Massachusetts Institute of Technology
Department of Economics
Working Paper Series

Labor- and Capital-Augmenting Technical Change

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Working Paper 03-02
December 2002

Room E52-251
50 Memorial Drive
Cambridge, MA 02142

December 2002

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Abstract

I analyze an economy in which firms can undertake both labor- and capital-augmenting technological improvements. In the long run, the economy resembles the standard growth model with purely labor-augmenting technical change, and the share of labor in GDP is constant. Along the transition path, however, there is capital-augmenting technical change and factor shares change. Tax policy and changes in labor supply or savings typically change factor shares in the short run, but have no or little effect on the long-run factor distribution of income.

Keywords: Economic Growth, Endogenous Growth, Factor Shares, Technical Change.

JEL Classification: O33, O14, O31, E25.

*I thank Rubén Segura-Cayuela for excellent research assistance, and Manuel Amador, Abhijit Banarjee, Olivier Blanchard, Chang Hsieh, Steven Lin, Robert Lucas, Torsten Persson, Mathias Thoenig, Jaume Ventura, two anonymous referees, and seminar participants at the Canadian Institute for Advanced Research, Harvard, MIT, the NBER Economic Fluctuations meeting, and Rochester for useful comments. I also gratefully acknowledge financial help from The Canadian Institute for Advanced Research and the National Science Foundation Grant SES-0095253.

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I. Introduction

Figures 1 and 2 show the shares of GDP accruing to labor in the U.S. and France over the past 80 years (with the remainder accruing to capital).[1] The first striking, but well known, pattern is that these factor shares show no trend in the long run (despite significant capital deepening during the same period). The second important observation is that there are large movements in the share of labor over periods as long as 10 or 20 years. For example, in both countries, there is a large increase in the share of labor after World War II. Almost all models of growth and capital accumulation, of both endogenous and exogenous types, explain the stability of factor shares using one of two assumptions: either the elasticity of substitution between capital and labor is taken to be equal to 1, or all technical change is assumed to be labor augmenting (Harrod neutral).[2]

With an elasticity of substitution between capital and labor equal to 1, i.e., with a Cobb-Douglas production function, the shares of capital and labor are pinned down by technology alone (as long as firms are along their factor demand curves). For example, suppose that the aggregate production function is $Y = AL^{\alpha}K^{1-\alpha}$ where K is capital, and L is labor. Then, the share of labor will always be equal to α. There are reasons to be skeptical that the Cobb-Douglas production function provides an entirely satisfactory approximation to reality, however. First, most estimates suggest that the aggregate elasticity of substitution is significantly less than 1.[3] Second, a production function with an elasticity of substitution of 1 does not provide a framework for analyzing fluctuations in factor shares, such as those shown in Figures 1 and 2.

[1] The French data are from Piketty (2001), and the U.S. data are from Piketty and Saez (2001), who in turn use the National Income and Product Accounts data.

[2] A third possibility is that the aggregate production function is $Y = F(K, H)$ where H is human capital, accumulating at the same rate as K, so that there is no "capital deepening". However, the rate of accumulation of human capital appears to be substantially less than that of physical capital. For example, in the U.S., average schooling of the workforce increased by about one year in every decade in the postwar era, which translates roughly to a 6 percent increase in the human capital of the workforce (e.g., Acemoglu and Angrist, 2000), compared to an approximately 4 percent a year growth in the capital stock between 1959 and 1998 (see The Economic Report of the President, 1999).

[3] For example, Nadiri (1970), Nerlove (1967) and Hamermesh (1993) survey a range of early estimates of the elasticity of substitution, which are generally between 0.3 and 0.7. David and Van de Klundert (1965), similarly estimate this elasticity to be in the neighborhood of 0.3. Using the translog production function, Griffin and Gregory (1976) estimate elasticities of substitution for nine OECD economies between 0.06 and 0.52. See also Eisner and Nadiri (1968) and Lucas (1969). Berndt (1976), on the other hand, estimates an elasticity of substitution equal to 1, but does not control for a time trend, creating a strong bias towards 1. Using more recent data, and various different specifications, Krusell, Ohanian, Rios-Rull, and Violante (2000) and Antras (2001) also find estimates of the elasticity significantly less than 1. Estimates implied by the response of investment to the user cost of capital also typically yield an elasticity of substitution between capital and labor significantly less than 1 (see, e.g., Chirinko, 1993, Chirinko, Fazzari and Mayer, 1999, and 2001, or Mairesse, Hall and Mulkay, 1999).

The patterns depicted in Figures 1 and 2 are consistent with a more general neoclassical production function, but require all technical change to be labor augmenting and to take place exactly at the same rate as the rate of capital deepening. More specifically, consider an aggregate production function of the form $Y = F(MK, NL)$. The assumption of labor-augmenting technical change implies that technical progress only increases N, and does not affect M—or in other words, it rotates the isoquants around the capital axis. A neoclassical production function with (purely) labor-augmenting technical change provides an attractive framework for macroeconomic analysis, since it is consistent not only with the long-run stability of factor shares, but also with medium-term swings in response to changes in capital stock, labor supply or technology. It is in fact the starting point of graduate textbooks on growth (e.g., Barro and Sala-i-Martin, 1995). However, this framework raises another important question: why is all technical change labor-augmenting? Or equivalently, why do profit-maximizing firms choose innovations that only increase N? Although starting with Romer's (1986, 1990) and Lucas' (1988) contributions a large literature has investigated the determinants of technological progress and growth, *the direction of technical change*—the reason why all progress takes the form of increases in N—has received little attention.

In this paper I investigate the forces that push the economy towards labor-augmenting technical change. I analyze an otherwise standard endogenous growth model where profit-maximizing firms can invest to increase both M and N in terms of the production function $Y = F(MK, NL)$. The only asymmetry is that capital, K, can be accumulated, while labor, L, cannot.[4] I show that in this economy all technical progress will be labor-augmenting along the balanced growth path. Hence, given the standard assumptions for endogenous growth, the result that long-run technical change must be labor-augmenting follows from profit-maximizing incentives. Consequently, in the long run, the share of capital and the interest rate remain stable, while the wage rate increases steadily due to labor-augmenting technical change and capital deepening. In some sense, this paper therefore provides a microfoundation for the basic neoclassical growth model with labor-augmenting technical change.

Notably, however, while the balanced growth path of this economy resembles the standard neoclassical model, along the transition path there is typically capital-augmenting technical change. That is, purely labor-augmenting technical change is only a long-run phenomenon.

I also show that as long as capital and labor are gross complements, i.e., as long as the

[4] The important assumption is that (efficiency units of) labor cannot be accumulated asymptotically, which appears reasonable with finite lives, since individuals will have on a limited time to invest in human capital. See Jones (2002) on this, and also footnote 2.

elasticity of substitution between these two factors is less than 1, the balanced growth path with purely labor-augmenting technical change is the unique asymptotic (non-cycling) equilibrium, and it is stable. The stability property is intuitive: the profitability of new capital-augmenting techniques is increasing in the share of capital in GDP—both a higher interest rate and a larger supply of capital increase the demand for new technologies that complement or use capital. Consequently, when the share of capital in GDP is large, there will be further capital-augmenting technical change. With the elasticity of substitution less than 1, these new technologies will reduce the share of capital, pushing the economy towards the BGP.

In addition to providing an explanation for why long-run technical change is labor augmenting, the framework presented here also suggests a reason for the long-run stability of factor shares despite major changes in taxes and labor market institutions. The neoclassical growth model with labor-augmenting technical change predicts a constant long-run share of labor, but this share should respond to policies which affect the capital-labor ratio. In contrast, I show that in the framework here with both capital-augmenting and labor-augmenting technical change, a range of policies will have no effect (or only second-order effects) on long-run factor shares: they will affect capital-deepening, but will also have an offsetting effect on capital-augmenting technical change. These results suggest that the framework here is not only useful as a microfoundation for the standard growth model, but for policy analysis as well: it points out that a number of comparative statics that assume capital-augmenting technical change away may give misleading answers.

It is useful to briefly outline the intuition for why long-run technical change will be labor augmenting. Suppose labor-augmenting progress takes the form of "labor-using" progress, that is, the invention of new labor-intensive goods.[5] Similarly, capital-augmenting progress corresponds to the invention of new capital-intensive goods. In this economy, new goods will be introduced because of future expected profits from their sale. When there are n labor-augmenting goods, the profitability of an additional labor-intensive good is proportional to $\frac{wL}{n}$ because each intermediate good producer will hire $\frac{L}{n}$ workers, and its profits are given by a markup over the marginal cost of production—the wage rate, w. Similarly, when there are m capital-intensive goods, profits from further capital-augmenting progress are proportional to $\frac{rK}{m}$, where r is the rental rate of capital. When technical progress relies on the use of scarce factors such as labor, long-run growth

[5]In principle, there are two ways to model labor-augmenting technical progress: as the introduction of new production methods that directly increase the productivity of labor, or as the introduction of new goods and tasks that use labor. Here, I discuss the first formulation. Later, I will show that the same results apply when labor-augmenting progress takes the form of "labor-enhancing" progress.

requires that further innovations build "upon the shoulders of giants", that is, increases in n and m have to be proportional to their existing levels.[6] The return to allocating further resources to labor-augmenting innovation is therefore proportional to $n \cdot \frac{wL}{n}$, while the return to capital-augmenting innovation is proportional to $m \cdot \frac{rK}{m}$. These two returns will be balanced for a specific factor distribution of income.

Furthermore, when the elasticity of substitution between capital and labor is less than 1, a high level of n relative to m implies that the share of capital is high compared to the share of labor. This will encourage more capital-augmenting technical progress and increase m. The converse applies when m is too high. Equilibrium technical progress will therefore stabilize factor shares.

Finally, capital accumulation along the balanced growth path implies that technical progress will increase n more than m.[7] Intuitively, there are two ways to increase the production of capital-intensive goods, via capital-augmenting technical change and capital accumulation, and only one way to increase the production of labor-intensive goods, through labor-augmenting technical change. Capital accumulation, therefore, implies that technical change has to be, on average, more labor-augmenting than capital-augmenting. In fact, the model implies a stronger result: with an elasticity of substitution between capital and labor less than 1, in the long run there will be no net capital-augmenting technical change, m will remain constant, and all technical change will be labor augmenting.

The ideas in this paper are closely related to the induced innovation literature of the 1960s and to Hicks' discussion of the determinants of equilibrium bias of technical change in *The Theory of Wages* (1932). Hicks wrote: "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive." (1932, pp. 124-5). Fellner (1961) expanded on this argument and suggested that technical progress was more labor-augmenting because wages were growing, and were expected to grow, so technical change would try to save on this factor that was becoming more expensive. In an important contribution, Kennedy (1964) argued that innovations should occur so as to keep the share of GDP accruing to capital and labor constant. Samuelson (1965), inspired

[6] Rivera-Batiz and Romer (1991) refer to this case as the knowledge-based specification. Empirical work in this area supports the notion of substantial spillovers from past research, e.g. Caballero and Jaffee (1993), or Jaffee, Trajtenberg and Henderson (1993).

[7] An important question is what n and m correspond to in practice. Although it is difficult to answer this question precisely within the context of a stylized model, it seems plausible to think of many of the major inventions of the 20th century, including electricity, new chemicals and plastics, entertainment, and computers, as expanding the set of tasks that labor can perform and the types of goods that labor can produce. In contrast, some of the early important technological improvements, such as the introduction of coke, the hot blast, the Bessemer process, can be viewed as capital-augmenting advances, since they reduced the costs of capital and other nonlabor inputs, see Habakkuk (1962, pp. 157-159).

by the contributions of Kennedy and Fellner, constructed a reduced form model where firms choose M and N in terms of the production function above in order to maximize the instantaneous rate of cost reduction. He showed that under certain conditions, this would imply equalization of factor shares. Samuelson also noted that with capital accumulation, technical change would tend to be labor-augmenting. Others, for example Nordhaus (1973), criticized this whole literature, however, because it lacked microfoundations: it was not clear who undertook the R&D activities, and how they were financed and priced.

My paper revisits this territory, but starts from a microeconomic model of technical change, as in, among others, Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a,b), and Aghion and Howitt (1992, 1998), where innovations are carried out by profit-maximizing firms. In contrast to these papers, and crucial for the analysis here, I allow for both labor- and capital-augmenting innovations.

The rest of the paper is organized as follows. The next section outlines the basic environment, and characterizes the asymptotic equilibria and the balanced growth path. Section III analyzes transitional dynamics, and shows that with an elasticity of substitution less than 1, the economy tends to a balanced growth path with stable factor shares and labor-augmenting technical progress. Section IV analyzes the consequences of a range of policies on the factor distribution of income. Section V investigates the implications of alternative formulations of the "innovation possibilities frontier," extends the model to allow for the production and R&D sectors to compete for labor, and also shows that the same results obtain with different formulations of technical change. Section VI concludes, while the Appendix contains all the proofs.

II. Modeling The Direction of Technical Change

A. The Environment

I consider an economy consisting of L unskilled workers who work in the production sector, and S "scientists" who perform R&D. The distinction between unskilled workers and scientists is adopted to ensure that the production and R&D sectors do not compete for workers. This is only to simplify the exposition, and will be relaxed in Section V.

I assume that the economy admits a representative consumer with the usual constant relative risk aversion (CRRA) preferences:[8]

$$\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \tag{1}$$

where $C(t)$ is consumption at the time t and $\theta \geq 0$ is the elasticity of marginal utility. When $\theta = 0$, the utility function in (1) is linear, and the representative agent is risk neutral. When $\theta \to 1$, the utility function becomes logarithmic. I drop time arguments when this causes no confusion (I use the time arguments in the proofs in the Appendix). The budget constraint of the representative consumer requires that consumption and investment expenditures are less than total income:

$$C + I \leq wL + rK + \omega_S S + \Pi, \tag{2}$$

where I denotes investment, w is the wage rate of labor, r is the interest rate, K denotes the capital stock, ω_S is the wage rate for scientists, and Π is total profit income. The resource constraint of the economy implies that

$$wL + rK + \omega_S S + \Pi = Y = \left[\gamma Y_L^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) Y_K^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where Y is an output aggregate produced from a labor-intensive and a capital-intensive good, respectively Y_L and Y_K, with elasticity of substitution ε, where $0 \leq \varepsilon < \infty$.

For simplicity, I assume that there is no depreciation of capital, so the change in the capital stock (and in the representative consumer's asset level) is given by

$$\dot{K} = I. \tag{4}$$

The labor-intensive and capital-intensive goods are produced competitively from constant elasticity of substitution (CES) production functions of labor-intensive and capital-intensive intermediates, with elasticity $\nu \equiv 1/(1-\beta)$:

$$Y_L = \left[\int_0^n y_l(i)^\beta di\right]^{1/\beta} \text{ and } Y_K = \left[\int_0^m y_k(i)^\beta di\right]^{1/\beta}, \tag{5}$$

where $y(i)$'s denote the intermediate goods and $\beta \in (0,1)$, so that $\nu > 1$ and different intermediate goods are gross substitutes.[9] This formulation implies that there are two

[8] The presence of two types of agents, scientists and workers, causes no problem for the representative consumer assumption since with CRRA utility functions these preferences can be aggregated into a CRRA representative consumer. See, for example, Caselli and Ventura (2000).

[9] Alternatively, preferences could be directly defined over the different varieties of $y(i)$, with identical results.

different sets of intermediate goods, n of those that are produced with labor, and m that are produced using only capital. An increase in n—an expansion in the set of labor-intensive intermediates—corresponds to *labor-augmenting technical change*, while an increase in m corresponds to *capital-augmenting technical change*.

Intermediate goods are supplied by monopolists who hold the relevant patent, and are produced linearly from their respective factors:

$$y_l(i) = l(i) \text{ and } y_k(i) = k(i), \tag{6}$$

where $l(i)$ and $k(i)$ are labor and capital used in the production of good i. Market clearing for labor and capital then requires:

$$\int_0^n l(i)\,di = L \text{ and } \int_0^m k(i)\,di = K. \tag{7}$$

To close the model, I need to specify the innovation possibilities frontier—that is, the technological possibilities for transforming resources into blueprints for new varieties of capital-intensive and labor-intensive intermediates. I assume that these blueprints are created by the R&D efforts of scientists, who are, in turn, employed by R&D firms. There is free-entry into the R&D sector. Once an R&D firm invents a new intermediate, it receives a perfectly enforced patent and becomes the perpetual monopolist of that intermediate. R&D firms have access to the following technologies for invention:

$$\frac{\dot{n}}{n} = b_l \phi(S_l) S_l - \delta \text{ and } \frac{\dot{m}}{m} = b_k \phi(S_k) S_k - \delta, \tag{8}$$

where b_l, b_k and δ are strictly positive constants and $\phi(\cdot)$ is a continuously differentiable and decreasing function such that $\phi(s)s$ is always increasing, and $\phi(0) < \infty$. S_l and S_k denote, respectively, the number of scientists working to discover new labor-intensive and capital-intensive intermediates, with the market clearing condition

$$S_l + S_k = S. \tag{9}$$

I also assume that the economy starts at $t = 0$ with $n > 0$ and $m > 0$.

Equation (8) implies a number of important features:

1. Technical change is *directed*, in the sense that the society (researchers) can generate faster improvements in one type of intermediates than the other. This feature will enable the analysis of whether equilibrium technical change will be labor- or capital-augmenting.

2. The fact that $\phi(\cdot)$ is decreasing means that there are intra-temporal decreasing returns to R&D effort; when more scientists are allocated to the invention of labor-intensive intermediates, the productivity of each declines. This might be, for example, because scientists crowd each other out in competing for the invention of similar intermediates. This decreasing returns assumption is adopted to simplify the analysis of transitional dynamics—when $\phi(\cdot)$ is constant, the behavior of S_l and S_k is discontinuous.

3. Research effort devoted to the invention of labor-intensive intermediates, $\phi(S_l) S_l$, leads to a proportional increase in the supply of these intermediates at the rate b_l, while the same effort devoted to the discovery of capital-using intermediates leads to a proportional increase at the rate b_k. The parameters b_l and b_k potentially differ since the discovery of one type of new intermediate may be "technically" more difficult than discovering the other type (the standard model with only labor-augmenting technical change can be thought as the special case with $b_k = 0$). I also assume that the crowding effect captured by the function $\phi(\cdot)$ is not internalized by individual R&D firms, so each R&D firm takes the productivity of allocating one more scientist to each of the two sectors, $b_l\phi(S_l)$ or $b_k\phi(S_k)$, as given when deciding which sector to enter. The results are identical when R&D firms act "non-competitively" and form global research consortiums, internalizing these crowding-out effects.

4. Each intermediate disappears at the rate δ, so that when there is no research effort devoted to a particular type of intermediates, its stock declines exponentially. With $\delta = 0$, the results are similar, but there will exist multiple balanced growth paths (see Proposition 4 below).

Notice that in (8) scientists are "standing on the shoulders of giants"—benefiting from knowledge spillovers from past research. This type of knowledge spillover is necessary for growth when technical change uses scarce factors, such as labor or scientists (see, e.g., Romer, 1990, Rivera-Batiz and Romer, 1991, or Barro and Sala-i-Martin, 1995). In fact, equation (8) is a direct generalization of the accumulation equation in the standard endogenous growth model where we would have $\dot{m} = 0$ by assumption, and $\dot{n}/n = b_l\phi(S) S$ (e.g., equation (3) of Romer, 1990). An additional assumption implicit in (8) is that a higher stock of knowledge accumulated in one sector benefits only that sector (i.e., a higher n increases the productivity of scientists working in the n-sector). I return to a discussion of this assumption later.

Finally, define S_l^* and S_k^* as the number of scientists required to keep the state of technology in each sector constant, i.e., $b_l\phi\left(S_l^*\right)S_l^* = \delta$ and $b_k\phi\left(S_k^*\right)S_k^* = \delta$. I impose:

Assumption 1: $S_l^* + S_k^* < S$.

which implies that there is enough scientists in the society to enable technological progress in both sectors.

B. Consumer and Firm Decisions

An equilibrium in this economy is given by time paths of factor, intermediate and good prices, w, r, ω_S, $[p_l(i)]_{i=0}^n$, $[p_k(i)]_{i=0}^m$, p_L and p_K, employment, consumption and saving decisions, $[l(i)]_{i=0}^n$, $[k(i)]_{i=0}^m$, $[y_l(i)]_{i=0}^n$, $[y_k(i)]_{i=0}^m$, C and I, and the allocation of scientists between the two sectors, S_l and S_k, such that $[y_l(i)]_{i=0}^n$, $[y_k(i)]_{i=0}^m$, C and I maximize the utility of the representative consumer given factor, intermediate and good prices; and $[l(i)]_{i=0}^n$, $[k(i)]_{i=0}^m$, $[p_l(i)]_{i=0}^n$ and $[p_k(i)]_{i=0}^m$ maximize profits of intermediate goods monopolists, S_l and S_k imply zero-profits for all R&D firms, and all markets clear.

I start with the optimal consumption path of the representative consumer, which satisfies the familiar Euler equation:[10]

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho), \tag{10}$$

where recall that r is the rate of interest. The consumption sequence $[C(t)]_0^\infty$ also satisfies the lifetime budget constraint of the representative agent (the no Ponzi game constraint):

$$\lim_{t\to\infty} K(t)\exp\left[-\int_0^t r(v)\,dv\right] = 0. \tag{11}$$

Consumer maximization gives the relative price of the capital-intensive good as:

$$p \equiv \frac{p_K}{p_L} = \frac{1-\gamma}{\gamma}\left(\frac{Y_K}{Y_L}\right)^{-\frac{1}{\varepsilon}}, \tag{12}$$

where p_K is the price of Y_K and p_L is the price of Y_L. To determine the level of prices, I choose the price of the consumption aggregate, Y, in each period as numeraire, i.e., $\left[\gamma^\varepsilon p_L^{1-\varepsilon} + (1-\gamma)^\varepsilon p_K^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1$, which implies that:

$$p_K = \left[\gamma^\varepsilon p^{\varepsilon-1} + (1-\gamma)^\varepsilon\right]^{\frac{1}{\varepsilon-1}} \text{ and } p_L = \left[\gamma^\varepsilon + (1-\gamma)^\varepsilon p^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}}. \tag{13}$$

[10] Equation (10) implies that when consumption grows at a constant rate, the interest rate will be constant, which is a well-known feature of CRRA preferences. It may therefore appear that these preferences ensure a stable interest rate in the long run. This is not the case, since there may not exist an equilibrium with a constant growth rate of consumption. Conversely, if preferences were not CRRA, there could never exist an equilibrium with a constant growth rate of consumption and constant interest rate.

Next, consumer maximization and the CES functions in (5) yield the following isoelastic demand curves for intermediates:

$$\frac{p_l(i)}{p_L} = \left(\frac{y_l(i)}{Y_L}\right)^{-\frac{1}{\nu}} \text{ and } \frac{p_k(i)}{p_K} = \left(\frac{y_k(i)}{Y_K}\right)^{-\frac{1}{\nu}}. \tag{14}$$

Given these isoelastic demands, profit maximization by the monopolists implies that prices will be set as a constant markup over marginal cost (which is w for the labor-intensive intermediates and r for the capital-intensive intermediates):

$$p_l(i) = \left(1 - \frac{1}{\nu}\right)^{-1} w = \frac{w}{\beta} \text{ and } p_k(i) = \left(1 - \frac{1}{\nu}\right)^{-1} r = \frac{r}{\beta}. \tag{15}$$

Since, from (15), all labor-intensive intermediates sell at the same price, equation (14) implies that $y_l(i) = y_l$, for all i, and since all capital-intensive intermediates also sell at the same price, $y_k(i) = k$ for all i as well. Then from the market clearing equation (7), we obtain

$$y_l(i) = l(i) = \frac{L}{n} \text{ and } y_k(i) = k(i) = \frac{K}{m}. \tag{16}$$

Substituting (16) into (5) and integrating gives the total supply of labor- and capital-intensive goods as:

$$Y_L = n^{\frac{1-\beta}{\beta}} L \text{ and } Y_K = m^{\frac{1-\beta}{\beta}} K. \tag{17}$$

These equations reiterate that n and m correspond to labor- and capital-augmenting technologies. Greater n enables the production of a greater level of Y_L for a given quantity of labor, and similarly an increase in m raises the productivity of capital.

Equations (14), (15), (16) and (17) give the wage rate and the rental rate of capital as:

$$w = \beta n^{\frac{1-\beta}{\beta}} p_L \text{ and } r = \beta m^{\frac{1-\beta}{\beta}} p_K. \tag{18}$$

Finally, using (12) and (17), the relative price of the capital intensive good is

$$p \equiv \frac{p_K}{p_L} = \frac{1-\gamma}{\gamma}\left[\left(\frac{m}{n}\right)^{\frac{1-\beta}{\beta}} \frac{K}{L}\right]^{-\frac{1}{\varepsilon}}. \tag{19}$$

The value of a monopolist who invents a new f-intermediate, for $f = l$ or k, is:

$$V_f(t) = \int_t^\infty \exp\left[-\int_t^s (r(\omega) + \delta)\, d\omega\right] \pi_f(v) dv, \tag{20}$$

where $r(t)$ is the interest rate at date t, δ is the depreciation (obsolescence) rate of existing intermediates, and

$$\pi_l = \frac{1-\beta}{\beta}\frac{wL}{n} \text{ and } \pi_k = \frac{1-\beta}{\beta}\frac{rK}{m} \tag{21}$$

are the flow profits from the sale of labor- and capital-intensive intermediate goods.

Scientists are paid a wage ω_S, and competition between the two sectors and free-entry ensure that this wage is equal to the maximum of their contribution to the value of monopolists in the two sectors. Recall that R&D firms do not internalize the crowding effects, so the marginal value of allocating one more scientist to the invention of labor-intensive intermediates is $b_l \phi(S_l) nV_l$, and for capital-intensive intermediates, it is $b_k \phi(S_k) mV_k$, where V_l and V_k are given by (20). Therefore, free-entry requires:

$$\omega_S = \max\{b_l \phi(S_l) nV_l, b_k \phi(S_k) mV_k\}. \tag{22}$$

Equation (22) implies zero expected profits for all firms at all point in time, so $\Pi = 0$ in (2).

An equilibrium in this economy is therefore a set of factor prices, w, r and ω_S that satisfy (18) and (22), good prices, $[p_l(i)]_{i=0}^{n}$, $[p_k(i)]_{i=0}^{m}$, that satisfy (15), intermediate production levels given by (16), output levels given by (17), sequences of aggregate consumption and investment levels that satisfy (10) and (11), and sequences of S_l and S_k that satisfy (22).

C. Asymptotic and Balanced Growth Paths

I define an *asymptotic path* (AP) as an equilibrium path that the economy tends to as $t \to \infty$, and does not include limit cycles.[11] In an AP, we can have either $\lim_{t\to\infty} \dot{C}(t)/C(t) = \infty$, i.e., consumption grows more than exponentially (explodes), or $\lim_{t\to\infty} \dot{C}(t)/C(t) = g_c$, i.e., the rate of consumption growth tends to a constant, possibly 0 (including the case where $\lim_{t\to\infty} C(t) = 0$ as a special case). A *balanced growth path* (BGP) is defined as an AP where output, consumption and the capital stock grow at the same finite constant rate, i.e., $\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = g$.[12]

This subsection will show that with $\varepsilon < 1$, only BGPs can be an AP, so if the economy is going to tend to a non-cycling path, this has to be a BGP. In contrast, with $\varepsilon \geq 1$, there exists asymptotic paths where consumption grows more than exponentially or grows at a different rate than capital.

To facilitate the analysis, it is useful at this point to define

$$N \equiv n^{\frac{1-\beta}{\beta}} \text{ and } M \equiv m^{\frac{1-\beta}{\beta}}.$$

[11] Unfortunately, I am unable to rule out limit cycles, except in the case with risk neutrality. See Section III.

[12] This definition is convenient for the purposes here. Some authors also refer to growth paths where consumption and capital grow at different rates as BGP.

which simplifies the notation below, and, together with (17), allows me to write output in a more compact way:

$$Y = \left[\gamma (NL)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(MK)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{23}$$

In addition, I define a normalized capital stock,

$$k \equiv \frac{MK}{NL}, \tag{24}$$

which is a direct generalization of the normalized capital stock defined in the standard growth models as capital stock divided by the effective units of labor. Here the numerator contains the "effective units of capital" as well, since there can be capital-augmenting technical change. Then, using (13), (18), (19) and (24), we can write the interest rate as:

$$r = R(M,k) \equiv \beta (1-\gamma) M \left[\gamma k^{-\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{1}{\varepsilon-1}}. \tag{25}$$

Also, define the "relative share of capital", σ_K, as[13]

$$\sigma_K = \frac{rK}{wL} = pk = \frac{1-\gamma}{\gamma} k^{\frac{\varepsilon-1}{\varepsilon}}. \tag{26}$$

The relationship between the relative share of capital and the normalized capital stock depends on ε, which is the elasticity of substitution between capital-intensive and labor-intensive goods. Equation (26) shows that ε is also the elasticity of substitution between capital and labor in this economy. In response to an increase in k, σ_K will also increase if $\varepsilon > 1$, and will decrease if $\varepsilon < 1$.

Now we can state (proof in the Appendix):

Proposition 1: With $\varepsilon < 1$, all APs are BGPs and feature purely labor-augmenting technical change, i.e., they have $\lim_{t\to\infty} \dot{M}(t)/M(t) = 0$.

This is the first important result of the paper. It demonstrates that with $\varepsilon < 1$, i.e., with labor and capital as gross complements, the only asymptotic (non-cycling) paths will feature purely labor-augmenting technical change. There will be research effort devoted to the invention of capital-intensive intermediates, but this is only to keep the state of technology in that sector at a constant level.

For completeness, the next proposition covers the cases with $\varepsilon > 1$ and $\varepsilon = 1$.

Proposition 2: • With $\varepsilon > 1$, there are three APs:

[13] Note that this "relative share of capital" leaves out the income accruing to scientists from the denominator.

1. $\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = g < \infty$ and $\lim_{t\to\infty} \dot{M}(t)/M(t) = 0$;
2. $\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = \infty$ and $\lim_{t\to\infty} S_k(t) = S$; and
3. $\lim_{t\to\infty} \dot{C}(t)/C(t) = g_c < \infty$, $\lim_{t\to\infty} \dot{K}(t)/K(t) = g_k < g_c$, and $\lim_{t\to\infty} S_k(t) = 0$.

- With $\varepsilon = 1$, there is a unique AP which is a BGP.

With the elasticity of substitution between capital and labor greater than 1, in addition to the BGP with purely labor-augmenting technical change, there is an equilibrium path where consumption grows faster than exponentially, and technical change is purely capital augmenting, and another equilibrium path where consumption grows at the constant finite rate greater than the rate of growth of the capital stock, and all technical change is labor augmenting. We will in fact see below that the BGP in this case is not stable, and the economy will tend to one of the two other APs. In the case with $\varepsilon = 1$, the aggregate production function is Cobb-Douglas, the type of technical change does not matter, and the only possible asymptotic equilibrium path is a BGP (which features growth of both M and N, but since $\varepsilon = 1$, both of these are neutral, i.e., neither capital nor labor augmenting).

D. Characterization of Balanced Growth Path

We saw above that with $\varepsilon < 1$, only a BGP with purely labor-augmenting technical change can be an AP. Now I show that there in fact exists a unique BGP as long as $\delta > 0$, and characterize the properties of this equilibrium path.

First note that from the Euler equation, (10), the BGP rate of interest has to be constant. Moreover, since from Proposition 1 $\dot{M}/M = 0$, equation (25) immediately implies that the price index for capital-intensive goods, p_K, and therefore, the relative price of capital-intensive goods, p, must remain constant.

In addition, in BGP, output, Y, the wage rate, w, and the capital stock, K, will all grow at a common rate, g. Furthermore, for p to remain constant, (12) implies that Y_L and Y_K should grow at the same rate. Therefore, with M constant, n has to grow at the rate $\beta g/(1-\beta)$ (or N has to grow at the rate g). We can then integrate equation (20), allowing for the depreciation of technologies at the rate δ, and the growth of w, K and n, to obtain the values of inventing labor- and capital-intensive goods as:

$$V_l = \frac{1-\beta}{\beta} \frac{wL/n}{r+\delta-(1-2\beta)g/(1-\beta)} \text{ and } V_k = \frac{1-\beta}{\beta}\frac{rK/m}{r+\delta-g}. \tag{27}$$

Notice that these values also grow at a constant rate along the BGP because w, K and n are growing. The denominator for V_l is different from that of V_k because its BGP growth rate is lower than that of V_k: n, which is in the denominator of π_l, grows along the balanced growth path, while m remains constant.

Recall that in BGP, p and m are constant, so there is no net capital-augmenting technical change. This implies $\phi(S_k) S_k = \delta/b_k$, i.e., $S_k = S_k^*$ as defined above. The remaining scientists will work on labor-augmenting technical change. The growth rate of the economy is therefore

$$g^* = \frac{1-\beta}{\beta}\frac{\dot{n}}{n} = \frac{1-\beta}{\beta}\left[b_l\phi\left(S - S_k^*\right)\left(S - S_k^*\right) - \delta\right]. \tag{28}$$

Assumption 1 ensures that $g^* > 0$.

The Euler equation (10) then gives the BGP interest rate as $r^* = \rho + \theta g^*$. The interest rate has to be higher when the growth rate is higher in order to convince consumers to delay consumption, and the elasticity of marginal utility, θ, determines how strong this effect needs to be.

Let $k = G(M)$ such that M and k are consistent with BGP (i.e., $r^* = R(M,k)$). It is clear from (25) that $G' > 0$—that is, there is a strictly increasing relationship between M and k. This is because a greater k implies a lower price of capital-intensive goods, so capital has to become more productive, i.e., M has to increase in order to keep the interest rate at r^*.

Next, let k^* be the level of normalized capital such that at this normalized capital stock and at $\dot{M}/M = 0$, R&D firms are indifferent between capital- and labor-augmenting technical change, i.e., $b_l\phi\left(S - S_k^*\right) nV_l = b_k\phi\left(S_k^*\right) mV_k$, or from equation (27),

$$\frac{b_l\phi\left(S - S_k^*\right) wL}{r^* + \delta - (1-2\beta)\, g^*/\left(1-\beta\right)} = \frac{b_k\phi\left(S_k^*\right) r^* K}{r^* + \delta - g^*}. \tag{29}$$

This implies that, at $k = k^*$, the relative share of capital, σ_K, must satisfy:

$$\sigma_K = b^* \equiv \frac{b_l\phi\left(S - S_k^*\right)\left(1-\beta\right)\left(\rho + \delta + \left(\theta - 1\right) g^*\right)}{b_k\phi\left(S_k^*\right)\left(\left(1-\beta\right)\left(\rho+\delta\right) + \left(\left(1-\beta\right)\left(\theta-1\right) + \beta\right) g^*\right)}, \tag{30}$$

with g^* given by (28). In other words, using equation (26), we have:

$$k = k^* \equiv \left(\frac{\gamma b^*}{1-\gamma}\right)^{\frac{\varepsilon}{\varepsilon-1}} \Longleftrightarrow \sigma_K = b^*. \tag{31}$$

Finally, let M^* be such that $k^* \equiv G(M^*)$, i.e., M^* is the level of capital-augmenting technology that is consistent with the equilibrium interest rate taking its BGP value when $k = k^*$. As a result, when $k = k^*$ and $M = M^*$, the interest rate will be equal to r^* and the relative share of capital will be b^*.

In BGP, $\dot{M}M = 0$, while $\dot{N}/N > 0$. Because of the depreciation of technologies, there must be both research to invent new labor-intensive and capital-intensive intermediates—if there were no research directed at capital-intensive intermediates, we would have $\dot{M}/M < 0$. This implies that firms working to invent both types of goods have to make equal profits, so we need conditions (29) and (30) to hold, i.e., $k = k^*$, which in turn requires that $M = M^*$ so that $r = r^*$.

We can therefore state (proof in the text):

Proposition 3: Suppose that $\varepsilon \neq 1$ and $\delta > 0$. Then there exists a unique BGP where $k = k^*$ as given by (31), $M = M^* = G^{-1}(k^*)$, $r = r^* = \rho + \theta g^*$, and output, consumption and wages grow at the rate g^* given by (28).

This proposition is the second main result of the paper. It characterizes the unique BGP, which features purely labor-augmenting technical change. In this BGP, most research is devoted to the invention of labor-intensive intermediates. There is just enough capital-augmenting technical change to keep the productivity of capital constant—that is, there is no *net* capital-augmenting technical change. As a result, despite growth and capital deepening, factor shares remain constant in the long run. Intuitively, when the relative share of capital is equal to $\sigma_K = b^*$, R&D firms are just indifferent between inventing capital-intensive and labor-intensive intermediates; so in equilibrium they allocate their effort between the two sectors precisely to keep the relative share of capital at b^*. We have already seen that when $\varepsilon < 1$, the BGP with purely labor-augmenting technical change is the only possible asymptotic equilibrium path. In addition, we will see also below that, under certain conditions, this BGP is dynamically stable, so starting from different initial conditions, the economy will tend towards this growth path.

Given the CRRA preferences, the conclusion that for a BGP with constant interest rate and growth rate, we need $M = M^*$—i.e., no net capital-augmenting technical change—is not surprising. What is important (perhaps surprising), however, is that such a BGP exists despite the possibility of capital-augmenting technical change.[14]

The results are similar in spirit when there is no technological depreciation, i.e., $\delta = 0$, but there are now many balanced growth paths. These paths have the same growth rate, g^* (given by (28) evaluated at $\delta = 0$), but different factor distributions of income. This

[14] We saw in Proposition 2 that with $\varepsilon > 1$, there are other equilibrium paths with capital-augmenting technical change. We will also will see in Sections III and V that the equilibrium path with purely labor-augmenting technical change is unstable and that for other formulations of the innovation possibilities frontier such a balanced growth path typically fails to exist.

reflects that the equilibrium correspondence is lower-hemi continuous, but not continuous, in δ at $\delta = 0$. Summarizing (proof in the text):

Proposition 4: Suppose that $\varepsilon \neq 1$ and $\delta = 0$. Then, there exists a BGP for each $M \geq M^* \equiv G^{-1}(k^*)$, where k^* is given by (30) and (31) with $\delta = 0$. In all BGPs, output, consumption, wages, and the capital stock grow at the same rate g^* given by (28) with $\delta = 0$, and the share of labor is constant. Each BGP has a different normalized capital stock, $k = G(M)$, and a different relative share of capital, σ_K.

