On Inflation and Output with Costly Price Changes: A Simple Unifying Result

Roland Benabou
Massachusetts Institute of Technology and National Bureau of Economic Research

Jerzy D. Konieczny
Wilfrid Laurier University

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by

Roland Benabou*
Massachusetts Institute of Technology
and
National Bureau of Economic Research

and

Jerzy D. Konieczny**
Wilfrid Laurier University

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* Department of Economics
Massachusetts Institute of Technology
Cambridge, MA 02 139
USA

** Department of Economics
Wilfrid Laurier University
Waterloo, Ont., N2L 3C5
Canada
I. Introduction.

We study the effect of inflation on the average output of monopolistic firms facing fixed costs of changing their nominal prices. The slope of this Phillips curve has been analyzed by several authors, usually with specific functional forms. As a result, the literature offers a somewhat confusing array of special results. In this paper we derive a simple, explicit formula which unifies all the effects identified in earlier literature.

Rotemberg (1983) finds, for constant elasticity demand functions and quadratic costs, that the average of log-output is not affected by inflation. Kuran (1986) finds effects of opposite signs for nonincreasing elasticity and concave increasing elasticity demand functions, assuming constant marginal costs. Naish (1987) uses linear and constant elasticity demand and cost functions to demonstrate the important role played by asymmetries in the profit function around its peak. Konieczny (1990) shows, more generally, that the effect depends not only on the skewness of profits, but also on the curvature of the demand function. He does not, however, provide a way of determining which of these two factors dominates and incorrectly claims that, at small inflation rates, the profit effect is negligible. Finally, Danziger (1988) stresses the role played by discounting at small inflation rates.

This paper provides a unifying but very simple treatment of all the effects described above, for the case where the costs of price adjustment are small. This framework, which seems to us the most relevant, allows the use of Taylor expansions to obtain a closed form solution (Proposition 2), which can easily be applied to any specification.
II. The Model.

We consider a monopolistic firm which produces a single, perishable good and expects the inflation rate to remain constant over time.\(^1\) The firm can change its price at any time but it incurs a cost, \(c > 0\); the new price must be decided upon, the information disseminated, etc. \(c\) also proxies for the adverse reaction of customers and competitors, not captured by the model.

It is convenient to express profit and demand functions in terms of the log of the real price. The firm's problem, at the time of the first price change, is to choose the sequence of times of adjustment, \(\{t_\tau\}\) and (logs of) real prices set at each adjustment, \(\{P_\tau\}\), so as to maximize the present value of its profits:

\[
V = \sum_{\tau=0}^{\infty} \left[ \int_{t_{\tau-1}}^{t_\tau} F(P_\tau - g(t-t_\tau)) e^{-rt} dt - c e^{-rt} \right]
\]

where \(F\) is the profit function, assumed to be strictly quasiconcave and \(C_3\); \(g > 0\) is the inflation rate and \(r\) is the discount rate. We shall denote by \(z^m\) the (log) real monopoly price, assuming: \(F(z^m) > 0 - F'(z^m) > F''(z^m)\). The optimal pricing policy is of the \((s,S)\) type (Sheshinski and Weiss, 1977): at each price change the firm chooses the nominal price so that the (log of) real price is \(S > z^m\); a new change occurs when inflation has eroded the real price to \(s < z^m < S\).\(^2\)

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\(^1\) Benabou (1991) analyzes firms which compete through customer search. In this case demand and profit functions are endogenous and the effect of inflation on output also depends on characteristics of the market such as search costs.

\(^2\) The case of deflation is obtained by permuting \(S\) and \(s\) throughout the paper.
The optimal \((s,S)\) policy satisfies the first order conditions:

\[(2a) \quad F(S) - rc = F(s)\]
\[(2b) \quad F(s) = rV\]

Strict quasiconcavity implies that the optimum is unique and satisfies:

\[(3) \quad F'(s) > 0 > F'(S) ; \quad dS/dg > 0 > ds/dg\]

We now consider an economy consisting of many such firms which stagger their price changes uniformly over time. Aggregate output is then equal to the average output of a firm over its price cycle:

\[(4) \quad \bar{Y} = \frac{g}{S-s} \int_0^{(S-s)/g} y(S-gt) dt = \frac{1}{S-s} \int_s^S y(z) dz\]

where \(y(\cdot)\) is the demand function, \(y'(\cdot) < 0\).

