QUALITY AND QUANTITY COMPETITION

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Non-price competition has been generally recognized as an important feature of oligopolistic industries. When the decision variables for each firm comprise not merely the quantity of output but also product quality, all of these choices present the usual oligopolistic interactions. Since the costs of enforcing a collusive decision will generally differ with respect to different choice variables, a variety of equilibria is conceivable, with collusion in some dimensions and competition in others. There are important related policy problems. It is often thought that non-price competition is excessively indulged in by firms in an industry (like the airlines) which successfully enforces price (or quantity) collusion. It has recently been mooted that increased competitiveness will bring benefits if legal and medical professions are allowed to advertise. While such issues have a long history in the literature on industrial economics, there have been very few attempts to pin them down by setting up precise models.

Spence (1975) and Sheshinski (1976) have recently analyzed quality and quantity choices by a monopolist, and Spence (1977) has extended this work to the setting of Chamberlinian monopolistic competition. This work compares the socially optimum choices of quantity and quality, and the extent of product diversity, with those in the relevant market equilibrium. Biases are found to depend on whether a quality increase raises or lowers the elasticity of demand for each product line. However, these comparisons do not address themselves to the issues posed above, where the effect of collusion in one dimension on competition in another is sought. This paper seeks to do that.

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We come up at once against another important choice variable, viz. the total number of products being produced in the industry. If this is competitively set, i.e. if there is free entry, then any power to control output or quality levels of the active firms might as well not be used. Entry will eliminate profit in any case, so given even small costs of collusive action, there is no point to it. If entry can be controlled, it becomes important to specify the objective. Where one member is one product, as in the professions, it seems sensible to maximize profit per product line. But where there is a fixed number of members and the number of products is being varied by having each member produce more or fewer lines, as for a fixed number of airlines setting flight schedules, the total profit becomes the relevant maximand. The outcome turns out to depend crucially on this distinction.

If collusive decisions are made to maximize profit per product, it is found that control of entry is tantamount to total collusive control. Once entry has been set at its collusive optimum level, competitive individuals will make quantity and quality choices that are the same as the collusive optima. This is essentially an 'envelope' property. On the other hand, if the collusive goals is total profit, equilibria for different combinations of collusion and competition differ. It is found that quality competition is used in an overcompensating manner if quantity competition is suppressed, but whether this extent of quality competition is excessive from a welfare point of view is ambiguous.

Non-price competition is regarded here in the sense of product quality. Advertising affects tastes, and its welfare effects must be treated differently; see Dixit and Norman (1976). The point is whether inframarginal changes in
consumers' surplus are to be included or excluded. But the positive, comparative static, aspects of the two models are identical, and all non-welfare propositions of this paper can be interpreted taking advertising as the non-price variable.
I. THE MODEL OUTLINED

The setting is very similar to that in Spence (1977) and Dixit and Norman (1976). There is a competitive numeraire commodity labelled 0, produced at constant cost, and an infinite range of potential products \( i = 1, 2, 3, \ldots \) in the industry being studied. The utility function is

\[
 u = G(\sum \phi(x_i, z_i)) + x_o
\]

where \( x_o \) is the quantity of the numeraire, and \( x_i, z_i \) the respective quantity and quality levels of good \( i \), for \( i = 1, 2, 3, \ldots \). The functions \( G \) and \( \phi \) are increasing, and \( \phi(0, z) = 0 \) for all \( z \). They are assumed to satisfy the second-order conditions for the problems we consider: these are sometimes not spelt out in detail for brevity. The inverse demand functions

\[
p_i = \partial u / \partial x_i
\]

are

\[
p_i = G'(s) \phi(x_i, z_i)
\]

where

\[
s = \sum \phi(x_i, z_i)
\]

Oligopolistic interactions among products are thus channelled through \( s \). It will be assumed throughout that the group is large enough to make the effect of any one \( x_i \) or \( z_i \) on \( s \) negligibly small, and therefore neglected in competitive settings. Of course collusive decisions consider all the \( x_i \) and \( z_i \) simultaneously, when the effect on \( s \) is not insignificant.

