STOCK EXCHANGE COMMISSIONS:
THE PRICING AND MARKETING OF SECURITIES

Ronald E. Grieson

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The views expressed in this paper are the author's sole responsibility and do not reflect those of the Department of Economics nor of the Massachusetts Institute of Technology.
STOCK EXCHANGE COMMISSIONS: THE PRICING AND MARKETING OF SECURITIES

Ronald E. Grieson*

While authors like Sharpe [5]; Samuelson [3]; Jensen [2]; Samuelson and Merton [4]; and Black and Scholes [1] have devoted much attention to the fundamental valuation of securities, mutual funds, warrants and options; little formal attention has been given to the setting of the number of shares and therefore the value per share of the stock of a given company.

This paper proposes to develop a model of the transaction cost minimizing per share price of securities. We then go on to compare the stock prices thus calculated with the level of stock prices existing today and discuss the reason companies may price their securities below transaction cost minimizing values (in order to increase brokerage firms' sales commissions, increasing promotion of the stock). Mention is also made of: the new negotiable commission rates; the existence of monopolistic rates which differ from the true costs of transactions in a non-proportional way, and the harmful effects and distortions which may result from the preceding.

I. The Present Structure of Commission Rates

At present brokerage fees are set by the exchange and not by competition thus not necessarily conforming to and perhaps lying above costs. The brokerage fee roughly consists of a fixed amount per hundred shares, or fraction thereof, and a percentage of the value of the transaction. The percentage of value portion of commission is higher for multiple round lot shares than for a transaction, of the same value, which involves one hundred or less shares even though there is a separate fixed commission of $6 per 100 shares and a per share odd lot fee.

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The maximum commission per hundred shares (the maximum minimum commission that is) is $71 per hundred shares for: a 100 share transaction of $4,778 or more; a 200 share transaction of $12,000 or more; a 300 share transaction of $19,230 or more; a 400 share transaction involving $31,250 or more; etc., all of which we will see are far below the transaction cost minimizing price per hundred shares. Hence purchasing 100 shares for $12,000 entails one-half of the commission involved in purchasing 200 shares having the same value. Purchasing a round lot of 400 shares selling at $31,250 involves four times the commission involved in purchasing 100 shares of the same value.

The commission on an odd lot is a maximum of $71 per hundred shares, reached for a 99 or less share transaction of over $5,000, plus 25c per share (when the per share price is over $55).

In addition there will be negotiated commissions for transactions involving $100,000 or more by April 1974. Presumably the negotiated commissions will be lower than the existing minimums and will be almost completely dependent upon value and only slightly (perhaps $6 per 100) dependent upon the number of shares.

In our models we will first assume that the value of desired transactions is determined by a probability density function, which is independent of the transactions cost that will be involved. In another model the value of transactions will be determined by a density function subject to the constraint that odd lots are avoided. Both are slightly unrealistic, defining the range of reality. The case in which the value of desired transactions, hereafter known as Case A, is influenced by brokerage commissions leads to higher optimal per share values than the case, B, in which the density function for desired transactions is independent of brokerage fees.
In either case we want to know the effect of the number of shares involved, in purchasing a given value of a security, upon the commission, since that is all the nominal price per share affects. The commission cost of an odd lot is dependent upon value plus 25¢ per share and hence the cost of increasing the number of shares is only 25¢ per share. In the case of round lots increasing the number of round lots, decreasing the per share price, raises the cost of commission by $71 per hundred shares. The third effect is that increasing the number of shares involved reduces the cost of 99 share transactions that go to 100 shares by $24.75. Later a special fractional share charge ranging from 1/2 to 5% will be added to the model.

II. The Stock Transfer Cost Minimizing Model

Case A) Assuming all transactions of over 100 shares are in round lots, even 100 share units, the total brokerage cost of all transactions in the security which are a function of a number or price of shares per unit time would be:

(continued on p. 3)
where: \( C \) is the total brokerage cost of all transactions per unit time:

\[ C = h(P^{-1})v + d \int_0^P \frac{V}{P} g(V) dV + b \int_{m}^{V_m} \frac{V}{P} g(V) dV \]

\[ = \frac{V_m}{P} \sum_{n=1}^{(n+1)P} \frac{1}{nP} \]

\[ = n = \frac{V_m}{P} \sum_{n=1}^{(n+1)P} \frac{1}{nP} \]

\[ = \frac{V_m}{P} \sum_{n=1}^{(n+1)P} \frac{1}{nP} \]

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All notation is the same. The model has all shares paying the odd lot commission which is rebated on the round lot portion of transactions.

III. The solution to the model given various probability density functions, \( g(V) \).

First let us assume a rectangular distribution \( g(V) = a = \frac{N}{V_m} \)

where \( N \) is the total number of transactions per unit time. Our cost function in Case A now becomes:
\( C = h(P^{-1})V + d \int_{0}^{P} \frac{V}{aP} \, dv + b \int_{P}^{V_m} \frac{V}{aP} \, dv \)

Integration and substitution gives:

\( C = h(P^{-1})V + \frac{a(d-b)}{2}P + \frac{abV_m^2}{2P} \)

Differentiating with respect to \( P \) yields

\( \frac{dC}{dP} = -h'(P^{-1})V + \frac{a(d-b)}{2} - \frac{abV_m^2}{2P^2} \)

Setting \( \frac{dC}{dP} = 0 \) and solving gives

\( P^* = V_m \sqrt{\frac{b}{(d-b)}} \)

Letting \( h'(P^{-1}) = 0 \), as is the case when the maximum \$71 per 100 shares commission is reached, it can easily be shown that \( P \) will be above the value at which \( h'(P^{-1}) = 0 \). Let \( P = \$100 \), then \( d = \$12.50 \) and \( b = \$14.40 \), hence \( d - b > 0 \).

If \( d \leq b \), \( V_m \to \infty \). At values for below infinity \( b \) goes to \$71, \( d \) to \$25 and \( h'(P^{-1}) = 0 \) etc.

But since \( V_m < \frac{P^*}{P} \) (3) becomes

\( C = d \int_{0}^{P_m} \frac{V}{aP} \, dv \)
Solving for \( \hat{P} \) yields

\[
\frac{abdV^2}{2P^2} = 0 \quad \text{or} \quad \hat{P} = \infty
\]

In practice \( \hat{P} = 100V_m \) so that no one need buy more than one share before reaching negotiated commissions.

The one objection which may be made to this calculation is that it ignores cost that might be incurred in purchasing less than one share.

Let us now add a special charge for purchasing less than one share:

\[
C = h\left(P^{-1}\right)V + e \int_0^{P/100} Vg(V)dV + d \int_0^P \frac{V}{pg(V)dV} + b \int_\frac{V}{p} \frac{V}{pg(V)dV} \quad \text{from (2)}
\]

where \( eV \) is the special \% charge on all transactions of less than one share.

Integrating and differentiating (9) \( W/r \) to \( P \) yields

\[
C' + -h\left(P^{-1}\right)V + \frac{eNP}{V_m(100)^2} + \frac{(d-b)N}{2V_m} - \frac{bNV_m}{2P^2} \quad \text{with } g(V) = a
\]

Letting \( C' = 0, h'(P^{-1}) = 0 \) and solving (10) for \( \hat{P} \) gives:

\[
\hat{P} = V_m \sqrt{\frac{b}{(d-b) + \frac{2eP}{100^2}}}
\]

Let \( e = 1.5\% = \frac{1.5}{100} \) hence (11) becomes:

\[
1.5\% \text{ is the usual mutual fund management fee and is thus higher than the extra brokerage cost of fractional shares would come to be.}
\]
(12) \[ P = \frac{V_m}{\sqrt{d-b + \frac{3P}{100^2}}} = \$15,333,000 \]

when \( V_m \) is set equal to the $100,000 negotiations floor

where \( b = $71 \), and \( d = $25 \).

But since \( P > V \) again (9) becomes:

\[
0 \leq \frac{P}{100} \leq \frac{V_m}{V_dV} \leq 0
\]

Again solving for \( P \) yields

(13) \[ P = \left( \frac{100V^2}{2e} \right)^{1/3} = \$437,500 \text{ with } e, \text{ the fractional share fee,} \]

equal to 1.5 percent; $500,000 with \( e = 1 \) percent; and $630,000 with \( e = .5 \) percent.

\[ \frac{d^2P}{dd^2} > 0. \text{ Increasing the odd lot fee raises } P. \]

Now let us look at Case B assuming \( g(V) = a = N/V_m \).

(15) \[
C = h(P^{-1})V + d \left( \frac{V}{P} + (b+d) \right) \sum_{n=1}^{n=m-P-1} \frac{(n+1)P}{nP} \\
+ \frac{m}{d} \left( \frac{V_m}{P} - P \right) + \frac{d}{P} \]

Integrating (15) gives

(16) \[ C = h(P^{-1})V + \frac{ba}{2} \left( \frac{m}{P} - P \right) + \frac{dV_m}{P} \]
Differentiating W/r to P yields

\[ C' = \frac{ba}{2} + \frac{baV^2_m}{2P^2} = 0 \]

which cannot be solved since \( P > V_m \) and thus become case (11) with \( P \geq 100V_m \).

All transactions above one share are negotiated. This result is intuitively obvious. Since \( V \) is continuous the probability of any transaction not involving a round lot is zero, everyone will have to pay the odd lot fee of 25¢ on at least one share and thus might as well only have to buy one share or less to pay it only once.

Now let us add the special fractional share charge, \( eV \), on all fractional share transactions to our Case B model.

\[ C = h(P^{-1}V) + \int_0^P \frac{V}{P}g(V)dV + (b+d)\int_0^V \frac{V}{pg(V)}dV \]

\[ - \sum_{n=1}^{V_m} \int_{nP}^{(n+1)P} g(V)dV + e\int_0^{V_m} Vg(V)dV - e \frac{P}{100} \sum_{n=1}^{100} \int_0^{\frac{nP}{100}} g(V)dV \]
Letting \( g(V) = a \) and integrating gives,

\[
C = h(p^{-1})V + \frac{baV^2}{2\left(\frac{m}{p} - P\right)} - adV + \frac{eaPV}{200}
\]

Differentiating \( W/r \) to \( P \) gives:

\[
\frac{dC}{dP} = -h'V - \frac{baV^2}{2\left(\frac{m}{p^2} + 1\right)} + \frac{eaV}{200}
\]

Setting \( \frac{dC}{dP} = 0 \) and letting \( h' = 0 \) yields

\[
\frac{x}{p} = \sqrt{\frac{bV_m^2}{eV_m^2 - b}} = \frac{V_m}{\sqrt{\frac{b}{eV_m^2 - b}}}
\]

Letting \( V_m = \$100,000, b = \$71 \) and \( e = 1/100 \), the solution is again imaginary since \( \frac{x}{p} > V_m \).

But since \( \frac{x}{p} > V_m \), (21) becomes:

\[
C = d \int_0^{V_m} \frac{V}{p} dV + e \int_0^{V_m} aV dV - \frac{eP}{100}
\]

Integrating, differentiating \( W/r \) to and solving for \( \frac{x}{p} \) yields:

\[
\frac{x}{p} = \sqrt{\frac{d \cdot 100 \cdot V}{e}} = 50 \sqrt{\frac{V_m}{e}}
\]

equal to \$158,000 with \( e = 1/100 \) and \$224,000 with \( e = 1/2\% \).

\$129,000 with \( e \) as high as 1.5\%.
\( \hat{p} \) is biased downward in Case B, since large share traders do not try to avoid odd lot and fractional share fees; as can be seen by comparison with Case A, \( \hat{p} \) is reduced by approximately 50% in this case. Case A and Case B lie to opposite sides of reality thus defining a narrow range.

We can now see that even under $100,000 negotiated commissions and assumptions which tend to make the cost minimizing \( \hat{p} \) lower than realistic, the prices per share come out at significantly over $1,290 per share for any corporation ever having a transaction of $100,000 in value (which would probably include almost every N.Y.S.E. stock), even with a one and one half percent fractional share charge. Even a very small new issue whose largest transaction is $10,000 would initially be priced at over $410 per share, with a 1 1/2% fractional share charge. IBM’s $400 per share price still seems to be less than 1/3 of the transaction cost minimizing value of a share even with the not yet realized $100,000 negotiated commission floor.

In order to check our results let us establish a set of assumptions as apparently biased toward low stock prices as possible. We will use Case B, in which large transactors do not in any way attempt to avoid odd lot and fractional share fees.

Second we will assume density functions of the form \( g(V) = \frac{S_1}{V^{1/2}} \)

where \( S_1 = \frac{N}{2 \sqrt{V_m}} \), and \( g(V) = \frac{S_2}{V} \), where \( S_2 = \frac{N}{\log V_m} \) which concentrates transactions near \( V = 0 \) and is thus more skewed in this direction than either Gamma (exponential), normal or rectangular distributions.

\textit{Whence} our Case B functions become:
Cost functions (24) and (25) cannot be integrated directly and must be solved by the use of numerical methods. Using numerical methods explained in Appendix I and solving for \( P \) in (24) and (25) yields \( P = V_m = \$100,000 \) in both cases with fractional share charges as high as 4 1/4% and 5% respectively.

Hence, the transactions cost minimizing price per 100 shares will be above the negotiated commission floor, down to \$100,000 by April 1974. The reason for this is that transaction cost minimization requires the price of an issue be set high enough so that no one ever incurs the commissions on a second hundred shares and so that as few shares as possible be purchased, while avoiding the fractional fee.

III. The Marketing of Securities

The most casual observation immediately demonstrates present stock prices to be on the order of 1/3 to 1/100 of the minimum values we have calculated. What could be the explanation of this phenomenon? The answer can lie in two directions: The present brokerage commissions are likely to be too high for multiple round lot transactions, while they might
be too low (relative to costs) for stocks which would be priced significantly above the maximum $71 commission point (reached at $P = $5,000 for 100 shares); and secondly the present brokerage commissions which are probably above costs for issues priced below $5,000 or so per hundred, thus including a profit which can be gained by selling or promoting the stock (and are relative more above costs per dollar of multiple hundred transactions the lower the value of a stock). Brokerage commissions may be below costs for very low value (under $500 or less) transactions, though such a subsidy would probably lead to the abuse of small investors.

Companies can thus choose how much incentive or economic profit they give brokers to promote, sell or recommend their stock, though the incentive is also strong for brokers to sell and de-recommend low priced stocks. High commissions per value of trades really encourage trading or activity in the stock.

The trading of a stock does have possible advantages for the corporation. It creates a market in the stock possibly preventing large fluctuations in the value of a thinly traded stock. Even if a stock's low price only encourages trading it creates an inventory effect and probably encourages both the general holding and knowledge of relatively unknown issues. All three of these may raise the value and reduce the level of price fluctuations of the issue.

IBM has priced their stock above any other security traded in 100 share round lots. IBM thus yields the minimum economic profit per dollar value of a transaction of any security. IBM would be likely to fear that any further price increase would reduce the little incentive brokers have to push the stock to zero and even negative if real costs do rise as a function of the value of a share about $360.
IV. The True Cost of Transactions

In looking at commissions on orders below $100,000\(^2\) we must decide whether costs are largely proportional to the number of shares (i.e. \(C = a \frac{V}{P} + a_1 V\)) or largely proportional to the value of the transaction. This proposition is very likely since the cost of a transaction is likely to be something like \(C = a_0 + a_1 V/P + a_2 V\), while fees are more in the order of \(C = \frac{V}{P}(b + h(P^{-1})l)\).

There is a $6.00 fee per hundred shares or fraction thereof on all transactions plus a higher percentage charge on multiple round lot transactions. Thus the changing of both higher fixed and percentage commissions on multiple round lot sales probably lead to commissions which are too high. It is also interesting to question why the percentage of value part of commissions declines with the value of the transaction. If part of the cost of a transaction varies with the value of the transaction it would seem that it ought be a constant portion of the transaction's value.

All in all it would seem that the cost of processing a transaction would probably decompose into three parts: a fixed charge for any transaction; a fixed cost per hundred or less shares; and a constant portion of value, independent of the number of shares. Likely values for these three parts might be $15 to $21 per transaction (the old maximum surcharge) plus $6.00 per hundred (as now charged) and 0.4% of the transaction's value, with no maximum commission per hundred shares. These values are of course speculative, though based on existing portions of commissions.

\(^2\)Again we must emphasize that $300,000 is actually the negotiation level at present and we should multiply all our calculations by approximately 3.
This declining percentage of commission is reminiscent of railroad policy of charging shipping rates set at declining percentage of the value of the cargo. The commission achieves monopoly profits, while not going high enough so as to drive customers away from the exchange. Can the reason be the need to meet fixed costs in a declining cost industry so that high value per share stocks subsidize very small transactions conducted at marginal costs? It seems not, since the commission depends so heavily on the number of shares, since that would mean everyone subsidized a few very small traders. The recent Wall Street paper work difficulties would seem to indicate constant or decreasing returns, thus limiting a possible reason for this procedure.

Our general analysis is confirmed by the fact that commissions on transactions over $500,000 in value are supposed to have declined by 50%, since the institution of negotiated commissions on such transactions.

Robert Haack, President of the New York Stock Exchange, has explained that when negotiated commissions were instituted on orders above $500,000 (in 1971), some firms actively sought business on terms stipulating that they would charge no commission at all on the portion of the transaction over $500,000. The Wall Street Journal further quotes him as saying:

"This leads me to the conclusion that there was enough profitability in their business below $500,000 to compensate them," he said, adding, "Our members are a lot of things but they aren't altruistic."

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V. Conclusions

The most striking conclusion to which this research leads is that the present commission rates contain large elements of arbitrariness bearing little relation to actual costs. Furthermore if corporations took the rates at face value the price per share of security would be increased drastically above present levels probably bringing about an end to the present system.

The present value of all securities is far below the lowest values that would minimize the costs of transactions even with negotiated fees commencing at $100,000, since the smallest value per 100 shares that would minimize commissions is at least the negotiated fee point (or the value of the largest transaction in the security whichever is smaller), even in the presence of transactions distributions which are extremely skewed toward very small investors and with fractional share charges of over 4%. From this point of view a company would want to price one share not 100 shares at the smallest typical transaction (even with the not yet attained $100,000 negotiated commission level many securities sell for less than one one-hundredth the commission minimizing value.

In addition, the present brokerage rates seem to be above costs for most multiple 100 shares transactions except for extremely low value shares. There is reason to believe that transactions costs are largely a high fixed cost plus a very small percent of value cost (lower than is typically charged) and that the cost per hundred of multiple hundred transactions is likely to be a small fraction of the fixed costs of the first 100 or less shares and much below the commission.

The above commission scheme may lead to the following distortions: monopoly profits, excess capacity and inefficiency in the brokerage industry; excessive
promotion of constant churning or trading; distortion toward low price, high commission, profit and promotion value issues; and possible unethical procedures in favor of large investors upon whom monopoly profits are great and against very small investors who provide small and perhaps negative economic profits.

VI. Recommendations

First, it would certainly appear that all brokerage fees might well be subject to negotiation and competition. If institutional or political factors prevent a move to full competition, then some revisions in the present minimum commission structure would seem justified. The reforms might include: 1) negotiated commissions on all transactions involving more than $100,000 or 1,000 shares; 2) the percentage value portion of commissions become the same for single or multiple hundred transactions, at something near or below the minimym 0.4% portion of value commission now charged; and 3) a fixed charge of $15 to $21 per transaction be instituted; plus a fee of $6 or slightly more per hundred shares or fraction thereof involved in the trade.
APPENDIX I

The following solutions to equations (24) and (25) were done with the aid of Professor F. B. Hildebrand of the M.I.T. Mathematics Department.

First we will assume $P > V_m$ and hence (24) and (25) become:

\[
(24')\quad C_1 = d \left( \int_0^{V_m} \frac{V_m S_1 V^{1/2}}{P} \, dV + e \int_0^{V_m} S_1 V^{1/2} \, dV - \frac{\epsilon P}{100} \sum_{n=1}^{n=100-1} \frac{V_m}{P} \right)
\]

\[
(25')\quad C_2 = d \left( \int_0^{V_m} \frac{V_m S_2}{P} \, dV + e \int_0^{V_m} S_2 \, dV - \frac{\epsilon P}{100} \sum_{n=1}^{n=100-1} \frac{S_2}{V} \right)
\]

Integrating, dividing them by $S_1$ and dropping the second term which becomes a constant gives, (letting $d = 25$, $b = 71$ and $e = 1/100$).

\[
\hat{C}_1 = \frac{50 V_m^{3/2}}{P} - \frac{2\epsilon P^{3/2}}{100^2} \sum_{n=1}^{n=100-1} \frac{V_m}{P} \cdot \left[ \frac{n(n+1)}{2} - \frac{1}{2} \right]
\]

\[
\hat{C}_2 = \frac{25 V_m}{P} - \frac{dP}{100} \sum_{n=1}^{n=100-1} n \log \left( \frac{n+1}{n} \right)
\]
Letting \( V_m \) equal the negotiation point \( 10^5 \) and defining \( W = \frac{10^7}{P} \), equations (26) can be expressed as:

\[
I_1 = (5/3 \times 10^{3/2})w - \frac{2 \times 10^{3/2}}{w^{3/2}} \sum_{n=1}^{w-1} n \Delta n^{1/2}
\]

(27)

\[
I_2 = \frac{w}{4} - \frac{1000}{w} \sum_{n=1}^{w-1} n \Delta \log n
\]

Using Sterling's Factorial Method:

\[
\sum_{n=1}^{w-1} n \Delta \mu n = \left[ (n-1)\mu n \right]_1^{w-1} - \sum_{n=1}^{w-1} \mu n
\]

(28)

\[
\sum_{n=1}^{w-1} n \Delta n^{1/2} \approx 1/3 w^{3/2} - w^{1/2} + 2/3, \quad \text{since} \quad \sum_{n=1}^{w-1} n^{1/2} \approx \int_1^{w} n^{1/2} \, dn
\]

and

\[
\sum_{n=1}^{w-1} n \Delta \log n = \log \left( \frac{w^w}{w!} \right)
\]

Thus,

\[
I_1 \approx 5/3 \times 10^{3/2} \left\{ w - (2/5 \times 10^4)[1 - \frac{3}{w} + \frac{2}{w^{3/2}}] \right\} = \min.
\]

\[
w^{5/2} - (6/5 \times 10^4)(w^{1/2} - 1) \approx 0,
\]

\[w \approx 104, \quad \text{whence} \quad P \approx 0.962 \times 105\]

and
\[
I_2 = \frac{w}{4} - \frac{1000}{w} \log \left( \frac{w}{w!} \right) \approx \frac{w}{4} - \frac{1000}{w} \log \left( \frac{e}{\sqrt{2\pi w}} \right), \text{ since } w! \sim \sqrt{2\pi w} \left( \frac{w}{e} \right)^w
\]

\[
\approx \frac{w}{4} - \frac{1000}{w} [w - 1/2 \log(2\pi w)] = \min.
\]

\[
w^2 - 2000[\log(2w)] \approx 0,
\]

\[
w \approx 105, \text{ and } P \approx 0.952 \times 105.
\]

Since \( P < V_m \) in (32) and (33), (29) becomes (34) derived from (27) and (28)

\[
\hat{C}_1 = \hat{C}_1 + \frac{141}{3} \left[ \frac{V_m^{3/2}}{P} - P^{1/2} \right] - 50P^{1/2} \sum_{n=0}^{V_m/P-1} n[(n+1)^{1/2} - n^{1/2}]
\]

\[
\hat{C}_2 = \hat{C}_2 + 71 \left[ \frac{V_m}{P} - 1 \right] - 25 \sum_{n=1}^{V_m/P-1} n \log \left( \frac{n+1}{n} \right)
\]

Hence equations (30) become:

\[
I_1 = 0.96w - 25 \sum_{n=1}^{10} n \log n - \frac{1000}{w} \sum_{n=1}^{w-1} n \Delta \log n
\]

\[
I_2 = \frac{19.2}{3} \times 10^{3/2} w - \frac{14.2 \times 10^{9/2}}{3w^{1/2}} - 5 \times 10^{9/2} \sum_{n=1}^{w-1} n \Delta n^{1/2} - \frac{102}{w^{1/2}} \sum_{n=1}^{w-1} n \Delta n^{1/2}
\]

The solutions to both of which are \( w < 100 \) and \( P > V_m \). Hence the solution will actually be \( w = 100, P = V_m \) for both distribution functions.
Diagramming the cost function (21) with transactions density functions

g(V) = a, g(V) = \frac{S_1}{V^{1/2}} \text{ and } g(V) = \frac{S_2}{V} \text{ gives.}

\[ C(21) \]

\[ g(V) = S_1 V^{1/2} \quad \text{or} \quad g(V) = S_2 V^{-1} = \frac{S_2}{V} \]

\[ g(V) = a \]

\[ \hat{p} > V_m \quad \hat{p} < V_m \]

\[ \frac{V_m}{p} \]

The next step is to solve for the values of e the fractional share charge that would cause the minimum C to occur at \( P < V_m \) with the various distributions. To do this we need only solve (32) and (33) for the value of e that would give \( w = 100 \).

\[ w^{5/2} - e(6/5 \times 10^6)(w^{1/2} - 1) \approx 0 \]

(33)

\[ w^2 - 2 \times 10^5 e[\log(2mw) - 1] \approx 0 \]
Solving yields a 4 1/4% per fractional share charge with \( g(V) = \frac{S_1}{V} \), 5% with \( g(V) = \frac{S_2}{V^{1/2}} \) and even more ludicrously high per share charge with a rectangular distribution. Hence \( P \geq V \) even with \( g(V) = \frac{S_1}{V} \) and \( e = 4 1/4\% \).

**APPENDIX II**

**Present Commission Rates**

1) On 100 Share Orders and Odd Lot Orders

<table>
<thead>
<tr>
<th>Money Involved in the Order</th>
<th>Minimum Commission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100--but under $800</td>
<td>2.0% plus $6.40</td>
</tr>
<tr>
<td>800--but under 2,500</td>
<td>1.3% plus 12.00</td>
</tr>
<tr>
<td>2,500--and above</td>
<td>0.9% plus 22.00</td>
</tr>
<tr>
<td></td>
<td>Odd Lot--$2 less</td>
</tr>
</tbody>
</table>

2) Multiple Round Lot Orders

<table>
<thead>
<tr>
<th>Money Involved in the Order</th>
<th>Minimum Commission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100--but under $2,500</td>
<td>1.3% plus $12.00</td>
</tr>
<tr>
<td>2,500--but under 20,000</td>
<td>0.9% plus 22.00</td>
</tr>
<tr>
<td>20,000--but under 30,000</td>
<td>0.6% plus 82.00</td>
</tr>
<tr>
<td>30,000--to and including $500,000</td>
<td>0.4% plus 142.00</td>
</tr>
</tbody>
</table>

3) Plus (for Each Round Lot)

- First to tenth round lot: $6 per round lot
- Eleventh round lot and above: 4 per round lot

The minimum commission on a 100 share order or an odd lot order need not be more than $65.00. The minimum commission per round lot within a multiple round lot order is not to exceed the single round lot commission computed in accordance with the rate for 100 share orders.

4) Odd Lots (less than 100 shares) N.Y.S.E.

Same as above, less $2 plus 25¢ for shares selling above $55 and 12.5¢ for shares under $55.


In Addition:


