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OPTIMAL POLICY INTERVENTION IN THE PRESENCE OF BRAIN DRAIN: EDUCATIONAL SUBSIDY AND TAX ON EMIGRANTS' INCOME*

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Apropos of the brain drain from the developing countries, Bhagwati (1972) has proposed that an income tax (supplementary to the immigration-country income tax) be levied on those who emigrate and that the revenue so collected be transmitted to the developing country, or developing countries en bloc, for developmental spending.¹

The welfare implications of such a tax were analyzed in an earlier paper of ours (1974), in the context of a model of the country of emigration, characterized by sticky wages and Harris-Todaro (1971) type of unemployment. In subsequent papers, the tax has been analyzed also in the context of modified models, still incorporating the unemployment phenomena, by Rodriguez (1975) and McCulloch-Yellen (1975).² Essentially, these models analyze the impact of the income tax on emigrants' incomes as arising primarily through the reduced differential between foreign (net-of-tax) and domestic salaries for the emigrant class of labour: this reduction in differential, in turn, reduces the expected and possibly the actual salaries in the home countries and thus generates consequences for education, unemployment, income and income distribution (among the educated, uneducated, employed and unemployed).

The present paper takes an altogether different theoretical approach to the analysis of the income tax on emigrants. We employ rather the frame-

¹This tax proposal has been explored from the viewpoint of its economic rationale, its revenue implications, and its legal (constitutional, tax and human rights) implications in Bhagwati and Partington (1976). The present paper can be seen as developing a yet further rationale for such a tax (in the version where the tax proceeds accrue to the country of emigration).

²For a review of these and other brain drain models in recent theoretical writings, see Bhagwati and Rodriguez (1976).
work of income taxation developed by Atkinson (1973) in his wellknown paper on the optimal linear income tax. There, the choice for individuals is the length of education, which affects their productivities after graduation so that education increases the earnings of individuals while simultaneously postponing the realization of these earnings. Atkinson studies, in the context of this model, the conflict between efficiency and equity: the income tax would redistribute income but also distort the educational choice and hence reduce efficiency.  

In a subsequent contribution, Hamada (1974) demonstrated that if an additional policy instrument, i.e. an educational subsidy, is introduced, the conflict between equity and efficiency in the Atkinson model can be virtually resolved.

In the present paper, we modify this model to allow emigration and the earning of incomes abroad. The choice for the individual is again the length of education which now affects also, via his earnings, the decision to migrate.

Section I sets out the basic model and the policy instruments considered: the (Atkinson) linear income tax on domestic earnings, the (Hamada) educational subsidy, the (Bhagwati) linear income tax on foreign earnings, and a lump-sum tax at the time of emigration.

Section II, using the simplification that there is only domestic education, then demonstrates that the use of the (Bhagwati) income tax on foreign earnings, in conjunction with the (Hamada) educational subsidy and the (Atkinson)

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3Note that lump-sum taxation to redistribute income is not being permitted; if it were, the conflict between efficiency and equity would naturally disappear! The conflict between efficiency and equity in the presence of factor mobility has also been discussed by Cooper (1973, page 54).
income tax on domestic earnings, enables again the virtual resolution of the conflict between efficiency and equity. Section III then analyzes the precise manner in which the absence of the income tax on foreign earnings would entail a sub-optimal situation for the country. Section IV then extends the analysis briefly to the case where education can be acquired abroad as well: a case that is certainly important in practice. We will not however explicitly consider the case where the education is foreign but the income is earned domestically (i.e. the case of the return of the foreign-trained native) or the case of to-and-fro migration (discussed in Bhagwati (1976)) where the income is earned both at home and abroad sequentially over the lifetime of the person.

I: The Basic Framework

Following the models of optimal income taxation, we shall assume that differences exist in the innate ability of the individuals of the home country. Let \( n \) be the index of innate ability and \( S \) be the length of education that an individual undertakes. The innate ability in the initial population is distributed by the density function \( p(n) \), which is normalized such that:

\[
\int_{0}^{\bar{n}} p(n) dn = 1
\]

where \( \bar{n} \) indicates the upper bound of the index of innate ability. Let the domestic earning of an individual be a function of his ability and his length of

---

4 The availability of the tax revenue for the country of emigration is thus absolutely critical in this analysis whereas the role of the tax on foreign earnings in the Bhagwati-Hamada (1973), Rodriguez (1975) and McCulloch-Yellen (1975) analyses was merely to reduce net earnings abroad and the question of where the tax revenue was disbursed was ignored in the analysis.

5 On this empirical fact, see Reubens in Bhagwati (1976).
education such that \( f(n, S) \) indicates the resulting earnings. Let \( f^*(n, S) \) be the foreign earnings of an individual who has received education at home. Similarly, let \( g^*(n, S^*) \) be the foreign earnings of an individual who has received education in the country of immigration. Naturally, all the earning functions are increasing functions of \( n \) and \( S \), while the marginal productivity of \( S \) is decreasing, so that:

\[
\begin{align*}
&f_n, f^*_n, g^*_n > 0, \\
&f_S, f^*_S, g^*_S > 0,
\end{align*}
\]

and

\[
\begin{align*}
&f_{SS}, f^*_{SS}, g^*_{SS} < 0,
\end{align*}
\]

with subscripts denoting the partial derivatives with respect to the subcripted variables. We will further assume, for simplicity, that the length of working period after education is identical at home and abroad, and designated as \( R \).

The ratio of emigrants in the population is assumed to depend on the difference between the discounted income stream at home and that in the country of immigration.\(^6\) Let \( m(I) \) be this ratio of emigrants in the population and \( I \) be the difference between the foreign (accumulated) income net of trans-

---

\(^6\) George Psacharopoulos has pointed out to us that, while it is plausible to write earnings as a function of innate ability, the econometric attempts at isolating this relationship have not been particularly successful so far.

\(^6\) In order that the individual maximization over time coincide with the socially optimal choice, we need to use the right discount rate. As discussed in Hamada (1973), the right discount rate should be the biological rate of interest, i.e., the growth rate of the population (plus the positive rate of labour-augmenting technical progress). We shall therefore assume that the government of the home country keeps the rate of interest equal to the biological rate of interest.
portation and adjustment costs and the domestic (accumulated) income. We will assume that:

\[ m(\Gamma) = 0 \quad \text{if} \quad \Gamma \leq 0 \]

and

\[ m(\Gamma) \geq 0, \quad m'(\Gamma) > 0 \quad \text{if} \quad \Gamma > 0. \]

It is also natural to require that:\(^7\)

\[ 1 \geq m'(\Gamma) \geq 0 \]

and

\[ \lim_{\Gamma \to \infty} m(\Gamma) = 1. \]

Finally, while it makes no difference to the analysis of the optimal policy mix in Sections II and IV, we will consider in Section III (where sub-optimal situations are discussed) that the opportunity to emigrate and earn foreign incomes is open only to those whose innate ability enables them to earn more than the average income. This assumption enables us to approximate the empirical reality where the highly skilled, professional manpower have far more easy access than the unskilled to the developed countries of immigration.

Having stated the essential characteristics of the analytical framework to be deployed in this paper, we now list and describe the policy instruments which will be considered in the analysis.

---

\(^7\) As a special case, we will consider: \[ m(\Gamma) = 1 \quad \text{if} \quad \Gamma > 0 \]
\[ = 0 \quad \text{if} \quad \Gamma \leq 0. \]
(1) **Linear Income Tax on Domestic Income**: Following Atkinson, we write the after-tax income in the form \([a + \beta f]\) where \(a > 0\), \(1 > \beta \geq 0\) and \((1 - \beta)\) is the marginal rate of taxation and \(a\) constitutes the uniform lump-sum payment to each individual.\(^8\)

(2) **Subsidy to Education**: Following Hamada, we assume that the subsidy \((G)\) is directly given to individuals enrolled in the educational process.\(^9\)

(3) **Linear Income Tax on Foreign Income**: Following Bhagwati, we consider also a linear income tax on emigrants so that their after-tax income can be written in the form \([a^* + \beta^* f^*]\) where the starred subscripts refer to emigrants. In principle, \(a\) and \(\beta\) need not equal \(a^*\) and \(\beta^*\) respectively.\(^9a\)

(4) **Lump-sum Emigration Tax at Time of Departure of Emigrants**: In the analysis below, a lump-sum tax will be considered at times as a supplement to the tax on foreign incomes and will be denoted by \(T\).

Next, we must choose our welfare criterion. First, we will adopt the so-called "nationalistic" criterion and evaluate the impact of our policies on the average income, the average utility or the minimum income of those left behind by the emigrants. On the other hand, we will also consider the average income, average utility and minimum income of the "augmented" set including the emigrants: a viewpoint that may be described as "partially internationalist" and which, in any event, seems to be more consistent with the view that skilled emigrants (who constitute the brain drain) today retain their affiliation to the country of

\(^8\)Since \(f\) is non-negative, \(a\) can at the same time be interpreted as the minimum income in the home country and hence would constitute the welfare objective according to the Rawls (1971) criterion, discussed below.

\(^9\)As shown by Hamada (1974), this subsidy enables the achievement of the coexistence of Pareto efficiency with "almost" perfect equality.

\(^9a\)The linearity of the income tax on foreign income is a restriction that is used here but is not a necessary feature of Bhagwati's tax proposal.
emigration.10

II. Optimal Policy Mix With Domestic Education

We now analyze the optimal policy mix in our model, so that the linear income tax on foreign incomes and the educational subsidy are both available as policy instruments. We will assume, here and in the next section, that education is available or undertaken only at home. The two basic decisions faced by the individual then are (i) the choice of the optimal length of education and (ii) the choice of earning domestic or foreign income.

(i) Choice of the Length of Education: First, consider the choice of the length of education. Concentrating on domestic earnings, an individual will be maximizing his lifetime income $I_d$ with respect to $S$. Now,

$$I_d = \int_{S}^{S+R} (\alpha + \beta f(n,S)) e^{-it} dt + G \int_{0}^{S} e^{-it} dt$$

$$= A(R)\{\alpha + \beta f\}e^{-iS} + G \int_{0}^{S} e^{-it} dt$$

(1)

where $A(R) \equiv \frac{1 - e^{-iR}}{i}, i$ is the rate of discount, and $G$ is the educational subsidy. Maximizing $I_d$ with respect to $S$, we then obtain the first-order condition:

$$A\{-i (\alpha + \beta f) + \beta f_s\} + G = 0$$

(2)

10 This viewpoint has been argued in Section I of Bhagwati and Rodriguez (1975) and in fact would appear to make also the brain drain tax, on emigrants' incomes in the country of immigration, a socially and politically more acceptable proposal. Cf. also Chapters 1 and 8 in Bhagwati and Partington (1976).
The second-order condition is satisfied with the assumption that $f_{SS} < 0$.

