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SEN I OR I TY RATIONING AND UN EMPLOYMENT

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by

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ABSTRACT

This paper lays a foundation of macroeconomic theories in that it explains why unemployed workers fail to exercise downward pressure on wages sufficiently to eliminate unemployment. It shows rationing of (un)employment by seniority effectively contains the downward pressure. Seniority, guaranteeing to current junior workers future employment and so future income at their present firm, gives rise to a high reservation wage for starting to work for other firms. Then, it is not in the interest of unemployed workers to put downward pressure on wages in an effort to be employed by other firms. They simply stay unemployed.
I. Introduction

It would not be an oversimplification to say it is the way the labor market is viewed that differentiates macroeconomic theories. If one takes a view that the wage rate is not a Walrasian market clearing price, macroeconomic theories in the Keynesian tradition follow. On the other hand, if one regards the labor market to be always in Walrasian equilibrium, macroeconomic theories supporting monetarists' views emerge.

Friedman (1976, p. 214) sharply criticizes the Keynesian notion of the labor market. He says that a theory attributing unemployment to a high non-market clearing wage rate is incomplete as it lacks an explanation why downward pressure on the wage rates does not materialize. Put in slightly more general terms, Keynesian theories ignore rational behavior on the part of unemployed workers and, in consequence, fail to consider the downward pressure on wages that the rational behavior of unemployed workers would imply.

Starting from the assumption of fixed wage and prices, the disequilibrium theory such as Barro and Grossman (1971) and Malinvaud (1977) does not go to the heart of Friedman's criticism. Important attempts to explain fixed (non-Walrasian) wages are the implicit contract theory and the efficiency wage hypothesis. Contract theory, once considered as a promising explanation of the fixed wage rate and lay-offs, has several undesirable properties as a theory of unemployment (for example, see Akerlof and Miyazaki, 1980, and Stiglitz, 1984). The efficiency wage hypothesis (such as Shapiro and Stiglitz (1984) and Yellen (1984)) may be an excellent theory of structural unemployment, but it seems much less acceptable as a theory of short-run employment fluctuations.
Criticizing the lack of rational behavior by unemployed workers and explanations for non-market clearing wages in the Keynesian tradition as ad hoc, economists in the classic tradition adhere to the notion that the labor market is always in the Walrasian equilibrium. They have developed the intertemporal substitution models (such as Lucas and Rapping (1969), Lucas (1972), and Kydland and Prescott (1982)) as an alternative explanation of the business cycle. Though consistent as a theoretical construct, this theory does not fit well with actual observations. Contrary to predictions of some intertemporal substitution models, for instance, consumption and labor supply move in the same direction (Altonji, 1982).

The present paper presents a model of the labor market which takes into full consideration the rational behavior of unemployed workers, and at the same time accommodates the Keynesian view of the functioning of an economy. The model allows for involuntary unemployment and fluctuations of employment in response to changes in aggregate demand.

This paper assumes some wages are set by unions. As it is frequently mentioned as a cause of unemployment, in itself this is not a novel idea in the theory of unemployment. However, whenever mentioned as a cause of unemployment, it has suffered from the same criticism that Friedman (1976) made against Keynesian theories. Unemployed workers created by a high union wage rate may become outsiders to other unions, and underbid union wages there. Alternatively, unemployed workers may join another union, and swell the membership of that union. Then, in an effort to gather a necessary support out of the enlarged membership, the union leadership would be forced to reduce its wage and to increase employment. In these ways, unemployed workers may put downward pressure on wages and create demand for themselves, even if they cannot reduce the wage rate of their present union after
ratification of its wage contract. This paper focuses explicitly on what kind of rationing scheme successfully diffuses the downward pressure on wages from unemployed workers. The focus represents the principal difference between the model of this paper and other models that assume union wage setting.

In the literature, especially the implicit contract literature, it is common to assume rationing of unemployment by lottery. This paper considers seniority rationing, which is a common practice in the U.S. I show that seniority rationing produces involuntary unemployment, and effectively restrains the downward pressure on wages created by unemployed workers.

The mechanism can be briefly stated as follows. In seniority rationing, workers who have worked longer for a firm have priority for jobs at the firm over those who have worked for the firm for a shorter time. As old workers with high seniority will retire in the future, current junior workers will become relatively more senior, and hence more likely (in fact certain) to be employed by their current firms. Seniority rationing guarantees future employment and so, in effect, guarantees future incomes to junior workers (uncertainty over future wages left aside). Consider a worker who is not employed by the firm with which he is presently associated. He may seek employment at other firms by becoming an outsider to another union, and underbidding wages there. If he succeeds in being employed by another firm, he will not be able to work for his presently associated firm in the future. Therefore, in the event that he can be employed by another firm, he must forego the guaranteed future union wage from his present firm. The future incomes implicitly guaranteed by seniority rationing thus create a sufficiently high reservation wage for working elsewhere -- even with negligible moving costs, no unemployment compensation, and no utility from leisure. Roughly speaking, the wage rate of a union becomes the reservation
wage of unemployed workers of that union. It follows that, if a firm employs outsiders of its union, it must pay to them a wage rate higher than union wage rates at other firms. On the other hand, a firm is willing to hire outsiders only when the wage rate of outsiders for the firm is lower than the wage rate of its union members. Then, if a union sets its union wage rate below other unions' wage rates, the wage rate its firm is willing to pay to outsiders of the union is lower than the wage rate that unemployed workers of other unions want to be paid if they work for the firm as outsiders. With such a wage policy, a union need not fear its firm hiring outsiders from other unions at lower wages. This gives a union some latitude in wage setting which allows the union to set its wage, to the benefit of its members in general, at a level that creates unemployment. This unemployment does not exercise the downward pressure on wages.

Even if a union does not exploit this latitude, unemployment results. For, if it does not do so, its wage rate, and hence the reservation wage of its unemployed members is higher than the wage of the other unions, and thus the wage other firms will pay to outsiders; this wage differential makes it in the interest of unemployed workers of a union not to try to be employed by other firms, but to stay unemployed.

Thus, given seniority, there is no way that unemployed workers can get themselves employed on their own initiative. It is only when unemployed workers and their present firms defy the ratified wage contract and the seniority rule that the unemployed workers come to be employed without an increase in (a favorable shift of) demand for labor.

Our model shares with Grossman (1983), Osborne (1984), and Solow (1985)\(^1\) the notion that some independence of firm-union wage-setting pairs from labor

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\(^1\)I learned from Lindbeck and Snower (1985) that they have contributions with the same notion. As their works are not readily available yet, I regret that I could not discuss their works here.
market conditions is responsible for unemployment. However, my model differs from their works in some important ways. First, they consider a particular firm-union pair in the labor market, while I consider the labor market consisting of multiple firm-union pairs. Consequently, Grossman, Osborne (in his short-run model), and Solow take the reservation wage of outsiders (the hiring cost of strike breakers in the Osborne model) as parametrically given. In contrast, the reservation wage is endogenously determined in my model. This difference is important, because one of the mysteries of unemployment is why the reservation wage of unemployed workers does not fall to zero, or to a sufficiently low level to create demand for them. Second, in the above-cited models, reasons for independence of a firm-union pair from the labor market condition are "technological," requiring no further elaboration. By contrast, the reason for independence in our model is economic. One of the focal points of this paper is to show the mechanism by which an artificial, non-technical arrangement of seniority creates such independence.

The paper is organized as follows. The first part of Section II presents the labor market dynamics under seniority rationing. Its second part presents a theorem that states the results outlined in this introduction. The third part gives an intuitive account of the theorem, and argues that the model of this paper lays a labor market foundation for Keynesian theories. Section III gives a formal proof of the theorem.
II. A Model of the Labor Market Dynamics Under Seniority

I examine a model with the following nine characteristics to explore the labor market dynamics and unemployment in an economy with seniority rationing of unemployment.

(i) Workers live four periods. They are identical except for the periods they are born in. The population is constant, and the economy is demographically stationary. Therefore, if the size of population is \( N \), \( \frac{N}{4} \) workers are born at the beginning of each period.

Workers start their working life immediately after their birth. Workers who are just born or have lived one, two, or three periods are referred to as zero, one, two, or three years old.

(ii) Each employment spell of a firm extends two periods. Wages are kept constant in each spell.

By (i) and (ii), each worker can experience, at most, two spells of employment.

(iii) There are only two firms: firms a and b. They are identical except for the timing of the start of their employment spells. They start them alternatively. Firm a begins its employment spells at odd periods, that is, the \( 2k + 1 \)th period. Firm b does so at even periods, namely, the \( 2k \)th period.
Needless to say, here and henceforth, $k$ denotes an integer. By (ii) and (iii), employment spells of firm a and b are from the beginning of period $2k + 1$ to the end of period $2k + 2$, and from the beginning of period $2k + 2$ to the end of period $2k + 3$, respectively. The differences in the timing are designed to capture the circumstance that workers, after being unemployed by a firm, seek employment opportunities at other firms, and hence that there is the time difference in applying to firms for a job.

(iv) An employed worker cannot leave his employing firm, and a firm cannot lay off its workers during an employment spell.

In other words, even if employment and wage conditions of another firm-union pair are preferable to either of the employment parties, both parties of employment cannot terminate their employment relation in the interim of an employment spell.

(v) Each firm has a union. All the workers born at period $2k + 1$ ($2k$) join the union of firm a (b) when they are born, though they may leave it later. A firm can employ non-union members, so that a worker need not be a union member to be employed.

I will refer to unions of firm a and b as union a and b, respectively. I will also refer to the memberships of union a and b as labor pool a and b, or the labor pool of firm a and b. By this definition, non-union workers do not belong to the labor pools. They are referred to as outsiders.

Firm b (a) does not necessarily employ all the newly born workers, namely, zero year old union b members, in its employment spell from the
beginning of period $2k \ (2k + 1)$ to the end of period $2k + 1 \ (2k + 2)$. Those workers who are not employed by firm b (a) in the spell will have two options at the beginning of period $2k + 1 \ (2k + 2)$ when they become one year old: They stay unemployed in labor pool b (a), or they become outsiders to union a (b) to be employed by firm a (b) in the employment spell from the beginning of period $2k + 1 \ (2k + 2)$. Thus, outsiders at period $2k + 1 \ (2k + 2)$ for firm a (b) are some of the one year old workers who were not employed by firm b (a) at the time of their birth, namely, period $2k \ (2k + 1)$.

I assume none of one year old unemployed workers at period $2k + 1 \ (2k + 2)$ joins union a (b) at the beginning of the period to be employed by firm a (b). This is merely for simplicity. Allowing them to do so complicates the model substantially, but causes no change in the conclusions.

(vi) A union unilaterally sets the wage rate for its members.

The wage rate determined by a union is not applicable to outsiders to the union.

The institutional assumption (vi) is admittedly a gross abstraction of negotiation procedure between a firm and its union. Further, (vi) implies an inefficient outcome to both parties (see Hall and Lilien, 1979, and McDonald and Solow, 1981). It is desirable to postulate a more sophisticated situation between a firm and a union as the wage setting circumstance. However, I believe that, while an economic mechanism to be elucidated in this paper could survive a more sophisticated wage setting assumption, a simple structure bought at the cost of crudeness and some abstraction from reality isolates the problem of job rationing from other complications of the wage and employment settings, and would help us to concentrate upon our problem.
(vii) The union leadership must obtain support of, at least, half of its membership for its policy. Otherwise, it will be ousted, or the contract made with management (more precisely, the wage set by the union under (vi)) cannot be ratified by the membership.

This is the assumption of the union democracy. Next comes the assumption of seniority rationing.

(viii) A worker who, whether employed or not, has stayed in a firm's labor pool longer has priority for employment over a worker who has stayed there for a shorter time.

In the industrial union contract, normally, a worker can accumulate seniority only when he is employed. However, here, a worker can do it even if he is unemployed, as is the case in the craft union arrangements. In this sense, this assumption is at odds with practices. However, one can accommodate this assumption to practices of the industrial union arrangements by the following setting. A firm nominally employs any job applicant and any worker in its labor pool just a second at the beginning of each employment spell. After that, a firm decides whether it will employ a worker in that employment spell. In this setting, any worker who entered in a labor pool two periods (one employment spell) earlier, but was not employed in an employment spell following the entry, has a higher seniority than an outsider and a newcomer to the labor pool.

An unemployed worker does not lose his seniority even if he seeks to be employed by another firm. A worker who sought it, but unsuccessfully, is still considered to have stayed in his original labor pool. I do not consider
a union or a firm to penalize an unemployed worker seeking a job at another firm by depriving him of seniority. The assumption of such penalty renders the problem of this paper trivial. Besides, penalizing such an act does not seem in wide practice.

I will refer to workers who have stayed in a labor pool for two periods (one employment spell) as senior workers of the pool (or the firm, or the union), and to workers who are union members; but have not yet stayed there for an employment spell as junior workers. Outsiders are not referred to as junior workers.

If a firm offers a job, it goes first to senior workers under (viii). Being non-union members, outsiders are not protected by seniority. Even if the outsider's wage is lower than the union wage, a firm must offer jobs first to senior workers under the seniority rule.

(ix) If an unemployed worker of a union turns out to actually work for another firm as an outsider, he must incur negligible moving costs.

As locations of firms are generally different, this assumption is natural. Of course, the other reasons such as psychological costs and requirements of new specific skills create, in effect, moving costs. An important point I want to stress with respect to (ix) is that, as Assumption 2 will specify later, the moving cost exists, but is so negligible as to make the wage rate net of the moving cost is always positive, and hence that the moving cost itself does not prevent any unemployed worker from working for another firm. The assumption of the negligible moving cost serves to ensure determinacy of the stationary state (see Remark 1 in Section III).
II-A. Labor Market Dynamics

Let me formulate the labor market dynamics involving the preceding nine characteristics. Let $P_{aj}(2k + 1)$ be the number of junior workers in union a at the beginning of period $2k + 1$. Junior workers of union a consist exclusively of the newly born workers. All the newborn workers at period $2k + 1$ join union a. None of the junior workers are one, two, or three years old. As was noted, one year old workers at $2k + 1$ are assumed not to join union a. Two year old workers in period $2k + 1$ joined union a at period $2k - 1$, i.e., at the time of their birth. By (viii), whether employed at that time by firm a or not, they are senior workers of union a if they belong to union a. Three year old workers are not eligible for a new spell of employment that extends two periods. Then, one has:

\[(1) \quad P_{aj}(2k + 1) = \bar{N}/4.\]

Let $P_{as}(2k + 1)$ represent the number of senior workers in union a at the beginning of the $2k + 1$ period. They are among those born in period $2k - 1$. Others, after having found themselves unemployed in period $2k - 1$, went to firm b in period $2k$, and were employed there at age one. They will stay there until the end of period $2k + 1$ by (iv). As they cannot then work for firm a in period $2k + 1$, they cannot be counted as potential employees of firm a, and their membership of union a is nominal even if they still belong to union a. Therefore, we consider that they left union a when they were employed by firm b. Let the number of these workers be denoted by $E_{b1}(2k)$. Hence, the following relation holds:
By definition, if \( P_a(2k + 1) \) represents the number of union a members,

\[
(3) \quad P_a(2k + 1) = P_{aj}(2k + 1) + P_{as}(2k + 1).
\]

In addition to union a members, outsiders to union a are also available for firm a as its labor force. Let \( Y_a(2k + 1) \) be the number of outsiders available for firm a at period \( 2k + 1 \) for its employment spell beginning from that period. They are some one year old workers whom firm b did not employ in its employment spell from period \( 2k \) when the workers were zero year old. No outsider is two or three years old for the same reason that none of the junior workers is two or three years old.

Now, I come to firm a's employment policy under seniority rationing. Let \( R(N) \) be the revenue of the firm when it employs \( N \) workers. Of course, \( R'(\cdot) > 0 \) and \( R''(\cdot) < 0 \) in its relevant range. Let \( L(\cdot) \) be the inverse function of \( R'(\cdot) \), that is, a function relating the marginal revenue product to the workforce of the firm. \( L'(\cdot) < 0 \) by \( R''(\cdot) < 0 \). Note all the workers, junior and senior workers or outsiders, are the same as the workforce. There are two wages for firm a at period \( 2k + 1 \): the union wage rate set by its union, \( \tilde{W}_a(2k + 1) \), and the wage rate for outsiders, \( \tilde{W}_a(2k + 1) \). All the workers are the same, and so all the outsiders offer the same wage. Let \( E_{a2}(2k + 2) \), \( E_{a0}(2k + 1) \), and \( E_{ay}(2k + 1) \) be firm a's employment for two year old (and so senior), zero year old (and so junior) workers, and outsiders.

Consider first firm a's employment policy under \( \tilde{W}_a(2k + 1) > \tilde{W}_a(2k + 1) \). In this case, union workers' wage being higher than that of outsiders, a firm employs no junior workers. But, it employs senior workers out of its union
members, since seniority rationing requires the firm to employ all senior workers first if it provides any employment to outsiders. Define $R_{as}(2k + 1)$ and $R_{ay}(2k + 1)$ as:

$$R_{as}(2k + 1) = \max_{0 \leq \xi \leq \xi_{as}(2k+1)} \{R(\xi) - W_a(2k + 1)\xi\}$$

and

$$R_{ay}(2k + 1) = \max_{0 \leq \eta} \{R(P_{as}(2k+1) + \eta) - W_a(2k+1)P_{as}(2k+1) - \bar{W}_a(2k+1)\eta\}.$$ 

$R_{as}(2k + 1)$ is the maximum profit if firm a's employment is restricted only to its senior workers. $R_{ay}(2k + 1)$ is the maximum profit if outsiders are allowed to be employed after all senior workers are employed. Then, one has:

(4) \quad E_{as}(2k + 1) = L(W_a(2k + 1)) \text{ and } \newline E_{a0}(2k + 1) = E_{ay}(2k + 1) = 0 \newline \text{if } W_a(2k + 1) > \bar{W}_a(2k + 1) \text{ and if } R_{as}(2k + 1) \geq R_{ay}(2k + 1).$

(5) \quad E_{a2}(2k + 1) = P_{as}(2k + 1), \newline E_{ay}(2k + 1) = L(\bar{W}_a(2k + 1)) - P_{as}(2k + 1) > 0, \text{ and } \newline E_{a0}(2k + 1) = 0 \newline \text{if } W_a(2k + 1) > \bar{W}_a(2k + 1) \text{ and } R_{as}(2k + 1) \leq R_{ay}(2k + 1).$

When $R_{as}(2k + 1) = R_{ay}(2k + 1)$ holds, either (4) or (5) holds.

If $W_a(2k + 1) < \bar{W}_a(2k + 1)$, there is no reason for firm a to employ outsiders. Hence, on account of (viii),
\[ E_{a2}(2k + 1) = \min \{L(W_a(2k + 1)), P_{as}(2k + 1)\}, \]
\[ E_{a0}(2k + 1) = \theta_a(2k + 1) \cdot \tilde{N}/4, \text{ and} \]
\[ E_{ay}(2k + 1) = 0 \]
\[ \text{if } W_a(2k + 1) < \tilde{W}_a(2k + 1). \]

Here, the employment rate of junior workers \( \theta_a(2k + 1) \) is defined as:

\[ \theta_a(2k + 1) = (L(W_a(2k + 1)) - E_{a2}(2k + 1))/P_{aj}(2k + 1). \]

If \( W_a(2k + 1) = \tilde{W}_a(2k + 1) \), junior workers and outsiders are treated equally.

\[ E_{a2}(2k + 1) = \min \{L(W_a(2k + 1)), P_{as}(2k + 1)\}, \]
\[ E_{a0}(2k + 1) = \theta_a(2k + 1) \cdot \tilde{N}/4, \text{ and} \]
\[ E_{ay}(2k + 1) = \theta_a(2k + 1) \cdot Y_a(2k + 1) \]
\[ \text{if } W_a(2k + 1) = \tilde{W}_a(2k + 1). \]

In this case, \( \theta_a(2k + 1) \) is defined as:

\[ \theta_a(2k + 1) = (L(W_a(2k + 1)) - E_{a2}(2k + 1))/(Y_a(2k + 1) + P_{aj}(2k + 1)). \]

Let \( E_{a1}(2k + 1) \) be the number of one year old workers employed by firm a in the employment spell from \( 2k + 1 \). Since all the one year old workers employed in firm a are outsiders, one has

\[ E_{a1}(2k + 1) = E_{ay}(2k + 1). \]
It might be convenient to define $\theta_a(2k + 1)$ even for $W_a(2k + 1) > \tilde{W}_a(2k + 1)$; that is,

\begin{equation}
\theta_a(2k + 1) = 0 \text{ if } W_a(2k + 1) > \tilde{W}_a(2k + 1).
\end{equation}

Demand and supply of outsiders must balance:

\begin{equation}
Y_a(2k + 1) \geq E_{ay}(2k + 1) \text{ with } \tilde{W}_a(2k + 1) = 0
\end{equation}

if the inequality holds strictly.

Equation (12) means that outsiders compete with each other to lower their wage, so that all the outsiders are employed.

Equations (1) through (11) define firm a's demand for junior and senior workers and outsiders, given the union wage rate, the outsider's wage rate, and the labor pool. (12) specifies determination of the outsider's wage rate.

The preceding explanation is concerned with firm a and union a. Similar explanations can be given to the pair of firm b and union b. Defining notations for b in the consistent manner with those for a, one can write equations for b corresponding to (1) through (12). If necessary, we refer to those equations corresponding to (1) through (12) by putting primes on those equation numbers. For instance, $P_{bj}(2k)$ denoting the number of junior workers in union b at period 2k, (1') refers to $P_{bj}(2k) = \bar{N}/4$. Failing to define all the variables for b and to write down equations for b would not cause confusion. I will apply the same conventions for other variables and equations to be introduced later.
Next, I turn to the behavior of one year old unemployed workers that determines the value of $Y_a(2k + 1)$. Let $U(\cdot)$ be the worker's utility function for one period. $U(\cdot)$ is increasing. Let $\beta$ be the subjective discount rate (the pure time preference). If a zero year old worker expects to consume $c_i$ at age $i$, his lifetime utility as of birth is $\sum_{i=0}^{3} U(c_i)\beta^i$. I assume workers do not save, that is, they consume all the incomes within the period in which they earn the incomes. Consider a worker who was born in period $2k$, but was not employed by firm $b$ in the employment spell starting from the beginning of period $2k$. At the beginning of period $2k + 1$, he can seek to be employed as an outsider by firm $a$. His benefit from being employed by firm $a$ in this way in periods $2k + 1$ and $2k + 2$ is $(1 + \beta)U(\bar{W}_a(2k + 1) - t)$ as of period $2k + 1$. (By (ii), the outsider's wage rates in period $2k + 1$ and $2k + 2$ are the same.) Here, $t$ is the moving cost mentioned in (ix). For analytical simplicity, I assume a worker pays the cost by installments; that is, he pays $t$ in each of two ensuing periods after the move.

Getting employed by firm $a$ is not costless to him. If he begins to be employed by firm $a$ at period $2k + 1$, he will work for the firm in periods $2k + 1$ and $2k + 2$. Then, he cannot work for firm $b$ in the employment spell from period $2k + 2$ to $2k + 3$. It must be noticed that, as equation (16) shows later, the worker will be certainly employed by firm $b$ in periods $2k + 2$ and $2k + 3$ under the seniority rule and the union democracy. Accordingly, he has to forego the guaranteed\(^2\) earnings at firm $b$ from $2k + 2$ to $2k + 3$. It follows that his opportunity cost of being actually employed by firm $a$ in the employment spell from period $2k + 1$ to $2k + 2$ is $(\beta + \beta^2)U(\bar{W}_b(2k + 2))$ as of period $2k + 1$.

\(^2\)We assume there is no uncertainty with respect to the future wage rates.
If an unemployed worker of labor pool b is to become an outsider for an employment opportunity at firm a, the utility from doing it must exceed the opportunity cost; that is, 
\[(1 + \beta)U(\tilde{W}_a(2k + 1) - t) > (\beta + \beta^2)U(W_b(2k + 2))\]
must hold. Note that, on account of (12), one can be certainly employed by firm a if one becomes an outsider, and therefore that the benefit from becoming outsiders is not an expected value, but 
\[(1 + \beta)U(\tilde{W}_a(2k + 1) - t).\]

Of course, some zero year old junior worker's of union b must have been unemployed at period 2k if there are some unemployed workers who become outsiders of union a at period 2k + 1. The above considerations lead to the following characterization of \(Y_a(2k + 1):\)

\[(13) \quad Y_a(2k + 1) > 0 \text{ iff the following two conditions are satisfied:} \]
\[
e_{b0}(2k) < \tilde{N}/4; \beta U(W_b(2k + 2)) < U(\tilde{W}_a(2k + 1) - t).\]

All the workers are the same. Hence, placed in the same conditions, unemployed workers behave in the same way. Hence,

\[(14) \quad Y_a(2k + 1) = \tilde{N}/4 - e_{b0}(2k) \text{ if } Y_a(2k + 1) > 0.\]

It must be noted that (13) and (14) allow for the case with \(\tilde{N}/4 > e_{b0}(2k)\) but \(Y_a(2k + 1) = 0.\) That is, unemployed workers may remain in labor pool b without trying to be employed by firm a.

I now turn to the objective of the union leadership. I presume that, given \(P_a(2k + 1),\) the leadership of union a solves the following constrained maximization problem:

---

3 Equation (12) means that outsiders compete with each other to lower the wage rate for them, so that all of them are employed.
(15) \[
\max U(W) \left\{ \theta_a(2k + 1)P_{aj}(2k + 1) + E_{a2}(2k + 1) \right\}
\]
subject to \[
P_a(2k + 1)/2 \leq \theta_a(2k + 1)P_{aj}(2k + 1) + E_{a2}(2k + 1)
\]
\[
\leq P_a(2k + 1).
\]

The solution of this problem, \(W\), is set to \(W_a(2k + 1)\).

Two remarks on (15) are in order. First, \(\theta_a(2k + 1)P_{aj}(2k + 1) +
E_{a2}(2k + 1)\) is total employment for union a's members. Hence, the constraint in (15) represents (vii). If a wage rate set by the union cannot ensure employment for no less than half of its members, the wage policy cannot be ratified. It must be noted that, by (1), (2), (3), the constraint in (15), and seniority rationing, all the senior workers are always employed:

\[
(16) \quad E_{a2}(2k + 1) = P_{as}(2k + 1).
\]

Second, it must be made clear which variables the union takes as given (expects unaffected by its wage policy) in solving (15). The union leadership takes \(P_a(2k + 1), W_b(2k)\), and \(W_b(2k + 2)\) as given. Given the wage rate of the other union, the union's wage policy affects the outsider's wage rate via the firm's employment policy. The union takes this effect into consideration in deciding its wage rate.

Let \(E_a(2k + 1)\) denote the number of workers firm a employs in its employment spell extending from period \(2k + 1\) to \(2k + 2\); namely,
\[
E_a(2k + 1) = E_{a0}(2k + 1) + E_{a1}(2k + 1) + E_{a2}(2k + 1).
\]
Let \(E(2k + 1)\) and \(E(2k + 2)\) denote the total employment in period \(2k + 1\) and \(2k + 2\);
\[
E(2k + 1) = E_a(2k + 1) + E_b(2k) \quad \text{and} \quad E(2k + 2) = E_a(2k + 1) + E_b(2k + 2).
\]
II-B. Stationary State

Analyzing movements of difference equations with such high order and complexity as those in the last subsection is beyond the ability of the author. Further, in the transitory phase to the stationary state, any level of employment could realize. In this sense, studying the transitory phase would not be a wise strategy in exploring effects of seniority rationing on employment. For these two reasons, I confine my attention only to the stationary state of the labor market dynamics in subsection II-A.

I denote the stationary state values of the variables in the last subsection by deleting period indices from them: for instance, \( P_a \) denotes the stationary state value of \( P_a(2k + 1) \).

I make the following assumptions:

**Assumption 1** \( U(W)L(W) \) is strictly concave, and reaches its maximum at \( W^* \).

Let \( L^* = L(W^*) \). By Assumption 1, \( U(W)L(W) \) is increasing for \( W < W^* \), and decreasing for \( W > W^* \).

In the subsequent part of this section, I consider shifts of \( L(\cdot) \). As \( L(\cdot) \) shifts, \( L^* \) generally changes. Then, \( L^* \) can be regarded as an index of location of \( L(\cdot) \), or, its shift parameter. More precisely, I take that the function \( L(\cdot; L^*) \) has the following property: Denote the solution of \( \max \limits_{W} U(W)L(W; L^*) \) by \( W^*(L^*) \); then, \( L(W^*(L^*) ; L^*) = L^* \) holds.

I define \( \bar{W}^0(L^*) \) and \( \bar{W}^1(L^*) \) as \( L(\bar{W}^0(L^*) ; L^*) = \bar{N}/4 \) and \( L(\bar{W}^1(L^*) ; L^*) = \bar{N}/2 \). It is proper to write \( L(\cdot ; L^*) \), \( R(\cdot ; L^*) \), \( W^0(L^*) \), \( W^1(L^*) \), and \( W^*(L^*) \), but I will write \( L(\cdot) \), \( R(\cdot) \), \( W^0 \), \( W^1 \), and \( W^* \) instead to make notations simpler. This shorthand should not cause any confusion.
Assumption 2 \( L(t) > \bar{N} \).

This assumption implies that the moving cost is so small that it in itself does not keep unemployed workers from moving to work for another firm. When the wage rate is below \( t \) so that it is not worth moving to work for another firm, demand for labor exceeds the total labor force. This is impossible. Therefore, Assumption 2 implies that the wage-rate net of the moving cost always makes working for another firm rewarding to the unemployed. It must be remarked that Assumption 2 also implies the wage rate is always positive.

When the wage rate is zero, there is excess demand for labor. \( W_a > 0 \) and \( W_b > 0 \) by Assumption 2.

Assumption 3 \( \beta = 1 \).

This assumption means that the pure rate of time preference is zero. I use this assumption to avoid lengthy, complicated mathematical arguments. Seniority rests upon individuals' appreciation of the future. Assumption 3, therefore, helps demonstrate economic mechanisms under seniority rationing most dramatically. Besides, in the stationary state, this assumption justifies another assumption that workers consume all income when they earn it.

Let \( f(x) \) be the function which satisfies the following:

\[
\max_{n \geq 0} \{ R(\bar{N}/4 + \eta) - f(x)\bar{N}/4 - x\eta \} = \max_{0 \leq \xi \leq \bar{N}/4} \{ R(\xi) - f(x)\xi \}.
\]

Apparently, \( f(x) = \bar{W}^0 \) for \( x \geq \bar{W}^0 \); and \( f(x) > x \) and \( f'(x) < 0 \) for \( x < \bar{W}^0 \). It is easy to observe that, given \( x \) as the outsider's wage rate, a union with
N/4 senior workers sets its wage rate at f(x), if it determines to provide employment only for its senior workers (see Lemma 3 for a detailed account). Let v(x) = U(f(x)). v(x)N/4 represents the value of the union objective function in such an event.

I assume the following:

**Assumption 4** If W* < W0 (L* ≥ N/4), v(W*)N/4 < U(W*)L(W*).

I must note that the assumption automatically holds when R(N; L*) = 

\[-(N - L^*/2)^2 + (L^*)^2/4]\]/2d for N < L^*/2 and \((L^*)^2/8d\) for L^*/2 < N with d > 0 and U(c) = c. When L* ≤ N/3 or N/2 ≤ L*, one can establish the theorem to follow without invoking this assumption (see the proof of Proposition 5).

When N/3 < L* < N/2, one still has an outcome similar to the theorem without the assumption; namely, unemployment cannot be eliminated in face of the rational behavior of unemployed workers, and the downward pressure on wages from them. However, in this case, outsiders may emerge unlike the case with Assumption 4. The reason for introducing the assumption is that it reduces the number of possible cases under N/3 < L* < N/2 to one, and thereby enables us not to get deeply involved in a taxonomatic argument, and then to simplify the argument and the theorem. Besides, the assumption allows the level of employment to change continuously in response to changes in L*.

The economic meaning of the assumption is as follows. f(W*) is the union wage rate when union a with N/4 senior workers allows firm a to employ outsiders under the outsider's wage W*. Hence, v(W*)N/4 is the maximum value the union a objective function can take when union a allows outsiders to be employed, and it has N/4 senior workers. When the outsider's wage is W*, it follows from (4) and (5), that union a can prevent outsiders from being
employed if union a sets its wage at a level slightly lower than $W^*$. The value of the union a objective function is slightly lower than $U(W^*)L(W^*)$ if it does so. Accordingly, Assumption 4 means that, when the outsider's wage rate is $W^*$, it is in the interest of union a with $\bar{N}/4$ senior workers not to let firm a employ outsiders.

Under Assumptions 1 through 4, the following theorem holds for the labor market dynamics of the last subsection.

**Theorem**  The stationary state values of $E$, $W_i$ and $\bar{W}_i$ ($i = a, b$) in the model of the last subsection are given by $E = \bar{N}/2$, $W_i = W^0$, $\bar{W}_i \leq W^0 + t$ if $L^* \leq \bar{N}/4$; $E = 2L^*$, $W_i = W^*$, $W^* \leq \bar{W}_i \leq W^* + t$ if $\bar{N}/4 < L^* < \bar{N}/2$; and $E = \bar{N}$, $W_i = W_1$, $W_1 \leq \bar{W}_i$ if $\bar{N}/2 \leq L^*$. Further, $Y_i = 0$ for $i = a, b$ holds; that is, no unemployed worker becomes an outsider to be employed by another firm.

Figure 1 graphically shows the level of employment indicated by the theorem. The theorem says that seniority rationing effectively contains the downward pressure on wages that would otherwise stem from frustrations of unemployed workers and undermine the union's wage policy eventually. Consequently, given seniority, unemployment results even in the face of the rational behavior of unemployed workers when demand for labor is not large. The level of unemployment increases as the demand increases.

A formal proof of the theorem is delegated to Section III. In the following subsection II-C, I will give an intuitive and verbal account of the theorem.

---

*The containment is perfect when $L^* < \bar{N}/3$, or when $L^* > \bar{N}/3$, and Assumption 4 holds as the theorem says; it is effective, but imperfect, in some cases with $L^* > \bar{N}/3$, but without Assumption 4.*
II-C. An Intuitive Account

A thorough and rigorous account of the theorem in an intuitive and verbal manner could not be given easily. In this account, I confine my attention to the case in which each union has $\bar{N}/4$ junior workers and $\bar{N}/4$ senior workers. With respect to this case, I will explain why unemployment results even in face of the rational behavior of unemployed workers. I restrict my account to this case in the hope that doing so gives a concise and illuminating account of the theorem and economic mechanisms under seniority rationing.

Let me first show that seniority rationing gives rise to a sufficiently high reservation wage of unemployed workers. For the simple reason that present senior workers retire in the future, present zero year old junior workers are certain that they will be senior workers in the future, and that they will, under the seniority rule, be employed by their presently associated firm two periods later (when they become two years old). Consider a one year old worker at period $2k + 1$ whom firm $b$ did not employ in period $2k$ when he was zero years old. He may go to firm $a$ for a job period $2k + 1$ as an outsider, and underbid the union $a$'s wage. If he thus succeeds in being employed by firm $a$ at age one (period $2k + 1$), he must work for that firm also at age two (period $2k + 2$), as the employment spell of firm $a$ is two periods long. Then, he will not be able to come back to firm $b$ period $2k + 2$ and to work for that firm in period $2k + 2$ and $2k + 3$ (at age two and three) as a senior worker. This means that he must forego the wage from future employment at firm $b$ guaranteed by seniority, once he is employed at age one by firm $a$.

---

5Seniority of workers and their ages are naturally correlated with each other in a higher degree.

6As noted earlier, we assume the future wage level is correctly forecast.
Therefore, the future wage at firm b plus the negligible moving cost becomes the reservation wage for starting to work for firm a at age one. Despite the small moving cost, no unemployment compensation, and no utility from leisure, future income implicitly guaranteed by the seniority rule creates a sufficiently high reservation wage. With the negligible moving costs, the guaranteed future income almost entirely accounts for the reservation wage.

The reservation wage is much smaller in the absence of seniority rationing, in which case the future employment and thus income from firm b are not guaranteed. I will discuss this later in more detail.

Next, consider the firm's employment policy. Under seniority rationing, a firm must give employment first to senior workers, but a firm treats junior workers and outsiders equally. Then, a firm can employ outsiders only when the wage rate for outsiders is lower than, or equal to, the union wage rate which applies to junior workers. On the other hand, by the definition of the reservation wage, the wage rate for outsiders paid by firm a (b) must be above the reservation wage of unemployed members of union b (a) if the unemployed members are to work as outsiders for firm a (b). It follows that, if a union sets its wage rate below the reservation wage of unemployed members of the other union, the union can expect that its firm does not employ outsiders, and need not fear the underbidding of wages by outsiders. The union can expect all jobs at its firm to go to its members.

Let me summarize the preceding argument by using some symbols for the convenience of subsequent arguments. Let $W_a$, $W_b$, and $t$ be the union a wage rate, the union b wage rate, and the moving cost. As I consider the stationary state here, period indices are unnecessary. When $P = 1$, for the reason stated, $W_b + t$ becomes the reservation wage of unemployed union b workers in the stationary state. If $W_a \leq W_b + t$, union a expects the level of
employment for its members to be \( L(x) \) as long as \( L(W_a) \) does not exceed the union's membership (\( \bar{N}/2 \) in the present account). Therefore, the union expects the value of its objective function to be \( U(W_a)L(W_a) \).

In subsequent accounts, I concentrate on the most interesting case of \( \bar{N}/4 < L^* < \bar{N}/2 \), namely, \( W^1 < W^* < W^0 \). Let me study the union a response to the union b's wage policy.

Consider first the case with \( W^* \leq W_b + t \). If union a does not set its wage above \( W_b + t \), union a need not fear firm a hiring outsiders from union b at a low wage rate, and the union expects the value of the union a objective function to become \( U(W_a)L(W_a) \). Hence, as \( U(x)L(x) \) reaches its maximum at \( x = W^* \), union a sets its wage rate at \( W^* \) if it sets its wage below \( W_b + t \).

If union a sets its wage above \( W_b + t \) when \( W^* \leq W_b + t \), the union a wage becomes higher than the reservation wage of unemployed members of union b, and hence higher than the wage for outsiders to union a. Then, only senior workers of union a can be employed out of union a members. As the number of senior workers are \( \bar{N}/4 \), employment for the union a members is no more than \( \bar{N}/4 \). On the other hand, by the union democracy, union a with \( \bar{N}/2 \) members must assure, at least, \( \bar{N}/4 \) employment for its members. Accordingly, the union sets its wage low enough to have all its senior workers employed. It follows that, if union a sets its wage above \( W_b + t \), the value of its objective function is \( U(W_a)\bar{N}/4 \). Suppose \( W_a \leq W^0 \). By Assumption 1, then, \( U(W_a)\bar{N}/4 \leq U(W^0)L(W^0) \) \( < U(W^*)L(W^*) \). Then, union a sets its wage rate at \( W^* \) (\( > W^0 \)) rather than \( W_a \). This is a contradiction. Therefore, if union a sets its wage above \( W_b + t \), \( W_a > W^0 \) must hold.

\(^7\) See Section III for other cases.
Suppose \( W_b + t \geq W^0 \) when \( W^* \leq W_b + t < W_a \). Then, the reservation wage of unemployed union b workers and hence the outsider's wage rate for union a is above \( W^0 \). \( W_a \) has been seen above \( W^0 \). With both the union wage rate and the outsider's wage rate for firm a being higher than \( W^0 \), firm a employs less than \( \bar{N}/4 \) workers, especially, less than half of union a members. This contradicts the union democracy. Therefore, if \( W_a > W_b + t > W^*, W^0 > W_b + t \).

Consider next the case with \( W_b + t < W^* \). In this case, if union a does not set its wage above \( W_b + t \), the value of its objective function is 
\[ U(W_a)L(W_a) \]  
It follows that, because of Assumption 1 and \( W_b + t < W^* \), if union a does not set its wage rate above \( W_b + t \), it sets its wage rate at \( W_b + t \). Therefore, if \( W_b + t \leq W^* \), \( W_b + t \leq W_a \) holds.

Figure 2 summarizes the preceding discussions of union a responses to the union b wage policy. For \( W_b < W^* - t \), \( W_a \) must be above segment \( A_1A_2 \); for \( W^* - t \leq W_b < W^0 - t \), \( W_a \) must be above the segment \( A_3A_4 \) or be equal to \( W^* \); for \( W^0 - t \leq W_b \), \( W_a \) must be equal to \( W^* \). Similarly, one can draw the response of union b to the union a wage policy in Figure 2. The figure shows that either \( W_a = W_b = W^* \), or, without loss of generality, \( W_a > W^0 \) and \( W_b = W^* \) holds at the stationary state equilibrium. If the former holds, \( E_i = L^* \) and \( W_i = W^* \) for \( i = a, b \), and so \( E = 2L^* \); and unemployment results in the amount of \( \bar{N} - 2L^* \). It must be emphasized that, with \( W_a = W_b \) and thus \( W_a < W_b + t \) (\( W_b < W_a + t \)), the outsider's wage rate for firm a (b) does not fall below \( W_a (W_b) \), and thus no unemployed workers of union b (a) can be elicited to become outsiders to union a (b).

The latter case of \( W_a > W^0 \) and \( W_b = W^* \) does not occur if \( L^* \leq \bar{N}/3 \) (see Proposition 5). Even if \( L^* > \bar{N}/3 \), the case does not occur if Assumption 4 holds. For, this assumption means, as was already mentioned, that, with
W_b = W*, it is not in the interest of union a to set its wage rate so as to allow outsiders from union b to be employed by firm a.

Even if the latter case of W_a > W^0 and W_b = W* holds, full employment does not realize. With W_a > W_b + t in this case, the wage rate of union a is higher than the reservation wage of unemployed workers of union b, and hence the outsider's wage for firm a. Therefore, firm a employs only senior workers out of union a members. Other workers employed by firm a are outsiders. Junior members of union a are not employed by firm a. The fact that W_a > W_b + t also means that the reservation wage of these unemployed union a members (W_a + t) exceeds the union b wage rate, and thus the outsider's wage rate payable by firm b. Then, the unemployed union a members do not become outsiders to union b. They stay unemployed. Thus, unemployment results in the amount of (\tilde{N} - L*)/2 in the latter case as well as in the former case.

The cause of unemployment even in face of the rational behavior of unemployed workers can be briefly expounded as follows. Guaranteeing future employment and hence, in effect, future income to unemployed workers, seniority rationing makes the wage rate of union b plus the moving costs the reservation wage of unemployed union b members for working at firm a. Firm a employs outsiders from union b only when their wage rate is lower than the union a wage rate. Then, as long as union a sets its wage rate below W_b + t, it need not consider the threat from outsiders and fear underbidding of wages by them. This latitude in the wage setting enables union a to set its wage high enough to create unemployment without causing unemployed workers to exercise downward pressure on wages.

---

\(^8\)See Remark 2 after Proposition 6 in Section III.
Even if it is not in the interest of unions to take advantage of this latitude, unemployment still results. For, union a's setting its union wage above $W_b + t$ forces firm a not to employ its zero year old junior members, and, at the same time, makes the reservation wage of the junior members higher than the wage firm b can pay to them as outsiders. Hence, the junior workers stay unemployed without exercising downward pressure on wages.

If workers observe the wage contract they ratified, the only way that unemployed members of a union can get themselves employed is to become outsiders to another union, or, though not formally analyzed for the sake of simplicity, to join another union. Given seniority, however, to do so is not to the advantage of at least some of the unemployed workers. Given seniority, therefore, there is no way for unemployed workers to get hired on their own initiative. Unemployment results without creating downward pressure on wages.

It must be noticed as a corollary of the above conclusion that unemployed workers and their presently associated firm must disregard the ratified wage contract and the seniority rule if employment is to increase without a positive shift of demand for labor. Consider that, instead of becoming outsiders, unemployed workers propose a wage rate lower than the union wage rate to the firm with which they are presently associated, and try to replace senior workers of their union by themselves as employees of the firm. If the firm accepts this offer, senior workers are also compelled to offer wage cuts. Thus, employment should increase. In this circumstance, the unemployed workers and the firm defy the wage contract and the seniority rule. Reasons for workers' and firms' observance of the contract and the seniority rule in face of unemployment might be, to cite casually, their desire to maintain their reputation as promise-keeping trade partners to the third parties, or to make future jobs stable. However, to explain the reasons requires a more
complicated model, and is out of the scope of this paper. I take the
observance for granted in this paper.

One can argue that a reduction in the workers' turnover rate is the key
in our mechanism of creating unemployment under seniority rationing. It is
known since Oi (1962) that moving costs associated with location changes and
specific skills reduce the turnover rate of workers. However, a decrease in
the turnover rate in our model is due not to the moving cost which is assumed
negligible, but to social arrangements of seniority. The moving cost in our
model is low enough not to prevent unemployed workers from working for another
firm; that is, by Assumption 2, $\tilde{W}_t - t$ are always positive. As already noted,
to penalize an unemployed worker for looking for a job at another firm by
depriving seniority of that worker is excluded in this paper, and is not
responsible for a decrease in the turnover rate in this paper.

An unemployed worker always gains from moving to another firm to work
there in our model. However, the gain from doing so is outweighed by the gain
from staying unemployed on account of seniority. This reduces the turnover
rate in our model.

II-D. The Lottery Rationing

A brief examination of a case without seniority rationing, but with
lottery rationing helps illuminate the significance of seniority rationing in
creating unemployment and in containing the downward pressure on wages from
unemployed workers. Consider the stationary state in which each union has $\tilde{N}/2$
workers. There is no distinction between senior and junior workers. For an
unemployed one year-old worker of union $b$ at period $2k + 1$, $2U(\tilde{W}_b)L(\tilde{W}_b)/(\tilde{N}/2)$
is the expected utility from staying in the labor pool of firm $b$ in periods
2k + 2 and 2k + 3. Note that \( L(W_b)/(\bar{N}/2) \) is the rate (probability) of employment at firm b in the periods. I also assume \( \beta = 1 \) here. Let \( \tilde{W}_b \) be defined by \( U(\tilde{W}_b) = 2U(W_b)L(W_b)/\bar{N} \). \( \tilde{W}_b + t \) is the reservation wage of one year old unemployed workers for working for firm a. This reservation wage is much lower than that in the presence of the seniority rationing even if the same wage rate of \( W_b \) is anticipated in the presence and absence of seniority rationing.

If union a sets its wage below \( \tilde{W}_b + t \), no outsiders from union b are employed, and so employment for union a members is \( L(W_a) \) as long as \( L(W_a) \) does not exceed \( \bar{N}/2 \). If union a sets its wage above \( W_b + t \), no union a member is employed by firm a. Hence, by Assumption 1, \( W_a = \max \{ W^1, \min \{ \tilde{W}_b + t, W^* \} \} \) must hold. Similarly, \( W_b = \max \{ W^1, \min \{ \tilde{W}_a + t, W^* \} \} \). The solution of these two equations is given by the solution of \( U(W - t) = 2U(W)L(W)/\bar{N} \) for \( i = a,b \). One can easily see that, if \( t > 0 \), \( W = W^1 \), and thus \( E > \bar{N} \). That is, without seniority, unemployment is negligible under the negligible moving costs. By contrast, we have found that unemployment is substantial even with the negligible moving costs if unemployment is rationed by seniority.

II-E. Keynesian Theories

As Friedman's criticism (1976) implies, economic theories in the Keynesian tradition do not take account of individual rationality on the part of unemployed workers. They cannot be complete without a specification of a rationing scheme of unemployment that would effectively diffuse downward pressure on wages that individual rationality of unemployed workers implies. Rationing by (subjective) constraints in disequilibrium analysis, such as Grandmont and Laroque (1976) and Malinvaud (1977), does not solve the problem,
because the analysis does not explain a mechanism by which subjective
constraints of different agents can be so nicely coordinated with each other
as to eliminate excess demand and supply. The analysis shows a logically
possible rationing scheme of unemployment, but there is no reason to believe
that the scheme is in practice.

The model in this paper takes account of individual rationality on the
part of unemployed workers, and yet shows that unemployment results. The
model demonstrates that seniority rationing of unemployment diffuses the
downward pressure on wages from unemployed workers effectively enough to
maintain unemployment. The model also shows that the labor market with
seniority rationing allows an economy to function in such a manner as to
accommodate the Keynesian view of macroeconomy. \( L^* \) may be interpreted as an
index of the strength of labor demand. If firms compete with each other
monopolistically, firms' demand for labor should depend upon the level of
aggregate demand for products. Then, the model implies that an increase in
aggregate demand would bring about a decrease in unemployment, justifying the
Keynesian scenario.

It must be emphasized that unemployment in the model of this paper is
involuntary. There is neither utility from leisure, nor unemployment
compensation in the model. Working always dominates leisure in workers'.preference. Accordingly, unemployed workers willingly work at the current
union wage rate if jobs are offered.

Some might argue that unemployment in the seniority rationing model is
yet voluntary unemployment. First, it is voluntary because the union which
represents unemployed workers causes unemployment. Second, the reason that
workers accept present unemployment in the current labor pools, and do not try
to find jobs outside the current labor pools, is because they choose future
employment in the current pools rather than present employment elsewhere. This is not a much different picture from that of the intertemporal substitution models which take the workers' choice of present leisure and future work in preference to present work and future leisure as the cause of a smaller labor input.

Both of these arguments are not appropriate. The first argument does not differentiate between a collective choice and an individual choice. In compliance with the common usage of the term "voluntary," it is from the viewpoint of an individual choice that this paper distinguishes between voluntary and involuntary unemployment. The second argument is also mistaken. In intertemporal substitution models, as an extension of Walrasian theory, working at the going wage rate at present decreases utility of a worker who does not work at present. However, in the seniority rationing model, working at the going union wage rate, if possible, always improves the welfare of unemployed workers. Further, a choice unemployed workers face is not a choice between present work and future work. It is a choice between jobs that will occupy the same period. If a worker born in period 2k is employed by firm a in period 2k + 1, he must work for the firm periods 2k + 1 and 2k + 2. If he is not employed by firm a, he will work for firm b periods 2k + 2 and 2k + 3 under seniority rationing. The employment spells overlap each other. A worker must choose one of them, as he cannot work for multiple firms at one time. This is the choice an unemployed worker faces in the seniority rationing model.
III. A Proof of the Theorem

Lemma 1 For \( i = a, b, Y_i = 0 \) means \( L(W_i) = E_{i2} + E_{i0} = E_{i2} + \theta_i P_{ij} \).

Proof By (12), \( Y_i = 0 \) implies \( E_{iy} = 0 \).

Suppose \( \bar{W}_i \geq W_i \). By (6), (7), (8), and (9), \( L(W_i) = E_{i2} + E_{i0} + E_{iy} = E_{i2} + E_{i0} = E_{i2} + \theta_i P_{ij} \). Next, suppose \( \bar{W}_i < W_i \). If \( R_{iy} > R_{is} \), by (5), \( E_{iy} > 0 \). A contradiction. If \( R_{ay} < R_{as} \), by (4), \( L(W_i) = E_{i2} \). If \( R_{ay} = R_{as} \), \( L(W_i) = E_{i2} \) by (4), (5), and \( E_{iy} = 0 \). In view of (11), then, \( L(W_i) = E_{i2} + \theta_i P_{ij} \) also for \( \bar{W}_i < W_i \). (Q.E.D.)

Lemma 2 \( Y_a > 0 \) implies \( W_a > \bar{W}_a \), \( Y_b = 0 \), \( P_{as} = \bar{N}/4 \), \( P_{bs} = \bar{N}/4 - Y_a \), and \( \bar{N}/4 < L(\bar{W}_a) \leq \bar{N}/3 < L(W_b) = \bar{N}/2 - 2Y_a < \bar{N}/2 \).

Proof When \( Y_a > 0 \), \( E_{ay} = Y_a > 0 \). (Otherwise, \( \bar{W}_a = 0 \) by (12). This together with (13) and \( W_b > 0 \) means \( Y_a = 0 \).) By (6), \( E_{ay} > 0 \) implies \( W_a \geq \bar{W}_a \).

Suppose \( W_a = \bar{W}_a \). Then, by (8) and \( Y_a = E_{ay}, \theta_a = 1 \). Therefore, by (8), \( E_{a0} = \bar{N}/4 \). By (13'), \( E_{a0} = \bar{N}/4 \) means \( Y_b = 0 \). Then, \( E_{b1} = 0 \) by (12'), and so \( P_{as} = \bar{N}/4 \). By (16), then, \( E_{a2} = \bar{N}/4 \). Therefore, by (1) and (9), \( L(W_a) = E_{a2} + \theta_a(Y_a + P_{aj}) = \bar{N}/2 + Y_a \).

On account of \( E_{ay} = Y_a \) and (10), \( E_{a1} = Y_a > 0 \), and so \( P_{bs} = \bar{N}/4 - Y_a \). By (14) and \( Y_a > 0 \), \( E_{b0} = \bar{N}/4 - Y_a \). It has been seen that \( Y_b = 0 \) under \( Y_a > 0 \) and \( W_a = \bar{W}_a \). By Lemma 1, \( Y_b = 0 \) implies \( L(W_b) = E_{b0} + E_{b2} = E_{b0} + P_{bs} = \bar{N}/2 - 2Y_a \).

Therefore, \( L(W_b) = 3\bar{N}/2 - 2L(\bar{W}_a) \). By (13) and \( \beta = 1 \), \( Y_a > 0 \) implies \( \bar{W}_a - t > W_b \), and so \( W_a = \bar{W}_a > W_b \). Hence, by \( L'(*) < 0 \), \( L(W_a) < 3\bar{N}/2 - 2L(\bar{W}_a) \). Therefore, \( L(W_a) < \bar{N}/2 \). This contradicts \( L(W_a) = \bar{N}/2 + Y_a > \bar{N}/2 \). Hence, \( Y_a > 0 \) must imply \( W_a > \bar{W}_a \).
On account of (13) and \( \beta = 1 \), \( W_a > \bar{W}_a \) and \( Y_a > 0 \) imply \( W_a - t > \bar{W}_a - t \) \( > W_b \), and hence \( W_a > W_b \). By a similar argument, \( Y_b > 0 \) implies \( W_b > W_a \).

Therefore, \( Y_b = 0 \) if \( Y_a > 0 \). Then, by (2), (10), and (12'), \( P_{as} = \bar{N}/4 \).

Further, \( P_{bs} = \bar{N}/4 - Y_a \) by (2'), (10'), and \( E_{ay} = Y_a \).

When \( W_a > \bar{W}_a \) and \( E_{ay} = Y_a > 0 \), \( L(\bar{W}_a) = P_{as} + E_{ay} = \bar{N}/4 + Y_a (> \bar{N}/4) \) on account of (4) and (5). On the other hand, by Lemma 1, \( Y_b = 0 \) implies \( L(W_b) = E_{b2} + E_{b0} = P_{bs} + (\bar{N}/4 - Y_a) = \bar{N}/2 - 2Y_a < \bar{N}/2 \). Then, \( 2L(\bar{W}_a) + L(W_b) = \bar{N} \). (13), \( Y_a > 0 \), and \( \beta = 1 \) imply \( U(\bar{W}_a - t) > U(W_b) \) and hence \( \bar{W}_a > W_b \). Therefore, \( L(\bar{W}_a) < \bar{N}/3 \) and \( L(W_b) > \bar{N}/3 \). (Q.E.D.)

Lemma 3 For \( i = a, b \), \( Y_i > 0 \) implies \( W^0 > \bar{W}_i \) and \( W_i = f(\bar{W}_i) > W^0 \).

Proof As was shown in the beginning of the proof of Lemma 2, \( Y_a > 0 \) means \( E_{ay} > 0 \). By Lemma 2, \( L(\bar{W}_a) > \bar{N}/4 \) and hence \( \bar{W}_a < W^0 \). This means \( f(\bar{W}_a) > W^0 \).

Suppose \( W_a > f(\bar{W}_a) (> W^0) \). By Lemma 2, \( P_{as} = \bar{N}/4 \). Then, the definition of \( f(\cdot) \) and the envelope theorem imply \( R_{ay} < R_{as} \), and so, by (4), \( E_{ay} = 0 \). A contradiction. Therefore, \( W_a \leq f(\bar{W}_a) \).

Suppose \( W_a < f(\bar{W}_a) \). In this case, the definition of \( f(\cdot) \) and the envelope theorem imply \( R_{ay} > R_{as} \). For the union a wage rate \( x \) slightly higher than \( W_a \), then, \( R_{ay} > R_{as} \) still holds. By \( \bar{W}_a < W_a < x \), it follows that, when \( x \) is the union a wage rate, union a expects, given \( \bar{W}_a \), that the level of employment for its members is \( \bar{N}/4 \), and so the value of the union a objective function is \( U(x)\bar{N}/4 \). With \( \bar{W}_a < W_a \), by (4) and (5), the value of the objective function at \( \bar{W}_a \) is \( U(\bar{W}_a)\bar{N}/4 \). By \( W_a < x \), \( U(W_a)\bar{N}/4 < U(x)\bar{N}/4 \). This contradicts the definition of \( W_a \), if the outsider's wage is kept constant when the union a

\[ \text{In the beginning of this proof, we have seen this.} \]
wage rate is changed from $W_a$ to $x$. Indeed, the outsider's wage rate for firm a is unchanged in that event, since, being the solution of $R'(N/4 + n) = \tilde{W}_a$, firm a's demand for outsiders at $\tilde{W}_a$ is kept constant by the rise in the union a wage rate from $W_a$ to $x$, and so is the outsider's wage rate.

Accordingly, $W_a = f(\tilde{W}_a)$. Similarly, $Y_b > 0$ implies $W_b = f(\tilde{W}_b)$. (Q.E.D.)

**Lemma 4** Let the union a wage rate be $x$. Given $x$, if union b sets its wage rate at $y$ which is lower than or equal to $x + t$, the union can expect that firm b employs no outsider, that the level of employment for its members is $L(y)$, and that the value of its objective function is $U(y)L(y)$. The same lemma holds when indices a and b are interchanged.

**Proof** Suppose firm b employs outsiders when union b sets its wage rate at $y$ ($\leq x + t$). Let $\tilde{y}$ denote the wage rate of outsiders to union b. Then, by (6), $y \geq \tilde{y}$. However, if $\tilde{y} \leq y$, $\tilde{y} - t \leq x$. It follows from (13) and $\varphi = 1$ that no unemployed worker of union a becomes an outsider of union b. Hence, if union b sets its wage rate at a level no higher than $x + t$, it expects no outsider to be employed by firm b, that, by Lemma 1, employment for its members should be $L(y)$, and that the value of its objective function should be $U(y)L(y)$. (Q.E.D.)

**Proposition 1** If $\bar{N}/4 \geq L^*$, $Y_i = 0$, $P_{is} = \bar{N}/4$, and $P_i = \bar{N}/2$ for $i = a, b$.

**Proof** Suppose $Y_a > 0$. By Lemma 2, $L(W^0) = \bar{N}/4 < L(W_b)$. Hence, $W^0 > W_b$. By the assumption of this proposition, $W^0 \geq \bar{W}$. Hence, $W^* \geq W^0 > W_b$.

By Assumption 1, $W^* \geq W^0 > W_b$ implies $U(W^0)L(W^0) = U(W^0)\bar{N}/4 > U(W_b)L(W_b)$.

By Lemma 2, $Y_a > 0$ implies $Y_b = 0$. By Lemma 1, $L(W_b) = E_{b2} + \theta_{b}P_{b1}$. 


Accordingly, \( U(W^0)L(W^0) > U(W_b)(E_{b2} + \theta_b P_{b3}) \). Further, by Lemma 2, 
\[ P_b = \bar{N}/2 - Y_a. \]
Then, by \( Y_a \leq \bar{N}/4, P_b/2 \leq L(W^0) = \bar{N}/4 \leq P_b \).

By Lemma 3, \( W_a > W^0 \). By Lemma 4, then, it follows from the above results that, given \( W_a \), union b expects the value of its objective function to increase from \( U(W_b)L(W_b) \) to \( U(W^0)L(W^0) \) if it sets its wage at \( W^0 \) rather than \( W_b \). This is a contradiction. Therefore, \( Y_a = 0 \). Similarly, \( Y_b = 0 \).
Together with (2) and (12), \( Y_i = 0 \) for \( i = a, b \) establishes Proposition 1. (Q.E.D.)

**Proposition 2** If \( \bar{N}/4 \geq L^*, E_i = \bar{N}/4, W_i = W^0 \), and \( \bar{W}_i \leq W^0 + t \) for \( i = a, b \), and \( E = \bar{N}/2 \).

**Proof** By Proposition 1, \( P_{iS} = P_{ij} = \bar{N}/4 \) and \( Y_i = 0 \) for \( i = a, b \). By Lemma 1, then, \( L(W_i) = E_{i2} + \theta_i P_{ij} \). By the constraints of (15) and (15'), this means \( \bar{N}/4 \leq L(W_i) \leq \bar{N}/2 \) and hence \( W^1 \leq W_i \leq W^0 \) for \( i = a, b \). It also means the value of the union i objective function is \( U(W_i)L(W_i) \).

By (4) through (9), if union a sets its wage rate at \( W^0 \), firm a provides \( \bar{N}/4 \) employment to union a members, and the value of the union a objective function becomes \( U(W^0)L(W^0) \), whether the outsider's wage is above or below \( W^0 \). If \( W_a < W^0 \), \( U(W_a)L(W_a) \) is smaller than \( U(W^0)L(W^0) \) on account of Assumption 1 and \( W^0 \leq W^* \) (or \( \bar{N}/4 \geq L^* \)). Therefore, \( W_a \geq W^0 \). On account of \( W_a \leq W^0, W_a = W^0 \). Similarly, \( W_b = W^0 \). Hence, \( E_i = \bar{N}/4 \) for \( i = a, b \) and \( E = \bar{N}/2 \).

By \( E_b = \bar{N}/4 \) and \( P_{bS} = \bar{N}/4, E_{b0} = 0 \) must hold. By (13), \( Y_a = 0 \), and \( \beta = 1 \), then, \( \bar{W}_a \leq W_b + t = W^0 + t \). (Q.E.D.)

**Proposition 3** If \( L^* \geq \bar{N}/2, Y_i = 0, P_{iS} = \bar{N}/4, \) and \( P_i = \bar{N}/2 \) for \( i = a, b \).
Proof Suppose $Y_a > 0$. By Lemma 2, $L(W_b) = \tilde{N}/2 - 2Y_a > \tilde{N}/4$ and $P_b = \tilde{N}/2 - Y_a$. Let $x$ be defined by $L(x) = P_b$. By $Y_a > 0$, $\tilde{N}/2 > L(x) > L(W_b) > \tilde{N}/4$, and so $W^1 < x < W_b < W^0$. By the assumption of $L^* > \tilde{N}/2$, $W^* \leq W^1$. By Lemma 3, $W_a > W^0$. Hence, $W^* \leq W^1 < x < W_b < W^0 < W_a$. By Assumption 1, this inequality implies $U(x)L(x) > U(W_b)L(W_b)$.

By Lemma 2, $Y_b = 0$. By Lemma 1, then, $L(W_b) = E_{b_2} + \theta_b P_{b_2}$. Therefore, $U(x)L(x) > U(W_b)(E_{b_2} + \theta_b P_{b_2})$. Further, by definition, $P_{b/2} \leq L(x) \leq P_b$.

By Lemma 4, it follows from $x < W_a$ that, if union $b$ sets its wage at $x$, given $W_a$, it expects employment for its members to be $L(x)$, and the value of its objective function to be $U(x)L(x)$. Hence, given $W_a$, union $b$ sets its wage at $x$ rather than $W_b$. This is a contradiction. Therefore, $Y_a = 0$. Similarly, $Y_b = 0$. $Y_i = 0$ for $i = a, b$ together with (2) and (12) establishes Proposition 3. (Q.E.D.)

Proposition 4 If $L^* > \tilde{N}/2$, $E_i = \tilde{N}/2$, $W_i = W^1$ and $W^1 \leq \bar{W}_i$ for $i = a, b$, $E = \bar{N}$.

Proof By Proposition 3, $P_i = \tilde{N}/2$ and $Y_i = 0$. By Lemma 1, then, $L(W_i) = E_{i_2} + \theta_i P_{i_2}$. By the constraints of (15) and (15'), therefore, $\tilde{N}/4 \leq L(W_i) \leq \tilde{N}/2$, and hence $W^1 \leq W_i \leq W^0$ for $i = a, b$. Further, the value of the union $i$ objective function is $U(W_i)L(W_i)$.

By the assumption of this proposition and Assumption 1, if $W^1 < W_a$, $U(W^1)L(W^1) > U(W_a)L(W_a)$. It follows from Lemma 4 that, given $W^1 \leq W_b$, union $a$ can expect that it can make its objective function equal to $U(W^1)L(W^1)$ by setting its wage at $W^1$. Therefore, $W^1 \geq W_a$. Accordingly, together with $W^1 \leq W_a$, $W^1 = W_a$. Similarly, $W_b = W^1$. Therefore, $E_i = \tilde{N}/2$ for $i = a, b$ and $E = \bar{N}/2$. 


In the present case, \( N/2 = L(W_a) = E_{a2} + \theta_a P_{aj} \) and \( P_{as} = \bar{N}/4 \).

By (1) and (16), \( \theta_a \bar{N}/4 = \bar{N}/4 \), and hence \( \theta_a = 1 \). Then, by (11), \( W^1 = W_a \leq \bar{W}_a \).

Similarly, \( W^1 \leq \bar{W}_b \). (Q.E.D.)

**Proposition 5** If \( \bar{N}/4 < L^* < \bar{N}/2 \), \( Y_i = 0 \), \( P_{iS} = \bar{N}/4 \), and \( P_i = \bar{N}/2 \) for \( i = a, b \).

**Proof** Suppose \( Y_a > 0 \). By Lemma 2, \( \bar{N}/4 < L(W_b) \) and hence \( W^0 > \bar{W}_b \). By Lemma 3, \( W_a > W^0 \). Therefore, \( W_a > W^0 > \bar{W}_b \). By the assumption of this proposition, \( W^0 > W^* \). By Lemma 2, \( Y_b = 0 \). Then, by Lemma 1, \( L(W_b) = E_{b2} + \theta_b P_{bj} \), and so the value of the union b objective function is \( U(W_b)L(W_b) \).

Suppose \( P_b/2 \leq L(W^*) \leq P_b \). By Lemma 4 and \( W_a > W^0 > W^* \), union b expects the level of employment to be \( L(W^*) \), and the level of its objective function to become \( U(W^*)L(W^*) \) if it sets its wage rate at \( W^* \). On account of Assumption 1, \( U(W^*)L(W^*) > U(W_b)L(W_b) \) if \( W^* = \bar{W}_b \). Hence, \( W^* = \bar{W}_b \), or \( P_b \leq L(W^*) \), or \( L(W^*) \leq P_b/2 \) holds.

Suppose \( P_b \leq L(W^*) \) holds. Let \( x \) be defined by \( L(x) = P_b \). In view of Lemma 2, \( \bar{N}/4 < L(W_b) = \bar{N}/2 - 2Y_a < P_b = L(x) < L(W^*) \). Hence, \( W^0 > \bar{W}_b > x > W^* \). By Assumption 1, \( U(x)L(x) > U(W_b)L(W_b) \). By \( W_a > W^0 > x \), it follows from Lemma 4 that union b expects setting its wage rate at \( x \), given \( W_a \), makes the value of its objective function \( U(x)L(x) \). This is a contradiction to the definition of \( \bar{W}_b \).

Suppose next \( L(W^*) \leq P_b/2 \). By the assumption of the present proposition and Lemma 2, \( \bar{N}/4 < L(W^*) \leq P_b/2 = (\bar{N}/2 - Y_a)/2 < \bar{N}/4 \). This is a contradiction. Accordingly, \( Y_a > 0 \) implies \( W_b = W^* \), and so \( L(W_b) = L^* \).

By Lemma 2, then, if \( L^* \leq \bar{N}/3 \), \( W_b = W^* \) cannot occur, and hence, \( Y_a = 0 \). This establishes the proposition for \( \bar{N}/4 < L^* \leq \bar{N}/3 \).
Consider the case with \( N/3 < L^* \). By Lemma 2, \( Y_a > 0 \) implies \( W_a > \bar{W}_a \).

By (4) and (5), only senior workers are employed out of union a members. By (16) and \( P_{as} = \tilde{N}/4 \), \( \bar{N}/4 \) union a members are employed by firm a. Hence, the value of the union a objective function is \( U(W_a)\bar{N}/4 \). By Lemma 3, \( W_a = f(\bar{W}_a) \).

Therefore, \( U(W_a)\bar{N}/4 = v(\bar{W}_a)\bar{N}/4 \).

By Lemma 2 and \( W_b = W^* \), \( \bar{N}/4 < L(\bar{W}_a) < L(W^*) \) and so \( W^0 > \bar{W}_a > W^* \). Then, in view of \( f'(x) < 0 \) for \( x < W^0 \) and Assumption 4, \( U(W_a)\bar{N}/4 = v(\bar{W}_a)\bar{N}/4 < v(W^*)\bar{N}/4 < U(W^*)L(W^*) \).

Besides, \( P_{a/2} = \bar{N}/4 < L(W^*) < \bar{N}/2 = P_a \) by the assumption of the present proposition and Lemma 2. Accordingly, it follows from Lemma 4 that, given \( W_b = W^* \), union a expects it can increase the value of its objective function by setting its wage rate at \( W^* \) instead of \( W_a \). This is a contradiction. Therefore, when \( N/3 < L^* \), \( Y_a = 0 \). This establishes the proposition. (Q.E.D.)

Proposition 6 If \( \bar{N}/4 < L^* < \bar{N}/2 \), \( W_1 = W^* \), \( E_i = L^* \), \( W^* \leq \bar{W}_1 \leq W^* + t \) for \( i = a, b \), and \( E = 2L^* \).

Proof By Proposition 5, \( P_{is} = \bar{N}/4 \) and \( P_i = \bar{N}/2 \). By Proposition 5, \( Y_i = 0 \).

By Lemma 1, then, \( L(W_i) = E_{i2} \theta_{i1} \).

Hence, by the constraints of (15) and (15'), \( \bar{N}/4 \leq L(W_i) \leq \bar{N}/2 \), namely, \( W^1 \leq W_i \leq W^0 \), and the value of the union i objective function is \( U(W_i)L(W_i) \).

Suppose \( W_a > W^* \). It follows from Lemma 4 and \( \bar{N}/4 = P_{b/2} \leq L(W^*) \leq P_b = \bar{N}/2 \) that, given \( W_a \), union b expects it can make the value of its objective function equal to \( U(W^*)L(W^*) \) by setting its wage rate at \( W^* \). It has been seen that the value of the union b objective function is \( U(W_b)L(W_b) \). It follows from Assumption 1 that this value of the union b objective function is smaller than \( U(W^*)L(W^*) \) if \( W_b = W^* \). Therefore, \( W_b = W^* \) must hold. By the same argument, \( W_b = W^* \) implies \( W_a = W^* \).
Suppose \( W_a < W^* \). If \( W_a < W_b + t \), by Lemma 4 and Assumption 1, union a expects setting its wage rate at \( \min \{ W_b + t, W^* \} \) rather than \( W_a \) increases the value of the union's objective function. Hence, \( W_b + t \leq W_a < W^* \). Then, \( W_b < W^* \). Similarly, then, \( W_a + t \leq W_b < W^* \). Therefore, \( W_a + 2t \leq W_a \). This is a contradiction on account of \( t > 0 \).

Accordingly, \( W_i = W^* \), \( E_i = L^* \) for \( i = a, b \), and \( E = 2L^* \). One can prove \( W^* \leq \tilde{W}_i \leq W^* + t \) similarly to \( \tilde{W}_i \leq W^0 + t \) and \( W^1 \leq \tilde{W}_i \) in Propositions 2 and 4.

(Q.E.D.)

Propositions 2, 4, and 6 complete the proof of the Theorem in Section II.

Remark 1 If \( t = 0 \), only \( W^1 < W_a = W_b \leq W^* \) holds rather than Proposition 5. Equilibrium under \( \bar{N}/4 < L^* < \bar{N}/2 \) becomes indeterminate when \( t = 0 \). The other propositions hold even when \( t = 0 \). Especially, Propositions 1, 3, and 5 hold; that is, the moving cost is not responsible for unemployed workers not seeking to be employed by another firm.

Remark 2 Without Assumption 4, \( Y_a = 0 \) does not necessarily hold for \( \bar{N}/3 < L^* < \bar{N}/2 \). By the proof of Lemma 3, \( 2L(\tilde{W}_a) + L(W_b) = \bar{N} \) holds when \( Y_a > 0 \). Further, from the proof of Proposition 5, \( L(W_b) = L^* \) holds in that event. Hence, \( E = L^* + (\bar{N} - L^*)/2 = (\bar{N} + L^*)/2 \) if Proposition 5 does not hold. \((\bar{N} - L^*)/2\) unemployment results.
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FIGURE 1