working paper
department
of economics

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No. 416 April 1986

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Paul Anthony Samuelson (born in Gary, Indiana in 1915) has made fundamental contributions to nearly all branches of economic theory. Besides the specific analytic contributions, Samuelson more than anyone else brought economics from its pre-1930's verbal and diagrammatic mode of analysis to the quantitative mathematical style and methods of reasoning that have dominated for the last three decades. Beyond that, his *Economics* (McGraw Hill, first edition, 1948, now in its twelfth edition, the first with a co-author, William D. Nordhaus) has educated millions of students, teaching that economics however dismal need not be dull.

Ten eminent economists describe and evaluate his work in their respective fields in Brown and Solow (1983). Others have written their evaluations of contributions in specific areas in Feiwel (1982). Arrow (1967) and Lindbeck (1970) provide useful overall reviews.


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1Department of Economics, MIT, and Research Associate, NBER. This article was written for the New Palgrave Dictionary of Economics. I am grateful to E. Cary Brown, Rudiger Dornbusch and Robert Solow for informative comments on a first draft.
first four volumes of the Collected Papers include 292 articles, most of them indeed scientific.

Bliss in his 1967 review of the first two volumes of the Collected Scientific Papers comments on the impossibility for anyone other than Samuelson of reviewing his work. The task has not been made any easier by the publication of another two volumes of collected papers, and by the 144 page summary of developments in economic theory since the Foundations in the 1983 enlarged edition. Rather than try to be comprehensive, I will describe the major analytic contributions in several areas, ending with macroeconomics where I also discuss Samuelson’s views and advice on economic policy. I conclude with a description of his role at and through MIT.

Although the topic-by-topic approach is unavoidable, the man and the economist is more than the sum of his contributions in several areas. The verve and sparkle of his style, the breadth of his economic and general knowledge, the mastery of the historical setting and the generosity of his hyphenated freight-train allusions to predecessors, are unique. Samuelson’s presidential address to the American Economic Association (1961:II.Ch.113) is a good sampler. (References to the Collected Scientific Papers (CSP) will give year of publication of the original article where needed, followed by volume number, and chapter and/or page number as needed.)

I. Background.

Samuelson has provided fragments of his autobiography in
"Economics in a Golden Age: A Personal Memoir" (1972:IV,Ch.278), and in biographical articles on contemporaries and teachers. He attended fourteen schools in Gary, Indiana, on the North Side of Chicago, in Florida, and then at Hyde Park High in Chicago. From Hyde Park High he entered the University of Chicago in January 1932, taking his first economics course from Aaron Director. "It was as if I was made for economics" (1972:IV, p.885). Milton Friedman and George Stigler were Chicago graduate students at the time. Jacob Viner's famous course in economic theory provided the sound non-mathematical microeconomics that any economist needs to truly understand the field. ((1972:IV,Ch.282); see also Bronfenbrenner (1982)).

In 1935 he moved to graduate school at Harvard, propelled by a fellowship that required him to leave Chicago, and attracted he claims by the ivy and the monopolistic competition revolution. Samuelson spent five years at Harvard, the last three as a Junior Fellow. It was the time of both the Keynesian and monopolistic competition revolutions and "Harvard was precisely the right place to be" (1972:IV,p.889). The teachers he mentions most are Hansen, Leontief, Schumpeter and E.B. Wilson, the mathematical physicist and mathematical economist.

His fellow students make up the larger part of the honor roll of early post-World War II United States economics (1972:IV,p.889). Among them was his wife of forty years, Marion Crawford, author of a well-known 1939 article on the tariff. Abram Bergson (particularly his 1938 article on the social welfare function) and Lloyd Metzler are most mentioned among his other fellow students. Samuelson was the dominant
presence among the students: Cary Brown in conversation describes the excitement as his papers were analyzed and absorbed by the graduate students.

The Keynesian revolution and Alvin Hansen had a greater impact on Samuelson's work and attitudes than the monopolistic competition revolution and Chamberlin. Chamberlin is barely mentioned in his reminiscences of Harvard and his only monopolistic competition article appeared in 1967 in the Chamberlin *festschrift*. (III.Ch.131). Much of Samuelson's work assumes perfect competition, but none of his macroeconomics or his policy advice gives any credence to the view that the macro-economy is better left alone than treated by active policy (except perhaps his views on flexible exchange rates).

His first published article "A Note on the Measurement of Utility" (1937:1,Ch.20) appeared when he was a twenty one year old graduate student. By 1938 the flow was up to five articles a year, a rate of production that has been maintained with perturbations for half a century. And of course, since 1948 he has produced a new edition of *Economics* almost every three years.

Samuelson moved to MIT as an Assistant Professor in 1940 and has remained there since. Harvard's failure to match MIT's offer at that time has been the subject of much speculation. Samuelson has been eager to find excuses for Harvard (1972:IV,Ch.278, footnote 11, p.896). His is not the best position from which to judge or to write freely; he has noted that academic life, and by implication the chairman of the Economics Department, Burbank, one of the few of whom Samuelson speaks
harshly in print, were not innocent of antisemitism in that pre-World War II era. Burbank was a political power in the Department and University. His attitude to mathematical economics can be gauged by the fact that indifference curves were outlawed in the introductory course he supervised.

It is hard to believe that even the Harvard of 1940 would have been unable to find room for an economist of Samuelson's already recognized stature unless a non-academic reason or reasons stood in the way. Among those reasons were antisemitism, his then brashness, and his brilliance: indeed Schumpeter is rumored to have told his colleagues that it would have been easier to forgive their vote if it had been based on antisemitism rather than the fact that Samuelson was smarter than they were.

The MIT Economics Department owes much to Harvard. Samuelson has been at MIT since 1940, virtually without a break. Except for a few months away on a Guggenheim, he has taken time off only in Cambridge, Mass. He proudly claims that he has never been in Washington for as long as a week—though he was a major adviser to President Kennedy. His only departure from academic economics came in 1944-45 when he worked at MIT's Radiation Laboratory. He became one of twelve MIT Institute Professors in 1966.

He has gathered all the honors the profession can offer: the first John Bates Clark medal (1947) for the best American economist under forty; the second Nobel Memorial Prize in Economics (1970); he has been President of the American Economic Association (1961), the
Econometric Society (1951), and the International Economic Association (1965-68); and he has been awarded numerous other prizes and honorary degrees.

Although many graduate students have passed through his classes and been profoundly affected by him, there is no Samuelson school of economics, no overarching grand design for either economics or the world that is uniquely his. It is for that reason that his contributions have to be discussed field by field. The nearest that he has come to proclaiming a vision is in the Foundations.

II. Foundations of Economic Analysis.

Foundations, published in 1947, is based on Samuelson's 1941 David Wells' Prize-winning dissertation. Foundations of Analytic Economics, subtitled "The Observational Significance of Economic Theory". Its themes are partially described by the subtitle and by the motto from J. Willard Gibbs, "Mathematics is a Language". The thesis, dated 1940, is very close in content to the Foundations.

The Foundations in places claims to be an attempt to derive empirically meaningful comparative equilibrium results from two general principles, that of maximization, and Samuelson's correspondence principle. The correspondence principle states that the hypothesis of dynamic stability of a system yields restrictions that make it possible to answer comparative equilibrium questions.

The maximizing theme recurs in Samuelson's 1970 Nobel Prize lecture "Maximum Principles in Analytic Economics" (III, Ch.130). The
point is not the now common view that only models in which everyone is relentlessly maximizing are worth considering. Rather it is that the properties of the maximum (for instance second order conditions) usually imply the comparative static properties of the system. Samuelson also invokes the generalized LeChatelier principle, which loosely interpreted states that elasticities are larger the fewer constraints are imposed on a system. Analogies from physics (and biology) figure prominently in Samuelson's analytic methods and explanations of his results.

The correspondence principle was intended to do for market or macroeconomic comparative statics what maximization did for the comparative statics of the individual or firm. The principle can be useful when the analyst knows something about the dynamic behavior of a system, but as noted by Tobin (1983), is ambiguous in that different dynamics may be consistent with the same steady state behavior.

The simplest example of the ambiguity can be seen in a demand-supply diagram where the supply curve is negatively sloped. Whether a tax on the good will increase or reduce price depends on which curve is more steeply sloped. Whether the market is stable or not depends on the same fact and whether quantity or price rises in response to excess demand--that is, whether dynamics are Marshallian or Walrasian.

In the introduction to the 1983 enlarged edition, Samuelson records correctly that the *Foundations* was better off for not sticking to its narrow themes. Substance keeps breaking in on the methodology. The treatment of the theory of the consumer and firm, developed in detail, does not differ in substance from that of Hicks in *Value and
Capital. But where Hicks hides the mathematics in appendices, Samuelson flaunts his in the text. Nonetheless Samuelson takes pains to provide economic insight, including interpretations of Lagrange multipliers as shadow prices. These portions of the *Foundations* apparently existed in 1937-38 and were written independently of *Value and Capital* (Bronfenbrenner, 1982, p.349), though not of course of Hicks and Allen (I,Ch.1.p.4).

The theory of revealed preference (see below) receives prominence, as do two chapters on welfare economics, and in Part II chapters on the stability of general equilibrium. A few pages on money in the utility function (pp. 117-124) remain authoritative. The mathematical appendices on maximization and difference equations have been useful despite an elliptical style that leaves many steps to be filled in by the user.

Samuelson's thesis is dated 1940: *Foundations* is the work of a 25-year old. There are signs of youth in the eagerness to proselytize for the new mathematical faith and its overreaching in trying to impose an entirely coherent theme on the material. But the book bears the unmistakeable mark of the master, in command of the economics of his material, at home with technique, and most remarkably for a young man in a hurry, thoroughly familiar and patient with the literature. It is, as Schumpeter no doubt remarked, a remarkable performance.

III. Consumer Theory and Welfare Economics.

Samuelson's first published paper (1937:I,Ch.20) set up a finite
horizon continuous time intertemporal optimization model of a consumer with additively separable utility function and exponential discounting, and derived the result that the profile of consumption is determined by the relation between the interest rate and rate of time preference. The focus is however the measurability of utility.

The theory of revealed preference, his major achievement in consumer theory, made its unnamed appearance in 1938 in "A Note on the Pure Theory of Consumer's Behaviour" in *Economica* (I, Ch.1; see also Houthakker, 1983, and Mas-Colell, 1982, for exceptionally lucid accounts). The purpose was to develop the entire theory of the consumer free of "any vestigial traces of the utility concept". (I, p.13). Rather than postulate a utility function, or as Hicks and Allen had done, a preference ordering, Samuelson imposed conditions directly on the choices made by individuals—-their preferences as revealed by their choices. The key condition was the weak axiom of revealed preference, applying to choices made in two situations, say zero and one. With prices and quantities of goods \( j \), \( j = 1, \ldots, n \) in situation \( i \) given by \( p_j^i \) and \( x_j^i \), the axiom is

\[
\sum_j p_j^0(x_j^1 - x_j^0) \leq 0 \text{ implies } \sum_j p_j^1(x_j^1 - x_j^0) \leq 0
\]

In words, if the individual chooses consumption bundle zero when he could have chosen the bundle one, he will not choose one when zero is available.

This minimal condition of consistency is shown to imply most of the conditions on demand implied by utility theory. But the symmetry
and negative definitiveness of the Slutsky matrix could not be established using the weak axiom. Equivalently, the issue was the so-called integrability of demand functions, with the question being whether the preference map could be recovered given enough observations on the individual's choices. Houthakker (1950) solved the problem, by proposing the strong axiom of revealed preference, namely that in a string of choices, in which A is not preferred to B, B is not preferred to C, ..., and Y is not preferred to Z, that A is not preferred to Z. In this case the demand functions are integrable and an entire preference map, satisfying the Slutsky conditions, can be recovered from the individual's choices.

The full equivalence between the properties of the demand functions of an individual and the preference ordering is the leading example of Samuelson's definition of the operational or observational significance of economic theory. Samuelson regards a theory as meaningful if it is potentially refutable by data. A single consumer could make a succession of choices that contradict the strong axiom. But the theory is not operational in the sense that a modern econometrician would want it to be: it does not apply to aggregate data, nor in the form in which Samuelson left it, does it apply to choices that are made in chronological time.

Revealed preference links the theory of demand, index numbers, and parts of welfare economics. The link between demand and index number theory comes in the Foundations (pp.147-148) recognition that the fundamental index number problem is to deduce from price and
quantity information alone whether an individual is better off. Using the weak axiom, Samuelson demonstrates the conditions under which, in a comparison of two situations, it is possible to say whether an individual is better off in one. (Foundations, pp.156-163) He argues that index numbers add no information on the essential question and indeed may be positively misleading in tempting the observer to attach significance to the numerical scale of measurement.

A similar concern no doubt motivates Samuelson's long-standing hostility to the use of consumer surplus measures. He has frequently argued that there is no need for the concept. He asserts in Foundations (p.197) that there is no need for consumer's surplus in answering, for example, the question of whether Robinson Crusoe, a socialist state, or a capitalist one, should build a particular bridge. That view may have been moderated over the decades: the 1985 Samuelson-Nordhaus Economics (p.418) states that the concept "is extremely useful in making many decisions about public goods--it has been employed in decisions about airports, roads, dams, subways, and parks" (bridges are conspicuously absent).

The revealed preference axiom comes into play too in Samuelson's "Evaluation of Real National Income" (1950:II.Ch.77), a largely negative report on the then new welfare economics that attempted to deduce from aggregate data criteria that would make it possible to say whether society was better off in one situation than another. Taking as he has since 1938, the viewpoint that a Bergsonian social welfare function is the best way of understanding social welfare issues, Samuelson showed
that no index-number type national income comparison between situations A and B could reveal whether society's feasible utility possibility frontier (a useful Samuelson innovation, apparently simultaneously invented by Allais) in A lies uniformly outside that of B. And, he argued, we could claim situation A is better than B only if that is the case.

In the Foundations (Chapter 8) Samuelson draws extensively on the Bergsonian social welfare function to elucidate definitively the notion of Pareto optimality and the "germ of truth in Adam Smith's doctrine of the Invisible Hand" (Foundations, 1983 edition. p.xxiv). Arrow (1967) is critical of Samuelson's failure to look behind the social welfare function, and of his failure to link it to actual policy decisions. Similar sentiments are conveyed along with a more complete evaluation of Samuelson's welfare economics in Arrow (1983). Samuelson (1967:III,Ch.167) asserts that the Bergson Social Welfare Function and the Arrow Constitution Function are distinct concepts, though the argument is difficult to follow.

The expected utility theorem shows Samuelson wrestling for decades with his doubts over the independence axiom (I:Ch.12,1950; Ch.13,1952;Ch.14,1952; Foundations,1983,pp503-518). Despite his tentative 1983 acceptance of the expected utility formulation, he notes with approval Machina's 1982 development of expected utility without the independence axiom. Of course, these doubts have not kept him from making creative use of the expected utility approach in models of portfolio choice and finance.
IV. Capital Theory.

The theory of capital and growth sections of the first four volumes of CSP account for 38 papers, the largest single category. Although capital theory is the branch of economics most vulnerable to Samuelson's comparative technical advantage, and although both his earliest papers are placed in that category in CSP (1937: I, Ch. 17, Ch. 20) the output in this area is concentrated in CSP III, covering the years from 1965 to 1970. Solow (1983) provides a fine review of this part of Samuelson's research, some of which he co-authored.

Among the early papers, the 1943 Schumpeter festschrift contribution "Dynamics, Statics, and the Stationary State" (I, Ch. 19) discusses the economics of the steady state and the possibility of a zero interest rate. Samuelson argues that a steady state with a zero real interest rate is possible if the rate of time preference of the infinitely lived individuals is zero; he has in mind a situation in which the marginal product of capital can be driven to zero. In this article (I, p. 210), as in his first paper (I, p. 216), Samuelson makes highly favorable reference to Ramsey, in contrast to the famous unflattering 1946 remark (II, p. 1528). The well-known argument that a zero rate of interest is impossible because income generating assets would have an infinite value is rejected, on the grounds that an infinite value is not a problem since assets could trade against each other at finite price ratios. Some second thoughts are presented in a 1971 paper (IV, Ch. 217); curiously Samuelson discusses the Schumpeter
issue entirely in a model with infinite horizon maximizers rather than
in an overlapping generations framework.

The modern contributions in CSP I include the famous 1958
consumption loans model, which will be examined in the macroeconomics
section, and the surrogate production function (1962: I Ch.28). As Solow
(1983) notes, much of the capital theory in CSP is related to
developments in Dorfman, Samuelson and Solow (1958), which itself grew

Notable among the contributions is a variety of turnpike
theorems. A turnpike theorem is conjectured in the 1949 Samuelson
memorandum, and fully worked out in the 1958 volume. The theorem states
that for any accumulation program, starting from an initial vector of
capital goods, and with specified terminal conditions, as the horizon
lengthens the optimal program spends an increasing proportion of its
time near the von Neumann ray; more generally in problems with
intermediate consumption, the economy spends time near the modified
golden rule. Several of the papers in the capital and growth section of
CSP III contain turnpike theorems. A periodic turnpike result is
reported in 1976 in (IV, Ch.224).

The surrogate production function was an attempt to justify the
aggregate production function as being consistent with an underlying
model with heterogeneous capital goods and production techniques, and
one type of labor. The article names and uses the factor price
frontier, noting that it had been used earlier by others, including
himself (in 1957: I.Ch.29). Samuelson shows that a downward sloping
factor price frontier is traced out in a competitive multi-capital goods multi-technique economy, with higher steady state wages accompanying a lower steady state interest rate. Further, this frontier has the same properties as in the one-sector model, with the slope of the factor price frontier equal to the capital labor ratio. The theorem is correct, but as noted by Solow, the conditions for it to obtain are special.

Under more general conditions, the famous reswitching result may occur in which a given technique of production that had been used at a low interest rate comes back into use again at a high interest rate (see the November 1966 Quarterly Journal of Economics). Reswitching implies that the one-sector neoclassical production function cannot be viewed as a general "as if" construct that describes the behavior of economies with several techniques of production. Cambridge, England critics of neoclassical capital theory viewed reswitching as a confirmation of the view that marginal productivity had nothing to do with distribution, since the same techniques of production might be used with two (or many) different distributions of income. Various criticisms are offered by Robinson (1975) and responded to with forebearance in CSP (1975:IV,Ch.216).

Samuelson started the surrogate production function article by denying the need for any concept of aggregate capital. That position would be strengthened by the reswitching result. However, as with so many useful constructs in economics, the concept of aggregate capital has survived the demonstration that its validity may be limited.
Neither Samuelson nor other neoclassics have been constrained by
reswitching from using one-sector production functions or marginal
productivity factor-pricing conditions.

The property that the slope of the factor price frontier is
equal to the capital-labor ratio is one example of the duality between
price and quantity that Samuelson began to emphasize in the Foundations
and has used repeatedly since. Foundations (p.68) contains the Roy’s
identity envelope condition that the derivative of the minimized cost
function of the firm with respect to the wage of factor i is the demand
for factor i. It also provides shadow price interpretations of Lagrange
multipliers. Samuelson has used duality in optimal growth and linear
programming problems ("Market Mechanisms and Maximization", 1949
(I,Ch.33) is a gem) and in CSP (1965:III,Ch.134).

V. Dynamics and General Equilibrium.

Chapters IX through XI of Foundations cover stability analysis
and dynamics, in both individual markets and the economy at large. The
basic assumption of this dynamics is the "law of supply and demand", that price rises in response to excess demand.

The impetus for the multi-market analysis came partly from
Hicks’ Value and Capital discussion of stability, in which there is no
explicit dynamical system. The Samuelson approach is general
equilibrium, though it does not start from the primitives of endowments.
As Hahn (1983) notes, the underlying microeconomics is not specified.
Samuelson nonetheless set the agenda of the next fifteen years for the
study of dynamics in a more explicitly general equilibrium framework, and most important, in a framework in which the issue of stability is precisely posed.

Explicit use of the law of supply and demand in theoretical work has fallen out of favor, though the Phillips curve can be interpreted as using that approach. The monopolistic competition wing of macroeconomics prefers to model price setting by firms and workers explicitly rather than rely on an auctioneer, and the equilibrium approach assumes prices are continuously at market-clearing levels. The older approach is used in disequilibrium macroeconomics, but is typically regarded as suspect.

Samuelson has not been a general equilibrium theorist in the sense of one striving for maximum generality. He has been general equilibrium in the sense opposed to partial equilibrium: he frequently works with models of the whole economy, in growth and capital theory, in trade and macroeconomics, and in his excursions into the history of thought.

The most micro-oriented of these general equilibrium contributions are the non-substitution theorem (1951:1, Ch.36) and factor-price equalization. The non-substitution theorem was presented at a 1949 conference, and was obtained independently by Samuelson and Georgescu-Roegen (I,p.521). Consider an economy where labor is the only primary factor, and where goods are used either for consumption or as input into the production of other goods. Suppose the production function for each good is neoclassical, permitting substitution among
factors of production, but there is no joint production.

The theorem is that relative prices in this economy are independent of demand, that is, are determined on the supply side alone. There is a single least cost way of producing each good, where cost is determined by direct and indirect labor requirements. Hahn (1983) provides a clear account of the theorem, and generalizations to dynamic systems with capital (1961: I, Ch. 37). The question in the system with capital is whether, given the interest rate, the relative price structure is unique. Conditions for uniqueness are discussed in Hahn. The link with the surrogate production function, published at about the same time, is clear. The nonsubstitution theorem is used also in Samuelson's discussions of Ricardo (1959: I, Chs. 31, 32).

VI. International Trade.

"[O]ur subject puts its best foot forward when it speaks out on international trade" (1969: III, p. 683), and some of Samuelson's best-known contributions are undoubtedly in this field. Jones' 1983 article describes Samuelson's considerable impact on trade theory: on the gains from trade; the transfer problem; the Ricardian model; the Heckscher-Ohlin-Samuelson model; and the Viner-Ricardo model.

Earliest among the well-known contributions is the 1941 Stolper-Samuelson result (II, Ch. 66) which uses the two-sector, two-country Heckscher-Ohlin model with identical production functions in the two countries to analyze the effects of the opening of trade, or the imposition of a tariff, on the wage. The result is that protection will
benefit the factor that is relatively (to the other country) scarce. Or, the opening of trade benefits the relatively plentiful factor. But the paper contains more than that result. As Jones (1983) notes, it introduces the basic elements of Heckscher-Ohlin theory for small scale trade models—and those models were the analytic core of real trade theory for decades.

Stolper-Samuelson flags the issue of factor-price equalization, the question of whether trade in goods alone can produce the factor price equalization that would obtain if factors were freely mobile. Ohlin claimed that trade would cause a necessarily incomplete tendency to equalization. Samuelson (1948 and 1949:II, Chs.67,68) showed in the Heckscher-Ohlin context conditions under which equalization would be complete: identical production functions in the two countries, no factor-intensity reversals, and similarity of the ratio of endowments (so that countries are not specialized in production). The paper was remarkable and surprising, and did not suffer from the happy co-incidence that a 1933 Abba Lerner contribution rediscovered by Lionel Robbins had independently reached the same conclusions in a similar model.

Factor price equalization in more generality is considered in the famous 1953 paper "Prices of Factors and Goods in General Equilibrium" (II, Ch.70), which caused a substantial literature including Gale-Nikaido (1965). It is striking that many of Samuelson's famous papers led to prolonged discussion of the exact conditions needed for his particular results to obtain: he opened more doors in economics than
he closed.

The transfer problem is an old issue in the literature that arose in the twenties, after World War II, and arises again in contemplation of the world debt crisis. Samuelson's 1952 and 1954 papers (II, Chs.74,75) are classics in this extensive literature, on the issue of whether a transfer from one country to another (such as German reparations) is likely also to worsen the terms of trade of the country making the transfer, which Samuelson describes as the orthodox presumption. In the modern context the orthodox presumption would be that the developing countries will have to suffer a terms of trade loss to run current account surpluses to reduce their indebtedness. Samuelson typically argues that there is no presumption about the terms of trade shift, though the orthodox presumption is more likely to hold where there are non-traded goods or impediments to trade (1971:III,Ch.163).

Samuelson's contributions to trade theory are classics: the contributions are basic, the models are tractable and fecund, the problems come from the real world as well as the literature, the articles continue to reward the reader. And they continue to be read.

VII. Finance.

Despite his longtime personal interest in capital markets, Samuelson's contributions to finance theory started only as he turned fifty. These papers are concentrated in CSP III and IV; the earlier ones are self-reviewed in "Mathematics of Speculative Price" (1972:IV,
Ch.240). Merton (1983) describes and evaluates six of Samuelson's favorite papers in finance, broadly defined to include his 1952 paper on expected utility and the independence axiom (I, Ch.14).

The two most important papers are "Proof that Properly Anticipated Prices Fluctuate Randomly", (1965:III, Ch.198) and "Rational Theory of Warrant Pricing", (1965:III, Ch.199). "Proof ..." provides a first precise formulation of the consequences for speculative prices of market efficiency. The theorem describes the behavior of the current price of a commodity for delivery at a given future date, e.g. June 1990 wheat. Assuming that speculators do not have to put up any money to enter the contract, the result is that the market price should be the expectation at each date of the June 1990 wheat price. Given rational expectations, there is no serial correlation in the changes in price. Hence "properly anticipated prices fluctuate randomly".

Samuelson says of this theorem that is now entirely basic: "This theorem is so general that I must confess to having oscillated over the years between regarding it as trivially obvious (and almost trivially vacuous) and regarding it as remarkably sweeping. Such perhaps is characteristic of basic results." (III,p.786).

Note what the theorem does not say, using the exchange rate as the example. The theorem is not that the exchange rate fluctuates randomly; predictable inflation or predictable business cycle fluctuations can cause predictable movements in the exchange rate. Rather it is the current price of foreign exchange at a given future date that fluctuates randomly. The notion that efficiency produces
random motion is itself fascinating. But far more important is the restriction on empirical behavior implied by efficiency that Samuelson derives in a well-defined context. Testing for efficiency of speculative markets has become a major industry.

"Rational Theory of Warrant Pricing" missed its target, but it is as Merton (1983) remarks, a near miss. Samuelson had pursued option pricing for well over a decade. He supervised Kruizenga's 1956 MIT dissertation on the topic, and was familiar with Bachelier's 1900 continuous time stochastic calculus calculation of rational option prices. Samuelson derived a partial differential equation for the option price that depends, among other variables, on the expected return on the stock and the required return on the option. The remarkable feature of the Black-Scholes solution to the problem is that the rational price of the warrant does not depend on the expected return on the stock, but rather on the risk-free rate. Nonetheless, the Samuelson differential equation can be specialized to the correct Black-Scholes equation.

Other contributions to finance theory include papers on diversification (1967:III,Ch.201), and on conditions under which mean-variance analysis can be justified (1970:III,Ch.203)--with continuous time models providing the best argument for the procedure.

VIII. Macroeconomics.

All the Samuelson contributions described to this point are firmly neoclassical. His work in macroeconomics presents a more mixed
picture. I take up in turn the early multiplier-accelerator model, which is not at all price-theory oriented, the neoclassical synthesis. Samuelson the policy adviser and commentator, and the entirely neoclassical consumption loans model.

The Multiplier-Accelerator Model: In a 1959 note (II.Ch.84) on the multiplier-accelerator model, Samuelson describes his contribution as being the algebraic generalization of a numerical example of Alvin Hansen's. The model (1939:II Chs.82,83) is a simple one in which current consumption is proportional to lagged output and investment is determined by the difference between current and lagged consumption (the accelerator). This implies a second order difference equation, which can generate asymptotic or oscillatory damped approaches to equilibrium, or oscillatory or non-oscillatory explosive paths for output. Although Frisch and Slutsky had already written on the ability of stochastic difference equations to mimic cycle-like behavior, Samuelson does not--except for a quotation from J.M. Clark that receives little emphasis--link his second order equation with a stochastic forcing term.

Samuelson (1939:II.p.1111), while emphasizing the simplicity of the algebraic analysis, argues for the empirical importance of the accelerator. This judgment has held up over time as accelerator effects continue to feature strongly in modern estimated investment functions. From the theoretical point of view, the multiplier-accelerator model is interesting for the lack of concern over microfoundations. Where a 1980's macroeconomist might agonize over the microfoundations of the consumption function, over the accelerator, or over the impact of
rational expectations of future output on investment. Samuelson proceeds constructively with a simple implicitly fix-price model. The famous 45-degree diagram popularized in *Economics*--and for several editions on the cover--forcefully emphasizes Samuelson's view that aggregate demand is the key determinant of output.

In the 1940 "Theory of Pump-Priming Reexamined" (II,Ch.85) he stipulates "the basic features of the private economy forming the environment within which governmental action must take place..."

(1) The economic system is not perfect and frictionless so that there exists the possibility of unemployment and under-utilization of productive resources. ..."

This view pervades Samuelson's macroeconomics. Indeed, when asked recently his view of the causes of wage and price stickiness, he replied that he decided forty years ago that wages and prices were sticky, that he could understand the behavior of the economy and give policy advice on that basis, that he had seen nothing since then to lead him to change his view on the issue--and that he had not seen a payoff to researching the question.

He was of course aware of the issues. An abstract of a paper presented at the 1940 meetings of the Econometric Society (II,Ch.88) describes a totally modern discussion of the question of whether general involuntary unemployment is impossible in a world of price flexibility. His penetrating 1941 review of Pigou's *Employment and Equilibrium* (II,Ch.89) outlines a simple classical model in which price flexibility through its effects on aggregate demand produces full employment even
with a constant real wage. This is not however Pigou's model: according to Samuelson Pigou adopts a model in which money wage flexibility is an alternative to active monetary policy. Samuelson never regarded the Pigou effect as being of real world significance (1963:II,Ch.115).

The Neoclassical Synthesis: Tobin (1983, p.197) describes the neoclassical synthesis as Samuelson's greatest contribution to macroeconomics. The synthesis is outlined in articles in the early fifties (1951:II,Ch.98;1953:II,Ch.99;1955:II,Ch.100) and developed in successive editions of Economics. It argues that monetary and fiscal policy can be used to keep the economy close to full employment, and the monetary-fiscal mix can be used to determine the rate of investment.

The synthesis represents the views of mainstream macroeconomics in the fifties and sixties, and perhaps in the seventies and even the eighties. Its activist spirit was evident in the Kennedy administration. Its acceptance must have been helped by the widespread use of Samuelson's Economics and by the many clones that preached its message.

Perhaps the most notorious component of the neoclassical synthesis is the 1960 Samuelson-Solow "Analytic Aspects of Anti-Inflation Policy" (II,Ch.102), which presents a United States Phillips curve. This article is frequently cited as containing the view that the Phillips curve presents society's long-run tradeoffs between inflation and unemployment.

It does not. The paper starts by discussing the difficulties of distinguishing cost-push from demand-pull inflation. Samuelson and
Solow then plot the scatter of percentage changes in average hourly earnings in manufacturing against the unemployment rate (the years plotted are not specified, but include the thirties). The discussion that follows considers alternative points on the Phillips curve as policy choices for the next few years. But the authors warn explicitly that the discussion is short-term, and that it would be wrong to think that the menu of choices represented by the Phillips curve "will maintain its same shape in the longer run. ... [I]t might be that ... low-pressure demand would so act upon wage and other expectations as to shift the curve downward in the longer run ..." (II,p.1352). This is though hardly a clear demonstration of the vertical long run Phillips curve--for Samuelson-Solow suggest that low demand might also cause the Phillips curve to shift up (a notion that many in Europe now find entirely believable)--but it is clear evidence that the authors were not guilty of believing the Phillips curve would stay put no matter what. In conversation, Samuelson has said that he was always the Kennedy administration pessimist about the long-run Phillips curve tradeoff.

The Policy Adviser and Commentator: Samuelson has long taken an active part in economic policy debates, through Congressional testimony, as consultant to the Treasury and the Fed, in his Newsweek column that ran every three weeks from 1966 to 1981, in other newspaper columns, public addresses, advice to candidates and Presidents, and in contributions at academic conferences and in symposia.

His views reflect the neoclassical synthesis, a disdain for rules rather than discretion in determining policy, and an almost
shameless eclecticism. He knows the macroeconomic numbers and can speak the language of policy discussions. He is a cautious forecaster, rarely committing numbers to print, preferring to decide on which side of the consensus to place his bets. His 1941 consumption function remains his only econometric work (II.Ch.87); he has said that the major disappointment in economics in the last forty years has been the failure of econometric evidence to settle disputes.

Macroeconomics is Samuelson's primary applied economics field, with finance the second. He keeps up with the current state of the macroeconomy, drawing on forecasts and empirical work of others. He is sceptical of individual forecasts though a law of averages permits him to put some trust in the mean or median forecast. His eclecticism makes his policy views less exciting than those of economists with a strong view of the way the world works— but he has never sought to be interesting rather than right. (This despite his 1962 (II,Ch.113,p.1509) comment on John Stuart Mill "It is almost fatal to be flexible, eclectic, and prolific if you want your name to go down in the history books...".)

Nonetheless, Samuelson's implied attitude to the applications of economics gives pause. As Arrow (1967) notes, his work reveals ambivalence about the relevance of neoclassical price theory. He shows no great faith that his microeconomics can be applied to the real world. No doubt comparative advantage plays a role in that attitude. But the theoretical sophistication he brings from microeconomics does not distinguish his macroeconomic policy advice and forecasting from that of
the pack; his neoclassical training is not seriously used in Samuelson's applied macroeconomics. Economics may be evidence however that he values simple microeconomics.

The Consumption Loans Model: In the classroom Samuelson has confessed that among his many offspring the consumption loans model (1, Ch. 21) is his favorite. The affection is amply rewarded: within macroeconomics the two-period lived overlapping generations structure of the model has been used in countless papers in which a tractable framework with an explicit time structure is needed. The original consumption loans model examined the role of money or bonds as institutions for making Pareto-improving trades feasible; the structure has been used subsequently to examine the dynamics of capital accumulation, the burden of the debt, Ricardian equivalence, social security, the role of money, the effects of open market operations, intertemporal substitution of leisure, labor contracts, government financial intermediation, and more.

The setup for the original model is one in which people live two periods, with utility functions defined over consumption in the two periods. Each young person receives an endowment of one nonstorable chocolate in period 1. In the absence of trade each person could consume only in period 1. Trades are possible in which the current young give part of their chocolate to the current old in return for chocolate to be received next period from the then young. But there is no double coincidence of wants, no direct way of making the bilateral trades.

