Stochastic Credit in Search Equilibrium

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Multiple equilibria play a central role in the recent bank run literature following the Douglas Diamond-Philip Dybvig paper\(^1\) (1983). In that literature, agents can move very quickly to withdraw funds. This paper examines whether there are similar multiple equilibria in a model where the underlying technology for bringing people together has everyone moving slowly. We look for easy credit availability and hard credit availability as two different steady state equilibria rather than the dynamic break in the position of the economy that is associated with bank runs. There is a second difference with the Diamond-Dybvig paper. Their model had an illiquid production technology as a basis for multiple equilibria. This paper bases illiquidity in credit and trading limitations rather than in the production technology, focusing on the endogenous aspects of liquidity.

I start with a model of trade with trading frictions and then introduce credit to examine its effects. Since my mathematical techniques do not permit me to analyze money and credit simultaneously, this is a barter model. I proceed by first presenting a barter model which is a simplification of a model that I have published earlier (1982). My only claim for the choice of this model is that it was very easy for me to work with, as I was rather familiar with its structure. I do not claim that it is a particularly good model for this purpose, but it was

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*This paper differs from my (1986) paper by the introduction of stochastic trade, a suggestion made by David Kreps. I am indebted to H. Ichimura and K. Murphy for helpful comments and to the National Science Foundation for financial support.

\(^1\)For a partial survey of this literature, see my (1985) paper.
readily at hand. Then I ask two questions. The model is set up in a world where not only is there no money because, let us say, nobody has thought of the idea, but also there is no credit because no one has thought of that idea. If someone thinks up the idea of credit, will credit be introduced into this economy? The central focus is on the conditions one must look for with pairwise trade for credit to be introduced into an economy like this. In particular I look for conditions such that credit is not introduced into this economy. That is, the continuous time steady state equilibrium without any credit remains an equilibrium with no actual credit taking place within the rules of credit introduction I describe.

Then I consider an economy where credit is readily available with the same sorts of information and penalty rules that go with the first economy. I look for parameters such that there is a steady state equilibrium with readily available credit. The step of bringing these two models together is to ask if there is an overlap in parameter space between those parameters which allow an equilibrium with no credit and those parameters which allow an equilibrium with readily available credit. We will find that there are many such parameters. This implies that this economy may have multiple equilibria: one equilibrium where the ready availability of credit is a self-fulfilling description of the state of the economy and another where the absence of credit is a self-fulfilling description of the economy. This implies that in a model not restricted to steady states one could construct all sorts of rational expectations dynamic paths (Diamond-Fudenberg (1986)). Some of these may well resemble things that happen in actual economies where credit availability changes rather rapidly and this has a major feedback on the production level in the economy. The underlying presumption here is that some aspects of the fluctuations in the economy come from credit feedback mechanisms.
Before getting into the details, let me give a flavor of what makes this work. Essential for credit is a belief in repayment by the borrower. The penalties for failure to repay affect the incentive to repay. I assume that failure to repay keeps the borrower out of the trading network. Anything making trade more valuable increases the incentive to repay. Ceteris paribus, generally available credit has this effect. Thus a greater credit limit may justify itself by the induced increase in the willingness to repay.

A second feedback loop is that when supplying credit, one is tying up one's purchasing power for some length of time. (This is often done through a chain of intermediaries, with the intermediaries often worried about liquidity.) One's interest in tying up funds depends in part on the different scenarios one sees happening, in particular the possibility that one might need or want purchasing power before the debt is repaid. If that is true, the creditor has the option of borrowing. The willingness of someone to provide credit thus depends in part on what he thinks are the prospects of getting credit himself at some point in the future. So if credit is perceived as hard to get, then lenders are relatively unwilling to provide credit. Similarly if credit is easy to get, lenders are relatively willing to provide credit, making it easy to get. This positive feedback mechanism affects the endogenous terms of credit. This is the natural feedback loop to model given the underlying structure of the economy that is modeled. It is not what I believe to be the most important feedback loop, although I think the important loops have the same character to them. There is a great deal of short term debt that is regularly rolled over. One's willingness to lend to someone who is regularly rolling over short term debt depends more on one's belief of their continued ability to roll over the short term debt than on one's own ability to borrow. I think that is the important feedback loop when
we get rather rapid cutoffs of credit to firms that were capable of borrowing considerable sums.

In addition to these two differences between equilibria with different credit limits, the difference in credit availability affects the stock of goods in inventory available for trade. Since this also affects the value of trade, multiple equilibria depend on the net force of these different effects.

1. Basic Model Without Credit

In order to have a model with both continuous time and discrete transactions, one needs to have a complicated purchase and storage technology or a preference structure that is different from the standard integral of discounted utility of instantaneous consumption. The alternative preferences I work with have the consumption good in an indivisible unit, which is consumed from time to time. I denote by \( y \) the utility that comes whenever one of these units is consumed. This is an instantaneous utility from a discrete consumption at an instant of time. That is a mathematically convenient approximation to the fact that it does take a while to consume goods. But we also do not go around consuming (or purchasing) nondurable goods continuously through the day. Similarly, production of consumer goods takes time but is modeled as an instantaneous process. (Modeling the length of time to complete production as a Poisson process permits a straightforward generalization of this class of models.) After production, the good is carried in inventory until it can be traded. Denote by \( c \) the labor disutility of instantly producing one unit of this good. All opportunities involve the same cost. Instantaneous utility thus satisfies

\[
U = y - c
\]

For viability of the economy we assume \( 0 < c < y \). Over time there is a sequence of
dates, \( t_i \), at which one will have opportunities either to acquire a unit to
consume or to produce a unit for trade. The preferences of the individual
(identical for all agents) are representable as the expected discounted sum of
the utilities associated with this random stream of discrete events as given in
equation (1-2).

\[
V = \sum_{i=1}^{\infty} e^{-rt_i} U_{t_i}
\]

The focus of this analysis is on trade, so it will not do to have this
economy collapse into autarchy, with people producing and promptly consuming what
they produce themselves. Therefore we add some restrictions. The first
restriction is that individuals never consume what they produce themselves. You
can think of it as a physical impossibility or an element of preferences—people
just do not like the good that they themselves produce. On producing a unit,
agents look for someone else who also has one unit with whom to barter. The
other restriction that will keep the model simple is that the inventory carrying
costs are such that one never carries more than one unit of good available for
trade. Thus an individual in this economy is in one of two positions. Either he
has no goods in inventory and is unable to trade or he has one unit of good in
inventory and is available to trade. In the former case the agent is looking for
an opportunity to produce. I spread opportunities out smoothly in time by
assuming a Poisson process with arrival rate \( a \) for the opportunity to give up the
labor disutility \( c \) and add one unit to inventory. This process goes on
continuously; there is no cost in being available to produce, there is merely a
cost in actually carrying out production. Of course once one has an opportunity
to produce, one still has a choice. One does not have to produce. If one has a
unit in inventory, one does not produce because one can not carry the good in
inventory. Without a unit in inventory, one looks ahead to the length of time it
will take to trade a unit if produced. The utility $y$ obtained when the good is traded one-for-one and consumed will happen some time in the future and will be discounted by the utility discount rate $r$. Therefore it will only be worthwhile to produce for trade if the process of carrying out a trade is fast enough relative to the utility discount rate and to the gap between the utility of consumption and the disutility of production.

I denote by $e$ the fraction of people with inventory for sale. If every opportunity is carried out, and that will be my first assumption, then $e$ is growing as all the people without goods for sale, the fraction $1-e$, carry out all of their opportunities. (Thus I normalize the implicit continuum of the population to one.) In addition, people will be meeting each other. They will carry out a trade whenever they have the opportunity. In a barter economy with no money and no credit, such a trade can be carried out only when both of the people meeting have inventory to trade. We are not concerned here with the double coincidence of their liking each other's goods. That is assumed to happen automatically. But we are concerned with a double coincidence in timing. Two people must come together at a time when they both have goods in inventory. They do not have the ability, the communications technology, to keep track of lots of potential trading partners and so instantly trade on completing production. The underlying idea here is that for many goods consumers are not searching for the good, they are searching for the good in the right size, color, and design. So retailers stock large quantities of goods that are held for consumers who do a great deal of shopping, not because it is hard to find out who is a supplier but because it may take some time to find one that has available precisely what is wanted.

We assume that this meeting process takes the simplest possible stochastic form of random meetings between individuals. These meetings are going on all of
the time. Any individual experiences a Poisson arrival of people at rate $b$. This is again a Poisson process with an exogenous technological parameter. But some of the people met have no inventory and can not be traded with. Some of the people met have inventory and can be traded with. So the rate at which goods can be traded is $b$, an endogenous variable depending on the stock of inventories in the economy. An economy with a high level of production will have strong incentives to produce for inventory because it is easy to meet people to trade with. Equation\ (1-3) is the differential equation for the behavior of inventories over time assuming that all production opportunities are carried out. (Below we give a sufficient condition for this behavior to be consistent.)

$$
\dot{e} = a(1-e) - be^2 \quad (1-3)
$$

That is each of the fraction $e$ with inventories faces the probability $be$ of having a successful trade meeting and being freed to seek a new opportunity. Each of the $1-e$ without inventories has the flow probability $a$ of learning of an opportunity. With all opportunities taken, the employment rate converges to $e_0$, the solution to $\dot{e} = 0$ in (1-3).

$$
2be_0 = (a^2 + 4ab)^{1/2} - a \quad (1-4)
$$

Note that $e_0$ is homogeneous of degree zero in $(a,b)$. Note also that as $b/a$ varies from 0 to $+\infty$ so does $be_0/a$. Equation\ (1-4) describes the steady state equilibrium at which I will evaluate the possibility of credit when I come to the next step.

In this steady state equilibrium we can calculate the expected discounted value of lifetime utility for those with and without inventory ($W_e$ and $W_u$ respectively) assuming that production opportunities are worth carrying out. (If they are not, $W_u$ is zero.) For each value, the utility rate of discount times value equals the expected dividend plus the expected capital gain.
\[ rW_e = be(y - \dot{W}_e + \dot{W}_u) \]  \hspace{1cm} (1-5) \\
\[ rW_u = a(W_e - W_u - c) \] \hspace{1cm} (1-6)

Those with inventory wait for the utility from consumption plus a change in status to being without inventory. Those without inventory wait for the disutility of labor plus a change in status. Note that the value equations are homogeneous of degree one in \((c,y)\) and homogeneous of degree zero in \((a,be,r)\) and so in \((a,b,r)\) given (1-4).

All projects will be taken if the capital gain from production, \(W_e - W_u\), exceeds the cost of a project. To have an equilibrium at \(e_o\), the economy must be productive enough to satisfy this condition, which I will call the breakeven constraint and denote by \((B_o)\). Subtracting (1-6) from (1-5), we can write this breakeven condition as

\[ \frac{W_e - W_u}{\mathcal{c}(y-c)} = \frac{be + ac}{r + a + be} \equiv \mathcal{c}_o(e) \] \hspace{1cm} (1-7)

We write the cost of a project that is just worth taking as \(\mathcal{c}_o(e)\). For later use we note that

\[ rW_u = a(c_o - c) = \frac{a(be(y-c) - rc)}{r + a + be} \] \hspace{1cm} (1-8)

\(W_u > 0\) is equivalent to \(c < W_e - W_u\). Solving (1-7) we see that willingness to produce for sale can be written as

\[ \frac{c}{y} < \frac{be}{r + be} \] \hspace{1cm} (1-9)

In Figure 1 we plot the breakeven condition relating \(c/y\) to \(b/a\) for given values of \(r/a\) where \(e\) in (1-9) is set equal to \(e_o\), given in (1-4) and dependent on \(b/a\). We have an equilibrium below the curve \(B_o\). That is, projects are worth undertaking if the arrival rate of trade opportunities is sufficiently large relative to the ratio of cost to value of a good. There are five parameters in this economy but with two normalizations there are really only three. There is
the utility of consumption and the disutility of labor. All we are really interested in is their relative size, c/y which is on the vertical axis. There are three flow rates per unit time, the utility discount rate, the arrival rate of production opportunities, the arrival rate of trading partners. Since we are free to measure time any way we want, again we have a normalization. I divide through by a so b/a is on the horizontal axis and I’ve drawn the curves for three different utility discount rates.

That completes the picture of the economy. It is simpler than my 1982 paper by having all of these projects cost the same. Of course there is another uniform equilibrium in this economy which we ignore. If nobody ever produces anything then it is obviously not worthwhile to produce for trade. That is a stable, robust equilibrium, but not interesting for our purposes. If there is no trade there is no possibility of introducing credit either. So we will ignore that equilibrium.

2. A Single Credit Transaction

We now wish to consider the introduction of credit to this barter economy, preserving the details of the search-trade technology and the simplicity of uniform inventory holdings. To do this we introduce two assumptions. The first is that repayment of a loan involves no transactions cost and represents consumable output. (It would be straightforward to add a transaction cost (in labor units) paid by either the borrower or the-lender.) That is, individuals have sufficient memory to costlessly find each other to complete the (delayed) barter transaction but this memory (or perhaps taste for variety) does not permit a new transaction at the same time, nor the opening of a regular channel of trade. Nor do two individuals without inventory enter into contracts for two
future deliveries.²

The second assumption is that credit terms are smoothly varied by changing the probabilities in a lottery but always involve repayment of a single unit. This simplifies keeping track of the state of the economy since all debtors will owe a single unit. Let us consider a pair of individuals who have come together in this no credit steady state equilibrium. One of them has a unit of the good to trade and the other one does not. The proposed trade begins with realization of a random variable. With probability \( p \), the inventory on hand is delivered for immediate consumption. Independent of the outcome of the random variable, the debtor promises that at his next opportunity to produce he will carry out production and deliver that good to the creditor. As we confirm below, there is always a probability of delivery, \( 0 < p < 1 \) such that this trade is (ex ante) mutually advantageous provided that the borrower always repays. However, unless the borrower is known to be totally honest, the lender must check whether it is in the borrower's interest to repay. We turn to this concern after examining the existence of a mutually advantageous probability \( p \). (A more complicated argument would consider subjective probabilities of total honesty.)

Assuming repayments the lender compares the probabilistic dynamic programing cost of giving up his unit of inventory with the utility gain from his own consumption adjusted for the expected waiting time. The trade is advantageous to the lender if

\[
p(\tilde{w}_e - \tilde{w}_u) < \frac{\tilde{g}_V}{r + a}
\]

(2-1)

The condition is that the probability of delivering the good times the value of a unit of inventory be less than the expected value of delayed payment. Delayed

²I suspect that costs of completing transactions could be used to justify the value of one delayed payment but not two. A need to inspect goods, plus symmetry in evaluations is an alternative route to justification.
payment is a Poisson process with arrival rate \( a \) and a payoff discounted at rate \( r \). The borrower needs to compare the certain cost of being in debt to the probabilistic gain of current consumption. The trade is advantageous to the borrower if

\[
W_u - \left( \frac{a}{r + a} \right) (W_u - c) \leq py
\]  

(2-2)

If he enters the trade, the debtor switches from the status of waiting for production (with value \( W_u \)) to waiting for the opportunity to repay his debt (at cost \( c \)) which will then restore him to the status of waiting for production.

Combining (2-1) and (2-2) there is a mutually advantageous trade if there is a value of \( p \), \( 0 < p \leq 1 \), satisfying

\[
\left( \frac{r}{r + a} \right) \frac{W_u}{y} + \left( \frac{a}{r + a} \right) \frac{c}{y} < p < \left( \frac{a}{r + a} \right) \frac{y}{W_u - W_u^*}
\]  

(2-3)

Using (1-8) we have

\[
\left( \frac{r}{r + a} \right) \frac{W_u}{y} + \left( \frac{a}{r + a} \right) \frac{c}{y} = \left( \frac{a}{r + a} \right) \frac{be + ac}{r + a + be} < \frac{a}{r + a}
\]  

(2-4)

Since the value of a unit of inventory is no greater than its value from instant trade we have \( W_u - W_u^* < y \) implying

\[
\frac{a}{r + a} < \left( \frac{a}{r + a} \right) \frac{y}{W_u - W_u^*}
\]  

(2-5)

Thus one can always find an interval of values of \( p \) satisfying (2-3).

The lender must ask whether the borrower has an incentive to repay this loan if made. The answer depends on the structure of penalties available for enforcing contracts. I assume one particular example of penalty. That there be inadequate incentive to repay I call the credit limit constraint. I would be unhappy if the results depended critically on the particular choice of penalties for refusal to pay since penalties vary enormously with institutional structure; that is, they are very sensitive to the way the model is set up.

I assume that it is observable to everybody whenever a production opportunity is carried out and that the legal system is available to enforce
delivery to the lender if one is carried out. But I assume that no one can observe whether there is in fact an opportunity which is not taken. So if a lender chooses not to pay back, he does that by ceasing production. In other words, not repaying a loan implies dropping out of the economy, going to the autarchic state which I have implicitly modeled as the origin. These rules do not conform with modern bankruptcy law, but have the advantages of simplicity and of easy construction of a consistent equilibrium.

Debtors will repay if it is worthwhile to pay the cost of production to remain in the economy. We will have an equilibrium with a zero credit limit if the value of being in the economy with zero inventory, \( W_u \), is less than the cost of repaying \( c \).

\[
(\text{CL}_0^*): \quad W_u < c. \tag{2-6}
\]

From the equation for \( W_u \), (1-8), the credit limit condition can be written as

\[
(\text{CL}_0^*): \quad c^* < \left( \frac{r+a}{a} \right) c \tag{2-7}
\]

Combining (2-7) with the breakeven constraint (1-7), we have an equilibrium if the parameters satisfy the inequalities

\[
c \leq c^* < \left( \frac{r+a}{a} \right) c \tag{2-8}
\]

Substituting from (1-8) we write the credit limit condition as

\[
(\text{CL}_0^*): \quad \frac{c}{y} > \frac{abe}{r(r + be) + abe + 2ar} \tag{2-9}
\]

Again, the condition is evaluated at \( e_o \) satisfying (1-4).

For \( r/a = .1 \), the shaded area in Figure 2 contains parameter values \((c/y, b/a)\) for which we have an equilibrium without credit: the credit limit condition is satisfied above the curve labeled \( \text{CL}_0^* \); the breakeven condition is satisfied below the curve labeled \( B_0 \). With a potential moral hazard problem, credit will appear at the no credit equilibrium as soon as someone has the idea if \((\text{CL}_0^*)\) is not satisfied. This will hold for \( c/y \) sufficiently small. Conversely, the economy
can be trapped in a no credit equilibrium if this condition is satisfied.

In Figure 3 we examine equilibrium for c/y equal to .8 and r/a equal to .1 by plotting c*/y, c/y, and (r+a)c/(ay) against b/a. There is an equilibrium without credit for b/a in the interval \((b^*_0, b^0)\).

3. Basic Model With Credit

We turn next to the situation where credit is always available. We assume that if you have no goods in inventory and if you are not in debt, then somebody with inventory is willing to lend to you, willing to provide you (stochastically) consumption in return for future delivery of goods. However, I will not get into the network of being willing to lend to someone because he is a creditor of someone else. I will just ignore all of that. Also we will look for parameters so that the (endogenous) credit limit is one. Thus there are three possible positions an individual can be in. (1) He can have a unit of good available for trade. He may or may not also be a creditor, but that is just future consumption, it does not affect his trading abilities. As before, e is the fraction of the population in this position. (2) He may not have a unit available to trade and also not be a debtor. We denote by u the fraction of the population in that position. Or, (3) he may be a debtor. The fraction of the population in that position is denoted by d. These people cannot borrow any more; they are up against their credit limit.

We now consider dynamics where credit is given by those with inventory to finance all potential transactions with nondebtors but no transactions with debtors. The fraction with inventory, e, drops by any contact with someone with inventory and drops with the probability p from a contact with a nondebtor. The number with inventory rises from any production by a nondebtor. The latter
lowers the fraction of nondebtor's without inventory. This fraction also rises whenever two agents with inventory trade and whenever a debtor produces. There is an expected change of \((p-1)\) in the number of nondebtor's without inventory from a trade involving credit. The number of debtors, \(d\), falls from production and rises from the acceptance of credit. Thus we have the differential equations

\[
\begin{align*}
\dot{e} &= -be(e+up) + au \\
\dot{u} &= -au + be^2 + ad + beu(p-1) \\
\dot{d} &= -ad + beu
\end{align*}
\]  

\(3-1\)

It is convenient to eliminate \(d\) from these equations, giving us the pair of differential equations:

\[
\begin{align*}
\dot{e} &= au - be^2 - beup \\
\dot{u} &= -au + be^2 + a(1 - e - u) + beu(p-1).
\end{align*}
\]  

\(3-2\)

Since \(\dot{e} + \dot{u} = a(1-e-u) - beu\) any intersection of \(\dot{e} = 0\) and \(\dot{u} = 0\) with \(e > 0\) and \(u > 0\) must have \(e + u < 1\).

In \(e - u\) space, we examine the phase diagram for \((3-2)\) inside the triangle \(e \geq 0, u \geq 0, e + u \leq 1\). Setting \(\dot{e} = 0\) we have

\[
\begin{align*}
u &= \frac{be^2}{a - bep}
\end{align*}
\]  

\(3-3\)

The relevant portion of this curve rises from the origin and is asymptotic to the line \(e = a/(bp)\). This is shown in Figure 4 for \(b = a\) and \(p = 1\).

Setting \(\dot{u} = 0\) and solving for \(u\) we have

\[
\begin{align*}
u &= \frac{be^2 - ae + a}{2a + be(1-p)}
\end{align*}
\]  

\(3-4\)

For \(e = 0\) and \(e = (a/b) + (1-p)/2\), we have \(u = 1/2\). The curve is decreasing as it leaves the vertical axis and has a unique turning point with \(e\) positive. The
curve is shown in Figure 4 for \( b = a \) and \( p = 1 \).

Equating (3-3) and (3-4) we have a cubic equation for \( e \). Defining \( a' = a/b \) we can write the cubic equation as

\[
e^3 + a'(1-p)e^2 + (a'^2 + a'p)e - a'^2 = 0.
\]

(3-5)

Define

\[
x = (3(a'^2 + a'p) - a'^2(1-p)^2)/3
\]
\[
z = (2a'^3(1-p)^3 - 9a'(1-p)(a'^2 + a'p) - 27a'^2)/27
\]
\[
A = (-z/2 + (z^2/4 + x^3/27)^{1/2})^{1/3}
\]
\[
B = (-z/2 - (z^2/4 + x^3/27)^{1/2})^{1/3}
\]

Then, we can write the equilibrium level, \( e_c \) as

\[
e_c = A + B - a'(1-p)/3
\]

(3-7)

Thus there is a unique intersection of (3-3) and (3-4). It is straightforward to check the stability of this equilibrium.

Implicitly differentiating (3-5), we see that \( e \) decreases with \( p \); that is, the greater the probability of delivery in a stochastic credit transaction the lower the steady state stock of inventory.

Eliminating \( p \) from (3-3) and (3-4) we have

\[
b e u = a(1-e-u).
\]

Thus \( u_c \) is decreasing in \( e_c \) and so increasing in \( p \).

Note from (1-3) that \((e_0, u_0)\) lies at the intersection of \( e = 0 \) drawn for \( p=0 \) and \( e-u = 1 \) in \( e-u \) space. Thus \( e_0 > e_c \) for all \( p \). We can have either sign for \( u_0 - u_c \).

Next we examine wealths under the assumption that all projects are carried out and credit is extended up to a credit limit of one. As above, the utility discount rate times the value of being in a position equals the expected flow of utility dividends and capital gains.
\[ rW_e = be(y - W_e + W_u) + bu\frac{ay}{r+a} - pW_e + pW_u \]  
(3-8)

\[ rW_u = be(y_p - W_u + W_d) + a(W_e - W_u - c) \]  
(3-9)

\[ rW_d = a(W_u - W_d - c) \]  
(3-10)

Following the notation used above, we write the cost of a project just worth taking as \( c^* \):

\[ c^*_u (e,u) = W_e - W_u \]  
(3-11)

\[ c^*_d (e,u) = W_u - W_d \]  

Subtracting (3-9) from (3-8) and (3-10) from (3-9) we have

\[ (r + a + be + pbu)c^* = bec^*_d + ac + bu\frac{ay}{r+a} + bey(1-p) \]  
(3-12)

\[ (r + a + be)c^*_d = bey + ac^*_u \]  
(3-13)

Solving from (3-12) and (3-13) we have

\[ (r + a + be + pbu - \frac{abe}{r+a+be})c^*_u = \frac{b^2e^2y^2}{r+a+be} + ac + bu\frac{ay}{r+a} + bey(1-p) \]  
(3-14)

\[ ((r + a + be + pbu)(r + a + be) - abe)c^*_d = (r + a + be + pbu)bey + a^2c + bua\frac{ay}{r+a} + abey(1-p) \]  
(3-15)

4. Terms of Credit

In order to determine the terms of credit, (the value of p) we use the Nash bargaining solution for a single credit transaction, assuming all other credit transactions occur with delivery probability \( p \); i.e., that position values satisfy (3-8) to (3-10). We want a fixed point in \( p \) so that the condition for the Nash bargaining solution is satisfied. Without this credit transaction the pair of agents who have met have values \((W_e, W_u)\). With a credit transaction with delivery probability \( p \), their values become
(17)

\[ (\dot{w} + ay/(r + a) - p(w - \dot{w}_u), \dot{w}_d + py). \]  (4-1)

Using the willingness to produce, (3-11), the gains from trade can be written as

\[ \left( \frac{\dot{y}}{r+a} - pc^*, py - c^* \right). \]  (4-2)

The Nash bargaining solution satisfies the maximization problem

\[ \text{Max} \ (py - c^*)(\frac{\dot{y}}{r+a} - pc^*). \]  (4-3)

Calculating the first order condition, we have

\[ y\left(\frac{\dot{y}}{r+a} - pc^*\right) = (c^*)(py - c^*). \]  (4-4)

Solving for \( p \) we have

\[ p = \frac{c^*_u c^*_d + ay^2/(r+a)}{2yc^*_u}. \]  (4-5)

For there to be a mutually advantageous trade the gains to trade to both parties must be nonnegative. Thus \( p \) must satisfy

\[ \frac{c^*_d}{y} \leq p \leq \frac{\dot{y}}{(r+a)c^*_u} \]  (4-6)

The Nash bargaining solution value of \( p \) is the mean of the two limits in (4-6). Thus we have a mutually advantageous trade provided

\[ c^*_u c^*_d \leq \frac{ay^2}{(r+a)} \]  (4-7)

There is no guarantee that the solution to (4-5) is less than one. However the equations are only valid when \( p < 1 \). Thus we restrict attention to parameter values having this property. There may be alternative equilibria corresponding to a negative interest rate and a lottery on repayment of the debt rather than a lottery on delivery of goods on hand.

5. General Credit Availability

For an equilibrium with a credit limit of one and a mutually agreeable probability of delivery \( p \), \( 0 < p < 1 \), we need to check three conditions. We must
check a breakeven condition: it is worth producing for inventory. We must check the moral hazard condition: it is worth repaying debt if you succeed in borrowing. And we must check the credit limit condition: lending to someone beyond the credit limit would violate the moral hazard constraint. In this section we examine the three conditions. To distinguish these conditions from those above we drop the subscript and write them as (CL), (MH), and (B).

We have an equilibrium with credit if the three conditions are met:

(B): \( c^* > c \)

(MH): \( c^*_d > c \)

(CL): \( W_d < c \)

Using (3-10) and (3-11) the credit limit constraint can be written alternatively as

(CL): \( c^*_d < \left(\frac{r+a}{a}\right)c \). \hspace{1cm} (5-2)

The conditions are summarized in Table 1. In Figure 5 we calculate the values of \( c_u^*/y \) and \( c_d^*/y \) as functions of \( b/a \) for \( r/a = .1 \) and \( c/y = .8 \). We have an equilibrium in the range \((b_c, b_c^*)\). Comparing Figures 3 and 5 we have two equilibria for a sizeable fraction of the values of \( b/a \) for which we have either a zero or one credit limit equilibrium.

Using (3-14) the breakeven constraint can be written as

(B): \( (r+be+pbe - \frac{abe}{r+a+be}) \frac{c}{y} \leq \frac{b^2}{r+a+be} \frac{e}{r+a} + \frac{bu}{r+a} + be(1-p) \) \hspace{1cm} (5-3)

We evaluate this condition at \( e_c \), satisfying (3-7) and \( u_c \) satisfying (3-3). Using (3-15) we can write the moral hazard constraint as

(MH): \( \left(\frac{(r+be+pbe)(r+a+be)}{y}\right)^c \leq \frac{(r+be+pbe)be}{(r+a)} + \frac{bu^2}{(r+a)} + abe \) \hspace{1cm} (5-4)

Again, we evaluate this condition at \((e_c, u_c)\).
Using (3-15), we can write the credit limit condition as

\[
(CL): \frac{c}{y} \geq \frac{(r + be + pbu)(r + a + be)(r + a)/a + r^2 + 2ar + \frac{bua^2}{r + a} + abe}{y^2 + c^*^2} = \frac{1}{2yc^*_o}(\frac{a}{a + r})
\]  

(5-5)

Again, we evaluate this condition at \((e_c, u_c)\).

In Figure 6 we show \(p\) as a function of \(b/a\) over the range of values for which we have an equilibrium with \(c/y = .8\) and \(r = .1\). Also shown in Figure 6 is the value of \(p\), denoted \(p_o\), which would satisfy the Nash bargaining solution if a single transaction were to take place at the no credit equilibrium if the credit limit constraint was irrelevant for a single transaction.

\[
p_o = \frac{y^2 + c^*^2}{2yc^*_o}(\frac{a}{a + r})
\]

(5-6)

A further difference between the two equilibria is in the stock of inventory available for trade. In Figure 7 we show \(e_o\) and \(e_c\) as functions of \(b/a\) for \(c/y = .8\) and \(r = .1\).
<table>
<thead>
<tr>
<th>Credit Limit</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous variables</td>
<td>$e_o$</td>
<td>$e_c, u_c$</td>
</tr>
<tr>
<td></td>
<td>$c^*_o$</td>
<td>$c^<em>_u, c^</em>_d$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td></td>
</tr>
<tr>
<td>Breakeven constraint</td>
<td>$c^*_o &gt; c$</td>
<td>$c^*_u &gt; c$</td>
</tr>
<tr>
<td>Moral Hazard condition</td>
<td>$c^*_a &gt; c$</td>
<td></td>
</tr>
<tr>
<td>Credit Limit condition</td>
<td>$\bar{w}_u &lt; c$</td>
<td>$\bar{w}_d &lt; c$</td>
</tr>
<tr>
<td></td>
<td>$c^*_o &lt; \left(\frac{r+a}{a}\right)c$</td>
<td>$c^*_d &lt; \left(\frac{r+a}{a}\right)c$</td>
</tr>
</tbody>
</table>
6. Discussion

For the values \( r = 0.1 \) and \( c/y = 0.8 \), we have found an equilibrium with a credit limit of zero over a range of values of \( b/a \) running from 0.5 to 3.4. For all of these values of \( b/a \) in excess of 1.2 there is another equilibrium with a credit limit of 1. Thus multiple equilibria are not merely possible in this model but are in some sense a common phenomenon. While the discrete nature of transactions and credit lend themselves to the finding of multiple equilibria, the pervasiveness of multiple equilibria suggests that similar findings may well hold in a smoother model.

It is interesting to contrast the two equilibria that exist for some values of \( b/a \). We see that \( p \) exceeds by 3 to 4 percentage points \( p_0 \), the credit terms that would hold in a zero credit limit equilibrium if a single transaction were not concerned with the moral hazard constraint. This represents a sizable difference in implicit interest rates and adds to the incentive to pay back one's debt in order to stay in the trading network. On the other hand, the stock of inventories drops by approximately 0.2, representing a large decrease in the rate of transactions. This decreases the advantages of being part of the trading network. Thus there are three effects combining to change the net incentive to pay back one's debt—the direct impact of the change in the credit limit, the improvement in the terms of credit, and the deterioration in the speed of transactions. The net feedback must be positive in order to sustain the second credit limit.

The absence of financial intermediaries in this model severely limits the lessons that might be drawn from it. While financial intermediaries will internalize some of the externalities that appear in the model they will not eliminate all of them if one preserves transactions and information limitations.
that are realistic. Thus the paper points to the value of continued exploration of models of credit that recognize that smoothly functioning credit markets are only part of the story of credit provision.

The basic idea pursued in this paper is that the credit structure is delicate. Obviously this is more heavily so for a pyramided credit structure than for the non-pyramided one I have analyzed here. Even with rational expectations, even if we rule out what in the United States is the cause of many bank failures, incompetence of bank managers, the structure of credit is fragile. If people begin to think that credit availability is a problem, then credit availability can become a problem. In a model looking only at steady states, this has something of a sunspot nature to it. There is no cause that I have modeled for people to have this sort of attitude. However, in an economy outside steady state where production levels were going up and down, then one could easily get feedback from production decisions onto beliefs about credit availability and then a feedback from credit availability onto profitability and production. So one has here what seems to me a very Keynesian mechanism. Positive feedbacks make the expectations about the future state of the credit market play a central role in the trajectory of the economy. This is most simply modeled by comparing alternative steady states, but in a dynamic setting the credit structure will play a major role in the dynamic movement of the economy.
References


FIGURE 4