An important special case will be one where \( \phi \) is iso-elastic with respect to \( x \):

\[
\phi(x, z) = \Lambda(z) x^{\alpha(z)}
\]

where \( 0 < \alpha(z) < 1 \). The price-elasticity of demand for each product is now \( 1/(1-\alpha(z)) \). Increased quality levels increase or decrease demand elasticity according as \( \alpha' > 0 \) or \( < 0 \).
The cost of production of good \( i \) is \( c(x_i, z_i) \). There are some economies of scale, implying the survival of only a finite subset out of the potential product range, and an oligopolistic market structure. An important special case will be the linear one:

\[
c(x, z) = f + ax + bz
\]

where \( f \) is the fixed cost, and \( a, b \) the respective constant marginal costs of quantity and quality.

Since the functional forms of \( \phi \) and \( c \) are assumed independent of \( i \), optima or equilibria will be symmetric. When a total of \( n \) products is active, with common levels \( x, z \) and \( p \) respectively for quantity, quality and price, we will have

\[
s = n \phi(x, z)
\]

\[
p = G'(s) \phi_x(x, z)
\]

and

\[
x = e - n c(x, z)
\]

where \( e \) is the endowment of the numeraire.

**Social optimum**

The object here is to choose \( n, x \) and \( z \) to maximize utility

\[
u = G(n \phi(x, z)) - n c(x, z) + e
\]

The first order conditions are easily seen to be

\[
G'(s) \phi(x, z) = c(x, z)
\]

\[
\phi_x(x, z) / \phi(x, z) = c_x(x, z) / c(x, z)
\]

and

\[
\phi_z(x, z) / \phi(x, z) = c_z(x, z) / c(x, z)
\]

We can interpret (10) and (11) as saying that \( x \) and \( z \) minimize \( c/\phi \), the cost per unit of the contribution of each product to \( s \). Then \( s \) is chosen to make its marginal benefit \( G'(s) \) equal to this cost.
It will prove useful to think in terms of a two-step procedure. Given $z$, (10) defines $x$ as minimizing $c/\phi$. Let this choice be $x = X(z)$, and the corresponding minimum value of $c/\phi$ be $M(z)$. Then (9) becomes

$$G'(s) = M(z)$$  \hspace{1cm} (12)

By the envelope theorem,

$$\frac{M'/M}{z} = c / c - \frac{\phi}{\phi} / z$$  \hspace{1cm} (13)

and therefore (11) becomes

$$M'(z) = 0$$  \hspace{1cm} (14)

The optimum can then be shown at the intersection of the two curves (12) and (14) in the $(z,s)$ space, as done by Spence (1977). This is the point $0$ in Figures 1 and 2, which appear later. The second order conditions are important in determining the shapes of the curves, but for sake of brevity they are not spelt out.

**Market equilibria**

There are three variables to be determined—$n$, $x$ and $z$—and the rules defining them depend on the market structure. If $n$ is collusively set, i.e. entry is controlled, this could be done to maximize profit per product,

$$\pi = G'(s) x \phi_x(x,z) - c(x,z)$$  \hspace{1cm} (15)

or total profit

$$\Pi = n G'(s) x \phi_x(x.z) - n c(x,z)$$  \hspace{1cm} (16)

depending on the nature of the industry as explained before. If $n$ is competitively set, i.e. there is free entry, then the marginal firm only just breaks even. Under symmetry, this amounts to $\pi = 0$ for each firm, or $\Pi = 0$.

Collusive choices of $x$ and $z$ are made in the obvious way, remembering that $s$ is a function of both. Non-cooperative choices are made by individual firms whose effects on $s$, being of order $1/n$, are being neglected. The
corresponding first order conditions are therefore derived treating s as constant.

Finally, we need a concept of equilibrium. We have a game in which each firm is a player, and the group as a whole is a separate player making any collusive decisions. The simplest setting is a Nash equilibrium, where each player regards the choices of all the others as fixed. An alternative of some interest is a leader-follower equilibrium, where the individuals react to the collusively set variables, and such reaction functions are calculated and used by the group in its collusive decisions. Both concepts will be used in the next section, but after that the focus will be on Nash equilibria. The leader-follower equilibrium attributes superior knowledge to the group as a whole, but it can be argued that a group with such knowledge will probably also possess the ability to exercise direct collusive control in the relevant decision. The Nash concept does not involve such informational asymmetry.
II. COLLUSION FOR MAXIMUM AVERAGE PROFIT.

Here the group and the individuals have the same maximand \( \pi \) defined by (15), but they maximize it from different perspectives. The group decisions recognize the oligopolistic interaction arising from the dependence of \( s \) on its choice variables as given by (6), but individuals neglect this and regard \( s \) constant. We consider various possible equilibria.

Full collusion

Now \( n, x, z \) are all collusively chosen to maximize \( \pi \). If \( G \) is a concave function, this will lead us to a choice of \( n \) as small as possible, i.e. \( n = 1 \). This seems unrealistic and uninteresting for most relevant industries e.g. the professions. We therefore assume that \( G \) has an initial convex section followed by a concave one, and that the resulting choice of \( n \) is large enough to allow us to treat it as a continuous variable.

We then have the first order conditions

\[
G''(s) \phi_x = 0
\]

\[
G''(s) n \phi_x + G'(s) \phi_x / \phi_x = 0
\]

\[
G''(s) n \phi_z + G'(s) \phi_z / \phi_z = 0
\]

These reduce to

\[
G''(s) = 0
\] (17)

\[
G'(s) \phi_x / \phi_x = c
\] (18)

\[
G'(s) \phi_z / \phi_z = c
\] (19)

Let \( s^* \) be the collusive average - profit - maximizing level of \( s \), and \( s_o \) the socially optimum one. Since \( G''(s_o) < 0 \) by the second order conditions, we have \( s^* < s_o \), i.e. the industry sub-utility \( s \) and the gross benefit \( G(s) \) are smaller in the collusive solution. However, the distribution of this between \( n, x \) and \( z \) is ambiguous.
For the social optimum, we can use (9) to rewrite (10) and (11) as

\[ G'(s_0) \phi_j = c_j \]  
\[ G'(s_0) \phi_k = c_k \]

We can draw these curves in the \((x,z)\) space and compare them with (18) and (19) where \( s = s^* \). The procedure is similar to that of Sheshinski (1976) and Spence (1977). However, this does not lead to any simple interpretable conditions, and any outcome of comparisons between the optimum and the collusive values of \( x \) and \( z \) seems possible.

We next consider the case where only entry is collusively controlled, and look at two possible kinds of equilibria.

**Nash equilibrium**

Given \( n \), and regarding the impact of their decisions on \( s \) as negligibly small, individuals choose \( x \) and \( z \) to maximize \( \pi \). The first order conditions are

\[ G'(s) \phi (x \phi_j) / \partial x = c_j \]
\[ G'(s) \phi (x \phi_k) / \partial z = c_k \]

These are just (18) and (19) looked at in a different way.

Given \( x \) and \( z \), the collusive choice of \( n \) yields the first order condition

\[ G''(s) \phi x \phi_j = 0 \]

These reduce to (17) - (19), so the Nash equilibrium with only entry collusively set coincides with the fully collusive solution.

**Leader-follower equilibrium**

Here, as before, the individuals choose \( x \) and \( z \) given \( n \) and regarding \( s \) fixed. The (18) and (19) can be thought of as defining the reaction functions \( x(n) \) and \( z(n) \). The group takes these into account, so the choice of \( n \) yields the first order condition
\[
G''(s) \left[ \phi + n \frac{\phi}{x} x'(n) + n \frac{\phi}{z} z'(n) \right] \\
+ \left[ G'(s) \frac{\partial (x \phi)}{\partial x} - c_x \right] x'(n) \\
+ \left[ G'(s) \frac{\partial (x \phi)}{\partial z} - c_z \right] z'(n) = 0
\]

Save in the exceptional case where the reaction functions are such that the first square bracket vanishes, this reduces to \(G''(s) = 0\), and then the full set of equations defining the equilibrium is just (17) - (19), the same as with full collusion.

The reader can similarly verify that adding either \(x\) or \(z\) to the side of collusive choices again yields Nash or leader-follower equilibria that are the same as the fully collusive one. In other words, in this setting the collusive control of entry amounts to full collusion.

This is a somewhat surprising answer to the question of whether advertising by doctors or lawyers will lead to greater competition or merely wasteful expenditure - we suggest that if the professional associations control entry to maximize income per member, it will make no difference at all. On reflection, the result should be clear as an 'envelope' property, and in fact the derivations above used such results.

\textbf{Regulation of entry}

It is of interest to examine how the reaction functions \(x(n), z(n)\) behave for values of \(n\) other than the collusive choice. This tells us the effects of regulating entry at a level other than the group's optimum. We differentiate (18) and (19) totally with respect to \(n\), remembering (6). Writing 
\[
F(x,z) = x^\phi (x,z)
\]
for brevity, we have
\[
\begin{bmatrix}
G' F_{xx} - c_{xx} & G' F_{xz} - c_{xz} \\
G' F_{zx} - c_{zx} & G' F_{zz} - c_{zz}
\end{bmatrix}
\begin{bmatrix}
x'(n) \\
z'(n)
\end{bmatrix}
\]
The coefficient matrix in the first term is negative definite by the second order conditions for the individuals' choice of x and z. Then at the group optimum value of n, we have $G'' = 0$ and therefore $x'(n) = 0 = z'(n)$. Close to this point, $G''$ will be small, and the full coefficient matrix will be dominated by the negative definite one above. We expect $F_{xz} > 0$ (i.e. increased quality raises marginal revenue for each product), and in the linear cost case $c_{xz} = 0$, making the off-diagonal terms positive. The sign pattern of the matrix is then $[+ - +]$, and that of its inverse, $[- - -]$. Thus the sign of $x'(n)$ or $z'(n)$ is the same as that of $G''$, i.e. positive for $n$ slightly too low and negative for $n$ slightly too large. In other words, we expect both reaction functions to attain local maxima for the choice of $n$ corresponding to the group optimum.

The intuitive reason can be seen by examining the expression (15) for $\pi$. The group optimum choice of $n$, being at a point where $G''(s)$ is changing from positive to negative values, has the highest $G'(s)$, and therefore the highest total revenue for each x and z. If higher marginal revenue goes hand in hand with higher total revenue, and the possibly offsetting cross effects of one variable on the marginal cost of the other are not too strong, then this leads to choices of higher x and z.

This says that higher quantity and quality levels cannot be achieved by regulating entry. Of course $x'(n)$ and $z'(n)$ are zero at the group optimum, and therefore $s$ can be increased (locally in the same proportion as $n$) by forcing entry beyond the collusive choice.
III. COLLUSION FOR MAXIMUM TOTAL PROFIT

Now the group and the individuals have different maximands, and simple envelope arguments do not apply. This forces us to restrict the scope somewhat. We consider only iso-elastic functions $\phi$, and sometimes further specialize to a linear cost function $c$. Also, only the Nash concept of equilibrium is studied.

Finally, note that when $n$ is chosen to maximize $\Pi = n \pi$, its choice will carry $s$ into the region where $G$ is concave. To simplify the argument and the diagrams, we omit complications arising in immaterial parts of the range of $s$, by assuming $G$ to be concave throughout. In fact we assume an iso-elastic $G$:

$$G(s) = s^{1-\varepsilon}/(1-\varepsilon), \ 0 < \varepsilon < 1$$  \hspace{1cm} (20)

This allows some explicit solutions without altering any relevant qualitative features of the problem.

With iso-elastic $\phi$, we have

$$x \phi (x,z) = \alpha(z) \phi(x,z)$$  \hspace{1cm} (21)

in the notation of (4), while iso-elastic $G$ yields

$$s G'(s) = (1-\varepsilon) G(s)$$  \hspace{1cm} (22)

The total profit becomes

$$\Pi = (1-\varepsilon) \alpha(z) G(s) - n c(x,z)$$  \hspace{1cm} (23)

Monopolistic competition

This follows Spence (1977). Given $n$, and regarding $s$ constant, each firm sets $x$ and $z$ to maximize $\pi$. The first order conditions are

$$G' \alpha \phi_x = c_x$$  \hspace{1cm} (24)

$$G' [\alpha \phi_z + \phi \alpha'] = c_z$$  \hspace{1cm} (25)
Then entry eliminates profit, i.e.

\[ G' \alpha \phi = c \]  \hspace{1cm} (26)

Dividing (24) by (26), we have

\[ \phi_x / \phi = c_x / c, \]

i.e. for given \( z \), the choice of \( x \) is as if made to minimize \( c/\phi \). This is formally the same as for the social optimum, where we wrote the relation as \( x = X(z) \) and the corresponding minimized \( c/\phi \) as \( M(z) \). Using these, we write (26) as

\[ G'(s) = M(z)/a(z) \]  \hspace{1cm} (27)

Dividing (25) by (26) and using (13), we find

\[ M'(z)/M(z) = \alpha'(z)/\alpha(z). \]  \hspace{1cm} (28)

The equilibrium occurs in \( (z,s) \) space at the intersection of the curves defined by (27) and (28). Two cases arise depending on the sign of \( \alpha' \):

these are shown in Figures 1 and 2. The latter, where \( \alpha' < 0 \), i.e. increased quality lowers the price elasticity of demand, may be thought to be the more plausible. The monopolistic competitor equilibrium is labelled 'A' in each case. Comparisons between the optimum and this equilibrium will be made, together with some other comparisons, in a moment. The lines and curves in the Figures are labelled by the corresponding equation numbers in the text.

**Full collusion**

The first order conditions are

\[ (1-\epsilon) \alpha G' \phi = c \]  \hspace{1cm} (29)

\[ (1-\epsilon) \alpha G' n\phi_x = n c_x \]  \hspace{1cm} (30)

\[ (1-\epsilon) [G' \alpha + \alpha G' n\phi_z] = n c_z \]  \hspace{1cm} (31)

Dividing (30) by (29) we have

\[ \phi_x / \phi = c_x / c, \]
so once again \( x = X(z) \) and \( c/\phi = M(z) \).

This turns (29) into

\[
G'(s) = \frac{1}{1-\epsilon} \frac{N(z)}{\alpha(z)}
\]

Dividing (31) by (29), we have, using (13).

\[
\frac{M'(z)}{M(z)} = \frac{1}{1-\epsilon} \frac{\alpha'(z)}{\alpha(z)}
\]

The intersection of these is at B in Figure 1 and 2.

**Collusion over entry and quantity**

Now \( n \) and \( x \) are chosen by the group to maximize \( \Pi \) given \( z \), while \( z \) is chosen by the individuals to maximize \( \pi \) given \( n \) and \( x \), and neglecting the effect on \( s \). The conditions for \( n \) and \( x \) are

\[
(1-\epsilon) \alpha G' \phi = c \quad (34)
\]

\[
(1-\epsilon) \alpha G' \phi_x = nc_x
\]

These yield \( \phi_x/\phi = c_x/c \) once again, and using familiar notation we have (32) for this equilibrium as well.

The choice of \( z \) satisfies

\[
G' \left[ \alpha \phi_z + \alpha' \phi \right] = c_z
\]

Dividing by (34) and using (13), we find

\[
\frac{N'(z)}{N(z)} = \frac{1}{1-\epsilon} \frac{\alpha'(z)}{\alpha(z)} + \frac{\epsilon}{1-\epsilon} \frac{\phi_z}{\phi}
\]

Where \( \phi_z/\phi \) is evaluated at \( (\lambda(z), z) \). The equilibrium, at the intersection of (32) and (35), is labelled C.

**Some comparisons**

We pause to compare the social optimum and the three types of equilibria obtained so far. The first point to note is that in all four, the choice of \( x \) given \( z \) occurs at the minimum of \( c/\phi \), the cost per unit of contribution to the industry sub-utility. This limited efficiency property is good, but leaves open the possibility that the choice of \( z \) itself might be suboptimal.
More can be said about the function \( x = X(z) \) in the case of linear cost as in (5). It is easy to derive

\[
\frac{1}{x} \frac{dx}{dz} = \frac{1}{1-\alpha} \left[ \alpha'/\alpha + \frac{c}{c_z} \right]
\]

where \( c_z = b \). This is clearly positive if \( \alpha' > 0 \). Also, \( \alpha' \) cannot be too negative. The inverse demand curve for each firm is

\[
p = G'(s) \phi = G'(s) \alpha \phi/x,
\]

and for this to be increasing in \( z \), we need

\[
\frac{\alpha'/\alpha + \phi_z/\phi}{1-e} > 0
\]

then the expression in brackets in (36) is

\[
> \frac{c_z}{c} - \frac{\phi_z/\phi}{c} = -\frac{M'(z)}{M(z)}.
\]

But when \( \alpha' < 0 \), Figure 2 shows that the region of interest is where \( M' < 0 \), so again \( dx/dz \) is positive there.

Contours of equal welfare can be drawn in \((z,s)\) space. These are horizontal through the line (14) and vertical through the curve, (12). It is then easy to see that when \( \alpha' > 0 \) the successive equilibria A, B and C move farther away from the optimum in all respects - having too high \( z \) and therefore \( x \), and having too low \( s \), and finally too low \( n = s/\phi(x,z) \).

In the case \( \alpha' > 0 \), therefore, forcing a fully collusive industry to allow only quality competition is not a desirable policy, but makes things even worse.

If \( \alpha' < 0 \), which is probably more realistic, the suboptimality of monopolistic competition is likewise magnified in full collusion: \( z, x \) and \( s \) are all too low in \( B \) as compared to \( A \), and in turn in \( A \) as compared to \( 0 \). A big difference is that allowing quality competition now decreases the bias in \( z \). We have from (37) that

\[
\frac{\alpha'/\alpha + \epsilon \phi_z/\phi}{1-\epsilon} > \frac{\alpha'/\alpha}{1-e}.
\]
so the level of z in C in fact exceeds that in A. When quality is the only dimension of competition, it is used in a way to overcompensate for the absence of other types of competition, and its equilibrium level exceeds that under full monopolistic competition. This may explain the common views concerning 'wastefully excessive' non-price competition in certain cartels. However, our analysis tells us that the welfare consequences of this are ambiguous, and C may lie on a higher welfare contour than A or on a lower one.

Only entry collusive

We turn to consider some other possible types of equilibria. One is where only the total number n of product lines is collusively set, while quantity and quality are competitive. In contrast to the similar case of Section II, the group objective is now the total profit.

Given n, the first-order conditions for the choice of x and z are

\[ G' \alpha_x = c_x \]  \hspace{1cm} (38)
\[ G' (\alpha_x + \alpha_z) = c_z \]  \hspace{1cm} (39)

Given x and z, the choice of n satisfies

\[ (1-\epsilon) \alpha G' \phi = c \]  \hspace{1cm} (40)

Dividing (38) by (40) we find

\[ (1-\epsilon) \frac{c_x}{\phi_x} = \frac{\phi_z}{\phi} \]  \hspace{1cm} (41)

This differs from the four solutions looked at above. Given z, the choice of x does not minimize \( c/\phi \). This may be thought to be an undesirable feature of this equilibrium. In fact for each z, the quantity will be too large compared to the cost-minimizing \( X(z) \). To see this, let us use (41) to define a new relation \( x = \xi(z) \). Since \( x = X(z) \) was defined by \( c_x/c = \phi_x/\phi \), and since \( 0 < \epsilon < 1 \), it is evident from Figure 3 that \( \xi(z) > X(z) \). Also, letting \( \mu(z) \) stand for the value of \( c/\phi \) when \( x = \xi(z) \), we clearly have \( \mu(z) > M(z) \).
Using this notation, we can write (40) as

\[
G'(s) = \frac{1}{1-e} \frac{\mu'(z)}{\alpha(z)}
\]  

(42)

Comparing this with (32), the curve which was relevant for equilibria with full collusion or for entry and quantity collusion, we see that the right hand side of (42) is always bigger, and hence the \( s \) defined by (42) is smaller for each \( z \). Thus (42) lies entirely below (32). It appears that the control of entry is being exercised in a manner that compensates for the loss of control over quantity.

Finally, dividing (39) by (40), we have

\[
\frac{\alpha'}{\alpha} = (1-e) \left( \frac{c}{c} / c - \frac{\phi_z}{\phi} \right)
\]  

(43)

where the right hand side is evaluated with \( x = \xi(z) \). To enable comparison with the equilibria considered before, we compare the right hand side in (43) with a similar expression evaluated at \( x = X(z) \). To do this, we must find out whether the expression is increasing or decreasing in \( x \). This can be done in special cases. For a linear cost function, \( c_z \) is constant, and so \( c_z / c \) is a decreasing function of \( x \). In the iso-elastic case,

\[
\phi / \phi = A'/A + \alpha' \log x
\]  

which is increasing in \( x \) if and only if \( \alpha' > 0 \). In case \( \alpha' > 0 \), therefore, we can be sure that the right hand side is decreasing in \( x \), and since \( \xi(z) > X(z) \), we have

\[
\frac{\alpha'}{\alpha} = (1-e) \left( \frac{c_z}{c} \right) \left| \xi(z) \right| - \left( \frac{\phi_z}{\phi} \right) \left| \xi(z) \right| < (1-e) \left( \frac{c_z}{c} \right) \left| X(z) \right| - \left( \frac{\phi_z}{\phi} \right) \left| X(z) \right| = (1-e) \frac{N'}{M} - \varepsilon \left( \frac{\phi_z}{\phi} \right) \left| X(z) \right| .
\]

Comparing this with (35), we see that the vertical line defined by (43) lies to the right of that defined by (35). Since (42) lies below (32), we conclude
that this kind of equilibrium lies in the region to the south-east of C in Figure 1, an even worse state of affairs from the point of view of welfare.

A clear conclusion does not seem possible in case $\alpha' < 0$.

**Entry and quality collusive**

Finally we consider the case where entry and quality are collusively set to maximize total profit, but quantity is left to competitive individual profit-maximization. This may be thought to capture the case of certain franchises.

Given $n$ and $z$, the choice of $x$ satisfies

$$G' \alpha \phi_x = c_x$$

while given $x$, the choices of $n$ and $z$ satisfy

$$(1-\epsilon) G' \phi = c$$

$$G' \alpha' \phi + (1-\epsilon) \alpha G' n \phi_z = c_z$$

Dividing (44) by (45), we have

$$(1-\epsilon) \frac{c_x}{c} = \frac{\phi_x}{\phi}$$

and therefore $x = \xi(z)$, $c/\phi = \mu(z)$ and

$$G'(s) = \frac{1}{1-\epsilon} \frac{\mu(z)}{\alpha(z)}$$

as in the previous case. Using (22) in (46) and dividing by (45), we find

$$\frac{\alpha'/\alpha}{\phi} = (1-\epsilon) \left( \frac{c_z/c - \phi_z/\phi}{z} \right)$$

where the right hand side is evaluated at $x = \xi(z)$. Again, in case $\alpha' > 0$, with linear $c$ and iso-elastic $\phi$, we can easily show that the right hand side is decreasing in $x$, and therefore

$$\frac{\alpha'/\alpha}{\phi} < (1-\epsilon) \frac{M'/M}{z}$$

i.e. (47) defines a line to the right of (33). Thus, in Figure 1, the equilibrium for this case would lie in the region to the south-east of $B$. Starting with full collusion and enforcing only quantity competition actually lowers welfare in this case.
IV. CONCLUDING COMMENTS

In this paper we have considered several possible cases of oligopolistic equilibria and compared them with each other taking into account both positive and welfare aspects. The strategic variables were entry, quantity and quality, and different circumstances and objectives governing their choice produced the different possible equilibria. Since these circumstances are amenable to policy controls, we were able to draw some inferences as to desirable and undesirable controls over oligopolistic industries. Some of these conclusions were rather surprising in the light of previous casual thinking on the subject.

We would not claim such theorising to be infallible in its applications. We are also aware that a great deal remains to be done. Clear answers could not be obtained for some of our cases due to analytical difficulties. More importantly, concepts of equilibria other than the Nash one are largely neglected. Nevertheless, we hope to have contributed to progress in the analysis of oligopolistic industries. We hope we have convinced some readers that the subject is not as hopeless a quest as is commonly believed, and that simple and rigorous economic thinking can be productively brought to bear upon it.
REFERENCES


case $\alpha' > 0$

Figure 1
case $\alpha^1 < 0$

Figure 2
Choice of $x$ for given $z$

Figure 3