Differentiating (4) and rearranging, we obtain:

\[(5) \quad \frac{d\bar{Y}}{dg} = \frac{2}{F'(s)(S-s)} \frac{dS}{dg} \left\{ \frac{y(s) + y(S)}{2} - \bar{Y} \right\} F'(s) - \left[ \frac{F'(s) + F'(S)}{2} [y(s) - \bar{Y}] \right]\]

Since both \(F'(s)\) and \(y(s) - \bar{Y}\) are positive, (5) makes clear that the effect of inflation on output depends on:

(i) the skewness of the profit function (represented by the asymmetry in marginal profits, \(F'(s) + F'(S)\)) which, through (2), determines the effect of inflation on the price bounds, \(s\) and \(S\); this will be called the profit effect;
(ii) the curvature of the demand function (represented the output difference \( y(s) + y(S) - \bar{Y} \)) which determines the output effects of those changes; this will be called the demand effect.

Following Konieczny (1990) we call \( F(\cdot) \) strongly left skewed if \( F'(z_1) < -F'(z_2) \) for every \( z_1 < z^m < z_2 \) such that \( F(z_1) = F(z_2) \), and strongly right skewed if the opposite inequality holds. If \( F(\cdot) \) is strongly left skewed then, as inflation rises, \( S - z^m \) increases slower than \( z^m - s \). As a result, each price cycle is skewed towards lower values, so that the profit effect tends to increase output (see (5) as well as (11) below). The opposite holds for \( F(\cdot) \) strongly right skewed.

While (5) reveals the qualitative effects at work, its implicit nature makes it rather opaque and unwieldy. Below we therefore derive, for the case of small \( g \) and/or \( c \), a simple, explicit formula which shows the effect of all underlying parameters.

**III. Small Adjustment Costs or Low Inflation: A Simple Formula.**

We now concentrate on the most relevant case, where \( g \) or especially \( c \) is small, and use Taylor expansions to derive closed-form solutions for \( s \), \( S \) and \( d\bar{Y}/dg \). To simplify the exposition we first consider the case where \( r = 0 \); a positive discount rate is incorporated in Section V below. The first order conditions become:

\[
\begin{align*}
(2a') & \quad F(s) - F(S) \\
(2b') & \quad F(s) - \bar{F}
\end{align*}
\]

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3 Numerical simulations in related models (Benabou (1991) and Dixit (1991)) show that such approximations are generally quite accurate over a wide range of parameters.
where $\bar{F}$ denotes average profits per unit of time, net of price adjustment costs. By $(2')$, both price bounds, $s$ and $S$, approach the monopoly price as $c$ or $g$ approach zero. We therefore define:

$$S - z^m = \delta \ll 1 \quad \text{and} \quad z^m - s = \tau \approx \alpha_1 \delta + \alpha_2 \delta^2$$

where, throughout the paper, $X \approx \sum_{i=0}^{n} \alpha_i x^i$, $\alpha_n \neq 0$, represents a Taylor expansion of order $n$ in $x$, i.e. stands for $X - \sum_{i=0}^{n} \alpha_i x^i < x^n$. Thus $\alpha_1 + \alpha_2 \delta$ represents the relative rate at which $\tau = z^m - s$ goes to zero in comparison to $\delta = S - z^m$, when $c$ or $g$ become very small.

Now, for all $z$ in $[s,S]$:

$$F(z) \approx F(z^m) + \frac{F''}{2} (z^m)(z-z^m)^2 + \frac{F'''}{6} (z^m)(z-z^m)^3$$

Expanding $(2a')$ to the third order in $\delta$ leads to $\alpha_1 = 1$, $\alpha_2 = -2a/3$, where $a \equiv -F'''(z^m)/2F''(z^m)$ measures the skewness of the profit function around $z^m$. Thus:

$$z^m - s \approx \delta (1 - 2a \delta /3)$$

Next, inserting $(6)$ and $(7)$ into $(2b')$ gives the following expression for $\delta$:

$$S - z^m \equiv \delta \approx \left[ \frac{3c g}{-2 F''(z^m)} \right]^{1/3}$$

as in Mussa (1981) and Rotemberg (1983); its interpretation is straightforward.

Finally, we return to output. Using $(7)$-$(8)$ to expand $(4)$ we get:
\[
\bar{Y} \approx y(z^m) + y'(z^m) a \delta^2 / 3 + y''(z^m) \delta^2 / 6
\]

and so:

\[
(9) \quad \bar{Y} \approx y(z^m) - y'(z^m) \frac{\delta^2}{6} \left[ \frac{F'''(z^m)}{F''(z^m)} - \frac{y''(z^m)}{y'(z^m)} \right]
\]

Proposition 1.

Let \( r = 0 \); then, for small adjustment costs or/and inflation rates \((g_c < - F''(z^m))\):

\[
(10) \quad \frac{d\bar{Y}}{dg} \approx A g^{-1/3} \left[ \frac{F'''(z^m)}{F''(z^m)} - \frac{y''(z^m)}{y'(z^m)} \right]
\]

where \( A = -y'(z^m) \left[ -\frac{c}{18F''(z^m)} \right]^{1/3} > 0 \).

IV. Interpretation and Examples.

The use of Taylor approximations yields explicit measures of the effect of skewness of profits and the curvature of demand on the average output. Assuming that \( F'''(z) \) does not change signs in the neighbourhood of \( z^m \), \( F(\cdot) \) is strongly left (right) skewed near \( z^m \) if \( F'''(z^m) < (>) 0 \). Thus two measures of the effect of skewness on price bounds are, from (6)-(8):

\[
(11) \quad \frac{s + S}{2} - z^m \approx \frac{a \delta^2}{3} \quad \text{or} \quad f'(s) + f'(S) \approx \frac{4a \delta^2}{3}
\]

The effect of changes in the price bounds on average output depends, in turn, on the curvature of the demand function. A measure of this effect is, from (7)-(9):
By Jensen's inequality, the demand effect tends to increase output if the demand function is convex, and to decrease it if demand is concave.

Proposition 1 shows how the net effect of inflation depends on the relative strength of the profit and demand effects. Konieczny (1990) claims that, at low inflation rates, the profit effect is negligible as compared to the demand effect, because the profit function is (log) quadratic, hence symmetric up to a third order approximation. It is clear from (11) and (12) that this is not correct: both effects are of order $\delta^2$. This is due to the fact that the relevant asymmetry is not that of profits, but of marginal profits, as can be seen from the term $F'(S) + F'(s)$ in (5).

We conclude this discussion by identifying the two effects for some of the most commonly used functional forms.

When the demand function is linear (in the log of real price) the demand effect is zero and so $\text{sign}(d\overline{Y}/dg) = \text{sign}(F'''(z^m))$. If the profit function is (log) quadratic the profit effect is zero and $\text{sign}(d\overline{Y}/dg) = \text{sign}(y''(z^m))$. Output is not affected by inflation if both conditions are met; this is the special case considered by Rotemberg (1983).

For demand functions linear in the real price: $y(z) = A_1 - A_2 e^z$, $A_1 > 0$, $A_2 > 0$, and either constant or linear marginal costs the profit effect dominates the demand effect, as the former equals $3A$ and the latter is $-A$; thus $\frac{d\overline{Y}}{dg} = 2A > 0$.

For isoelastic demand and cost functions: $y(z) = A_1 e^{-\beta z}$, $\Phi(y) = y^a$ where $\Phi(\cdot)$ is the cost function and $\alpha > 0$, $\beta > 1$, the profit effect is $A(1 - \beta - \alpha \beta)$; the demand effect is $A \beta$ and so $d\overline{Y}/dg = A(1 - \alpha \beta)$. The profit effect tends to decrease output while the
the demand effect operates in the opposite direction. The profit effect is stronger for \( \alpha \beta > 1 \); this is true in particular for constant and for increasing marginal costs \( (\alpha \geq 1) \). For the demand effect to dominate, marginal cost must decrease fast enough \( (\alpha < 1/\beta) \).

V. A General Formula with Discounting.

When the discount rate is positive, firms care more about profits closer to the beginning of each cycle than those in the later phase. Therefore, in comparison to the case considered in section III, they set the initial real price, \( S \), closer to the profit maximizing level, \( z^m \). The price is then allowed to deteriorate more before the next adjustment.\(^4\)

Discounting therefore creates a discontinuity at \( g = 0^+ \), resulting in an upward jump in average output \( \bar{Y} \); see Danziger (1988). As to \( d\bar{Y}/dg \) (at any given \( g > 0 \)), it is negatively affected by discounting. This is because, by (2), \( F'(S) dS/dg = F'(s) ds/dg \).

Since discounting makes \( S \) closer to \( z^m \), for small \( c \) or \( g \) it reduces \( |F'(S)| \) relative to \( F'(s) \). Therefore, as the inflation rate increases, the initial price increases more than the terminal one falls. This raises the average price and lowers average output (Danziger (1988), Konieczny (1990)).

To show precisely how discounting combines with the profit and demand effects we generalize Proposition 1 to the case of a positive discount rate. We let \( g \) take any given positive value, and consider the case where \( c \) is small. (2b) can be rewritten as:

\[
(13) \quad \int_s^S F(z) e^{(z-z^m)r/g} dz - F(s) \left[ e^{\delta r/g} - e^{-\tau r/g} \right] r/g - c g e^{\delta r/g}
\]

\(^4\) In particular, as \( g \to 0 \), \( S \) tends to \( z^m \) while \( s \) remains bounded away since \( F(S) = F(s) + rc \).
where, as before, \( S - z^m = \delta < 1; \) \( z^m - s = \tau \approx \alpha_1 \delta + \alpha_2 \delta^2. \) Expanding again the first order conditions (2a) and (13) to the third order in \( \delta \) we obtain: \( \alpha_1 = 1, \ \alpha_2 = (2/3)(r/g - a). \) Hence:

\[
\tau \approx \delta + \frac{2}{3} \left( \frac{r - a}{g} \right) \delta^2 \quad \text{or} \quad z^m - s + S \approx \frac{\delta^2}{3} \left( \frac{r}{g} - a \right)
\]

where \( \delta \) is still given by (8). Expanding average output as before leads to:

\[
\bar{Y} \approx y(z^m) - y'(z^m) \frac{\delta^2}{6} \left( \frac{2r}{g} + \frac{F'''(z^m)}{F''(z^m)} - \frac{y''(z^m)}{y'(z^m)} \right)
\]

Equations (14)-(15) clearly show that positive discounting lowers the average price and raises average output, as argued above. As to its effect on the slope of the output-inflation tradeoff, it is given by:

**Proposition 2.**

If the cost of price adjustment is small \( (gc < -F''(z^m) \text{ and } rc < -F''(z^m)) \), then:

\[
\frac{d\bar{Y}}{dg} \approx A g^{-1/3} \left( \frac{F'''(z^m)}{F''(z^m)} - \frac{y''(z^m)}{y'(z^m)} - \frac{r}{g} \right)
\]

where \( A \) is the same as in Proposition 1.

This result embodies all three factors which affect the slope of the inflation-output tradeoff, solely in terms of the underlying parameters of the model. It incorporates

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\(^5\) A detailed proof is available from the authors upon request.
Proposition 1 as a special case and also shows that, for small enough inflation rates, output falls with inflation regardless of the strength of the demand and profit effects.

VI. Conclusions.

We analyzed the effect of inflation on the average output of monopolistic firms facing a small fixed cost of changing nominal prices. Using Taylor expansions, we derived a closed form solution which can be applied to any specification. This extremely simple formula allows us to evaluate the relative impact of the three factors which affect the inflation-output tradeoff: the asymmetry of the profit function, the convexity of the demand function, and discounting. It thus provides a unifying framework which substantially clarifies the previous results of this literature.
References:


