PREDATION WITHOUT REPUTATION

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1. **INTRODUCTION**

Can firms rationally engage in predatory pricing? That is, can they in equilibrium choose to cut prices to lower their rivals' profits and thus induce them to leave the market?

That question has been a long-standing point of controversy. While some economists have claimed that predation has occurred, others, notably McGee [1958], have argued that predation would never be rational. Recently, Kreps-Wilson [1982] and Milgrom-Roberts [1982a] have shown that if with small probability a firm is "crazy" and enjoys predation, then even if the firm is "sane" it may choose to prey if entry occurs. In these papers, predation is an investment in a reputation for "crazy" play. The fact that predation reduces opponents' current profits is not crucial to the reputation-effects story; what matters is that the predator signals that it is likely to prey in the future. Thus these models do not quite correspond to the view that predation aims at lowering opponents' profits. Nevertheless, reputation effects may be a good model of predation in some cases of long-term competition (These models require many periods in order for a small probability of craziness to make a difference). The reputation-effects models are similar in spirit to Milgrom-Robert's [1982b] paper on limit pricing, in which an established firm tried to mislead entrants about its costs by manipulating the pre-entry price. Here, the emphasis was on entry-deterrence rather than on post-entry predation.
This paper offers a new explanation of predatory pricing which does not depend on reputation effects. We assume that the entrant is uncertain of its future profitability, and uses its current profit to decide whether or not to remain in the market. Moreover, the established firm can take some competitive actions which are not (fully) observed by the entrant, such as secret price cutting. Because these actions lower the distribution of the entrant's realized profits, they increase the probability of exit and so the established firm will compete more than it would if the entrant were certain to remain active. This is predation, in the sense of lower prices (or higher expenditures on non-price competition) than would have occurred were exit not a factor. In equilibrium, the established firm "preys", reducing both its own and the entrant's profits, even though the entrant is not "fooled" by the predation and leaves the market only when it would have left without predation. Nevertheless, predation may be of some benefit to the predator, because it lowers the expected profitability of entering the market. The fact that the established firm can, on the margin, increase the probability of exit by predation makes the threat to prey credible, thus discouraging entry.

Our model is an example of a kind of signalling equilibrium we call "signal jamming". The predator's characteristics are common knowledge; it changes its actions from the full-information ones not to signal its own information but rather to "jam" or interfere with the inference problem faced by the entrant. (while in our case "nature" is the one who sends the signal which is jammed, signal jamming can also occur when the signal is sent by a third player.) Other examples of signal jamming equilibria can be found in Holmstrom's [1982] paper on marginal incentives, and in Riordan's [1985] paper on "dynamic conjectural variations", which we discuss in more detail in the
next section.

After developing to our signal-jamming model, we show how to embed the traditional long-purse story of predation in a model with rational actors. We do so by appealing to the work of Gale-Hellwig [1983] on optimal debt contracts with costly monitoring to show that if the entrant loses enough equity it may be unable to raise capital for profitable investments. Thus we offer two models of "predation without reputation", one old and one new.

2. **THE SIGNAL-JAMMING MODEL**

We focus on the simplest case with only two periods and perfect correlation of the random terms over time. Later we will discuss the extensions to more periods and partial correlation. Signal jamming is based on the entrant’s need to infer its future profitability from its current returns. We model this by assuming that the entrant is uncertain of its (per-period) fixed costs, and is unable to observe these costs directly. We motivate this unobservability by alluding to “agency problems” which prevent the firms management from eliciting cost information from its managers.

We have in fact derived such an information blockage from a model in which managers' utility functions are as in Hart [1983] and managers face moving costs in changing jobs. However, this paper studies predation not agency, so we have not included a formal justification for the unobservability of fixed costs. An alternative explanation of the entrant's relying on its current profits for information would be that the entrant's demand depends on a random parameter whose value is unknown to the entrant as in Riordan [1985]. Riordan developed a two-period model of entry accommodation with demand shocks and quantity competition. In the resulting equilibrium, each firm's second-period output is decreasing in the observed
first-period price and thus, is decreasing in its opponent's (unobserved) output. These "negative dynamic conjectural variations" lead the firms to choose larger first-period outputs than in Static Nash equilibrium. Riordan's paper differs from ours in its focus on entry accomodation, as opposed to entry deterrence, and in the comparative statics it considers. Our specification of fixed costs, rather than demand, as the uncertain variable leads to a more tractable model, because with this specification the first-period observations influence only the entrant's decision whether to remain active, and not its marginal trade-offs and second-period price. This allows us to fold back the second-period equilibrium, replacing it with the expected payoffs, and conclude that the entrant will remain active if its first-period profit is sufficiently high.

We will call the incumbent "firm one" and the entrant "firm two". For the moment let the first period be "period A" and the second period "period B", so that, for example, firm one's second-period price is $p_B^1$. Period B will soon be folded back and we will then be able to drop the superscripts. Let us introduce some more notation. Let $\Pi_i^1(p_1^1,p_2)$ denote firm i's per period gross profit function. Let $\alpha$ denote the entrant's fixed cost in the first (and second) period. $\alpha$ is distributed on $[\hat{\alpha},\check{\alpha}]$ with cumulative distribution function $F(\alpha)$ and continuous density $dF(\alpha)=f(\alpha)d\alpha$. As firm one's exit is not an issue, we can normalize its fixed cost to be zero. If the entrant stays in period B, the two firms obtain their Bertrand-Nash profits ($\hat{\Pi}_1^1,\hat{\Pi}_2^2-\alpha$). So under complete information, the entrant will stay if and only if $\hat{\alpha}^<\alpha=\check{\alpha}^2$. If it leaves, then the incumbent makes its monopoly

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1As suggested by the work of Bulow-Geanakoplos-Klemperer [1985] and Fudenberg-Tirole [1984], this conclusion is reversed if firms compete in prices, not quantities, and their products are differentiated.
profit $\Pi^1(p^n, \infty)$, where $p^n$ is the incumbent's monopoly price. Firm $i$ tries to maximize its expected total profit $V^i$, where:

$$V^1 = \begin{cases} 
\Pi^1(p^A_i, p^A_2) + \Pi^1(p^B_i, p^B_2) & \text{if firm two remains active in period } B; \\
\Pi^1(p^A_i, p^A_2) + \Pi^1(p^m_i, \infty) & \text{otherwise}; 
\end{cases}$$

$$V^2 = \begin{cases} 
\Pi^2(p^A_i, p^A_2) - \alpha + \Pi^2(p^B_i, p^B_2) - \alpha & \text{if firm two remains active in period } B; \\
\Pi^2(p^A_i, p^A_2) - \alpha & \text{otherwise}. 
\end{cases}$$

(To simplify notation, we ignore discounting).

Let subscripts denote partial differentiation, so that, for example, $\Pi^1_{12}$ is the cross partial of $\Pi^1$ with respect to $p_1$ and $p_2$. We will assume:

A1) The $\Pi^i$ are three times continuously differentiable, and for $p_1$ and $p_2$ such that $\Pi^i > 0$, $\Pi^i_{11} < 0$, $\Pi^i_{1j} > 0$, $\Pi^i_{j} > 0$, and $\Pi^i_{11} \Pi^2_{22} > \Pi^1_{12} \Pi^2_{12}$.

Let $R_i(p_j)$ be firm $i$'s static reaction function; that is, $R^i_2(p_1)$ solves:

$$1) \quad R^i_2(p_1) = \arg\max_{p_2} \Pi^2(p_1, p_2).$$

The reaction functions are differentiable and slope up from (A1). Moreover, the last part of (A1) ensures that the reaction functions intersect at most once. If, as we assume, such an intersection exists, it will be the (unique) pure-strategy equilibrium of the static Bertrand game.

