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ON SILENT SPRINGS AND MULTIPLE ROOTS
Cost-Benefit Methods and Environmental Damage

Matthew D. Edel

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massachusetts
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50 memorial drive
Cambridge, Mass. 02139
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The views expressed in this paper are the author's sole responsibility and do not reflect those of the Department of Economics, nor of the Massachusetts Institute of Technology.
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Cost-Benefit Methods and Environmental Damage
by Matthew Edel*

This article considers the application of cost benefit analysis to investments which deteriorate the environment in the long run. Internal rates of return with multiple roots, generally considered theoretical curiosities, exist for these investments, and may provide a useful paradigm for their understanding. Theoretical consideration of investment criteria for the multiple roots case is presented, along with a classification of the forms of damage which may give rise to multiple roots.

Journalistic and scientific discussions of "ecological catastrophe", pollution, slum formation, and other problems of the human ambiance have focused popular concern on the environment. Yet economic analysis has only been applied to these issues to a slight degree. The economic theory of environmental damage, currently based on two powerful concepts - externalities and population pressure - is still far from complete.

Models of overpopulation allow comprehension of many ecological problems. An unfavorable balance between man and space or natural resources is at the root of many of these difficulties, either directly through exhaustion of or diminishing returns to resources, or indirectly through the negative effects of technologies originally designed to offset diminished returns. However, both simple land-constraint systems and low-level equilibrium traps lead to conclusions uniformly favorable to any production-increasing technologies. If, however, increased production depends on chemicals which damage the environment, a crisis not predictable from
Ricardian or Malthusian models may occur. A means to consider these temporary offsets to population pressure is needed.

Models of external diseconomies fill part of this need. The failure of utility-maximizing or profit-maximizing individuals to spill over unto others as they would have others spill over unto them explains much in the way of congestion, neighborhood decay, pollution of streams and air, and underinvestment in public goods including conservation projects. These models point to what may be a necessary condition for the prevention of further blight and possible ecological catastrophe: the imposition of controls and/or managed incentives (fines or subsidies) by society over individual decision makers, or the internalization of externalities through direct social ownership of potentially damaging industries or activities. However the explanation and the proposed solution are not sufficient. Men have been known to let their own farms erode, public transport may pollute the air, and water pollution has appeared despite state ownership in the Soviet Union. Even where protection of the environment is a public goal and government has the power to act, problems remain of which future damages should be avoided at the expense of current consumption or other forms of material accumulation.

To determine which external diseconomies to avoid, or whether some offset to diminishing returns is too costly in the long run, requires some means of imputing values to future side effects. This requires discounting future effects to present value terms or, alternatively, comparison of the alternatives in terms of rates of return. The flows of future effects, in this case, take on particular form: they may be highly positive in some near future (usually after an initial negative period of
investment), but after some future point, they become negative. Discounting such flows raises conceptual problems, notably the non-uniqueness of the rate of return. These problems have been discussed sufficiently for their treatment as the theoretical curiosity they have been considered. J.F. Wright, who has analyzed them most explicitly, states that "the multi-solution case is rather unlikely." Their further discussion here may be justified by the commonness of final flows of negative returns in conservation problems.

II

Delayed environmental damage can take several forms. Their enumeration not only demonstrates the occurrence of multiple roots; it can also suggest an organization for theoretical and empirical research in environmental economics. A lack of information, as much as lack of theory, has held back research in this area. With the increase in scientific investigation of ecological problems, more data will undoubtedly be generated, but a method of ordering this information for economic research will still be needed. A typology is a first step toward such a framework. At least three forms of environmental damage—contamination, overdurability and depletion—appear distinguishable. Investments producing these may in turn be considered part of a wider range of investments which produce inflexible outputs, and whose consequences have only recently come under investigation through the use of "putty-clay" vintage models of growth. These models, however, generally assume old capital can be scrapped without cost, and that external effects are not created in any proportion to the productive process. A classification of environmental damage may
thus also have some use in directing theoretical economics.

A. CONTAMINATION. When a productive process introduces some damaging materials into the environment, or creates other damaging side effects, contamination may be said to occur. Air and water pollution, the breeding of pests, and the raising of noise levels can all be treated within a static model of external diseconomies. However, many of the effects of contamination are delayed, creating a need for intertemporal comparison. For example, the causes of air pollution interact cumulatively. Different chemicals may accumulate slowly in the air until critical levels are reached, or they may undergo photochemical reactions in the atmosphere, eventually producing dangerous gasses.

The effects of pollution on health may also be delayed. This is true of carcinogenic agents, and of other damaging elements as well:

Chronic bronchitis, which in great Britain is established as a specific disease entity, is a good example. It develops over a long period of time, and can become crippling through a combination of many factors—air pollution, smoking, repeated and recurring bouts with infectious agents, occupational exposures—all affected, perhaps, by an hereditary predisposition. ...There is probably no single cause, but there is sufficient evidence that air pollution can and does contribute to its development.  

In the very long run, the accumulation of pollutants in the air may have even more dire ecological consequences. It has been suggested that rising levels of carbon dioxide due to increased fuel consumption, may cause a "greenhouse effect", trapping heat in the atmosphere, with serious effects on climate.
Water pollution, also, may work only slowly. Chlorinated hydrocarbons used as insecticides may accumulate slowly to a point at which they begin to destroy life. Thermal pollution, or chemical contamination may also build up slowly. Sometimes initial pollution will cause abnormal growth of aquatic vegetation.

As these plants die and decay, 'secondary' pollution may affect the taste and odor of the water. In certain types of algae 'blooms', a strychnine-like poison may be generated.6

Delayed damage characterizes other types of contamination as well. Radioactivity is an obvious example, but even "noise pollution", which might at first appear totally non-durable, may give rise to negative terminal benefits to an investment, if it is repeated exposure to high noise levels which causes physiological or psychological damage.

B. OVERDURABILITY. When the capital used in a productive process itself becomes a costly nuisance over time, the result may be termed overdurability. Conceptually, overdurability may be distinguished from contamination as follows: If the continued existence of a capital good which may not be removed (except perhaps at a high cost) causes damage after the capital ceases to be used, this is overdurability, whereas in contamination as defined here, the causing of new damage is dependent on continued use of the capital in production. In practice, the line between the two may not always be clear: whether slag heaps, tin cans or automobile graveyards are examples of overdurability or contamination might depend on a definition of capital. However, the distinction between the
two forms of damage may have some use both in the specification of alternative growth models, and in determining when a tax on production might provide a proper incentive to avoid damage.\(^7\)

The best known example of overdurability is slum formation. Old buildings may continue to earn some commercial return even when their external costs exceed private rent, and even if totally unutilized, they are costly to demolish. Furthermore, the proximity of other aging and unsightly buildings reduces the incentive to demolish for new construction and obsolescent transportations and location patterns.\(^8\)

Overdurability of human capital may also be possible. The skills and habits necessary for the operation of an industry may later make its workers less easily trainable for new activities than would be totally untrained workers. This effect has been described frequently in the extreme cases of coal mining and slave-plantation agriculture.\(^9\) Similarly, members of obsolete occupations have at times imposed costs on the rest of society through a political claim on subsidies, or, in the case of unemployed military castes, posed threats to the overall functioning of society. There are, of course, instances on record where members of groups which had lost old functions turned to new entrepreneurial activities in a highly productive manner. But the overdurability of skills and attitudes may still be a cost found today. The current debate over the conversion of military facilities and weapons engineering skills to civilian uses is a case in point, as are criticisms of formal education which charge that it develops narrow habits of obedience or special-
isation, which increase short-term productivity at the expense of adapt-
ibility in the long run.¹⁰ Institutional overdurability is also pos-
sible: those social or political forms which foster development at one
point may impede it later on.

C. DEPLETION The depletion of natural resources, so that they are
not available for later production, takes different forms depending upon
whether the resources are renewable if used at less than some critical
rate (as are soils, forests, or animal species), or nonrenewable, as are
fossil fuels and nonrecuperable minerals. Destruction of renewable re-
sources always may be described in terms of a flow of benefits which is
positive at first, but becomes highly negative over time.

Soil erosion, deforestation, the creation of dust bowls and the
leaching and laterization of tropical soil all result from overexploita-
tion of land. In any of these cases, some level of productivity increase
is probably consistent with conservation, but some increase in production
beyond a point of safety will leave eventual yields below the level that
existed before the first investment was made. Waterlogging and salinity
that can result from irrigation projects are other examples.¹¹

The destruction of species can occur not only through overfishing
or unconstrained hunting. It may also occur through side effects of in-
secticides which destroy not only pests but also their natural predators,
allowing a later infestation of pests, as occurred in one well-known case
in the Cañete Valley of Peru. Even the successful breeding of new crops,
and the widespread distribution of high-yield hybrids, may lead to the
abandonment of planting of old varieties, thus extinguishing many genetic
strains which might at some later time have been useful for breeding purposes, if for example they possessed resistance to diseases not present where the original hybrid was developed. 12

While in any of these cases, increased utilization of resources beyond a certain point may lead to such severe reduction in later production that the discounted social value of the project is negative at all but some extremely high discount rate, this is not likely to be true in the case of nonrenewable natural resources. If the total supply of a mineral or a fossil fuel is limited, more production in the short run will lead to less production being possible later on. Returns will be positive once the initial investment is completed but will drop to zero sooner if initial exploitation is more rapid. Thus the stream of benefits from increasing initial production may have multiple roots. However, if the total amount of production over all time is not changed by more rapid mining, it will only be at a negative discount rate that the present value of the stream of benefits would turn negative. At any positive discount rate, the same amount sooner is better than the same amount later, and only the initial mining costs could offset the incentive to produce more faster.

In the case of nonrenewable resources therefore, a project approved at some positive interest rate cannot be rejected at a lower positive discount rate unless the earlier more rapid production involves a lower value use of the product than that under slower exploitation. This may occur if there are diminishing returns to use of the material within any time period, if more rapid extraction increases waste, or if technological change or population and industrial growth make the need for
the material greater in later periods. However, technological development of substitutes or of means of reusing scrap may be an offsetting technological force. In these cases, too, the later costs may be taken into account in private decisions to produce, because they will be reflected by expected price increases, unless either later needs for the resource take the form of increased external economies from its use, or increased waste through rapid extraction is not an internal diseconomy to the producer. Under these circumstances, an increment to production may have an internal rate of return greater than the market interest rate, but have a negative present value at a lower social discount rate. This situation may not be common. Existence of a nonrenewable resource may not call for regulation or for penalty taxes, except in special cases. Even the normal case, however, does not call for subsidies to speed depletion, such as now exist in the United States.

III

The frequency of benefit streams which turn negative after a number of years gives reason to re-examine cost-benefit assumptions for the case of multiple roots to the rate of return. As is well-known, the internal rate of return for marginal efficiency of capital is defined as that rate of discount \( r \) which equates present value (PV) to zero, when in each year \( t \), there is a flow of benefits less costs \( V_t \) and

\[
(1) \quad PV = \sum_{t=0}^{t=\infty} \frac{V_t}{(1 + r)^t}.
\]

If \( V_t \)'s are first negative, and thereafter all positive, then at discount rates
greater than the internal rate of return, PV will be negative, while for lower discount rates, present value is positive.

If however, the stream of benefits is first negative, then positive, and then negative again, setting PV equal to zero and solving for r yields an equation with two roots, which may or may not be real. If two unequal real roots exist, they are the limiting values of the range of interest rates within which the project has a positive present value. At higher rates than the greater root of r, the first years of negative returns will have so much weight as to make the project uneconomical. However, at rates below the lesser root of r, the years farthest into the future will enter so heavily as to make the value negative again. Only in the intermediate range will the years of positive benefits dominate and the project be approved.

The case of multiple roots has been described in the literature primarily as an argument for using present values rather than an internal rate of return to compare investments. Thus Hirshleifer, Feldstein and Flemming, who argue for present value criteria mention the possibility of multiple roots. Wright and Henderson, in supporting the internal rate of return, argue that multiple roots are unusual.

The existence of multiple roots need not however be an argument for the present value approach. Given two real roots, use of the present value calculation at some given discount rate may be the more deceptive method, in that normally stipulation of one such rate indicates that projects with positive present values at that rate would be advisable also at lower rates. An internal rate calculation which presented both upper
and lower bound rates might prevent inadvertent ignorance of the consequences of an overestimated discount rate.

Multiple roots have also been cited as possible on discounting streams of differences between the returns to two projects, only one of which may be constructed. The roots of such streams, Irving Fisher's "rate of return over costs", may be multiple. For such cases, Feldstein and Flemming have pointed out that the project preferable at a higher rate of discount may not be the one preferred at a lower rate. For that reason, they argue that in present value calculations, if ambiguity is to be avoided, the discount rate must be precisely specified. For example, if the discount rate is meant to reflect 'social time preference' one might be tempted to put limits on it and say that at least one can be sure that what is acceptable at the upper limit is admissible, but this is not true. If social preference is really lower in the range it might require that the project be rejected. This consideration will hold in any case of multiple roots, but Feldstein and Flemming did not consider any case but the return on the difference between two investments. Their formulation thus leaves open questions of what discount rate should be specified, for calculation of the present value or for comparison with the internal rates of return, in cases of environmental depletion.

IV

The evaluation of flows of benefits which eventually become negative raises different problems depending on whether or not a permanent social rate of discount is agreed to independently of the market interest rate, and whether or not total investment must be rationed.

A. In the simplest case, suppose the level of investment is variable,
and that some social discount rate has been determined by a political consensus which binds future generations, or is established on the basis of some ethical principle or technocratic consideration. That such a rate can ever be agreed to is by no means certain. If it is established, then it can easily be used to discount the flow of returns, including benefits, and external costs can be discounted at the social rate of time preference. Positivity of the present value is the criterion for acceptance. If there is doubt as to the appropriateness of the social rate, sensitivity analyses may be conducted, with the proviso that both upward and downward variation must be investigated. (Alternatively, the two roots of the internal rate of return may be calculated, for consideration of whether the social rate of discount lies between them).

If the decision to be considered is not a government or public investment, but rather the degree to which a private investor must be fined or forced to compensate the public for future damages his investment will do, these damages should be discounted at this social rate, to induce a private decision in accord with social considerations at the time the investment is made.

Thus far the analysis is similar to any other discounting procedure. However, differences between the present case and the single root case arise in the treatment of risk and uncertainty. These elements are important in any decision concerning environmental hazard. Much scientific work on contamination and depletion is still highly conjectural. In this situation, however, the familiar means of taking account of risk, by raising the shadow
discount rate, will be counterproductive. A normal risk premium will let more projects through. One solution might be to lower the social rate of discount used in the calculations, which might entail use of a negative discount rate. Where the social rate is used merely to veto projects (as in the capital rationing situation described below), this reverse risk premium may be appropriate. However if present value, or present value per dollar invested, is to be the principal means of evaluating projects, this approach could lead to grave distortions when equal risk deductions are applied to projects with unequal lives.

A conceptually better method of evaluation would impute probabilities to different possible outcomes when risk is involved. If this can be done, it is possible to use as basis for evaluation of each project any of a number of measures: the expected value of the present value (or of the rate of return), the most probable value, the lowest possible value, or a set of possible values each with its probability. Which evaluation method to choose will depend on the circumstances. If the project does not bulk large in the economy, and if no risk of major disaster, is involved, evaluating projects at their expected values will maximize the expected value of the entire investment program. If the probabilities of different outcomes are not really known, and different values posited depend not on different possible states of nature, but rather on different sets of postulates by the scientist or economist, the result pointed to by the largest number of estimating methods might be used. However, when there is a possibility of ecological catastrophe, there would seem to be no rational alternative to risk aversion. Either the minimization of probability of destruction, the choice of the program which will lead to the least damage in the worst
possible case (maximum strategy), or the setting of maximum acceptable probabilities of catastrophes could be an acceptable decision rule in this case. (An equivalent method would involve increasing the negative value imputed to catastrophes, and using expected value criteria.)

E. A second case is that in which ethics or consensus decree that all investments yielding at a given social discount rate would be desirable, but in which funds are not sufficient to allow all such investments. This situation may arise in an economy with a central plan stating that investment should take place on all projects whose returns are positive at the social rate of discount, unless this drives current consumption below some minimum acceptable level. The availability of such a surplus above subsistence, out of current production plus foreign aid, may be a limiting factor in an underdeveloped country, where projects with returns higher than a politically acceptable time preference rate might at any point require an investment many times greater than the GNP for their simultaneous completion. If this occurs, a social opportunity cost of capital, higher than the social discount rate, may exist. A similar divergence between a social rate of discount and a market rate of interest has been posited by Marglin to arise from the admittance of other people's consumption into individual utility functions. 18 The existence of taxes may separate the social opportunity cost of capital from the rate of time preference, as well (although this case yields results somewhat more complex than are discussed below). 19 Finally, rates of return on government and on private investments may also differ in an
economy in which the government may tax or borrow up to some limit.