The socially efficient choice of $S$ however is given by:

$$A(-if + f') = 0$$  \hspace{1cm} (3)

Thus, in order to make the individual choice of $S$ consistent with the socially efficient choice, we should have a tax-subsidy scheme such that:

$$C = A\alpha = \alpha(1 - e^{-IR})$$  \hspace{1cm} (4)

As explored by Hamada (1974), an economy without brain drain will then "almost" attain the first-best solution, with efficient educational length and equality, by reducing $\beta$ close to zero while keeping (4) intact.

On the other hand, if emigration to earn foreign incomes is introduced, a potential emigrant will maximize the following if he chooses to go abroad after the end of education:

$$I_e = \int_S^{S+R}(\alpha* + \beta*f*(n,S)) e^{-it} \, dt - Ce^{-iS} + G\int_0^S e^{-it} \, dt$$  \hspace{1cm} (5)

where $I_e$ is the life-time income with foreign earnings and $C$ is the once-and-for-all cost of emigration. This gives the first-order condition:

$$A(-i (\alpha* + \beta*f*) + \beta*f*) + iC + G = 0$$  \hspace{1cm} (6)

On the other hand, the socially optimal choice of $S$ is now given by:

$$A(-i f* + f') + iC = 0$$  \hspace{1cm} (7)

To reconcile the individual and socially efficient choice of $S$, we then obtain the following necessary condition, by multiplying (7) by $\beta*$ and subtracting it from (6):

$$G = \alpha*(1 - e^{-IR}) - (1 - \beta*)iC$$  \hspace{1cm} (8)
In order that our tax-subsidy scheme achieve the efficient choice of
the length of education for both emigrants as well as non-emigrants, (4) and
(8) should be simultaneously satisfied. Therefore, we must have:

\[ \alpha(1 - e^{-i\theta}) = \alpha^*(1 - e^{-i\theta}) - (1 - \beta^*)iC \]
or

\[ \alpha^* = \alpha + \frac{i}{1 - e^{-i\theta}} \frac{(1 - \beta^*)C}{\alpha + \frac{(1 - \beta^*)C}{A(R)}} \]  

(9)

If the adjustment cost C is zero, therefore, the same intercept must be chosen
both for the domestic linear income tax and for the linear income tax on
foreign income.

(ii) Decision to Migrate: There remains now the problem of the choice
between staying at home and going abroad. One has to choose the tax scheme
therefore in such a way as not to disturb the proper choice in this regard.
In fact, we shall prove that the linear income tax on foreign income, combined
with educational subsidy and the domestic income tax satisfies this requirement
provided that \( \beta^* \) is set equal to \( \beta \).

The total condition for the socially efficient choice for deciding to
emigrate is:

\[
\left[ \int_{\bar{S}}^{S+R} f^* e^{-it} dt - Ce^{-i\bar{S}} \right] - \int_{\bar{S}}^{S+R} f e^{-it} dt > 0
\]  

(10)

where both \( S \) and \( \bar{S} \) are chosen to satisfy the marginal condition for securing
the efficiency of the length of education.

On the other hand, individuals receiving educational subsidy \( G \) and facing
the domestic tax schedule \( (\alpha + \beta f) \) and the foreign tax schedule \( (\alpha^* + \beta^* f^*) \)
which satisfy (9), will emigrate or not, depending on their evaluation of the
following:

\[ \Gamma = \left[ \int_{\tilde{S}}^{S+R} (\alpha + \beta f) e^{-i\tau} dt - Ce^{-i\tilde{S}} + G \int_{0}^{\tilde{S}} e^{-i\tau} dt \right] \]

\[ - \left[ \int_{\tilde{S}}^{S+R} (\alpha + \beta f) e^{-i\tau} dt + G \int_{0}^{\tilde{S}} e^{-i\tau} dt \right] \]

which, in view of (9), equals:

\[ \alpha \left[ \int_{\tilde{S}}^{S+R} e^{-i\tau} dt - \int_{S}^{S+R} e^{-i\tau} dt \right] \]

\[ + \left[ \int_{\tilde{S}}^{S+R} \frac{(1 - \beta)}{A} Ce^{-i\tau} dt - (1 - \beta) Ce^{-i\tilde{S}} \right] \]

\[ + G \left[ \int_{0}^{\tilde{S}} e^{-i\tau} dt - \int_{0}^{S} e^{-i\tau} dt \right] \]

\[ + \beta \left[ \int_{\tilde{S}}^{S+R} f e^{-i\tau} dt - Ce^{-i\tilde{S}} - \beta \int_{S}^{S+R} f e^{-i\tau} dt \right] \]

It is easy to see that the second bracket vanishes. Moreover, if we substitute here \( G = \alpha(1 - e^{-i\tau}) \), it can be seen that:

\[ \alpha \left[ \int_{\tilde{S}}^{S+R} e^{-i\tau} dt - \int_{S}^{S+R} e^{-i\tau} dt \right] + G \left[ \int_{0}^{\tilde{S}} e^{-i\tau} dt - \int_{0}^{S} e^{-i\tau} dt \right] = 0 \]

Therefore the criterion for the individual decision on emigration reduces to:

\[ \Gamma = \beta \left[ \int_{\tilde{S}}^{S+R} f e^{-i\tau} dt - Ce^{-i\tilde{S}} \right] - \beta \int_{S}^{S+R} f e^{-i\tau} dt \quad (11) \]

Therefore, if and only if \( \beta^* \) is set equal to \( \beta \), permitting the fulfillment of (10) and (11) simultaneously, will the criterion for the optimal decision for emigration be satisfied.

The combination of the three policies—the Atkinson tax on domestic income, the Hamada educational subsidy and the Bhagwati tax on foreign income—at appropriate values will then enable efficient choices to be made regarding the
length of education and the decision to migrate. At the same time, as in
Hamada's (1974) analysis of the economy without the brain drain, the setting
of $\beta (= \beta^*)$ virtually equal to zero would also reconcile efficiency with al-
most perfect equality. As before, $\beta (= \beta^*)$ cannot be set equal to zero, or
everything would collapse; but it can be made virtually equal to zero, imply-
ing that the marginal rate of taxation is virtually close to unity and that
"as if" equality of incomes has been reached among all members (including the
emigrants) of the population of the country of emigration.

Recalling then that while $\beta = \beta^*$, we have $\alpha = \alpha^*$ only if migration
cost is ignored, for socially efficient choices, we would then have to give
emigrants a lump-sum subsidy at the point of departure, equal to $\left(\frac{1 - \beta^*}{A}\right) C$
--see (9)-- if the migration cost $C$ is not negligible. Thus, we can state
the following:

Proposition I:

By a combination of linear income taxation of domestic and foreign incomes,
and educational subsidy, the home country can achieve Pareto efficiency and "al-
most" total equality. The marginal rate of taxation for the two income taxes
should be identical, while the intercept for the foreign taxation should be dif-
ferent from that for domestic income taxation by virtue of the migration cost.

It is interesting that, in this optimal policy mix, it does not matter
whether the welfare criterion relates to per-capita income, average utility or
Rawlsian minimum income. For, once $\beta$ (or $\beta^*$) is taken close enough to zero
in order to attain the efficient solution with "almost" total equity, the in-
come of everybody, at home and abroad, will be virtually equalized. Therefore
the maximization of per-capita income implies at the same time the utilitarian
and the Rawlsian solution. Identically, the distinction between the welfare
of those remaining behind and the welfare of these plus the emigrants will also cease to be significant.

Fixed Life-Span Prior to Retirement:

We now consider briefly a slightly modified version of the model where we assume a fixed total lifespan before retirement, which we denote by $\bar{R}$. Then the optimal tax-subsidy scheme in the case without emigration turns out simply, as shown by Hamada (1974), to be:

$$G = \alpha$$

With emigration possible, however, emigrants are maximizing the following with respect to $\bar{S}$:

$$\int_{\bar{S}}^{\bar{R}} (\alpha + \beta*\bar{f}*) e^{-it} dt - Ce^{-i\bar{S}} + G \int_{0}^{\bar{S}} e^{-it} dt$$

The first-order condition for the choice of their length of education is therefore:

$$-(\alpha + \beta*\bar{f}*) e^{-i\bar{S}} + \beta*\bar{f} \int_{\bar{S}}^{\bar{R}} e^{-it} dt + iC e^{-i\bar{S}} + Ce^{-i\bar{S}} = 0 \quad (12)$$

On the other hand, the socially optimal length of education requires that:

$$-f* e^{-i\bar{S}} + f \int_{\bar{S}}^{\bar{R}} e^{-it} dt + iC e^{-i\bar{S}} = 0 \quad (13)$$

Suppose now that $G$ is set equal to $\alpha$, in order to guarantee the socially optimal choice for the length of education for the non-emigrants. Then in order that (13) gives an identical value of $\bar{S}$ to that derived by (12), we must have:

$$G = \alpha* - (1 - \beta*) iC$$

or

$$\alpha* = \alpha + (1 - \beta*) iC \quad (14)$$

Thus, if the migration cost is negligible, we again must equate $\alpha$ to $\alpha*$. 
However, it can be shown that the total condition for the other decision, i.e. whether or not to emigrate, is no longer satisfied by choosing $\beta^*$ equal to $\beta$. Emigrants will now evaluate the following expression:

$$
\Gamma = \left[ \int_S^R (\alpha + \beta f^*) e^{-it} dt - Ce^{-i\delta} + G \int_0^S e^{-it} dt \right] - \left[ \int_S^R (\alpha + \beta f) e^{-it} dt + G \int_0^S e^{-it} dt \right]
$$

Considering $G = \alpha$ and (13), we obtain the relationship:

$$
\left[ \int_S^R \alpha e^{-it} dt + G \int_0^S e^{-it} dt \right] - \int_S^R (1 - \beta^*)iCe^{-it} dt = \int_S^R \alpha e^{-it} dt + G \int_0^S e^{-it} dt
$$

