KEYNESIAN MODELS OF UNEMPLOYMENT

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Why is there persistent unemployment? This question is as relevant today as when it was first posed over forty years ago. Since then Keynes and his followers have provided an answer, of sorts, in the theory of effective demand. The standard textbook Keynesian model shows how the level of unemployment is determined given a fixed level of the nominal wage and the money stock. But why is the nominal wage fixed? The textbook answer is that it is institutionally fixed, or that workers suffer from money illusion.

Such answers seem unsatisfying to economists who would like to explain such wage stickiness in terms of rational behavior. Six years ago the path breaking volume of Phelps, et al. ( ) appeared and attempted to provide explanations of wage (and price) stickiness in terms of optimizing behavior of individual agents. Since then, there have been many other contributions to the microfoundations of macroeconomics.

In this paper I will survey a few recent contributions to the theory of unemployment, and provide some simple algebraic examples of models to explain wage stickiness. Many of the original papers presented partial equilibrium models, but here I will always close up the models by requiring that the output market clear. It will become clear that this requirement itself tends to determine the level of unemployment. The micro-models of the labor market serve only to determine the level of wages.

The emphasis throughout will be on presenting simple theoretical models. Emphasizing simplicity rather than realism helps to clarify the nature of the
underlying phenomenon, but it can only be viewed as a preliminary step in providing a complete description of the nature of unemployment.
Keynesian Unemployment

The basic framework for this paper will be the simplest form of the standard Keynesian model. Let \( y \) be real output, let real investment demand be fixed at \( I \), and let real consumption demand be given by a linear consumption function, \( C(y) = cy \). For equilibrium in the output market we must have the demand for output equal to the supply of output:

\[
    cy + I = y \tag{1}
\]

Hence the equilibrium level of output is given by:

\[
    y = \frac{I}{1-c} \tag{2}
\]

Suppose that \( m \) identical firms produce output from labor via a standard neoclassical production function. In a symmetric equilibrium we must have

\[
    mf(\ell) = y = \frac{I}{1-c} \tag{3}
\]

The aggregate demand for labor is thus \( L = m\ell \). We denote the aggregate supply of labor by \( N \). It is clear that if \( I \) is "too small", \( L \) can easily be less than \( N \). The result is unemployment.

Let us imagine now that the labor market is competitive, and let \( \ell_d(w) \) be a representative firm's profit maximizing demand for labor as a function of the real wage \( w = W/P \). Then \( w \) must satisfy the condition that:

\[
    mf(\ell_d(w)) = y = \frac{I}{1-c} \tag{4}
\]

The above equation determines the equilibrium values of output, unemployment and the real wage in terms of the exogenous investment demand. It is clear that the "cause" of unemployment is insufficient effective demand. The puzzle has always been how this situation can persist. Is there no adjustment mechanism that will restore full employment?

The usual story is that full employment will be restored eventually through the adjustment of the nominal wage and price levels. If there is unemployment, firms will find it profitable to lower their nominal wage and hire
unemployed workers. If investment demand were to remain fixed, this would have no effect on output or unemployment. Since workers' real income declines, consumption demand declines, the nominal price level falls, and equilibrium would be re-established at the same levels of output and unemployment as before.

However, it is generally agreed that a change in the nominal price level would tend to stimulate investment through the so-called Keynes effect. If the nominal price level falls, people will need less money for transactions. Hence more money will be available to finance investment. This increased supply of loanable funds will tend to lower the interest rate and stimulate investment. The process continues until full employment is re-established.

Now, there are certain problems in the above story. It is not at all clear that the interest rate can fall low enough to stimulate investment sufficiently. In this case one may want to appeal to some sort of real balance effect to provide the necessary stimulus to effective demand. However, it is clear that both the Keynes effect and the real balance effect rely upon a decline in the nominal price and wage level. If the nominal wage is "institutionally given" or "sticky downward", and effective demand is too low, unemployment can persist indefinitely.

But why should nominal wages be sticky? If there are unemployed workers who are willing to work at a lower wage, firms should be able to increase their profits by hiring such workers at a lower wage. Why does this not occur? How can a sticky nominal wage be compatible with rational behavior on the part of workers and firms?

There have been several recent attempts to answer this question. At a very general level they all have the same structure: the authors examine various institutional features of the labor market and show how the equilibrium
wage is the solution to some optimization problem by workers, firms, or both. This is, after all, what economists mean by rational behavior.

The contribution of this paper is as follows:

1. I will survey several of these recent papers and provide simplified versions of the various models. Hopefully this will allow the basic features of the models to be better understood.

2. In several cases the original papers involved partial equilibrium models. In these cases I will close the models by adding a description of the output market. The basic insight of the Keynesian model described above is that unemployment is caused by insufficient effective demand. Hence, considerations involving the output market are certainly relevant. In each case, closing the model tends to simplify, not complicate, the basic structure.

3. Finally, I will present one new model of unemployment which is based on some ideas about imperfect information.

I have one final remark about the formulation of the various models. It is clear that workers and firms set nominal wages, but they are presumably interested in real wages. Since the price level can be taken to be exogenous, the optimization problems will often be posed in real terms, but the reader should remember that from the viewpoint of the individual agent it is the nominal wage which is being chosen.
Speculation by Workers

One class of models argues that the stickiness of the wage rate is due to the fact that unemployed workers will not accept employment at a lower wage. Workers would rather speculate on getting a job at the current or higher wage elsewhere. This is the basic feature of most of the models in Phelps et al. ( ), and comprises the basic insight of the subsequent search theory literature.

A simple version of this phenomenon has been described by Negishi ( ), ( ).

Suppose that all workers have identical vonNeumann Morgenstern utility functions for income. The working day is institutionally fixed so that the utility of employment at a real wage $w = W/P$ is given by $u(w)$. We will normalize utility so that the utility of unemployment is zero. Workers believe that their probability of employment depends on the wage they quote, $w$, and on the level of aggregate employment $e = L/N$. Denote this probability by $\pi(w,e)$. Hence the expected utility of maintaining a reservation wage $w$ is given by $\pi(w,e)u(w)$.

When will workers be made worse off by lowering their reservation wage? Workers will not lower their wage when:

$$\pi'(w,e)u(w) + \pi(w,e)u'(w) \geq 0$$  \hspace{1cm} (5)

This is essentially the condition derived in Negishi ( ).

Let us now consider the conditions characterizing an unemployment equilibrium. First, we have the standard condition for the clearing of the output market:

$$mf(\ell_d(w)) = I/(1-c)$$  \hspace{1cm} (6)

Secondly, we have the condition that workers do not desire to lower their reservation wage:

$$\pi(w,e)u'(w) + \pi'(w,e)u(w) \geq 0$$  \hspace{1cm} (7)
Thirdly, we need a condition that the workers' perception of the probability of employment is consistent with the aggregate frequency of employment. This is clearly given by:

$$\pi(w,e) = L/N = m\ell_d(w)/N$$  \hspace{1cm} (8)

Hence, the condition for downward stickiness in the wage rate is simply given by:

$$\pi'(w,L/N)u(w) + (L/N)u'(w) \geq 0$$  \hspace{1cm} (9)

The conditions (6) and (9) completely characterize an unemployment equilibrium.

Note carefully the structure of Negishi's argument: (1) the level of employment is determined entirely by effective demand; (2) the real wage is determined by profit maximization on the part of firms; (3) if this real wage is consistent with the speculative beliefs of workers, there will be no tendency for the wage to fall.

But why won't the real wage tend to rise? Negishi argues that competition among workers leads to a kinked probability function: if a worker raises his reservation wage, other workers will not follow suit and he will have zero probability of becoming employed. On the other hand, if he lowers his reservation wage, other workers will follow suit so that his probability of employment is less than one. The decision to maintain a given wage depends on the workers' subjective assessment of the magnitude of this effect; that is, it depends on $\pi'(w,e)$.

It is not completely clear why workers should behave in this manner. In many ways it seems more natural to assume that $\pi(w,e)$ is a smooth function, and that workers choose a reservation wage $w^*$ that maximizes their expected utility. Hence $w^*$ would satisfy

$$\pi'(w^*,L/N)u(w^*) + (L/N)u'(w^*) = 0.$$  \hspace{1cm} (10)
But now notice the following: the wage that firms are willing to pay workers, \( \hat{w} \), must satisfy the effective demand condition that:

\[
mf(\ell_d(\hat{w})) = \frac{1}{1-c}
\]  

(11)

There is no particular reason why \( w^* \) should equal \( \hat{w} \). What would happen in such a case?

Suppose that the workers' reservation wage were less than the firms' reservation wage. Then why don't firms lower their nominal wage offers to workers? Such a policy would tend to move the economy back towards full employment through the nominal price level effects described earlier.

On the other hand, suppose that the firm's reservation wage is less than the worker's reservation wage. Then the employment rate is zero, and the only way that equation (10) can hold is if \( \pi'(w^*,0) = 0 \); that is, if workers believe that lowering their reservation wage will not affect their probability of employment even when 100 per cent of the work force is unemployed! There seems to be no particular reason to impose such a restriction on \( \pi \) on an a priori basis. The difficulty is that \( \pi \) is an exogenously given function in Negishi's formulation of the problem. There is no mechanism by which workers' beliefs about lowering their reservation wage can be made to adjust to reality.

Futia ( ) offers a model that suggests a way out of this difficulty. In this model workers all have identical utility functions but there is a whole distribution of reservation wages. Workers who have a high reservation wage have a low probability of employment, but when they do find employment, they receive a large income. Workers who have a low reservation wage are frequently employed but at a low wage. Hence when workers wish to determine whether it would be desirable to change their reservation wage, they simply have to examine the objective circumstances of other workers who are quoting such a wage.
The introduction of a distribution of wage solves the problem posed by the Negishi model, but it introduces some problems of its own. How is it that a distribution can persist in the face of profit maximizing firms? Futia's answer is that each firm only samples a small subset of the labor market. Each firm hires the cheapest workers first from its own pool of labor. Workers who have a low reservation wage get employed often, but even workers with high wages occasionally get lucky and end up in a pool where their reservation wages are low compared to the other workers.

Let $L(w)$ be the continuous distribution function describing how many workers have reservation wages that are less than the wage $w$. Then the above operation of the labor market is summarized by a function giving the probability of employment at wage $w$ as a function of the number of workers quoting a wage less than $w$ and the aggregate demand for labor. This function will be denoted by $\pi(L(w), m\ell_d)$. 

How is the demand for labor determined? In Futia's model firms are profit maximizers on the average: they choose a demand for labor each period that maximizes profits at the overall average wage quoted, $\tilde{w}$. Hence, if $\ell_d(w)$ is the firm's competitive demand function for labor, each firm will demand $\ell_d(\tilde{w})$ workers each period. It will pay whatever wage is necessary to get this many workers on a period by period basis, but in the long run firms pay an average wage of $\tilde{w}$.

The equilibrium conditions in Futia's model are as follows. First, the output market must clear

$$mf(\ell_d(\tilde{w})) = I/(1-c)$$

(12)

Secondly all workers must achieve the same expected utility no matter what their reservation wage:

$$\pi(L(w), m\ell_d(\tilde{w}))u(w) = u_o$$

all $w$ 

(13)
Here the utility of unemployment has been normalized to be zero. Finally, firms' expectations about the average wage bill are confirmed:

$$\bar{w} l_d(\bar{w}) = \int \pi(L(w), l_d(\bar{w})) l(w) \, dw$$

(14)

(Here $l(w)$ is the density function of $L(w)$.)

Futia proves that for any choice of exogenous investment demand, $I$, there is a distribution of wages that solves this problem. An outline of his proof is as follows: first fix the level of utility $u_o$. Then construct a wage distribution $L(w)$ that satisfies equation (13). The only property that a distribution function need have is that it be monotonic. As $w$ increases, $u(w)$ increases, so $\pi(L(w), l_d)$ must decrease to maintain equality in equation (13). But to make $\pi$ decrease we need to make $L(w)$ larger. Hence $L(w)$ will indeed be monotonic.

Now consider equation (14). If $u_o = 0$ the expected utility of employment is equal to the utility of unemployment. Hence the average wage must be very small. If $u_o$ is very large, the expected utility of employment is quite large so the average wage must be large. A continuity argument shows that there must exist a choice of $u_o$ that makes the average wage equal to $\bar{w}$. This completes the argument.

The most serious problem with the Futia model is with its description of firm behavior. Workers behave rationally and "search" for firms that will employ them at a high wage, but firms behave passively and accept whichever workers come their way. A more realistic model would be one where firms actively search for workers. But if firms search a large enough portion of the labor market, it becomes correspondingly more difficult to support a distribution of wages.

Instead of setting a demand for labor and paying whatever wages are necessary to get that amount of labor, suppose firms set their reservation wage at $\bar{w}$, and hire however many workers are willing to accept that wage.
Then the probability of employment at any wage greater than \( \bar{w} \) is zero and the distribution collapses. If there is unemployment at this wage firms will attempt to exploit it by lowering their nominal wage and we are back in the old story. If firms have enough information about the labor market, and set wages so as to maximize profits, a distribution of wages will not persist.

On the other hand, a wage distribution can persist if some employers do not act in a competitive, profit maximizing manner. Hall ( ) has suggested that approximately 25 per cent of the work force is employed by the government and other sectors of the economy that are highly unionized. In these sectors wages are set in a non-competitive manner, and consequently, these wages can easily be different from wages set in the competitive sector of the economy.

Suppose, as is natural, that the non-competitive sector (hereafter called "the government") sets a nominal wage, \( \bar{w}_g \), that is higher than the competitive nominal wage, \( \bar{w}_c \). Then unemployed workers may prefer to speculate on the chance of getting a government job at a high wage rather than accept employment in the competitive sector at a low wage.

Hall ( ) presents a search theory model of this type, as well as some estimates of the magnitude of this effect. He references some previous work by Eaton and Neher ( ) who explored a similar model. Here I will present a somewhat more unrealistic but conceptually simpler model which conveys the basic point in a somewhat clearer manner.

Suppose that there are \( G \) government job openings per month and \( N \) people in the labor force. Let \( L \) be the supply of labor to the competitive sector, and \( N-L \) the supply of labor to the government sector. Workers have two options: (1) they can accept a job in the competitive sector at wage \( \bar{w}_c \), or they can "search" or "wait" for a government job at wage \( \bar{w}_g \). Let \( \pi \) be the
probability of getting a government job. In equilibrium, the expected utility of these two strategies must be the same:

$$u(W_c/P) = \pi u(W_g/P)$$  \hspace{1cm} (15)

Again we have normalized the utility of unemployment to be zero. The equilibrium value of $\pi$ should be the frequency of successful government employment; that is, in equilibrium, $\pi$ should equal $G/(N-L)$. Inserting this in the above equation and solving for $L$ gives:

$$L(W_c/P, W_g/P) = N - \frac{u(W_g/P)G}{u(W_c/P)}$$  \hspace{1cm} (16)

This is a supply function of labor to the competitive sector. Adding the equilibrium condition for the output market, we have the following conditions:

$$mf(L_d(W_c/P)) = y = 1/(1-c)$$  \hspace{1cm} (17)

$$mL_d(W_c/P) = L(W_c/P, W_g/P)$$  \hspace{1cm} (18)

Consider what happens if we lower the government nominal wage. In the first instance, we would expect this to reduce unemployment, since the speculative alternative has been made less attractive. However, this is not the case in the final analysis. Look at the above conditions: the demand for labor is determined by the real wage which itself is determined by effective demand. If the government nominal wage falls, the competitive nominal wage and the nominal price level also falls. All real variables remain the same and the level of unemployment is unchanged.

In the above argument I have tacitly assumed that the government followed a monetary policy that keep the interest rate and thus the demand for investment constant. An alternative assumption is that the government keeps the supply of money fixed in nominal terms. Then the fall in the price level would reduce interest rates, stimulate investment, and increase employment: the standard effect of reducing the nominal price level.
However, notice the chain of causality: lowering the government wage increases employment, but only by stimulating effective demand. Other ways of stimulating effective demand would work just as well.
Implicit Contracts

Consider a firm facing a randomly fluctuating demand for its product. If the firm chooses a wage and employment level after it observes each realization of demand, wages and employment will also presumably fluctuate, and this in turn will cause workers' incomes to fluctuate. If workers are risk averse, they would like to smooth out this fluctuating income stream; if firms are risk neutral they would be willing to accommodate the workers in this respect in return for a lower average wage bill. This is the basic insight of the implicit contract theory of wage rigidity. Bailey ( ) provided the first rigorous study of such phenomena.

Here we will consider a simplified version of a model due to Azariadis ( ) that attempts to describe this sharing of risk between firms and workers.

Suppose that there are only two states of nature. State 1 occurs with probability \( \pi \); in this state investment demand is \( I_1 \), the price level is \( Q_1 \), and the price of the product of the individual firm we are considering is \( P_1 \). State 2 occurs with probability \( (1-\pi) \) and the levels of these same three variables are \( I_2, Q_2, \) and \( P_2 \). The object of the firm is to pick the levels of its nominal wage, \( W \), and employment, \( \ell \), for each of the two periods; we will call this choice an employment contract and denote it by \( (W_1, \ell_1, W_2, \ell_2) \). The firm is constrained by the fact that it must offer a contract that gives the workers just as much expected utility as they could get elsewhere in the economy. We assume that the objective of the firm is to maximize real expected profits subject to this constraint:

\[
\max \pi [P_1 f(\ell_1) - W_1 \ell_1] / Q_1 + (1-\pi) [P_2 f(\ell_2) - W_2 \ell_2] / Q_2
\]

s.t. \( \frac{\pi \ell_1}{N} u(W_1/Q_1) + (1-\pi) \frac{\ell_2}{N} u(W_2/Q_2) \geq u_0 \) \( \ell_1 \leq N \) \( \ell_2 \leq N \)
Some remarks on the constraint are in order. We are denoting the labor force facing the firm by $N$. When the constraint is met with equality, $N$ will presumably be one $m^{th}$ of the aggregate labor force. As before we assume that the aggregate supply of labor is inelastic so that $N$ is independent of $W$. The probability of being in state 1 is $\pi$, and the probability of being hired if we are in state 1 is $\ell_1/N$. Hence $\pi \ell_1/N$ is the total probability of being employed in state 1. We have again normalized the utility of unemployment to be zero so the constraint simply expresses the requirement that the expected utility of the employment contract be at least as great as the exogenously given utility level $u_o$.

The first order conditions for this problem are:

$$
\pi \left[ P_1 f'(\ell_1) - W_1 \right]/Q_1 - \frac{\lambda \pi}{N} u(W_1/Q_1) + \mu_1 = 0
$$

(19)

$$(1-\pi) \left[ P_2 f'(\ell_2) - W_2 \right]/Q_2 - \frac{\lambda (1-\pi)}{N} u(W_2/Q_2) + \mu_2 = 0
$$

(20)

$$
- \frac{\pi \ell_1}{Q_1} - \frac{\lambda \pi \ell_1}{NQ_1} u'(W_1/Q_1) = 0
$$

(21)

$$
- (1-\pi) \frac{\ell_2}{Q_2} - \frac{\lambda (1-\pi) \ell_2}{NQ_2} u'(W_2/Q_2) = 0
$$

(22)

The last two constraints can be rearranged to give:

$$
\lambda = \frac{N}{u'(W_1/Q_1)} = \frac{N}{u'(W_2/Q_2)}
$$

(23)

This in turn implies the interesting result that the real wage is constant in each period: $W_1/Q_1 = W_2/Q_2 = w$. This fact was first derived in a similar context by Bailey ( ) who pointed out the simple intuition behind it: no matter what $(\ell_1, \ell_2)$ is, workers are always made better off by being payed the average wage and firms are indifferent. Hence the optimal choice of $(\ell_1, \ell_2)$
must involve a constant real wage. Note carefully that it is the real wage that will be constant; implicit contract theory does not provide a justification for constant nominal wages.

What is the nature of the optimum employment pattern? Bailey and Azariadis have little to say on this; however Sargent ( ) has provided a very surprising answer: in this model full employment is optimal!

To see this, we argue by contradiction. Let \((W_1, \ell_1, W_2, \ell_2)\) be a (supposedly) optimal contract with \(\ell_1 < N\) and \(\ell_2 < N\). Then suppose the firm were to pay instead the average real wage:

\[
\bar{w} = \frac{\pi \ell_1 + (1-\pi) \ell_2}{N} w
\]

Consider the contract \((\bar{W}_1, N, \bar{W}_2, N)\). The expected utility of this contract to workers is clearly higher than the expected utility of the contract \((W_1, \ell_1, W_2, \ell_2)\). The average real revenue of the firm is at least as high, even if it 'hoards' the extra labor. The average real labor bill to the firm is:

\[
\bar{w}N = \frac{\pi \ell_1 + (1-\pi) \ell_2}{N} w N
\]

\[= \pi \ell_1 w + (1-\pi) \ell_2 w
\]

This is exactly the real labor bill under the contract \((W_1, \ell_1, W_2, \ell_2)\). Hence the full employment contract dominates the unemployment contract.

We now substitute \(N\) for \(\ell_1\) and \(\ell_2\) into the expected utility constraint, and find that \(u(w) > u_o\). If all firms in the economy can offer contracts, this constraint should hold as an equality: \(u(w) = u_o\). Apparently the level of the equilibrium real wage depends on \(u_o\). But how is \(u_o\) determined? Presumably \(u_o\) should involve consideration about the nature of the alternatives to employment in a given firm.
What about the aggregate output market? According to the usual argument we should have:

\[
m f(N) = I_1/(1-c) = I_2/(1-c)
\]

But these conditions are clearly contradictory when \( I_1 \neq I_2 \). There are several ways out - inventories and so on - but the easiest way is just to assume that the firm hoards \( N \) workers, but that it only puts some of them to work. If the firms really are competitive profit maximizers this sort of excess capacity situation may be quite a strain; there is a continual incentive to cheat and produce more output. As we have seen this would tend to move the aggregate price level and adjust aggregate demand, thereby compensating for the demand fluctuations.

There is something very peculiar about this model. We started out to explain unemployment and sticky nominal wages. We ended up with fixed real wages and full employment! The reason is that the contract model presented here shares two important features with the simple competitive model of the labor market: (1) all workers are perfect substitutes; and (2) workers are willing to work at any wage. If we have these two features it must always be optimal for the firm to lower its wage in the face of unemployment. The speculation theories discussed in the last section relaxed the second feature; the heterogeneity models to be discussed in the next section will relax the first feature. By relaxing these two features one can get a consistent model of unemployment; but if both of them are present it seems that full employment will be a necessary result.

In his original model, Azariadis assumed that the supply of labor depended on the real wage due to the usual labor-leisure tradeoff. He showed that under this assumption unemployment may result. The basic mechanism is this: in some
states of nature, the price facing the firm may be so low that it would want to employ very few workers. The workers may prefer to substitute leisure for employment in such a state, and the firm would be happy to oblige them. Azariadis derives necessary and sufficient conditions for such unemployment contracts to be optimal.

This type of unemployment is entirely voluntary. There is some question as to whether it should even be called unemployment; after all, the voluntarily determined supply of labor is equal to the demand for labor in each state. Why should such "unemployment" be a matter of social concern?
Heterogeneity in the Labor Market

In the speculation and implicit contract models, workers were reluctant to lower their reservation wages in the face of unemployment. Casual empiricism suggests that firms are also often reluctant to lower wages during periods of unemployment. It would seem that if unemployed workers are willing to work for a lower wage it would pay firms to lay off their own workers and hire new ones at such a lower wage. If they do not do this, it must be because new workers are not perfect substitutes for old workers. In this section I will describe two models that exploit this basic insight.

Transactions and training costs have long been a favorite explanation of why firms do not lower the wage in the face of unemployment. In this view unemployed workers may be willing to work for a lower wage, but not sufficiently low to induce firms to undertake the additional costs of hiring and training them. Although this story is an old one, there are surprisingly few formal models of this phenomenon. Recently Salop ( ) has presented a nice model of unemployment due to such factors.

Let $E$ be the number of employed workers of a representative firm. Suppose that the quit rate of employed workers is given by some function of the ratio of the firms' real wage offer, $w$, and the real wage available elsewhere in the economy, $\bar{w}$. Thus when the firm sets a wage $w$ it loses $q(w/\bar{w})E$ workers each month. Salop assumes that $q(\bar{w}/\bar{w}) = q(1)>0$ because of non-pecuniary search among firms.

In equilibrium $E$ must remain constant, so the number of new workers hired each month must be $N = q(w/\bar{w})E$. The new workers require a real expenditure of $T(N)$ dollars which is thought of as the current value of the training costs incurred. Following Salop we write the maximization problem facing the firm:

$$\max_{E} f(E) - wE - T(N)$$

$$N = q(w/\bar{w})E$$
The first order conditions for this problem are given by:

\[ f'(E) - w - T'(N)q(w/\bar{w}) = 0 \]  
(29)

\[ -E - T'(N)q'(w/\bar{w})E/\bar{w} = 0 \]  
(30)

The second equation implies

\[ T'(N) = -\bar{w}/q'(w/\bar{w}) \]  
(31)

Substituting this into the first equation and rearranging gives us

\[ f'(E) = w[1 + \bar{w}q(w/\bar{w})/wq'(w/\bar{w})] \]  
(32)

This is a standard monopsony result. In a symmetric equilibrium \( w \) must equal \( \bar{w} \). Then the above equation can be solved for \( w \) to give:

\[ w = f'(E)/(1 + q(1)/q'(1)) \]  
(33)

Salop closes his model by requiring a zero profit condition in the output markets which fixes entry by firms and hence the level of aggregate supply. Here I will take a more Keynesian route, and close the model by appending the effective demand equation:

\[ mf(E) = y = I/(1-c) \]  
(34)

This equation determines employment and equation (33) determines the real wage as a function of this level of employment and a parameter of the labor "supply" function. Notice an important feature of this model. If \( q'(1) = \infty \), we are back in the competitive case. Hence the reluctance of the firm to lower its wage is in the end due to the elasticity of the supply of labor. This supply is in turn the solution to a speculation problem on the part of workers: thus, speculation plays an important role even in this type of model.

Salop describes the unemployment result as being due to the fact that the firm must control two markets with one wage: "The firm faces an internal labor market for experienced workers and an external market for new applicants. Since the firm has only a single wage rate with which to economize on labor in two markets, the single wage is generally unable to clear both markets simultaneously." Salop ( ).
But why must there be a single wage? Why can't the firm pay a starting wage, and then promote workers when they have amortized their training costs? If this structure is allowed, and unemployed workers are willing to accept any starting wage, the nominal wage will again be bid down and the Keynes effect story takes over.

In Salop's model, unemployed workers are not perfect substitutes for employed workers since they require an expenditure for training and transactions cost. In the next model unemployed workers are not perfect substitutes for employed workers because of quality differentials among workers.

Consider the problem facing a firm when it wants to reduce its labor force. It can either lower the wage and induce workers to leave, or it can lay off workers that it chooses. If workers vary in quality, a firm will of course lay off the worst workers first. In general it will pay the firm to both lower the wage and lay off workers.

Now consider the situation when unemployment is high. Since each firm lays off its worst workers first, the quality mix in the unemployment pool will be worse than the quality mix of the employed workers. Will these unemployed workers exert a downward pressure on the wage? If a firm lowers its wage it looses some of its current workers and can only replace them by workers who are on the average of lower quality or engage in costly search to locate workers of better quality. Hence there is a cost to lowering wages. Of course, there is also a benefit, namely the reduced wage bill. In equilibrium, the benefits balance the costs and the wage remains fixed.

This particular story is, I believe, original. However, the basic concept is related to Akerlof's ( ) lemon market paper. Akerlof shows that when there is imperfect information concerning quality variation in a market, there may not exist equilibrium prices that clear the market. Here we
ask the question whether there exist market configurations that support an "excess supply" equilibrium. Greenwald ( ) and Weiss ( ) have explored lemon models of the labor market, but as far as I can tell, have not examined the general equilibrium feedback that leads to an unemployment equilibrium.

Suppose that there are two types of workers, "good" workers and "bad" workers, or "experienced" and "inexperienced" workers, or what have you. Let \( g \) be the number of good workers hired by a representative firm, and let \( b \) be the number of bad workers hired. The output of the firm will be \( f(g + e_b) \) where \( e < 1 \) is the relative effectiveness of the bad workers.

As in the Salop model \( q(w, \bar{w}) \) is the quit rate of workers. In equilibrium both the size and the quality composition of the workforce must remain constant. Hence each period \( n_g = q(w, \bar{w})g \) good workers and \( n_b = q(w, \bar{w})b \) bad workers must be rehired. We will assume bad workers can be costlessly acquired; however locating a good worker requires search. Let \( C(n) \) be the (expected) costs of locating \( n \) good workers.

The profit maximization problem of the firm is to choose \( w, g \) and \( b \) so as to solve the following problem:

\[
\max f(g + e_b) - w (g + e_b) - C(q(w, \bar{w})g)
\]

The first order conditions are:

\[
f'(g + e_b) - w - C'(n_g)q(w, \bar{w}) = 0 \quad (35)
\]

\[
f'(g + e_b)w - w = 0 \quad (36)
\]

\[-(g + e_b) - C'(n_g)q'(w, \bar{w})g/\bar{w} = 0 \quad (37)\]

Let \( a = g/(g + e_b) \) be the per cent of good workers in the firm. Then the last condition gives us:

\[
- C'(n_g) = \frac{(g + e_b) \bar{w}}{g q'(w, \bar{w})} = \frac{\bar{w}}{a q'(w, \bar{w})} \quad (38)
\]
Inserting this into the first equation gives us:

\[ f'(\ell_e + \epsilon \ell_b) - w - \frac{\bar{w}}{a} \cdot \frac{q(w/\bar{w})}{q'(w/\bar{w})} = 0 \]  \hspace{1cm} (39)

As before, the demand for effective labor units is given by effective demand:

\[ m f(\ell_e + \epsilon \ell_b) = y = I/(1-c) \]  \hspace{1cm} (40)

The wage is given by the condition that it equals the effective marginal product of a bad worker. (Here we are assuming that \( I \) is chosen large enough so that \( \ell_b > 0 \) in order to get an interior solution.)

\[ w = f'(\ell_e + \epsilon \ell_b)e \]  \hspace{1cm} (41)

Finally the quality-mix in the firm must adjust so as to satisfy the first condition. Some simple algebra yields:

\[ a = e/(e-1) \]  \hspace{1cm} (42)

where \( e = q'(1)/q(1) \) is the elasticity of the quit rate when \( w = \bar{w} \).

A potential problem with the above model is the question of why both good and bad workers are paid the same wage. It would seem that firms would compete for good workers by offering higher wages. However, from the viewpoint of the firm good workers are already receiving higher wages in the sense that the cost of a good worker is the sum of his wage and the search costs. In equilibrium, firms are just indifferent between hiring a bad worker off the streets or searching for a good worker. On the other hand once a good worker is hired, the search costs are sunk costs and good workers may attempt to threaten to leave in order to win higher wages. In the long run, one might expect to see wage differentiation after an initial period of apprenticeship. Thus we might think of this model as applying only to the apprentice part of the market.

Furthermore, there is a definite incentive among workers to attempt to acquire signals to identify themselves as good workers. This allows firms to
economize on search costs and raises the wage level. On the other hand as long as it is harder to find good workers than it is to find bad workers, the lemon feature should be present.
Summary

We have explored six models of unemployment. In each one the structure was the same: effective demand determines the level of unemployment, while the nominal wage remains stable because it is the solution to some optimization problem by workers or firms.

Each of the stories focusses on only one aspect of the employment problem. An important empirical problem is to try to estimate the magnitude of the various effects. It is important to understand how these magnitudes must be measured. Statements are often made that relate a contribution of an effect to the level of unemployment; eg: "Search costs can explain only 1 per centage point of the unemployment rate..." or, "Transactions costs can explain only .5 per centage points of the unemployment rate." These statements are completely misleading. In the Keynesian model effective demand is supposed to explain one hundred per cent of the unemployment rate. The correct form of the above remarks would be: 'Search costs can explain at most 10 per cent of the elasticity of wages with respect to unemployment.' , or "Transactions costs can explain only five per cent of the wage response to unemployment." Hopefully, we will eventually have empirical studies that can provide estimates such as these.
Footnotes:

1. Of course one could use a more complex model such as a version of the IS-LM model. I use the "Keynesian cross" model only because it is the simplest.

2. By $\pi'(w,e)$ I mean $\partial \pi(w,e)/\partial w$.

3. In Futia's original paper, the condition corresponding to (12) here used government expenditure financed by a profits tax rather than investment as a source of exogenous expenditure.

4. We suppose that government employees produce no output so that $W_G$ is effectively a transfer payment.

5. Other references are Okun ( ) and Hashimoto ( ).

6. In Azariadis's formulation of the problem, the expected utility constraint was written as an equality. In his terminology the contract considered here would be "infeasible". Azariadis, ( ), p. 1188.

7. See Spence ( )
References