In any of these cases, a project may have a positive present value at one interest rate and a negative one at the other. For example, when there is a subsistence constraint, if the project's present value is positive at the social rate, but negative at the opportunity cost of capital, the project must be postponed; this is a common occurrence. If, however, there are multiple roots, the value may be positive at the opportunity cost of capital, but negative at the social rate of discount. A decision rule in this situation might reject all projects with negative present values at the social rate of time preference, and then select among the remaining projects according to their rates of return. If some projects had negative benefits in their later years, but these were small enough that the lower root were below the social rate of time preference, the higher root would in these cases be used as the internal rate of return for choices among acceptable projects.

A more accurate criterion might take into account the effect of reinvestability of the higher initial returns from a project which eventually caused environmental damage. Such a compound criterion would be analogous to the compound criteria proposed by Marglin, Feldstein, and others for cases in which private and public reinvestment rates and rates of return differ.20

In the present case, the compound criterion would compare the return on the reinvestment with the later contamination or deterioration caused. On the assumption that the environmental effects enter directly
into utility as does consumption, then if $B(t)$ is the benefit net of internal costs of the project in year $t$; if $c$ is the propensity to consume out of these; if $i$ is the rate of return on alternative investments, or the social opportunity cost; $r$ is the social rate of discount; $D(t)$ the environmental damage in year $t$; and $K$ the initial cost of capital, then:

$$\text{PV} = \sum_{t=0}^{\infty} \left\{ \frac{[cB(t) + (1-c)B(t)(i/r) - D(t)]}{(1+r)^t} \right\} - K.$$  

If all of the output is consumed, the formula would reduce to the simpler

$$\text{PV} = \sum_{t=0}^{\infty} \left\{ \frac{(Bt - Dt)/(1+r)^t}{(1+r)^t} \right\} - K.$$  

On the other hand, if the initial investment considered were made by the government, and government spending itself partially displaced consumption and partially displaced private investment with an annual return of $i$, then Marglin's criterion for opportunity cost could be directly added to the formula. If $c$ also represented the propensity to reduce consumption arising from a dollar of taxes, then $K$ would be multiplied by $[(1-c)(i/r) + c]$ in equation (2).

A similar calculation of returns could be carried out by the government to determine whether projects which private investors sought to carry out should be permitted. Undesirable projects could be vetoed.
under zoning, sanitary or other regulatory powers. In this case, it might be assumed that all of the private investment directed into the project might still take the form of investment, although at lower rates of profit, if the veto were invoked. In that case, the appropriate value of capital to use would be $K$ multiplied by $i/r$, the social value of the returns from each dollar of investment, in the case of the simpler equation (3). The multiplier in the reinvestment case of equation (2) would be $(i/r)[c + (1-c)(i/r)]$.

If the government does not have regulatory power, but can enact a penalty tax on private activities which cause social damage, it may create incentives against projects whose social present value is discounted at $r$, and net of returns to alternative uses of capital is negative. This can be done simply enough by discounting future environmental effects back to the date on which they occur, using discount rate $r$, and levying a tax equal to the present value of the damages at that time. In the case of pollution, in which the nuisances are created by production of different amounts of output, during the life of the project, the investor would use these annual costs as data in determining whether to make the investment. In the case of an investment which caused damage at some future time irrespective of its level of utilization, the tax would have to be levied at the time of construction. Discounting all of the negative effects from the distant future back to the date of construction at the social rate of return does not yield a tax which equalizes the private and the social present discounted values of the project, inasmuch as the private investor will be discounting his gains at the market interest
rate, and comparing these with the tax. Therefore the optimality of the tax may not be obvious. However, it does equalize private discounted value with the social discounted value plus the normal gap between private and public returns which exists in this case of dual discount rates. Thus optimal allocation from the social viewpoint will be induced.

The levying of a tax at the time of construction or production, rather than at the time the damage actually causes costs, is necessary to ensure collectability of the sum, and hence its consideration in the allocation decision. If the penalty can only follow the suffering of damages, as is true when civil suits by the parties harmed are the only remedy, the investor plans to withdraw his profits and let the investment itself, if it is incorporated, go bankrupt, long before the damage is felt. The abandonment of slum buildings does not make them any less a nuisance. If the government does collect a penalty tax on those projects which cause some eventual deterioration but are still optimal to carry out, it may also be noted that financial investment of the proceeds by government, at the going interest rate i rather than the social rate of return r, would yield it enough funds in the future to compensate individual victims of the damage, if the victims are individuals, and still run a surplus. However, under the assumption of this second case, that total capacity is rationed, the government cannot, of course, increase the well being of the future society beyond that attainable through the optimal investment program, as insured through the imposition of proper in-
centives through the tax itself.21

C. A third case is that in which no independent social discount rate is agreed to. Thus the social opportunity cost of capital remains as the only discount rate for evaluation of projects or establishment of penalty taxes. The case is only complicated if, as is often true in economic development, the rate of return to capital is expected to fall over time. In this case, if an investment decision were made on the basis of a current discount rate, at some future time when the rate has fallen, later generations may find that from their viewpoint the investment is unwarranted and should have been unwarranted from the beginning. As a positive theorem about economic attitudes, rather than as a normative rule, this case may be important in explaining why conservation issues have often led men to castigate their forefathers with uncommon scorn. The fall in social rates of time preference will not change posterity's evaluation of any past investments that have been made whose benefit streams do not terminate in negative flows, although it may change the assessment of investment opportunities originally passed by. It is, generally, easier to be indignant about past sins of commission than those of omission.

As a normative guideline, falling rates of discount, if they are predicted accurately, can be used to calculate present values of projects. The procedure is complex, but feasible. As Feldstein and Flemming point out for the single-root case, calculation of internal rates of return will lack significance in this case, for want of a single interest rate with which to compare the internal rates.
However, there remains the possibility of using some form of sensitivity analysis, remembering that in the case of multiple-root investments, sensitivity to downward as well as upward variations in the rates of time preference will be necessary.

Any of these cases underlines the importance of the case of multiple roots as a paradigm, and the importance of the social rate of discount for calculations. Normally, if the private rate of interest, or some other opportunity cost of capital is believed to overstate the social rate of time preference, that private rate or opportunity cost may still be used with the knowledge that it may be short-changing the future somewhat through underinvestment. But there is little reason to fear this underinvestment will be disastrous to posterity; instead it is felt that use of the higher rate serves the role of surrogate for a premium for risk. Recognition of the multiple roots case, and of the potentially great magnitude of negative effects in the distant future changes this picture. Use of too high an interest rate for discounting can lead to an overinvestment in projects that can damage the environment. Risk is increased, and there is no grounds for presupposition that the damage of miscalculation will be relatively minor. Theoretical arguments over the proper social rate of discount to use must thus be taken quite seriously.

The effects of neglecting future external costs may perhaps also be explored through the construction of growth models in which environmental damage is included. The classification of environmental damage into three categories—contamination, overdurability,
and depletion—in addition to providing a tentative framework for empirical research, suggests some different assumptions under which this task of modeling might be begun. Whether these theoretical conclusions can affect the actual course of environmental damage is, naturally, a more difficult question. The institutional structures of federalism and private enterprise in the United States make a direct application unlikely. There may, however, be centrally planned economies in which the danger of neglecting environmental conservation in a drive to industrialize may be countered through a greater awareness of the problem by technicians. Even in the United States, furthermore, a better understanding of the damage done may feed pressure for reforms or appropriate institutional change.
* The author thanks Conrado Luhrs, Jerome Rothenberg, Peter Diamond and Martin Feldstein for comments. The author is Assistant Professor of Economics at Massachusetts Institute of Technology.


7. In a vintage growth model, overdurability might be represented by a cost stemming from existence of capital which has passed out of use, and which,
in such models, is usually ignored once it is left obsolete as too costly to operate. Contamination costs might be taken as a function of a weighted average of present and some proportion of past production, on the assumption that contaminants do pass out of the environment at some rate.


17. The problems are basically the same if there are greater numbers of roots, due to more alterations between positive and negative returns in different years, or if there is a single, real root in the case of an investment with no initial cost, but eventual negative returns following its years of positive production. In this case, the root is the lowest discount rate at which the project is worthwhile. Wright mentions this possibility ("Notes on the Marginal Efficiency of Capital", *Op. Cit.*., p. 125), but states "this is not usually thought of as a species of investment—it is a type of 'banking' operation like that of the tax-farmer who collects taxes and only remits them to his government after a sensible delay". Obviously, however, the case also has meaning for considering social costs of environment-depleting activities which require no initial investment.


21. Investment by the government of the tax proceeds in financial assets may, however, enable it to overcome a problem of raising public revenues which might arise if contamination or overdurability raised the eventual need for public goods.
22. The analysis presented in this paper stems in part from discussions with Cuban economists as to whether or not it was advisable for Cuba to set policies to limit eventual factory smoke and vehicular air pollution, at a time when factories and automobiles are still rare.
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