Therefore, we derive:

$$
\Gamma = \beta^* \left[ \int_S^R f^* e^{-it} dt - Ce^{-i\delta} \right] - \beta \int_S^R f e^{-it} dt - (1 - \beta^*)Ce^{-i\bar{R}}
$$

The criterion $\Gamma$ does not therefore coincide with the socially optimal criterion

$$
\int_S^R f^* e^{-it} dt - Ce^{-i\delta} - \int_S^R f e^{-it} dt,
$$

even with $\beta^* = \beta$, by the amount of $-(1 - \beta^*)Ce^{-i\bar{R}}$. One possible way of resolving this difficulty is to give a lump-sum subsidy of

$$
(1 - \beta^*)Ce^{-i\bar{R}} \cdot e^{-i\delta} = (1 - \beta^*)Ce^{-i(\bar{R} - \delta)}
$$

at the time of the departure to emigrants.

III: Welfare Analysis in the Absence of the Tax on Foreign Incomes

To gain further insight into the optimality properties of the tax on
foreign incomes, it is necessary now to investigate the effects of the inability to tax foreign incomes in our model. Here, we have to choose between assuming that only the linear income tax on domestic incomes operates and assuming instead that the educational subsidy is also being utilised. The following analysis is predicated on the latter assumption, so that we will assume that the home country is using two policy instruments: the (Atkinson) income tax on domestic earnings and the (Hamada) educational subsidy, but that the (Bhagwati) tax on foreign earnings is not available as a policy instrument. We will further conduct the analysis on the assumption that the educational subsidy is being used at the level required to render the choice of the length of education socially optimal. (Needless to say, the analysis can be appropriately modified if the reader should wish to consider other assumptions, e.g. that only the linear income tax on domestic incomes is being used.)

Note first that the use of the educational subsidy, to neutralise the distorting effect of the domestic income tax on the length of education undertaken by non-emigrants, leads to two other types of distortion: distorted choice of length of education by emigrants, and distortion in the decision to migrate. This is seen as follows.

The choice of the length of education by migrants, if it is to be socially efficient, must satisfy the marginal condition:

\[ A \{ -if^* + f^*_{S*} \} + IC = 0 \] (7)

But in the presence of the educational subsidy \( G \) that optimizes the choice of length of education for non-emigrants, the actual decision to migrate will be made so as to satisfy the condition:

\[ A \{ -if^* + f^*_{S} \} + IC + G = 0 \] (15)
By comparing (15) and (7), we can see therefore that the presence of educational subsidy without the tax on foreign incomes will prolong the period of education for emigrants beyond the socially optimal level.

This distortion is, of course, curable provided that emigrants are taxed so as to offset fully the accumulated subsidy at the time of departure. That is, if a lump-sum tax equalling

$$\int_0^S Ge^{-it} dt$$

is imposed at the time of departure, the optimal choice of education for emigrants will be adjusted to the socially optimal level.

Next, consider the other decision: whether or not to emigrate. The socially optimal decision to migrate or not should be made according as

$$\Gamma_0 = \left[ \int_{\frac{S}{S+R}} f* e^{-it} dt - Ce^{-iS} \right] - \int_S^{S+R} fe^{-it} dt$$

(10)

is positive or negative, $S$ and $\bar{S}$ being chosen optimally here. However, the actual decision will be made by the criterion

$$\Gamma_1 = \left[ \int_{\frac{S}{S+R}} f* e^{-it} dt - Ce^{-iS} \right] - \left[ \int_S^{S+R} (a+bf)e^{-it} dt + \int_0^S Ge^{-it} dt \right]$$

(16)

(where it is assumed that a compensation scheme is adopted to neutralize the effect of the educational subsidy on $\bar{S}$.)

Thus even though we assume that the distorting effect on the choice of the length of education by non-migrants and migrants is suitably removed, the absence of the tax on foreign incomes will lead to a divergence between the socially optimal and the individual decision on migration. In fact, the com-
parison of (10) and (16) shows that, if the emigration is available only to those with higher-than-average domestic incomes (an assumption listed earlier as an empirically relevant phenomenon), the incentive to migrate will be stronger than is socially optimal since the assumption on the possibility of emigration will make the value of the second bracket in (16) smaller than the value of the second term on the R.H.S. of (10) as long as $\beta < 1$.

In the analysis in the rest of this Section, therefore, we will operate with the assumption that the use of the educational subsidy plus the levying of the lump-sum tax at the point of departure on emigrants will have removed the distorting effect, of the absence of the tax on foreign incomes, on the choice of the length of education by migrants and non-migrants. However, as seen above, the situation with regard to the decision to migrate will remain suboptimal and therefore we must now proceed to analyse this suboptimal situation in greater depth below. From a welfare viewpoint, we will compare this situation with the optimal situation where the tax on foreign incomes is available and is being utilised along with the other policy instruments, as discussed in the preceding Section, so as to secure efficiency and "almost" perfect equity. Furthermore, we will choose two successive criteria for comparison: the average per-capita incomes in the two situations, and the Rawlsian minimum incomes in the two situations. \(^{11}\)

(1) **The Per-Capita Income Criterion:**

Take first the simpler case where we consider the per-capita incomes of all individuals, including the emigrants. We can see immediately that the dis-

