidity to offer at date 1. Thus the international liquidity of the public bonds is closely tied to the international liquidity of the private sector. The other potential buyer of the bonds is the domestic private sector itself. Since this sector does not have short horizons, it can always take advantage of the high return on the bonds by borrowing abroad (a capital inflow) to purchase the public bonds. However, once again, the capacity of the private sector to do so is limited by its international liquidity. In both cases it is the liquidity of the private sector that determines the outcome of sterilization.

3.3.1 Liquidity Conservation

Lemma 3 (Liquidity Bias and Aggregate International Illiquidity)

*If the government sterilizes so that the private sector is internationally illiquid, $d_{0,f} = w$, then foreigners will hold no domestic claims and restrict their holdings to international claims.*

Proof: see appendix.

This is the date 0 effect of future suspension of convertibility and market illiquidity. If the domestic private sector has no international liquidity to offer a foreigner when he sells domestic claims at date 1, then the foreigner’s date 1 liquidity bias extends back to date 0. The foreigner anticipates that there will be a suspension of convertibility at date 1 and
From the previous proposition, we know that sterilization is ineffective if \( d_{0,f} < w \). Taking the other case \( (d_{0,f} = w) \), we can rewrite the program as,

\[
\max_{k,b} \quad k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1}) \\
\text{s.t.} \quad c(k) + \frac{b}{L_0} = w.
\]

The first order conditions for this program yield,

\[
L_0 c'(k) = \frac{R + r \frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}}
\]

Now optimal policy will be such that \( k < k' \). For this to be the case, we must have that \( c'(k) < c'(k') \). \( L_0 \) satisfies,

\[
\frac{1}{L_0} \frac{R + r \frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}} < c'(k') = \frac{R + r \frac{\Delta}{L_1}}{\Delta + L_1'}
\]

Rewriting this yields

\[
L_0 > L_1 \frac{R + r \frac{\Delta}{L_1} \Delta + L_1'}{R + r \frac{\Delta}{L_1} \Delta + L_1'}.
\]

Consider the market clearing condition for \( L_1 \).

\[
L_1 = L_0 \frac{rK + B - T}{2B}
\]

which we can rewrite as,

\[
L_1 \frac{B}{L_0} + L_1 \frac{B}{L_0} = rK + B - T.
\]

Substituting in the government's budget constraint and rewriting,\(^21\)

\[
L_1 = L_0 \frac{rK}{B}
\]

Whereas,

\[
L_1' = \frac{rK'}{W - D_{0,f}}.
\]

If \( k < k' \) then \( \frac{B}{L_0} = W - c(K) > W - D_{0,f} \). Therefore, \( L_1 < L_1' \) and \( B > 0 \). Combining this with (8), we can conclude that \( L_0 > L_1 \).\(^22\)

\(^21\)The budget constraint for the government is that,

\[
T + L_1 \frac{1}{L_0} B = B.
\]

When \( L_0 = L_1 \), the budget balances without having to raise taxes. When \( L_0 > L_1 \), it must be that \( T > 0 \) to pay the interest on the government debt.

\(^22\)We also need to make sure that \( \Delta > L_1 \) after the intervention so that we are still in the region where both liquidity constraints are binding.

\[
rK < \frac{\Delta}{L_0} B = \Delta(W - c(K))
\]

Since \( K < K' \), if the condition is satisfied at the decentralized solution, it must also be satisfied at the central planner's solution.
bank action at both date 0 and date 1.

When sterilization is successful, the private sector is not the marginal international liquidity provider. The central bank takes over this job. Indeed, since $L_1$ falls, the private sector has little incentive to do so. Is it possible to arrive at scenario where both, $L_1$ falls so that the private sector has little incentive to liquidity provision, and the private sector remains the marginal liquidity provider? If so, the private sector would free-ride by cutting its liquidity provisioning (relative to the case of no-intervention) and on net the economy would lose international liquidity. In this section we show that this scenario is a very real possibility when domestic markets are illiquid. We shall demonstrate conditions under which the capital inflow accompanying sterilization leads not just to a purchase of government bonds but also to increased lending to the domestic private sector.

At date 1, government bonds are exchanged for international liquidity by both domestic distressed firms and any potential foreign investors. Distressed firms sell in order to receive funds for investment, while foreign holders sell in order to exit the market. We now reintroduce a friction in this transaction ($\alpha > 0$).

The illiquid secondary market makes it harder to liquidate the two period government bond and exchange it for international liquidity. Transactions costs must be paid, there are search costs involved in making the exchange, and potentially even rents must be paid to market makers. This will further raise the required return for holding bonds at date 0, and will increase the domestic illiquidity index, $s_d$, at date 1.

3.4.1 Backfiring Policy

Suppose that the government sells bonds at date 0 in an attempt to sterilize, but does not intervene sufficiently so that the private sector is still internationally liquid at date 0. $B$ government bonds are sold at interest rate of $L_0$. Consider the program for a firm choosing to hold $b$ bonds at date 0,

$$\text{(P7)} \quad \max_{k, d_0, f, b} \quad (w - d_{0,f})(\Delta + L) + k(R + r\frac{\Delta}{L_1}) + (b - T)(1 - \alpha)\frac{\Delta}{L_1}$$

subject to

$$d_{0,f} \leq w$$

$$c(k) + \frac{b}{L_0} = d_{0,f}.$$

Lemma 4 (Government Interest Rates)

The date 0 interest rate on government bonds will always exceed the date 1 interest rate. $L_0 > L_1$.

This lemma is easy to verify. Purchasing one government bond costs $\frac{L_0}{L_1}$ units of international liquidity. At date 1, if the firm is distressed the bond can be sold for $\frac{1 - \alpha}{L_1}$ and the proceeds invested to yield output at date 2 of $\Delta \frac{1 - \alpha}{L_1}$. If the firm is not distressed, the bond
Rewriting this expression, yields

\[ \frac{rK - T}{L_1} + \left( \frac{B}{L_1} - \frac{B}{L_0} \right) = W + \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}. \]

The loss to the government of intervening by issuing bonds at date 0 at the high rate of \( L_0 \) and purchasing bonds at date 1 at the lower rate of \( L_1 \) must be made up in taxes. Thus, rewrite this expression by substituting in, \( T = B - \frac{L_1}{L_0} B \), as,

\[ \frac{rK}{L_1} = W + \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}. \] (10)

Now imagine that the private sector exactly offsets the government’s reserve accumulation. That is to say, suppose that for every unit that \( \frac{B}{L_0} \) rises, the private sector takes on an extra unit of date 0 debt. In this case, the term in the parentheses on the right hand side would be unchanged by sterilization. However if this is the case, and \( B > 0 \), it must be that \( L_1 \) falls, since the supply of liquidity to purchase private sector assets has risen by \( \frac{B}{L_1} \). This is why sterilization backfires. The government supports private sector assets and hence lowers corporate borrowing costs.

The case where the government bonds are fully liquid highlights the key role played by illiquid markets in the previous conclusion. Suppose that \( \alpha = 0 \), so that government bonds can be sold at date 1 without any friction. In this case, distressed firms sell \( B \) bonds at the price of \( L_1 \) at date 1. Thus append \( \frac{B}{L_1} \) to the left hand side of (10) to arrive at,

\[ \frac{rK}{L_1} + \frac{B}{L_1} = W - D_{0,f} - \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}. \]

The government’s reserves go towards purchasing back the bonds that it issued at date 0. Sterilization does not bring additional support for corporate assets, and hence \( L_1 \) is unaffected.

Let us now state this result more formally.

**Proposition 5 (International Liquidity Loss)**

Consider the case where \( \alpha > 0 \). Let \( (k', d_{0,f}', L'_1) \) be equilibrium choices and prices to (P7) and (9) when \( B = 0 \) and let \( (k, d_{0,f}, b, L_0, L_1) \) be equilibrium choices and prices to (P7) and (9) when \( B > 0 \) but \( d_{0,f} < w \). Then, we have that,

\[ L_1 < L'_1 \]

\[ k > k' \]

\[ d_{0,f} > d_{0,f} + \frac{B}{L_0} \]

Proof: see appendix.
Proposition 6 (Sterilization with Illiquid Markets) There exists a sterilization policy of \( B > 0 \) such that the resulting equilibrium of \((k, d_{0,f}, b, L_0, L_1)\) is Pareto superior to \((k', d'_{0,f}, L'_1)\), as long as,

- (i) Domestic markets are sufficiently liquid (i.e. small \(\alpha\)), and
- (ii) In the resulting equilibrium, \( d_{0,f} = w \), so that the private sector is internationally illiquid.

Optimal policy requires that, in equilibrium, the government raises date 0 interest rates, \( L_0 > L_1 \), and the date 1 interest rate falls relative to no-intervention, \( L_1 < L'_1 \).

See the appendix for the proof. The result can be understood by studying figure 4. The figure traces out welfare for the decentralized equilibrium as a function of \(k\) \((U^{\text{PRIV}})\), and welfare for the equilibrium assuming that the central planner intervenes via sterilization, and forces the economy to incur the secondary market illiquidity costs \((U^{\text{CP}})\).

As in proposition 2, it is clear that the central planner values international liquidity higher than the private sector, and domestic liquidity less. Thus the benefit of intervention is that it moves the private sector away from a sub-optimal choice. That is, \(K^{\text{CP}} < K'\). However intervention has a cost, since firms must sell their bonds into an illiquid market. Thus, it is clear that intervention always lowers the welfare function when \(\alpha > 0\).

Intervention is beneficial as long \(U^{\text{CP}}(K^{\text{CP}}) > U^{\text{PRIV}}(K^{\text{PRIV}})\). But this depends on the size of the externality versus the cost of intervention. The private sector always chooses an inefficient point - point C on the figure. The cost of intervention is that it lowers welfare.
must be that $L_0 > L_1$. Hence, atempting to sterilize in an illiquid bond market can have large detrimental effects on the quasi-fiscal deficit, even without the rewards of a higher domestic corporate borrowing rates and a slowdown in aggregate demand.

Needless to say, this deterioration in the fiscal situation can be worsened if the reserves of international liquidity are not targeted back to the private sector in an efficient fashion at date 1. This will occur if the government receives a return less than $L_1$ on its international reserves.

4.2 Excessive Short Term Capital Flows

The observed shortening in the maturity of capital flows following sterilization is particularly interesting from our point of view. The step in arriving at this result from our model is in defining short and long term debt in terms of their insurance features. One can think of long term debt as short term debt plus rescheduling insurance. A simple extension of the model presented in section 2 shows that agents undervalue the insurance component of long term debt as long as $L_1 < \Delta$, which is the case when domestic financial markets are underdeveloped. The result is akin to our previous result on the undervaluation of international liquidity.

Suppose that only a fraction $1 - \psi$, where $0 < \psi < 1$, of $w$ is directly pledgeable to foreigners at date 1. Debt that is taken on against this $1 - \psi$ of international liquidity will be viewed as short term debt. Now suppose that the rest, $\psi w$, can be seized by foreigners at date 2, however doing so requires payment of a monitoring cost of $0 < \epsilon < (\Delta - 1)$ at date 2. A domestic firm has two choices. (A) It can take on one period debt up to the limit of $(1 - \psi)w$ at the international interest rate of one. Then at date 1, if it needs the funds, it can roll this over and take on additional debt of $\psi w$. However, the interest rate on this additional debt will obviously be above one to compensate the foreign lenders for bearing the monitoring cost. (B) It can take on long term debt against the full $w$, in which case the foreign lenders will always pay the monitoring cost to seize the additional $\psi w$. Thus, domestics face an upward sloping term structure of borrowing.

With option A, only firms that are distressed at date 1 will take on the additional debt and draw down $\psi w$. With option B, on the other hand, all firms will have pledged their extra-collateral ex-ante, and the intact firms will sell the corresponding international funds at date 1 to the distressed firms. The latter option is clearly socially preferable, since the social value of an extra unit of liquidity is $\Delta - 1 > \epsilon$. The problem, as before, is that the return to intact firms is only $L_1 - 1$, which could well be below $\epsilon$ if domestic financial markets are illiquid. If this last inequality holds, the equilibrium is one where no firm values

\footnote{Diamond (1991) develops a model of debt maturity structure based on liquidity risk. The sketch of our model is related but the maturity structures depends on aggregate liquidity risk.}
In order to lend to the corporate sector at date 0, bankers require domestic collateral from corporations and need to hoard $\mu < 1$ units of domestic money between dates 0 and 1 per unit of debt’s face value. They are born with $M^0$ units of high powered money. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also $\frac{M^0}{\mu}$.\(^{28}\)

The central bank fully backs $M^0$ with international reserves, which are exchange for money at date 1. Thus, the nominal exchange rate at date 1, $E_1$, is equal to one regardless of the exchange rate system prevailing at date 0. Banks are not needed at date 1, so bankers can participate directly in the financial markets without holding any money.

Taken together, these assumptions create a transmission mechanism for domestic monetary policy via the “lending channel.”\(^{29}\) A central bank that contracts or expands $M^0/\mu$ can affect the amount of lending from bankers to the corporate sector. Our assumptions are designed to isolate the impact of this mechanism on the date 0 problem of taming the boom, as it is apparent that at date 1 the Central Bank will attempt to ensure perfect domestic aggregation of international collateral.

5.2 Monetary Policy in a Flexible Exchange Rate System

We remove bonds for now and fix $M^0$, so monetary policy takes the form of a tightening in reserve requirements, $\mu$. None of our main conclusions is affected by reintroducing bonds and implementing the monetary contraction via open market operations instead (see below).

At date 0, bankers lend as much as they can to firms as long as $L_0 > L_1$. That is, they supply:

$$X_0^s = \min \left[ \frac{M^0}{\mu}, W \right].$$

(11)

Let $X_0^{dh}$ denote the potential demand for loans, defined as the demand that would arise if $L_0 = L_1$. Then the amount lent, $X_0$, is:

$$X_0 = \min[X_0^s, X_0^{dh}]$$

(12)

\(^{28}\)These steps can be disentangled more finely. For example, separating bankers from savers, the story goes as follows: Banks face a reserve requirement on taking deposits from savers. Let $\mu$ be the reserve requirement. Then, given $M^0$ banks can only take in deposits of $\frac{M^0}{\mu}$. We additionally assume that banks cannot raise funds from savers in any other way than by taking deposits. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also $\frac{M^0}{\mu}$. Last we assume that the $rK$ of domestic liquidity that firms create is only tradeable among the corporate sector and banks. That is loans to a firm with some domestic liquidity can only be made by other firms (for example, via asset sales, trade credit, or mergers), or from savers through the banking sector.

\(^{29}\)For discussion and evidence of the lending channel in the U.S., see for example, Kashyap, Stein and Wilcox (1993).
5.3 Monetary Policy in a Fixed Exchange Rate system

Suppose now that the exchange rate is fixed at one at date 0 as well, and the central bank stands ready to swap international reserves for domestic money at the private sector’s will.

There are two basic scenarios to consider in this fixed exchange rate system. In the first one, foreigners do not value domestic money as collateral.\(^{31}\) It is apparent that in this case bankers will offset any monetary contraction by selling their \(X_1\) to the central bank in exchange for domestic money. They will do so for as long as \(L_0 > L_1\) and \(X_1 > 0\). Thus, as in Mundell-Flemming, monetary policy is futile. This changes once \(X_1 = 0\) for then the bankers are constrained in the same sense as firms were in section 3 in the scenario where sterilization worked. The “holy trinity” of open economy macroeconomics establishes that only two out of the following three are possible: effectiveness of monetary policy, control of the exchange rate, and free capital mobility. It is the endogenous canceling of the latter that gives back its powers to monetary policy.\(^{32}\)

The other limit case to consider is when high-power money is part of international collateral. In that case, neither \(X_0\) nor \(X_1\) can be affected by monetary policy, thus monetary policy is useless. This case highlights an important aspect of the policy considerations we have stressed throughout the paper: In order to be successful in preventing an external crisis, the policymaker needs to be able to “hide” some of the private sector’s international liquidity at date 0. This, it will not be able to do if it attempts it be selling highly internationally liquid instruments to its private sector.

6 Final Remarks

A central consideration of macroeconomic policy in emerging economies is external-crisis prevention. In practice, the main macroeconomic tool utilized for such purpose is the sterilization of capital inflows. However, existing models are not particularly well suited to study this policy. On one hand, the sovereign debt literature identifies the aggregate international constraint as limiting external debt repayments, and therefore links external crises to this aggregate constraint. But as it is designed to answer a different question, this literature suppresses domestic financial markets and is therefore unable to study outcomes of sterilization policy. On the other hand, the Mundell-Fleming framework directly addresses sterilization policy. However, as it essentially ignores all aspects of the external financial constraint and instead emphasizes the monetary aspect of sterilization, it is best suited

\(^{31}\)Recall that bankers need to hold the money at date 0 in order to make the loans. Hence, this assumption does not mean that foreigners wouldn’t accept money as a method of payment at date 0, but that money in banks’ hands does not count as collateral.

\(^{32}\)Reisen (1993) argues that the “holy trinity” does not apply when the central bank uses reserve requirements rather than open market operations. We do not find support for such claim in our model.
asset and liability management policies – and its effects on the corporate sector. Liquidity management, in practice, has more layers. Argentina has considered liquidity requirements in the banking sector. Central banks often respond to inflows by increasing domestic reserve requirements. Until recently, Chile required the private sector to hold liquidity against short term external financing. Each of these actions results in liquidity provisioning at a more decentralized level. When is liquidity provisioning by the banking system more or less effective than that of the central bank? In assessing the international liquidity of a country, should we equally weight the holdings of the central bank and those of the domestic banking system? These are important questions that require, among other things, enriching the framework to include a domestic banking sector. This remains something we are working on.

Similarly, while sterilization —and international liquidity management in general, including external public debt management— may be the tool of choice in the short run, long term solutions to the problems we have highlighted are not cyclical but structural in nature. Our framework not only illustrates the second best options and policy problems, but also points at domestic financial underdevelopment as the primitive source of concern. It is important when thinking about second best solutions to also ask whether they will have any long run effects on the primitive problem. Taxing capital flows, for example, while obviously appropriate from the second best point of view, and even useful as a companion to sterilization, loses appeal once one thinks in terms of the medium and long term development of financial markets. Flexible exchange rates may have an advantage over fixed - for Mundell-Fleming reasons - but they may have long run detrimental effects on financial markets.

Regardless of the specific answers to these concerns, it appears to us that there is an increasing realization that a modern debate on issues such as the advantages and drawbacks of dollarization, capital flows taxation, liquidity requirements, and so on, ought to consider the asset markets aspects of the problem, and that the structure we have proposed here is a useful tool for such a task. We are currently exploring some of these structural problems in ongoing work.
since they will surely be repudiated at date 2. Thus, $B_f = 0.34$

Case II: Suppose the foreigner goes to a domestic investor and offers $B_f$ bonds for international liquidity. Since $d_{0,f} = w$, domestics have none, and can only get international liquidity by tendering bonds to the central bank. Suppose a domestic tenders $B$ bonds to the central bank, shows import goods of $B/L_1$ and then receives $B/L_1$ reserves. However these reserves will exactly pay for the import goods, hence the domestic will be left with no liquidity to offer foreigners.

Finally let us show that only distressed firms will sell bonds to the central bank for its international reserves.

Case III: Either distressed or intact firms can tender $rK + B - T$ to the government for $B + B_f$ of reserves. If intact firms tender, they onsell the imported goods to distressed firms in exchange for some of $rK + B - T$ of distressed firms. Suppose an intact firm tenders one domestic claim, it receives $1/L_1$ import goods, which it sells to the distressed firm for $L_1$ domestic claims. Thus it is indifferent between tendering and not. Assume that it does not.

Distressed firms receive all imported goods totalling, 
\[
\frac{rK + B - T}{L_1}
\]

\[\text{From (7), this is exactly equal to } \frac{B + B_f}{L_0} \text{ which is all of the government's reserves. Thus when only distressed firms tender, this is an equilibrium.}\]

\[34\text{We have made two unrealistic assumptions here. First, we have said that foreigners cannot take the imported goods and liquidate them outside the country for international liquidity. If there is any liquidation cost in this transaction, it is easy to see that foreigners would prefer not to hold domestic claims, since they must always bear this cost, while domestics never bear the cost. The more interesting case is that of over-invoicing. During periods of capital-controls domestic firms routinely over-invoice their imported goods. That is they claim higher prices than actual ones for their goods, thereby getting their hands on more valuable international reserves. Suppose that a firm can get away with over-invoicing by a multiple of } M > 1. A firm that tenders one unit of domestic claim receives $\frac{1}{M}L_1$ import goods, and $\frac{M-1}{M}L_1$ international reserves. The import goods can be sold to a distressed firm, for $\frac{1}{M}$ domestic claims. The international reserves can now be sold to a foreigner to redeem some of the foreigner's bonds. Foreigners selling $B_f$ bonds can receive at maximum $\frac{M-1}{M}B + B_f$ reserves. Thus let,}

\[
L_{1,f} = \frac{B_f}{\frac{M-1}{M}P_B(B + B_f)}
\]

\[\text{This is the discount that foreigners sell their bonds at. In equilibrium, foreigners will hold only enough bonds so that } L_{1,f} = L_1. \text{ This means that a fraction } \frac{M-1}{M} \text{ of international reserves can be promised away to foreigners by the private sector at date 0. Over-invoicing creates a leak in the system.} \]
Consider an equilibrium with \( L_1 \leq L'_1 \) and \( K \leq K' \) - where at least one of the inequalities is strict. From the firm’s first order conditions,

\[
c'(k) = \frac{R + r \frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}} < \frac{R + r \frac{\Delta}{L'_1}}{1 + \frac{\Delta}{L'_1}} = c'(k').
\]

Since \( c(k) \) is strictly convex, this is a contradiction. The case with \( L_1 \geq L'_1 \) and \( K \geq K' \), can be ruled out by the same logic. Hence the only equilibrium is, \( L_1 < L'_1, k > k' \), and \( d_{o,f} > d'_{o,f} + \frac{R}{L_0} \).

**A.5 Proof of Proposition 6**

Proposition 6 derived optimal policy when secondary markets were illiquid.

At date 1, the private sector sells the government bonds in return for the international liquidity. When \( \alpha > 0 \), this transaction suffers a real cost. Thus intervention can be costly because it requires the private sector to sell more bonds into an illiquid market. Policy can result in a Pareto improvement as long as this cost is not too high. In the extreme case when \( \alpha = 0 \), policy always leads to Pareto improvement.

First let us define \( U^{PRIV} \) as,

\[
(P8) \quad \max_{k,d_{o,f}} U^{PRIV} \equiv (w - d_{o,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1})
\]

s.t. \( d_{o,f} \leq w \)

\[ \quad c(k) = d_{o,f}. \]

The first order condition for this program is,

\[
c'(k') = \frac{R + r \frac{\Delta}{L_1}}{\Delta + L_1}
\]

The optimal choice is denoted \( k' \) to refer to the no-intervention point. The market clearing condition remains that of (5).

Suppose that a central bank offered \( B \) bonds for sale at the interest rate of \( L_0 \), but bearing the illiquidity cost of \( \alpha \). This program must be altered as follows:

\[
\max_{k,d_{o,f},b} (w - d_{o,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1})
\]

s.t. \( d_{o,f} \leq w \)

\[ \quad c(k) + \frac{b}{L_0} = d_{o,f}. \]

Since we require that the central bank sell enough bonds so that \( d_{o,f} = w \), we can rewrite as:

\[
\max_{k,b} k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1})
\]

s.t. \( c(k) + \frac{b}{L_0} = w. \)
References


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