A2) The reaction functions $R_1$ and $R_2$ have a (unique) intersection $(\hat{p}_1, \hat{p}_2)$.

The following assumption is for technical convenience.
A3) The density \( f(a) \) is smooth and bounded away from zero.

Let \( \hat{\pi}^1 = \pi_1(p_1, p_2) \). In the last period the incumbent has no incentive to prey, and if the entrant is active it will receive \( \hat{\pi}^2 - \alpha \). With complete information, the entrant would then choose to remain active if \( \alpha < \hat{\alpha} \) and would exit otherwise. For our problem to be interesting the exit decision must depend on \( \alpha \), which we assume in (A4).

\[
\hat{\alpha} < \alpha < \hat{\alpha}.
\]

(A4) implies that in the Bertrand equilibrium the entrant has positive output, (otherwise its profit would be negative for any positive fixed cost) and thus that at that equilibrium its payoff is sensitive to the incumbent's price:

\[
(\hat{A}4') \quad \hat{\pi}_1^2(p_1, p_2) > 0.
\]

Let us now state the informational assumptions. Firm two's fixed cost is not directly observable by any firm. Furthermore, firm two does not observe (even ex-post) the price charged by firm one; firm one may or may not observe firm two's price (for consistency we may as well assume that it does not). All other variables are common knowledge.

The assumption that firms do not observe their rival's price deserves some comment. We have in mind the following situation: the firms sell to a small number of customers and their discounts under the list price are not public information (see e.g., Scherer [1980], page 222). Alternatively we could think of firm one engaging in "non-price predation", say by increasing its (unobservable) advertising expenditures.
Now we can work out the necessary conditions for equilibrium play in the first period. The entrant will not be misled by the incumbent's predation, and thus will leave the market exactly when the net profit reveals that $\alpha$ is above $\alpha$. Call this cut-off net profit $N^\wedge$. Moreover, the entrant can do no better than choose $p_2^A$ to maximize its first-period profit that is, $p_2^A$ will be on the entrant's reaction curve. Finally we must determine the incumbent's first-period price, given that the entrant will leave the market whenever $\Pi^2 < N^\wedge$. If the incumbent sets $p_1^A$ so that the probability of exit is strictly between zero and one, then the optimal choice of $p_1^A$ sets the marginal first-period gain from increasing $p_1^A$ equal to the marginal change in the probability the entrant stays active times the value of inducing exit. (We know that in a pure strategy equilibrium, the probability of exit is the same as under complete information, and therefore is strictly between 0 and 1.) If we define this latter value to be $\Delta = (\Pi^1(p_1^m,^\wedge)-\Pi^1)$, we have the following necessary conditions for first-period equilibrium prices $(p_1^*, R_2(p_1^*))$ and exit rule "leave if net profit is less than $N^\wedge$"

\begin{align}
(2a) \quad N^\wedge &= \Pi^2[p_1^*, R_2(p_1^*)] - \alpha \\
(2b) \quad \Pi_1^1(p_1^*, R_2(p_1^*)) &= \Delta \cdot f(\alpha) \cdot \Pi_1^2(p_1^*, R_2(p_1^*)).
\end{align}

The first equation says that in equilibrium the entrant is not fooled, and the second equation is the first-order condition for the incumbent's choice of $p_1^A$. We wish to provide conditions ensuring that (i) a solution to the system (2a), (2b) exists, and (ii) such a solution is in fact an
equilibrium. We give sufficient conditions for (i) in terms of exogenous variables, but our conditions for (ii) to be obtained can only be checked after computing a candidate equilibrium satisfying (2a), (2b). For this reason, and for ease of exposition, we treat (i) and (ii) separately. The Appendix presents an example with quadratic pay-offs and a uniform distribution which satisfies all the assumptions and conditions we will invoke. While this is the only example we have solved, we present the general case in the text because we feel that doing so makes clear which features of quadratic-uniform example are important for our results.

The incumbent's first-period choice problem will have a maximum at prices satisfying (2b) if it is locally concave, that is if:

\[(3) \quad \Pi_{11}^{1} - A \cdot (f\Pi_{11}^{2} - f'(\Pi_{11}^{2})^2) < 0.\]

(A5) Equation (3) is satisfied (at least when the first-order condition 2b is satisfied).

(A5) holds if \( \Pi \) is quadratic and the density \( f \) is uniform.

To ensure that the system (2a), (2b) has a solution we must rule out a corner solution at \( p_{1}^{A} = 0 \):

\[(A6) \quad \lim_{p_{1} \to 0} \Pi_{11}^{2}(p_{1}, R_{2}(p_{1})) = 0.\]

(A6) says that the marginal loss to the entrant caused by a decrease in the incumbent's price when the entrant responds optimally is small when the incumbent's price is low. The intuition is that for sufficiently low \( p_{1} \), the entrant will choose not to produce, and will thus not be affected by the incumbent's price.
Lemma: Under (A1) to (A6) there is at least one solution \((p_1^*, N^*)\) to (2a), (2b).

Proof: Equation (2b) is independent of equation (2a), so to find an equilibrium we can first find a \(p_1^*\) that satisfies (2b) and then plug it into (2a) to find \(N^*\). Both sides of (2b) are continuous, the left-hand side of (2b) exceeds the right for small \(p_1\) from (A6), and when \(p_1^*\) is the Bertrand price \(p_1^*\) the inequality is reversed from (A4'). Thus there is a \(p_1^*\) that solves (2b).

Q.E.D.

A given solution \((p_1^*, N^*)\) need not be an equilibrium because we have not yet checked that the incumbent's choice problem is globally concave. Equations (2b) and (3) hold only for \(p_1\) which result in a probability of exit strictly between zero and one. If \(\Pi^2(p_1, R_2(p_1^*)) - \alpha < \Pi^2(p_1^*, R_2(p_1^*)) - \alpha\), firm two will observe an "impossibly small" first-period profit, i.e., a profit lower than it expected could occur in equilibrium. We will specify that such observations make firm two exit with probability one. Thus in this region firm one's profit is \(\Pi^1(p_1, R_2(p_1^*)) + \Delta\). Since \(\Pi^1\) is concave in \(p_1\), and \(\Pi^1(p_1^*, R_2(p_1^*))\) is positive, \(\Pi^1\) cannot have a maximum in this region. However, we need an additional assumption to ensure that the incumbent does not prefer to deviate to prices so high that exit never occurs. (Otherwise no pure strategy equilibrium would exist as we know that in equilibrium some exit must occur). Once again, as \(p_1\) does not influence the entrant's exit decision, the best the incumbent can do is to maximize its first-period
profit, which yields at most $\Pi^1(R_1(R_2(p^*_1)), R_2(p^*_1))$. Thus we require

(A7) $\text{Prob}(\alpha < \bar{\alpha}) \Delta^* \Pi^1(p^*_1, R_2(p^*_1)) > \Pi^1(R_1(R_2(p^*_1)), R_2(p^*_1))$.

(A7) is a condition that includes the endogenous variable $p^*_1$. It holds immediately if the incumbent's best response $R_1(R_2(p^*_1))$ is lower than the price that just induces certain entry. This price, which we call $d(p^*_1)$, is given by

(4) $\Pi^2(d(p^*_1), R_2(p^*_1)) - \Pi^2(p^*_1, R_2(p^*_1)) = \bar{\alpha} - \alpha$.

If $R_1(R_2(p^*_1)) < d(p^*_1)$, then the incumbent's payoff function is single-peaked. This will be the case, loosely speaking, if $\bar{\alpha}$ is large so that $\alpha$ has a wide support. This will also be the case in the "noisy" version of the model we discuss at the end of the section. If $R_1(R_2(p^*_1)) > d(p^*_1)$, then the incumbent's payoff has two local maxima, but if $\Delta$ is "large enough" then $p^*_1$ maximizes the incumbent's payoff.

Proposition 1: A solution $(p^*_1, R_2(p^*_1))$ to \{(2a), (2b)\} which satisfies (A7) is an equilibrium.

We do not know whether an equilibrium exists when (A7) is not satisfied. Given (A7), we have found an equilibrium, and can do (local) comparative statics. (The qualifier "local" is needed because we have not shown that the equilibrium is unique.) Examining equation (2b), we see that increasing the value of exit, $\Delta$, increases the incumbent's incentive to prey, and thus might be expected to lower $p^*_1$. This intuition need not be correct
given our assumptions so far. Assumption (A5) guaranteed that the partial derivative with respect to \( p_1 \) of the incumbent's first-order condition (2b) was negative, but for comparative statics (and the uniqueness of equilibrium) we need this to be true of the total derivative. This total derivative is given in equation (4).

\[
(4) \quad [\Pi_{11}^1 - f\Pi_{11}^2] + (\Pi_{12}^1 + \Pi_{12}^2 \Delta f) R_2'.
\]

The sign of the term in square brackets is indeterminate while the second term is positive. Signing equation (4) would require assumptions on the third derivatives of \( \Pi^2 \), in order to bound \( R_2' \). Instead for comparative statics we assume that (4) is negative.

**Proposition 2:** At an equilibrium where equation (4) is negative,

\[
\frac{dp_1^*}{d\Delta} < 0.
\]

We remind the reader that our many assumptions and conditions are satisfied in the quadratic-uniform example of the Appendix, for which the equilibrium is unique and Proposition 2 applies.

One might wonder as well how \( p_1^* \) changes with the density \( f(\alpha) \). The term \( f(\alpha) \) enters equation (2b) multiplicatively with \( \Delta \), so that the key is how the density changes at the cutoff level. For most densities, this question is difficult to answer. However, for the uniform distribution considered in the Appendix, decreasing the variance of \( f \) while keeping \( \alpha \) inside its support ("more information") increases \( f(\alpha) \) and thus lowers \( p_1^* \). Thus the better the information the more predation occurs, so that the equilibrium is discontinuous at perfect information. Two comments are in order on this result. First, the result requires keeping the probability
that \( \alpha \) exceeds a constant, to ensure that (A7) is satisfied. If we think of a "general" class of uniform distributions, as we shrink the support we are "unlikely" to converge down to \( \alpha \); instead we would expect to converge to a non-predation equilibrium in which either the entrant always leaves (\( \alpha \) below the support) or always stays (\( \alpha \) above it). Second, given a "narrow" uniform distribution centered at \( \alpha \), the result is really not surprising. If we consider starting at the full-information equilibrium price, a small decrease in the incumbent's price is fairly inexpensive, because \( \Pi_1^*=0 \), yet yields a large increment in exit, because the entrant thinks there's lots of information in its low-variance profit. In equilibrium \( p_1^* \) must decrease enough to compensate for the high "efficiency" of predation.

The possible discontinuity of \( p_1^* \) in the support of \( \alpha \) is the opposite of a result in Holmstrom's [1982] paper on managerial incentives. In Holmstrom's model, a manager expends effort to mislead the market about her ability, because effort and ability are confounded in the manager's observed performance. Holmstrom shows that as the variance of the manager's ability decreases (the equivalent of shrinking the support of \( \alpha \)) the manager expends less effort. As Holmstrom's model has transitory randomness in performance ("noise") as well as random abilities, one might speculate that the discontinuity we found would disappear if we introduced noise. This is not the case. We have considered a variant of the quadratic-uniform model of the Appendix with a uniformly distributed noise term added to first-period profits. In that model, \( p_1^* \) is independent of the support of \( \alpha \) once that support is sufficiently small, but still less than the Bertrand price, so
that the discontinuity persists. The discontinuity would also occur with Holmstrom's normal-normal stochastic specification, provided that the critical level \( \hat{\alpha} \) was the median of the support of \( \alpha \). Otherwise, the comparative statics considered by Holmstrom, namely decreasing the variance of \( \alpha \), would lead either to \( \Pr(\alpha < \hat{\alpha}) = \epsilon \) or \( \Pr(\alpha < \hat{\alpha}) = 1 - \epsilon \), \( \epsilon \) small, and in either case there would be no predation with little uncertainty about \( \alpha \). The requirement that \( \hat{\alpha} \) be the median of \( \alpha \) is the analog of our shrinking the support of \( \alpha \) uniform distribution without changing the probability that \( \hat{\alpha} \) exceeds \( \alpha \). We conclude that, with or without noise, when there is good but not perfect information about \( \alpha \) then predation may well be heavy if it occurs at all.\(^2\)

In equilibrium, predation does not "fool" the entrant, and thus the entrant leaves the market in exactly the same states as it would without predation. However, the predation does lower the entrant's profits. Thus if we now work backwards to consider the entry decision, the entrant will enter less often than if predation did not occur. Continuing to work backwards, we can add a third period to the beginning of the game. If the shock \( \alpha \) is perfectly correlated over all three periods, then predation will only occur in the first period, and thus there will be the same amount of exit as without predation. If however, the shocks are imperfectly correlated, then there will be predation in both the first and the second periods. Because of the second-period predation, there will be more exit at

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\(^2\)We thank Bob Gibbons for encouraging us to consider the relationship of our model to Holmstrom's, and for working out the equilibrium in Holmstrom's model with uniform distributions of ability and noise. Holmstrom's model differs from ours in an additional way not mentioned in the text; in Holmstrom, the "payoff" to signal-jamming is the change induced in the expected value of \( \alpha \), and not the change in the probability that \( \alpha \) exceeds \( \hat{\alpha} \). Thus, when ability has a tight support, there is little payoff to signal-jamming, while in our model, the payoff is independent of the support of fixed costs. For this reason, discontinuities at the full-information limit do not arise in Holmstrom's model, even with our uniform-uniform specification.
the end of the first period than without predation, for the same reason there was less entry in the two-period model. With three periods and imperfect correlation, firms (or more precisely "types" of the entrant) will enter and then exit who would have stayed in the market without predation.

3. An alternative theory of predation without reputation: the long purse story

We have shown that a firm may want to jam its opponent's information to encourage the latter's exit. The long purse story is a more traditional explanation of predation without reputation. If firms have financial constraints, they have an incentive to reduce their opponents' current profits. The preyed-upon firms may then be forced to exit (or more generally reduce their activity) because of the impossibility of renewing capital or financing new projects.

Until now the long purse story has remained very informal, and as a consequence some authors have doubted about its validity. Indeed in a world of perfect information such predation cannot occur. Bygones are bygones, and a firm's loss of money has no impact on its prospects, so current predation will not affect future financing and is useless. In other words, in a perfect information world firms do not face financial constraints. (See McGee [1958]; see also Benoit [1983], who provides a game-theoretic analysis of predation with exogenous limits on the losses that firms can sustain and remain active). Recent research on borrowing under asymmetric information has provided a foundation for the assumption of financial constraints. We here simply demonstrate that the long purse story is a direct spinoff of this research and we compare it with the signal-jamming story. Consider the
simplified version of the one-period model developed by Gale-Hellwig [1983](see also Diamond [1984]):

A debtor has a potential project with random payoff (gross of investment) \( \tilde{\Pi} > 0 \). The cumulative distribution function is \( F(\tilde{\Pi}) \). This project requires a capital investment \( K \). Assume \( E(\tilde{\Pi}) > K \). The debtor has initial wealth \( E < K \). As we will see below, the debtor finances as much as he can through his own funds. To finance the rest, he can sign a contract with a (competitive) bank. The bank pays a cost \( a > 0 \) if it decides to monitor the debtor's ex-post payoff \( \tilde{\Pi} \). Furthermore bankruptcy laws prevent the bank from taking more money from the debtor than the latter owns. Gale and Hellwig have shown that the optimal contract between the debtor and the bank takes the simple "standard debt contract" form: Either the debtor decides to reimburse a fixed amount \( R \) to the bank and there is no auditing, or the latter audits and then confiscates \( \tilde{\Pi} \). The debtor then defaults if and only if \( \tilde{\Pi} < R \).