---

\(^{11}\) The utilitarian criterion creates analytical complexities and is therefore omitted from the paper. We suspect that the solution with it lies in the middle of those for the per-capita and minimum incomes criteria.
tortion on the decision to emigrate (discussed above, as stemming from the lack of identity of (10) and (16)) implies loss of income and hence a decline in the per-capita income in the suboptimal situation resulting from the absence of the tax on foreign incomes. 12

Consider next the "nationalistic" criterion: i.e. the comparison between the two situations of the per-capita income of those who remain behind in the home country. The per-capita income of those remaining behind in the situation without the tax on foreign incomes can be written as: 13

\[ \frac{\int f(1 - m(T))dP(n)}{\int (1 - m(T))dP(n)} \]

Taking next the "first-best" situation and writing per-capita income therein as 'y', we see immediately that:

\[ y = \int f(1 - m(T))dP + \int fm(T)dP \geq \int fdP \geq \frac{\int f(1 - m(T))dP}{\int (1 - m(T))dP} \] (17)

where the inequality will hold because of our assumption that only those who earn more than average income can emigrate. Furthermore, in this "first-best" situation, recall that everyone in the home country gets virtually equal income (with \( \beta \) virtually close to zero). Hence it follows from (17)

12 Even from the totally cosmopolitan view, i.e. from the Paretian criterion for the world as a whole, this leaves room for a distorting effect caused by the emigration possibility and domestic income taxation. However, a full treatment of this problem would require explicit consideration of income taxation by the foreign government. It is quite likely that a game like situation would emerge. Cf. Hamada (1966) on the double taxation problem of capital movements.

13 For a given value of \( T \), the value of the expression will remain unchanged.
that the per-capita income of those remaining behind will be definitely smaller in the situation without, than in the situation with, the tax on foreign earnings.

(2) The Rawls Criterion:

We turn next to the Rawls criterion which concentrates on the income of the least advantaged. Since, under our assumptions, the least income is always obtained by those who remain at home, we do not need to distinguish anymore between the case where the emigrants are excluded from the welfare analysis and the case where they are not. Furthermore, in the "first-best" situation with the optimal policy mix in force, all incomes will be virtually equalized at \( \alpha \); and in the case without the tax on foreign incomes again we can take \( \alpha \) as the least, minimum income that will be available to each individual in the economy. Hence, our problem reduces in this subsection to comparing the values of \( \alpha \) that can be attained in the two situations that are to be contrasted for their welfare implications for the least advantaged.

Recalling that, in the suboptimal situation, the lengths of education of both emigrants and non-emigrants are adjusted to the socially optimal level via use of educational subsidy and the lump-sum tax at the time of departure of emigrants, the budget constraint of the government with the biological rate of interest \( i \) can be written as:

\[
\alpha \int_S^{S+R} e^{-it} dt \int (1 - m(\Gamma)) dP + G \int_0^S e^{-it} dt + \int (1 - m(\Gamma)) dP = (1 - \beta) \int_S^{S+R} e^{-it} dt \int f \cdot (1 - m(\Gamma)) dP
\]

Since \( G \) is adjusted to neutralize the effect on \( S \), we have:

\[
G = \Delta \alpha = \alpha (1 - e^{-iR})
\]
Substituting (4) into (18), and dividing both sides by $A(R)e^{-iS}$, we then obtain:

$$\alpha \int (1 - m(\Gamma))dP = (1 - \beta) \int f(1 - m(\Gamma))dP,$$

or

$$\alpha = (1 - \beta) \frac{\int f(1 - m(\Gamma))dP}{\int (1 - m(\Gamma))dP} \tag{19}$$

for the sub-optimal situation without the tax on foreign incomes.

By analogous reasoning for the "first-best" case with use of the tax on foreign incomes, and therefore with $\beta = \beta^*$, we get:

$$\alpha = (1 - \beta) \left\{ \int f(1 - m(\Gamma))dP + \int f^* m(\Gamma)dP \right\} \tag{20}$$

where $m(\Gamma)$ takes the optimal value. Note that the total income received by the individuals originating from the home country is constant irrespective of the value of $\beta$, so that the expression in the brace in equation (19) is constant. Therefore we can draw the relationship between the value of $\beta$ and $\alpha$ as the solid line in Figure 1.

Now, recall that the average income in the absence of the tax on foreign incomes is less that that in the "first-best" situation, i.e.

$$\frac{\int f(1 - m)dP}{\int (1 - m)dP} < \int f(1-m)dP + \int f^* mdP \tag{21}$$

And, while $m$ is itself a function of $\beta$, (21) holds for any combination of $\alpha$ and $\beta$. Hence, it follows that the relationship between $\alpha$ and $\beta$ in the suboptimal situation without the tax on foreign incomes is dominated by the relationship in the "first-best" situation with the tax, as illustrated in

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14 Of course, we cannot attain the best solution where $\beta = 0$; we can only approximate the value by making $\beta$ approach zero.
Figure 1 by the dotted curve for the suboptimal situation which lies below
the solid line for the "first-best" situation. Hence, on the Rawls criterion, as well, it follows that the situation without the tax on foreign incomes is inferior to the situation with the tax.

