TRADE AND PRODUCTION PATTERNS UNDER UNCERTAINTY

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Number 78  August 1971
I. Introduction

What happens to the propositions of trade theory when the traditional assumption of certainty is relaxed? In the present paper we direct this question to the classical or Torrens-Ricardo version of the theory. The classical theory is a most suitable guinea pig since the strong assumptions on which it is based yield implications for trade and production patterns which, compared to those of the more fashionable Heckscher-Ohlin theory, are ambiguous.

Of course, we cannot take over the classical theory just as it is. It must be suitably modified so that uncertainty may be grafted in a non-trivial way. The modifications we have chosen relate to the timing and short-run reversibility of decisions concerning the allocation of labour to competing activities. We do not think that this modest dose of dynamics would have been found unpalatable by Torrens or Ricardo.

Suppose then that:

(i) There are just two trading countries, each composed of identical consuming-producing units which may, however, differ from country to country.

(ii) Each country is capable of producing two tradeable commodities. With each commodity there is associated a no-joint-products activity vector in which a single primary factor, labour, and possibly the other commodity appear as inputs. The commitment of labour to each activity
must be made one period before production takes place; in the period during which production takes place, labour may be withdrawn from an activity, but it cannot be transferred to the competing activity. In short, _ex ante_ labour is mobile between activities, _ex post_ it is immobile. However, actual input and output are contemporaneous.

(iii) Each country produces in addition a purely domestic or non-tradeable commodity. The output of this commodity is, like Cournot's spring water, an exogenously determined constant, beyond the control of the individual consuming-producing units, and does not absorb labour or either tradeable good. The supply of this commodity to the consumers is random and it affects their choices concerning the producible commodities. This is how uncertainty enters the model.

(iv) Labour is internationally immobile and in fixed aggregate supply.

(v) Either all product markets are spot, that is, all contracts to buy and sell are contemporaneous with the exchange itself, or all product markets are forward, that is, all contracts are made one period before the exchange takes place.

The assumptions are not quite those of Torrens and Ricardo. In particular, we have departed from the strict letter of classical trade theory in recognizing both purely domestic and intermediate goods and in introducing a lag between the commitment of labour and the associated output of each of the two tradeable commodities. Nevertheless, in the absence of uncertainty (i.e. when the supply of the exogenous commodity is constant) our assumptions yield all the familiar classical conclusions.
In particular, the relevant production possibility frontiers are straight lines based on free mobility of labour between activities, i.e. based on long run possibilities. The comparative advantage in production will then play its full role as in classical theory.

We shall show, however, that under uncertainty and imperfect mobility the implications of the classical theory are no longer valid. In particular, the comparative advantage in production has little predictive value concerning the patterns of trade or specialization. To dramatize our analysis we will show that uncertainty may completely reverse the patterns of trade and specialization under certainty. This will be shown both for spot and future markets.

These results have some implications in addition to the purely theoretical ones. They imply that the planning patterns of trade or specialization cannot be based on static considerations alone. They also imply that one should not generally expect to be able to explain empirical patterns of trade in terms of some crude notion of comparative advantage, as was for example the case with the controversy concerning the Leontief Paradox.

II. The Equilibrium of a Single Country in Isolation
(Spot Markets)

We begin by drawing a distinction between short-run and long-run production possibilities. Consider Figure 1. If the entire labour force were allocated to the production of the first tradeable commodity, the net output of that commodity would be \( X_1 \).
and the (negative) net output of the second tradeable commodity would be \( X_2 \), where \(-X_2\) is the amount of the second commodity needed to produce \( X_1 \) of the first commodity.* The vector \( OA_1 \) then represents the first activity normalized on the total labour force. Similarly, if the entire labour force were devoted to the second activity, the net outputs of the two commodities would be represented by the point \((X_1, X_2)\) and the second activity by the vector \( OA_2 \). For an open economy, the "long-run" locus of net production possibilities is the straight-line segment \( A_1A_2 \); for a closed economy, the long-run locus is that part of \( A_1A_2 \) cut off by the axes, that is, \( B_1B_2 \). During any particular time period, however, the allocation of labour between industries is fixed by decisions of the preceding period. Labour cannot be immediately transferred from one activity to another; at most it can be withdrawn from an activity and allowed to stand idle, thus ensuring some saving of the intermediate input. Suppose that one-half of the labour force is committed to each activity, so that the long-run production point is \( P^0 \). Then, for an open economy, the locus of short-run production possibilities is \( E_1P^0E_2 \), where \( E_1P^0 \) is parallel to \( OA_2 \) and \( E_2P^0 \) is parallel to \( OA_1 \); for a closed economy, the locus is \( D_1P^0D_2 \). (Of course, there is a different locus of short-run production possibilities for each allocation of labour, that is, for each choice of \( P^0 \) on \( A_1A_2 \).)

We consider now the equilibrium of a single country in isolation, say country A. Since all consuming-producing units in A are alike, we may introduce a single utility function \( U(C_1, C_2; C_3) \)

*See Figure 1
and interpret $C_i$ either as aggregate consumption of the $i$th commodity or as a constant multiple of consumption or as a constant multiple of consumption by the typical unit. The function $U$ is assumed to be not separable with respect to $C_3$. Under conditions of technological certainty we may set $C_3 = x_3^0$, where $x_3^0$ is the exogenously-given constant output of the purely domestic good, and thus obtain a "partial" utility function of the amounts consumed of the two tradeable commodities, that is, $U(C_1, C_2; x_3^0)$. Then, superimposing a family of indifference curves on Figure 1, we find that the community reaches a production-consumption equilibrium at $p_0$, where the locus of long-run production possibilities $b_1B_2$ forms a tangent to the indifference curve $I_0$. The loci of short-run production possibilities play no role in the determination of an uncertainty-free equilibrium.

In order to maintain comparability of the certainty and uncertainty models we introduce uncertainty in such a way as to have the production technology of the two tradeable commodities unaffected. For this purpose we suppose that the exogenously given supply of the third commodity is random, taking the values of $X_3^R$ and $X_3^S$ (R and S for "Rain" and "Shine") with probabilities $W^R$ and $W^S ( = 1 - W^R)$, $0 < W^R < 1$. The utility may then be written as $U(C_1, C_2; C_j^3)$ where $C_j^3 = x_j^3$ and $j = R, S$. For $X_3 = X_3^R$ we shall have a partial indifference map based on curves such as $I^R_{1R}$ in Figure 1, while for $X_3 = X_3^S$ the entire indifference map will change and we shall have curves such as $I^S_{1S}$. 


Given the choice of $p^0$ by the economic units the relative price of the first commodity in terms of the second ($p$) will be determined by the demand of the given value of $X^1$, and we shall have $p^R$ and $p^S$. This represents a short-run equilibrium. In a long-run the point $P^0$ itself is to be determined optimally.

Now what do we mean precisely by the long-run equilibrium of the economy? We mean (a) that all individual units behave in the same way, since they are all alike and face the same conditions. (b) that there exists a fixed and known probability distribution of relative commodity prices $p^R$ and $p^S$ with probabilities $q^R$ and $q^S$ respectively. (c) At any price $p^j$ ($j = R, S$) the economic units maximize $U(C, X; X^1)$ subject to the budget constraint (as under certainty). (d) At any given price the market is cleared. Thus in Figure 1 this forces the price $p^R$ to be identical with the slope of $D_2 P^0$. In fact it follows from the foregoing that if $OT^R$ and $OT^S$ are the expansion paths at $p^R$ and $p^S$ than for a feasible equilibrium $P^0$ must be located on the segment $F_1 F_2$. (e) The point $P^0$ is chosen so as to maximize the expected (maximal) utility at the given distribution of $p^1$. Thus denoting the maximal utility at $p^1$ and $P^0$ by $U^*(p^1, P^0)$, the point $P^0$ is chosen so as to maximize

$$U^R U^*(p^R, P^0) + U^S U^*(p^S, P^0)$$

for given $p^R$ and $p^S$.

Thus the long-run equilibrium is one of stationary uncertainty and consistent expectations. When we deal with a single country in isolation, which consists of identical units one can show that the foregoing stationary equilibrium is also Pareto optimal and identical with the optimum attained in an Arrow-Debreu model with contingent
prices.

III. The Trading Equilibrium (Spot Markets)

Let us now introduce a second country, say B, and the possibility of free trade in the first and second commodities. To keep the argument as simple as possible, we suppose that country B is small in relation to A, in the sense that B could sell any part of its maximum output of either commodity at the A-prices $p^A$ or $p^S$. The technology of country B is displayed in Figure 2. Denoting by $s(P_iP_j)$ the absolute value of the slope of any straight-line segment, referred to the horizontal axis, we assume that *

$$s(E_2P_0) < p^R = s(E_2P_0) < s(A_1A_2) < s(E_1P_0) = p^S < s(E_1P_0)$$

It follows that the price ratio facing country B fluctuates above and below the level indicated by $s(A_1A_2)$.

As a further simplification, we suppose that in country B there is no technological uncertainty at all. (Of course, B, a price taker, must still cope with uncertainty concerning prices.) The partial utility function of B is then $\hat{U}(\hat{C}_1, \hat{C}_2; \hat{x}_3^0) = \hat{U}(\hat{C}; \hat{x}_3^0)$, with $\hat{x}_3^0$ a constant. If $F_1P_0\hat{E}_2$ is the locus of short-run production possibilities in country B then at prices $p^R$ and $p^S$ the equilibrium consumption vectors are indicated by points $\hat{P}^R$ and $\hat{P}^S$, respectively. The equilibrium points change if the commitment of labour changes.

*See Figure 2
In the long-run equilibrium the small country chooses its $P^i$ so as to maximize the expected utility at the $p^i$ prices determined in the large country. (It may be noted that for the small country this stationary equilibrium with trade is different than the one that would prevail in an Arrow-Debreu model of contingent markets.)

In the case of certainty we know that the relevant production slopes are $s(A_1A_2)$ and $s(A_1\hat{A}_2)$ and if the latter is larger then country $B$ will specialize in the second commodity. The same will be true if both $p^R$ and $p^S$ are smaller than $s(A_1\hat{A}_2)$. However, if $p^i$ fluctuates above and below $s(A_1\hat{A}_2)$ then country $B$ will ordinarily diversify its production between the two industries. This does not mean however that its production will be dominated by its 'comparative-advantage' in the second commodity. To take an extreme case it is even possible that country $B$ will completely specialize in the first commodity (in which it has a comparative disadvantage under certainty).

IV. An Example

In the present section we develop a numerical example of complete trade reversal. Let the utility function of country $A$ be

$$U(C_1,C_2;C_3) = \log C_1 + \delta(C_3) \log C_2$$

where

$$\delta(C_3) > 0, \quad \frac{d}{dC_3} \delta(C_3) > 0, \quad \frac{d^2}{dC_3^2} \delta(C_3) < 0$$
so that $U$ is strictly concave in all of its arguments exhibiting constant relative risk aversion. The equation of the locus of long-run production possibilities is, say,

\[(5) \quad X_2^* = \alpha - \beta X_1^*\]

and the budget constraint is

\[(6) \quad C^R_2 + p^R C^R_1 = X_2^* + p^R X_1^* \quad \text{when} \quad X_3 = X^R_3\]

\[C^S_2 + p^S C^S_1 = X_2^* + p^S X_1^* \quad \text{when} \quad X_3 = X^S_3\]

The demand functions derived from (3), (5) and (6) are

\[(7a) \quad C^R_1 = \frac{\alpha + (p^R - \beta)X_1^*}{p^R[1 + \delta(X^R_2)]}, \quad C^R_2 = \delta(X^R_3) p^R C^R_1\]

and

\[(7b) \quad C^S_1 = \frac{\alpha + (p^S - \beta)X_1^*}{p^S[1 + \delta(X^S_3)]}, \quad C^S_2 = \delta(X^S_3) p^S C^S_1\]

Substituting from (7) into (3), and taking account of (5), we obtain

\[(8) \quad E\{u\}_j \equiv R, S \sum_j \left[ \log \frac{\alpha + (p^j - \beta)X_1^*}{p^j(1 + \delta^j)} + \delta^j \frac{\alpha + (p^j - \beta)X_1^*}{1 + \delta^j} \right] \equiv V(X_1^*)\]

Hence

\[(9) \quad \frac{d}{dX_1^*} V(X_1^*) = \sum_j \frac{\delta^j(1 + \delta^j)(p^j - \beta)}{\alpha + (p^j - \beta)X_1^*}\]

and it can be verified that $\frac{d^2 V}{dX_1^*^2} < 0$. At an interior maximum, expression (9) is equal to zero; hence
\[ \text{opt } x^* j = \frac{a \sum_j w^j (1 + \delta^j) (p^j - \beta)}{-(p^R - \beta)(p^S - \beta) \sum_j w^j (1 + \delta^j)} \]

We assume, as in Figure 1, that \( p^R < \beta < p^S \); hence the denominator of (10) is positive. We also assume, as in Figure 1, that \( p^R \) and \( p^S \) correspond to the slopes of the locus of short-run production possibilities and that the slopes of \( \partial T^R \) and \( \partial T^S \) are given, by (7), as \( \delta^S p^R \) and \( \delta^S p^S \), respectively. To ensure the existence of a feasible solution for the economy as a whole, we require that

\[ \frac{\delta^S p^S}{x^* 1} < \frac{x^* 2}{x^* 1} < \delta^R p^R \]

Applying (5) and (10) to (11), we obtain

\[ \frac{\delta^S p^S}{w^R (1 + \delta^R)(p^R - \beta)(-p^S) + w^S (1 + \delta^S)(p^S - \beta)(-p^R) < \delta^R p^R}{w^R (1 + \delta^R)(p^R - \beta) + w^S (1 + \delta^S)(p^S - \beta)} \]

Suppose now that

\[ \omega_s = 1 - \omega_R = 0.83, \delta^R = 6, \delta^S = 1, p^R = 1, p^S = 3, \beta = 2 \]

It may be verified that these values are consistent with (12). They therefore yield an internal equilibrium of country A between the two expansion paths.

Let us turn to the small country, B. The utility function of B is assumed to be

\[ U(C_1, C_2; C_3) = \log C_1 + \delta \log C_2 \]
For R, δ^R = δ^S = δ; hence the counterpart to (10) is

\[
\begin{align*}
\text{opt } X^*_1 &= - \frac{\alpha \sum_j w^j (p^j - \hat{\beta})}{(p^R - \beta)(p^S - \hat{\beta})} \\
&= - \frac{\alpha(p - \hat{\beta})}{(p^R - \hat{\beta})(p^S - \hat{\beta})}
\end{align*}
\]

(15)

where \( \bar{p} = \sum_j w^j p^j \). As in Figures 1 and 2, we require that

\[
p^R < \beta < \hat{\beta} < p^S
\]

(16)

Suppose further that neither productive process involves intermediate inputs, so that the locus of long-run production possibilities in country B is confined to the non-negative quadrant and the maximal value of \( \hat{\lambda}^* \) is

\[
\hat{\lambda}^*_{1m} = \frac{\alpha}{\hat{\beta}}
\]

(17)

For a corner solution at \( \hat{\lambda}^*_{1m} \) it is necessary and sufficient that

\[
\text{opt. } \hat{\lambda}^*_{1} \geq \hat{\lambda}^*_{1m} \quad \text{or, in view of (15) and (17), that}
\]

\[
\frac{- \hat{\beta}(p - \hat{\beta})}{(p^R - \hat{\beta})(p^S - \hat{\beta})} \geq 1
\]

(18)

From (13) we calculate that \( \bar{p} = 2.66 \). Suppose now that \( \hat{\beta} = 2.1 \).

The left-hand side of (18) is then 1.18 and the inequality is satisfied. Since \( p^R = 1 \) and \( \beta = 2 \), \( \hat{\beta} = 2.1 \) also satisfies (16). Hence country B specializes completely in the production of the first commodity, in spite of the fact that \( \hat{\beta} > \beta \).
V. An Alternative Model Incorporating Future Markets

The foregoing analysis rested on the assumption that, while the allocation of labour is determined under conditions of uncertainty, trade itself is conducted \textit{ex post} or spot, under conditions of complete certainty. In the present section we swing to the other extreme and assume that all contracts to exchange the two traded commodities are entered into \textit{ex ante} and involve the Arrow-Debreu type of contingent claims on future goods. In our earlier model we allowed trade within given states of nature but not across states of nature. We now introduce the latter possibility by defining commodities 1 and 2 in terms of their physical characteristics, location and state of nature and by assuming that all contracts to buy and sell are entered into before the actual production of the commodity. The third commodity is now supposed to be not subject to exchange.

The assumptions of Arrow and Debreu are, of course, unrealistic.\textsuperscript{2} Nevertheless they are attractive because they are simple and because, from a welfare point of view, they represent an idealization of the real world. Indeed, since the risk model of Arrow and Debreu preserves so many features of the model of risk-free competitive equilibrium one might suppose that it is incompatible with trade reversal of the type discussed earlier.

We begin by studying A (the large country) in isolation. For simplicity we now consider the extreme case in which neither commodity 1 nor commodity 2 is needed as an intermediate input in the
product on of the other commodity. Then the locus of short-run production possibilities is rectangular, as in Figure 3. We denote by \( p_{1}^{R} \) and \( p_{1}^{S} \) the prices contracted now to be paid next period for the delivery next period of a unit of the \( i \)th commodity in the two alternative states of nature. Then \( p_{1}^{R} + p_{1}^{S} \) is the price to be paid for the certain delivery of a unit of the \( i \)th commodity. Every economic unit is endowed with the same utility function \( V(C^{R}, C^{S}, C_{1}^{R}, C_{1}^{S}) \) which, we suppose, takes the special von Neumann form

\[
V = w^{R}U(C^{R}, C_{1}^{R}) + w^{S}U(C^{S}, C_{1}^{S})
\]

where \( C_{1}^{J} \) denotes the vectors \([C_{1}^{1}, C_{2}^{1}]\).

If today the economic unit sells \( X_{1}^{*} \) on the futures market, it receives tomorrow an income of

\[
I = \sum_{i=1}^{2} (p_{1}^{R} + p_{1}^{S})X_{1}^{*}
\]

where \( X_{2}^{*} = \alpha + \beta X_{1}^{*} \) along the locus of long-run production possibilities (\( A_{1}A_{2} \) of Figure 3). We consider now the problem of maximizing \( V \) with respect to \( C^{R} \) and \( C^{S} \) for given \( X_{1}^{*} \) and, to this end, introduce the Lagrangean

\[
L = w^{R}U(C^{R}, C_{1}^{R}) + w^{S}U(C^{S}, C_{1}^{S}) - \lambda \left[ \sum_{i=1}^{2} (p_{1}^{R}C_{1}^{R} + p_{1}^{S}C_{1}^{S}) - I \right]
\]

The first order conditions for an interior maximum are
where \( U_i^R = \partial U(C_1^R, C_2^R, X_3^R) / \partial C_1^R \), etc. Differentiating the optimal value of \( L \) with respect to \( X^*_1 \) and applying the appropriate envelope theorem, we obtain

\[
\frac{\partial}{\partial X^*_1} (\text{opt.} L) = \lambda \frac{\partial I}{\partial X^*_1} = I((p_1^R + p_1^S) + \beta(p_2^R + p_2^S))
\]

Thus for an interior solution it is necessary that the prices satisfy

\[
\frac{\partial I}{\partial X^*_1} = 0 \quad \text{or}
\]

\[
\beta = \frac{p_1^R + p_1^S}{p_2^R + p_2^S}
\]

a condition with an obvious counterpart under certainty. Finally, the equilibrium of the economy is determined by (22), (24) and the market clearing conditions

\[
C_1^R = C_1^S = C_1^* \quad i = 1, 2
\]

Including (5) we have altogether nine equations and nine variables: \( C_1^R \) and \( C_1^S \) (4 variables), \( X^*_1 \) (2 variables) and three price ratios. (In Sections II and III, because of the separation of the states of nature, it was possible to choose two separate numéraires; here we may choose only one.)
Let us now re-introduce country B. Since B is small in relation to A, all prices are determined in the manner just described. For B we retain our earlier assumption that each commodity is needed in the production of the other. Thus $a_{1j}$ is the amount of good $i$ required per unit of good $j$. The loci of short-run production possibilities are typified therefore by $E_1^P E_2^P$ in Figure 4 where $s(E_2^P) = \frac{a_{21}}{a_{12}}$ and $s(E_1^P) = \frac{1}{a_{12}}$. That figure is drawn on the further assumption that

$$0 < \frac{p_1^R}{p_2^R} < \frac{a_{21}}{a_{12}} < \frac{1}{\frac{p_1^S}{p_2^S}} < \infty$$

Let the equation of the locus of long-run production possibilities be

$$\hat{X}_2^R = \hat{a} - \hat{\beta} \hat{X}_1^*$$

and denote by $\hat{X}_{2m}^R$ the value of $\hat{X}_2$ at $E_2^P$ and by $\hat{X}_{1m}^S$ the value of $\hat{X}_1$ at $E_1^P$. It can be calculated that

$$\hat{X}_{1m}^R = \frac{\hat{a}_{12}}{1 - \hat{a}_{12} \hat{a}_{21} \hat{a}} \left[ -\hat{a} + (\hat{\beta} - \frac{\hat{a}_{21}}{a_{12}}) \hat{X}_1^* \right]$$

$$\hat{X}_{2m}^R = \frac{1}{1 - \hat{a}_{12} \hat{a}_{21}} \left[ \hat{a} - (\hat{\beta} - \frac{\hat{a}_{21}}{a_{12}}) \hat{X}_1^* \right]$$

$$\hat{X}_{1m}^S = \frac{\hat{a}_{12}}{1 - \hat{a}_{12} \hat{a}_{21}} \left[ \hat{a} - (\hat{\beta} - \frac{1}{\hat{a}_{12}}) \hat{X}_1^* \right]$$
The income of country B is

\[ \hat{I} = \sum_{i=1}^{2} (p_i R \hat{X}_{i1} + p_i S \hat{X}_{i1}) \]

(29)

(The short-run transformation function containing \( \hat{X}_{i1}^R \) and \( \hat{X}_{i1}^S \) depends, of course, on \( \hat{X}_{i1}^* \).)

Suppose now that the utility function of B is of the same general form as that of A, that is, \( \hat{V}(C^R, C^S, \hat{X}_{31}^R, \hat{X}_{31}^S) \), and let us maximize \( \hat{V} \) subject to given \( \hat{X}_{i1}^* \). This can be done in stages: first, \( \hat{I} \) is maximized; then \( \hat{V} \) is maximized, given \( \hat{I} \). From the first stage we obtain \( \hat{V}(\hat{X}_{i1}^*) \), and from the second \( \hat{V}(\hat{I}) \). Thus

\[ \frac{d\hat{V}}{d\hat{X}_{i1}^*} = \frac{d\hat{V}(\hat{I})}{d\hat{I}} \cdot \frac{d\hat{I}(\hat{X}_{i1}^*)}{d\hat{X}_{i1}^*} \]

(30)

Now \( d\hat{V}(\hat{I})/d\hat{I} \) is obviously positive; hence the economic unit will continue to increase (decrease) \( \hat{X}_{i1}^* \) as long as \( d\hat{I}(\hat{X}_{i1}^*)/d\hat{X}_{i1}^* \) is positive (negative). We may concentrate therefore on the second stage of maximization.

We note that the maximization of \( \hat{I} \) with respect to \( \hat{X}_{i1}^R \) and \( \hat{X}_{i1}^S \), given \( \hat{X}_{i1}^* \), involves two separable constraints, which may be written in general form as
\[ \phi(\hat{\lambda}^R; x^*_1) = 0 \]
\[ \phi(\hat{\lambda}^S; x^*_1) = 0 \]

It follows that if \( I \) is maximized with respect to \( \hat{\lambda}^R \) and \( \hat{\lambda}^S \) then \( \Sigma p_1^R x_1^R \) must be maximized subject to \( \phi(x^R; x^*_1) = 0 \) and \( \Sigma p_1^S x_1^S \) must be maximized subject to \( \phi(x^S; x^*_1) = 0 \). Thus, given \( x^*_1 \), we may independently maximize the income components associated with each of the two states of the world. From Figure 4, it is clear that in the first state the optimal production point is \( \hat{p}_2 \) and that in the second state the optimal point is \( \hat{p}_1 \). The net amounts produced are given by (28) above. Substituting from (28) into the expression for income, we obtain

\[
\max_{x_1^R, x_1^S} I = \text{const.} + \frac{1}{1 - \hat{\alpha}_{12} \hat{\alpha}_{21}} \left[ (\hat{\beta} - \hat{\alpha}_{21}) (p_1^R \hat{\alpha}_{12} - p_2^R) \right. \\
\left. + (\hat{\beta} - \frac{1}{\hat{\alpha}_{12}}) (p_2^S \hat{\alpha}_{21} - p_1^S) \hat{\alpha}_{12} \right]^x_1^* \\
= \text{const.} + k^{x_1^*}
\]
say, where, by the construction of Figure 4,

\[ \hat{\beta} - \hat{\alpha}_{21} > 0, \quad p_1^R \hat{\alpha}_{12} - p_2^R < 0, \quad \hat{\beta} - \frac{1}{\hat{\alpha}_{12}} < 0, \quad p_2^S \hat{\alpha}_{21} - p_1^S < 0 \]

and \( 1 - \hat{\alpha}_{12} \hat{\alpha}_{21} > 0 \).

We note now that since \( k \) is constant the small country must always specialize completely, as in the classical model (but in contrast to
Figure 4
the conclusions of Section III). We note also that the sign of \( k \) depends not only on \( \hat{B} \) and the prices but also on the parameters \( \hat{a}_{ij} \) of the locus of short-run production possibilities. Thus the direction of specialization in B depends partly on the ease of short-run adjustment in that country and is not simply dependent on the relative magnitudes of \( \beta \) and \( \hat{B} \). It follows that an example of trade reversal can be constructed.

We now provide such an example. Suppose that \( \hat{B} > \beta \), so that, under certainty, country B specializes in the production of the second commodity. Suppose further that there exists in country A an interior equilibrium, so that \( \beta = (p_1^R + p_1^S)/(p_2^R + p_2^S) \). Then

\[
\hat{B} > \frac{p_1^R + p_1^S}{p_2^R + p_2^S}
\]

(33)

in addition, from Figure 4,

\[
\frac{p_1^R}{p_2^R} < \hat{a}_{21} < \hat{B} < \frac{1}{\hat{a}_{12}} < \frac{p_1^S}{p_2}
\]

(34)

Now if country B is to specialize in the production of the first commodity then \( k \) must be positive, that is,

\[
(\hat{B} - \hat{a}_{21})(p_1^R \hat{a}_{12} - p_2^R) + (\hat{B} - \frac{1}{\hat{a}_{12}})(p_2^S \hat{a}_{21} - p_1^S) \hat{a}_{12} > 0
\]

(35)

Thus the problem of constructing an example of trade reversal reduces to that of finding positive values of \( \hat{B} \), \( \hat{a}_{ij} \), \( p_1^R \) and \( p_1^S \) which satisfy (33), (34) and (35). We offer the following example:
In conclusion we note that, simply by considering $X_3$ as a random preference parameter, it is possible to interpret our models in terms of uncertainty about tastes. On this interpretation, however, the plausibility of (1) as a criterion of country A's welfare is much reduced, and we do not wish to emphasize the possibility.
Footnotes

(1) See Arrow [1] and Debreu [2].

(2) See, for example, Stigum [3].
References


