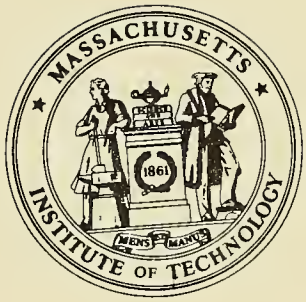

working paper
department
of economics

ON REANALYSING THE HARRIS-TODARO MODEL: POLICY RANKINGS IN THE CASE OF SECTOR-SPECIFIC STICKY WAGES*

Jagdish Bhagwati and T. N. Srinivasan

Number 99 February 1973

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

ON REANALYSING THE HARRIS-TODARO MODEL: POLICY RANKINGS IN THE CASE OF SECTOR-SPECIFIC STICKY WAGES*

Jagdish Bhagwati and T. N. Srinivasan

Number 99 February 1973

*The authors are Professors of Economics at M.I.T. and the Indian Statistical Institute, respectively. This paper was written while Srinivasan was Visiting Professor of Economics at M.I.T. The research underlying the paper has been supported by the National Science Foundation. The computations were carried out by David McClain. The basic results on second-best policies in this paper have been developed and established by us elsewhere, in a companion paper for a forthcoming (1973) issue of Sankhya, in memory of the late Professor P. C. Mahalanobis, being edited by C. R. Rao.

The views expressed in this paper are the authors' sole responsibility and do not reflect those of the National Science Foundation, the Department of Economics nor of the Massachusetts Institute of Technology.

In a pioneering paper, Harris and Todaro (1970) introduced a model with two sectors, manufacturing (urban) and agriculture (rural), a (sticky-) minimum wage in manufacturing and consequent unemployment. They also introduced a labour allocation mechanism under which, instead of the usual equalisation of actual wages, the actual rural wage was equated with the expected urban wage; the latter was defined as the (sticky-) minimum wage weighted by the rate of employment, so that, unlike in the standard rigid-wage models of trade theory -- e.g. Haberler (1950), Bhagwati (1968), Johnson (1965), Lefeber (1971) and Brecher (1970) -- , the unemployment resulting from the minimum wage is to be construed as specific to the urban sector.

In the context of this model, Harris and Todaro analyse two policies: (1) a wage subsidy policy in the manufacturing sector (alone); and (2) a labour-mobility restriction policy. They argue that the former, as also the latter, can be used to improve welfare, defined as a function of available goods in the usual way; but that, to attain the optimal first-best solution, both policies are necessary. The authors express regret at the necessity of using migration restrictions in view of the "ethical issues involved in such a restriction of individual choice and the complexity and arbitrariness of administration" and end their exercise with the sentiment that (p. 138):

"All of the above suggests that altering the minimum wage may avoid the problems of taxation [to finance the wage subsidy in manufacturing], administration, and interference with individual mobility attendant to the policy package just discussed. Income and wage policies designed to narrow

the rural-urban wage gap have been suggested by D. P. Ghai, and Tanzania has formally adopted such a policy along with migration restriction. In the final analysis, however, the basic issue at stake is really one of political feasibility and it is not at all clear that an incomes policy is any more feasible than the alternatives."

We contend in this paper that this dilemma is false and rests on the fact that the authors fail to realise that:

(1) a uniform wage subsidy, regardless of the sector of employment, will yield the optimal, first-best solution;

(2) equivalently, a wage subsidy in manufacturing plus a production subsidy to agriculture will yield the optimal, first-best solution;

(3) in either case, no resort to "ethical compromises" in the direction of sanctioning migration restrictions will be necessary;

(4) proposition (2) implies that the authors' frequent assertion that the traditional prescription to use shadow pricing of labour (i.e., a wage subsidy in employment) is inapplicable to their model is erroneous and their error stems from confusing this prescription with the prescription that the wage subsidy be given for employment in the manufacturing sector alone; and

(5) proposition (2) also implies that the authors' contention that two policies are necessary to attain the first-best optimum is not valid unless one construes a general wage subsidy to constitute two policies when there are two sectors employing labour.

In demonstrating these propositions, we also note that the Harris-Todaro analysis is vitiated by the fact that their formal model has a

demand function which is not related to the utility function in their (later) welfare analysis, so that their analytical system is open to the possibility of being overdetermined. We therefore rewrite their model, with the utility function explicitly incorporated into the model and eliminating the "additional" demand equation of Harris and Todaro.

Since the basic errors of Harris and Todaro relate to the first-best optimal-policy characterisation, we begin with analysis of the first-best, optimal policy in the model.* However, we also take the opportunity to extend the analysis in Section II to two second-best policy measures: i) wage subsidy in manufacturing and ii) production subsidy to agriculture. both of which policies can be shown to be equivalent, singly or in combination, to all other conceivable policy interventions in the model. However, rather than prove these results with rigor -- we have done this elsewhere (1973) in a companion paper -- we produce numerical examples in the Appendix to establish and illustrate the least intuitive among them.

I: The Model

We may now restate the Harris-Todaro model. First, there are two production functions:

$$X_A \leq f_A(L_A) \tag{1}$$

$$X_M \leq f_M(L_M) \tag{2}$$

where X_A and X_M are the output levels of agriculture (rural sector) and manufactures (urban sector) respectively and L_A , L_M are the labour-input levels in the two sectors. The functions are strictly concave. The labour supply is fixed and assumed to be unity by choice of units:

$$L_A + L_M \leq 1 \tag{3}$$

---

*Some errors of detail are picked up by us later in this paper.

We then have a standard, social utility function:

$$U = U(X_A, X_M) \tag{4}$$

where U is concave with positive marginal utilities for finite $[X_A, X_M]$.

For a fully competitive economy, the resulting Pareto-optimal equilibrium can be shown in Figure (1) at S where the production possibility curve DE is tangent to SS' and

$$\frac{U_1}{U_2} = \frac{f'_M}{f'_A} \tag{5}$$

with U_1/U_2 equal to the negative of the slope of SS', and U_1 and U_2 representing the partial derivatives of U with respect to X_A and X_M respectively.

But we now assume that the wage in manufacturing is fixed as a minimum, so that for this competitive economy, we must have:

$$f'_M(L_M) \geq \bar{w} \tag{6}$$

If we then assume that this constraint is binding at S, the first-best optimal solution is inadmissible and unemployment ensues. The system could then have been characterized nonetheless by the equalization of <u>actual</u> wages in the two sectors. Harris and Todaro, however, chose to rewrite the wage-equalization equation in terms of the <u>expected</u> wage in manufacturing, defined as the actual wage there weighted by the rate of employment, so that the critical equilibrium conditions in their model, relevant for our analysis, are

$$f'_M = \bar{w} \tag{7}$$

$$\frac{U_1}{U_2} f'_A = \bar{w} \frac{L_M}{1-L_A} \tag{8}$$

where the total labour force is assumed to be one by choice of units and where consumption and production price of the agricultural good is identical and equal to U_1/U_2.

With $\bar{w}$ specified, (7) and (8) can be solved for L_M and L_A, using the two production functions. The laissez faire equilibrium, with unemployment ($L_M < 1 - L_A$), will then lie in Figure (1) along RK (where X_M and hence L_M are fixed at the value that makes $f'_M = \bar{w}$) at Q. (It may be emphasized that the laissez faire equilibrium would so lie along RK even if actual wages were equalized in the two sectors: nothing critical to our interests hangs on the expected-wage wrinkle in the Harris-Todaro analysis.)

As for the available policy instruments (that use the price-mechanism as distinct from direct allocation mechanisms) in this model, we note now the following:

(i) laissez faire;

(ii) wage tax-cum-subsidy in manufacturing; and

(iii) production tax-cum-subsidy.