The next, interesting question is whether this dotted curve, characterizing the suboptimal situation, will take a maximum value at some $\hat{\beta}$ such that $\hat{\beta} > \epsilon > 0$. Now, in general, $m(\Gamma)$ is a function of $\alpha$ and $\beta$ and we cannot say a priori whether the dotted curve will have a maximum value in the interior of $0 < \beta < 1$. Thus, define

$$\varphi(\alpha, \beta) = \frac{\int f(1-m)d\mathcal{P}}{\int (1-m)d\mathcal{P}}$$

Then:

$$\frac{d\alpha}{d\beta} = \left[-\varphi(\alpha, \beta) + (1 - \beta)\left(\frac{\partial \varphi}{\partial \alpha} \frac{d\alpha}{d\beta} + \frac{\partial \varphi}{\partial \beta}\right)\right]$$

i.e.

$$\left[\frac{\partial \alpha}{\partial \beta} = -\varphi(\alpha, \beta) + (1 - \beta)\frac{\partial \varphi}{\partial \beta}\right] / \left[1 - (1 - \beta)\frac{\partial \varphi}{\partial \alpha}\right]$$

It can then be shown that:

$$\frac{\partial \varphi}{\partial \alpha} > 0, \quad \frac{\partial \varphi}{\partial \beta} > 0$$

since $m'(\Gamma) > 0$, $\frac{\partial \Gamma}{\partial \alpha} < 0$, $\frac{\partial \Gamma}{\partial \beta} < 0$ and we have assumed that the emigrants have more-than-average earnings. Hence, if the effect of $\frac{\partial \varphi}{\partial \beta}$ is strong, we cannot preclude the case where the optimal value of $\beta$ is positive and, for some positive $\epsilon$ (as in the broken curve in Figure 1), we have:

$$\hat{\beta} > \epsilon$$
FIGURE 1
An Example:

In fact, we can construct an example where $\beta$ will have a positive value. Assume that the earning functions take the following form:

\[ f(n, S) = nS \]
\[ f^*(n, \bar{S}) = \lambda n\bar{S} \]

Then it can be easily calculated that

\[ S = \frac{1}{4} \quad \bar{S} = \frac{1}{4} \]

are the socially optimal lengths of education. Assume next that innate ability is distributed by the discrete distribution taking only two values:

\[ \tau(n_1) = \frac{9}{10} \quad n_1 < n_2 \]
\[ \tau(n_2) = \frac{1}{10} \]

Also assume that the cost of transportation and adjustment is negligible and that all individuals migrate if their accumulated income abroad exceeds their accumulated income at home:

\[ C = 0, \]
\[ m(\Gamma) = 0 \quad \text{if } \Gamma \leq 0, \]
\[ m(\Gamma) = 1 \quad \text{if } \Gamma > 0. \]

The accumulated income of the non-migrants is:

\[ I_d = I_0^S Ge^{-it}dt + \int_0^{S+R} (\alpha + \beta nS)e^{-it}dt \]

where \( G = \alpha(1 - e^{-iR}) \),
which reduces to:

\[ I_d = A(\alpha + \beta n S^{-i^2}) \]  

(22)

On the other hand, the accumulated income abroad can be written as:

\[ I_f = A\alpha n S^{-i^2} \]

Therefore:

\[
\Gamma \equiv I_f - I_d \\
= A \left\{ (\lambda - \beta)n S^{-i^2} - \alpha \right\} \\
= A \left\{ (\lambda - \beta) \frac{n}{i^2} - \alpha \right\}
\]

Therefore, if \( \alpha < (\lambda - \beta) \frac{n_2}{i^2} \), then all individuals with \( n_2 \) will go abroad.

Accordingly, the budget constraint of the government is written as:

\[ \alpha = (1 - \beta) \frac{n_1}{i^1} \]  

(23)

if \( \alpha < (\lambda - \beta) \frac{n_2}{i^2} \)

and

\[ \alpha = (1 - \beta) \left( \frac{9}{10} n_1 S + \frac{1}{10} n_2 S \right) = \frac{1 - \beta}{i} \cdot \frac{9 n_1 + n_2}{10} \]  

(24)

if \( \alpha \geq (\lambda - \beta) \frac{n_2}{i^2} \)

Suppose now that \( n_1 = \frac{10}{9} \), \( n_2 = 30e \), and \( \lambda = \frac{2}{3} \), where e is the base of the natural logarithm. Then it is easy to draw the trade-off curve for the society between \( \beta \) and \( \alpha \), representing equations (23) and (24), as in Figure 2.
\[ a = (\lambda - \beta) \frac{n_2}{i e} \]

\[ (1 + 3e)/i \]

\[ 10(1 + 3e)/i(29-3e) \]

\[ \frac{10}{9i} \]

\[ 0 \]

\[ \frac{19-3e}{29-3e} \]

\[ \frac{2}{3} \] (\( = \lambda \))

\[ \beta \]

FIGURE 2
Above the line $\alpha = (\lambda - \beta) \frac{n_2}{1e}$, equation (24) is valid, while under the same line equation (23) is valid. When the government reduces the value of $\beta$ to increase equality, this serves to increase $\alpha$ as long as individuals with ability $n_2$ stay at home. After $\beta$ is reduced to a value lower than $(19 - 3e)/(29 - 3e)$, all the skilled labor emigrates and the society can only enjoy the product of individuals with ability $n_1$. Therefore the government should choose the value of $\beta$ equal to $(19 - 3e)/(29 - 3e)$ in this second-best situation.\(^{15}\)

IV: Optimal Policy Mix With Foreign Education

Finally, we extend the analysis of Section II to the case where education is undertaken abroad. For simplicity, we assume that those who study abroad will also work abroad and that the choice to emigrate therefore arises before education is undertaken at all. The three alternatives open to an individual now are therefore: (i) study at home and work at home; (ii) study at home and work abroad; and (iii) study abroad and work abroad.

The choice between the first two alternatives was discussed thoroughly in Section II. There we saw that the policy combination of

$$G = \alpha(1 - e^{-1R})$$

for non-emigrants

$$\beta + 0$$

\(^{15}\)Perhaps the artificial nature of this example may indicate the robustness of the procedure of reducing $\beta$ to zero in normal situations. For, in this example, $n_2$ is much larger than $n_1$. And, since $\lambda < 1$, the Pareto-optimal situation is where emigration is not existent. The tax to implement equality induces the brain drain in this example.
and
\[
\begin{align*}
G &= \alpha(1 - e^{-iR}) \\
\alpha^* &= \alpha - \left(\frac{1 - \beta^*}{A}\right)c \\
\beta^* &= \beta + 0
\end{align*}
\]

for emigrants with domestic education

permitted the attainment of both the optimal lengths of education and the optimal decision whether or not to migrate, while also leading to "almost" total equity. Therefore we have now to consider only the choice between (iii) and one of (i) or (ii), because the choice between (i) and (ii) can be adjusted optimally.

Recall then that we denote by \( g^*(n, S^*) \) the earnings (net of taxes by foreign governments) of the emigrants who have studied abroad for length \( S^* \). The socially optimal condition for the choice of the length of \( S^* \) is readily given by:

\[
A e^{-iS^*} (-ig^* + g^*_S^*) = 0
\]

where \( A = \Lambda(R) = \frac{1 - e^{-iR}}{i} \).

Now, denote the subsidy for foreign education by the home country as \( G^* \), and the income schedule for foreign incomes as \( \alpha^{**} + \beta^{**}g^* \). Then the individual choice for the length of education abroad is given by

\[
A e^{-iS^*} \left\{ -i(\alpha^{**} + \beta^{**}g^*) + \beta^{**}g^*_S^* \right\} + G^* e^{-iS^*} = 0
\]

In order to reconcile (26) with (25), we then need to have:

\[
G^* = \alpha^{**}(1 - e^{-iR})
\]
Next, let us consider the total condition relating to the decision to study and work abroad, as between choices (i) and (iii). The socially optimal criterion for this decision to study and work abroad is:

\[
\Gamma_2 = \left[ \int_{S^*}^{S+R} g e^{-it} dt - C_0 \right] - \int_{S}^{S+R} f e^{-it} dt
\]

where \( C_0 \) is the cost of going abroad. On the other hand, the potential emigrants are considering, given the linear tax on foreign incomes:

\[
\Gamma_3 = \left[ \int_{S^*}^{S+R} (\alpha^{**} + \beta^{**} g) e^{-it} dt - C_0 + \int_{0}^{S^*} G^{**} e^{-it} dt \right] - \left[ \int_{0}^{S+R} (\alpha + \beta f) e^{-it} dt + \int_{S}^{0} G e^{-it} dt \right],
\]

\[
= \left[ A(\alpha^{**} + \beta^{**} g e^{-iS}) - C \right] - \left[ A(\alpha + \beta f e^{-iS}) \right]
\]

after substituting (27) and (4). \( \Gamma_3 \) can be further transformed to

\[
\Gamma_3 = \left\{ \beta^{**} \left[ A e^{-iS} - C_0 \right] - \beta A e^{-iS} f \right\} + \left\{ A \alpha^{**} + (1 - \beta) C_0 \right\} - A \alpha
\]

Now, choose \( \alpha^{**} \) and \( \beta^{**} \) to satisfy the following conditions:

\[
\alpha^{**} = \alpha - \frac{(1 - \beta) C_0}{A}
\]

(28)

\[
\beta^{**} = \beta
\]

(29)

and we reconcile the socially optimal and the individual decisions on the choice to study and work abroad, the former depending on \( \Gamma_2 \) and the latter on \( \Gamma_3 \).

Therefore, by choosing \( \alpha \), \( \alpha^* \) and \( \alpha^{**} \) to satisfy (9) and (28), while equating \( \beta \), \( \beta^* \) and \( \beta^{**} \), we can simultaneously make individuals choose the optimal lengths of education and also decide optimally among the three alterna-
tives (i), (ii) and (iii) distinguished above. We can therefore state the following proposition:

**Proposition II:**

By a suitable choice of the different intercepts of the domestic income tax on domestic and on foreign incomes for those educated at home and those educated abroad, and by equating the marginal rates of the three tax schedules and letting them approach unity, a national economy can attain the virtually first-best solution: Pareto-efficiency with "almost" total equality of income at home and abroad.  

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16 Note further that, since we are specifying only the relative magnitudes between $\alpha$, $\alpha^*$ and $\alpha^{**}$, we still have a degree of freedom to decide their absolute values. The absolute value would be determined, of course, by the budget constraint of the government.

17 Putting $B$'s equal to zero is, of course, the same as making the marginal tax rates unity.
References


