SEARCH THEORY
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Walrasian analysis relies on the premise that an adequate model of resource allocation can be built with the assumption that the coordination of traders is instantaneous and costless. In contrast, Search Theory is the analysis of resource allocation with specified, imperfect technologies for informing agents of their trading opportunities and for bringing together potential traders. The modeling advantages of assuming a frictionless coordination mechanism, plus years of hard work, permit Walrasian analysis to work with very general specifications of individual preferences and production technologies. In contrast, search theorists have explored, one by one, a variety of very special allocation mechanisms, in combination with very simple preferences and production technologies. Pending the development of more general theories, it is necessary to examine the catalogue of different analyses that have been completed.

Paralleling the Walrasian framework, we will proceed by examining first individual choice and then equilibrium with different coordination technologies. There are a large number of variations on the basic search-theoretic choice problem. We will explore one set-up in detail, while mentioning some of the variations that have been developed in the literature. The mechanism for the coordination of trade can be

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distinguished in two different directions: information gathering about opportunities, and arrangement of individual trades. One simple case is where information gathering is limited to visiting stores sequentially, which combines the transaction costs of collecting goods with the information gathering costs. Alternatively, there can be an information gathering mechanism which is independent of the process of getting into position to order and receive delivery of the good. We will begin by analyzing equilibrium models where the only information gathering is associated with purchasing opportunities and then look at the modification of results that comes from supplementing this mechanism with an additional device for information spread.

Once two potential trading partners have come together there remains a variety of mechanisms for determining whether they trade and the terms on which they trade if they do. Among these are price setting on a take-it-or-leave-it basis, idealized negotiations where mutually advantageous trades are always carried out at a price determined by some bargaining solution, and more realistic negotiation processes that recognize the time and cost of negotiation, the possibility of the failure to find a mutually advantageous trade, and the possible arrival of alternatives for one or the other of the trading partners. We will explore the first two mechanisms.

One final distinction in the literature is between one-time purchases of commodities and on-going trade relations. Infrequently purchased consumer goods are the classic example of the former, while the employment relationship is the classic example of the latter.
Introducing on-going relationships permits the exploration of delayed learning of the quality of the match and associated rearrangements through quits and firings. Intermediate between these two cases is the situation such as that of frequently purchased consumer goods, where past trades facilitate future trades but do not bring about the closeness of an employment relationship. We shall discuss mainly the one-shot purchase.

The discussion of individual choice and partial equilibrium will be given in terms of a consumer purchase. The parallel discussion of labor markets is only briefly mentioned.

I. Individual Choice

Consider a consumer in a store who is deciding whether to make a purchase rather than visiting another store with a price which is not yet known. We denote by $U(p,1)$ the utility that the consumer will receive (net of purchase costs) if the purchase is made at the first store at a price equal to $p$. This assumes an ability to purchase the optimal number of units of the commodity given the constant price per unit of $p$. If the second store is visited and the purchase is made there at price $p$, the utility is $U(p,2)$. This utility is less than $U(p,1)$ because of the cost of visiting the second store and possibly because of utility discounting and the time delay associated with visiting a second store. We assume that the entire purchase is made at a single store. Let us assume that it is impossible to return to the first store, and that there are no other stores that can be visited. Ignoring the possibility of making no purchase and no further searches, the alternative to purchasing in store
one at price \( p \), is a single visit to store two where the price will be
drawn from a (known) distribution which we denote \( F(p) \). The purchase
should be made in store one if the utility of purchase there is at least
as large as the expected utility of purchase in store two:

\[
U(p^*, 1) \geq \int U(p, 2) dF(p) \tag{I-1}
\]

Implicit in this formulation is the lack of any new information reaching
the shopper between times 1 and 2 which alter the second period utility
function, and so give an advantage to delayed purchase. As long as the
distribution of prices in store two is viewed by the consumer as being
independent of the price in the first store, the rule in (I.1) can be
applied to determine behavior in store one. Thus there is a cutoff
price, \( p^* \), given by (I.2):

\[
U(p^*, 1) = \int U(p, 2) dF(p). \tag{I-2}
\]

For prices above \( p^* \), optimal behavior calls for visiting the second
store, while for prices below \( p^* \), optimal behavior calls for making a
purchase in the first store. Thus \( p^* \) is the cut-off price. Implicit in
this formulation is the assumption that it is not possible or not
desirable to make some purchase in store one and the remaining purchase
in store two. While this assumption is true for many consumer goods it
is certainly not true for all of them. Without this assumption one has a
problem that resembles portfolio choice and which has not been explored
in the literature. A similar analysis applies to the search for high
quality.
If the consumer does not know with certainty the distribution of prices in the second store, the consumer's beliefs about those prices may depend upon the price observed in the first store. We write the subjective distribution of prices in the second store, conditional on an observed price of \( p_1 \) in the first store as \( F(p;p_1) \). The purchase will be made in the first store if \( p_1 \) satisfies the inequality:

\[
U(p_1,1) \geq \int_{x} U(x,2) dF(x; p_1)
\]  

(1-3)

With no restriction on the beliefs of the consumer as to the structure of prices found in both stores, the set of prices resulting in a purchase in store one does not necessarily satisfy the cut-off price rule. For example, if either a high or a low observed price implies the same price in both stores, while an in-between price in store one implies a low price in store two, then the consumer will purchase in store one at the highest and lowest prices but not purchase there at an intermediate price. This might be the case if an intermediate price is a signal of a price war. If one restricts the information content of the price found in store one to the implication of a greater likelihood of similar prices in store two, the optimality of a cut-off price rule is restored. For the remainder of this essay we will restrict analysis to the case of known distributions. The caveats implicit in this simple counterexample should be kept in mind when thinking of applications of the theory to follow.

Returning to the set-up with a known distribution, we can add to the options of the shopper by giving him the possibility of returning to the
first store after observing the price in the second store. Denote by $U(p,3)$ the utility that is realized if this option is followed. The utility function $U(p,3)$ is less than $U(p,2)$, which, in turn, is less than $U(p,1)$. Given this additional option the behavior in the second store is to make the optimal decision between the known choices of buying in the second store or returning to the first store. Therefore it will pay to purchase in the first store in the first period if the price there, $p_1$, satisfies the inequality:

$$U(p_1,1) \geq \int \max[U(p,2), U(p_1,3)] dF(p)$$ (I-4)

That is, a purchase will be made in store one if utility there is higher than the expected value of utility with optimal behavior in choosing between the second store and returning to the first store. This is a particularly simple example of the backwards induction that can be applied to solve the sequential choice problem if there is a finite horizon to the shopping problem. If we consider behavior in the first store, (I.0.4) again yields the cut-off price rule if the utility functions reflect constant search costs and a discount rate that is independent of the price observed.

We now specialize the example by assuming additive, constant search costs and utility discounting with a discount factor $R$. That is, $U(p,2)$ equals $RU(p,1) - c$, with $R \leq 1$. Returning to the choice problem without a return to store one, let us denote by $V(p_1)$ expected utility on observing the price $p_1$ in store one, given optimal behavior:
\[ V(p) = \text{Max}[U(p), -c + R \int U(p)dF(p)] \quad (I-5) \]

The value of being in a store that has price \( p \) is the larger of (i) the utility from making optimal purchases at that price, and (ii) the expected utility if the search cost \( c \) is paid and the purchase is made in the second store. By having \( F(p) \) independent of \( p \), we are in a setting of sampling with replacement rather than sampling without replacement from a known distribution. Using this function \( V \), we can describe choice in the first period of a new three period search problem with no return to previous stores. The optimal rule is to purchase if

\[ U(p^*) \geq -c + R \int V(p)dF(p) \quad (I-6) \]

That is, purchase is made in the first period if the achievable utility there is at least as large as that achievable with optimal behavior, beginning with a visit to a randomly selected second store. The latter utility is the discounted expected optimized utility minus the search costs of the visit, recognizing that the second period choice is again a choice between a purchase and a search in the following period. The choice rule given in (I.6) again shows cut-off price behavior for period one choice. However, the cut-off price will be higher in the second period than what it was in the first period because of the reduction in available options as the end of the search process comes closer.

Denoting the cut-off prices in the two periods by \( p_1^* \) and \( p_2^* \), they satisfy the two equations:
\[ U(p_1^*) = - c + R \int V(p) dF(p) \quad (I-7a) \]

\[ U(p_2^*) = - c + R \int U(p) dF(p) \quad (I-7b) \]

\( V(p) \) is at least as large as \( U(p) \) since it represents the choice between purchase and searching again. Thus \( p_1^* \leq p_2^* \), with a strict inequality in problems that may involve repeated search; that is, in cases where the search cost and discount factor are not so large as to always imply a purchase in the current store.

There are additional reasons why cut-off prices for purchase could be rising over time or, equivalently in a job search setting, why reservation wages would be falling over time. In many settings, it is plausible that the search cost \( c \) is rising over time. The utility of making a purchase or finding a job might not be constant over time. In the wage setting, this could arise both from declining wealth being used to finance consumption while searching for a job and from the shortening period over which any job might be held.

A known finite horizon for the end of search is an implausible assumption in many settings. In addition, with many periods, the backwards induction optimization process is cumbersome to use for describing individual choice. It is fortunate therefore that there is one infinite horizon case which is straightforward to analyze, that of a stationary environment. In this setting a parallel analysis to that in (I.7) is a straightforward application of dynamic programming principles. With the assumption of a stationary environment the cut-off price in any period is going to be the same, period after period. Let us denote by \( p^* \)
the cut-off price and by $V$ the optimized expected value of utility after paying the search cost to enter the first store but before observing the price. Then $V$ equals the utility of purchase if a purchase is made in the first period plus the probability of not making a purchase in the first period times the discounted optimized utility from facing the same problem one period later after paying search cost $c$:

$$V = \int_{0}^{p^*} U(p)dF(p) + [1-F(p^*)] [-c-RV]$$

(Solving \(I-8\) for $V$ we have:

$$V = \int_{0}^{p^*} U(p)dF(p) - c[1-F(p^*)]$$

$$V = \frac{\int_{0}^{p^*} U(p)dF(p) - c[1-F(p^*)]}{1 - R[1-F(p^*)]}$$

The optimal $p^*$ maximizes $V$ and can be calculated by differentiation. More intuitively, we can note that a purchase just worth making will give the same utility as that from waiting to search again:

$$U(p^*) = -c + RV$$

(Rearranging terms, we can write the implicit equation for $p^*$:

$$(1-R)U(p^*) = -c + R \int_{0}^{p^*} [U(p) - U(p^*)]dF(p)$$
Using this first order condition we can analyze the comparative statics of optimal search behavior. Naturally, the cut-off price increases if the search cost increases or if the discount factor becomes smaller. Interestingly, an increase in the riskiness of the distribution of prices (holding constant mean utility from a randomly selected price, \( U(p)dp \)) makes search more valuable and so lowers the cut-off price. This result follows from the structure of optimal choice -- decreases in low prices make search more attractive while increases in high prices are irrelevant since no purchase will be made at high prices. Analysis of the relationship between the expected number of searches and the distribution of prices is complicated since it depends on the shape of the distribution of prices.

Thus far we have assumed that all stores are ex ante identical; that is, that a choice to search is a choice to draw from the distribution \( F(p) \). In many problems one can choose where to search. In that case, one is choosing which distribution \( F(p) \) to draw from or, if there are limited draws allowed from a particular distribution, the sequence of distributions from which the prices should be drawn. Interestingly, the reservation prices which tell whether to purchase or to draw again from a given distribution also serve to rank distributions.

In the set-up of the choice problem used so far we have been working with discrete time, with the arrival of one offer in each time period. There are two straightforward generalizations. First, one might have the
opportunity to receive more than one offer in any period, with the number of offers received being a (possibly stochastic) function of search costs. Then, one can model the first order condition for search intensity. Second, the process of attempting to locate individual purchase opportunities might have a stochastic rather than a determinate time structure. The simplest such model has the arrival of purchase opportunities satisfying the Poisson distribution law. That is, at any moment of time there is a constant flow probability of an offer arriving, any such offer being an independent draw from the distribution of available prices. Let us denote by \( a \), the arrival rate of these offers; and by \( c \), the constant search cost from being available to receive these offers. Utility is discounted at the constant (exponential) rate \( r \). One can derive the optimal cut-off price and the optimized level of expected utility by analyzing the discrete time process as above and passing to the limit. As an alternative that may have more intuitive content, let us think of the opportunity to make a purchase as an asset, where \( V \) now represents the value of that asset. The utility discount rate plays the role of an interest rate in asset theory. The asset is priced properly when the rate of discount times the value of the asset equals the expected flow of benefits from holding that asset. The expected flow of benefits is the gain that will come from making a purchase at a price below the cutoff price rather than continuing to search, adjusted for the probability of such an event happening, less search costs. Thus asset value satisfies

\[
rV = a \int_0^{p^*} (U(p) - V) dF(p) - c \quad (1-12)
\]
It is worthwhile to make any purchase which gives a higher utility than that associated with continued search. Thus the cut-off price satisfies

\[ \textit{r}U(p^*) = \int_{0}^{p^*} (U(p) - U(p^*))dP(p) - c \]  

(1-13)

Again one can introduce search intensity by having the Poisson arrival rate be a function of the search cost. In the equilibrium discussions below we will use the choice problem in the form (1.13).

Implicit in the discussion thus far is the assumption that a purchase is the end of the search process. In the labor setting this is equivalent to the assumption that taking a job is the end of search. However, in practice individuals frequently shift from job to job with no intervening period of unemployment. One can model the choice of job recognizing the possibility of continued search while holding a job. Such an analysis must consider the rules that cover compensation between the parties in the event of a quit or firing, with no compensation and compensatory damages being the two simple situations that have been analyzed in the literature. The search for a better job is only one aspect of turnover. Also, one can model the process of learning about the quality of match in a particular job as a function of the time on the job and the stochastic realization of experience. With a shadow value for quitting to search for a new job, one then has a second aspect of the theory of turnover.

The formulation of job taking given above has been combined with data on individual experience to examine empirically the determinants of
the distribution of spells of unemployment. Since this essay focuses on equilibrium, and the empirical literature has not examined the determinants of the distribution of opportunities, we will not explore this sizeable and interesting literature, nor the estimates of the effect of unemployment compensation on the distribution of unemployment spells. For an example, see Kiefer and Neumann (1979).

An implicit assumption in the model given above is that no other information is received during the search process. In practice, individuals are simultaneously searching for many different consumer goods and often for jobs and investment opportunities as well. The relations among search processes coming from the arrival of information and the random positions coming from simultaneous search for many different goods have not been explored in the literature. Focusing on search for a single good, we have added several new factors to the theory of demand, particularly the cost of attempting to purchase elsewhere and the knowledge and beliefs of shoppers about opportunities elsewhere. In practice, these are likely to be important determinants of demand.

II. Equilibrium with Bargaining

The theory of individual choice above is a simple example of the complex problem that people face when making decisions about information gathering and purchases over time. That simplicity is part of having an individual choice theory that can be embedded in an equilibrium model. To have an equilibrium model compatible with this theory we need to model the determination of two endogenous variables: the arrival rate of purchase opportunities and the distribution of prices at which
transactions can take place. In this section, we consider prices that satisfy the bargaining condition of equal division of the gains from trade. In the next section we will consider prices set on a take-it-or-leave-it basis by suppliers. In both cases we preserve the simplification that there are no reputations either of soft bargaining or low price setting that affect the arrival rate of potential customers to a trader. We begin by assuming that all buyers are identical and all sellers are identical. This case brings out the role of search in determining the level of prices. Below we will consider determinants of the distribution of prices.

Axiomatic bargaining theory relates the division of the gains from trade to the threat points of the two bargainers and the shape of their utility functions. To avoid complications from the latter, we assume that a single unit is purchased. Thus utility from purchase equals a constant, u, minus the price paid. We also assume that each seller has a single unit to sell. The utility from a sale is simply the price received. One might think of this as a homogeneous used car market. To divide equally the gains from trade, the difference between the utility position with the trade and the utility position without it are equalized for the two parties. The value of purchasing at price p is u−p; the alternative expected utility should there be no trade, is V, the optimized expected utility from continued search. We ignore the sufficient condition for search to be worthwhile ($V \geq 0$).

We restrict ourselves to an economic environment where all trades take place at the same price. With a degenerate distribution of prices, the cut-off price is strictly larger than the equilibrium price since an
early purchase saves search costs. Therefore, we can rewrite the value equation (I-12) as

\[ rV = a(u-p-V) - c \]  \hspace{1cm} (II-1a)

or

\[ (u-p-V) = \frac{rV+c}{a} = \frac{r(u-p)+c}{r+a} \]  \hspace{1cm} (II-1b)

Thus, the gain from buying a car at price \( p \) now rather than later, \( u-p-V \) can be expressed in terms of the parameters of the trade process. For suppliers, the utility from a sale is \( p \). The gain from selling now rather than later is the price received less the value of having a car for sale. The advantage of having the car while awaiting a sale can be incorporated in the search cost. (For the present, we ignore the determination of the level of supply, which depends on alternative uses of cars and the time of sellers.) We add \( d \) and \( s \) subscripts to distinguish demanders and suppliers of this commodity. The assumption of equal division of the gains from trade then implies

\[ u_d-p-V_d = \frac{r(u_d-p)+c_d}{r+e_d} = \frac{r p + c_s}{r+e_s} = p - V_s \]  \hspace{1cm} (II-2)

We have assumed the same utility discount rate for both parties. Thus we have a relationship between the equilibrium price, the arrival rates of trading opportunities, the search costs, and the utility from purchase. Solving (II.2) for the equilibrium price, we have:
Without direct search costs \( (c_d = c_s = 0) \), the position of the price between the seller's reservation price of zero (given the sunk cost of having a car for sale) and the demander's reservation price, \( u_d \), depends on the relative ease of finding alternative trading partners. As it becomes very easy to find buyers \( (a_s \) becomes infinite), the price goes to \( u_d \). Alternatively, as it becomes very easy to find suppliers \( (a_d \) becomes infinite), the price goes to 0. Furthermore, an increase in one's search cost pushes the price in an unfavorable direction. In this extremely simplified setting, (II.3) brings out the new element that search theory brings to equilibrium analysis, namely the dependence of equilibrium prices on the abilities of traders to find alternatives. Implicit in Walrasian theory is the idea that a perfectly substitutable trade can be found costlessly and instantaneously. In this restricted sense, there is no consumer surplus in a Walrasian equilibrium.

To complete the theory we need to determine the two endogenous arrival rates of trading partners. Assuming a search process without history, these depend on the underlying technology for bringing together buyers and sellers and the stocks of buyers \( (N_d) \) and sellers \( (N_s) \). The technology relates the two arrival rates to the numbers of buyers and sellers in a way that is consistent with the accounting identity between the numbers of purchases and of sales:
To complete the theory, we must examine the determinants of the stocks of buyers and sellers. This theory can be based on the stocks of traders or the flows of new traders. One extreme example of supply and demand is that the steady state stocks of buyers and sellers are exogenous. One then inserts the solution to (II.4) in the price equation (II.3).

An alternative extreme to perfect inelasticity is the assumption of perfectly elastic supplies of buyers and sellers at given reservation values for search, \( \overline{V}_s \) and \( \overline{V}_d \). From (II.1) and the equality of the gains from trade, in equilibrium the arrival rates will have to satisfy

\[
\frac{a_d}{a_s} = \frac{r\overline{V}_d + c_d}{r\overline{V}_s + c_s} \tag{II-5}
\]

Assuming values that are consistent with the existence of equilibrium with positive trade, the equality of gains from trade (II.2) implies

\[
p = \frac{u_d + \overline{V}_s - \overline{V}_d}{2} \tag{II-6}
\]
Thus, to have equilibrium with the equal division bargaining solution and perfectly elastic supplies of buyers and sellers, the numbers of traders actively searching adapts to give this simple formula.

For a market with professional suppliers it becomes plausible to consider the case of inelastic demand ($N_d$) and perfectly elastic supply ($V_s$). If we assume further that demanders visit suppliers at a rate independent of the number of suppliers, then $a_d$ is a parameter. Solving (II.2) for $p$ in terms of the exogenous variables we now have

$$p = \frac{V_s(r + a_d) + ru_d + c_d}{2r + a_d} \quad (II.7)$$

In this case the response of price to an increase in the perfectly elastic reservation utility of suppliers is $(r + a_d)/(2r + a_d)$, which is less than one. The speed of the search process relative to the interest rate determines the extent to which search equilibrium is different from Walrasian equilibrium. In a labor setting, an analog to (II.7) shows how unemployment compensation affects wages by changing search cost.

**Efficiency** There are two decisions implicit in the model above - whether to enter the search market and whether to accept a particular trade opportunity. The decision to enter a search market, like the choice of search intensity, affects the ease of trade of others. There is nothing in the process that determines prices which reflects the externalities
arising from the impact of changed numbers on the opportunities to trade.
Thus, in general, equilibrium will not be efficient and one has the
possibility of both too much entry and too little entry.

In order to explore the efficiency of the choice of which trading
opportunities to accept, we need to introduce a reason for waiting for a
better deal in the future. This can be done by introducing differences
in traders or differences in matches between preferences of demanders and
goods on sale. However formulated, we have the proposition that the
marginal trade generates no surplus whatsoever to the two trading
partners making that trade, yet the marginal trade changes the search
environment of others. This involves externalities of exactly the same
kind as the entry decisions already discussed. Again, in general,
equilibrium is not efficient.

Individual Differences There are many patterns of differences among
demanders in their evaluations of different goods. We explore two simple
cases which have been dubbed quality differences and variety differences.
With quality differences, all demanders have the same utility evaluation
of goods. One asks how the price of a good varies with the quality of
the good. With variety differences, all demanders have the same
distribution of utility evaluations of the set of goods in the market,
but demanders disagree as to which is better. There is then an issue of
"matching" preferences with goods. One asks how the price in a
transaction varies with the quality of the match.
We use \( q \) as the index of universally agreed on quality, and denote by \( p(q) \) the price paid in a transaction for a good of quality \( q \). By suppressing all other differences, we have a unique price for a transaction of a good of any particular quality. We denote by \( V_s(q) \) the optimized net value to a supplier of having a unit of quality \( q \) for sale. Parallelizing (II.1), we can calculate the net gain to a supplier of selling his unit. This gain, \( p(q) - V_s(q) \), satisfies

\[
p(q) - V_s(q) = \frac{rp(q) + c_s}{r + s}
\]

(II-8)

For the demander, we denote by \( V_d \) the value of entering the search market to make a purchase, and by \( u_d(q) \) the utility, gross of purchase price, of purchasing a unit of quality \( q \). Parallelizing (I.12), the utility discount rate times the value of being a demander is equal to the net flow of gains from search. The gross flow of gains from search equals the arrival rate of purchase opportunities times the expected gain from a purchase. The expected gain is the utility of owning the good less the price that has to be paid for the good less the shadow value of being a searcher. Denoting the distribution of qualities available in a randomly selected trade encounter by \( F(q) \), the value of being a demander satisfies

\[
rV_d = a_d \int_{q_1}^{q_2} [u_d(q) - p(q) - V_d]dF(q) - c_d
\]

(II-9)
A full equilibrium analysis of this model would require determination of the distribution $F(q)$ as well as the arrival rates $a_d$ and $a_s$. $V_s(q)$ would play an important role in determining $F(q)$. Such a model could consider investment in human capital with a search labor market. We will not carry out such an analysis, but focus merely on the relative prices $p(q)$, given the distribution $F(q)$. Note that this problem is kept simple by the uniformity of product evaluations, which results in a consumer purchasing the first unit encountered, just as in the uniform case analyzed above. In any particular encounter the gains from trade are shared equally between buyer and seller. Using (II.8) and (II.9) to eliminate $V_d$ and $V_s(q)$ in the equal gain condition (II.2) we have the equilibrium price function

$$\frac{2r + a_s}{r + a_s} p(q) = u_d(q) + \frac{c_d}{r + a_d} - \frac{c_s}{r + a_s}$$

$$-a_d \int_{q_1}^{q_2} [u_d(z) - p(z)] dF(z)$$

$$\frac{r + a_s}{r + a_s}$$

This generalization of the price in a uniform case, (II.3) shows a price that rises with the quality of a good but more slowly than its utility evaluation.
\[ p'(q) = u'_d(q) \frac{r + a_s}{2r + a_s} \]  

(II-11)

Again, the speed of the search process relative to the interest rate determines the magnitude of deviation from the Walrasian result that with identical demanders all transactions result in the same utility level \( (p'(q) = u'_d(q)) \).

With pure quality differences, all consumers have the same expected utility from search, while suppliers have an expected utility from search which varies with the quality of good for sale. In a symmetric variety model, both demanders and suppliers have the same expected utility from search. The variable \( q \) now represents the quality determined by the particular match of demander and good. We view the distribution of these qualities, \( F(q) \), as given and the same for all demanders and all goods. Implicitly we are assuming random matching between demanders and different goods. It is now the case that a sufficiently poor match will not result in a trade. We denote by \( u_d(q) \) the utility evaluation, gross of purchase price, of buying a good when the quality of a match is \( q \), and by \( p(q) \) the price in such a match. The value of search for supplier satisfies

\[ rV_s = a_s \int_{q_1}^{q_2} [p(q) - V_s]dF(q) - c_s \]  

(II-12)

where \( q_1 \) is the lower bound of match qualities at which it is mutually
advantageous for buyer and seller to carry out a trade. The quality \( q_1 \) satisfies the condition that \( p(q_1) \) is equal to \( V_s \). The value of search for a demander continues to satisfy (II.9). The assumption that all mutually advantageous trades are taken implies that \( q_1 \) also equates the gain from a purchase \( u_d(q_1) - p(q_1) \) with the utility from search \( V_d \).

Equating the gains from trade for buyer and seller and solving for the price we have

\[
2p(q) = u_d(q) - \frac{a_d}{q_1} \int u_d(z) - p(z) dF(z) - c_d \quad \text{(II.13)}
\]

\[
2p(q) = u_s(q) - \frac{s_s}{q_1} \int p(z) dF(z) - c_s \quad \text{and} \quad \frac{a_d}{q_1} \int u_d(z) - p(z) dF(z) - c_d
\]

In this setting the price increases with the quality of a match at half the rate at which the utility of demanders increases with the quality of the match, a smaller rate than in the pure quality case, (II.11).

Recapitulating our analysis of search equilibrium with bargaining, we have seen two themes. The first is how the search for trading partners introduces an additional element in the determination of trading prices: namely, the relative ease of the two potential trading partners in finding alternative traders. Secondly, the presence of a costly trade coordination mechanism is naturally replete with externalities as the availability of traders affects the trading opportunities of others.
Implicit in the model used in this section is the assumption that negotiation is instantaneous while search is slow. A fascinating recent literature is beginning to explore equilibrium for models where the negotiation process is an explicit game of exchanging bids which can be interrupted by the arrival of an alternative trading partner (c.f., Rubinstein and Wolinsky (1985)).

III. Equilibrium with Price Setting

In contrast to the bargaining theory used above, we now consider a situation where prices are set on a take-it-or-leave-it basis by suppliers of a commodity. This rule of (not) bargaining over prices gives a potential for monopoly power to the supplier. The fundamental question addressed by search theory in this setting is the limiting effect of the search for alternative opportunities on this monopoly power. We begin with the assumption that the only source of price information is to visit suppliers sequentially one at a time. We assume many identical suppliers and free entry. This implies equal profitability of different pricing strategies followed by different stores in equilibrium. If all buyers have identical positive search costs and demand curves which have a unique profit maximizing price, then the unique equilibrium is the price that would be set by a monopolist. The result assumes a sufficient number of suppliers that a cut in price by one of them is not seen as sufficient to induce buyers to search for the low price. When considering a price cut, a firm must forecast the gradual learning of the new distribution or, more simply (but less realistically) a known new distribution of prices with the identity of
the price cutter not known. This extreme result comes from the uniformity of trading opportunities. The best a buyer can do is wait to make exactly the same deal in the future. Therefore a buyer is always willing to pay a little bit more today than he has to pay in the future. Thus the demand curve for the individual seller coincides with the underlying demand curve for the good in the neighborhood of the equilibrium price. Even though this result is limited to unrealistic cases, it is interesting that the price is independent of the cost of search and of the speed of the search process, as long as search is not costless and instantaneous.

Once we introduce differences in demanders, either because of differences in underlying characteristics or from differences in their history of past purchases, then the equilibrium can involve a distribution of prices and the structure of that distribution will depend upon the search costs. In this case, consumers care about the characteristics of other consumers since these characteristics affect price setting behavior. Similarly, differences among suppliers will permit the search technology to affect the equilibrium price distribution. Given the pervasive reality of price distributions in retail markets, it has been natural for the literature to concentrate on developing ways of generating equilibria without uniform prices.

**Information Gathering** When the only way to find out about the price charged in an individual store is to visit it, price information is gathered one observation at a time. Separating the gathering of price information from going to stores to collect goods does not necessarily
change the underlying model. If price information is still solicited one observation at a time, the cost of going to purchase the good can be deducted from the utility of acquiring it, leaving the model unchanged. However, the separation of information gathering from the collection of goods opens up the possibility of information gathering which sometimes generates observations one at a time and sometimes two or more at a time. This possibility destroys the single price equilibrium in the model of identical buyers and sellers with free entry. To see this result, note that profit per sale is continuous in price but, with uniform prices, the number of sales is discontinuous in price since a slight decrease in price wins all sales when a firm's price is one of two that are learned simultaneously. With positive profit made on each sale it would always pay to decrease price slightly below the uniform price of all other suppliers. With constant costs the competitive price is not a possible equilibrium either. In this case a price increase gains profits when one is the only price quote while losing zero profit sales when one is not the only price quote.

In this setting there is necessarily a distribution of prices in equilibrium. Without price reputations, a store can choose any price it wants without affecting the flow of information about that store. Therefore, with identical firms the equilibrium will satisfy a zero profit condition. There will be low price high volume stores and high price low volume stores. One way to complete this model is to allow purchasers a choice of intensity of search which stochastically generates varying numbers of price quotations per period. We examine three additional models -- price guides, advertising, and word of mouth.
Price Guide  In this extension of the model we continue to have consumers seek out price information one price at a time. In addition, consumers can purchase a guide to lowest cost shopping, with the purchase cost varying across consumers. A consumer who purchases such a guide is automatically directed to one of the lowest price stores; a consumer who does not, follows the search procedure described above. Assuming free entry of identical firms with U-shaped costs and an equilibrium where some consumers purchase the price guide and some do not but otherwise consumers are identical, we have a two-price equilibrium. Some of the stores will set the price at the competitive equilibrium price. These stores will sell to all consumers who purchase price information and those sequential shoppers who are lucky enough to find one of these stores on their first shopping visit. The remaining stores will set their prices higher, equal to the cut-off price for shopping consumers or the profit maximizing price for selling to such a consumer, whichever is lower. The fraction of stores of the two kinds and the aggregate quantity of stores per consumer are then determined by the zero profit conditions for the two kinds of pricing strategies. When more consumers purchase the price guide, there will be more stores setting the competitive price and a drop in the cut-off price of searching consumers. This external benefit to searching consumers implies the inefficiency of the original equilibrium. A very slight subsidization of the cost of purchasing the price guide involves a second order efficiency cost to the purchase of guides, no effect on firms (which have zero profits), and a first order gain to searching consumers.
Advertising  It is obviously counterfactual to have all the information flows in the economy resulting from actions by shoppers. Advertising is a pervasive modern phenomenon. We continue to assume that stores have no price reputations. If the form of advertising is direct communication of prices to individual consumers, we can construct a model that again results in a distribution of prices. Stochastic communication from stores to consumers naturally generates a distribution of the number of price quotes that consumers receive. Any specific model of the stochastic structure of attempted communication will generate a distribution of numbers of price quotes learned by consumers. Free entry and zero profits then imply a particular equilibrium distribution of prices provided consumers receive a single price quotation some of the time and more than one quotation other times.

Word of mouth  It is natural to model both the seeking of price information and the distribution of price information as costly activities. However, some price information gets spread among consumers as a costless activity, part of the pleasure of discussing life. The presence of word of mouth communication in addition to sequential shopping naturally alters equilibrium.

The natural way to model word of mouth price communication brings price reputations into the model, since the prices set in one period affect communications about stores in future periods when their prices might be different. In order to isolate the effect of word of mouth we consider a very artificial model. Stores set prices which must hold for two periods. Consumers shop in the first or second period but are
otherwise identical. In the first period, there is only sequential search, visiting stores one at a time as modeled above. Between the first and second periods there are random contacts between first period shoppers and second period shoppers. In this way, each second period shopper has received information about the price in some positive number of stores. We assume the word of mouth process is such that some people hear of only one store, while others hear of at least two. In this setting there will be a distribution of prices, with the structure of the distribution depending on the details of the word of mouth process. This analysis can be extended by having shoppers tell not only of the prices they paid, but also of prices they have heard about from others. Both types of communication require a model of memory. The density of stores has different effects on equilibrium prices for different models of memory. This line of approach has been used in a setting of search for quality rather than low price to argue that doctor's fees can be higher where there are more doctors per captia (Satterthwaite (1979)).

Recapitulating the analysis of search equilibrium with price setting but no price reputation, we have seen two themes. One is the tendency for even low cost search to generate sizeable amounts of monopoly power because of similar incentives for all suppliers. The second is a tendency for equilibrium to have a distribution of prices. Since price distributions are a widespread phenomena in decentralized economies, it is reassuring that the theory produces such distributions.
IV. Additional Issues

The discussion of equilibrium in the previous two sections was the search analogue to a competitive equilibrium. It was assumed that there were many small firms, the behavior of which was adequately approximated by ignoring their impacts on certain aspects of equilibrium. Search theory has also examined equilibria with small numbers of firms. It has been observed that in a search environment it may pay a monopolist to have a distribution of prices across his outlets rather than a single price as a method of discriminating among consumers, even though the need to search for a low price adds to the cost of purchase of his good (Salop (1977)). In a duopoly or oligopoly setting, it is natural to consider randomized pricing strategies which again give rise to a distribution of prices (Shilony (1977)). This may be one of the many factors that go into the empirical fact of sales by retail outlets.

The technology of shopping in the models above is extremely simple. Nothing has been done to marry the underlying search issues with some of the realities of the geographic distributions of consumers and firms and the normal travels of shoppers. Similarly, very little has been done to model the search basis for the role of intermediaries.

Price Reputations All the models mentioned above omit or severely limit the intertemporal links in profitability that arise from price reputations. This is a major hole in the existing literature. Significant progress in this area probably will have to await the discrimination of cases in which optimal strategies (whether determinate or stochastic) are stationary, from those in which optimal strategies
involve building up a reputation which is then run down. In such a setting analysis will be very sensitive to the assumptions made about consumer knowledge both of existing prices, and of price strategies followed by firms. It would be nice to have both an empirical evaluation of the level of consumer ignorance about opportunities, and a theoretical structure capable of examining the relationship between equilibrium and the extent to which consumers are accurately informed.

Conclusion Walrasian theory assumes that consumers are perfectly informed about the prices of all commodities in the economy. This assumption is central for the law of one price, that a given homogeneous commodity sells at the same price in all transactions in a given market. This assumption is also central for a variety of inequalities on prices, limiting price differences to be less than transportation costs. These inequalities are consequences of the absence of opportunities for arbitrage profits. In order to make a rigorous arbitrage argument, there must be simultaneous purchase and sale of a homogeneous commodity at different prices net of transportation costs. If the purchase and sale are at different times, there is likely to be risk involved for the would-be arbitrageur. Similarly, a proper arbitrage argument requires homogeneous commodities. It is improper to apply arbitrage arguments to labor markets for example, although migration arguments may lead to similar conclusions. In search theory with a known distribution of prices, there is a cost to finding any trading partner and possibly a large cost to finding one willing to trade at some particular price. This idea captures one aspect of the appropriate limitations on the extent of arbitrage arguments.
Realistically, one must recognize that infrequent traders are usually ill-informed about the distribution of prices in the market. This introduces two important changes in the basic theory. One is that gathering information changes beliefs about the distribution of prices, as well as revealing the location of possible transactions. The second is the incentive created for sellers to locate those whose beliefs make them willing to transact at high prices. The differences between the search for suckers and the hunt for the highest value use of resources has not been clearly drawn in the literature, yet this distinction is both valid and important for evaluating the functioning of some markets. Search-based theory and empirical work have a long way to go until we have satisfactory answers to a number of allocation questions that are totally ignored in a Walrasian setting. Nevertheless, the theory has already shown how some informational realities can seriously alter the conclusions drawn from Walrasian theory.

It would have been highly duplicative to have reviewed search theory of the labor market as well as that of the retail market. For a survey of labor search theory and a partial guide to the literature, see Mortensen (1984). Individual patterns of unemployment spells are the key empirical fact requiring revision of the Walrasian paradigm.

The failure of the profession, thus far, to produce a satisfactory integration of micro and macroeconomics based on the Walrasian paradigm (with or without price stickiness) raises the thought that such an integration might come out of search theory. For a presentation of this view and discussion of some applications of search ideas to macro unemployment issues, see Diamond (1984).
References