The intuition for the multiplicity of BGPs is simple: without depreciation, all that is required for a BGP is that labor-augmenting improvements should be more profitable than capital-augmenting improvements, i.e. $V_k \leq V_l$, and this can happen for a range of capital (labor) shares.

III. Transitonal Dynamics

The previous section established the existence of a unique balanced growth path (when $\delta > 0$) with a constant interest rate, stable factor shares and purely labor-augmenting technical change, very much resembling the textbook growth model. Nevertheless, balanced growth would be of limited interest if, starting from an arbitrary capital stock and factor distribution of income, the economy did not tend to this BGP. I already showed in Proposition 1 that no other APs are possible, but this, by itself, is not sufficient to establish stability, since there can also be limit cycles. I now discuss transitional dynamics in this economy. Unfortunately, the transitional dynamics are rather difficult to analyze. So I will establish analyze local stability, and then prove global stability in a special case.

The key result is that when the elasticity of substitution between capital and labor is less than 1, i.e., when $\varepsilon < 1$, transitional dynamics will take the economy towards the unique BGP with purely labor-augmenting technical change. Along the transition path, however, there will also be net capital-augmenting technical change—that is, M will also change. In contrast, when $\varepsilon > 1$, the economy will tend to an AP that is not a BGP (explosive growth or different asymptotic growth rates of consumption and capital) .

A. Local Stability

The key result in this section is that:

Proposition 5: Suppose $\delta > 0$. Then the BGP characterized above is locally saddle-path stable when $\varepsilon < 1$, and unstable when $\varepsilon > 1$.

This proposition is proved in the Appendix. The argument is standard: around the BGP, the equilibrium behavior is approximated by four linear differential equations in M, $k \equiv MK/NL$, S_k, and $c \equiv C/K$. The first two of those are state variables, while the latter two are control variables. I show in the Appendix that, with $\varepsilon < 1$, the set of linear differential equations has two positive and two negative eigenvalues, and is thus locally saddle-path stable.

The intuition for local stability can be obtained from equations (30) and (31). BGP requires $\sigma_K = b^*$. With $\varepsilon < 1$, the relative share of capital, σ_K, is decreasing in k. If $\sigma_K > b^*$, then $b_k \phi(S_k^*) mV_k > b_l \phi(S - S_k^*) nV_l$, and there will be more capital-augmenting technical change than along the BGP. This implies that k will increase.[15] But because σ_K is decreasing in k, the economy will approach the BGP. Clearly, this argument applies in reverse when $\varepsilon > 1$, and the economy moves away from the BGP, even when it starts arbitrarily close to it.

Finally, when $\varepsilon = 1$, the economy is identical to a standard endogenous growth model with a CobbDouglas production function, and the BGP is locally (and globally) stable.

B. Global Stability with Risk Neutrality

I next characterize the global stability properties in the special case where $\theta = 0$, i.e., where the representative agent is risk neutral. I also assume that negative consumption is allowed. This immediately implies that the interest rate, r, always has to be equal to the discount rate ρ, and removes the Euler equation of the representative consumer, (10), and the capital stock (and therefore k) also becomes a control variable. This ensures that at all points in time $\rho = R(k, M)$ where $R(k, M)$ is given by (25). In other words, the relationship $k = G(M)$ has to hold at all points in time, and as before, G is strictly increasing in M, with $k^* = G(M^*)$. These properties imply (proof in the Appendix):

Lemma 1: With $\theta = 0$, the transitional dynamics of the economy are given by

$$\frac{\dot{M}}{M} = \psi(M) \qquad (32)$$

where $\psi(M^*) = 0$, and when $\varepsilon < 1$, $\psi(M) \lesseqqgtr 0$ for all $M \gtreqqless M^*$ and when $\varepsilon > 1$, $\psi(M) \gtreqqless 0$ for all $M \gtreqqless M^*$.

This lemma implies that transitional dynamics can be represented by Figures 3 and 4 for the cases with $\varepsilon < 1$ and $\varepsilon > 1$, respectively. Inspection of these figures immediately

[15] Unfortunately, this is not true globally, since, in general, k may not increase despite the fact that $\dot{m}/m > 0$, because we can have $\dot{K}/K < 0$. Hence, the argument here is a local one.

implies that the BGP is globally stable when $\varepsilon < 1$, and globally unstable when $\varepsilon > 1$. The intuition is the same as in the last subsection: with $\varepsilon < 1$, when M and k are above their BGP levels, there will be capital-augmenting technical change, reducing both towards their BGP levels. Because the dynamics of k are pinned down by the behavior of M via the equation $k = G(M)$, we can also rule out limit cycles, and the result is one of global stability. In contrast, with $\varepsilon > 1$, levels of M and k greater than M^* and k^* lead to further increases, taking the economy towards the asymptotic path with capital-augmenting technical change and explosive growth, while $M < M^*$ leads to the AP with purely labor-augmenting technical change, and consumption growing faster than capital. Finally, as noted above, the economy with $\varepsilon = 1$ always converges to the unique BGP.

The next proposition summarizes these results (proof in the text):

Proposition 6: With $\theta = 0$, the BGP is globally (saddle-path) stable when $\varepsilon \leq 1$, and unstable when $\varepsilon > 1$.

IV. Policy and Comparative Dynamics

In this section, I analyze the effect of policy on the factor distribution of income in the basic model of Section II. The main result of this analysis is that the long-run factor distribution of income is independent of fiscal policy and labor market policy, and approximately independent of the discount rate (the savings rate). This result contrasts with the implications of the standard growth model with only labor-augmenting technical change, where such policies would affect the long-run factor distribution of income. In many OECD countries, tax and labor market policies have changed substantially over the past 100 years (see Persson and Tabellini, 2003, on tax policies, and Saint-Paul, 2000, on labor market policies). Figures 1 and 2 show large medium-run changes in the factor distribution of income, but no long-run changes. The long-run stability of factor shares is difficult to reconcile with the standard model with only labor-augmenting technical change, but is in line with the predictions of the framework outlined here.[16] To simplify the discussion, in this section, I focus on the case with $\varepsilon < 1$.

A. Changes in Capital Income Taxation and Discount Rates

First consider taxation of capital income at some rate τ. Assume that the proceeds from capital income taxation are distributed lumpsum to consumers. This implies that

[16] Obviously, long-run stability of factor shares in response to policy changes is consistent with the Cobb-Douglas production function, but with such a production function, we cannot explain/analyze short-run and medium-run swings in factor shares as those shown in Figures 1 and 2.

the budget constraint of the representative agent changes to

$$C + I \leq wL + (1-\tau) rK + \omega_S S + \Pi + T,$$

where r is that pre-tax interest rate and T is the lumpsum redistribution to consumers from the proceeds of taxation. The government budget constraint implies that $T = \tau r K$. Clearly, the resource constraint of the economy is identical to (3). The Euler equation of the representative consumer is similar to (10), except that the relevant interest rate is the after-tax one, $(1-\tau) r$, thus $\dot{C}/C = ((1-\tau) r - \rho)/\theta$.

For comparison, first consider the case of exogenous labor-augmenting technical change, where $\dot{M} = 0$, and $N = e^{gt}$. The BGP growth rate is now exogenously given at g, which will also be the BGP growth rate of consumption. Therefore, from the Euler equation (10), the BGP after-tax interest rate still has to satisfy $r^* = (\rho + \theta g)/(1-\tau)$. Since the pre-tax interest rate must equal the marginal product of capital, this also implies:

$$\beta (1-\gamma) M \left[\gamma \hat{k}^{-\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{1}{\varepsilon-1}} = \frac{\rho + \theta g}{1-\tau},$$

where $\hat{k}$ is the BGP value of the normalized capital stock in this case. This equation immediately implies a decreasing relationship between τ and $\hat{k}$ (recall that, by assumption, there is only labor-augmenting technical change, so M is constant). As long as the elasticity of substitution, ε, is less than 1, an increase in the rate of capital income taxation reduces the BGP value of the normalized capital, and through this channel, increases the share of capital income in GDP (the case with $\varepsilon > 1$ would give the reverse). Only in the case where $\varepsilon = 1$, i.e., when the production function takes the Cobb-Douglas form, is the long-run factor distribution of income independent of the rate of capital income taxation.

Now consider the case where both capital-augmenting and labor-augmenting technical change are allowed and endogenous. Equation (10) still determines the rate of growth of consumption, but for balanced growth we need to have both $\dot{M} = 0$ and $\dot{N} > 0$, which implies that there should be both capital-augmenting and labor-augmenting innovations (otherwise we would have $\dot{M}/M < 0$). Since firm profits depend on the pre-tax interest rate, equilibrium still requires $\sigma_K = b^* \iff k = k^*$. Therefore, the long-run factor distribution of income is unaffected by capital income taxation. In addition, in BGP consumption must grow at the rate g^* as given by (28), and so the Euler equation (10) implies that the pre-tax interest rate has to satisfy $r = (\rho + \theta g^*)/(1-\tau)$, and is therefore an increasing function of the rate of capital income taxation. Since $k = k^*$, to ensure both capital- and labor-augmenting research, M has to increase to raise the interest rate, and since $k \equiv MK/NL$, is also implies that capital to effective labor ratio, K/NL, also falls. Therefore, with endogenous capital- and labor-augmenting technical change, capital income taxation *reduces* the capital-labor ratio, but *creates an exactly offsetting capital-augmenting technical change*, and leaves the long-run factor distribution of income unchanged.

Next, consider a change in the discount rate ρ. The analysis is analogous. In the standard model with only labor-augmenting technical progress, this will change the savings rate, the capital-labor ratio and the factor distribution of income. In contrast, in the framework here, long-run equilibrium still requires $\sigma_K = b^*$, so that it remains profitable to undertake R&D towards both types of technologies. However, now there will be an effect on the factor distribution of income because the change in ρ will also influence the BGP interest rate faced by consumers, r, and through this channel, it will change b^* and k^*. Inspection of equation (30) immediately shows that this effect disappears when the BGP growth rate is zero. Similarly, when g^* is small, this effect will be second order. Therefore changes in the discount rate will generally have small or second-order effects on the factor distribution of income.

B. Labor Market Policy

Next to analyze labor market policy in a simple way, suppose that the government imposes a (binding) minimum wage $\bar{w}$, and moreover, indexes this minimum wage to the level of income. In particular, assume that

$$\bar{w} = \chi^{-1} (rK + wL),$$

where $\chi > 0$ and L is the level of employment, which is now determined endogenously.[17] Since the minimum wage is binding, the equilibrium wage rate has to be $w = \bar{w}$ at all points in time. Multiplying both sides of this equation by L, and rearranging we obtain the quasi-labor supply curve, relating employment, L, to the relative share of capital, σ_K:

$$L = \chi^{-1} (1 + \sigma_K)^{-1}. \tag{33}$$

I refer to (33) as the quasi-labor supply curve of the economy, since any equilibrium has to be along this curve.

As before, BGP requires $\dot{M}/M = 0$, thus $\sigma_K = b^*$ and $k = k^*$. Therefore, the long-run share of capital in GDP will be unchanged—irrespective of the equilibrium level of employment. An increase in χ will immediately reduce employment, however, and via this channel raise k (recall that $k \equiv MK/NL$). In the case where the elasticity of

[17] This expression makes the minimum wage proportional to the sum of capital and labor income rather than total income, which also includes scientists' earnings. This is only to simplify the expressions, without any substantive implications.

substitution between capital and labor, ε, is less than 1, the labor share will also increase. Subsequently, the economy adjusts back to BGP starting with $k > k^*$. Throughout this process, the share of labor in GDP falls, and returns to its initial level in BGP. Since (33) relates employment to the share of labor in GDP, employment also falls steadily during this adjustment process.

This result is interesting in light of the developments in many European labor markets over the past several decades. For example, Blanchard (1997) documents that both unemployment and the labor share in a number of continental European economies rose sharply starting in the late 1960s. Both Blanchard and Caballero and Hammour (1998) interpret this as the response of these economies to a wage-push; the militancy and/or the bargaining power of workers increased because of changes in labor market regulations taking place over this time period, or because of the ideological effects of 1968. This wage-push translated into higher wages and lower employment. During the 1980s, we see a different pattern: unemployment in these countries continues to increase, but the labor share falls sharply. Blanchard documents that the decline in the labor share cannot be explained by capital-labor substitution, and conjectures that it may have been due to "biased technical change". The framework presented here is consistent with these patterns: in response to a wage-push shock, i.e., an increase in χ, both the share of labor in GDP and unemployment increase. Then as technology adjusts, k returns to its BGP value, k^*, and employment falls further. The fall in k is accompanied by an offsetting decline in M, which corresponds to capital-biased technical change.[18]

V. Discussion and Extensions

The analysis so far has established that in a natural model with potentially labor- and capital-augmenting technical change, there is a unique balanced growth path equilibrium with no net capital-augmenting technical change, stable factor shares and a constant long-run interest rate. Moreover, as long as capital and labor are gross complements (i.e., the elasticity of substitution is less than 1), the economy converges to this BGP. This analysis relied on a number of assumptions. For example, technical change took the form of invention of new goods; R&D was carried out by scientists so that the production and R&D sectors did not compete for labor; and productivity in the R&D sector depended on the number of existing goods, with spillovers from past research. I now clarify which of these assumptions are important for the substantive results. We will see that the only

[18] Because the elasticity of substitution, ε, is less than 1, a decline in M corresponds to "capital-biased" technical change. See Acemoglu (2002) for a discussion of the relationship between factor-augmenting and factor-biased technical change.

important assumption for the results is the form of the innovation possibilities frontier (or the form of spillovers from past research).

