THE THEORY OF EFFECTIVE PROTECTION
AND RESOURCE ALLOCATION*

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*This paper grew out of an attempt at a systematic evaluation of the theory of effective protection in light of the earlier work of Ramaswami and Srinivasan (1969) and it dedicated to the memory of the late Ramaswami. The research of the former author has been supported by the National Science Foundation.
The theory of effective protection has recently been evolved to develop a concept of protection which, in the presence of traded factors of production, would be able to answer the same set of questions that nominal tariffs on commodities do in their absence.

With tariffs on inputs and outputs clearly affecting the profitability of, and hence the resource utilisation in, alternative processes in an economy, the concept of the effective rate of protection (ERP) has come to be applied to processes of production, or more precisely to the domestic value added in these processes. This reflects, in turn, the basic concern of trade theorists in the resource-allocational impact of tariff structures.

This paper examines the ERP concept critically and derives a set of sufficient conditions under which it correctly predicts the resource-allocational impact of a tariff structure. In light of this analysis, it also pinpoints the rationale underlying the Ramaswami-Srinivasan "impossibility theorem" (1971) which states that there is in general no ERP index that will predict resource flows once substitution between domestic and imported inputs is admitted.

In a companion paper, we examine the validity of the ERP concept under the assumption of monopoly power in trade and also explore further the Ramaswami-Srinivasan model to establish parametric conditions under which the ERP concept could successfully predict resource allocation. We also examine there the recent claims that the ERP-rankings of activities are tantamount to a chain of 'comparative advantage.'
I: The ERP Concept

The concept of the effective protective rate (ERP) seems to have been independently evolved by Barber (1955) and by Travis (1952). It was, however, developed and put into effective circulation by several economists working independently of one another: Corden (1966) and Johnson (1965) in pioneering theoretical contributions, and Balassa (1965) in the context of empirical work on tariff measurement.

There has subsequently been a phenomenally rapid growth of analytical literature. The bulk of it has been in the mould of partial-equilibrium analysis as was indeed the case with the Corden, Johnson and Balassa papers (cf. Anderson and Naya, 1969; and Clark Leith, 1968). However, the more important work has attempted analysis of the problem, using general-equilibrium models, as is indeed the only correct approach: chief among the contributors in this area have been Tan (1970), Ramaswami and Srinivasan (1971), Corden (1969) and Ruffin (1969). At the same time, a flood of empirical literature has appeared, with calculations far outpacing the clarification and legitimation of the concept (cf. Balassa, 1965; Basevi, 1966, Lewis and Guisinger, 1968) and thus almost creating a vested interest against the conclusions of general-equilibrium analysis, to be discussed below, which are generally devastating to the ERP concept's analytical legitimacy and practical utility!

The ERP concept was developed presumably by analogy with the nominal tariff concept. Thus, if we define:

\[ V_j = \text{value added at domestic prices per unit output of } j \text{ in the process; and} \]

\[ V'_j = \text{value added at international prices per unit output of } j \text{ in the process}, \]
then the analogy with the nominal tariff suggests that we should define ERP in the j-process as:

\[
(1 + \text{ERP}) V_j'' = V_j
\]

\[
\therefore \text{ERP} = \frac{V_j'' - V_j}{V_j} \quad (1)
\]

so that it follows that the ERP measures the proportionate increment in value added between the free-trade and the tariff situation (the former implying \(V_j''\) and the latter, \(V_j\)).

The same formula can be rewritten in two alternative forms. Letting international unit prices be \(P_j^F\) and \(P_i^F\) for the jth output and the ith input respectively, and using the standard Leontief terminology to represent input-coefficients as \(a_{ij}\) (i = 1, ..., n), we can write:

\[
V_j'' = P_j^F - \sum_{i=1}^{n} P_i^F a_{ij}
\]

\[
V_j = P_j^F (1 + t_j) - \sum_{i=1}^{n} P_i^F (1 + t_i) a_{ij}
\]

\[
\text{ERP} = \frac{P_j^F t_j - \sum_{i=1}^{n} P_i^F a_{ij}}{P_j^F - \sum_{i=1}^{n} P_i^F a_{ij}}
\]

provided we make two critical assumptions: that the international prices are fixed (the small-country assumption) and that the \(a_{ij}\)'s are unchanging between the free-trade and the tariff situation. Then, assuming international prices to be unity of choice of units, we can rewrite (1) as follows:

\[
\text{ERP} = \frac{\sum_{i=1}^{n} a_{ij} t_i}{\sum_{i=1}^{n} a_{ij}}
\]

\[
1 - \sum_{i=1}^{n} a_{ij}
\]

\[
(2)
\]
This defines the ERP in terms of the tariff rates and the imported factor-coefficients. There also follow the well-known results (cf. Corden, 1966) that:

if \( t_j = t_i \), then \( ERP = t_j = t_i \);

if \( t_j > t_i \), then \( ERP > t_j > t_i \); and

if \( t_j < t_i \), then \( ERP < t_j < t_i \).

We could also rewrite the ERP-formula directly in terms of domestic factor payments:

\[
V_j = \sum_{i=1}^{n} r_i f_{ij}
\]

where \( i = 1, \ldots, n \) are the domestic factors employed in the \( j \)th process, \( f_{ij} \) are the domestic factor inputs-coefficients, and \( r_i \) is the rental of factor \( i \) at equilibrium factor prices in the free-trade situation;

\[
V_j = \sum_{i=1}^{n} r_i^d f_{ij}^d
\]

where \( r_i^d \) is the rental of factor \( i \) at equilibrium factor prices in the tariff situation and \( f_{ij}^d \) are the domestic factor input-coefficients in the tariff situation.

Then:

\[
ERP = \frac{\sum_{i=1}^{n} (r_i^d f_{ij}^d - r_i f_{ij})}{\sum_{i=1}^{m} r_i f_{ij}}
\]

As soon as formula (1) is thus translated into formulae (2) and (3), it becomes clear that whenever \( a_{ij} \)'s can change between the free-trade and the tariff situations, thanks to substitution possibilities, we can choose from (at least) three alternative ERP definitions. In terms of (1), these are defined as:
(i) "Free Trade" (Coefficients) ERP: In this case, the $a_{ij}$'s of the free-trade situation are used to evaluate both $V_j$ and $V'_j$: thus, in the tariff-situation, in evaluating $V_j$, we use the free-trade $a_{ij}$'s but tariff-inclusive domestic prices:

(ii) "Tariff" (Coefficients) ERP: In this case, the $a_{ij}$'s of the tariff situation are used to evaluate both $V_j$ and $V'_j$: thus, in the free-trade situation, in evaluating $V'_j$, we use the free-trade prices but the tariff-situation coefficients; and

(iii) "True" ERP: In this case, suggested by Naya and Anderson (1969), the $a_{ij}$'s of both situations are used: the tariff-situation $a_{ij}$'s for evaluating $V_j$ and the free-trade $a_{ij}$'s for evaluating $V'_j$.

It may be re-emphasised that all three definitions yield identical results when the $a_{ij}$'s are invariant.

In this context, it is useful to note that the ERP definitions (i) and (ii) are only approximations to the "true" definition (iii) and are to be discussed only insofar as the "true" definition is theoretically proper but empirically unuseable.* In addition to these three "traditional" ERP definitions, we will introduce later in our analysis another, "new" definition based on the "price of value-added", due to Corden (1969).

*Recall, for example, the problem in consumer theory of ranking the commodity bundles bought by a consumer in two periods according to his preferences when both the relative prices and total expenditures differ between periods. This can be done when two measures of real expenditure of one of the periods (obtained by deflating the money expenditure by the Laspyere and Pasche index of prices of that period relative to the other period as base) both show a fall or rise as compared to the other period. Anderson and Naya (1969) have a useful, partial-equilibrium analysis of the ERP problem in this context.
II: **The Objective and Structure of the Analysis with the ERP Concept**

The formulation of the ERP concept, in either formula (1) or formula (3), is in terms of the increment in value added which characterises the tariff-situation *vis-à-vis* the free-trade situation.

That the activity which is characterised by a higher (proportionate) increment in value added, and hence by a higher ERP, should also have attracted greater domestic resources seems intuitively plausible: and hence the analysis has been centered on this question.

However, resource-allocation may be construed in two different ways, both of which are considered in this paper. We may ask:

(i) whether the gross-output changes move in the same direction as ERP ranking: i.e. whether \((x/y)\) increases, for example, in the tariff-situation if \(ERP_x > ERP_y\); and

(ii) whether the domestic factors move between activities in the same direction as ERP rankings; i.e. whether \(K_x^x\) and \(L_x^x\) increase, and \(K_y^y\) and \(L_y^y\) decrease, for example, in the tariff-situation, if \(ERP_x > ERP_y\). **

We next note that, since tariffs can be imposed on both outputs and the traded factors/inputs, the analysis of whether ERP-rankings can predict resource-allocation (in either of the two senses distinguished above) can distinguish between two polar cases: (i) where there is an output-tariff; and (ii) where there is an input-tariff. ***

\* \(x\) and \(y\) denote the levels at which processes \(x\) and \(y\) are respectively operated. \(ERP_x\) and \(ERP_y\) represent the effective protective rates on the processes \(x\) and \(y\) respectively.

\**\(K\) and \(L\) are the given, domestic factor supplies; the suffixes indicate the employment of these factors in processes \(x\) and \(y\).

\**\(Tan\) (1970) uses output-tariffs in his analysis whereas Ramaswami and Srinivasan (1970) take input-trade-subsidy in their analysis.
Our procedure in this paper will, therefore, be to analyse in a general-equilibrium framework the feasibility of predicting resource-allocational effects via ERP-ranking, using both the notions of resource-allocational effects distinguished above and also distinguishing between output-tariffs and input-tariffs.

In doing so, we use a succession of production models, while not specifying trade or consumption explicitly. Clearly this is enough since our purpose is merely to analyse the resource-allocational impact of the tariff situation: and, with international prices given, the problem reduces merely to an examination of the production model chosen.

Further note that in the traditional production model of trade theory, non-traded domestic factors produce traded commodities; and gross-outputs are the same as value-added. However, in the ERP theory, we depart from the traditional model in one essential respect: one or more factors must be traded. Hence their prices are given but their quantities become dependent variables, in contrast to the traditional model where all factors have their quantities exogenously specified but their prices endogenously determined. Thus, all the production models which are used to explore the ERP theory must allow for one or more factors whose prices are exogenously fixed and whose quantities are dependent variables: and this would suffice to capture the essential feature of the ERP problem. Hence, the phrase "traded" factors can equally be changed to: "fixed-price, variable-quantity" factors in the models, denoting their coefficients per unit output as $a_{ij}$'s.

We also initially confine our analysis to production models where the primary domestic factors are directly employed in producing outputs which are traded, instead of producing intermediates which in turn are also
employed in producing these final outputs.* This also rules out models where a product is both a final-demand and an intermediate good.** Finally, we rule out the possibility of the "traded" factors being identical with "domestic" factors: the two are treated as exclusive categories.***

Note again that the analysis is conducted throughout on the assumption of given international prices.

*Thus we will not initially consider a model such as the following:

\[
\begin{align*}
X &= X(K_x, L_x, Q_x, M_x) ; 
K_Q + K_x + K_y &\leq K ; \\
Y &= Y(K_y, L_y, Q_y, M_y) ; 
L_Q + L_x + L_y &\leq L ; \\
\zeta &= \zeta_x + \zeta_y = \zeta(Q_Q, L_Q)
\end{align*}
\]

where \(X\), \(Y\), and \(\zeta\) are two final outputs and an intermediate respectively, produced by primary domestic factors, \(K\) and \(L\), and imported factor, \(M\).

**Such a model would be

\[
\begin{align*}
X &= X(K_x, L_x, Y_x) ; 
K_x + K_y &\leq K ; \\
Y &= Y(K_y, L_y, X_y) ; 
L_x + L_y &\leq L ; \\
X_n &= X - X_y \\
Y_n &= Y - Y_x
\end{align*}
\]

where \(X\) and \(Y\) are the net final-demand availabilities of commodities \(x\) and \(y\), after intermediate use (\(Y_x\) and \(X_y\)) has been accounted for.

***Hence, we cannot have a model such as the following:

\[
\begin{align*}
X &= X(K_x, L_x) \\
Y &= Y(K_y, L_y) \\
K_x + K_y &= K_d + K_f \\
L_x + L_y &= L_d + L_f
\end{align*}
\]

where \(K_d\) and \(L_d\) are the given domestic stocks of factors \(K\) and \(L\), and \(K_f\) and \(L_f\) are the quantities of imported (exported) factors.
III: Successive Models Sufficient for ERP Theory

We begin by formulating models which provide sufficient conditions for the ERP-ranking to predict resource-allocational effects. In the next section, however, we consider the Ramaswami-Srinivasan "impossibility" theorem on this question.

Model I: Traditional Model: Two Primary, Domestic Factors Producing Two Traded Goods

We begin with the traditional model, which is essentially a special case of the models we will consider presently, where $a_{ij}$'s = 0. It is therefore also a case where we can allow only for output-tariff changes: with imported factors/inputs at zero level, there are necessarily no input-tariffs to be investigated.

The traditional production model is basically the following:

$$X = X(K_x, L_x)$$  \hspace{1cm} (I.1)
$$Y = Y(K_y, L_y)$$  \hspace{1cm} (I.2)
$$K_x + K_y \leq \bar{K}$$  \hspace{1cm} (I.3)
$$L_x + L_y \leq \bar{L}$$  \hspace{1cm} (I.4)

The foreign commodity price ratio $P^f_x / P^f_y$ is specified exogenously and constant for this small country. Under free trade this will also equal the domestic price ratio $P^d_x / P^d_y$. Perfect competition, competitive pricing and the standard restrictions on production functions obtain.

National tariffs on commodities $x$ and $y$ ad valorem rates $t_x$ and $t_y$ will result in \[
\frac{P^d_x}{P^d_y} = \frac{P^f_x}{P^f_y} (1 + t_x) \frac{P^f_x}{P^f_y} (1 + t_y) \]

* We know then that $\frac{d(X/Y)}{d(P^d_x/P^d_y)} > 0$ at points of

*In this two-commodity model, it is clear that one of the two 'tariff' rates would have to be a trade subsidy because, with trade, one commodity must be exported and one imported. For simplicity, however, we talk of tariff rates in the text.
incomplete specialisation in production. We also know that \( \frac{d(L_x)}{d(P^d)} > 0 \) and \( \frac{d(K_x)}{d(P^d/y)} > 0 \). After-tariff \( \frac{P^d_x}{P^d_y} \) is greater than free-trade \( \frac{P^d_x}{P^d_y} \) if \( t_x > t_y \). If we can then show that the increment in \( \frac{P^d_x}{P^d_y} \) (i.e. \( t_x > t_y \)) also implies that \( ERP_x > ERP_y \), then ERP theory is clearly valid in this model.

This is indeed the case as:

Unit value added at free-trade prices = \( P_x'P_y \)

Unit value added at tariff-inclusive prices = \( P_x(1+t_x), P_y(1+t_y) \)

\[ ERP_x = \frac{P_x(1+t_x)-P_x}{P_x} = t_x; \text{ and} \]

\[ ERP_y = \frac{P_y(1+t_y)-P_y}{P_y} = t_y \]

\[ ERP_x > ERP_y \]

Hence, we have deduced that if \( ERP_x > ERP_y \), then \( d(K_x) > 0, d(L_x) > 0 \) and \( d(X/Y) > 0 \), so that the resource-allocational effects in both the ways defined earlier are predictable from ERP rankings in the traditional model. Note also that, in the absence of traded inputs, value added in each activity equals the value of output of that activity. Therefore, the units of \( x \) and \( y \) themselves serve as physical units of value added in their production.

Model II: Imported Input \( a_i \)'s Constant and Strictly Positive and Two Primary, Domestic Factors:

Let us now take a production model with primary, domestic factors with substitution among themselves and imported inputs that are used in fixed proportion to output:

\[ X = \min [X(K_x, L_x), A_{ix}/a_{ix}] \quad \text{(II.1)} \]

\[ Y = \min [Y(K_y, L_y), A_{iy}/a_{iy}] \quad \text{(II.2)} \]
where \( a_{ix} \) and \( a_{iy} \) are the fixed imported-input coefficients; \( K_x, K_y, L_x, L_y \) represent domestic factor inputs; and \( A_{ix}, A_{iy} \) represent imported inputs.

In this model, it is obvious that the price-ratio relevant for resource allocation is \( \frac{P^n_x}{P^n_y} \) where \( P^n_x, P^n_y \) are net realisation per unit of output of \( x \) and \( y \) respectively: i.e. \( P^n_x = P^d_x - a_{ix} P^d_x, P^n_y = P^d_y - a_{iy} P^d_y \) where \( P^d_x, P^d_y \) are domestic prices per unit of \( x \) and \( y \) respectively and \( P^d_{ix}, P^d_{iy} \) are domestic prices per unit of imported inputs used in the production of \( x \) and \( y \). It is also clear that:

\[
\frac{\frac{d}{d\left(\frac{P^n_x}{P^n_y}\right)}(X/Y)}{\frac{d}{d\left(\frac{P^n_x}{P^n_y}\right)}(P^n_x/p^n_y)} > 0, \quad \frac{\frac{dK_x}{d\left(\frac{P^n_x}{P^n_y}\right)}}{\frac{dL_x}{d\left(\frac{P^n_x}{P^n_y}\right)}} > 0 \quad \text{and} \quad \frac{\frac{dL_y}{d\left(\frac{P^n_x}{P^n_y}\right)}}{\frac{dL_y}{d\left(\frac{P^n_x}{P^n_y}\right)}} > 0.
\]

If we now show that, after tariff, \( \frac{P^n_x}{P^n_y} \) will be larger compared to its free trade value if and only if \( ERP_x > ERP_y \), we would have shown the predictive ability of ERP. This is indeed the case.

For,

\[
\left(\frac{P^n_x}{P^n_y}\right)_{\text{tariff}} > \left(\frac{P^n_x}{P^n_y}\right)_{\text{free-trade}}
\]

is equivalent to:

\[
\frac{\left(\frac{P^n_x}{P^n_y}\right)_{\text{tariff}}}{\left(\frac{P^n_x}{P^n_y}\right)_{\text{free-trade}}} > \frac{\left(\frac{P^n_y}{P^n_y}\right)_{\text{tariff}}}{\left(\frac{P^n_y}{P^n_y}\right)_{\text{free-trade}}}.
\]

The latter inequality, however, is equivalent to \( ERP_x > ERP_y \). This follows because \( \frac{P^n_x}{P^n_y} \) represents the domestic value added per unit level of process \( x(v) \) and hence \( \left(\frac{P^n_x}{P^n_y}\right)_{\text{after tariff}} - 1 \) equals \( ERP_x \).

*This model, and the validity of ERP theory in its framework, were formally developed earlier by Tan (1970) and are implicit in the work of Corden (1966).*
Note that we have incorporated the effects of output as well as input tariffs in this analysis. Hence we have shown that the ERP theory succeeds, for both output and input tariffs, in predicting correctly both the gross-output and primary-resource allocational effects.

It is important to note again that the imported input is used in fixed proportions with output and hence the physical unit of measurement of output serves also as the physical unit of measurement of value added. [Of course, unlike in the traditional Model I, price of value added is not the price per unit of output but the price net of imported inputs.] And this accounts for the success of ERP theory in this model.

Model III: One Primary Domestic Factor and One Imported Input with Substitution Among them, Producing Two Traded Goods:

When we specify one domestic and one imported factor, with substitution among them, we get a knife-edge model, which we now proceed to analyse. Thus consider the following model:

\[ X = X(L_x, I_x) \]  
\[ Y = Y(L_y, I_y) \]  
\[ L_x + L_y \leq L \]

where \( I_x \) and \( I_y \) represent the imported-factor utilisation in goods \( x \) and \( y \) respectively and \( L \) is now the only primary, domestic factor.

With \( \frac{P_x}{P_y} \) given exogenously, it follows that, in a perfectly competitive situation, the real wage of labour in activities \( x \) and \( y \) will be determine* and knife-edge specialisation on either commodity \( x \) or \( y \) will follow

---

*With \( \frac{P_x}{P_y} = \frac{\partial x}{\partial I_x} \) and given \( \frac{P_y}{P_x} \), \( \frac{\partial x}{\partial I_x} \) is determined; therefore, given constant returns to scale, \( \frac{\partial x}{\partial L_x} \) is also determined. So is \( \frac{\partial y}{\partial L_y} \). Therefore, given \( \frac{P_x}{P_y} \), labour will go to whichever activity that gives the higher real wage, except for a borderline Ricardian case.
except for a borderline, Ricardian possibility.

In this model, the physical unit of output clearly cannot serve as a unit for measurement of value added since substitution between domestic and foreign input is feasible in production. Therefore, the three measures of ERP outlined earlier, depending as they do on output units for measurement of value added, cannot be expected to be of help in predicting the resource allocation effects of a tariff structure: and indeed this is so. However there is a natural physical unit for value added in this model: namely, the unit of the only domestic factor of production, labour. Once ERP is defined in terms of the price of labor, i.e., the wage rate, it is of predictive value in both the gross-output and primary-factor allocational senses distinguished by us.

These conclusions can be illustrated by means of a simple example. Let us take a Cobb-Douglas production function for each of the two activities:

$$X = L_x^a I_x^{1-a}$$  (III.4)

$$Y = L_y^a I_y^{1-a}$$  (III.5)

Then, assuming: (i) that the tariff is levied only on the imported input so that the tariff-inclusive price per unit of this input is $P_i$, (ii) the world prices of $x$ and $y$ are unity by choice of units, we can solve for the marginal value product of labour in each activity as a function of $P_i$. Thus:

$$w_x(P_i) = a \left( \frac{1-a}{P_i} \right)^{a_x}$$  (III.6)

$$w_y(P_i) = a \left( \frac{1-a}{P_i} \right)^{a_y}$$  (III.7)
All the available labour will be devoted to the production of \( x(y) \) if

\[ w_x(P_i) > (<) w_y(P_i). \]

Also, given \( a_x \neq a_y \), \( w_x(P_i) \) and \( w_y(P_i) \) will cross each other at unique \( P_i^* \) given by:

\[
\frac{a_y}{a_x} \left( \frac{1-a_y}{a_y} \frac{1-a_x}{a_x} \right)^{\frac{1-a_y}{a_y} \frac{1-a_x}{a_x}} \frac{1-a_y}{a_y} \frac{1-a_x}{a_x}
\]

(III.8)

The \( w_x(P_i) \) and \( w_y(P_i) \) schedules are drawn in Figure (1), their intersection being at \( P_i^* \). In free trade, the economy is in equilibrium at \( P_i = OF \), and the specialization in production is in commodity \( y \) as \( w_y \)(at A) > \( w_x \)(at B). With a tariff on the imported input 'i', leading to domestic, tariff-inclusive \( P_i = OT \), the specialisation shifts to commodity \( x \) as \( w_x \)(at C) > \( w_y \)(at D). Thus all the available labour is shifted to commodity \( x \) and the relative output of \( x \) shifts from zero to infinity. Let us now examine the performance of our measures of ERP as predictors of this shift.

It is immediately obvious that the "true" measure is totally useless. For, the fact that there is only one domestic factor implies that value added per unit of output (in the post-tariff as well as in the free-trade situation) is nothing but the wage cost per unit of output. Since the units of output have been so chosen that the price per unit is unity, the wage cost per unit of output is also the wage share in the cost per unit. Given a Cobb-Douglas production function, this share is a constant. Hence \( ERP_x = ERP_y = 0 \) according to the "true" measure! It is thus of no predictive value whatever.

It is equally easy to construct a numerical example where either the post-tariff or the free-trade measure fails to predict the resource shift. For instance, choose \( a_x = 1/4, a_y = 1/2 \). Let free trade \( P_i = 1 \) and the post-tariff \( P_i = 1/2 \) so that the imported input enjoys an ad valorem subsidy at the rate of 50 percent. Under free trade, \( w_y = 1/4 \) and \( w_x = 27/256 \) and hence
Figure (1)
production is specialised in \( y \). After subsidy \( w_y = 1/2 \) and \( w_x = 27/32 \) so that \( w_x > w_y \) and production is specialised in \( x \). Now it can be checked that the post-subsidy imported input per unit of output is \( 3/2 \) in the production of \( x \) and \( 1 \) in the production of \( y \). Using these post-subsidy input coefficients we get:

<table>
<thead>
<tr>
<th>Process</th>
<th>Value added per unit of output</th>
<th>ERP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Post-subsidy prices</td>
<td>Free-trade prices</td>
</tr>
<tr>
<td>( y )</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( x )</td>
<td>1/4</td>
<td>-1/2</td>
</tr>
</tbody>
</table>

Thus \( \text{ERP}_y > \text{ERP}_x \) and this will lead us to expect no shift in labour and hence in gross output, contrary to the actual shift from \( y \) to \( x \). Of course, in this numerical example ERP based on free trade input coefficient will predict resource movement correctly. But the reader can easily construct an example where this measure of ERP also fails as a resource-allocational predictive device.

But we can manage to rescue a concept of ERP which will suffice to predict resource movement in this model. We do this by noting that the natural physical unit for value added in this model is labour. Defining ERP, therefore, in terms of its price, we must clearly get the correct prediction. Thus, if there is initially specialization in commodity \( y \), this must imply \( w_y > w_x \) and if labour and hence output must shift to commodity \( x \), this must imply that in the post-tariff situation \( w_x > w_y \). And, of course, this reversal in inequality in wages is possible only if \( w_x \) is raised proportionately more by the tariff change than \( w_y \). But this is only another way of saying that we must have \( \text{ERP}_x > \text{ERP}_y \), where the ERP is defined as the proportionate increment in the "price of value added" in the process thanks
to the tariff change.*

But it must be noted that, while our present model permits us to define a measure of ERP which correctly predicts resource-allocational effects of the tariff structure, the reason being that we have a natural physical unit of value added in the model thanks to labour being the only primary domestic factor, this is of no use from a practical point of view. Presumably one wants to construct an ERP index so as to be able to predict the resource-allocational impact of a tariff structure without having to solve the entire general equilibrium system. But, defining ERP in terms of the proportional increment in the "price of value added" means just that ERP's cannot be computed without solving the general equilibrium system first! For, we have to know the marginal value product of labour in each process before and after the change in the tariff structure if we are to calculate the ERP's as redefined in this subsection: and if we know these, then we have already solved the general equilibrium system and hence know already the resource-allocational shifts, so that the ERP calculation is altogether redundant from that point of view.

Two further observations are in order.

*Note however that it is quite possible in this knife-edge model for the tariff structure to shift in a way which merely accentuates the profitability of a process on which the economy is already specialised in the initial situation: hence, the ERP measure would predict a resource shift which does not occur. However, this is hardly a qualification as all resources are already employed in the process with the higher ERP and this is nothing but a "corner" situation. Nonetheless, from a practical point of view, it is important to remember that it would be wrong to infer from the ERP ranking that resources had actually been pulled towards the higher-ERP process by the tariff structure: all that can strictly be claimed is that resources will not have been pulled in the direction of the process with the lower ERP.
(i) A weak relaxation of the present model, in which the analysis above still applies, is where there are more than one primary factor but they are used in an identical, fixed proportion (i.e. one man to one spade) throughout the economy.

(ii) While our analysis has been presented, using an input-tariff, it applies equally to the other polar case of an output tariff. We briefly indicate, for Cobb-Douglas production functions again, how the ERP measure, in terms of proportionate increment in the price of value added, would still work for a change in the output tariff. Using identical Cobb-Douglas production functions, (III.4) and (III.5), we now choose units such that $P_i$ and $P_x$ are unity.

It then follows that:

$$w_x = a_x(1-a_x)^a_x$$  \hspace{1cm} (III.9)

$$w_y(P_y) = a_y(1-a_y)^a_y \cdot \frac{1-a_y}{1-a_y} = a_y(1-a_y)^a_y \cdot \frac{1-a_y}{1-a_y}$$  \hspace{1cm} (III.10)

Therefore, there exists a unique $P_y$, say $P_y^*$, such that:

$$w_y(P_y^*) = w_x$$

If $P_y > P_y^*$, then $w_y(P_y) > w_x$ and all $L$ will move to industry $y$; and if $P_y < P_y^*$, then all $L$ will move to industry $x$. Thus, when a tariff on industry $y$ increases $P_y$ from below $P_y^*$ to above $P_y^*$, the specialisation in production, and in domestic-factor ($L$) allocation, will shift from industry $x$ to industry $y$.

This is seen via Figure (2). There, the schedules $w_x$ and $w_y(P_y)$ are drawn, for the case of Cobb-Douglas production functions. Three alternative
Figure (2)
shapes of \( w_y(p_y) \), depending on whether \( 2 \xi < 1/2 \), are illustrated. Taking the case where \( \xi = 1/2 \), if we assume that a tariff on \( y \) shifts \( p_y \) from \( OX \) across \( P^* \) to \( OZ \), then the economy will shift from specialisation on \( x \) to specialisation on \( y \). Under free-trade, labour will go to industry \( x \) because its equilibrium wage is higher there than in \( y \): \( ES > QS \); under protection, on the other hand, it will go to industry \( y \): \( RZ > FZ \). At the same time, on the ERP definition in terms of proportionate increment in the "price of value added," \( ERP^*_x > ERP^*_y \):

\[
\frac{RZ - QS}{QS} > \frac{ZF - ZF}{ZF} \quad \text{or} \quad \frac{RZ}{QS} > \frac{ZF}{ZF} = 1.
\]

Hence, ERP theory holds for output price-changes as well.*

Model IV: Substitution between "Value-Added" and Imported Factors:

We now assume two domestic factors, one imported factor, and two outputs. However, we assume the production functions to be of the following special form:

\[
X = X[G_x(K_x, L_x), I_x], \quad \text{(IV.1)}
\]

\[
Y = Y[G_y(K_y, L_y), I_y], \quad \text{(IV.2)}
\]

\[
K_x + K_y \leq \bar{K}, \quad \text{(IV.3)}
\]

\[
L_x + L_y \leq \bar{L}, \quad \text{(IV.4)}
\]

\( X[\quad, \quad], Y[\quad, \quad], G_x(\quad, \quad), G_y(\quad, \quad) \) are each assumed to be homogenous of degree one in its two arguments. Thus, there is no direct substitution among the domestic factors \( (K \text{ and } L) \), on the one hand, and the imported factor

*This knife-edge model has other, analytically-interesting properties. For example, note that, if factor-intensity reversals can occur (thanks to the two industries having CES production functions with different elasticities of substitution, for example), they will not be ruled out because the economy has a given factor endowment: the fact that the supply of factor 'i' is variable permits the economy to go from one side of the factor-intensity crossover point to the other.
(I), on the other hand. However, unlike in Model II, \( a_{ix} \) and \( a_{iy} \) will not be fixed but will vary now as a function of the domestic \( (P_x/P_{i_1})^* \). It follows that the equilibrium ratio of the two domestic factors employed in production in each activity will depend only on the price-ratio of these factors (and not as well on the relative price of the imported input).

Differentiating the model successively with respect to output and input tariff change, and solving for output and primary factor utilisation in the production of each of the two goods, we can readily show that the three ERP definitions fail to give accurate resource-allocation predictions. However, the ERP definition in terms of proportionate increment in the price of value added will work accurately in predicting the primary-factor-allocation effect of the tariff structure but not the gross-output change.

---

*In this sense, Model IV is a generalization of Model II. On the other hand, it is also a generalization of Model III in another sense. In Model III, the imported input-coefficients \( (a_{ij}'s) \) varied with the relative price of output to import input \( (P_x/P_{i_1}, P_y/P_{i_1}) \). But there was only one domestic factor, its wage constituting value-added. Now, the structure of the production function basically remains the same: but this domestic factor, in turn, is a product of two primary domestic factors: \( K \) and \( L \). Thus,

\[
X = x(V_x, I_x) \\
Y = y(V_y, I_y)
\]

is now turning into:

\[
X = x(V_x, I_x) ; V_x = G_x(L_x, K_x) \\
Y = y(V_y, I_y) ; V_y = G_y(L_y, K_y)
\]

and \( V_x, V_y \) can then be construed as "value-added" factors, (produced by the domestic factors \( K \) and \( L \)), which enter the production functions of the two outputs, \( X \) and \( Y \). This model has been introduced into the ERP literature by Corden (1969) and extensively discussed by him (1971).

**Corden (1969) incorrectly seems to list this property, and the earlier property that the imported input per unit of output depends only on the price of this input relative to the price of the output, as independent restrictions on the nature of substitution between imported and domestic factors in his analysis. However, as stated above, and as proven in the Appendix, the latter restriction implies the former.
To see this clearly, note that in our present model, with substitution among the domestic and imported factors constrained in the highly restrictive fashion already defined, \( G_x = G_{x}(K_x, L_x) \) and \( G_y(K_y, L_y) \) can be construed as "value-added" production functions; and, given linear homogeneity, a natural physical unit for the "value-added product" in each process, could be defined as \( G_x(1, 1) \) and \( G_y(1, 1) \) respectively. Having defined these units, we can describe the solution of the model as follows. Given the price per unit of output of a process and its imported input price, the ratio of the value-added product to the imported input (in production) is determined by equating the marginal value product of the imported input to its price. Simultaneously the "price" of the value-added product in this process also emerges—it is simply the marginal value product of the "value-added product." Having determined the "prices" of value-added products in each process, their ratio uniquely determines the domestic factor allocations just as in the traditional, Model I. And hence if a tariff structure raises the "price" of the "value-added product" in one process relatively to the other, that process gains both domestic factors at the expense of the other. This is the same as saying that if \( ERP_x > ERP_y \) (where ERP's are defined in terms of the "price" of value-added product) then process \( x \) will gain domestic factors at the expense of process \( y \).

But this does not necessarily mean that the output of process \( x \) goes up. This can be readily explained. Suppose the tariff structure raises the ratio of the value-added product to the imported input in both processes. If the output of the value-added product in the process that gains domestic

*The linear homogeneity of \( X[ , ] \) and \( Y[ , ] \) is crucial for these two calculations.
factors rises by a smaller percentage than the rise in the ratio of the value-added product to the imported input in this process, then the total use of the imported input in this process must fall. Thus this process gains domestic factors while losing imported inputs. Hence its output need not rise.

Note thus that, as with Model III, an "appropriate" definition of ERP can be devised, i.e. the proportionate increment in the "price of value-added," which will correctly predict resource-allocational effects (although now only for primary, domestic factor allocations) but that this definition again requires that the general equilibrium system be solved before it can be computed. Hence we are again in the situation where the ERP index that works is no longer one which can be used in lieu of the solution to the general equilibrium system and hence one may well ask what use it is when we already know the resource allocational shifts once the general equilibrium system has been solved. In brief, the information required for the correct ERP computation includes the answer to the question which, once the ERP index is computed, that index will help to answer!

If we now take stock of the analysis so far, we have considered a succession of models where some ERP index worked. In all these cases, however, we could work with either a natural, physical unit in which we could measure value added (Model III) or a de facto physical unit in which we could measure value added (Model IV) or a derived unit, related to output, in which we could measure value added (Models I and II). Furthermore, whereas in the case of Models I and II, we could also use the "traditional" ERP definitions listed in Section I, in the case of Models III and IV, which permitted substitution among traded and domestic factors of production, we had to resort entirely to the "new" ERP definition in terms of the proportionate increment in the "price of value added": and this turned out to be open to the objection
that it required information whose availability/calculation made the ERP calculation unnecessary anyway.

We now proceed to discussing a model, developed by Ramaswami and Srinivasan (1971), which leads to an "impossibility theorem": there exists no definition of ERP which can correctly predict (in every case) the resource-allocational impact of a tariff structure.

V: The Ramaswami-Srinivasan "Impossibility" Theorem

Ramaswami and Srinivasan (1971) considered a model which permits generalized substitution among one imported and two domestic factors, all three of which produce two final outputs. This model is readily formalised as follows:

\[ X = X(K_x, L_x, I_x) \]  \hspace{1cm} (V.1)

\[ Y = Y(K_y, L_y, I_y) \]  \hspace{1cm} (V.2)

\[ K_x + K_y \leq \bar{K} \]  \hspace{1cm} (V.3)

\[ L_x + L_y \leq \bar{L} \]  \hspace{1cm} (V.4)

This model differs from Model IV only insofar as it allows for generalised substitution between the imported factor (I) and the primary, domestic factors (K and L).

They showed by means of a numerical example that, in this case, it is impossible to predict the way domestic factors will move and relative outputs will change, following on the imposition of a tariff on the imported factor, without bringing into consideration the endowments of the domestic factors. Thus if the domestic factor endowments took one set of parametric values, then
the same tariff on the imported factor, with the same production functions and foreign prices, would pull both domestic factors to one process whereas another set of parametric values for the domestic factor endowment had pulled these factors to the other process. Thus they succeeded in showing that no ERP index could be devised which could invariably predict resource-allocation changes resulting from tariff imposition on imported inputs (and by analogy on outputs) which ignored the domestic factor endowments.

Computing the three "traditional" ERP measures in Section I, they underlined this conclusion by showing how each would fail to predict correctly. In their numerical example, the functional form of one of the two production functions ruled out (as is the case for both production functions stated in the model here) the possibility of defining a physical, de facto or derived unit in a way which would permit resort to the "new" ERP definition in terms of proportionate increase in the "price of value added." Hence, that definition was also inappropriate.

VI: Conclusions

Our analysis leads us to conclude, somewhat nihilistically, that a measure of ERP which will unfailingly predict the domestic resource shift consequent on a change in tariff structure does not exist in general. Even

*It may also be useful to contrast the Ramaswami-Srinivasan model with Model IV from yet another angle. In Model IV, any change in the tariff structure changes the equilibrium ratio of the domestic factors in the same direction in both processes so that one process gains or loses both domestic factors due to the change in the tariff structure. In the Ramaswami-Srinivasan model, on the other hand, a subsidy on the imported input can change, and in their example does change, the domestic factor ratio in opposite directions for the two processes. This means that the aggregate endowment of primary, domestic factors has to be brought in to determine the resource-allocation impact. Note therefore that we could devise alternative restrictions, to that embodied in Model III, on the nature of substitution between imported and domestic factors which would ensure that both primary factors move in the same direction in both activities and hence enable us to predict resource-allocational shifts with a suitable ERP definition.
in the highly-restrictive situations where a measure can be shown to exist, generally the information required for its computation subsures the answer to the prediction problem which ERP computation is supposed to provide.

This nihilistic conclusion is reinforced by three further observations:

(1) Recent studies, by Cohen (1969) and Guisinger and Schydlovsky (1970), of the relationship between the (calculated) nominal tariffs and ERP's in a number of empirical studies have shown that a remarkably high correlation exists between them: thus raising the question whether it is useful to spend vast resources on calculating ERP's when nominal tariffs seem to be adequate proxies for them anyway.

(2) The most that, in a multi-commodity world, the ERP's could tell us ideally was that, if the different processes were ranked by their ERP's in a chain, the highest-ERP process would have gained resources and the lowest-ERP process would have lost them, in relation to the pre-trade situation.* As with nominal tariffs, the scope of purely "qualitative economics" does not go beyond this, so that once again the vast empirical effort required in making up the ERP numbers seems grossly disproportionate to what can be done to predict actual resource-allocation impacts of the tariff structure without resort to the full general-equilibrium solution.

(3) It also needs to be emphasised that the recent attempt at arguing that the constancy of the (imported-factor) \( a_{ij} \)'s is a reasonable restriction because raw materials do not substitute with domestic factors and are in a fairly fixed proportion to output is based on a false equation of the imported factors with intermediates and raw materials: most economies import capital goods and these do substitute with (domestic) labour quite generally. And, indeed, it is not at all uncommon for there to be substitution between

* However, as our companion paper shows, even this much cannot be asserted for Model III, for example.
intermediates and primary factors, though admittedly this is less important in practice than the substitution among the primary factors, capital and labour.

It seems clear that while the original stimulus to the ERP theory was given by the question as to whether an index could be devised to predict resource allocation correctly in the presence of traded factors with their attendant complication of tariff structures, this is a dead-end. Where does ERP theory go from here then?

Two fresh questions seem to us to be of interest to explore at this stage:

(1) Insofar as neither nominal-tariff nor ERP rankings are adequate, in theory, to predict the resource-allocational effects of tariff structures correctly when the general (Ramaswami-Srinivasan) case is considered, can we still rank the two measures by their relative performance in this regard under alternative, "probable" parametric situations?

(2) Since current tariff negotiations, as in the Kennedy Round, involve across-the-board tariff cuts in nominal tariffs, what economic differences would ensue (e.g. the impact on the welfare of the tariff-cutting trading nations) if the tariff cuts were made instead in ERP's?

*For examples, see Ramaswami and Srinivasan (1971).
APPENDIX

(I)
Let \( H(K,L,M) \) be the production function where \( K, L \) are domestic primary factors, \( H \) is homogenous of degree one and concave. Let \( x = \frac{K}{L} \), \( y = \frac{M}{L} \) and \( h = \frac{H}{L} \). Then it is clear that \( h = h(x,1,y) = h(x,y) \). If \( H \) is of the form \( F[G(K,L),M] \), where \( F \) and \( G \) are homogenous of degree one in their arguments, then \( h(x,y) = y f\left(\frac{g(x)}{y}\right) \) where \( f(z) = F(z,1) \) and \( g(x) = G(x,1) \).

It is shown below that the condition that the input coefficient of imported input, i.e. \( \frac{v}{h(x,y)} \), depends only on its price relative to output price implies \( h(x,y) \) is of the form \( v f\left(\frac{g(x)}{y}\right) \).

Proof: Under competitive conditions, the marginal product of the imported input, i.e. \( h_2(x,y) \) (where the subscript 2 denotes the partial derivative with respect to the second argument) equals the ratio of the price of the imported input to the price of output. We can therefore restate the above condition equivalently as:

\[
\frac{h(x,y)}{y} = J[h_2(x,y)]
\]
or

\[
h(x,y) = v J[h_2(x,y)] \tag{1}
\]

where \( J \) is an increasing function.

Let \( Z = h_2(x,y) \). Differentiating both sides of (1) partially with respect to \( x \) we get (with subscript \( i \) denoting the partial derivative of a function with respect to its \( i \)th argument):


\[ Z = J(Z) + y J_1(Z)z_2 \]

or
\[ \frac{1}{y} = \frac{J_1(Z)}{Z - J(Z)} z_2 \quad (2) \]

Let \( P(Z) = \int \frac{J_1(Z)}{Z - J(Z)} \, dZ \). It is clear that \( P(Z) \) is an increasing function of \( Z \) since \( P_1(Z) > 0 \) because \( J_1(Z) > 0 \) and \( Z > J(Z) \). Then (2) implies:

\[ \frac{\partial}{\partial y} (\log v) = \frac{\partial}{\partial v} \{ P(Z) \} \quad (3) \]

Integrating, we get
\[ P(Z) = \log \frac{y}{g(x)} = - \log \frac{g(x)}{y} \]

where \( g(x) \) is the "constant" of integration. Inverting this function and substituting in (1) we get:

\[ h(x,y) = y J[p^{-1} \{- \log \frac{g(x)}{y}\}] \]

\[ = y f \left( \frac{g(x)}{y} \right) \quad (4) \text{Q.E.D.} \]

(II)

It is shown below that condition (1) implies that the ratio of the marginal products of the two domestic factors depend only on their ratio \( x \). This means that \( x \) gets determined uniquely once the price-ratio of the domestic factors is specified.

Proof: Differentiating (1) partially with respect to \( x \) and \( y \) respectively we get:

\[ h_1(x,y) = y J_1(Z)z_1 = y J_1(Z)h_{21} \quad (5) \]

\[ h_2(x,y) = J(Z) + y J_1(Z)z_2 = J(Z) + y J_1(Z)h_{22} \quad (6) \]

\[ h - xh_1 - yh_2 = -y J_1(Z)[x h_{21} + y h_{22}] \quad (7) \]

Now \( h_1 \) is the marginal product of \( K \) and \((h-x h_1-y h_2)\) is the marginal product
of L. Hence from (5) and (7) we get that ratio of the marginal product of the two domestic factors. Thus:

\[
\frac{h-x_1-y_2}{h_1} = - \frac{(x_{21}+y_{22})}{h_{21}}
\]  

(8)

or

\[
\frac{h_{21}}{h_1} = - \frac{(x_{21}+y_{22})}{h-x_1-y_2}
\]

or

\[
\frac{\partial}{\partial y} \log h_1 = \frac{\partial}{\partial y} \log (h-x_1-y_2)
\]

or

\[
\frac{\partial}{\partial y} \log \left(\frac{h-x_1-y_2}{h_1}\right) = 0
\]

(9)

(and integrating)

or

\[
\frac{h-x_1-y_2}{h_1} = e(x)
\]

(10)

where \(e(x)\) is a function only of \(x\).

Q.E.D.
REFERENCES


