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Wages with escalator clauses are flexible nominal wages; wages without escalator clauses are flexible real wages. Accordingly, the relevant consideration in analyzing the effects of wage indexation on macroeconomic stability is whether sticky real wages or sticky nominal wages are more conducive to stability.

There are at least two prevalent views on the effects of wage indexation on economic stability, by which is meant the variability of output and prices. One, associated with Friedman (1974) and Giersch (1974), is that wage indexation stabilizes real output. In making this argument Friedman points to the 1967-70 period in the United States during which, he claims, sticky nominal wages in conjunction with rising prices led to an excessive expansion of output. Both Giersch and Friedman suggest that in 1974 a contractionary monetary policy would have had smaller output effects with

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1. With apologies to Herbert Giersch (1974, p. 6), who notes "Wages with escalator provisions are flexible wages".
indexed than non-indexed wages. The second argument is that wage indexation exacerbates real economic instability by reducing the responsiveness of the economy to disturbances that require real wage changes. Bernstein (1974) points to the oil and food shocks of 1973-4 in arguing for the potentially destabilizing effects of wage indexation. The ending of Finnish indexing in 1968 and the Israeli government's successful attempt to abrogate or at least delay wage indexing in 1974, each following a devaluation, can also be cited in support of the latter view, as can the current (end of 1975) Belgian government proposal to suspend indexing.

The analysis of this paper shows that each view is correct under appropriate circumstances: real output tends to be more stable in an indexed than in a non-indexed system when disturbances to the system are primarily nominal and persistent; output tends to be more stable in a non-indexed than in an indexed system when disturbances are primarily real.  

The recent discussion of wage indexation has taken place in a short-run context in which it is desired to reduce the rate of inflation over the next few years. The paper accordingly discusses the short run impacts on output and the price level of a change in the money stock in indexed and non-indexed systems respectively. But indexation is a relatively complicated instrument to introduce into the economy and it is not likely that it can be turned on and off at will by the monetary authority or other economic policy makers. Therefore the paper also examines the steady-state behavior of non-indexed and indexed economies that are subjected to random real and nominal disturbances each period.

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2 This result is also obtained by Gray (1976). I am informed by Harry Johnson that Keynes reached such a conclusion in discussing Australian indexation. A similar point was made by an Israeli Committee of Experts in 1964, whose membership included Ephraim Kleiman and David Levhari [see Brenner and Patinkin (1975)]. Recent analyses of the effects of wage-indexation are those by Arak (1975), Cukierman and Razin (1975) and Parkin (1975). Related papers are by Barro (1975), Brenner and Patinkin (1975) and Grady and Stephenson (1975).
Two important features of wage indexation should be pointed out before we proceed to the analysis. First, wage indexing exists in most economies. In the absence of legal impediments, the extent of indexation of contracts is endogeneous, and apparently related to the rate of inflation, though theoretical considerations would suggest that the variance of the inflation rate is the major determinant. That endogeneity suggests that the extent of indexing not only affects, but also is affected by, macroeconomic stability, and further raises the question of whether the private economy produces the optimal amount of indexing. In this paper we do not treat indexation as endogeneous but instead compare two economies in which wages are not and are indexed respectively. However, we recognize that a complete theory of the effects of indexation on macroeconomic stability will have to begin from the work contract by analyzing the circumstances under which wage contracts are indexed, embodying such contracts in a complete macro model, and then examining the consequences of the prohibition of indexation on stability.

Second, the form wage-indexing takes in practice is not the same as the method of indexing for debt instruments in which the payment for a given period is adjusted ex post on the basis of the realized price change for the period; instead, wage indexing typically applies to long term contracts and adjusts the wage for later periods of a contract in accordance with the realized behavior of the price level over the preceding periods. In a typical

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3 See the appendix to Giersch (1974) for a summary of international experience, and Perna (1973) and January issues of the *Monthly Labor Review* for United States experience.

4 However, on an international basis, the mean rate of inflation and the variance of inflation are positively correlated. See Jaffee and Kleiman (1975).

5 The one attempt I am aware of to do this is by Gray (1975).
indexed wage contract the nominal wage for a given period is known at the beginning of the period; further, the period is likely to be three months or longer, though it doubtless shortens with increases in the variance of the inflation rate. The analysis of Sections IV-VI below is designed to reflect this feature of wage indexation: in that analysis labor contracts are assumed to last two periods and differ between indexed and non-indexed economies according to whether the nominal wage for the second period of each contract is allowed to adjust to price changes realized since the beginning of the contract. In each period, though, the nominal wage is predetermined in that it is known at the beginning of the period.

This paper contains three models of wage indexing, each of which suggests that whether wage indexing is stabilizing or not depends on the nature of the disturbances affecting the economy and the collection of which suggests that the basic results of the paper are robust. Each model has three elements: wage setting behavior; an output supply equation (or a markup equation); and an aggregate demand equation. The first model - Case Zero - focuses as directly as possible on the question posed in the first paragraph above by examining the consequences for output variation of fixed nominal and real wages in an extremely simple model economy disturbed by real and nominal shocks. The second model, presented in Section II contains a simple Phillips curve for wage determination and is included because the model, indexing aside, is a standard textbook macro model.

The third model provides the major theoretical innovation of the paper. It uses the basic result of research on the labor contract and stylized facts

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6 In Israel, the indexation adjustment usually occurs one year after the start of the initial contract, but the period has been as short as three months; in the U.S. in 1972, 2 million of the 4.3 million workers covered by escalation clauses received adjustments after a year and 1.7 million every three months. (Sparrough and Bolton (1973) p.7).

7 Edmund Phelps suggested the inclusion of Case Zero, and Michael Parkin recommended the use of Figure 1 below.
about wage indexing to build a model with overlapping two-period labor contracts to study the effects of indexing on stability. Recent research on the wage contract is reviewed in Section III and used in the subsequent specification of wage adjustment equations. The model of a non-indexed economy is examined in Section IV and that of an indexed economy in Section V. The behavior of the two economies is compared in Section VI. The source of the basic conclusions is discussed in Section VII, together with the results of an extension of the model to include an interest elastic demand for money that are presented in an Appendix. Concluding comments are contained in Section VIII. Since it is clear that wage indexing makes no difference when inflation is correctly anticipated, the assumption throughout is that prices fluctuate around a stationary price level; fluctuations could as well be thought of as occurring around a rising price trend.

I. Case Zero

The three elements of each of the models of this paper are wage setting behavior, an output supply equation, and an aggregate demand equation. In all the models output is assumed to be a decreasing function of the real wage:

\[ Y_t = P_t - W_t + u_t \]

where \( Y_t \) is the level of real output, \( P_t \) the logarithm of the price level, \( W_t \) the logarithm of the wage rate, and \( u_t \) is (until Section IV) a serially uncorrelated real or aggregate supply disturbance term with expectation zero and variance \( \sigma_u^2 \). The disturbance \( u \) should be thought of as representing the effects of weather, etc. on real output. An alternative interpretation of (1) is that it is a markup equation with the markup depending on the level of output.

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8 Although (1) appears to make negative output levels possible, the reader can prevent that possibility by adding a sufficiently large constant to the right hand side, or else may view (1) as a relationship that applies to deviations of output from a specified level.
Aggregate demand is given by a simple velocity equation:

\[ M_t - P_t = Y_t + v_t \]

where \( M_t \) is the logarithm of the nominal money stock and \( v_t \) is a serially uncorrelated nominal or aggregate demand disturbance with expectation zero, variance \( \sigma_v^2 \) and is uncorrelated with \( u_t \). The strong simplifying assumption implicit in (2) is that velocity is independent of the nominal interest rate. This assumption is relaxed in the Appendix which discusses a Keynesian interpretation of (2) as the reduced form of a two equation system embodying goods and money market clearing conditions.

Since in this paper we are chiefly interested in the effects of indexing per se and not in the interactions of indexing and policy, we shall assume throughout that monetary policy is passive in that \( M_t \) is constant at the level \( M \).

The third element of the model is wage-setting behavior. In the non-indexed case, the nominal wage is assumed to be predetermined at a level \( \bar{W} \) that is expected to produce a specified real wage and level of employment. In the indexed case, the real wage is assumed to be set at the same expected level \( \bar{w} = (\bar{W} - \bar{P}) \) that is also expected to produce the same level of employment as \( \bar{W} \). The indexed wage, in other words, responds to the current price level while the nominal wage does not.

Using the prescripts NI for non-indexed and PI for price-indexed, the resultant behavior of output is

\[ \text{NI}_t = \frac{1}{2} \left[ M - \bar{W} + u_t - v_t \right] \]

\[ \text{PI}_t = -\bar{w} + u_t \]

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9 Although we shall generally refer to the disturbances as real or nominal, the reader should feel free to substitute "supply" for "real" and "demand" for "nominal".

10 This latter topic is analyzed in Fischer (1975).

11 This is precisely the analysis of Gray (1976).
Now suppose there is a nominal or demand disturbance $v_t$ that tends to reduce the price level. In the non-indexed system we see from (3) that an increase in $v_t$ increases the real wage (since $\tilde{W}$ is fixed) and tends to reduce output. In the price indexed system, by contrast, the nominal wage adjusts with the price level, the real wage is unaffected, and as (4) shows, output is unaffected. Hence wage indexing is stabilizing for the behavior of output if disturbances are nominal.

Next suppose there is a positive real or supply disturbance, $u_t$. In (4) we see that $u_t$ has its full effect on output since whatever effects the real disturbance has on the price level have no effect on the real wage. But if wages are not indexed, we see from (3) that the effects of the real disturbance on output are mitigated. How? The increase in output tends to reduce the price level and that, given the nominal wage, tends to increase the real wage - hence the effects of the initial real disturbance on output are reduced. Thus wage indexing is destabilizing for the behavior of output if disturbances are real.

Figure 1 illustrates these results. It represents the aggregate demand (DD) and supply (SS) functions in the $(Y,P)$ plane. The aggregate demand curve [equation (2)] is the same in both cases. Figure 1a shows the non-indexed case for which the aggregate supply function is, from (1) with $\bar{W}_t = \bar{W}$,

$$ Y_t = P_t - \bar{W} + u_t. \tag{1a} $$

Figure 1b is the indexed case in which the aggregate supply function from (1) with $(\bar{W}_t - P_t) = \bar{w}$, is

$$ Y_t = -\bar{w} + u_t. \tag{1b} $$
Aggregate supply in the indexed case does not respond to the absolute price level. The curves in Figure 1 are drawn for $u_t = 0 = v_t$. The reader may gain understanding of the basic results by shifting the supply and demand curves by letting $u_t$ and $v_t$ respectively take on non-zero values.

![Figure 1a: Non-indexed](image1a)

![Figure 1b: Indexed](image1b)

**FIGURE 1**

Note that indexing totally removes the effects of nominal disturbances on output while non-indexing only partially offsets the effects of real disturbances on output. In general, where both types of disturbances are possible, the question of whether the system is more stable with or without indexing turns on the size of the variances of the disturbances, and it is possible to develop an analysis of the optimal extent of indexing from the viewpoint of stability, as Gray (1976) does.

To summarize this section: indexing shields real output from nominal disturbances by permitting constancy of the real wage in the face of such disturbances; indexing does not permit the stabilizing effect on real wages that would otherwise result from price level movements when disturbances are real. This simple analysis points to the essence of the results contained in the remainder of the paper.
II. A Simple Phillips Curve Model

In this section we replace the wage setting behavior of the above model with a postulated Phillips curve, the fundamental notion of which is that wages respond to aggregate demand. The stabilizing effects of indexing on the response of output to nominal disturbances emerges clearly from the analysis. The output supply equation (1) and the aggregate demand equation (2) are retained.

A. Indexed Wages

We write the Phillips curve for an indexed economy as:

\( W_t = W_{t-1} + \alpha (Y_{t-1} - Y^*) + \beta (P_t - P_{t-1}) \quad \alpha > 0, \quad 1 > \beta > 0 \)

where \( Y^* \) is the full employment level of output and \( \beta \) is a coefficient representing the degree of indexing, that will later generally be assumed equal to unity. Equation (5) asserts that the nominal wage responds to lagged aggregate demand, \( Y_{t-1} \), and also to the realized one period rate of inflation. For \( \beta=1 \), the real wage changes in response to lagged aggregate demand. Notice that we are here not treating the nominal wage as predetermined and are allowing it to respond to the current price level. This does not accord with the stylized facts outlined in the introduction but is a simple way of allowing the nominal wage to adjust more rapidly to prices in the indexed system than in the non-indexed system of Section II B below.

We will later want to compare the dynamic responses of the indexed and non-indexed systems to disturbances. Accordingly we use (1), (2) and (5) to solve for the levels of output and prices as functions of their own lagged values and current and lagged values of the disturbances. We obtain:

\( Y_t \quad [2-\beta] = \alpha Y^* + Y_{t-1} [(1-\beta) + (1-\alpha)] + u_t - u_{t-1} - v_t (1-\beta) + v_{t-1} (1-\beta) \)
(7) $y_t = ay_t^* + (1-\alpha) y_{t-1} + u_t - u_{t-1}$ for $\beta = 1$

(8) $p_t [2-\beta] = \alpha(M-\gamma^*) + p_{t-1}[(1-\beta) + (1-\alpha)] - u_t + u_{t-1} - v_t + v_{t-1}(1-\alpha)$

(9) $p_t = \alpha(M-\gamma^*) + (1-\alpha) P_{t-1} - u_t + u_{t-1} - v_t + v_{t-1}(1-\alpha)$ for $\beta = 1$.

It is obvious from (7) that full indexation shields output from the effects of nominal disturbances, as is the case in the model of Section I, and for the same reason. We defer further discussion of (6) - (9) to Section II C below.

We want to use (6) and (8) also to calculate the steady state variances of output and prices in the system given by (1), (2) and (5). That is, we are asking what the variances of output and prices are once the economy has been operating sufficiently long for the effects of any particular initial conditions to have worn off. The steady state variances of output, $\sigma_y^2$, and the price level, $\sigma_p^2$, are given by (10) and (11) respectively:

(10) $\sigma_y^2 = 2(\zeta_1\zeta_2)^{-1} \{ \sigma_u^2 + (1 - \beta)^2 \sigma_v^2 \}$

$= \sigma_{11}^2 \sigma_u^2 + \sigma_{12}^2 \sigma_v^2$

(11) $\sigma_p^2 = (\zeta_1\zeta_2)^{-1} \{ 2\sigma_u^2 + \sigma_v^2 [\alpha(1 - \beta) + (2 - \alpha)] \}$

$= \sigma_{21}^2 \sigma_u^2 + \sigma_{22}^2 \sigma_v^2$
where \( \zeta_1 = 2 - \beta \)

\[
\zeta_2 = (2-\alpha) + 2(1-\beta)
\]

It is necessary for stability of the system that \( \zeta_2 \) be positive (this can also be seen by examining (6) and (7)) and we shall henceforth assume \( 2 > \alpha \): this stability condition restricts the effects of lagged output on the current real wage and thus on current output. It is immediate from (10) that nominal disturbances have no effect on the variance of output if wages are fully indexed (\( \beta = 1 \)) and also clear from (11) that, for \( \beta = 1 \), nominal disturbances are consequently fully reflected in the price level. Further comments are postponed to Section II.D.

B. Non-Indexed Wages

The only change made from the Phillips curve model of the indexed economy in studying the non-indexed economy is to replace the wage equation by an expectational Phillips curve in which the wage rate responds not to the actual price level, but rather to the expected price level

\[
W_t = W_{t-1} + \alpha(Y_{t-1} - Y^*) + b(t_l P_t - P_{t-1}) \quad 1 > b > 0
\]

where \( t_l P_t \) is the price level expected at the end of period \( t-1 \) to prevail in period \( t \).

We shall assume that expectations are formed rationally so that

\[
t_l P_t = (2-b)^{-1} [a + W_{t-1} + \alpha Y_{t-1} - bP_{t-1}]
\]
where $a - M - \alpha Y^*$. The major reason for using rational expectations in the present context is that such expectations are unbiased; we are accordingly certain that none of the results obtained turn on expectational errors that differ in anything other than an unavoidable way between indexed and non-indexed systems.

Once more, we shall want to study the dynamic impacts of real and nominal disturbances on output and prices and accordingly solve (1), (2), (12) and (13) to obtain expressions for $Y_t$ and $P_t$ respectively in terms of their own lagged values and the values of the disturbances:

\[(14) \quad 2Y_t = \frac{2aY^*}{2-b} + 2Y_{t-1} [2-b]^{-1} [(1-a) + (1-b)] + u_t - \frac{2}{2-b} u_{t-1} - v_t + \frac{2(1-b)}{2-b} v_{t-1}\]

\[(15) \quad Y_t = \alpha Y^* + (1-\alpha) Y_{t-1} + \frac{u_t}{2} - \frac{u_{t-1} - v_t}{2} \quad \text{for } b = 1.\]

\[(16) \quad 2P_t = \frac{2a}{2-b} [M-Y^*] + 2P_{t-1} [2-b]^{-1} [(1-a) + (1-b)] - u_t + \frac{2}{2-b} u_{t-1} - v_t + \frac{2(1-\alpha)}{2-b} v_{t-1}\]

\[12 \quad \text{For discussion of the hypothesis, see Barro and Fischer (1976).}\]
(17) \[ P_t = \alpha(M-\gamma^*) + (1-\alpha) P_{t-1} - \frac{u_t}{2} + \frac{v_{t-1}}{2} - u_t + u_{t-1} - \frac{v_t}{2} + (1-\alpha)v_{t-1} \]

We now use (14) and (16) to solve for the asymptotic variances of output, \( S_Y^2 \), and the price level, \( S_P^2 \), in the non-indexed economy. The variances are:

\[(18) \quad S_Y^2 = (\xi_3)^{-1} \left\{ \sigma_u^2 [b^2 + 4\alpha] + \sigma_v^2 [b^2 + 4\alpha (1-b)] \right\} \]

\[= a_{11} \sigma_u^2 + a_{12} \sigma_v^2 \]

\[(19) \quad S_P^2 = (\xi_3)^{-1} \left\{ \sigma_u^2 [b^2 + 4\alpha] + \sigma_v^2 [2(\alpha-b)^2 + 4\alpha(1-\alpha)] \right\} \]

\[= a_{21} \sigma_u^2 + a_{22} \sigma_v^2 \]

where \( \xi_3 = 4\alpha [2(1-b) + (2-\alpha)] \)

It is necessary for stability that \( \xi_3 \) be positive which, given our earlier assumption that \( 2 > \alpha \), requires \( \alpha > 0 \).
C. The Short-Run Responses of Output and Prices to Disturbances.

In order to examine the dynamic responses of output and prices to disturbances, we write $Y_t$ and $P_t$ as functions of current and lagged values of the disturbances. We shall examine only the cases $\beta = 1 = b$, i.e., we use (7), (9), (15) and (17), leaving the general case for the reader. Using once again the PI and NI prescripts for "price-indexed" and "non-indexed" respectively, we obtain:

\[ (20) \quad PI Y_t = u_t - \alpha \sum_{i=0}^{\infty} (1-\alpha)^i u_{t-1-i} \]

\[ (21) \quad NI Y_t = \frac{u_t}{2} - \frac{1+\alpha}{2} \sum_{i=0}^{\infty} (1-\alpha)^i u_{t-1-i} - \frac{1}{2} \sum_{i=0}^{\infty} (1-\alpha)^i v_{t-i} \]

\[ (22) \quad PI P_t = -u_t + \alpha \sum_{i=0}^{\infty} (1-\alpha)^i u_{t-1-i} - v_t \]

\[ (23) \quad NI P_t = -\frac{u_t}{2} + \frac{(1+\alpha)}{2} \sum_{i=0}^{\infty} (1-\alpha)^i u_{t-1-i} - \frac{1}{2} \sum_{i=1}^{\infty} (1-\alpha)^i v_{t-i} \]

\[ \]

13 Since lag coefficients are notoriously sensitive to specification, the short run responses discussed below should be viewed as broadly representative of the dynamic impacts of disturbances rather then as the precise patterns that would emerge in indexed and non-indexed economies.
FIGURE 2: Dynamic Multipliers for Effects of a Unit Real Shock on Output ($\alpha=1/2$)
We discuss first the impacts of nominal shocks, the \( v_t \). From (20) we observe again that indexing protects real output from nominal shocks and from (22) that with indexing the full impact on prices of a nominal shock occurs in the period of the shock. In the rational expectations context it is probably most reasonable to think of a monetary stabilization policy as a nominal shock and we accordingly see the basis for the view that a deceleration of the inflation rate is more easily achieved (comparing (22) with (23) and with less real consequences (comparing (20) with (21)) with indexing than without. Equation (21) shows that without indexing nominal shocks have an effect on output that is distributed over time and (23) makes it clear that the effects of a nominal shock on prices are distributed over time in the non-indexed system - though the initial impact on the price level of a nominal shock is smaller in the non-indexed than in the indexed system, precisely because in such a case the nominal shock also affects output. Notice also from (23) that in the non-indexed system a nominal shock that increases the price level on impact will tend to produce a lower price level in subsequent periods.

Second consider the dynamic impacts of real shocks: in this case the impacts of real shocks on output in each system are the mirror image of the effects of prices, so we discuss only the impact of a real shock on output. Figure 2 shows the dynamic multipliers for a real shock in the two systems for the case \( \alpha = \frac{1}{2} \). In each system the impact effect of a real shock on output is positive but the lagged effects are negative. In the indexed case the lagged effects are negative because the initial increase in output increases the real wage in subsequent periods. In the non-indexed case the lagged effects are negative for the reason outlined in the preceding sentence and also because the real wage rises initially due to a price level fall that occurs in the first period.

While the impact effect of a real shock on output is larger in the non-indexed system, lagged effects on output are (absolutely) smaller (for \( \alpha < 1 \)). Thus for \( \alpha > 1 \) it is clear that output is more stable without than with indexation in the
face of real shocks, but for $\alpha < 1$, a clearly specified stability criterion is needed to compare stability of the two systems. Such a criterion is provided in Section II D below.

Accordingly this section confirms the basic results of the initial simple model with respect to the role of indexing in dealing with nominal or demand shocks. However, the comparison between the full dynamic impacts of real shocks on output and prices is ambiguous for $\alpha < 1$. It is clear that monetary stabilization affects the price level more rapidly and with less real consequences in an indexed than in a non-indexed system -- and it is probably this argument that is behind the recent support for indexation.

D. Asymptotic Stability Comparisons

We use (10), (11), (18) and (19) to compare the steady state variances of the levels of output and prices in the two systems. The asymptotic variance is a convenient measure of how far away from steady state values $Y$ and $P$ are likely to be when the two systems are subjected to repeated shocks. Using the relevant equations and again setting $\beta=1=b$:

\[
\begin{align*}
\text{PI}_Y - \text{NI}_Y^2 &= (\alpha_{11} - \alpha_{11}) \sigma_u^2 + (\alpha_{12} - \alpha_{12}) \sigma_v^2 \\
&= \left[ \frac{2}{2-\alpha} - \frac{1+4\alpha}{4\alpha(2-\alpha)} \right] \sigma_u^2 - \frac{1}{4\alpha(2-\alpha)} \sigma_v^2 \\
&= \frac{4\alpha-1}{4\alpha(2-\alpha)} \sigma_u^2 - \frac{1}{4\alpha(2-\alpha)} \sigma_v^2
\end{align*}
\]
Consider first $(\alpha_{ll} - a_{ll})$ which gives the relevant comparison for the effect of real shocks on output: for large $\alpha (>.25)$ real shocks have a greater impact on output in the indexed system and for $\alpha < .25$ real shocks have a smaller impact on output in the indexed system. The reason can be seen in Figure 1 and from (21) and (23): although the initial impact of a real shock on output is always larger in the indexed system, the lagged impacts will be smaller in the indexed than in the non-indexed system for small $\alpha$. The variance measure of fluctuations weights deviations in either direction equally so that all that is relevant is the sum of squares of the two curves in Figure 1 from the zero axis. At the limit, as $\alpha$ goes to zero, the indexed system is stable and the non-indexed system unstable as there are no forces in the non-indexed system driving the real wage back to the appropriate value.

The same analysis is relevant for the effects of real disturbances on prices.

The comparative effect of nominal disturbances on output is clear: nominal disturbances have a greater impact on output in the non-indexed system. The relative effects of nominal disturbances on prices depends on the value of $\alpha$: for $.13 \leq \alpha \leq 1.85$ nominal disturbances have a greater impact on prices in the indexed system - outside those limits the non-indexed system has less stable prices.
E. Summary

The basic result of this section is that, from the viewpoint of the stability of output, indexing is fully appropriate in an economy subject predominantly to nominal disturbances, but may not be appropriate in an economy subject to real disturbances. However, the ambiguity of some of the results, the essentially ad hoc nature of the wage equations, and particularly the absence of one feature of actual wage indexing pointed out in the introduction - that indexed wages provide automatic adjustments in wages in the later periods of a contract but do not actually adjust to the current price level - make it desirable to study the wage equation and contracting more carefully.

III. The Labor Contract

Recent research on the optimal labor contract has emphasized differences in risk aversion between workers and employers. Facing a worker with a choice between a stream of income from a spot labor market and income derived from a contract with a risk-neutral employer, Baily (1974) shows that a risk averse worker would, in effect, be willing to buy insurance from the employer by agreeing to a fixed wage contract, even with some probability of his being laid off. The employer could thus obtain a given amount of labor at lower expected cost by offering a fixed wage contract than by varying the wage in accord with labor market conditions. Grossman (1975) points out that the fixed wage contract with variable employment is dominated for the worker by one with constant income, and that, in the absence of default costs, the type of contract analyzed by Baily and also by Azariadis (1975) is not viable in that one side or the other would always find it profitable to repudiate the contract: if the spot market wage was below the contract wage, the employer would repudiate; if the contract wage were below the spot wage, the worker would repudiate. By incorporating the possibility of default, Grossman shows that workers may prefer the fixed-wage contract to the fixed income contract.
Although the above-mentioned studies do not explicitly discuss indexing, they imply that wage contracts would be indexed, since workers are concerned with real and not nominal income. Such a conclusion is also implied for workers whose entire income is from labor by a result due to Shavell (1975) who shows that in contracting between a risk-averse individual and one who is risk-neutral, the payments function would leave the former with constant income. Partial indexing could obtain if both parties were risk averse. Using a model in which firms minimize the expected costs of deviations of labor input from desired levels, plus an assumed fixed per contract cost of contracting, Gray (1975) produces labor contracts in which the degree of indexing is dependent upon the source (real or nominal) of macroeconomic disturbances.

The fundamental result of the work on the labor contract, then, is that contracts attempt to maintain constancy of the real wage (in a stationary economy subject to random disturbances). We shall also incorporate in our models the existence of long-term labor contracts of finite duration, although we do not provide the analysis which explains the existence of such contracts. In any period, some workers will be in the first year of contracts drawn up at the end of the previous period, some workers will be in the second year of contracts, etc. Typically, wages in the pre-existing contracts will be changing in accordance with a formula fixed at the previous contract date. We now investigate the implications of alternative indexed and non-indexed contracts with their respective wage formulae for macro-economic stability.

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14 See Stiglitz (1975) for a discussion of firms behaving as risk-aversers.

15 See Williamson, Wachter and Harris (1975) for an interesting examination of labor contracts.

16 Obviously not all labor works on long-term contracts; for purposes of this paper it is necessary only that some labor do so.
IV. Non-Indexed Labor Contracts

We have now set the stage for the introduction of the third model used to study indexing. As before, the model consists of an output supply equation, an aggregate demand equation, and wage setting behavior. The innovation of the model is in wage-setting and the consequent output supply equation. All labor contracts run for two periods. In the non-indexed case, the contract specifies a nominal wage for each period. The model differs from those of Sections I and II in another respect: the real and nominal disturbances are no longer treated as serially independent.

In period t, half the firms are operating in the first year of a labor contract drawn up at the end of (t-1) and the other half in the second year of a contract drawn up at the end of (t-2). Given that the wage is predetermined for each firm, and assuming employment is determined by the demand for labor, the aggregate supply of output is:

\[ y_t^S = \frac{1}{2} \sum_{i=1}^{2} (P_t - t-iW_t) + u_t \]

where \( t-iW_t \) is the wage to be paid in period t as specified in contracts drawn up at the end of period (t-i). The properties of the real disturbance term will be specified below.

The aggregate demand equation (2) is repeated here for convenience:

\[ y_t = M_t - P_t - v_t \]

where the properties of the nominal disturbance \( v_t \) remain to be specified. The inclusion of an interest elastic velocity function is discussed in the Appendix.
Next we consider wage-setting. The labor contract drawn up at time \( t \) specifies nominal wages for times \( (t+1) \) and \( (t+2) \). Assuming, in accordance with the discussion of Section III, that contracts are drawn up to maintain constancy of the expected real wage, we specify:

\[
(28) \quad t-i^W = t-i^P \quad i = 1,2
\]

where \( t-i^P \) is the price level expected at the end of period \( (t-i) \) to prevail in period \( t \). To prevent misunderstanding it should be noted that the use of a one-period contract, and not a two-period contract, is optimal from the viewpoint of minimizing the variance of the real wage; there must be reasons other than stability of the real wage, such as the costs of frequent contract negotiations and/or wage-setting, for the use of long-term contracts. It should be noted that in this model, the goal of maintaining constancy of the real wage and that of maintaining constancy of labor income both imply \( (28) \); \( (28) \) could also be thought of as the result of a process of wage-setting in which each worker tries to set a wage equal to that of others at full employment, as in the Phelpsian process modelled by Parkin (1975).

It is now assumed that each of the disturbances follows a first-order autoregressive scheme:

\[
(29) \quad u_t = \rho_1 u_{t-1} + \varepsilon_t \quad |\rho_1| < 1
\]

\[
(30) \quad v_t = \rho_2 v_{t-1} + \eta_t \quad |\rho_2| < 1
\]

\[17\] No substantive results would be changed by the inclusion of a constant in \( (28) \) to reflect an equilibrium real wage other than \( e \).
where \( \varepsilon_t \) and \( \eta_t \) are serially and mutually uncorrelated stochastic terms with means zero and variances \( \sigma^2_\varepsilon \) and \( \sigma^2_\eta \) respectively. It should be clear that if the disturbances are not serially correlated, and given that the nominal wage is predetermined in both the indexed and non-indexed systems, then there will be no difference in the behavior of the two systems. To attempt to maintain constancy of the real wage the nominal wage would be set at the same level each period in both systems. It is only when current price and output movements convey information about future movements - when disturbances are serially correlated - that any differences between the behavior of the systems can arise.\(^{18}\) It should accordingly be appreciated that the assumption of serial correlation of disturbances is central to the analysis of the difference between the systems presented in the remainder of the paper.

Finally, we shall again assume that expectations are determined rationally. Using (28), and eliminating \( Y_t \) between (26) and (27), which is equivalent to assuming the price level adjusts to equate aggregate supply to demand:

\[
(31) \quad 2P_t = M + 1/2 \sum_{i=1}^{2} t-iP_t - (u_t + v_t)
\]

Using rational expectations, and noting that \( E_t[ t-1P_t ] = t-2P_t \),

we obtain:

\[
(32) \quad t-2P_t = t-2(u_t + v_t) = t-2W_t
\]

\(^{18}\) This was not true in the previous models because there the nominal wage could react to the current price level in the indexed system while it could not do so in the non-indexed system.
where \( X_{t-j} \) denotes the expectation of variable \( X_t \) taken at time \( (t-j) \), except for \( W \), and where \( M \), the logarithm of the money supply, has for convenience been set at zero.

Substituting (32) and (33) into (31) we obtain

\[
(34) \quad 2P_t = -(u_t + v_t) - \frac{1}{3} \left( \rho_1 u_{t-1} + \rho_2 v_{t-1} \right) - \frac{2}{3} \left( \rho_1^2 u_{t-2} + \rho_2^2 v_{t-2} \right)
\]

\[
(34') \quad 2P_t = -\left( \varepsilon_t + \eta_t \right) - \frac{4}{3} \left( \rho_1 \varepsilon_{t-1} + \rho_2 \eta_{t-1} \right) - \frac{2}{3} \left( \rho_1^2 \varepsilon_{t-2} + \rho_2^2 \eta_{t-2} \right) - 2 \left( \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} + \sum_{i=1}^{\infty} \rho_2^i \eta_{t-i} \right)
\]

Then using (34) in (26):

\[
(35) \quad 2Y_t = u_t - v_t + \frac{1}{3} \left( \rho_1 u_{t-1} + \rho_2 v_{t-1} \right) + \frac{2}{3} \left( \rho_1^2 u_{t-2} + \rho_2^2 v_{t-2} \right)
\]

\[
(35') \quad 2Y_t = \varepsilon_t - \eta_t + \frac{4}{3} \rho_1 \varepsilon_{t-1} - \frac{2}{3} \rho_2 \eta_{t-1} + 2 \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} + \sum_{i=1}^{\infty} \rho_2^i \eta_{t-i}
\]

Examining (34') we see that all current and lagged disturbances affect the price level. From (35'), however, we notice that only the current nominal disturbance and that disturbance lagged once affect output. All earlier nominal disturbances were known at the time the oldest existing labor contract
was drawn up and the nominal wage has accordingly adjusted to remove any real effect of such disturbances. The term $\eta_{t-1}$ affects current output because the real wage in the second year of the oldest existing contract is affected by that nominal disturbance. All current and lagged real disturbances, by contrast, affect output.

Equations (34') and (35') will be used in Section VI below to examine the dynamic responses of prices and output in the non-indexed system to disturbances. They are also used to calculate the asymptotic variances of the price level and output:

\begin{align}
\sigma_p^2 &= \sigma_e^2 \left[ \frac{1}{4} + \frac{4}{9}\rho_1^2 + \frac{\rho_1^4}{1 - \rho_2^2} \right] + \sigma_\eta^2 \left[ \frac{1}{4} + \frac{4}{9}\rho_2^2 + \frac{\rho_2^4}{1 - \rho_2^2} \right] \\
\sigma_y^2 &= \sigma_e^2 \left[ \frac{1}{4} + \frac{4}{9}\rho_1^2 + \frac{\rho_1^4}{1 - \rho_1^2} \right] + \sigma_\eta^2 \left[ \frac{1}{4} + \frac{1}{9}\rho_2^2 \right]
\end{align}

V. Indexed Wages

The formula by which wages are indexed is obviously one of the major determinants of the effects of indexing on stability. In this section we use an indexing formula, "general indexing", which minimizes the variance of the real wage, subject to the nominal wage being predetermined. However, because the implied indexing formula is unlike most seem in practice we discuss in Section VII the implications of an alternative "price indexing" formula in which wages adjust solely on the basis of realized inflation.
An essential difference between indexed and non-indexed contracts is that wages for a subsequent period in indexed contracts can be adjusted in accord with events which occur after the signing of the contract. Given that the wage to prevail in period $t$ can be adjusted on the basis of events occurring up to and in period $(t-1)$, it is clear that the nominal wage which minimizes the variance of the real wage is

$$t^{-i}W_t = t^{-i}P_t$$

Any contract, of any length, could specify (38) as the wage adjustment formula. The contract can be taken to be of length $m$, where $m$ is determined by considerations not involving the variance of the real wage.

Now replace (26), the supply equation by

$$Y_t = \frac{1}{m} \sum_{i=1}^{m} (P_t - t^{-i}W_t) + u_t$$

$$= P_t - t^{-i}P_t + u_t$$

and retain

$$Y_t = M_t - P_t - v_t$$

Using (27) and (39), rational expectations, and assuming $M_t = 0$, we obtain

$$t^{-i}P_t = W_t = bP_{t-1} + cY_{t-1} - dW_{t-1}$$
where \( b = (\rho_1 + \rho_2) \), \( c = (\rho_2 - \rho_1) \)

Alternatively

\[
(41) \quad t-l^1 p_t = W_t = b \sum_{i=0}^{\infty} (-\rho_1)^i (p_{t-1-i})
\]

\[
+ c \sum_{i=0}^{\infty} (-\rho_1)^i (y_{t-1-i})
\]

For \( .5 = \rho_2 = \rho_1 \), we obtain

\[
(42) \quad W_t = \sum_{i=0}^{\infty} (.5)^i [ p_{t-1-i} - p_{t-2-i} ]
\]

which approximates a contract indexed to the rate of inflation.

In general, however, it is clear that (40) and (41) are not similar to contracts indexed to the price level, nor for that matter, do they produce wage setting equations which look much like those used in typical Phillips curves, except for \( \rho_1 < 0 \). For \( \rho_1 < 0 \), (40) perhaps resembles some form of profit-sharing contract.

Using (40), (39) and (27) we solve for the price level in terms of current and lagged disturbances:

\[
(43) \quad p_t = -\frac{1}{2} (\epsilon_t + \eta_t) - \sum_{i=1}^{\infty} \rho_1^i \epsilon_{t-i} - \sum_{i=1}^{\infty} \rho_2^i \eta_{t-i}
\]
The price level is accordingly affected by all current and lagged disturbances. Using the same equations to solve for the level of output:

\[
Y_t = \frac{1}{2} (\epsilon_t - \eta_t) + \sum_{1}^{\infty} \rho_1^l \epsilon_{t-l}
\]

The level of output is unaffected by lagged nominal disturbances since these are embodied in the setting of the nominal wage.

The asymptotic variances of the price level and output are obtained by using (43) and (44):

\[
\sigma_p^2 = \sigma_\epsilon^2 \left[ \frac{1}{4} + \frac{\rho_1^2}{1 - \rho_1^2} \right] + \sigma_\eta^2 \left[ \frac{1}{4} + \frac{\rho_2^2}{1 - \rho_2^2} \right]
\]

\[
\sigma_y^2 = \sigma_\epsilon^2 \left[ \frac{1}{4} + \frac{\rho_1^2}{1 - \rho_1^2} \right] + \frac{\sigma_\eta^2}{4}
\]

VI. Short-Run Responses to Disturbances and Steady-State Variances in the Multi-Period Labor Contract Model.

We now draw together the analyses of the last two sections. To compare short run responses to disturbances we use (34') and (35') and (43) and (44). Table I summarizes the relevant information concerning the dynamic multipliers.
TABLE I.

Dynamic Multipliers for Real and Nominal Disturbances in Multi-Period Contract Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Disturbance</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>NI&lt;sup&gt;y&lt;/sup&gt;</td>
<td>Real</td>
<td>1/2</td>
</tr>
<tr>
<td>GI&lt;sup&gt;y&lt;/sup&gt;</td>
<td>Real</td>
<td>1/2</td>
</tr>
<tr>
<td>NI&lt;sup&gt;y&lt;/sup&gt;</td>
<td>Nominal</td>
<td>-1/2</td>
</tr>
<tr>
<td>GI&lt;sup&gt;y&lt;/sup&gt;</td>
<td>Nominal</td>
<td>-1/2</td>
</tr>
<tr>
<td>NI&lt;sup&gt;p&lt;/sup&gt;</td>
<td>Real</td>
<td>-1/2</td>
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<td>Real</td>
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<tr>
<td>NI&lt;sup&gt;p&lt;/sup&gt;</td>
<td>Nominal</td>
<td>-1/2</td>
</tr>
<tr>
<td>GI&lt;sup&gt;p&lt;/sup&gt;</td>
<td>Nominal</td>
<td>-1/2</td>
</tr>
</tbody>
</table>

The results are clear. The only period in which there is any difference in response pattern is the first lagged period. The reason should
also be clear. There cannot be any differential response to a current
perturbation because the nominal wage is predetermined in both cases. Nor
can there be any differential response to perturbations two periods or more old,
since all contracts are renegotiated over that time span in both systems.
The only difference arises from the assumed inability to renegotiate the
nominal wage for the second year of the two period non-indexed contract.
That short-run stickiness of the nominal wage tends to reduce the effects
on output of real perturbations by creating changes in the real wage. By
the same token, that stickiness of the nominal wage transmits nominal
disturbances to real income in the non-indexed system in period one that
are not transmitted in the indexed system.

Thus, once again, viewing a monetary stabilization policy as a nominal
disturbance, that policy has more rapid effects on prices and smaller
effects on output in the indexed than in the non-indexed system. The
dynamic multipliers of Table I should be regarded as illustrative of a
general point: if non-indexed labor contracts are for longer than two
periods, then there will be differences in the dynamic responses over
periods as long as the contract period, with the difference in general
being that output responses to nominal disturbances will be smaller in the
indexed system and output responses to real disturbances will be larger in
the indexed system.

The comparisons among asymptotic variances are in accord with the
short run results. Using (36), (37), (45) and (46):

\[
\begin{align*}
(47) \quad & NIT - G1 Y = \sigma_{\varepsilon}^2 \left[ - \frac{5}{9} \rho_1^2 \right] + \sigma_{\eta}^2 \left[ \frac{\rho_2^2}{9} \right] \\
(48) \quad & NIT - G1 P = \sigma_{\varepsilon}^2 \left[ - \frac{5}{9} \rho_1^2 \right] + \sigma_{\eta}^2 \left[ - \frac{5}{9} \rho_2^2 \right]
\end{align*}
\]
Thus the non-indexed system has a more stable price level than the indexed system; the non-indexed system has more stable output in the face of real disturbances and less stable output in the face of nominal disturbances. These results confirm the fundamental result of Section I - that indexing stabilizes output in the face of nominal disturbances.

VII. Further Considerations.

In this section we shall first provide a further explanation for the strong results of Sections IV - VI and then proceed to discuss elements omitted from the analysis that might affect those results.

The models of this paper indicate that wage indexing destabilizes output in the face of real disturbances while stabilizing output in the face of nominal disturbances. The source of these results is simple: the models of Section IV and V assume that the nominal wage is set in such a way as to try to maintain constancy of the real wage and/or labor income; our criterion of stability has been the variance of output. That wage rate, set at the end of period (t-1), which minimizes the variance of the real wage (the indexed wage of Section V) is not the wage which minimizes the variance of output.\(^{19}\) Thus, the same conflict that has been invoked to explain the existence of unemployment explains the results of this paper. Nonetheless, we should recognize that the professional presumption is that private contracts, freely entered into, lead to desirable outcomes in the absence of externalities. Accordingly, the welfare question of the desirability of the stability of real output requires further analysis, which is beyond the scope of this paper.

There are five major considerations that could alter the fundamental conclusions: interest elasticity of velocity; the use of alternative

\(^{19}\) We leave it as an exercise to find the indexing formula for wages that minimizes output variance.
indexing formulae; expectational errors; the consideration of relative wage and price changes; and inflexibility of the absolute price level.

In the Appendix we present a Keynesian-type model in which the demand for goods is a function of the real interest rate, the demand for money a function of the nominal rate, and expectations are formed rationally. The aggregate demand or velocity equation (2) of this paper then becomes a function of the expected rate of inflation. Depending on whether contracts are indexed or not, current price and output are differentially affected by current disturbances - in contrast to the results depicted in Table I. In particular, in an indexed system in which nominal wages are relatively flexible, a current nominal disturbance creates the expectation of a higher price level next period if nominal disturbances are positively correlated than would be expected if wages were sticky. Such a disturbance accordingly has a greater effect on the current price level in the indexed than in the non-indexed system and thus a greater current effect on output in the indexed system. It therefore appears possible that real output could fluctuate more in the indexed system than in the non-indexed system in response to nominal disturbances, although it is shown in the Appendix that does not happen in the model studied there.

Second, we noted in Section V that the general indexing formula is unlike most seen in practice. A more usual type of indexing formula is of the nature:

\[ t-i W_t = t-i W_{t-1} + P_{t-1} - P_{t-i} \]

in which the wage paid in period \( t \) on a contract made at the end of \( (t-i) \) is the wage specified for the first year of the contract adjusted for inflation over the intervening period. Such contracts stabilize output in the face of nominal disturbances only if the nominal disturbances tend to persist - in other words for large positive values of \( \rho_2 \) in (30). They will destabilize output if nominal disturbances are transitory or if disturbances are predominantly real. Thus the indexing formulae used in
practise may be a far cry from the idealized general indexing formula of Section V and are certain to be less stabilizing in the face of nominal disturbances than that formula. They may by good fortune provide a stabilizing influence against real shocks but there is no guarantee of that.

Third, we consider expectational errors. One of the benefits of indexing is that it allows nominal wages to react to actual rather than expected events. To the extent that the resultant behavior of the real wage is appropriate for stability of output, indexing will accordingly produce more stable real output. For a contract of given length and with an appropriate indexing formula, an indexed contract produces real wages closer on average to those anticipated than does a nominal contract. Of course, it is always possible, as in the models of this paper, that the expectational errors of non-indexed contracts are actually stabilizing - as they will be for unanticipated real disturbances. However, without a good theory of expectational errors, it is not in general possible to conclude how the smaller errors that generally occur with indexing affect the stability of output, unless indexed contracts are drawn up with the aim of maintaining stability of output.

Fourth, consider relative wage and price changes. Suppose that some wages are indexed and some not. Then, in response to disturbances, relative wages change, at least on contracts of a given vintage. Such changes can themselves exacerbate unemployment by making it more profitable to search longer for the higher wages.\textsuperscript{20} Indexing of wages in one sector - such as government employment - and not others might accordingly have output effects in a manner not considered in this paper. Similarly, this one-good model does not analyze the effects of indexation on relative price variability\textsuperscript{21} and the consequences, if any, of such variability for macroeconomic stability.

\textsuperscript{20} This point is inspired by a recent paper by Robert Hall (1975).

\textsuperscript{21} Relative price variability is the focus of Barro's (1975) indexing paper.
A priori, it is not clear how such considerations would affect our results. Finally, the aggregate price level has been assumed to adjust instantly to equate the supply of and demand for output; the effects of price inflexibility have not been considered. It would seem that price indexing cannot be of consequence if the absolute price level is fixed so that it is unlikely the results would be revised by limiting the flexibility of prices.

VIII. Concluding Comments.

This final section comments on a number of issues connected with wage indexing, some of which can be analyzed using the models of the preceding section, before summarizing the basic results.

1. Stabilizing monetary policy: In the non-indexed economy of Section IV, monetary policy, even when fully anticipated, can play a role in stabilizing output. In that economy, output stabilization requires money stock increases in response to real disturbances that tend to increase the price level and money stock decreases in response to nominal disturbances tending to increase the price level. In the generally indexed system, pre-announced monetary policy has no effect on output which fluctuates more than in the non-indexed system. If wages are indexed by any other formula there is room for an active stabilizing monetary policy.

Since monetary authorities sometimes want to use monetary policy to affect real output - at which times they would prefer non-indexed wages - and at other times want to reduce the inflation rate without affecting output - at which times they would prefer indexed wages - there is no presumption that either system would always be preferable to the monetary authority. There is probably a temptation to believe that indexing can be encouraged and discouraged at the appropriate times but that temptation should be avoided. Those factors that make wages and prices sticky and give

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22 This point is analyzed in Fischer (1975)
monetary policy real effects also mean that indexation is slow to be introduced and probably slow to be abandoned. Wage indexation is not a short-run stabilization device even though its presence or absence affects the success of short-run stabilization policy.

2. Indexing the money stock: Increasing the money stock in response to the lagged inflation rate is usually, but not always destabilizing in the models of this paper - the serial correlation properties of the disturbances may be such as to prevent instability if the money stock is indexed. However, in a more general context, if the money stock is indexed by means of interest payments on money, that does not rule out potentially stabilizing open market operations.23

3. The use of special indexes and partial indexing: The question of the price index to be used for wage indexation has previously received attention.24 Following the general principle of not allowing nominal wages to adjust for real disturbances, an index excluding import prices and indirect taxes, such as a price index of domestic value added might be appropriate. The models of this paper do not bear directly on the issue through it is clear that a general indexing formula in the sense of Section V could be constructed which would adjust nominal wages only in response to nominal disturbances. Such a formula would imply tying wages to the lagged value of velocity; partial indexation in which wages adjust less than proportionately to prices is not equivalent to this formula.

4. Identification of disturbances: This is not the first paper in which results have depended on the source of economic disturbances.25 At this stage we do not have a good notion of the relative variances of real and nominal disturbances. Since the identification of disturbances also requires the classification of policy actions as offsetting disturbances or creating them, the problem is not simple.

23 This point is due to Franco Modigliani.

24 For instance, by the Israeli Committee of Experts referred to in footnote 2 above. I have also benefitted from the views of Franco Modigliani on this question.

25 See Fischer (1974) and Poole (1970) for previous examples.
5. Conclusion: The fundamental results of the paper are that wage indexation stabilizes real output if the disturbances impinging on the economic system are nominal and destabilizes output if disturbances are real. Further, the short run response of the economy to monetary shocks and monetary stabilization provides greater and more rapid price level effects and smaller real effects in an indexed than in a non-indexed system. Thus monetary policies designed to reduce the inflation rate will operate more rapidly and with less impact on output in an economy with indexed wages than in a non-indexed economy. By the same token, expansionary monetary policies will have a smaller real effect and a more rapid effect on prices in an indexed than in a non-indexed economy.
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APPENDIX: Indexing and the Interest Elasticity of the Demand for Money.

In this appendix we examine the consequences for the basic results of the paper of having an interest elastic demand for money. The reason for undertaking the enquiry is the suspicion that current nominal disturbances will have differential impacts on prices in indexed and non-indexed systems and may thus have differential impacts on output if the demand for money is interest elastic.

To keep the appendix to a reasonable length, the exposition is terse. Familiarity with the non-indexed and indexed systems of Sections IV and V is assumed. Variables retain the same definitions. The aggregate supply equations of the text are retained, as is the assumption that the nominal wage is set to maintain constancy of the real wage.

1. The aggregate demand or velocity equation (2) of the text is replaced by IS and LM curves:

\[ Y_t = a_1 Y_t + b_1 r_t + c_1 (M_t - P_t) + \epsilon_{1t} \quad l > a_1 > 0, \ b_1 < 0, \ c_1 > 0 \]

where \( r_t \) is the logarithm of the real interest rate, and

\[ M_t - P_t = a_2 Y_t + b_2 (r_t + t^{P_{t+1}} - P_t) + \epsilon_{2t} \quad a_2 > 0, \ b_2 < 0 \]

where \( t^{P_{t+1}} - P_t \) is the expected one period inflation rate, and \( \epsilon_{1t} \) and \( \epsilon_{2t} \) are mutually uncorrelated random disturbances. Substituting (2) into (1) and eliminating \( r_t \), we obtain
(3) \[ M_t - P_t \left[ 1 + \frac{b_2 c_1}{b_1} \right] + b_2 P_t = \gamma_t \left[ a_2 + (1-a_1) \frac{b_2}{b_1} \right] + b_2 t P_{t+1} + \varepsilon_{2t} - \frac{b_2}{b_1} \varepsilon_{1t} \]

Setting \( M_t = 0 \), we write

\[ (3') \quad \alpha P_t = -\beta \gamma_t - b P_{t+1} - \nu_t \]

where

\[ \alpha = 1 + \frac{b_2 c_1}{b_1} - b_2 > 0 \]

\[ \beta = a_2 + (1-a_1) \frac{b_2}{b_1} > 0 \]

\[ b = b_2 < 0 \]

\[ \nu_t = \varepsilon_{2t} - \frac{b_2}{b_1} \varepsilon_{1t} \]

We henceforth assume \( \nu_t \) has the same properties as \( \nu_t \) in equation (30) of the text, i.e.

\[ (4) \quad \nu_t = \rho_2 \nu_{t-1} + \eta_t \quad \left| \rho_2 \right| < 1 \]

It is assumed that both \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) have the same coefficient of serial correlation.
2. Consider first the general indexing case of Section V in which output is determined by

\[ y_t = p_t - t^{-1} p_t + u_t \]

with the properties of \( u_t \) described by (29) of the text.

Substituting (5) into (3'):

\[ (\alpha + \beta) p_t = \beta t^{-1} p_t - b(t+1) - v_t - \beta u_t \]

Using a trial solution of the form (7) to obtain the rational expectation

\[ p_t = \sum_{i=0}^{\infty} \Pi_i u_{t-i} + \sum_{i=0}^{\infty} \omega_i v_{t-i} \]

and selecting out the stable solution for \( p_t \) (an assumption that sometimes masquerades under the guise of "ruling out speculative bubbles"), we find

\[ \Pi_0 = -[(\alpha + \beta)(\alpha + b \rho_1)]^{-1} \alpha \beta \quad \Pi_1 = -[(\alpha + \beta)(\alpha + b \rho_1)]^{-1} \beta ^2 \rho_1 \]

\[ \Pi_i = 0 \quad i = 2,3, \ldots \]
\[ (9) \quad \omega_o = -\alpha \left[ (\alpha + \beta) (\alpha + \beta \rho_2) \right]^{-1} \quad \omega_1 = -[(\alpha + \beta) (\alpha + \beta \rho_2)]^{-1} \beta \rho_2 \]

\[ \omega_i = 0, \quad i = 2, 3, \ldots \]

Accordingly

\[ (10) \quad G\Pi_t = \prod_{o} \varepsilon_t - (\alpha + \beta \rho_1)^{-1} \beta \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} + \omega_o \eta_t - (\alpha + \beta \rho_2)^{-1} \sum_{i=1}^{\infty} \rho_2^i \eta_{t-i} \]

and

\[ (11) \quad G\pi_t = (1 + \prod_{o}) \varepsilon_t + \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} + \omega_o \eta_t \]

The last two equations (10) and (11) provide the dynamic responses of prices and output to disturbances. Asymptotic variances are:

\[ (12) \quad G\Pi \sigma_P^2 = \sigma_{\varepsilon}^2 (\alpha + \beta \rho_1)^{-2} \beta^2 \left[ (\alpha + \beta)^{-2} \alpha^2 + (1 - \rho_1^{-2})^{-1} \rho_1^2 \right] \]

\[ + \sigma_{\eta}^2 (\alpha + \beta \rho_2)^{-2} \left[ (\alpha + \beta)^{-2} \alpha^2 + (1 - \rho_2^{-2})^{-1} \rho_2^2 \right] \]
and

\[ G_1^2 \sigma_Y^2 = \sigma \varepsilon^2 \left[ \left( (\alpha + \beta) (\alpha + b \rho_1) \right)^{-2} \left[ \alpha^2 + (\alpha + \beta) b \rho_1 \right]^2 + \left( 1 - \rho_1 \right)^{-1} \rho_1 \right] \]

\[ + \sigma \eta^2 \left[ (\alpha + \beta) (\alpha + b \rho_2) \right]^{-2} \]

We return to (10) - (13) later.

3. Next we proceed to two period nominal wage contracting - the model of Section IV of the text. The output equation becomes

\[ Y_t = P_t - \frac{1}{2} (t-1 P_t + t-2 P_t) + u_t, \]

resulting in

\[ (\alpha + \beta) P_t = \frac{\beta}{2} t-1 P_t + \frac{\beta}{2} t-2 P_t - b P_{t+1} - \beta u_t - v_t \]

Now we use a trial solution for the rational expectations equations for \( P_t \):

\[ P_t = \sum_{i=0}^{\infty} \theta_i u_{t-i} + \sum_{i=0}^{\infty} \xi_i v_{t-i} \]
Again selecting out the stable solution for $P_t$, we obtain:

\begin{equation}
\theta_0 = -\beta [(\alpha + \beta)(2\alpha + \beta)(\alpha + \beta \rho_1)]^{-1} [\alpha(2\alpha + \beta) + \beta \rho_1]
\end{equation}

\begin{equation}
\theta_1 = -\beta^2 \rho_1 [(\alpha + \beta)(2\alpha + \beta)(\alpha + \beta \rho_1)]^{-1} [\alpha - \beta \rho_1]
\end{equation}

\begin{equation}
\theta_2 = -\beta^2 \rho_1^2 [(2\alpha + \beta)(\alpha + \beta \rho_1)]^{-1}
\end{equation}

\begin{equation}
\theta_i = 0 \quad i = 3, 4, \ldots
\end{equation}

\begin{equation}
\xi_0 = - [(\alpha + \beta)(2\alpha + \beta)(\alpha + \beta \rho_2)]^{-1} [\alpha(2\alpha + \beta) + \beta \rho_2]
\end{equation}

\begin{equation}
\xi_1 = -\beta \rho_2 [(\alpha + \beta)(2\alpha + \beta)(\alpha + \beta \rho_2)]^{-1} [\alpha - \beta \rho_2]
\end{equation}

\begin{equation}
\xi_2 = -\beta^2 \rho_2^2 [(2\alpha + \beta)(\alpha + \beta \rho_2)]^{-1}
\end{equation}

\begin{equation}
\xi_i = 0 \quad i = 3, 4, \ldots
\end{equation}

Hence
\begin{align}
(19) \quad NIP_t &= \theta_o \varepsilon_t - 2\alpha\beta\rho_1 \left[ (2\alpha + \beta)(\alpha + \beta_1) \right]^{-1} \varepsilon_{t-1} - (\alpha + \beta_1)^{-1} \frac{1}{2} \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} \\
&\quad + \varepsilon_o \eta_t - 2\alpha\rho_2 \left[ (2\alpha + \beta)(\alpha + \beta_2) \right]^{-1} \eta_{t-1} - (\alpha + \beta_2)^{-1} \frac{1}{2} \sum_{i=1}^{\infty} \rho_2^i \eta_{t-i} \\
\text{and} \\
(20) \quad NI Y_t &= \left( \theta_o + 1 \right) \varepsilon_t - \left[ \alpha\beta\rho_1 \left[ (2\alpha + \beta)(\alpha + \beta_1) \right]^{-1} \rho_1 \right] \varepsilon_{t-1} + \sum_{i=1}^{\infty} \rho_1^i \varepsilon_{t-i} \\
&\quad + \varepsilon_o \eta_t - \alpha\rho_2 \left[ (2\alpha + \beta)(\alpha + \beta_2) \right]^{-1} \eta_{t-1}
\end{align}

The asymptotic variances are:

\begin{align}
(21) \quad NIP^2 &= \sigma_\varepsilon^2 \left[ \theta_o^2 + 4\alpha^2\beta^2 \rho_1^2 \left[ (2\alpha + \beta)(\alpha + \beta_1) \right]^{-2} + (\alpha + \beta_1)^{-2} \rho_1^4 (1-\rho_1^2)^{-1} \right] \\
&\quad + \sigma_\eta^2 \left[ \varepsilon_o^2 + 4\alpha^2\rho_2^2 \left[ (2\alpha + \beta)(\alpha + \beta_2) \right]^{-2} + (\alpha + \beta_2)^{-2} \rho_2^4 (1-\rho_2^2)^{-1} \right]
\end{align}

\begin{align}
(22) \quad NI Y^2 &= \sigma_\varepsilon^2 \left[ (1 + \theta_o)^2 + \left[ \alpha\beta\rho_1 \left[ (2\alpha + \beta)(\alpha + \beta_1) \right]^{-1} \rho_1 \right]^2 + \rho_1^4 (1-\rho_1^2)^{-1} \right]
\end{align}
4. Short-Run Responses to Disturbances.

(i) Effects of current nominal disturbances on output and price. From (11) and (20) we note that a unit current nominal disturbance \( \eta_t = 1 \) affects current output by the amounts \( \omega_0 \) and \( \xi_0 \) in the indexed and non-indexed systems respectively. It can be shown that both \( \omega_0 \) and \( \xi_0 \) are negative. Further

\[
(23) \quad \omega_0 - \xi_0 = \left[ (2\alpha + \beta)(\alpha + \beta)(\alpha + \beta \rho_2) \right]^{-1} b \beta \rho_2
\]

Since \( b \) is negative, we know if \( \rho_2 > 0 \) that a current nominal disturbance has a smaller effect on output in a non-indexed than in an indexed system. The reason has already been given in the text. A current nominal disturbance that increases the price level can also be expected to increase next period's price level if \( \rho_2 > 0 \). The implied expected inflation will increase velocity and thus affect the current price level. In the non-indexed system wages are relatively sticky and next period's price should be expected to move less than in the indexed system. Accordingly the current price level too moves less in the non-indexed system and there is a smaller impact on output of a nominal disturbance than in the indexed system.

It is clear from (10) and (19) that the comparative effects of a current nominal disturbance on prices is also given by (23) and the above verbal discussion applies exactly.

(ii) Effects of lagged nominal disturbances on output and prices. In the generally indexed systems lagged nominal disturbances have no effects on output whereas the first lagged nominal disturbance does have an impact on output in the non-indexed system. The reason is clearly that the lagged
disturbance affects the real wage of those currently in the second year of their contracts. Interestingly, the cumulative impact of a nominal disturbance is greater for $\rho_2 > 0$ with non-indexed than with indexed wages, i.e.,

\[
(24) \quad \omega_o - \xi_o + \alpha \rho_2 \left[ (2\alpha + \beta)(\alpha + b \rho_2) \right]^{-1} > 0
\]

and since each effect is negative, the cumulative impact of a given nominal disturbance on output is clearly larger in the non-indexed than in the indexed system.

Examination of (10) and (18) also shows that the $\eta_{t-1}$ disturbance has the greater impact on the current price level in the indexed system, and that disturbances lagged two or more periods have precisely the same effect on price in both systems.

(iii) **Effects of current real disturbances on output and price.** From (11), (20) and (23), making the appropriate substitutions and since $(\Theta_o + 1)$ and $(\Pi_o + 1)$ are both positive we know that a current real disturbance has a greater effect on current output in the non-indexed than in the indexed system if $\rho_1 > 0$. The reasoning is precisely the same as that contained in paragraph 4 (i) above. This time the fact that prices do not move as much in the non-indexed system offsets the effect of the output disturbance by less than in the indexed system. By putting in the future we obtain an interesting reversal of roles of indexing and non-indexing with regard to the current impacts of disturbances. Note from (10) and (19) that a current real disturbance moves price more in the indexed than in the non-indexed system.

(iv) **Effects of lagged real disturbances on output and price:** Lagged real disturbances have a differential impact on output and price in the two
systems only in period \((t-1)\). The disturbance \(\varepsilon_{t-1}\) has a larger (positive) impact on output in the indexed than in the non-indexed system (for \(\rho_1 > 0\)), but the relative magnitude of the price effect is indeterminate. The comparative cumulative effect of a given real disturbance on output is of indeterminate sign, but for plausible parameter values it appears that a given real disturbance produces smaller cumulative output effects (for \(\rho_1 > 0\)) in the non-indexed than in the indexed economy. We obtain

\[
\theta_o + 1 + \rho_1 - \alpha \beta \rho_1 \left[ (2\alpha + \beta)(\alpha + b \rho_1) \right]^{-1} - (1 + \Pi_o + \rho_1)
\]

\[
= \frac{-\beta \rho_1}{\alpha (\alpha + \beta) - b \beta} \left[ (\alpha + \beta)(2\alpha + \beta)(\alpha + b \rho_1) \right]^{-1}
\]

For small or any plausible values of \(b\) this expression is negative given \(\rho_1 > 0\). Thus while the initial impact of a real disturbance is larger in the non-indexed than in the indexed system, its cumulative effects are smaller in the non-indexed system.

5. **Asymptotic Variances:** In terms of the asymptotic variance, output is, for small (and plausible) values of \(b\), more stable in the face of real disturbances in a non-indexed system and is definitely more stable subject to nominal disturbances in the indexed system. Prices are more stable in the non-indexed system in the face of both types of shocks for positive serial correlation of the errors.

6. **Conclusions:** The analysis of the appendix shows that the dynamic pattern of the response of the systems to given shocks changes when the demand for money is interest elastic - in particular, a current nominal shock may have a greater current output effect in the indexed than in the non-indexed system - but the asymptotic properties of the model are largely unaffected by the change.