A. The Innovation Possibilities Frontier

There are two important assumptions embedded in this innovation possibilities frontier (8): first, R&D uses a scarce factor (scientists, or labor as in the next subsection). Second, there is a specific form of spillovers from past research; an increase in n raises the productivity of R&D in the n-sector, but not in the m-sector. I refer to this as *state-dependence*, since the relative productivities of R&D in the two sectors depend on the state of the system, (n, m).

Let us now relax each of these two assumptions on the form of the innovation possibilities frontier. The alternative to an R&D sector using scarce labor is what Romer and Rivera-Batiz (1991) refer to as the lab-equipment model where the final good (or capital) is used for R&D. For example, we could have (implicitly setting $\phi(\cdot) = 1$ in terms of (8) to simplify the notation):

$$\dot{n} = b_l X_l - \delta n \text{ and } \dot{m} = b_k X_k - \delta m, \tag{34}$$

where X_l and X_k are the R&D expenditures in the two sectors in terms of the final good, and the resource constraint needs to be modified to $C + I + X_l + X_k = Y$. The important point is that long-run growth is now possible without knowledge spillovers from past research, because R&D does not use any scarce factors—only the final good. Consequently, there is also no state-dependence, since the relative productivity of R&D in the two sectors is always constant.[19] The rest of the setup remains unchanged.

Much of the analysis so far applies,but the free-entry condition into R&D now requires $b_l V_l = 1$ and $b_k V_k = 1$, since one unit of final output is used to invent b_l labor-intensive or b_k capital-intensive goods. Therefore, BGP requires:

$$b_k \frac{rK}{m} = b_l \frac{wL}{n}. \tag{35}$$

This condition is not consistent with balanced growth, however. For the interest rate to remain constant, we need $\dot{m} = 0$, and w, n and K to grow at the same rate. But the BGP condition (35) implies that K and m will grow together. Therefore, with the innovation possibilities frontier given as in the lab-equipment specification, there exists no BGP (though there exist other APs with constant growth of consumption).

[19] Equation (34) is equivalent to the formulation of the innovation possibility frontier I used in Acemoglu (1998) in the context of technical change directed at skilled and unskilled labor. See Acemoglu (2002) for a more detailed discussion of the implications of different forms of the innovation possibilities frontier.

Next, let us return to the formulation with scientists undertaking R&D and spillovers from past research, but modify (8) to remove state-dependence (and again set $\phi(\cdot) = 1$):

$$\dot{n} = b_l n^{\phi} m^{1-\phi} S_l - \delta n \text{ and } \dot{m} = b_k n^{\phi} m^{1-\phi} S_k - \delta m. \tag{36}$$

In (36), there are still spillovers from past research to ensure long-run growth in this case. But there is no state-dependence: R&D in one of the sectors affects both sectors equally in the future. This contrasts with (8) where current research for the invention of labor-intensive goods increases the productivity of R&D for labor-intensive goods in the future, but not for capital-intensive goods. Free-entry into R&D now requires $b_l V_l = \omega_S$ and $b_k V_k = \omega_S$, which leads to equation (35) as a BGP condition. As a result, in this case also, there is no BGP.

Therefore, the BGP with purely labor-augmenting technical change is only consistent with an innovation possibilities frontier with a strong degree of state-dependence. Intuitively, balanced growth with capital accumulation requires the profitability of inventing new capital-intensive goods not to increase faster than the profitability of inventing new labor-intensive goods—so that in equilibrium firms are happy to undertake only labor-augmenting improvements. Since capital accumulation increases the profitability of research towards capital-augmenting technologies, a strong form of state-dependence in the R&D technology, whereby labor-augmenting technical change raises the profitability of further research towards labor-augmenting technologies, is necessary to balance this effect.[20]

B. Competition For Labor Between Production and R&D

In the baseline model, there are two types of workers, unskilled labor and scientists, with scientists specializing in R&D and thus no feedback from the relative price of labor to growth. This assumption was made only for simplicity, and I now modify the model to

[20] Is an innovation possibilities frontier in with a strong degree of state-dependence, like (8), plausible? Unfortunately, I am not aware of any direct investigation of this issue. The data on patent citations analyzed by, among others, Jaffe, Trajtenberg and Henderson (1993), Trajtenberg, Henderson and Jaffe (1992) and Caballero and Jaffe (1993), may be relevant in this context. These papers study subsequent citations of patents by other innovations. A citation of a previous patent is interpreted as evidence that a current invention is exploiting information generated by the previous invention. This corresponds to some degree of spillover from past research. One can therefore use patent citations data to investigate whether there is state-dependence at the industry level. Industry level state-dependence corresponds to patents being cited in the same industry in which they originated. Results reported in Table 1 in Trajtenberg, Henderson and Jaffe (1992) suggest that there is some amount of industry state-dependence. For example, patents are likely to be cited in the same three-digit industry from which they originated. Nevertheless, it is currently impossible to investigate state-dependence at the factor level. This is because, although we have information about the industry for which the patent was developed, we do not know which factor the innovation was directed at.

allow the production and R&D sectors to compete for labor. To simplify the discussion, let us again focus on the case with $\phi(\cdot) = 1$. Equation (8) then changes to

$$\frac{\dot{n}}{n} = b_l L_l - \delta \text{ and } \frac{\dot{m}}{m} = b_k L_k - \delta, \tag{37}$$

where L_l is the number of workers employed in R&D for labor-intensive goods, L_k is the number of workers employed in R&D for capital-intensive goods, and L is workers employed in production. Normalizing total labor supply to 1, the labor market clearing condition is $L + L_l + L_k = 1$. In this framework, new goods are invented by workers employed in the R&D sector, so the production and R&D sectors compete for workers. Most of the analysis from Section II applies, but the free-entry condition into R&D now relates the value of a new innovation to the wage rate (rather than the wage for scientists), and equation (22) is replaced by $w = \max\{b_l n V_l, b_k m V_k\}$. In BGP, M needs to remain constant, so there has to be some research devoted to inventing new capital-intensive intermediates to balance depreciation. Thus, we need $b_l n V_l = b_k m V_k = w$, with $b_l n V_l$ and $b_k m V_k$ given by (27) above. The condition $b_l n V_l = w$ immediately implies that in BGP: $\tilde{r} + \delta - (1 - 2\beta)\tilde{g} = b_l \frac{L}{\beta}$, where $\tilde{r}$ and $\tilde{g}$ are the BGP interest and growth rates. Now using the Euler equation for consumption, (10), we have

$$(2\beta + \theta - 1)\tilde{g} + \delta + \rho = b_l \frac{L}{\beta}. \tag{38}$$

Furthermore, in BGP we again have $\dot{m}/m = 0$, which implies $L_k = \delta/b_k$, and $\dot{n}/n = \beta\tilde{g}/(1-\beta)$, and also $L_l = \delta/b_l + \beta\tilde{g}/(1-\beta)\, b_l$. Using the market clearing condition for labor, and equation (38), the long-run growth rate of the economy is therefore given by:

$$\tilde{g} = \frac{(1-\beta)\beta b_k b_l - (1-\beta)\beta b_k(\rho + \delta) - (1-\beta) b_l \delta}{(1-\beta)\beta b_k \theta + \beta\delta - (1-\beta)\beta(1-2\beta) b_k}.$$

The rest of the analysis is unchanged. In particular, BGP requires $\dot{m} = 0$, hence $\sigma_K = b^*$ and stable factor shares. As before, along the balanced growth path, technical change is purely labor-augmenting, with research towards capital-augmenting goods only to keep the net productivity of capital constant. In this case, transitional dynamics are more complicated, however, because both the number of production workers and the speed of technical progress change along the transition path.

C. Different Forms of Technical Progress

Labor-augmenting technical change has so far been interpreted as "labor-using" change, that is, the introduction of new goods and tasks that use labor. I now show that the results of the above analysis generalize to different formulations of the technological change process, including a model of technological progress with new varieties of machines, and one where technical change takes the form of quality improvements as in Grossman and Helpman (1991a,b) and Aghion and Howitt (1992). In both cases, long-run technical change will be labor-augmenting, with no net capital-augmenting technical change.

To discuss the consequences of technical change resulting from the invention of new machines, let me modify the basic framework such that the two goods have the following production functions:

$$Y_L = \frac{1}{1-\beta}\left(\int_0^n z_l\left(j\right)^{1-\beta} dj\right) L^\beta \text{ and } Y_K = \frac{1}{1-\beta}\left(\int_0^m z_k\left(j\right)^{1-\beta} dj\right) K^\beta,$$

where $z_l(j)$ this the quantity of the j-th machine complementing labor, and $z_k(j)$ is the quantity of the j-th machine complementing capital. This is the model used in Acemoglu (2002), for the case where the two factors are not accumulable, and more details on the solution can be found there. Notice that n and m are now the numbers of different types of machines complementing these two factors. These machines are supplied by monopolists, while producers of Y_L and Y_K are competitive. I assume that machines depreciate at the rate $\bar{\delta} > 0$, and the cost of producing a new machine is normalized to 1 in terms of the final good. The demand for these machines is straightforward to derive from profit maximization: $z_l(j) = (p_L/\chi_l(j))^{1/\beta} L$ and $z_k(j) = (p_K/v)^{1/\beta} K$, where $\chi_l(j)$ and $\chi_k(j)$ denote the user cost of machines. Since the demand curves for machines are isoelastic, the profit-maximizing monopoly price of machines is a constant markup over marginal cost, which is $r + \bar{\delta}$, the interest rate plus the depreciation rate. Therefore, $\chi_l(j) = \chi_k(j) = \left(r + \bar{\delta}\right) / (1-\beta)$.[21] Next, from market clearing, factor prices are

$$w = \frac{\beta}{1-\beta}\left(\frac{1-\beta}{r+\bar{\delta}}\right)^{1/\beta} p_L^{1/\beta} n \text{ and } r = \frac{\beta}{1-\beta}\left(\frac{1-\beta}{r+\bar{\delta}}\right)^{1/\beta} p_K^{1/\beta} m.$$

These equations imply that the profits of technology monopolists are:

$$\pi_l = (1-\beta)\left(\frac{1-\beta}{r+\bar{\delta}}\right)^{1/\beta} \frac{wL}{n} \text{ and } \pi_k = (1-\beta)\left(\frac{1-\beta}{r+\bar{\delta}}\right)^{1/\beta} \frac{rK}{m}. \tag{39}$$

These profits are identical to those in (21), except for the constant and the fact that they depend on the interest rate. Next, assuming the innovation possibilities frontier is given by (8), we can see that BGP requires $\sigma_K = b^*$, and we obtain exactly the same results

[21] The monopolist will originally produce z_l (or z_k) units, and then replace the machines that have depreciated.

as in Sections II and III. This demonstrates that whether technical change is modeled as the introduction of new labor-intensive and capital-intensive goods or as the invention of labor-enhancing and capital-enhancing machines is immaterial.

The second possibility is one where technical progress takes the form of firms moving up the quality ladder (vertical innovations), which differs from the other two formulations because it features creative destruction: new goods/machines replace old ones. Suppose the two goods are now produced competitively with the production functions

$$Y_L = \frac{1}{1-\beta}\left(Q_L^{\beta} {z_L}^{1-\beta}\right) L^{\beta} \text{ and } Y_K = \frac{1}{1-\beta}\left(Q_K^{\beta} {z_K}^{1-\beta}\right) K^{\beta}, \tag{40}$$

where z_L and z_K are quantities of machines that complement labor and capital, and Q_L and Q_K denote the qualities of these machines. Technical progress results when an R&D firm discovers a new vintage of labor-complementary machines, with productivity $Q'_L = (1+\lambda)Q_L$, where $\lambda > 0$, or a new vintage of capital-complement the machines, with productivity $Q'_K = (1+\lambda)Q_K$. This R&D firm would be the monopoly supplier of this vintage, and it would dominate the market until a new, and better, vintage arrives. I assume that a scientist who works to discover a new vintage of Q_L (or Q_K) is successful at the flow rate b_l (or b_k). Notice that this assumption already builds in knowledge-based spillovers that were required for the existence of a BGP before: research on a vintage of quality Q_L leads to proportionately better machines, so the greater is Q_L, the greater is the resulting improvement in the "level" of productivity (i.e. λQ_L).

Without loss of generality, I assume here that machines depreciate fully after use, and normalize the marginal cost of producing z to $1/(1+\lambda)$. I also assume that λ is small enough that the leading monopolist will set a limit price to ensure that the next best vintage breaks even (see, for example, Grossman and Helpman, 1991b). Since the marginal cost of production is $1/(1+\lambda)$, the limit price is $\chi_K = \chi_L = 1$. Hence, $z_L = p_L^{1/\beta} Q_L L$ and $z_K = p_K^{1/\beta} Q_K K$. Substituting these into (40), it is straightforward to verify that the equilibrium interest and wage rates are: $r = \beta(1-\beta)^{-1} p_K^{1/\beta} Q$ and $w = \beta(1-\beta)^{-1} p_L^{1/\beta} Q_L$, and profits are given by an equation similar to (21) or (39). By standard arguments, the BGP values of inventing new (higher) quality intermediate goods are

$$V_l = \frac{\lambda w L}{(1+\lambda)(r+\delta_l)} \text{ and } V_k = \frac{\lambda r K}{(1+\lambda)(r+\delta_k)},$$

where δ_l and δ_k are the endogenous rates of creative destruction. From the above assumptions, we have $\delta_l = b_k S_l$ and $\delta_k = b_k S_k$. Similar reasoning to before implies that only $V_l \geq V_k$ is consistent with stable factor shares. Therefore, along the BGP, there will only be labor-augmenting technical change, i.e., $S_l = S$ and $S_k = 0$. Because I have not introduced technological obsolescence (in addition to the endogenous creative destruction already present in these models), BGP requires $V_l \geq V_k$ rather than $V_l = V_k$, so there is now a range of labor shares consistent with BGP.

VI. Conclusion

Almost all existing models of economic growth rely on one of two assumptions: either the production function is supposed to be Cobb-Douglas (an elasticity of substitution between capital and labor exactly equal to 1), or all technical change is assumed to be labor-augmenting. Much evidence suggests that the elasticity of substitution is less than 1. Moreover, a framework with an elasticity of substitution exactly equal to 1 does not enable an analysis of medium-run changes in factor shares. A model with purely labor-augmenting technical change is more attractive, but poses the question of why there are no capital-augmenting technological improvements. It also suggests that the long-run factor distribution of income should be a function of tax and labor market policies and of the savings rate, while in the data, the long-run factor distribution of income appears to be stable despite changes in these variables.

This paper studied the determinants of the direction of technical change in a model where the invention of new production methods is a purposeful activity. Profit-maximizing firms can introduce capital- and/or labor-augmenting technological improvements. The major result is that, with the standard assumptions used to generate endogenous growth, long-run technical change will be purely labor-augmenting. Along the balance growth path, the economy looks like the standard model with a steadily increasing wage rate and a constant interest rate. Therefore, the framework here offers a microfoundation for the standard neoclassical growth model with (exogenous or endogenous) labor-augmenting technical change. But, it also shows that away from the balanced growth path, there will typically be capital-augmenting technical change. Furthermore, I showed that a range of policies that affect the long-run factor distribution of income in the standard model have no long-term effects in this model.

It has to be noted that the results here hold under a very specific form of the innovation possibilities frontier, and the discussion in subsection V.A indicated that with other forms, a balanced growth path with purely labor-augmenting technical change typically fails to exist. Work on why this form of the innovation possibilities frontier is plausible, or why technical change may be labor-augmenting with other plausible forms is a fruitful area for future research.

VII. Appendix: Proofs

Throughout this appendix, I use the notation $\lim_{t\to\infty} x(t) = x$ and $x(t) \to x$ interchangeably. In addition, I first state the following result which will be useful in some of the proofs:

Lemma A1: Let

$$\Delta(t) \equiv \frac{m(t) V_k(t)}{n(t) V_l(t)}.$$

Suppose that $\varepsilon < 1$. Then, $\lim_{t\to\infty} k(t) = 0 \Longrightarrow \lim_{t\to\infty} \Delta(t) = \infty$, $\lim_{t\to\infty} \dot{M}(t)/M(t) = (1-\beta)(b_k\phi(S)S - \delta)/\beta$, and $\lim_{t\to\infty} \dot{N}(t)/N(t) = -(1-\beta)\delta/\beta$.
And $\lim_{t\to\infty} k(t) = \infty \Longrightarrow \lim_{t\to\infty} \Delta(t) = 0$, $\lim_{t\to\infty} \dot{M}(t)/M(t) = -(1-\beta)\delta/\beta$, and $\lim_{t\to\infty} \dot{N}(t)/N(t) = (1-\beta)(b_l\phi(S)S - \delta)/\beta$.

Suppose that $\varepsilon > 1$. Then, $\lim_{t\to\infty} k(t) = \infty \Longrightarrow \lim_{t\to\infty} \Delta(t) = \infty$, $\lim_{t\to\infty} \dot{M}(t)/M(t) = (1-\beta)(b_k\phi(S)S - \delta)/\beta$, and $\lim_{t\to\infty} \dot{N}(t)/N(t) = -(1-\beta)\delta/\beta$.
And $\lim_{t\to\infty} k(t) = 0 \Longrightarrow \lim_{t\to\infty} \Delta(t) = 0$, $\lim_{t\to\infty} \dot{M}(t)/M(t) = -(1-\beta)\delta/\beta$, and $\lim_{t\to\infty} \dot{N}(t)/N(t) = (1-\beta)(b_l\phi(S)S - \delta)/\beta$.

Proof of Lemma A1: We have from (27) that

$$\Delta(t) = \int_t^\infty \frac{r(v) K(v)}{w(v) L(v)} dv = \int_t^\infty \frac{\exp\left[-\int_t^v (r(\omega)+\delta) d\omega\right] r(v) K(v)}{\exp\left[-\int_t^v (r(\omega)+\delta) d\omega\right] w(v) L(v)} dv = \int_t^\infty k(v)^{\frac{\varepsilon-1}{\varepsilon}} dv.$$

As $k(t) \to 0$, all $k(s) \to 0$ for all $s \geq t$. When $\varepsilon < 1$, this implies $\Delta(t) \to \infty$, and therefore $S_l(t) \to 0$ and $S_k(t) \to S$. This immediately gives $\lim_{t\to\infty} \dot{M}(t)/M(t) = (1-\beta)(b_k\phi(S)S - \delta)/\beta$ and $\lim_{t\to\infty} \dot{N}(t)/N(t) = -(1-\beta)\delta/\beta$. The other cases follow analogously.

Proof of Proposition 1: The proof will show that paths with $\lim_{t\to\infty} \dot{C}(t)/C(t) = \infty$ cannot be equilibria, and that any equilibrium path with $\lim_{t\to\infty} \dot{C}(t)/C(t) = g$ must be a BGP with purely labor-augmenting technical change, i.e., $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = g$ and $\dot{M}(t)/M(t) = 0$.

First, I will prove that $\lim_{t\to\infty} \dot{C}(t)/C(t) = \infty$ cannot be an equilibrium. To derive a contradiction, suppose this is the case. Then from the budget constraint, (2), we need $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \infty$, which I will show is not possible. To start with, take the case where $\lim_{t\to\infty} k(t) = \infty$, and note that, from (23), output can be written as:

$$\lim_{t\to\infty} Y(t) = \lim_{t\to\infty} N(t) L \left[\gamma + (1-\gamma) k(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \lim_{t\to\infty} N(t) \gamma^{\frac{\varepsilon}{\varepsilon-1}} L,$$

since, with $\varepsilon < 1$, $k(t)^{\frac{\varepsilon-1}{\varepsilon}} \to 0$ as $k(t) \to \infty$. Therefore, $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \dot{N}(t)/N(t) < \infty$. Next, take the case where $\lim_{t\to\infty} k(t)$ =constant, then clearly, $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \dot{N}(t)/N(t) < \infty$. Finally, consider the case where $k(t) \to 0$, in which case, $\left[\gamma + (1-\gamma)k(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \to 0$, so $\lim_{t\to\infty} \dot{Y}(t)/Y(t) < \dot{N}(t)/N(t) < \infty$. As a result, in neither case is $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \infty$ possible, so $\lim_{t\to\infty} \dot{C}(t)/C(t) = \infty$ cannot be an equilibrium.

This implies that all APs must have $\lim_{t\to\infty} \dot{C}(t)/C(t) = g$. Next I will show that $\lim_{t\to\infty} \dot{C}(t)/C(t) = g$ also necessitates $\lim_{t\to\infty} \dot{M}(t)/M(t) = 0$, and that in this case $\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = g$, which will complete the proof.

First, recall that for consumption to grow at the constant rate, we need. from the Euler equation (10), the interest rate $r(t) = \beta M(t) p_K(t)$ to remain constant. Recall also that

$$p_K(t) = \left[\gamma(1-\gamma)^{\varepsilon-1} k(t)^{-\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\varepsilon}\right]^{\frac{1}{\varepsilon-1}}, \tag{A1}$$

which implies

$$\frac{\dot{p}_K(t)}{p_K(t)} = -\frac{1}{\varepsilon}\frac{\gamma(1-\gamma)^{\varepsilon-1} k(t)^{-\frac{\varepsilon-1}{\varepsilon}}}{\gamma(1-\gamma)^{\varepsilon-1} k(t)^{-\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\varepsilon}}\frac{\dot{k}(t)}{k(t)} \tag{A2}$$

Constant interest rate, i.e.. $\dot{r}(t) = 0$. then requires:

$$\frac{\dot{M}(t)}{M(t)} = -\frac{\dot{p}_K(t)}{p_K(t)}.$$

To derive a contradiction, first suppose that $\dot{M}(t)/M(t) < 0$, then $\dot{p}_K(t)/p_K(t) > 0$, which. from (A2). implies $\dot{k}(t)/k(t) < 0$. or $k(t) \to 0$. But then from Lemma A1, $k(t) \to 0$ implies $\lim_{t\to\infty} \dot{M}(t)/M(t) > 0$. giving a contradiction with $\dot{M}(t)/M(t) < 0$.

Next suppose that $\dot{M}(t)/M(t) > 0$. which requires $\dot{p}_K(t)/p_K(t) > 0$. which, from (A2), implies $\dot{k}(t)/k(t) > 0$, or $k(t) \to \infty$. But then, from Lemma A1. we have $\Delta(t) \to 0$. Thus, $\dot{M}(t)/M(t) < 0$, contradicting the supposition that $\dot{M}(t)/M(t) > 0$.

Therefore, we must have $\dot{M}(t)/M(t) = 0$, and $\dot{p}_K(t)/p_K(t) = 0$, and thus. $\dot{k}(t)/k(t) = 0$. This also implies that $\lim_{t\to\infty} \dot{K}(t)/K(t) = \lim_{t\to\infty} \dot{N}(t)/N(t) = g^*$ with g^* as given by (28) in the text. Then. we immediately have that $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = g^*$, and thus $\lim_{t\to\infty} \dot{C}(t)/C(t) = g^*$, completing the proof.

Proof of Proposition 2: *The case with* $\varepsilon > 1$. First, it is clear that the BGP with $\lim_{t\to\infty} \dot{C}(t)/C(t) = g^*$ and $\lim_{t\to\infty} \dot{M}(t)/M(t) = 0$ outlined in Proposition 1, where

$\lim_{t\to\infty} \dot{k}(t)/k(t) = 0$, is still possible. Moreover, the same argument as in the proof of Proposition 1 implies that there exists no other AP with $\lim_{t\to\infty} \dot{k}(t)/k(t) = 0$.

Second, consider the case where $\lim_{t\to\infty} \dot{k}(t)/k(t) > 0$, i.e., $k(t) \to \infty$, then Lemma A1 with $\varepsilon > 1$ immediately implies that $\lim_{t\to\infty} \dot{M}(t)/M(t) = (1-\beta)(b_k\phi(S)S - \delta)/\beta$, and $\lim_{t\to\infty} \dot{N}(t)/N(t) = -(1-\beta)\delta/\beta$. Next (23) with $\varepsilon > 1$ implies that $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = \dot{M}(t)/M(t) + \dot{K}(t)/K(t)$, and $r(t) \to \hat{r}(t) \equiv \beta M(t)(1-\gamma)^{\varepsilon/(\varepsilon-1)} = \infty$. Thus $\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = \infty$ is an AP.

Finally, consider the case where $\lim_{t\to\infty} \dot{k}(t)/k(t) < 0$, i.e., $k(t) \to 0$. Then Lemma A1 gives $\lim_{t\to\infty} \dot{M}(t)/M(t) = -(1-\beta)\delta/\beta$ and $\lim_{t\to\infty} \dot{N}(t)/N(t) = (1-\beta)(b_l\phi(S)S - \delta)/\beta$. (23), in turn, together with $k(t) \to 0$ and $\varepsilon > 1$, implies that

$$\lim_{t\to\infty} Y(t) = \lim_{t\to\infty} N(t) L \left[\gamma + (1-\gamma) k(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \lim_{t\to\infty} N(t)\, \gamma^{\frac{\varepsilon}{\varepsilon-1}} L.$$

Thus $\lim_{t\to\infty} \dot{Y}(t)/Y(t) = (1-\beta)(b_l\phi(S)S - \delta)/\beta < \infty$, but then $\lim_{t\to\infty} \dot{C}(t)/C(t) = \infty$ is impossible from (2).

So we must have $\lim_{t\to\infty} \dot{C}(t)/C(t) = g_c < \infty$. Then, for consumption to have a constant growth rate, the Euler equation (10) requires the interest rate to remain constant. From (A2) with $k(t) \to 0$, this requires $\dot{k}(t)/k(t) = \varepsilon \dot{M}(t)/M(t) = -\varepsilon(1-\beta)\delta/\beta$, or $\lim_{t\to\infty} \dot{K}(t)/K(t) = (1-\beta)(b_l\phi(S)S - \varepsilon\delta)/\beta$, giving us another AP, with a constant rate of consumption growth, which is, however, not a BGP, since $\lim_{t\to\infty} \dot{K}(t)/K(t) < \lim_{t\to\infty} \dot{C}(t)/C(t)$.

The case with $\varepsilon = 1$. As $\varepsilon \to 1$, the aggregate production function becomes Cobb-Douglas, $Y \equiv B(NL)^{\gamma}(MK)^{1-\gamma}$, and the equilibrium relative share of capital is always $\sigma_K = (1-\gamma)/\gamma$. Therefore, the equilibrium allocation of research effort is given by

$$\frac{1-\gamma}{\gamma} = \frac{b_l\phi(S-S_k)(\rho+\delta+\theta g^* - g_m)}{b_k\phi(S_k)(\rho+\delta+\theta g^* - g_n)}$$

at all points in time, where $g_n = b_l\phi(S-S_k)(S-S_k) - \delta$, $g_n = b_k\phi(S_k)S_k - \delta$, and $g^* = \gamma g_n + (1-\gamma) g_m$. By standard arguments, the unique equilibrium path has $\lim_{t\to\infty} \dot{K}(t)/K(t) = \lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{Y}(t)/Y(t) = g^*$.

Proof of Proposition 5: First, exploiting the definition of k given by (24), and using (2), (8), and (17), we obtain:

$$\begin{aligned}
\frac{\dot{k}}{k} &= \frac{\dot{K}}{K} + \frac{\dot{M}}{M} - \frac{\dot{N}}{N}, \\
&= \frac{\left[\gamma(NL)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(MK)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - C}{K} + \frac{1-\beta}{\beta}\left(b_k\phi(S_k)S_k - b_l\phi(S_l)S_l\right),
\end{aligned}$$

$$= M\left[\gamma k^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}-c \tag{A3}$$
$$+\frac{1-\beta}{\beta}\left(b_k\phi\left(S_k\right)S_k-b_l\phi\left(S-S_k\right)\left(S-S_k\right)\right).$$

where recall that $c\equiv C/K$. Linearizing around the BGP, we have the first linear differential equation of the four-equation system:

$$\frac{\dot{k}}{k}=a_{ks}\left(S_k-S_k^*\right)+a_{km}\left(M-M^*\right)+a_{kc}\left(c-c^*\right)+a_{kk}\left(k-k^*\right). \tag{A4}$$

with $a_{ks}>0$, $a_{km}=\left[\gamma\left(k^*\right)^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}>0$, $a_{kc}=-1<0$ and $a_{kk}<0$. Similarly, using the definition $c\equiv C/K$, the Euler equation (8), and equations (10) and (25):

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{\dot{C}}{C}+\frac{\dot{K}}{K} \\ &= \frac{1}{\theta}\left(\beta(1-\gamma)M\left[\gamma k^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{1}{\varepsilon-1}}-\rho\right) \\ &\quad -M\left[\gamma k^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}+c\end{aligned} \tag{A5}$$

Linearizing around the BGP gives the second linear differential equation:

$$\frac{\dot{c}}{c}=a_{cm}\left(M-M^*\right)+a_{cc}\left(c-c^*\right)+a_{ck}\left(k-k^*\right), \tag{A6}$$

with $a_{cc}=1>0$, and

$$a_{cm}=\frac{1}{\theta}\beta(1-\gamma)\left[\gamma\left(k^*\right)^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{1}{\varepsilon-1}}-\left[\gamma\left(k^*\right)^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}=a_{cm}'-a_{km},$$

where $a_{km}>0$ is defined above, and $a_{cm}'>0$.

In addition, a_{ck} is the derivative of $\left\{\left[\gamma k^{-\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\right]^{\frac{1}{\varepsilon-1}}\left[\frac{1}{\theta}\beta(1-\gamma)-\gamma k^{-\frac{\varepsilon-1}{\varepsilon}}-(1-\gamma)\right]\right\}$, where both terms have negative derivatives, hence $a_{ck}<0$.

Next, recall that (8) gives:

$$\frac{\dot{M}}{M}=\frac{1-\beta}{\beta}\left[b_k\phi\left(S_k\right)S_k-\delta\right].$$

Linearizing this relationship, we obtain the third differential equation:

$$\frac{\dot{M}}{M}=a_{ms}\left(S_k-S_k^*\right). \tag{A7}$$

with $a_{ms}>0$.

Finally, recall that in BGP, we have:

$$b_l \phi (S - S_k) nV_l = b_k \phi (S_k) mV_k$$

Define $\phi_l \equiv \phi (S - S_k)$ and $\phi_k \equiv \phi (S_k)$ to simplify the notation, and differentiate the above equation to obtain:

$$-\frac{\dot{S}_k}{S_k} e_k + \frac{\dot{n}}{n} + \frac{\dot{V}_l}{V_l} = \frac{\dot{S}_k}{S_k} e_l + \frac{\dot{m}}{m} + \frac{\dot{V}_k}{V_k}. \tag{A8}$$

where $e_k \equiv \phi' (S_k) S_k / \phi (S_k) < 0$ and $e_l \equiv \phi' (S - S_k) S_k / \phi (S - S_k) < 0$. Next differentiating (20), we have

$$rV_l - \dot{V}_l = \pi_l - \delta V_l \text{ and } rV_k - \dot{V}_k = \pi_k - \delta V_k, \tag{A9}$$

with π_l and π_k given by (21). Therefore,

$$\begin{aligned}
\frac{\dot{S}_k}{S_k} &= -[e_k + e_l]^{-1} \left[\frac{\dot{m}}{m} - \frac{\dot{n}}{n} + \frac{1-\beta}{\beta} \left(\frac{wL}{nV_l} - \frac{rK}{mV_k} \right) \right] \\
&\simeq -[e_k + e_l]^{-1} \left[\phi_k S_k - \phi_l (S - S_k) + \frac{1-\beta}{\beta \zeta} \phi_k (b^* - \sigma_K) \right],
\end{aligned}$$

where the second step uses the fact that we are in the neighborhood of the BGP, and ζ is a constant proportional to the BGP ω_S / wL ratio given by,
$\zeta \equiv \phi_l [\rho + \theta g^* + \delta - (1 - 2\beta) g^* / (1 - \beta)]^{-1}$. Also recall that $[e_k + e_l]^{-1} < 0$. Therefore, our fourth differential equation is:

$$\frac{\dot{S}_k}{S_k} = a_{ss} (S_k - S_k^*) + a_{sk} (k - k^*), \tag{A10}$$

with $a_{ss} > 0$ and $a_{sk} > 0$.

Expressing these four equations together, we have

$$\begin{pmatrix} \dot{S}_k / S_k \\ \dot{M} / M \\ \dot{c} / c \\ \dot{k} / k \end{pmatrix} \simeq \begin{pmatrix} a_{ss} & 0 & 0 & a_{sk} \\ a_{ms} & 0 & 0 & 0 \\ 0 & a_{cm} & a_{cc} & a_{ck} \\ a_{ks} & a_{km} & a_{kc} & a_{kk} \end{pmatrix} \cdot \begin{pmatrix} S_k \\ M \\ c \\ k \end{pmatrix} \tag{A11}$$

Since M and k are state variables and S_k and c are control variables, saddle-path stability requires the determinant in (A11) to have two positive and two negative eigenvalues. To find these eigenvalues, write:

$$\det \begin{pmatrix} a_{ss} - \lambda & 0 & 0 & a_{sk} \\ a_{ms} & -\lambda & 0 & 0 \\ 0 & a_{cm} & a_{cc} - \lambda & a_{ck} \\ a_{ks} & a_{km} & a_{kc} & a_{kk} - \lambda \end{pmatrix} = 0,$$

which gives the following forth-order polynomial:

$$\begin{aligned} \lambda^4 - [a_{ss} + a_{cc} + a_{kk}]\,\lambda^3 + [a_{ss}a_{kk} - a_{kc}a_{ck} - a_{ks}a_{sk}]\,\lambda^2 & \\ - [a_{ms}a_{km}a_{sk} - a_{ss}a_{kc}a_{ck} + a_{ss}a_{cc}a_{kk}]\,\lambda & = 0. \\ + a_{ms} \cdot a_{sk} \cdot (a_{cc} \cdot a_{km} - a_{kc} \cdot a_{cm}) & \end{aligned}$$

The four roots, λ_1, λ_2, λ_3 and λ_4 give the eigenvalues. By standard arguments we have that these four roots satisfy:

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = a_{ms} \cdot a_{sk} \cdot (a_{km} \cdot a_{cc} - a_{kc} \cdot a_{cm})$$

Now using the fact that $a_{cc} = 1$, $a_{kc} = -1$, and $a_{cm} = a'_{cm} - a_{km}$, we have

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = a_{ms} \cdot a_{sk} \cdot a'_{cm}.$$

Since $a_{cm} > 0$. $a_{sk} > 0$, and $a'_{cm} > 0$, we have that $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 > 0$. So we must have either four positive roots, or four negative roots, or two positive and two negative roots. Next also note that

$$\lambda_1 \cdot \lambda_2 + \lambda_1 \cdot \lambda_3 + \lambda_1 \cdot \lambda_4 + \lambda_2 \cdot \lambda_3 + \lambda_2 \cdot \lambda_4 + \lambda_3 \cdot \lambda_4 = \underset{(+)\cdot(-)}{a_{ss} \cdot a_{kk}} \quad \underset{-(-)\cdot(-)}{-a_{kc} \cdot a_{ck}} \quad \underset{-(+)\cdot(+)}{-a_{ks} \cdot a_{sk}} < 0,$$

which means that we can have neither four positive roots nor four negative roots, establishing that there must be some positive roots, and some negative roots. Therefore, there must be two positive and two negative roots, so the system is locally saddle-path stable.

With $\varepsilon > 1$, we have $a_{sk} < 0$, thus $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = -a_{ms} \cdot a_{sk} \cdot a'_{cm} < 0$, and we have three positive and one negative eigenvalues, and the BGP equilibrium is not locally stable.

Proof of Lemma 1: I will prove this lemma in two steps. First, I will show that if $b_l\phi_l^* n(t) V_l(t) \leq b_k\phi_k^* m(t) V_k(t)$ where $\phi_l^* \equiv \phi(S - S_k^*)$ and $\phi_k^* \equiv \phi(S_k^*)$, then $b_l\phi_l^* n(t') V_l(t') \leq b_k\phi_k^* m(t') V_k(t')$ for all $t' \geq t$. Next I will show that transitional dynamics can be represented by (32), where $\psi(M^*) = 0$, and $\psi(M) \lesseqqgtr 0$ for all $M \gtreqqless M^*$ for $\varepsilon < 1$ and $\psi(M) \gtreqqless 0$ for all $M \lesseqqgtr M^*$ for $\varepsilon > 1$.

Recall that $k(t) = G(M(t))$ at all t, with $G(\cdot)$ strictly increasing. Next recall that $b_l\phi_l^* n(t) V_l(t) = b_k\phi_k^* m(t) V_k(t)$, when $\sigma_K = b^*$, or when $M = M^*$ and $k = k^*$. Also in this case, $\dot{M} = 0$ and $\dot{N}/N = g^*$. Now I will show that these properties imply that, for $\varepsilon < 1$, $\sigma_K(t) > b^*$ if and only if $b_l\phi_l^* n(t) V_l(t) \leq b_k\phi_k^* m(t) V_k(t)$. Moreover, these imply $b_l\phi_l^* n(t') V_l(t') \leq b_k\phi_k^* m(t') V_k(t')$ for all $t' \geq t$.