The zero-profit condition for the bank is:

\[
(5) \quad \Pi^B = (1 - F(R))R + \int_0^R \tilde{\Pi} F(\tilde{\Pi}) - aF(R) - (K - E) = 0.
\]

\( R \) is the lowest root (if any) satisfying equation (5). If there is no root to equation (5), the debtor cannot find a bank to finance the project: one can then posit \( R = +\infty \).

The debtor's extra profit from this project is then:

\[
(6) \quad \Pi^D = \max\{0, -E + \int_0^R (\tilde{\Pi} - R) dF(\tilde{\Pi})\}.
\]

A straightforward analysis of equation (5) and (6) shows that

a) \( R \) is greater than \( (K - E) \) and is nonincreasing with the loan \( (K - E) \),
b) the debtor's profit \( \Pi^D \) is nondecreasing with \( E \).
Let us show that when $E$ is sufficiently small, for a sufficiently big, there is no loan contract which is acceptable to both parties. To this purpose subtract (6) from (5). One sees that if $\underline{R}$ is the lowest $R$ such that $\Pi^D=0$, then

$$
\Pi^B = (E(\bar{n}) - K) - aF(\underline{R}),
$$

i.e., the bank's profit is equal to the complete information aggregate profit minus the expected auditing cost. Note that $\underline{R}$ does not depend on $a$. Now if $a$ is "big", $\Pi^B$ is negative at $\underline{R}$. If we assume that $\Pi^B$ is an increasing function of $R$ \(^3\), then the bank stops lending before the debtor decides not to apply for a loan. If $a$ is "small", $\Pi^B$ is positive at $R$, and the debtor may decide not to apply even though its project could be financed. The conclusion of this study is that the lower the debtor's wealth, the less likely is the financing of the project.

To see how we can base a long purse story on this, consider a two-period model of competition between two firms in which a) the second period investment of firm two, say, must be financed through debt, b) debt contracts are short-run (one-period) debt contracts and c) the random variables influencing profits are not correlated over time. c) is not crucial and just serves to make sure that no predation through signal jamming or through more traditional signalling (limit pricing, war of attrition, reputation models) can occur. Then it may pay for firm one to prey in the first period. Doing so reduces firm two's equity in the second period and therefore makes firm two's investment less attractive to the firm and to the banks.

\(^3\)This need not be the case, but it is true, for example, if $F(\bar{n})=1e^{-k\bar{n}}$. 
So, using the Gale-Hellwig model, we have built a consistent model of the long purse story. A firm with more cash flow has a higher probability of staying in the market. Of course more complex models would give a more sensible description of the long-purse story. For instance the extreme observability assumption of the bankruptcy model should be relaxed. And mainly one would expect predation to often occur when a small firm enters a market (at least as far as the long purse story is concerned). Then, in the terminology of the above model, this firm (firm two) will certainly need financing from the first period on. This strengthens the case for the study of long-run debt contracts: in the above we had one-period debt contracts in a two-period model. We are confident that future research on financing under asymmetric information will lead to a more coherent theory of the long purse story.