The structure of the model also implies the following equivalences:

(iv) a wage tax-cum-subsidy in agriculture is equal to policy (iii);

(v) a uniform wage tax-cum-subsidy in all employment is a combination of policies (ii) and (iii);

(vi) for a closed economy, a consumption tax-cum-subsidy is equivalent to policy (iii), i.e. a production tax-cum-subsidy;* and

(vii) for an open economy, a tariff (trade subsidy) policy would, as usual, be equivalent to policy (iii), i.e. a production tax-cum-subsidy policy, plus a consumption tax-cum-subsidy policy.**

One final point may be noted. Our analysis does not explicitly distinguish between a closed and an open economy. Since it relates essentially to the production equilibrium in the economy, and since it allows the utility function to be linear or nonlinear, it can be interpreted as applying either to a closed economy or to an open economy with given terms of trade.***

---

*Thus, let $\pi_p = \frac{\bar{w}L_M}{(1-L_A)f'_A}$ be the production price of the agricultural good and $\pi_c = \frac{U_1[X_A,X_M]}{U_2[X_A,X_M]}$ be the consumption price of the agricultural good. The production tax-cum-subsidy is then $\frac{\pi_p-\pi_c}{\pi_c}$; and the consumption tax-cum-subsidy is $\frac{\pi_c-\pi_p}{\pi_p}$.

**Thus, if π^* is the international price of the (importable) agricultural good, a tariff at ad valorem rate t would imply: $\pi^*(1+t) = \pi_p = \pi_c$.

***The analysis would have to be amended to bring in the foreign reciprocal demand function explicitly into the formal model if we were to consider the case of a country with monopoly power in trade. For a "small," open economy, the analysis in the text for a linear utility function would be applicable without modification.

II: Optimal Policy Intervention

It is easy to see that the first-best optimum can be reached in this model by the use of a uniform wage subsidy or, equivalently, by the use of a wage subsidy in manufacturing and a production subsidy to agriculture.

(A): Thus, let

$$s^* = \bar{w} - f'_M(L^*_M)$$

be the wage subsidy (financed by appropriate lump-sum taxation) in manufacturing. If this subsidy is also extended to employment in agriculture, we should write the equilibrium condition in production as:

$$f'_M = \bar{w} - s^* \tag{9}$$

$$\pi^*_c f'_A = \bar{w} - s^* \tag{10}$$

where $\pi^*_c = \dfrac{U_1(X^*_A, X^*_M)}{U_2(X^*_A, X^*_M)}$ is the consumption price (equal to the producer's price $\pi^*_p = \dfrac{\bar{w}}{f'_A}$) of the agricultural good. It is clear then that the constraints of the model are met (i.e. the wage rate in manufacturing is at $\bar{w}$ and the wage rates are equalized at the producer's prices in both sectors) and full-employment optimal equilibrium is reached with wage subsidy at level s^* in both sectors. Thus, in Figure (2) (which illustrates for a closed economy case), the resulting full-employment, optimal equilibrium is at S, with $\pi^*_c = \pi^*_p$, (and the domestic, marginal rates of transformation in production and in consumption are equal at S).

(b): Alternatively, we could have used a wage subsidy in manufacturing (alone) at level s^* and combined it with a production subsidy in agriculture. Thus, with

$$\pi_p^* = \frac{\bar{w}}{f_A'(L_A^*)}$$

as the producer's price of the agricultural good, and π_c^* as the consumer's price of the agricultural good, as before, we have:

$$t^* = \frac{\pi_p^* - \pi_c^*}{\pi_c^*}$$

as the optimal subsidy to agriculture. With the optimal values for s^* and π_p^*, we then have:

$$f_M' = \bar{w} - s^* \tag{11}$$

$$\pi_p^* f_A' = \bar{w} \left(= \bar{w} \cdot \frac{L_M^*}{1-L_M^*} \right) \tag{12}$$

and, once again, we note that the constraints in the model are met, and full-employment, optimal equilibrium is reached with wage subsidy in manufacturing at level s^* and production subsidy to agriculture at rate t^*.

However, while the equilibrium is again optimally at S, it is characterized now by $\pi_p^* \neq \pi_c^*$ (though of course the domestic, marginal rates of transformation in production and substitution in consumption remain equal to each other and identical to that under the uniform wage-subsidy policy at S).

Hence we have established the validity of criticisms (1)-(5) levelled at the Harris-Todaro analysis at the outset of this paper.

