
SHADOW PRICES FOR PROJECT SELECTION IN THE PRESENCE OF DISTORTIONS: EFFECTIVE RATES OF PROTECTION AND DOMESTIC RESOURCE COSTS

T. N. Srinivasan and Jagdish N. Bhagwati

Massachusetts Institute of Technology
50 Memorial Drive
Cambridge, Mass. 02139
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Until recently, theorists of trade and welfare have, by and large, ignored the ever-increasing literature on project evaluation. This is puzzling since the bulk of the project evaluation literature attempts to derive shadow prices to replace the market prices that, in distorted situations, clearly will not reflect true opportunity costs whereas the major advances in the welfare theory of international trade have consisted precisely in the analysis of issues in trade and welfare when the market is characterized by a number of alternative endogenous or policy-imposed distortions.¹

The only attempts to date by international economists in the direction of project evaluation of a sort were by the proponents of the so-called DRC (domestic resource cost) and ERP (effective rate of protection) measures.² The question principally addressed by these proponents was the following: if the DRC's (or ERP's) were calculated for a distorted situation with tariffs and the current and potential industries/projects were ranked in terms thereof, would this enable one to infer that, in the non-distorted optimal situation, the industries with lower DRC's would expand while those with higher DRC's would contract? As is now well-understood, the answer to this question is in the negative. For example, Findlay (1971), using a

¹ Cf. Bhagwati and Ramaswami (1963); Johnson (1965); Bhagwati, Ramaswami and Srinivasan (1969); Bhagwati (1971); and the numerous writings of Kemp, Findlay, Corden, Magee, Brecher and several trade theorists.

² In the absence of distortions, ranking of projects/activities by these measures obviously reduces to the same thing. This is because \( ERP = \frac{DVA - FVA}{FVA} \) and \( DRC = DVA \) where DVA is value added at domestic prices and FVA is value added at international prices, so that \( ERP = (DRC - 1) \). In terms of the conceptual distinctions introduced in Section II later, we should state that the DRC in this argument is defined as \( DRC_{IX} \).
model of a small country with two final goods and two intermediates used in fixed proportions in the final goods, and with all tradeables, has demonstrated precisely that the optimal free trade solution may bring into production an industry which is characterized by the highest DRC in the distorted situation while eliminating an industry characterized by a lower DRC in the distorted situation.¹

But project analysis asks a somewhat different question: if there are existing distortions which cannot be removed, what prices does one use for project evaluation? As it happens, this question can also be related to DRC's provided one uses appropriately-derived shadow prices: such that, when such appropriate DRC's are estimated for existing activities and the marginal project, the project would be correctly rejected if its DRC was higher than unity (i.e. higher than that for the existing activities). It can also be shown, as is done below, that ERP's [which use shadow commodity prices, i.e. international or, in the Little–Mirrlees (1969) language, "border" prices for commodities, but use actual market prices for factors] will not provide the correct pricing valuation, nor will alternative measures that have been proposed and/or are used in practice.

As our departure point, we take the simple model of trade theory, with primary factors producing traded goods (including the project output), with no intermediates and with fixed international prices for the goods. This is also the model deployed by Findlay and Wellisz (1976) in an elegant paper on the subject of shadow prices for project evaluation, whose analysis we parallel in some respects, while complementing and "correcting" it in critical ways.² Following them, we will focus the analysis in Section I on trade distortions: i.e. tariffs and trade subsidies on the traded goods, while however treating also the case of endogenous factor market distortions in

¹ Identically, for "lower (higher) DRC" read "lower (higher) ERP". For definition of DRC in this context, refer to the preceding footnote. ² Earlier analyses by Joshi (1972), Lal (1974) and Corden (1974) should also be mentioned.
Section III.

I: The Model and Derivation of Shadow Prices

As stated above, we consider the usual trade-theoretic model with two primary factors, \( k \) and \( l \), producing two traded outputs, \( X_1 \) and \( X_2 \) that enjoy fixed international prices \( p_1^* \) and \( p_2^* \). The "small" project being considered will produce commodity \( X_3 \), at fixed international price \( p_3^* \). It is assumed that the planner is working with a well-behaved social utility function.

The problem of project analysis then is to evolve suitable prices, for the primary factors and output (\( X_3 \)) in the project, which would enable the analyst to decide whether the project should be accepted or rejected.

The problem would be straightforward indeed if there were no distortions in the system: the correct valuations of the primary factors would clearly be those in the market, as reflected by the international price-ratio \( \frac{p_1^*}{p_2^*} \), and the correct valuation for \( X_3 \) would be the international price \( p_3^* \). But the situation we must now introduce is one where the domestic price-ratio between commodities \( X_1 \) and \( X_2 \) is distorted by a tariff and/or trade subsidy and it is further assumed that this distortion must be taken as given. The problem then, as noted by Findlay-Wellisz (1976, p. 545) is "an inherently second best one" in which "the criterion for acceptance of the project is whether or not it will increase the value of total production at world prices as compared with the existing situation, assuming that the distrotional policy on the existing goods continues unchanged."¹

¹ Provided that inferior goods are ruled out, there is of course a monotonic relationship between welfare and the distance of the availability locus (at international prices) from the origin. Hence, we can disregard, without error, the fact that tariffs and/or trade subsidies will distort consumption as well as production.
In applying this criterion for a "small" project, we note first that the introduction of the project will use labour and/or capital that are withdrawn from their present use. As such, the answer to the question whether or not the project (producing X₃) will increase the value of production at world prices is the same as to the question whether the world price of a unit of output of the project exceeds or falls short of its cost of production as obtained by evaluating the labour and capital used in producing X₃ at their shadow prices i.e. at prices that equal their marginal contribution in their existing use to the value of total production at world prices.

Turn now to Figure (1). AB is the production possibility curve, defined on commodities X₁ and X₂. At free trade, production would be at P*(X₁*,X₂*) reflecting the international commodity prices. However, with trade distortion, the commodity price-ratio is more favourable to commodity X₂ and production is at P(Ŷ₁,Ŷ₂). Now, the planner is assumed unable to correct the situation directly, so that the commodity price-ratio, the factor price-ratio and factor proportions for X₁ and X₂ are to be held fixed at their respective values at P(Ŷ₁,Ŷ₂). Denote then the corresponding input coefficients as (k₁,Y₁) and (k₂,Y₂) and factor rentals as w and r.

Now, as noted above, the second-best shadow prices of labour (w*) and capital (r*) in this situation must equal the change in the quantities of X₁ and X₂ output, evaluated at international prices p₁* and p₂*, resulting from a marginal change in labour and capital respectively, starting at P(Ŷ₁,Ŷ₂) and maintaining the distorted commodity price-ratio for production decisions.¹ Thus, defining W=p₁*Ŷ₁+p₂*Ŷ₂ and the total availability of

¹ The notation w*, r* is used here because the 'hat' refers to the distorted situation and the 'star' to the evaluation of output change at international prices.
capital and labour as $\tilde{K}$ and $\tilde{L}$ respectively, it is clear that the shadow price of labour will be $\frac{d\tilde{w}}{d\tilde{L}}$ and that of capital will be $\frac{d\tilde{w}}{d\tilde{K}}$, where the derivatives must be evaluated for the distorted situation. This is readily done as follows. First, since capital supply is fixed ($\tilde{K}$), we have:

$$\frac{\hat{k}_1}{d\tilde{L}} + \frac{\hat{k}_2}{d\tilde{L}} \frac{d\tilde{X}_2}{d\tilde{L}} = 0$$

and, for labour, the corresponding equation is:

$$\frac{\hat{k}_1}{d\tilde{L}} + \frac{\hat{k}_2}{d\tilde{L}} \frac{d\tilde{X}_2}{d\tilde{L}} = 1$$

$$\frac{d\tilde{X}_1}{d\tilde{L}} = \frac{-\hat{k}_2}{\hat{k}_1 \hat{k}_2 - \hat{k}_2 \hat{k}_1}$$

and

$$\frac{d\tilde{X}_2}{d\tilde{L}} = \frac{\hat{k}_1}{\hat{k}_1 \hat{k}_2 - \hat{k}_2 \hat{k}_1}$$

Hence, the shadow price of labour, defined as:

$$\hat{\omega}^* = p_1 \frac{d\tilde{X}_1}{d\tilde{L}} + p_2 \frac{d\tilde{X}_2}{d\tilde{L}}$$

is seen to be equal to:

$$\hat{\omega}^* = \frac{p_2 \hat{k}_1 - p_1 \hat{k}_2}{\hat{k}_1 \hat{k}_2 - \hat{k}_2 \hat{k}_1}$$

(1.1)
Similarly, we can see that the shadow price of capital is:

\[ \hat{\gamma}^* = \frac{p_1^* \hat{k}_2 - p_2^* \hat{k}_1}{\hat{k}_2 - \hat{k}_1} \]

It is readily seen that these are also the values of \( \hat{\omega}^* \) and \( \hat{\gamma}^* \) that satisfy the equations:  

\[ p_1^* = \hat{\omega}^* \hat{k}_1 + \hat{\gamma}^* \hat{k}_1 \]  

\[ p_2^* = \hat{\omega}^* \hat{k}_2 + \hat{\gamma}^* \hat{k}_2 \]

Now, it is easy to see that the shift in outputs, as labour (capital) is withdrawn from P, maintaining the distortion and hence the distorted commodity price-ratio, is yielded by the corresponding Rybczynski line. So, assuming that \( X_1 \) is K-intensive at P, i.e. \( \hat{k}_1 > \hat{k}_2 \), one can see, in Figure (2), that the economy will move from P down line PB' as labour is reduced, up line PQ as labour is increased, up PA' as capital is reduced, and down PR as capital is increased. It equally follows, from the evaluation of these shifts at the international (rather than the distorted) commodity price-ratio, that \( \hat{\omega}^* \) will be negative if the international price line is steeper than PB', i.e. \( \frac{p_1^*}{p_2^*} > \frac{\hat{k}_1}{\hat{k}_2} \) and \( \hat{\gamma}^* \) will be negative if the international price line is flatter than PB', i.e. \( \frac{p_1^*}{p_2^*} < \frac{\hat{k}_1}{\hat{k}_2} \); and that non-negative values for \( \hat{\omega}^* \) and \( \hat{\gamma}^* \) will obtain only when the international price-ratio is in

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1 This is, in fact, the procedure suggested for deriving shadow factor prices by Diamond and Mirrlees (1976) in their analysis of a similar problem.
Figure (2)
the range spanned by PB' and PA'.

That it is possible for $\hat{\omega}^*$ or $\hat{\gamma}^*$ to be negative would appear to be a paradox. For, it of course implies, for instance, that when (say) $\hat{\omega}^* < 0$, it would pay society to implement a project with zero output ($X_3$) and positive labour input: i.e. that if labour were withdrawn from existing production, thanks to the project, this will increase the value of such production at international prices. But then this paradox is only yet another instance of "immiserizing growth" the presence of the marginal labour is immiserizing, given the distortion; and thus the paradox is readily resolved.

In their derivation of shadow factor prices for the above problem, however, Findlay-Wellisz (1976) bypass this possibility of negative factor prices by deriving these prices instead via the solution to the following programming problem:

$$\text{Minimize } [\hat{\omega}^* L + \hat{\gamma}^* K]$$

s.t. $l_1 \hat{\omega}^* + k_1 \hat{\gamma}^* < p_1^*$

$$l_2 \hat{\omega}^* + k_2 \hat{\gamma}^* < p_2^*$$

$$\hat{\omega}^*, \hat{\gamma}^* > 0$$

This is the dual to the following primal:

$$\text{Maximize } [p_1^* X_1 + p_2^* X_2]$$

s.t. $l_1 X_1 + k_2 X_2 \leq L$

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1 Cf. Bhagwati (1958); Johnson (1967) who deals with the precise distortion in our model here; and Bhagwati (1971) who states the general theory of immiserizing growth that explains and ties together the different instances of immiserizing growth.
As such, the Findlay-Wellisz procedure amounts to (cf. Figure 2) deriving the shadow factor prices corresponding to the international prices but subject to a "feasible" production possibility curve defined by A'PB'.

These Findlay-Wellisz shadow prices, \((\hat{w}^*, \hat{y}^*)\), are clearly yielded by putting the international price-ratio tangent to A'PB', in the usual way, and are illustrated to advantage in Figure (3).

Figure 3 is the all-too-familiar Samuelson diagram and needs no explanation. Now, movement along the unrestricted production possibility curve APB in Figure 2 corresponds to movement along the curve QPR in Figure 3, relating the commodity price-ratio to the corresponding factor price-ratio. Similarly, movement along the restricted production possibility curve A'PB' in Figure 2 corresponds in Figure 3 to following the y-axis in the fourth quadrant from \(\infty\) upto the point S where OS=\(\hat{k}_1/\hat{k}_2\), then along the curve SPNZ upto Z [where N is at a distance \(\hat{\lambda}_{1}/\hat{\lambda}_{2}\) from the x-axis] and then following a straight line parallel to the x-axis. The (restricted) curve SPNZ, depicting \(w/y\) as a function of \(p_1/p_2\), can be shown to be increasing and concave, with a common tangent with the (unrestricted) curve QPR at P. Thus, the Findlay-Wellisz shadow price-ratio \(\hat{w}^*/\hat{y}^*\) will be infinite for \(p_1^*/p_2^* \geq \hat{k}_1/\hat{k}_2\) and zero for \(p_1^*/p_2^* \leq \hat{\lambda}_{1}/\hat{\lambda}_{2}\), while taking positive values in the range spanned by \(\hat{k}_1/\hat{k}_2\) and \(\hat{\lambda}_{1}/\hat{\lambda}_{2}\).

Clearly, therefore, the Findlay-Wellisz procedure for deriving the second-best shadow factor prices excludes the possibility of deriving negative values which our correct procedure can yield and is evidently inappropriate.
Figure (3)
Their procedure would happen to yield shadow prices that coincide with the correct ones yielded by our procedure only when \(\hat{w}^*/\hat{\gamma}^* > 0\), i.e., in Figures 1 and 2, only for the parametric case where the international price-ratio lies in the range spanned by PA' and PB'. For the parametric cases where the international price-ratio lies outside of this range, the Findlay-Wellisz procedure would yield a shadow factor price-ratio, \(\hat{w}^*/\hat{\gamma}^* = 0\) or \(\infty\) according to whether the production specialization, corresponding to the international price-ratio, occurred in Figure 2 at B' (on X₁) or A' (on X₂): but this would be the correct shadow factor price-ratio only if the initial distorted situation were at B' or A' respectively, instead of at P as initially hypothesised for the problem at hand! The Findlay-Wellisz procedure is therefore critically inappropriate to the problem at hand.

To put the same point in another way, the Findlay-Wellisz procedure could be made accurate, i.e. the basic flaw just stated could be eliminated, if we were to assume that if the possibility of negative shadow price for a factor were parametrically present, that factor would be "thrown away" directly. This would be tantamount to saying, in Figure 2, that if the international price-ratio led to specialization at B' (on X₁), and hence implied a zero Findlay-Wellisz shadow price for labour \((\hat{w}^*/\hat{\gamma}^* = 0)\), the initial distorted situation at P would be shifted by direct policy intervention to B'. In this event, if the project (producing X₃) was considered as from B', clearly the correct shadow price for labour would indeed be zero! But then, in salvaging the Findlay-Wellisz procedure in this way, we would be "distorting" the interpretation of our second-best problem away from its statement as the derivation of second-best shadow factor prices at P, the initially-distorted, directly-unalterable situation.¹

¹ An alternative analysis of the inappropriateness of the Findlay-Wellisz procedure, in programming terms, is provided in the Appendix; naturally, it only corroborates what is stated in the text above.
II: ERP's, DRC's et. al.

We have thus deduced, in the preceding section, the precise shadow prices that must be used, in a distorted situation, for project appraisal. We are therefore now in a position to cast light on the inconclusive and confusing debate among the ERP and DRC proponents—as typified, for example, by the controversy in the *Journal of Political Economy* among Bruno (1972), Krueger (1972) and Balassa-Schydlowsky (1968), (1972) as to their relative merits as techniques of project appraisal. As careful reading of this debate will unmistakably reveal, the first priority in this area is to define one's concepts unambiguously.

Since these and other economists distinguish among direct and indirect inputs, thus including intermediates which were not included in the analysis in Section I above, we should first state that our project-acceptance criterion, suitably amended, is the following:

\[ p_3^* \geq k_3^* Y^* + \xi_3^* \hat{w}^* + f_1 p_1^* \]  

(II.1)

where it is now assumed that \( X_1 \) is used in project \( (X_3) \) with coefficient \( f_1 \) per unit output of \( X_3 \) and where \( k_3, \xi_3 \) and \( f_1 \) are assumed fixed so that one is essentially treating each process as a project. What the criterion says, of course, is that the project, to be accepted, must produce output which, when evaluated at international prices, exceeds or equals the cost of production evaluated at the (second-best) shadow factor prices. Now, note that the RHS of (II.1) is written in a form that includes the primary and intermediate inputs. But, it can equivalently be written in the form including direct plus indirect primary factors, i.e. by decomposing intermediates into primary factors:
Now, noting that the DRC concept implies that one is measuring the domestic resources used in an activity to produce a unit of foreign exchange, we can distinguish sharply among the following, alternative concepts that correspond, in one way or another, to the concepts that are often apparently used indistinguishably in the literature.

Note, initially, that by first-best we will refer to factor valuations, \((\hat{w}^*, \hat{\gamma}^*)\), corresponding to the first-best optimal situation at \(P^* (x_1^*, x_2^*)\) in Figure 1. By second-best, we will denote instead the factor valuations, \((\hat{\hat{w}}^*, \hat{\hat{\gamma}}^*)\) that reflect the second-best optimal situation, given the distortion. Finally, by "private", we will denote the factor valuations, \((\hat{w}, \hat{\gamma})\), that actually obtain in the distorted situation at \(P\).

Next, we should also note that the debate includes additionally a distinction between "direct plus indirect" versus only direct primary factors. Hence, we will distinguish between "total" measures which refer to gross values [i.e. taking into account gross cost of production (of, say, \(x_3\)] which therefore takes into account direct and indirect (a la Leontief) use of primary factors] and "direct" measures which refer to net values [i.e. to the last stage of production, as it were]. Again, for "total" measures, we will distinguish among two ways of formulating them: either we can take the use of direct and indirect primary factors, or we can take direct primary factors plus the intermediates.\(^1\) The former, we will denote as the "decomposed" (into primary factors) measure; the latter, as the (direct) "intermediates" measure.

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\(^1\) Thus, if we are dealing with the garment industry on a total basis, we can decompose the intermediate factors into primary factors producing them or take the factors directly into account.
(-valuation) measure.

We can now state the alternative concepts in regard to the project producing $X_3$, with brevity, noting that, in the denominator of all the formulae set out below, commodities ($X_1, X_2$ and $X_3$) are always valued at their international prices.\footnote{This clarification is necessary since the DRC's are defined for the project, whose output $X_3$ carries international price $p_3^*$ which is also the domestic price.}

I: **DRC$_I$: First-best, Total, Decomposed Measure:**

Here, we have the evaluation of domestic resources at first-best shadow wage and rental, $(w^*, \gamma^*)$, corresponding to the situation where the international commodity prices obtain domestically and therefore the distortions have been eliminated. These are also the shadow prices suggested by Bacha and Taylor (1971). In this case, we define:

$$DRC_I = \frac{(k_3 + f_1 k_1)\gamma^* + (\ell_3 + f_1 \ell_1)w^*}{p_3^*}$$  \hspace{1cm} (II.3)

for the project, using the total, direct plus indirect, decomposed primary-factor-use formulation.

II: **DRC$_{II}$: First-best Total, Intermediates Measure:**

Here, II.3 modifies, for the project, to:

$$DRC_{II} = \frac{k_3 \gamma^* + \ell_3 w^* + f_1 p_1^*}{p_3^*}$$  \hspace{1cm} (II.4)

III: **DRC$_{III}$: First-best, Direct, Intermediates Measure:**

Here, we shift to net valuation, to yield:
DRC_{III} = \frac{k_3 \gamma^* + \lambda_3 \omega^*}{p_3^* - f_1 p_1^*} \tag{II.5}

IV: DRC_{IV}: Second-best, Total, Decomposed Measure:

Here, we utilise second-best shadow prices, with gross value of output and decomposed primary factor use:

DRC_{IV} = \frac{(k_3 + f_1 k_1) \gamma^* + (\lambda_3 + f_1 \lambda_1) \omega^*}{p_3^*} \tag{II.6}

V: DRC_{V}: Second-best, Total, Intermediates Measure:

Here, we have the equivalent of DRC_{IV}:

DRC_{V} = \frac{k_3 \gamma^* + \lambda_3 \omega^* + f_1 p_1^*}{p_3^*} \tag{II.7}

VI: DRC_{VI}: Second-best, Direct, Intermediates Measure:

Here, we have:

DRC_{VI} = \frac{k_3 \gamma^* + \lambda_3 \omega^*}{p_3^* - f_1 p_1^*} \tag{II.8}

VII: DRC_{VII}: Private, Total, Decomposed Measure:

Here, we have:

DRC_{VII} = \frac{(k_3 + f_1 k_1) \gamma + (\lambda_3 + f_1 \lambda_1) \omega}{p_3^*} \tag{II.9}

VIII: DRC_{VIII}: Private, Total, Intermediates Measure:

Here, we have (using intermediates at domestic prices):
\[
\text{DRC}_{\text{VIII}} = \frac{k_3 \hat{Y} + \ell_3 \hat{\omega} + f_1 \hat{p}_1}{p_3^*}
\]

(II.10)

IX: DRC_{IX}: **Private, Direct Measure:**

Here, we then have:

\[
\text{DRC}_{\text{IX}} = \frac{k_3 \hat{Y} + \ell_3 \hat{\omega}}{p_3^* - f_1 p_1^*}
\]

(II.11)

Finally, we can write down the effective rate of protection (ERP) measure, which is always direct, as follows:

X: ERP:

\[
\text{ERP} = \frac{p_3^* - f_1 \hat{p}_1}{p_3^* - f_1 p_1^*}
\]

(II.12)

Note that the numerator in (II.11) refers to the evaluation of domestic primary factors *via* the valuation of output and intermediates at actual (rather than shadow) prices \(^1\) whereas the denominator represents the valuation at shadow (i.e. "border" or international) prices. Hence it is readily seen that in terms of our present terminology, the numerator implies the evaluation of direct, domestic factors at private prices and hence, since \((k_3 \hat{Y} + \ell_3 \hat{\omega}) = (p_3^* - f_1 \hat{p}_1)\), \(\text{ERP} = \text{DRC}_{\text{IX}}\).

\(^1\) Note, of course, that \(p_3^*\) is identical with \(p_3\) as the project output \((X_3)\) is assumed to be free from distortion.
Now, the relevant question before us is whether, if a project is accepted by our (correctly-derived) criterion, it will also be accepted if we were instead to compute the ERP or DRC for it and for the existing activities and then rank it correspondingly vis-a-vis these other activities. In short, would the ERP, and the DRC, be less for an acceptable project \((X_3)\) than for the existing activities \((X_1 \text{ and } X_2)\)?

To answer this question, note first the fact that, for the existing activities \((X_1 \text{ and } X_2)\) at first-best or second-best shadow factor prices, the DRC's must necessarily be unity. It is equally evident that the DRC's at the private factor prices will differ from unity. Thus, we have \(\text{DRC}_{I} \text{ to } \text{DRC}_{VI} = 1, \) \(\text{DRC}_{VII} = \text{DRC}_{VIII} \neq 1\) and \(\text{DRC}_{IX} = \text{ERP} \neq 1\).

By comparing the above with our project acceptance criterion, we then see right away that, if we do have the distorted situation, the measures \(\text{DRC}_{IV} \text{ to } \text{DRC}_{VI}\) will be unity for the existing activities and less than unity for the project if the project is acceptable. Hence the DRC's using appropriately derived, second-best shadow factor prices will lead to a correct acceptance/rejection of a project.

However, it is equally evident that neither the DRC's using the first-best shadow prices of factors (i.e. \(\text{DRC}_{I} \text{ to } \text{DRC}_{III}\)) nor those using private market prices of factors (i.e. \(\text{DRC}_{VII} \text{ to } \text{DRC}_{IX}\)) can, as a general rule, lead to the correct acceptance/rejection of a project. In particular, it is clear that the ERP measure, like \(\text{DRC}_{IX}, \) is quite inappropriate to the

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1 For an interesting analysis of the problem as to when a project accepted (rejected) by the incorrect use of first-best factor prices would be rejected (accepted) by the correct use of second-best factor prices, see Findlay-Wellisz (1976). Note however that this analysis is based on their inappropriate procedure for deriving second-best factor prices and therefore should be correspondingly recast.
task.¹

It is also evident that it makes absolutely no difference whether one takes total or direct DRC measures, as long as second-best shadow prices are used for project appraisal as indeed they ought to be when the initial distorted situation has to be taken as given.²

If, on the other hand, the total DRC measure is used in the absence of second-best shadow prices, e.g. the private measure DRCVIII', this will clearly be inappropriate. But then so will the direct DRC measure, DRCIX, and hence ERP.³

¹ Balassa and Schydowsky (1972) contend that, in view of the problem about shadow prices that the DRC proponents have always noted, the ERP measure be replaced by a so-called "social" ERP measure! Quite aside from the fact that it is somewhat strange to hold onto an inappropriate concept by tagging on new prefixes to it, the so-called "social" ERP, to be correct, must be converted into DRCVI. But this implies revaluing domestic factor inputs directly at the second-best shadow prices in the numerator whereas the essence of the ERP approach surely is to arrive at the numerator indirectly as the difference between the values of outputs and inputs. Clearly, it is therefore strange to call DRCVI a "social" ERP unless one's intention is to retrieve oneself from an error at the price of being peculiar.

² This conclusion would seem to bear out Bruno's (1972) counter-criticism of Balassa-Schydlowsky (1968) who make much of the distinction between total and direct measures in the presence of distortions; Bruno seems indeed to be thinking of DRC's with second-best shadow prices being used, describing them as "ex-post, social" DRC's.

³ This would also seem to contradict the Balassa-Schydlowsky (1972) assertion that the total measures yield incorrect conclusions while the direct measures do not. If the fabric industry (X₁) is protected, then the appraisal of a fabric project (X₃) will be generally erroneous whether one uses the total measure DRCVIII or the ERP. Thus, the assertion that merely taking the last-stage fabric project by itself and evaluating the garment input at its international price would be enough, i.e. that ERP (or its equivalent, DRCIX) would be correct, is false and ignores the fact that the tariff (subsidy) on the garment requires that second-best shadow prices for factors have to be correctly derived and used. And, as argued immediately earlier, if such second-best shadow prices are used, it is irrelevant whether one uses total or direct measures anyway.
Finally, the question has been raised in this ERP vs DRC debate: what if the introduction of the garment project leads (via a rule for example which requires that domestic fabrics must be used) to the licensing and creation of a tariff-protected fabric industry?¹ If such is indeed the case, we should naturally wish to redefine and consider, as a project, the vertically-integrated project involving both the garments and the fabrics that are produced for the garments. And then, the correct project appraisal would be along exactly the same lines as before, with DRC_{IV} to DRC_{VI}, all using second-best shadow factor prices, providing the correct method for doing project appraisal for this re-defined project.²

¹ Such a rule (or variations thereof) can be found in the context of input-substituting industrialization in many less developed countries. Cf. Bhagwati and Desai (1970) and Bhagwati and Srinivasan (1975) for India and Bhagwati (1977) for more extended discussion of such rules and the associated policies of "automatic" protection.

² In this evaluation, therefore, the fabrics would again enter the calculation at international prices.
III: Alternative Factor Market Distortions
and Second-Best Shadow Factor Prices

In this section, we briefly extend our analysis to three standard factor-
market distortions, deriving second-best shadow prices in each case in the man-
er set out in Section I. The three distortions are: (a) a sector-specific
sticky wage; 1 (b) a generalized sticky wage; 2 and (c) a wage differential be-
{}
{}tween sectors. 3

A: Sector-Specific Sticky Wage:

Consider a typical two-sector model of the Harris-Todaro variety. Here,
the minimum wage is set in the manufacturing sector, producing $X_2$, in terms
of $X_2$ at $\bar{w}$. The workers from the agricultural sector, producing $X_1$, migrate
to the manufacturing sector until the agricultural wage equals the expected
manufacturing wage. The expected wage is defined as the sticky manufacturing
{}
{}wage, $\bar{w}$, multiplied by the probability of a worker in the manufacturing sec-
tor obtaining employment therein. This probability, in turn, is assumed equal
to the ratio of actual employment ($L_2$) in manufacturing to the total labor
{}
{}force there, (i.e. $L=L_1$).

---

1 This distortion was brought into analytical discussion by Harris and
{}
{}Todaro (1970); the "sector-specificity" and its critical importance, were
{}
{}noted and analyzed in Bhagwati and Srinivasan (1974) and in Srinivasan and
{}
{}Bhagwati (1975).

2 This is the distortion where the sticky, actual wage exceeds the shadow
{}
{}wage but the sticky wage applies universally across sectors. The major pa-
ers on this distortion, initially analyzed by Haberler (1950), are by Lefeber
{}

3 Among the principal positive analyses of the distortion when the same fac-
{}
{}tor must be paid for differentially by different sectors are those by Hagen
{}
{}(1958), Herberg-Kemp (1971), Bhagwati-Srinivasan (1971), Jones (1971) and
{}
{}Magee (1976); the welfare analyses are by Hagen (1971) and Bhagwati-Ramaswami
{}
{}(1963).

4 The model as set out in Harris-Todaro (1970) is misspecified on the de-
{}
{}mand side. See therefore the correct specification, as set out in Bhagwati-
{}
{}Srinivasan (1974) and followed here.
Assuming perfect competition and the production functions in the two sectors to be strictly concave functions of employment, and denoting the latter by \( F_1 \) and \( F_2 \) and the international price-ratio as \( p_1^*/p_2^* \) as before, we can now write the Harris-Todaro equilibrium as:

\[
F_2'(L_2) = \bar{w} \tag{III.1}
\]

\[
\frac{p_1^*}{p_2^*} \cdot \frac{1}{F_1'(L_1)} = \bar{w} \cdot \frac{L_2}{L-L_1} \tag{III.2}
\]

Since the availability of foreign exchange in this model is given by

\[
Z=F_2 + \frac{p_1^*}{p_2^*} \cdot F_1 \ , \text{ the second-best shadow price of labour is clearly:}
\]

\[
\bar{w}^* = \frac{\frac{dZ}{dL}}{\frac{F_1'}{F_1'-\left(L-L_1\right)F_1''}} = \frac{p_1^*}{p_2^*} \cdot \frac{F_1'}{\left(L-L_1\right)F_1''} \tag{III.3}
\]

With \( F_1'' < 0 \) by strict concavity of \( F_1 \), and \( \bar{L} > L_1 \), we then see that the second-best shadow wage for labour is less than the agricultural wage which, in turn, is less than the manufacturing wage. Note also that the shadow wage is positive, instead of zero, despite the unemployed labour; this is because any withdrawal of labour from the labour force (\( L \), while initially reducing unemployment, will simultaneously raise the expected wage in manufacturing and hence result in reduction of agricultural employment and output.

B: Generalized Sticky Wage:

Shift now to the model where the wage is sticky across the two sectors at the level \( \bar{w} \). Assuming then that commodity \( X_2 \) is capital-intensive (i.e. \( \frac{K_2}{L_2} > \frac{K_1}{L_1} \)), we now get:
where $\frac{F_2^{K}}{L_2} - \frac{K_2^{K}F_2}{L_2} \geq \bar{w}$

(III.4)

$$\frac{F_2}{L_2} - \frac{K_2}{L_2} = \frac{F_1}{L_1} - \frac{K_1}{L_1}$$

(III.5)

We can then see that, in terms of Figure 4, the production possibility curve is APB, P representing the point at which $\left\{\frac{F_2}{L_2} - \frac{K_2}{L_2} \cdot F_2^{K}\right\} = \bar{w}$. At points to the left (right) of P,$\left\{\frac{F_2}{L_2} - \frac{K_2}{L_2} \cdot F_2^{K}\right\} > (<) \bar{w}$. It is evident then that, with the minimum wage constraint, the feasible production possibility curve will be APQ where PQ is the Rybczynski line (for variations in labour) and, at points on PQ other than P, there is unemployed labour. Let the capital-labour ratios at P then be $\bar{K_2}/\bar{L_2}$ and $\bar{K_1}/\bar{L_1}$.

Now, when the international price-ratio $p_1^{*}/p_2^{*}$ yields tangency along AP, the market and shadow wages will be naturally identical, and will exceed $\bar{w}$ if the tangency is off P. For the price-ratio tangent to APB at P, the production equilibrium however may be anywhere between P and Q, the different production equilibria implying different labour availabilities. Therefore, for this tangential price-ratio, the shadow and actual wages will be $\bar{w}$ for production at P, whereas the actual wage will be $\bar{w}$ but the shadow wage will
Figure (4)
be zero for points other than P on PQ. Finally, for all commodity price-ratios steeper than the price-ratio tangent at P, there will be complete specialization on $X_2$ at Q and the corresponding actual wage will be $\bar{w}$ while the shadow wage will be zero.

Hence, unlike in the sector-specific wage stickiness case, the unemployment of labour can indeed be taken to imply a zero shadow wage for labour. However, associated with this, the shadow rental of capital will exceed its market rental: so that the standard prescription of putting the wage of unemployed labour equal to zero but using the market rental of capital is erroneous.

C: The Wage-Differential Case

Take finally the distortion where the wage in $X_2$ is a multiple $\lambda$ of that in $X_1$. In this case, it is well known that the production possibility curve will shrink to AQB, in Figure 5. Furthermore, AQB need not be concave to the origin, the market equilibrium need not be unique for any commodity price-ratio, and the commodity price-ratio will not equal the marginal rate of transformation along AQB.

Let the market equilibrium in the initial, distorted situation be at Q. Then, we can derive the two Rybczynski lines, QB' (for variations in labour availability) and QA' (for variations in capital availability), assuming as earlier that $X_2$ is capital-intensive.

Now, the international price-ratio equals the ratio of marginal products

---

1 At points other than P on PQ, furthermore, the shadow rental of capital will be the average product of capital in $X_2$ at P along the curve APB, higher than its market value which will equal the marginal product.

2 At Q also, the shadow price of capital will continue to be the average product of capital in manufacturing at point P, since at Q only the manufactured good, $X_2$, is produced using all the available capital and the same technique as at P.

3 For these and other pathologies, see Bhagwati and Srinivasan (1971) and Magee's excellent survey (1976).
of capital in producing \( X_2 \) and \( X_1 \) with the techniques corresponding to \( Q \) (i.e. \( p_1^* / p_2^* = F_{K_2}^2 / F_{K_1}^1 \), the latter derivatives as at \( Q \)). On the other hand, the slope of \( QB' \) (measured against the vertical axis) will equal the ratio of the corresponding average products.

It follows then that the international price-line would be flatter that \( QB' \) and steeper than \( QA' \), given the capital-intensity of \( X_2 \) relative to \( X_1 \), provided there were no wage differential \( \lambda \). However, in the presence of the wage differential, the international price-line may well be steeper (flatter) than \( QB' \) (\( QA' \)), with the wage in \( X_2 \) exceeding that in \( X_1 \) by factor \( \lambda \>(1), the condition for this "reversal" of relative slopes of the price-ratio and the Rybczynski line being that \( X_2 \) cease to be capital-intensive relative to \( X_1 \) if the factor-intensities were compared on a differential-weighted basis.\(^1\)

It is then easy to see that, as in Section I, the second-best shadow wages of labour, i.e. \( \frac{p_1^* \bar{K}_2 - p_2^* \bar{K}_1}{F_2 \bar{F}_1} \), or the shadow rental on capital, i.e. \( \frac{\bar{K}_2 \bar{L}_1 - \bar{K}_1 \bar{L}_2}{F_2 \bar{F}_1 - F_1 \bar{F}_2} \)

\( \frac{p_2^* \bar{L}_1 - p_1^* \bar{L}_2}{F_2 \bar{F}_1 - F_1 \bar{F}_2} \), will be negative when such reversal of relative slopes exist; and, once again, the Findlay-Wellisz procedure of deriving shadow prices would yield an incorrect zero wage (rental).

\(^1\) Jones (1971) calls the differential-weighted intensities the "value" as against the Samuelsonian "physical" factor-intensities.
IV: Concluding Remarks

A few concluding observations are in order. First, it is clear that, for the distortions that we have examined, the criterion of "border-pricing", recommended by Little and Mirrlees (1969) in their celebrated Manual is clearly the correct one, provided of course that the primary factors are priced at appropriate shadow rates (as indeed Little-Mirrlees would seem to appreciate).

Second, while our results on project appraisal have been shown to be successfully convertible into appropriately-defined DRC's, this is not the same thing of course as having shown that these were precisely the DRC definitions (as against the many others that we have distinguished) that one or more of the DRC proponents, in the project-appraisal debate among the DRC and ERP proponents, had in mind.

Third, while we have confined our analysis to "small" projects, drawing infinitesimal resources away from the existing distorted situation, it is equally clear from our analysis that the results will hold also for "large" projects. Given the Rybczynski-line properties of the different models, the shadow prices of factors will be identical for small and large shifts of factors into the project.  

Fourth, we might as well note explicitly that our analysis could be readily extended to models involving non-traded goods; this would permit the introduction of the exchange rate in a meaningful manner into the analysis. On the other hand, the extension to models with many goods and factors, or to sector-specific factors, is not merely readily done; it will introduce no special insights that qualify what has been learnt from the present paper.

1 "Very large" projects may however take one away from the Rybczynski line and modify, in turn, the shadow prices.

2 For example, the latter is done readily, using the Jones (1971a) model where each of two sectors has a specific factor. The project \( X_3 \) can then be thought of as drawing one or both of these specific factors and/or the mobile, non-specific factor(s) from the existing, distorted situation.
Finally, note that we are implicitly assuming that, in respect of projects which will be chosen under shadow prices but not under actual, market prices, the resulting losses are covered in some non-distortionary way. However, if the losses can be covered only by some form of distortionary taxation, then the shadow prices (for both inputs and outputs) have to be calculated reflecting this fact. It is also clear that implicit in our analysis is the assumption that problems of income distribution and savings can be tackled through the deployment of appropriate non-distortionary instruments. Obviously, if this is not possible, the shadow prices will have to be calculated afresh by introducing additional constraints which reflect the feasible set of public policy instruments.
Appendix

We can set up the derivation of shadow prices in the second-best situation as a programming problem as follows. Given the market-determined input coefficients corresponding to the tariff-distorted output prices, choose the output levels $X_1$ and $X_2$ (denoting both the activities and their levels by the same symbols) in such a way as to maximize the availability of foreign exchange. I.e.,

Maximize $p_1^* X_1 + p_2^* X_2 + p_3^* X_3$

subject to $\hat{k}_1 X_1 + \hat{k}_2 X_2 + \hat{k}_3 X_3 = \bar{K}$ \hspace{1cm} (A.1)

$\hat{k}_1 X_1 + \hat{k}_2 X_2 + \hat{k}_3 X_3 = \bar{L}$ \hspace{1cm} (A.2)

$X_1, X_2, X_3 \geq 0$ \hspace{1cm} (A.3)

By setting A.1 and A.2 as constraining equalities, we are modeling a situation in which introducing the project is the only way of taking resources (capital and labor) away from activities $X_1$ and $X_2$. Clearly the optimal solution $(\hat{X}_1^*, \hat{X}_2^*, \hat{X}_3^*)$ to this problem is characterized as follows. Let $\lambda_1$ and $\lambda_2$ be the shadow prices of capital (constraint A.1) and labor (constraint A.2) respectively. Then:

$p_1^* \leq \lambda_1 \hat{k}_1 + \lambda_2 \hat{k}_2$ with equality holding if $X_1^* > 0$ \hspace{1cm} (A.4)

$p_2^* \leq \lambda_1 \hat{k}_2 + \lambda_2 \hat{k}_2$ with equality holding if $X_2^* > 0$ \hspace{1cm} (A.5)

$p_3^* \leq \lambda_1 \hat{k}_3 + \lambda_2 \hat{k}_3$ with equality holding if $X_3^* > 0$ \hspace{1cm} (A.6)

Note that there is no sign restriction on $\lambda_1$ and $\lambda_2$ since A.1 and A.2 are equalities. Now, since we have an initial feasible solution $\{\hat{X}_1 > 0, \hat{X}_2 > 0, X_3 = 0\}$, we can arrive at the optimal solution by starting the simplex
procedure at the initial feasible solution and solving then for the simplex multipliers \( \hat{\lambda}_1 \) (=\( \hat{r}^* \)) and \( \lambda_2 \) (=\( \hat{w}^* \)) by treating A.4 and A.5 as equalities. If with these values, A.6 then turns out to be satisfied, the initial feasible solution is indeed optimal and the project should not be introduced. This is exactly equivalent to evaluating the project through the second-best shadow prices as derived in the text; and either of these prices can be negative.

Suppose, however, that one admits other (direct) ways of disposing of factors than (the indirect one of) using them in the project. Then the constraints A.1, A.2 and A.3 should be replaced by:

\[
\begin{align*}
\hat{k}_1 x_1 + \hat{k}_2 x_2 + k_3 x_3 + s_1 &= \bar{K} \\
\hat{\lambda}_1 x_1 + \hat{\lambda}_2 x_2 + \ell_3 x_3 + s_2 &= \bar{L} \\
x_1, x_2, x_3, s_1, s_2 &\geq 0
\end{align*}
\]

where \( s_1 \) and \( s_2 \) are the so-called slack activities which use up a unit each of capital and labor respectively and produce nothing. The optimal solution to this problem \((\hat{x}_1^*, \hat{x}_2^*, \hat{x}_3^*, s_1^*, s_2^*)\) is then characterized by two constraints in addition to A.4 to A.6:

\[
\begin{align*}
0 \leq \lambda_1 &\text{ with equality holding if } s_1^* > 0 \\
0 \leq \lambda_2 &\text{ with equality holding if } s_2^* > 0
\end{align*}
\]

Thus, the introduction of the slack activities makes the shadow prices, \( \lambda_1 \) and \( \lambda_2 \), non-negative.

As before, we can start the simplex procedure here with the initial solution \( \{\hat{x}_1 > 0, \hat{x}_2 > 0, s_1 = 0, s_2 = 0\} \) and derive the simplex multipliers,
\[ \hat{\omega}^* \text{ and } \hat{r}^* \text{, that we derived earlier. Now, if A.7 and A.8 are satisfied along with A.4 to A.6, then the initial feasible solution is indeed optimal and the project should not be introduced. Of course, if A.7 and A.8 are satisfied, this means that } \hat{\omega}^* \text{ and } \hat{r}^* \text{ are non-negative; and the criterion for accepting a project (as a welfare-improving project) is the same as earlier: i.e. } \]

\[ p_3^* > \hat{r}^* k_3 + \hat{\omega}^* \lambda_3. \]

Suppose, however, that \( \hat{r}^* \) (or \( \hat{\omega}^* \)) turns out to be negative, so that the initial solution is not optimal since A.7 (or A.8) is then not satisfied. Then, the slack activity \( S_1 \) (or \( S_2 \)) will be eligible for introduction into the basis; and the project will also be eligible for introduction into the basis if it happens that \( p_3^* > \hat{r}^* k_3 + \hat{\omega}^* \lambda_3 \). To see this concretely, let \( \hat{\omega}^* \) be introduced into the basis. In the analysis in the text (Section I), this amounted to moving to the feasible solution:

\[ X_1 = \frac{-K}{\hat{\omega}^*} , X_2 = 0 , X_3 = 0 , S_1 = 0 , S_2 = L - \frac{\hat{\lambda}_1}{k_1} \cdot \hat{\omega}^* . \]

Now, a set of \( \lambda_1 \) and \( \lambda_2 \) must be calculated, treating A.4 and A.8 as equalities, so that we then obtain \( \lambda_1 = p_1^*/\hat{\omega}^* \) and \( \lambda_2 = 0 \). Clearly then it will not be optimal to introduce the project if \( p_3^* < \frac{p_1^*}{k_1} \cdot k_3 \), i.e. if the capital-rental costs, at \( \gamma^* = \lambda_1 \) valuation, exceed the international value of output; the labor input into the project will be valued at zero because \( \hat{\omega}^* = \lambda_2 = 0 \). We thus arrive at the Findlay-Wellisz criterion, of course, for project appraisal.

Now, it is easy to see that a project which passes our test for acceptance, i.e. \( p_3^* > \hat{r}^* k_3 + \hat{\omega}^* \lambda_3 \), may still fail the Findlay-Wellisz test, i.e. \( p_3^* < \frac{p_1^*}{k_1} \cdot k_3 \). And, as the above analysis demonstrates, our test is the correct one if there is no direct option available for drawing factors away from \( X_1 \) and \( X_2 \) at the initial, distorted situation. Indeed, it should be
noted that if this direct option were available, in a situation in which $w^*$ (say) is negative, it would be worthwhile to exercise it even if there was no project ($X_3$) available! In other words, it would be optimal to directly shift the initial position from $X_1 = \hat{x}_1$, $X_2 = \hat{x}_2$ to $X_1 = \frac{K}{k_1}$, $X_2 = 0$ by shifting an amount $\left\{\hat{L} - \frac{K}{k_1} \hat{x}_1\right\}$ from use in industries $X_1$ plus $X_2$. 
References


