working paper
department
of economics

ON THE NEAR OPTIMALITY OF LINEAR INCENTIVES

Peter Diamond
Massachusetts Institute of Technology
94-29 Aug. 1994

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139
ON THE NEAR OPTIMALITY OF LINEAR INCENTIVES

Peter Diamond
Massachusetts Institute of Technology

94-29 Aug. 1994
On the Near Optimality of Linear Incentives

Peter Diamond
Massachusetts Institute of Technology

Abstract

Managers make efforts and choices. Efficient incentives to induce effort focus on the signal extraction problem of inferring the level of effort. Efficient incentives for choices line up the relative payoffs of principal and agent. With choices much more important than the variation in the cost of inducing effort, the optimal payment schedule tends toward proportionality. The argument holds if the control space of the agent has full dimensionality, but not otherwise.

If the agent's choices are a complete set of fair gambles and insurance, the proportional payoff schedule is no more expensive than other schedules that induce effort.
On the Near Optimality of Linear Incentives

Peter Diamond

Managers are called on to make efforts and to make choices. In designing incentives to induce effort, efficiency focuses on the signal extraction problem associated with inferring the level of effort made (Hart and Holmstrom, 1987). In contrast, to encourage appropriate choices, efficient incentive design tends to line up the relative payoffs of principal and agent. The presence of choices therefore alters the design of incentives in the standard formulation by introducing an additional factor. In many cases the choices are much more important than the variation in the cost of inducing effort. In this case the payment schedule tends toward proportionality. In Section I, this point is made in a simple model with three states of nature where it is proven that as the cost of effort shrinks (relative to gross payoffs), the schedule of payoffs to the agent converges to a linear function of gross payoffs. This argument is made in a three state model where there is a single variable describing choice and a single relative payment control variable for the principal. In Section II, a four state model is used to argue that the same logic holds if the control space of the agent has full dimensionality, but not otherwise. If the principal has more degrees of freedom in setting incentives than the agent has degrees of freedom in responding, the extra degrees of freedom are associated with additional first order conditions that block the argument that leads to linearity. Similarly, in a setting with information on the performance of other firms, if the choice set of the agent has full dimensionality, as the cost of effort shrinks relative to gross payoffs, the optimal incentive schedule converges to one that ignores relative performance. In the basic model presented, effort choice is a zero-one variable. In the Appendix, the model is reexamined for the case that effort is a continuous variable.

1 I am grateful to Daron Acemoglu, Bengt Holmstrom, and Jim Poterba for helpful comments and the National Science Foundation for financial support.
2 For both choices and efforts, incentives based on outcomes can be supplemented by incentives based on observable aspects of agent behavior, such as gross inefficiencies.
3 For other approaches to deriving a linear optimal payment schedule, see Holmstrom and Milgrom, 1987 and Laffont and Tirole, 1986.
4 The dimensionality of the control space is the analog in this setup to the "multitask" perspective in Hart and Holmstrom, 1987.
In the event that the choices available to the agent are a complete set of fair gambles and insurance, a different argument is possible. If the agent is free to rearrange probabilities across payoffs in any way that preserves the expected gross payoff to the principal, the proportional payoff schedule is no more expensive than other schedules that induce effort. This argument is presented in Section III.

I Model

We make the following assumptions. The principal is risk neutral. The agent is risk neutral, but can not receive a negative payment. There is no individual rationality constraint (the expected amount of payment is sufficient to induce supply). Assume there are three states of nature, with payoffs to the principal of \( x_3 > 0, x_2 > 0, x_1 = 0 \). Assume that the principal chooses three payments to the agent satisfying \( s_3 \geq 0, s_2 \geq 0, s_1 = 0 \). The payment schedule is linear in gross payoffs if \( s_2/s_3 = x_2/x_3 \).

Effort

First we review the standard model, where there is only an effort choice. In the standard formulation of this problem, expending effort changes the probability vector of the gross returns from \([1-f_2-f_3, f_2, f_3]\) to \([1-g_2-g_3, g_2, g_3]\). With the usual formulation, the principal’s problem can be stated as

\[
(1) \quad \text{Maximize } (x_2-s_2)g_2 + (x_3-s_3)g_3 \\
\text{s. t. } s_2(g_2-f_2) + s_3(g_3-f_3) \geq c, \\
s_i \geq 0 \text{ for all } i.
\]

Given the linearity of this problem the optimal scheme can pay compensation in just one state of nature. The optimum is found by finding the state or states (other than state 1) for which \( g_i/f_i \) is

5 Adding a constant to these three payoffs makes no change in the analysis.
6 In a one period model, one can not consider issues in the appropriate way to measure the return to the principal. Thus, whether payments to managers should depend on some measure of accounting profits or current or future stock market value, or some combination of these is not addressed. Similarly, the choice of a single period schedule of payments to the agent does not allow consideration of the use of options as opposed to other methods of compensation.
7 The principal has no reason to use a payment that is the same in all states of nature, since it encourages neither more effort nor correct choices. Thus this restriction amounts to requiring no payment lower than the payment in the state with the lowest gross payoff.
a maximum and paying enough compensation in that state to satisfy the incentive compatibility constraint. If \( g_2/f_2 = g_3/f_3 \), compensation can be spread across both states; otherwise a linear payment schedule is not optimal.

Effort and Choice

We modify this model by assuming that expending effort generates a set of possible distributions of gross revenues. The agent is then free to (costlessly) choose any element in the set. To analyse this problem, we work backwards, beginning with the agent's choice of the probabilities of payoffs. Denote the probability of state two by \( g \). We assume that \( g \) is a choice variable (within some range), with \( h(g) \) \((h'<0, h''<0)\) being the (maximal) induced probability of state three. Over the feasible range of values of \( g \), we assume that \( h' \) varies sufficiently to result in an interior solution. The probability vector of the three states is \([1-g-h(g), g, h(g)]\). Since there is no payoff in state 1, the payoff to the agent is \( s_2 g + s_3 h(g) = s_3 ( (s_2/s_3) g + h(g)) \). Thus, the agent's choice depends only on the ratio of payoffs in states two and three which we denote by \( s = s_2/s_3 \). Let us write the chosen level of \( g \) as \( g^*(s) \). Then \( g^*(s) \) maximizes \( sg+h(g) \) and is defined by the first order condition

\[
(2) \quad s + h'(g^*) = 0.
\]

Differentiating (2), we see that \( g^* \) is monotonically increasing in \( s \).

Conditional on effort being expended, we can write expected gross revenue as a function of the relative payoffs, \( R(s) \), as

\[
(3) \quad R(s) = x_2 g^*(s) + x_3 h(g^*(s)).
\]

Differentiating (3) and using (2), we note that \( R' \) is zero if and only if \( s \) gives a linear payoff, \( s = x_2/x_3 \).

\[
(4) \quad R'(s) = x_2 g''(s) + x_3 h'(g^*(s)) g^*(s)
\]

\[= (x_2 - x_3 s) g^*(s).\]

Thus the expected gross revenue, \( x_2 g + x_3 h(g) \), is maximized at \( s \) equal to \( x_2/x_3 \). We note that with proportional increases in \( x_2 \) and \( x_3 \), we have a proportional increase in \( R' \).

If no effort is made by the agent, the probability vector is \([1-f_2-f_3, f_2, f_3]\). We assume that it is worthwhile to induce effort. Effort can be induced by using payoffs to the agent that give the agent an expected return at least as large as the cost of making effort, \( c \).

---

8 With the assumption that \( h \) is concave, the maximization problem of the agent is concave and there is a unique solution.
Thus we can induce effort with any payoff ratio that satisfies
\[(6) \quad s g^*(s) + h(g^*(s)) - f_2 - f_3 > 0.\]
That is, for a given ratio, \(s\), that satisfies this inequality, there is a value of \(s_3\) sufficiently large so that the effort inducement constraint \((5)\) is satisfied. For payoff ratios satisfying the condition in \((6)\), by using \((5)\), the expected cost of just inducing effort can be written as:

\[(7) \quad C(s) = s_2 g^*(s) + s_3 h(g^*(s))\]
\[= c\frac{sg^*(s) + h(g^*(s))}{sg^*(s) + h(g^*(s)) - sf_2 - f_3}.\]
We note that \(C(s)\), and so \(C'(s)\), are proportional to \(c\).

We can now state the problem of the optimal incentive structure as

\[(8) \quad \text{Maximize } R(s) - C(s).\]
The first order condition for this problem is

\[(9) \quad R'(s^*) = C'(s^*).\]
As noted above, preserving the ratio of \(x_2\) and \(x_3\), \(R'\) is proportional to \(x_3\), while \(C'\) is proportional to \(c\). Thus, as the ratio of the cost of effort to gross revenues, \(c/x_3\), decreases to zero, \(C'(s)/R'(s)\) tends to zero unless \(s\) converges to \(x_2/x_3\). Thus, \(s^*\) tends to \(x_2/x_3\); the optimal incentive structure converges to a linear structure. With the cost of inducing effort going to zero (relative to gross payoffs), both \(s^*_2/x_2\) and \(s^*_3/x_3\) tend to zero, but their ratio is well-defined, tending to one.

II Generalizations

Four states
In the three state model analyzed, there is a single variable describing choice for the agent and a single relative payment control variable for the principal. This structure prevents the principal from having additional degrees of freedom for

---

9 If we did not assume risk neutrality for the agent, we would be examining the utilities associated with each state rather than just the payoff in each state. These utilities might be state-dependent, \(u_i(s_j)\), for example if the agent has career concerns and different states impacted differently on future opportunities. As long as the cost of bearing risk associated with the agent's risk aversion is small relative to gross payoffs, we would expect a similar sort of convergence for utilities.
encouraging effort beyond the incentives for choice. It is the restriction in the dimensionality of agent choice in some formulations that results in a basically different structure of incentives. To examine this balance between the choice variables of the agent and the control variables of the principal we contrast two different formulations of the four state problem.

Now assume that there are 4 states. Assume that the gross payoffs satisfy \( x_1 = 0, x_i > 0, i=2, 3, 4 \). Similarly, assume that \( s_1 = 0, s_i >=0, i=2, 3, 4 \). If we assume that the range of choice of the agent has full dimensionality (given the constraint that probabilities add to one), then we assume that the agent can choose probabilities in two states. That is, the agent can choose both \( g_2 \) and \( g_3 \). For any choice in the feasible range (and it is assumed that there is a positive range for each variable) there is an implied (maximal) level of probability of state 4, written as \( h(g_2, g_3) \). We further assume that the shape of this function is such that the optimum involves a unique interior choice. Then we have the same first order conditions as before and the argument goes through in the same way, leading to the conclusion that the optimal choices of \( s_2/s_4 \) and \( s_3/s_4 \) converge to \( x_2/x_4 \) and \( x_3/x_4 \).

In contrast, if we assume that there is a single control variable for the agent, \( g_2 \), then the probabilities in the other states are all functions of this variable, \( h_3(g_2) \) and \( h_4(g_2) \). That is, in the case examined just above, the four probabilities are \( [1-g_2-g_3-h(g_2, g_3), g_2, g_3, h(g_2, g_3)] \). In the case of a single control variable, the probabilities are \( [1-g_2-h_3(g_2)-h_4(g_2), g_2, h_3(g_2), h_4(g_2)] \). In the case where the control variables do not span the space of states of nature, the principal has another degree of freedom in setting payments for the agent, and the argument above does not go through. That is, choice depends on the sum \( s_3 h_3(g_2) + s_4 h_4(g_2) \), not on the two terms separately. Thus, there is a continuum of values of \( s_3 \) and \( s_4 \) that result in the same choice and the selection of the particular combination of \( s_3 \) and \( s_4 \) is made solely to minimize the cost of inducing effort.

Relative performance

The model can be extended to the situation where information is available on the performance of other firms in the industry. The critical question remains that of dimensionality - whether the agent has the ability to modify the correlation between the returns to the principal and the returns to other firms. When full dimensionality remains present, the linearity argument goes through, implying that the optimal incentive scheme converges to one that ignores relative performance.

To review this formally, assume that the gross payoff to the principal can be high or low and the gross payoff to the industry can be high or low. Thus there are four states, given by the two-by-two matrix of low and high returns for the principal and the industry. With a zero payment in the event that the principal’s payoff is low while the industry payoff is high, the principal has
three controls, or two relative controls. If the agent’s action space is of full dimensionality, the agent can choose two payoff probabilities. In this case the argument above goes through, with the same result, that in the limit the payoff schedule should ignore the performance of the industry. The problem with trying to use such information to hold down the cost of inducing effort is that it affects the expected gross payoff as the agent responds to the incentive structure by altering the correlation of the principal’s payoff with that of the industry.