III: Second-Best Policy Intervention

The two second-best policies which then can be considered are: (1) a wage tax-cum-subsidy to manufacturing (considered by Harris-Todaro at some length); and (2) a production tax-cum-subsidy (not considered by Harris-Todaro, although their "migration-restriction" policy is the "quota-equivalent" thereof).

Wage Subsidy in Manufacturing: We only sketch here briefly the analysis of this policy as the Harris-Todaro results are generally correct.* With s as the wage subsidy in manufacturing, the equilibrium is now characterized by:

$$f'_M = \bar{w} - s \tag{13}$$

$$\frac{U_1}{U_2} f'_A = \bar{w} \cdot \frac{L_M}{1-L_A} \tag{14}$$

Clearly, given $\bar{w}$ and s, (13) and (14) can be solved for L_M and L_A. We can then demonstrate (1973) that:

(1) starting from a laissez faire equilibrium (s = 0), on RK at Q in Figure (3), increasing s means shifting the production equilibrium Q steadily north;

(2) the locus of successive production equilibria, mapped out by increasing s, must reach full employment (at an s_{max}) on the production possibility curve: such a locus being QH;**

---

*We have also developed the second-best analysis at much greater length, and with formal rigor, in the companion paper (1973), referred to earlier. Instead, we give numerical examples in the Appendix to illustrate the major propositions listed here.

**Harris and Todaro incorrectly argue (1970, page 134) that the full-employment equilibrium with a wage subsidy in manufacturing can be inside DE, off the production possibility curve. They forget that labour is the only factor in the model, in effect; they seem to have erred by relying on analogy with the standard two-factor model.

(3) the full-employment equilibrium may be inferior welfarewise to laissez faire - a proposition which we illustrate with a numerical example in the Appendix;

(4) a wage subsidy will necessarily improve welfare (i.e. $\frac{dU}{ds} > 0$ at $s = 0$); and

(5) the second-best wage subsidy need not be characterized by full-employment, so that tradeoff possibilities between increased welfare (via a standard social utility function of the type deployed by Harris and Todaro, and in this paper) and reduced unemployment may be pertinent.

Production Subsidy to Agriculture: For the case where the policy instrument is a production subsidy to agriculture, the equilibrium conditions are clearly rewritten as:

$$f_M' = \bar{w} \tag{15}$$

$$\pi_p f_A' = \frac{\bar{w}\, L_M}{1-L_A} \tag{16}$$

where, as before, π_p is the producer's price of the agricultural good and the implied production subsidy is $\frac{\pi_p - U_1/U_2}{U_1/U_2}$.

Clearly, given π_p and $\bar{w}$, we can solve for L_M and L_A. It is also then easy to show that:

(1) starting from a laissez faire equilibrium ($\pi_p = U_1/U_2$), on RK at Q in Figure (3), increasing π_p will steadily shift the production equilibrium to the right along QR untill full employment is reached at $\bar{\pi}_p$ at R;

(2) the equilibrium at R is also the second-best optimal equilibrium, so that the full-employment, second-best equilibrium is reached when

$\pi_p = \bar{\pi}_p$ and there is an implied production subsidy to agriculture; and

(3) the second-best wage subsidy in manufacturing cannot be ranked uniquely with the second-best production subsidy to agriculture - as illustrated by a numerical example in the Appendix.

IV: Concluding Remarks

Where do the "migration-restriction" policies of Harris and Todaro fit in?

If one is willing to contemplate direct, physical allocations, one can clearly reach the first-best, optimal solution, S in Figure (1), by assigning the corresponding labour to the two sectors (L_A^* and L_M^*) and enforcing the rule that all labour be employed regardless of private profitability (thus yielding X_A^* and X_M^*). The Harris-Todaro policy package for reaching S, consisting of a wage subsidy in manufacturing and migration-restrictions is thus a "mixed" package: one policy being of the price-mechanism variety and the other of the direct-physical-mechanism variety. One could equally turn this mixed-combination package on its head and have manufacturers forced to employ all available labour and let a production subsidy to agriculture allocate the labour force at the optimal values (L_A^* and L_M^*).

Nothing can be said, in principle, about the relative suitability of all these equivalent alternatives without bringing in other considerations, including the ethical considerations mentioned by Harris and Todaro, to introduce asymmetries/nonequivalences among them.

Finally, as for second-best policies, we might be able to justify the Harris-Todaro concentration on the wage subsidy to manufacturing policy,

as against a uniform wage subsidy policy, on feasibility grounds. It may well be that the government's capacity to intervene is confined to the (modern), urban sector and that a wage subsidy in agricultural employment is infeasible. This is, however, a question of empirical import; and it does not really justify the exclusion from the theoretical analysis of the first-best price-mechanism-variety intervention.

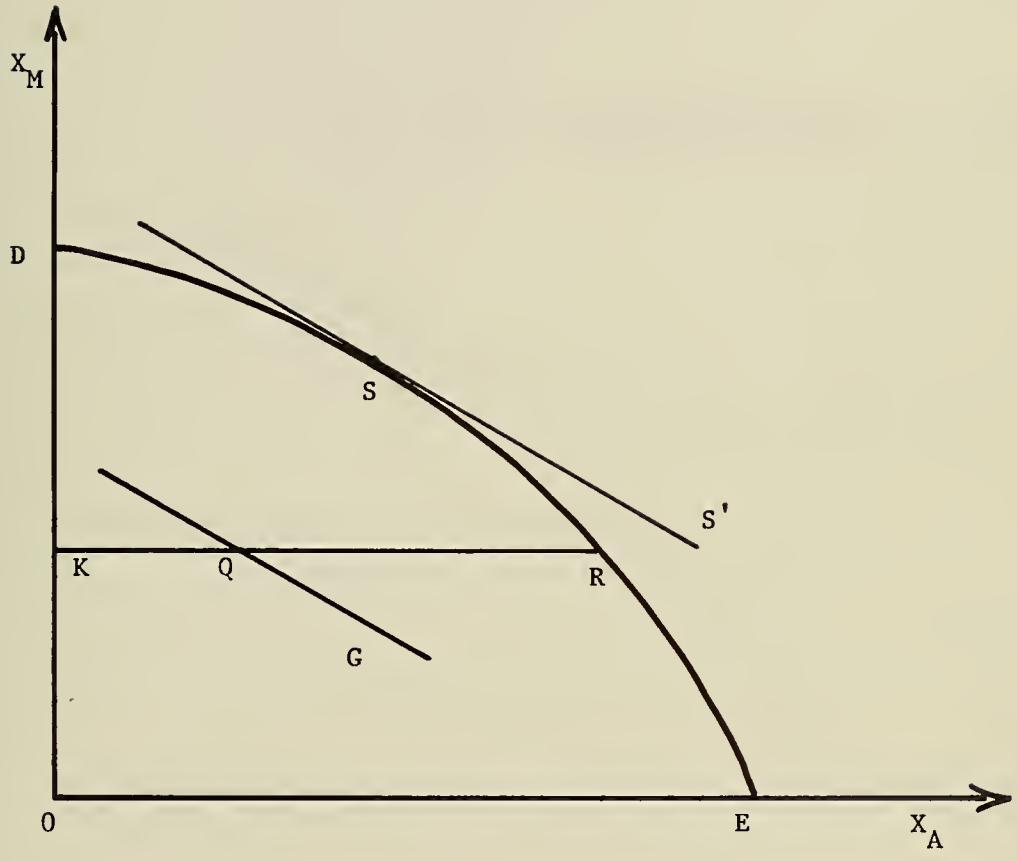

Figure (1)

DE is the production possibility curve when wage rigidity is absent. With the wage rigidity constraint, equilibrium production under laissez faire can lie only along RK instead of RD, because equilibrium on RD (excluding R), as at S, implies wage in manufacturing below the minimum wage. Q is the laissez-faire production point under price-ratio QG, under the wage constraint. For simplicity, the diagram depicts the price-ratio at S and Q to be identical, implying either a linear utility function for a closed-economy or a "small," open economy with unchanging terms of trade. The formal analysis in the text is not restricted to linear utility functions; but it does not apply, without amendment, to a "large" open economy with monopoly power in trade.