We ought to compare the signal jamming and the long-purse stories. Signal jamming, like the previous models of predation, relies on the predator's influencing beliefs and the correlation of uncertainty over time. The long purse story does not require such correlation; instead predation affects tangible variables (wealth levels). Another, related, difference is that predation may occur and at the same time not be successful in the signal jamming model. It seems, from the above model and the work of Benoit [1983], that predation to affect financial constraints occurs only if it is successful, at least with some positive probability. Lastly, what differentiates these two stories from the previous theories of predation (limit pricing, war of attrition and reputation models) is that the predator

"Although this does not mean that it faces a "quieter" market: if firm one chooses to prey after all, predation may be tougher in order to be successful."
does not try to convey any information about itself. So, as we mentioned in the introduction, no reputation is involved.
This Appendix shows that (A1) through (A7) are satisfied in the case of quadratic payoffs $\Pi^i$ and a uniform distribution over $\alpha$, and solves explicitly for the equilibrium. We derive the quadratic payoffs from price competition within a Hotelling model with differentiated products on each end of a unit interval, linear transportation costs $t$, and constant average cost $c$. In this model, static profits are given by

$$
\Pi^i(p_i, p_j) = \begin{cases} 
(p_i - c) & \text{if } p_j > p_i + t, \\
\frac{(p_i - c)(p_j - p_i + t)}{2t} & \text{if } p_i + t > p_j > p_i - t, \\
0 & \text{otherwise.}
\end{cases}
$$

It is easy to check that these payoffs satisfy (A1). Firm two's static reaction curve if $R_2(p_1) = \max(c, \frac{p_1 + t + c}{2})$. The reaction curves have a unique intersection, at which both firms' prices are above cost so that (A4') is satisfied. We assume that $\alpha$ is uniformly distributed on $[\bar{\alpha}, \bar{\alpha}]$, with the static Bertrand profit, $\alpha = (\bar{\alpha} + \bar{\alpha})/2$.

We now proceed to solve for firm one's first-period equilibrium price. Recall equation (2b),

$$
(2b) \quad \Pi_1^1(p_1, R_2(p_1^*)) = \Delta f(\bar{\alpha}) \cdot \Pi_1^2(p_1, R_2(p_1^*)).
$$

In this example equation (2b) becomes

$$
(7) \quad \frac{-3}{4t} (p_1 - c - t) = \Delta f(\bar{\alpha}) \cdot \frac{p_1^* + t - c}{4t}, \text{ so that}
$$

...
assuming that the first-order condition (2b) is sufficient for the incumbent's maximization. Since \( f' = II_{11}^2 = 0 \), equation (3) is negative, so that the incumbent's choice problem is locally concave and (A5) is satisfied. (A6) is satisfied but irrelevant because \( p_1^* > 0 \). The important condition is (A7): given that the entrant expects \( p_1^* \), does the incumbent prefer to deviate to a price above \( d(p_1^*) \)? Now condition (A7) depends on \( \Delta \), the value of entry deterrence, which in the model presented so far is infinite, because there is no outside good. Introducing an outside good of value \( u \) means that a consumer at distance \( y \) will buy from a monopolist only if \( p + ty < u \); so that by varying \( u \) (but keeping it high enough that the static Bertrand equilibrium is unaffected) we can make \( \Delta \) as large as we please.

Moreover, for \( \Delta \) large enough we can show (A7) is satisfied, so that we do have a (unique) equilibrium.

Now we turn to the comparative statics. As \( \Delta \) increases, \( p_1^* \) decreases and converges to \( c - t \), the price at which the incumbent captures the entire market. Changing \( f(\alpha) \) is more complicated because there are many ways to do it. Consider increasing \( a \) and decreasing \( \bar{a} \) so as to keep \( \text{Prob}(\alpha < \bar{a}) \) a constant and in particular maintaining \( \underline{a} < \alpha < \bar{a} \). In this case \( p_1^* \rightarrow c - t \) so that the "better the information" the lower the price. Now the reader may be concerned that this surprising result is not an equilibrium because the global concavity conditions are not satisfied for very concentrated distributions. However, if \( f(\bar{a}) \) is a constant, the incumbent's first-period
loss to playing $p_1^*$ converges to $-t$ and playing $p_1^*$ yields him $\text{Prob}(a>a) \cdot \Delta$ in expected second-period profits. If $\Delta$ is sufficiently large then it pays to invest in signal-jamming in the first-period, rather than deviating to $R_1(R_2(p_1^*))$, which maximizes first-period profit but induces no exit.
References


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