Perks

The model above does not have explicit representation of decisions by the agent that directly affect the agent’s utility at the cost of expected gross returns. Excessive numbers of limousines and jets are representative examples of such actions. More expensive are new corporate headquarters, or relocation of corporate headquarters for the pleasure of the agent. Since there is no limit on the ability to waste and since it is inevitable that top management will have only a small share of the variation of returns in a large firm, the presence of such possible actions calls for attempts to monitor them directly. Thus, just as the principal sets the cash reimbursement schedule of the agent and needs to check that the agent does not receive more cash compensation than the agent is entitled to, so the principal needs to monitor the noncash reimbursement of the agent. With sufficiently effective monitoring, the argument above should remain, so that complicated reimbursement schedules, as called for by the model with only effort, remain unnecessary, and possibly ineffective.

Another decision that involves large returns is that of replacement of the agent by another agent. The model above has not considered such a possibility, one that complicates the analysis by having nonfinancial returns associated with replacement.

III Fair gambles and insurance

In the usual formulation of the principal-agent problem, (1), the optimal schedule selects the state (or states) for which \( g_i/f_i \) is a maximum and pays enough compensation in that state to induce effort. If state \( i \) is the lowest cost state, then, paralleling the argument leading to (7), the cost of inducing effort is \( c g_i/(g_i-f_i) = c(g_i/f_i)/(c(g_i/f_i)-1) \). Note that the function \( c z/(z-1) \) is decreasing in \( z \). Thus, only if \( g_2/f_2 = g_3/f_3 \), can the optimum include compensation that is paid in both states.

In this formulation, expending effort changes the probability vector of the gross returns from \( [1-f_2-f_3, f_2, f_3] \) to \( [1-g_2-g_3, g_2, g_3] \). In the analysis above, we added a choice variable that was only available if effort was expended. We now consider the case where choice is available whether or not the agent expends effort. However, we now assume that the choice set of the agent is defined by the ability to take all possible fair gambles and fair insurance policies. That is, we assume the agent can rearrange the
probabilities of the different states in any way that preserves the expected gross payoff to the principal. In this case, we argue that no payoff schedule can induce effort at a lower expected cost than the proportional schedule. We proceed by first examining schedules that pay the agent in just one state, then considering all alternatives. We consider the case of n states, with \(0=x_1<x_2<x_3<\ldots<x_n\).

Payoff in one state

Let us denote the mean gross payoffs without and with effort by \(m_f\) and \(m_g\). With a proportional payoff schedule, gambles that do not change the expected return to the principal, do not change the expected payoff to the agent. Thus we can ignore fair gambles in evaluating the proportional schedule and note, from the argument leading to (7), that the cost of inducing effort is

\[
C = \frac{cm_g}{m_g-m_f} = \frac{cm_g/m_f}{((m_g/m_f)-1)}.
\]

Next consider a payoff schedule that gives compensation only in state i. The agent wants to concentrate as much probability as possible on this state. If \(x_i\) exceeds the mean return, the agent maximizes the expected payoff by taking gambles so that all probability is concentrated on states i and 1. If \(x_i\) is less than the mean return, the agent maximizes the expected payoff by taking gambles so that all probability is concentrated on states i and n. Let us denote the probabilities of payoffs after such gambles by \(f'\) and \(g'\) in the cases that effort is not and is expended. Then we have two cases (ignoring the state with a payoff precisely equal to the mean, in which case the probability of such a state can be set to 1).

\[
\begin{align*}
(11) & \quad \text{If } x_i > m_f, \text{ then } f'_i = m_f/x_i. \\
& \quad \text{ If } x_i < m_f, \text{ then } f'_i = (x_n-m_f)/(x_n-x_i).
\end{align*}
\]

The same rule holds for \(g'\).

To induce effort using a schedule that pays \(A\) in state i, we note that \(A\) must satisfy

\[
(12) \quad A(g'_i-f'_i) = c.
\]

Thus, the expected cost satisfies

\[
(13) \quad C = Ab'_i = c(g'_i/f'_i) = c(g'_i/f'_i)/((g'_i/f'_i)-1).
\]

We note that the cost, \(C\), is decreasing in \(g'/f'\).

We consider three cases as \(x_i\) exceeds \(m_g\), lies between \(m_f\) and \(m_g\), and is less than \(m_f\). In these three cases, we have:

\[
(14) \quad \text{If } x_i > m_g > m_f, \quad g'_i/f'_i = m_g/m_f.
\]
If $m_f < x_i < m_g$, $g'_i/f'_i = (x_n-m_g)x_i/(x_n-x_i)m_f < m_g/m_f$.

If $x_i < m_f < m_g$, $g'_i/f'_i = (x_n-x_i)/(x_n-m_f) < m_g/m_f$.

Comparing (13) and (10) and using (14), we conclude that costs are at least as high with payoff to the agent in a single state as with proportional payoffs. In the first case, we have the same cost as with proportional payoffs. In the other two cases, costs are increased by concentrating the payoff on a single state. With fair gambles, the agent is better able to take advantage of gambles with the lower expected return without effort.

General case

Schedules that pay the agent in just one state are at least as expensive as the proportional schedule. To extend the argument to more complicated schedules, we show that costs are not raised by changing a schedule reducing the number of states with a payoff to no more than two. A parallel argument to that above completes the proof.

We begin with the agent’s problem after exerting effort:

(15) Maximize $g'\bar{s}_2s_2+g'\bar{s}_3s_3+\ldots+g'\bar{s}_ns_n$

subject to $g'\bar{s}_2s_2+g'\bar{s}_3s_3+\ldots+g'\bar{s}_ns_n = m_g$,

$g'\bar{s}_2s_2+g'\bar{s}_3s_3+\ldots+g'\bar{s}_ns_n <= 1$,

$g'_i >= 0$ for all $i$.

As we will argue, the linearity of this problem implies that an optimum can be found that puts probability weight on no more than two states. In turn, this implies that costs are not increased and effort not discouraged by setting payoffs equal to zero in states in which the agent puts no probability if effort is taken.

If some probability is put on state 1, then the sum of probabilities on the other states is less than 1 and the second constraint is not binding. In this case, any state that has a positive probability has the same relative payoff, $s_i/x_i$. In this case, the principal can select just one such state and pay only in that state, without changing the cost to the principal. Since there is a payoff in only one state, the argument above shows that such a payoff schedule can not do better than the proportional schedule. This argument also works at the knife edge where no probability is put on state 1 and the constraint is not binding.

Thus, we are left with the case that no probability is put on state 1 and the constraint on probabilities adding to one is binding. For any state receiving positive probability, the derivative of the Langrangian expression is equal to zero. Denoting the two Lagrange multipliers by $L_1$ and $L_2$, for states receiving positive probability, we have
Note that (16) implies that the states receiving positive probability have different relative payoffs, $s_i/x_i$. Thus shifted probabilities that preserve expected gross payoff change the expected payoff to the agent. Therefore, by selecting the right pair of states, the agent can not do better with positive probabilities on more than two states than by restricting probability to 2 states, one with gross payoff below $m_g$ and one above $m_g$. If the state with the lower gross payoff has a lower relative payoff than the state with the higher gross payoff, the agent will do better using state 1 and the state with the higher payoff. But this involves a payoff in just one state, and does not cost less than the proportional schedule.

Thus we are left with the case that there are positive payoffs in two states and the state with the lower gross payoff has at least as high a relative payoff as the state with the higher gross payoff. It remains to calculate the expected cost in this case. Denote the state with the lower gross payoff by $i$ and its probability by $g'$, with the state with the higher payoff being $j$ and its probability $1-g'$. Then, from the constraint on expected gross returns, we have

$$(17) \quad g' = \frac{(x_j-m_g)}{(x_j-x_i)}. \tag{17}$$

The mean return without effort, $m_f$, might lie in the interval between $x_i$ and $x_j$, or might be less than $x_i$. In the former case, denoting the no-effort probabilities by $f'$ and $1-f'$, we have:

$$(18) \quad f' = \frac{(x_j-m_f)}{(x_j-x_i)}. \tag{18}$$

In this case, the cost of just inducing effort is

$$(19) \quad s_j - (s_j-s_i)g', \tag{19}$$

where

$$(20) \quad (s_j-s_i)(f'-g') = c. \tag{20}$$

Thus the cost of just inducing effort using payoffs only on states $i$ and $j$ is

$$(21) \quad s_j - (s_j-s_i)g' = s_j - cg'/(f'-g'). \tag{21}$$

This is minimized at $s_j = 0$, implying we are back in the case with payment in a single state, which does not cost less than the proportional schedule.

In the remaining case, $m_f < x_i$. Since the relative payoff is larger on $x_i$ than $x_j$, the agent would put no probability weight on $x_j$ if effort is not induced. Thus the principal will minimize cost by setting $s_i$ to zero, returning us again to a case with a payoff in
only one state of nature. Thus we can conclude that the proportional payoff schedule does at least as well as any other schedule.

IV Concluding remarks

In many settings, the cost of inducing managers to work hard is far less important than encouraging them to make the right choices from the set made available by their hard work. For example, the managers of large firms have effort costs which are very small compared to the range of possible profits of the firms, which can be in the billions. In modelling this situation, it was assumed that agents were risk neutral, but could not be paid a negative amount. This structure was designed to capture several aspects of large firms. One is that the wealth of management is indeed small relative to the value of the firm. Thus there is no way that the payoff to management can vary as a significant fraction of gross payoffs to the firm without having very large expected payoffs to the management. Thus the focus is on the structure of payoffs to the agent relative to gross payoffs, not the level of the ratio. Convergence of the payoffs to the agent to zero, relative to gross payoffs, is an implication of large firms without a large enough set of suitably wealthy individuals.

One assumption of the basic model is that agents make optimal choices. If the variation in returns were sufficiently small, agents might not bother making optimal choices, since choosing might not be costless, as was assumed. The convergence of the optimal schedule to one that is linear with positive slope, rather than to a flat salary, makes this concern not seem important.

Assuming the agents are well-enough paid in the worst state to be risk neutral reflects two presumed aspects of a more basic model. One is that it is probably efficient for a large firm to absorb the risk of agents as a way of holding down the cost of inducing a supply of suitable agents. Second is that the risk aversion of agents will lead to poor choices for a risk neutral principal. Thus significant minimal payoff takes care of the risk aversion problem (except that of losing the job) at relatively small cost to the firm. Given the assumptions made, a proportional payoff schedule (in utilities) will be nearly optimal. If, as also assumed, managers are paid sufficiently well to be risk neutral, then a proportional payoff schedule will be nearly optimal.

The second argument made in the paper is that the ability of managers to alter the probability structure of expected payoffs gives management an opportunity to take advantage of some nonlinearities in the payment schedule, implying that a nonlinear structure would result in higher expected costs to the principal. Thus, when the ability to manipulate probabilities is large enough, the proportional schedule is optimal even if the cost of inducing effort is significant.
Appendix: Continuous effort

The discussion above was made simpler by the discrete nature of the effort choice. In this section, I briefly turn to the case of a continuous adjustment of effort. Let us denote effort by e and its cost to the agent by ce. First we review the problem if there is no choice variable, only an effort variable. In this case, we write the probabilities as functions of effort, h_i(e). In this setting, the agent maximizes s_2h_2(e) + s_3h_3(e) - ce. This gives the first order condition for the agent's choice

A1 \ s_2h'_2 + s_3h'_3 = c.

From the first order condition, we can write the chosen level of effort as a function of the payments, e^*(s_2, s_3). We note that the derivatives of e^* with respect to payments are proportional to the values of h':

A2 \ e^*_i = -h'_i/(s_2h''_2 + s_3h''_3).

We can now state the principal's problem as

A3 \ Max (x_2 - s_2)h_2(e^*(s_2, s_3)) + (x_3 - s_3)h_3(e^*(s_2, s_3)).

The first order conditions for the optimal incentive schedule are

A4 \ -h_2 + (x_2-s_2)h'_2e^*_2 + (x_3-s_3)h'_3e^*_2 = 0,
     \ -h_3 + (x_2-s_2)h'_2e^*_3 + (x_3-s_3)h'_3e^*_3 = 0.

Using (A1), we can rewrite the first order conditions, (A4), as:

A5 \ -h_2 + (x_2h'_2 + x_3h'_3 - c)e^*_2 = 0,
     \ -h_3 + (x_2h'_2 + x_3h'_3 - c)e^*_3 = 0.

or, using (A2):

A6 \ -h_2(s_2h''_2 + s_3h''_3) + (x_2h'_2 + x_3h'_3 - c)h'_2 = 0,
     \ -h_3(s_2h''_2 + s_3h''_3) + (x_2h'_2 + x_3h'_3 - c)h'_3 = 0.

Thus, unless h'_i/h_i is the same for both states, the optimum offers a payment in only one state of nature. This is similar to the situation with a discrete choice of effort level.

Now let us assume that the probability of state three is a function of both the effort undertaken and the choice of probability of state two, h(g,e).\(^{10}\) In this setting, the agent

\(^{10}\) We are ignoring the impact of effort on the available range of values of g.
maximizes $s_2 g + s_3 h(g, e) - ce$. This gives the pair of first order conditions for the agent's effort and choice:

A7 $s_3 h_e = c,$

$$s_2 + s_3 h_g = 0.$$ 

From the first order conditions, we can write the choice variables as functions of the payments, $g^*(s_2, s_3)$ and $e^*(s_2, s_3)$.

We can now state the principals problem as

A8 $\text{Max } (x_2 - s_2) g^*(s_2, s_3) + (x_3 - s_3) h(g^*(s_2, s_3), e^*(s_2, s_3)).$

The first order conditions for the optimal incentive schedule are

A9 $-g^* + (x_2-s_2) g^*_2 + (x_3-s_3)(h g^*_2 + h e^*_2) = 0,$

$$-h(g^*, e^*) + (x_2-s_2) g^*_3 + (x_3-s_3)(h g^*_3 + h e^*_3) = 0.$$ 

Following the same tack as previously, let us group the first order conditions between terms relating to choice and those relating to effort:

A10 $x_2 g^*_2 + x_3 h g^*_2 = g^* + s_2 g^*_2 + s_3 (h g^*_2 + h e^*_2) - x_3 h e^*_2,$

$$x_2 g^*_3 + x_3 h g^*_3 = h(g^*, e^*) + s_2 g^*_3 + s_3 (h g^*_3 + h e^*_3) - x_3 h e^*_3.$$ 

Now using the first order conditions for the agent's problem, we can write this as:

A11 $(x_2 - x_3(s_2/s_3)) g^*_2 = g^* + s_3 h e^*_2 - x_3 h e^*_2,$

$$(x_2 - x_3(s_2/s_3)) g^*_3 = h(g^*, e^*) + s_3 h e^*_3 - x_3 h e^*_3.$$ 

or

A12 $(x_2 - x_3(s_2/s_3)) g^*_2 = g^* + (s_3 - x_3) h e^*_2,$

$$(x_2 - x_3(s_2/s_3)) g^*_3 = h(g^*, e^*) + (s_3 - x_3) h e^*_3.$$ 

If the right hand sides of these two equations go to zero, then $s_2/s_3$ converges to $x_2/x_3$. But the right hand sides of equations (A12) are precisely the first order conditions for the agent's efforts in the model without choice, equation (A4). So, we have a similar conclusion to that above - when the variation in the cost of inducing optimal effort becomes small relative to gross payoffs, the incentive schedule converges to linear.

The exploration of sufficient conditions for the effort choice not to be important at the margin is more complicated when effort is a continuous variable than when it is a zero-one variable, and I will not examine it in detail. In the latter case, it was sufficient
to have the cost of effort become small relative to gross payoffs, \( c/x_3 \) go to zero. In this case, one needs to have assumptions that limit the increase in effort, since \( ce/x_3 \) need not go to zero when \( c/x_3 \) does. Thus, we would want to make the plausible assumption that \( h_e \) goes to zero at a finite level of \( e \). We might also want to rule out the possibility that changes in effort have important effects on the slope of the tradeoff between probabilities in both states. That is, we might also want to assume that \( h_{ge} \) goes to zero as well.

References