Now comes the ostensible point of the model: the social
contrivance of money makes trade possible, and its introduction is a Pareto-improving change given the pattern of endowments. The consumption loans model has been much criticized as a model of money, because it implies the velocity of circulation is one per generation. Equivalently, the criticism is that the model describes money as effectuating intergenerational transactions whereas in practice other assets, such as bonds, serve that role. (Patinkin (1983) discusses the consumption loans model and also Samuelson's excursions into the history of monetary thought)

This is certainly correct. But the significance of the consumption loans model is not its rationale for the existence of money. Rather the model has been so influential and popular because it provides a simple tractable general equilibrium structure for modelling intertemporal problems with life-cycle maximizing individuals. The earlier examples prove how easily the general structure can be adapted. It can also be adapted to more periods of life (in the original article Samuelson extended lifetimes to three periods), with fifty period lifetime models being easily solvable on computers. Its strength lies in the elegance and robustness with which it captures the essential point that finite lived individuals exist in an infinitely lived economy (we are each but not all dead in the long run).

Samuelson has remarked that "there is but one grand concept of general equilibrium and it was Walras who had the insight (and luck) to find it" (1962:II,p.1502) A similar comment can be made about the structure of the consumption loans model.
IX. Samuelson and MIT.

MIT had famous economists before Samuelson, Francis A. Walker, third president of MIT (1881-99) and first president of the American Economic Association (1886-92) and Davis R. Dewey, president of the AEA (1909) and editor of the American Economic Review (1911-40). But the modern era, in which the Department of Economics has risen to world-wide prominence and an exalted ranking within the profession begins with the arrival of Samuelson in 1940. Brown and Solow (1983) describe the MIT Department of the thirties, and the transformation that nearly began in 1941 after Samuelson arrived and the first Ph.D. class, including Lawrence Klein and George P. Shultz, was about to get under way. World War II intervened, and it was only in the late forties and early fifties that the faculty and the Ph.D. program reached full strength.

The MIT department and Ph.D. program have been consistently among the best in the world since the early sixties. The names of the faculty members are well-known. Equally remarkable is the collection of eminent economists who are MIT Ph.D.'s, whose names are legion but whom it would only be invidious to begin to list.

Samuelson's role in this success was pivotal but not domineering. His research habits (including sheer hard work), the open-door policy for students (a lesser burden for someone of whom the students were in awe than for others) and fellow faculty, his absolute refusal to use authority instead of reason in faculty meetings, his zest for conversation about economics, economists, and all else, made him a
role model for a department where co-operation and friendliness have been extraordinary. He helped shape the department but he did not dictate its shape; he told one of his young co-authors that as a young man he decided that at age forty he would stop taking initiatives in the department, at fifty he would venture an opinion only when asked, and at sixty would stop attending faculty meetings. Within the margin of error allowed to economists, he held to that resolution.

Samuelson the teacher played a lesser role. His world-wide fame (and that of other faculty members) doubtless was a major reason many of the outstanding students were there. But, at least in the last two decades, he supervised relatively few theses. His method of supervision was ideally suited to better students, for he would ask broad questions and give general guidance rather than involve himself in details.

His classroom lectures in the period 1966-69 when I heard them were not a model of organization. His advanced theory lectures were given in the first class of the day and it was always possible to tell whether the traffic had been bad that day by whether his hand-written mimeographed lecture notes were available at the beginning of the lecture or only later. The time until the notes arrived was taken up by stories setting the historical background for the problem, and anecdotes about the protagonists. The day he lectured en route to deliver his contribution to the Irving Fisher festschrift (1967:III,Ch.184) was especially memorable, though word filtered back from New Haven that his Yale audience was less than enchanted by the stories. His students were not surprised to find in his Nobel lecture (1970:III,Ch.130) both that...
he had been warned that the lecture was to be serious, and that he started a story with that warning.

His lectures were simply not designed for the novice. But they were superb for those with some background. He explained finer points, threw out open questions, made unexpected connections between topics, and communicated the zest with which he approaches economics.

X. Concluding Comments.

Among the missing from this list of Samuelson's contributions are his work in the history of thought, his methodological articles, the famous public goods theorem, the recent work on mathematical biology, the informative and entertaining biographies of contemporaries, the frank self-evaluations, and *Economics*.

The extraordinary success of *Economics* is something of a mystery, for the book is not easy— as witness the fact that simpler texts that follow Samuelson's structure have found a large market. *Economics* is a multi-level book that in its appendices, footnotes, and allusions goes far beyond elementary economics. Depending on what students retain from their economics courses, *Economics* may have done much to raise the level of public discourse about economic policy.

Samuelson's self-evaluations, as in "Economics in a Golden Age" (1972:IV, Ch.278), must have shocked many readers. The typical self-effacing scientist does not include stories of Newton and Gauss in his intellectual autobiography. Reflection leads to a different perspective: it would have been easy for Samuelson not to "tell the
truth and shame the devil" (1972:IV,p.881). But how much more interesting it is to have the account of how Samuelson views his own achievements.

Samuelson was described in 1967 as "knocking on the door ... of the pantheon of the greats ..." (Seligman, p.160). He may have been let in by now. But the final word has to be left to Franco Modigliani, who, after all the speeches at the 1983 party at which Samuelson was presented with the Brown-Solow festschrift, walked over to the seated Samuelson, wagged his finger at him, and said "You", and after a pause "You have enriched our lives".
REFERENCES.