Suppose that $\sigma_K(t) > b^*$, and to derive a contradiction, suppose also that $b_l\phi_l^* n(t) V_l(t) > b_k\phi_k^* m(t) V_k(t)$. First if this is the case, we will have $b_l\phi(S - S_k(t)) n(t) V_l(t) = b_k\phi(S_k(t)) m(t) V_k(t)$ for some $S_k(t) < S_k^*$ (or $b_l\phi(S) n(t) V_l(t) > b_k\phi(0) m(t) V_k(t)$), and thus $\dot{m} < 0$. Second, from (A9), we can only have $\sigma_K(t) > b^*$ and $b_l\phi_l^* n(t) V_l(t) > b_k\phi_k^* m(t) V_k(t)$ if $b_l\phi_l^* n(t) \dot{V}_l(t) > b_k\phi_k^* m(t) \dot{V}_k(t)$. Third, note that for $\Delta t \to 0$, we have

$$\begin{array}{ccccccc} b_l\phi_l^* n(t+\Delta t) V_l(t+\Delta t) & = & b_l\phi_l^* n(t) V_l(t) & + & b_l\phi_l^* \dot{n}(t) V_l(t) & + & b_l\phi_l^* n(t) \dot{V}_l(t) \\ > & & > & & > & & > \\ b_k\phi_k^* m(t+\Delta t) V_k(t+\Delta t) & = & b_k\phi_k^* m(t) V_k(t) & & b_k\phi_k^* \dot{m}(t) V_k(t) & & b_k\phi_k^* m(t) \dot{V}_k(t) \end{array}$$

Now recall that $b_l\phi_l^* n(t) \dot{V}_l(t) > b_k\phi_k^* m(t) \dot{V}_k(t)$ implies $\dot{m} < 0$, which in turn, from $k(t) = G(M(t))$, implies $\dot{k}(t) < 0$. So $\dot{\sigma}_K(t) > 0$. Therefore, we must have $b_l\phi_l^* n(t') V_l(t') > b_k\phi_k^* m(t') V_k(t')$ at all $t' > t$, and consequently $\dot{M}(t)/M(t) < 0$, and therefore, $\dot{k}(t)/k(t) < 0$, and $\lim_{t\to\infty} k(t) = 0$. But Lemma A1 implies that in this case $\lim_{t\to\infty} m(t) V_k(t)/n(t) V_l(t) = \infty$, giving a contradiction. Therefore, whenever $\sigma_K(t) > b^*$, we must have $b_l\phi_l^* n(t) V_l(t) \leq b_k\phi_k^* m(t) V_k(t)$. But whenever $\sigma_K(t) > b^*$, from $k(t) = G(M(t))$, we have $k(t) < k^*$ and therefore $M(t) < M^*$. This establishes that the law of motion of the economy can be represented simply by $\dot{M}(t)/M(t) = \psi(M(t))$. In addition, recall that $\dot{M}(t) = 0$ requires $\sigma_K(t) = b^*$, therefore $M(t) = M^*$ and $k(t) = k^*$, establishing that $\psi(M^*) = 0$. Finally, it has already been shown that in the case with $\varepsilon < 1$, when $\sigma_K(t) > b^*$, we have $\dot{M}(t)/M(t) > 0$. Since $\sigma_K(t) > b^*$ is equivalent to $M(t) < M^*$, we have that $M(t) < M^* \Leftrightarrow \dot{M}(t)/M(t) > 0$. And similarly, when $\sigma_K(t) < b^*$, $M(t) > M^*$, we have $\dot{M}(t)/M(t) < 0$, proving that $\psi(M) \lesseqqgtr 0$ for all $M \gtreqqless M^*$. The arguments for $\sigma_K(t) < b^*$ and for the case where $\varepsilon > 1$ are analogous.

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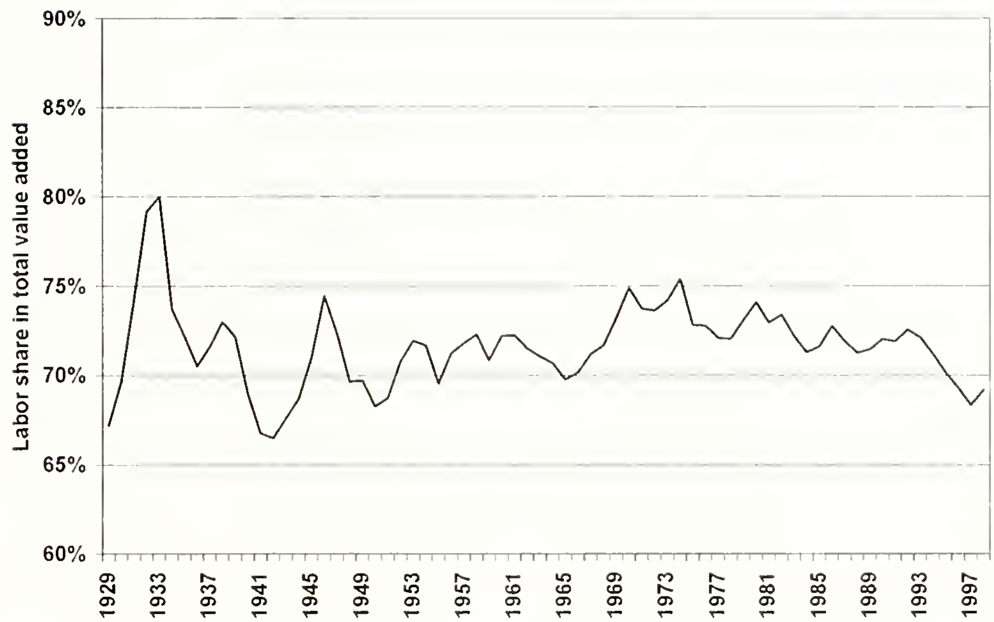

Figure 1: Labor share in total value added in the U.S. corporate sector from Piketty and Saez (2001). Source: National Accounts, NIPA Table 1,16.

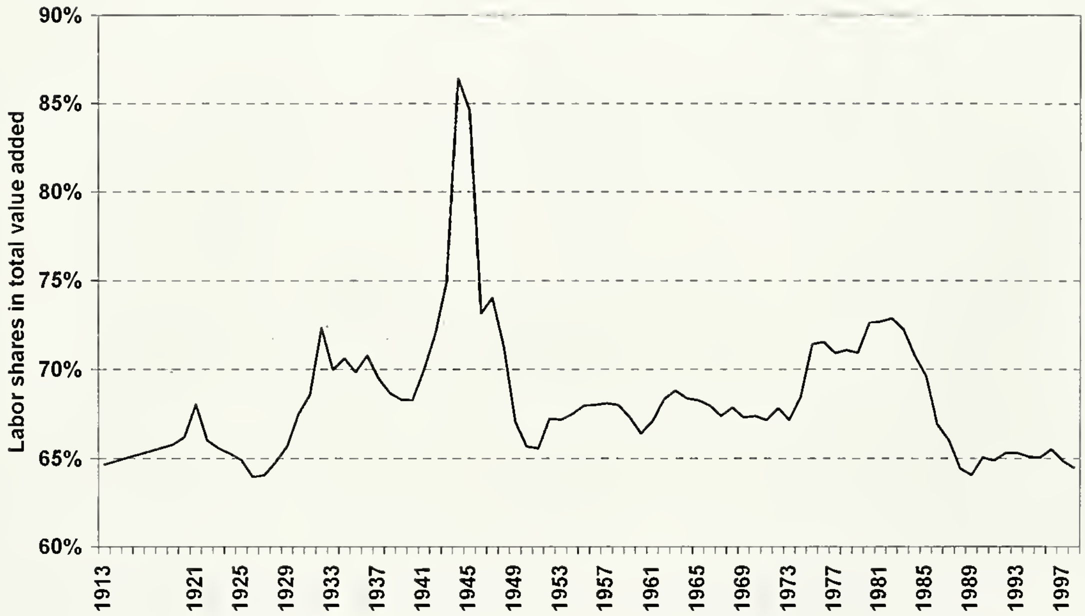

Figure 2: Labor share in total value added in the French corporate sector. From Piketty (2001) based on French National Accounts.

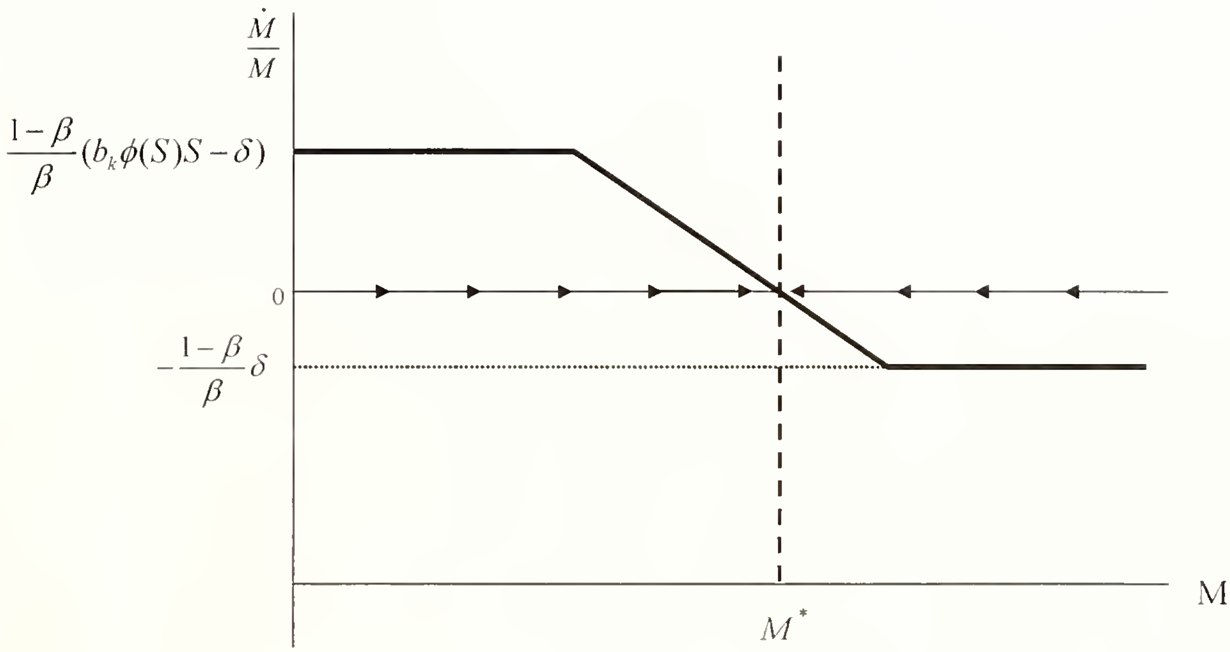

Figure 3: Transitional dynamics with risk neutrality and $\varepsilon < 1$.

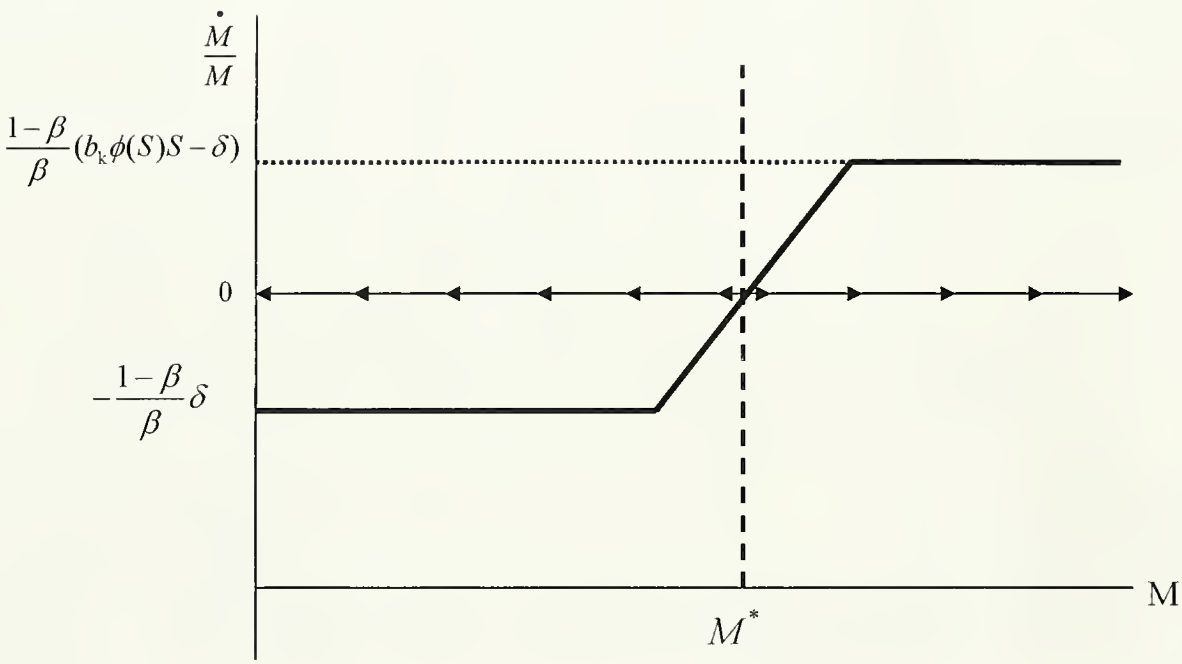

Figure 4: Transitional dynamics with risk neutrality and $\varepsilon > 1$.