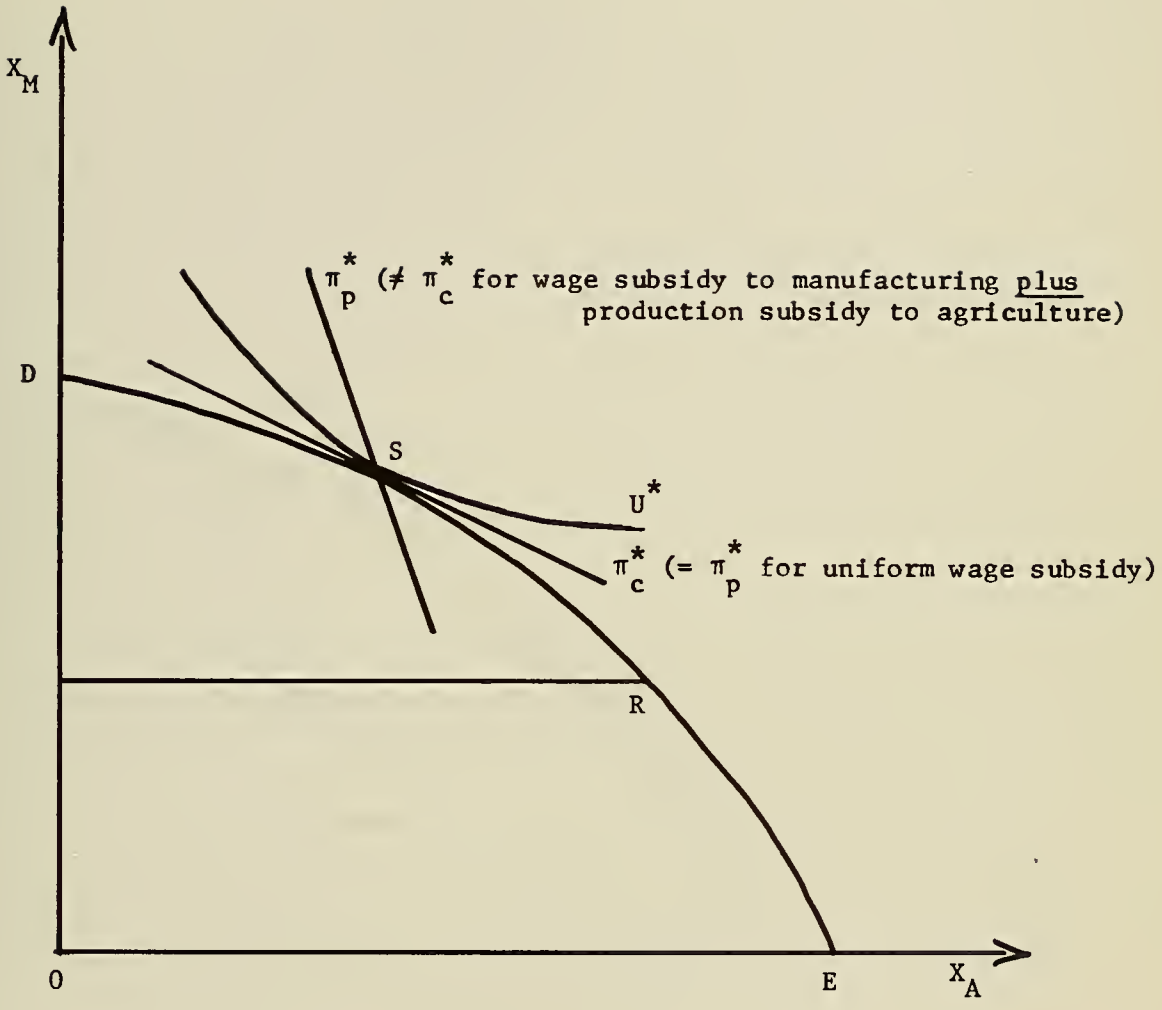

Figure (2)

S is the first-best, optimum for a closed economy, with the social utility curve U^* tangent to the production possibility curve DE. A suitable, uniform wage subsidy to both sectors, A and M, will equate the consumption and production prices with the domestic rates of transformation in production and substitution in consumption at S. A suitable wage subsidy to manufacturing plus production subsidy to agriculture will not equate the consumption and production prices but will equate the two rates of substitution in consumption and transformation in production at S with each other and with the consumption price alone.

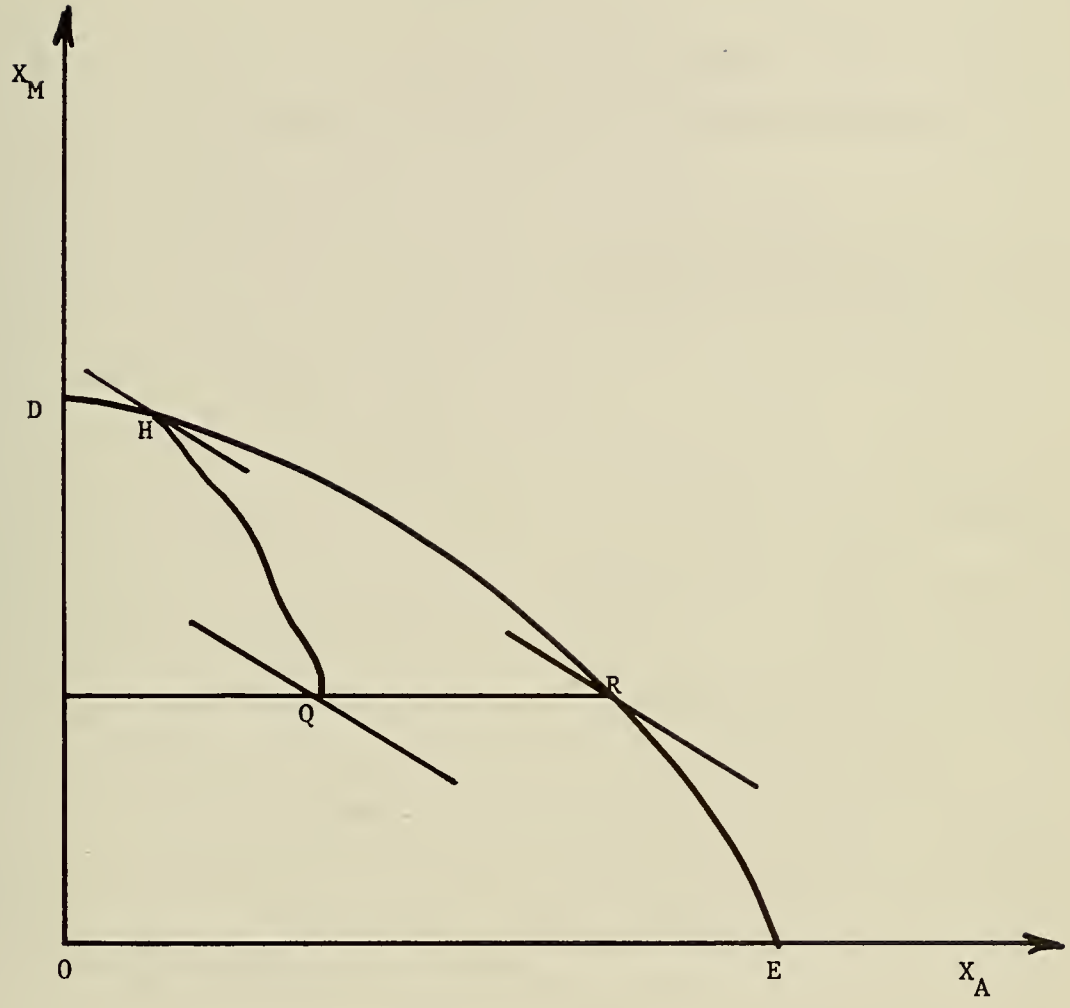

Figure (3)

QH is the locus of production equilibria, traced out by increasing the wage subsidy in manufacturing from s(0) to s_{max} yielding full-employment at H. QR is the locus of production equilibria, traced out by increasing the production subsidy to agriculture.

Appendix

In this Appendix, we produce numerical examples to show that:

(A): A full-employment-yielding wage subsidy in manufacturing may be inferior to laissez faire.

(B): The second-best wage subsidy in manufacturing may be inferior or superior to the second-best production subsidy to agriculture: the two policies cannot be uniquely ranked.

Let us consider the following production and utility functions: $f_A(L_A) = L_A^{0.75}$, $f_M(L_M) = L_M^{1/2}$, $U = pX_A + X_M$. Let p take two alternative values 1.5 and 0.5. Let the specified minumum wage (in terms of manufactured good) in manufacturing be twice the equilibrium wage associated with the first-best optimum. The following table gives the equilibrium factor allocations, output and welfare associated with each of the following policies: (1) first-best optimum, (2) laissez faire, (3) second-best wage subsidy to manufacturing, (4) full employment wage subsidy to manufacturing, and (5) second-best production subsidy to agriculture.

It is seen that when p = 0.5, the second-best optimum wage subsidy to manufacturing happens to be the full-employment wage subsidy, and it dominates the second-best production subsidy (to agriculture) whereas, when p = 1.5, the second-best production subsidy dominates the second-best wage subsidy. Further, the full-employment wage subsidy is inferior to laissez faire.

		p = 1.5	p = 0.5
Minimum Wage = $(L_M^*)^{-1/2}$		2.363709	1.119195
First-Best Optimum	L_A	0.821017	0.201660
	X_A	0.862510	0.300929
	L_M	0.178983	0.798340
	X_M	0.423064	0.893499
	U	1.716828	1.043963
Laissez Faire Equilibrium	L_A	0.908222	0.499286
	X_A	0.930345	0.593967
	L_M	0.044746	0.199585
	X_M	0.211532	0.446749
	U	1.607048	0.743733
Second-Best Wage Subsidy Equilibrium	L_A	0.904517	0.012604
	X_A	0.927497	0.037617
	L_M	0.046600	0.987396
	X_M	0.215869	0.993678
	U	1.607114	1.012486
Full-Employment Wage Subsidy Equilibrium	L_A	0.051314	0.12604
	X_A	0.107814	0.037617
	L_M	0.948684	0.987396
	X_M	0.974004	0.993678
	U	1.135726	1.012486
Second-Best Production Subsidy Equilibrium	L_A	0.955254	0.800415
	X_A	0.966249	0.846226
	L_M	0.044746	0.199585
	X_M	0.211532	0.446749
	U	1.660906	0.869862

References

Bhagwati, J., 1968, The Theory and Practice of Commercial Policy, Frank Graham Memorial Lecture (1967), Princeton, Special Papers in International Economics, No. 8.

Bhagwati, J. and T. N. Srinivasan, 1973, "The Ranking of Policy Interventions Under Factor Market Distortions: The Case of Sector-Specific Sticky Wages and Unemployment," Sankhya.

Brecher, R., 1970, Minimum Wages and the Theory of International Trade, Ph.D. Thesis, Harvard University.

Haberler, G., 1950, "Some Problems in the Pure Theory of International Trade," Economic Journal.

Harris, J. and M. Todaro, 1970, "Migration, Unemployment and Development: A Two-Sector Analysis," American Economic Review.

Johnson, H. G., 1965, "Optimal Trade Intervention in the Presence of Domestic Distortions," in Baldwin, et.al., Trade, Growth and the Balance of Payments (Essays in Honor of G. Haberler), North-Holland.

Lefeber, L., 1971, "Trade and Minimum Wage Rates," in J. Bhagwati, et.al., (ed.) Trade, Balance of Payments and Growth (Essays in Honor of C. P. Kindleberger), North-Holland.

Date Due

DEC 15 '75

OCT 20 '76

DEC 10 '76

JAN 01 '77

JAN 01 '77

MAR 25 '77

JUN 02 '77

NOV 17 '77

DEC 27 '77

AUG 04 '78

SEP 28 '78

DEC 5 '79

MAY 13 '79

JUN 12 '79

MAR 29 '80

MAR 19 '81

JUL 8 '88

OCT 18 '88

DE 27 '88

NOV 14 1991

NOV 23 2007

Lib-26-67