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FEB 21 1997

Wage Inequality and Segregation by Skill[1]

Michael Kremer* and Eric Maskin**

August 2, 1996

Abstract

Evidence from the United States, Britain, and France suggests that recent growth in wage inequality has been accompanied by greater segregation of high- and low-skill workers into separate firms. A model in which workers of different skill-levels are imperfect substitutes can simultaneously account for these increases in segregation and inequality either through technological change, or, more parsimoniously, through observed changes in the skill-distribution.

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[1]We thank Daron Acemoglu, V.V. Chari, Benjamin Friedman, Lawrence Katz, Thomas Piketty, Sherwin Rosen, Kenneth Troske, and Finis Welch for suggestions; Andrew Bernard, Eli Berman, Steve Davis, Brad Jensen, Francis Kramarz, Steve Machin, and Ken Troske for suggestions and data; and Sergei Severinov and Charles Morcom for exceptionally helpful research assistance. We are grateful to the Center for Economic Studies, U.S. Bureau of the Census, for providing data. We acknowledge research support from the National Science Foundation. All opinions, as well as mistakes, are our own, and not those of the U.S. Bureau of the Census.

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Introduction

One of the most disturbing economic trends of the last fifteen to twenty years has been a growing wage gap between high- and low-skill workers in several industrialized economies, including that of the United States. This trend has been well documented. Katz and Murphy [1992], for example, find that, between 1979 and 1987, wages of men with a high school education and 1-5 years of experience fell by twenty percent, while those of male college graduates rose by ten percent.

This paper provides evidence that this increase in inequality has been accompanied by growing segregation of workers by skill. That is, over time, it has become less common for high- and low-skill workers to work in the same firm. Economic activity has shifted from firms such as General Motors, which use both high- and low-skill workers, to firms such as Microsoft and McDonald's, whose workforces are much more homogeneous. For example, in France, between 1986 and 1992, the correlation of log wages among workers in the same establishment rose from 0.36 to 0.44, the correlation of experience rose from 0.11 to 0.16, and the correlation of seniority grew from 0.24 to 0.31. In the United States, the correlation between the wages of manufacturing production workers in the same plant rose from 0.76 in 1975 to 0.80 in 1986, and the correlation of a dummy variable for being a production worker rose from 0.195 in 1976 to 0.228 in 1987.

We propose a simple model that can account for the simultaneous increases in inequality and in segregation, as well as for the absolute decline in wages of low-skill workers. In the model, workers of different skill-levels are imperfect substitutes, and output is more sensitive to skill in some tasks than in others. What happens in such a model when, as the evidence bears out (see below), the dispersion and mean of the skill-distribution increase? We show that a rise in skill-dispersion causes firms that once hired both high- and low-skill workers to specialize in one level or the other, thereby fostering segregation (Proposition 3). Although, in this model, an increase in mean skill-level can enhance the wages of the poor -- thus formalizing the so-called "trickle-down" principle (Proposition 1) -- we demonstrate that this principle relies crucially on a "tight" distribution of skills. Once the skill-distribution becomes sufficiently dispersed -- as it is now likely to be in the United States -- a further increase in mean skill-level raises the wages of high-skilled workers but causes those of poorly-skilled workers to decline, thereby aggravating inequality (Proposition 2).

Note that this last implication contrasts with a widespread view of the labor market articulated by Schmitt [1992]: "Rising returns to skills in the face of large increases in the supply of skilled labor suggest a substantial shift in labor demand in favor of skilled workers." Whereas labor economists such as Schmitt see rising mean-skill levels as deepening the mystery of increasing inequality, our analysis suggests this rise in mean-skill may have helped cause the increase in inequality.

The model's implication that increases in skill-dispersion promote segregation of workers by skill is consistent not only with the time-series data cited above, but also with evidence that, among U.S. states, those with wider dispersion in educational attainment are more segregated by education in employment. Casual empiricism suggests that developing countries, with wide dispersion of skill, are particularly prone to dualism.

We noted above that there is evidence that both the mean and the dispersion of the skill-distribution have increased in the economies studied in this paper. It has been argued (see Kahn and Lim [1994]) that skill-biased technological change may have raised the effective skill-level of high-skill workers, which would have had the posited effects on the distribution of skills. Of course, such technological change is difficult to measure. However, there is also documentation of increases in the mean and dispersion of observable skills levels (see Juhn, Murphy, and Pierce [1993]). These shifts have been fairly modest, but our model shows how they could have been magnified into significant changes in the income distribution and degree of segregation.

The remainder of the paper is organized as follows. Section I offers a sketch of our model and the theoretical results. Section II presents evidence that the United States, Britain, and France have become more segregated in recent years. Section III develops our theoretical propositions in detail. Section IV provides evidence that U.S. states with greater skill dispersion are more segregated by skill, as the model predicts. Section V interprets recent increases in inequality and segregation in light of the model. The conclusion discusses policy implications and speculates on the possibility that segregation may spur inequality by reducing opportunities for low-skill workers to learn from higher-skill co-workers, giving rise to a vicious cycle of increasing inequality and segregation.

I. A Sketch of the Theory

A model consistent with the stylized facts about segregation and inequality discussed above must incorporate three ingredients. Specifically, it should be the case that

(i) workers of different skills are imperfect substitutes for one another;

(ii) different tasks within a firm are complementary;

and

(iii) different tasks within a firm are differentially sensitive to skill.

To see why (i), imperfect substitutability, is needed, let us examine a model in which perfect substitutability obtains. Specifically, consider workers of two skill-levels q' and $q = 2q'$. Then, in every firm, a worker of skill q can be replaced by two of skill q', with no other operational change. Clearly, q-workers will be paid exactly twice as much as q'-workers regardless of the distribution of skills in the economy. Moreover, such a model makes no prediction about what skill-levels we will see in a firm: there could as easily be one worker of skill q as two of skill q'. To make any headway in explaining either inequality or segregation, therefore, imperfect substitutability of skill is required.

In fact, there is a body of work within the labor economics literature that assumes such imperfect substitutability (see Sattinger [1993] for a review). There is also empirical evidence to justify the assumption; see Katz and Murphy [1992] and Murphy and Welch [1992].

To understand the role of assumption (ii), complementarity, consider a firm in which there are two tasks -- a "g" task and an "h" task -- but no complementarity between them. Specifically, take a production function in which, if a q-worker is hired for the g-task and a

q'-worker for the h-task, output is

(1) $$f(q,q') = g(q)+h(q').$$

Different skills are imperfect substitutes in this formulation: only one worker can be hired for each task, and so the sort of two-for-one substitution contemplated above is ruled out. This means that relative wages may depend on the distribution of skills; if, for example, the number of q-workers rises relative to that of q'-workers, the former workers' wages are likely to experience a relative decline. Thus, unlike the first model, this formulation may say something about inequality. However, it still implies nothing about the skill-composition of firms. In particular, because tasks are not complementary, the optimal choice of skill for the g-task is independent of that for the h-task.

Finally, to understand the need for (iii), differential skill-sensitivity, consider a production function in which different tasks are complementary but identical in their sensitivity to skill:

(2) $$f(q,q') = qq'.$$

This is the production function used by Becker [1981] in his analysis of assortative marriage, and also by Kremer [1993] in his study of labor markets and development. It has the implication that there is always <u>complete</u> segregation of skill within firms: q-workers are matched only with other q-workers, etc. To see this, imagine that there are two q-workers and two q'-workers. Then either the q-workers can be matched with each other and the same for the q'-workers (<u>self-matching</u>) or else there can be two matches consisting of one q- and one q'-workers each (<u>cross-matching</u>). But self-matching leads to total output of $q^2+q'^2$

which is greater than the output of $2qq'$ resulting from cross-matching. Hence, regardless of the distribution of skills, production function (2) implies that we should see only self-matching, and so the evolution from General Motors to Microsoft/McDonald's discussed above could not have taken place. We shall therefore introduce an asymmetry between tasks by working with a production function of the form,

$$f(q,q') = q^c q'^d, \tag{4}$$

where $0<c<d$. By redefining the unit of skill, we can rewrite this as

$$f(q,q') = qq'^e, \quad e>1. \tag{3}$$

For ease of computation (although none of the qualitative results depend on this), we shall, in fact, take e=2, so that

$$f(q,q') = qq'^2. \tag{5}$$

We can think of the q'-task (which is relatively sensitive to skill) as the "managerial" task and the q-task (which is relatively skill-insensitive) as the "assistant's" role. Hence, let us rewrite the production function as

$$f(q_a,q_m) = q_a q_m^2. \tag{6}$$

Production function (6) has the implication that if a firm employs workers of skills L and H ($L < H$), it is more efficient for the H-worker to be the manager (since $LH^2 > HL^2$). This production function is closely related to one used by Rosen [1981, 1982], Miller [1983], and Lucas [1978]:

(7) $$f(q_1,\ldots,q_r,q_m) = q_m f(\sum_{i=1}^{r} q_i),\ f'>0\ ,f''<0,$$

where q_m is the skill of the manager, q_i is the skill of subordinate i, and r, the number of subordinates, is a choice variable. Under production function (7), there is no substitutability of skill at the managerial level, but several low-skill subordinates can be substituted for one high-skill subordinate. Hence, in equilibrium, low-skill workers become subordinates, and high-skill workers become managers. The higher a manager's skill, the more employees he supervises.

By contrast, our production function (6) imposes imperfect substitutability on subordinates as well as on managers. (In this sense, it combines the approaches inherent in (2) and (7).) This has the implication that, in equilibrium, a manager of one firm may be of the same or lower skill than a subordinate in another firm (even though, as already noted, managers are more highly skilled than subordinates within the same firm). For example, an M.B.A. might choose either to accept an entry-level position with high-skill colleagues at an investment bank, or a more senior position at a less prestigious firm.

We consider a one-good economy in which there is an exogenous distribution of workers of different skill. We take this distribution as given because, although ultimately it too is endogenous, it is likely to change more slowly than the wage and matching patterns of interest to us. We suppose that there is an indefinite supply of potential firms, all of which have the production function (6). Hence, this is a competitive economy with constant returns to scale.

Our theoretical results (Propositions 1-3 in Section III) examine how the competitive equilibrium wage distribution and the pattern of skill-levels within firms are affected by shifts in the distribution of skills. Here we give a simplified account of these results.

Proposition 3 shows that, as the dispersion of skills increases in the economy, the relative dispersion of skill within firms falls. To get a feel for this, suppose that there are just two skill-levels, L and H, where $L < H$. Now, given two workers of skill L and two workers of skill H, it is more efficient for the workers to cross-match (L-workers match with H-workers) rather than self-match (workers of the same skill-level match) if and only if

$$L^3 + H^3 < 2LH^2. \tag{8}$$

Thus because competitive equilibrium is efficient, we will have equilibrium cross-matching if and only if (8) is satisfied, i.e.,

$$H < \frac{1 + \sqrt{5}}{2} L. \tag{9}$$

Imagine that the economy begins with a skill-distribution in which L and H are fairly close in value. From (9), we will have cross-matching in equilibrium, i.e., each firm -- following the G.M. pattern -- will employ both low- and high-skill workers. (This cannot be literally true if there are different numbers of L- and H-workers in the economy. If, say, there are more H's than L's, all the L's will be matched with H's, and the remaining H's will be self-matched.)

Suppose now that the dispersion of skills increases. This can be modeled by supposing that H increases and L decreases. Eventually (9) will no longer hold, at which point the economy will re-align so that there is <u>only</u> self-matching. That is, each firm will have only

H-workers (Microsoft) or only L-workers (McDonald's).

To understand this result less formally, note that there are two forces at work in determining the equilibrium matching pattern. On the one hand, the asymmetry of the tasks in the production function militates in favor of cross-matching between workers of different skills, in which the more highly skilled worker is assigned the managerial task. On the other hand, the complementarity between tasks promotes self- (i.e., assortative) matching. Now, when tasks are perfectly symmetric, only the second effect is present, and we have perfect assortative matching. For any given degree of asymmetry between tasks, however, there will be some deviation from self-matching. That is, unequally skilled workers will be cross-matched up to the point in which the difference in their skills is so great that the second effect overwhelms the first. At that point, as in the above two-point example, the economy switches to self-matching. Proposition 3 generalizes this logic to the case of many skill-levels.

Let us turn to the issue of wages. Proposition 1 demonstrates that if the distribution of skills has sufficiently low dispersion, then an increase in the mean skill-level reduces inequality in the sense that the wages of low-skill workers rise, whereas those of high-skill workers decline. Proposition 2 establishes that the opposite is true if the distribution of skills is sufficiently highly dispersed. That is, with a diffuse skill-distribution, an increase in the mean skill-level aggravates wage inequality.

To see why these propositions hold in a simple case, consider an economy with three skill-levels, $L < M < H$, and let us suppose that H and L satisfy (9). For easy computation, assume that there are equal numbers (say, x each) of L- and H-workers and that the number of M-workers is at least $2x$. Because H and L satisfy (9), we know that it is inefficient for L- and H-workers to be self-matched. In fact, we claim that both L- and H-workers should be matched with M-workers. The alternative would be for the L- and H-workers to be matched with each other. But notice that, since $L < M$ and $M < H$,

$$LH^2 + M^3 < LM^2 + MH^2. \tag{10}$$

Condition (10) implies that total output would be raised by taking an L- and an H-worker who are matched with each other and rematching both with M-workers (who were previously self-matched). Now, because there is perfect competition and constant returns to scale, firms make zero profit in equilibrium. Hence workers must absorb all revenue. Since there are more than $2x$ M-workers, at least one pair must be self-matched in equilibrium, in which case their firm's revenue is M^3. Therefore, the equilibrium M-wage is

$$w(M) = \frac{M^3}{2}. \tag{11}$$

Thus, the zero profit requirement implies that the equilibrium L-wage is

$$w(L) = LM^2 - w(M) = LM^2 - \frac{M^3}{2}. \tag{12}$$

Similarly,

$$w(H) = MH^2 - \frac{M^3}{2}. \tag{13}$$

To model the idea of an increase in the mean skill-level, let us examine a small increase in M. The derivatives of $w(L)$ and $w(H)$ with respect to M are, respectively,

$$2LM - \frac{3}{2}M^2 \tag{14}$$

and

$$H^2 - \frac{3}{2}M^2. \tag{15}$$

The assumption of "low" dispersion of skill -- the assumption treated by Proposition 1 -- corresponds in this three-skill model to the condition that $H \approx L$. In this case, (14) is positive, whereas (15) is negative. But this means that $w(L)$ rises and $w(H)$ falls as M increases, which is what Proposition 1 asserts.

By contrast, it is readily shown that if

$$L < \frac{3}{4}M < \frac{\sqrt{6}}{4}H, \tag{16}$$

which corresponds to the higher dispersion of Proposition 2, then (14) is negative and (15) is positive. Hence $w(L)$ falls while $w(H)$ rises when M increases, as required by Proposition 2.

To understand Proposition 1 at a more intuitive level, notice that when $L \approx M$ the share of the revenue going to the M-worker in an L-M match (i.e., $M^3/2$) rises with M more slowly than does the total product LM^2. Hence an increase in M confers a positive benefit on L: the net marginal revenue left over after the M-worker is paid for his higher product is positive. This can be viewed as a manifestation of the "trickle-down" principle, the theory that propounds that some of the fruits generated by improving the productivity of middle- and upper-class workers will accrue to the poor.

In our model, however, the trickle-down principle pertains only to skill-distributions with low dispersion. If L and M are far enough apart, a further increase in M causes the benefits from cross-matching between L and M to deteriorate relative to those from self-matching. Therefore since $w(M)$ rises, $w(L)$ must fall. At the same time, this increase in M enhances the quality of a match between M and H (provided M and H were far enough apart to begin with), raising $w(H)$. Hence, an increase in mean skill-level causes the wage gap between H- and L- workers to increase, as Proposition 2 establishes.

Notice that our model has the feature that workers of median skill (M-workers in the above example) can serve in equilibrium as either managers or assistants, depending on whom they are matched with. This is a property that generalizes to the case of many skill-levels (see Section III). We will speculate on its political and social implications in the conclusion.

II. Rising Segregation by Skill

We offer evidence that segregation of workers by skill has increased in the United States, Britain, and France. Subsection II.A defines the index we use to measure segregation, and Subsection II.B shows that the index has increased in these three countries when skill is measured by any of three proxies: wages, worker classification, and experience.

II.A. The Segregation Index

Sociologists have used an index of correlation to measure segregation by dichotomous variables, such as race [Bell, 1954; Robinson, 1950]. In this section, we propose an index of segregation that generalizes this measure of correlation so that it can be applied to variables taking on many values. We also adjust it to take account of the concentration of high-skill workers that would have been expected purely from chance. (Ellison and Glaeser [1994] develop an index of geographic concentration designed to address similar issues.) Appendix I discusses how confidence intervals can be constructed around the estimated segregation index.

Suppose that there are J firms (or plants) in the sample indexed by $j = 1, \ldots, J$. Let Z_j be the set of workers in firm j and z_j the number of workers in this firm. Given a measure of skill, q, such as wages, education, or worker classification, denote the mean skill-level in the sample by $\bar{q}$ (*i.e.*, $\bar{q} = \sum_{j=1}^{J} \sum_{i \in Z_j} q_i / N$, where N is the total number of workers in the sample).

Define ρ as our index of correlation or segregation:

(17)
$$\rho = \frac{\sum_{j=1}^{J} \sum_{i \in Z_j} (q_i - \bar{q}) \sum_{k \in Z_j} (q_k - \bar{q}) / z_j}{\sum_{j=1}^{J} \sum_{k \in Z_j} (q_k - \bar{q})^2},$$

where workers are indexed by i and k (so that q_i is the skill-level of worker i). A correlation of zero indicates that all firms have the same skill-mix of workers, and a correlation of one indicates complete segregation, in which all workers within a firm have the same q. Note that this index is invariant to affine transformations of the units in which skill is measured.[2]

The index of correlation, ρ, is equivalent to one minus the variance of q within firms divided by the overall variance of q, and thus to the R^2 obtained by regressing q on a series of firm dummies. To see this, add and subtract the mean $\bar{q}^j$ within each firm j to obtain:

(18)
$$\rho = \frac{\displaystyle\sum_{j=1}^{J}\sum_{i\in Z_j}\left[\frac{\left([q_i-\bar{q}^j]+[\bar{q}^j-\bar{q}]\right)\displaystyle\sum_{k\in Z_j}\left([q_k-\bar{q}^j]+[\bar{q}^j-\bar{q}]\right)}{z_j}\right]}{\displaystyle\sum_{j=1}^{J}\sum_{k\in Z_j}\left([\bar{q}-\bar{q}^j]+[\bar{q}^j-q_k]\right)^2}.$$

[2]In the case of a dummy variable, the correlation specializes to the index used by Bell [1954] and Robinson [1950],

$$\rho = \frac{\displaystyle\sum_{j=1}^{J} z_j(\bar{D}^j-\bar{D})^2}{N\bar{D}(1-\bar{D})},$$

where D denotes the dummy variable. Note that this index normalizes the variance across plants of the proportion of workers for whom the dummy variable equals one by dividing by the variance of the dummy variable in the total population. Without normalization, segregation by production-worker status, for example, would appear higher in an economy with five firms consisting only of non-production workers and ninety-five consisting only of production workers than in an economy with one firm consisting of only non-production workers and ninety-nine firms consisting of production workers. After normalizing, however, we find that both economies have the maximum segregation index of one.

After deleting the terms which equal zero, we can simplify to

(19)
$$\rho = \frac{\sum_{j=1}^{J} z_j(\bar{q}^j - \bar{q})^2}{\sum_{j=1}^{J} z_j(\bar{q}^j - \bar{q})^2 + \sum_{j=1}^{J}\sum_{k\in Z_j} (\bar{q}^j - q_k)^2} = \frac{s_b^2}{s_b^2 + s_w^2} = \frac{s_b^2}{s_T^2},$$

where s_w^2 and s_b^2 represent the variance of q within and between firms, respectively, and s_T^2 represents the total variance of q in the economy. For example,

(20)
$$s_b^2 = \sum_{j=1}^{J} z_j(\bar{q}^j - \bar{q})^2 / N.$$

One virtue of this index is that, since ρ depends only on the variance of q in the population and on the variance of mean q between firms, it can be calculated using separate data sources for workers and firms. This is useful if data linking employees and their firms are unavailable, as in the United States.

Note that if firms are of finite size, there will be some correlation between the skill-levels of workers in the same firm, even if workers are assigned to firms randomly. That is, some firms will happen to receive workers of above-average skill, and others will receive workers of below-average skill. If all firms consisted of K workers, then, with random assignment, the variance of mean skill between firms would be $1/K$ times the over-all variance of skill between workers, and the expected correlation would thus be $1/K$.

To compare the extent of segregation across economies with different average firm sizes, or to determine whether an economy exhibits more segregation than would be expected purely from chance, it may be helpful to use the adjusted R^2 as a corrected index of segregation:

$$\rho_c = 1 - \frac{s_w^2\,/(N-J)}{s_T^2\,/(N-1)} = 1 - (1-\rho)\left(\frac{N-1}{N-J}\right) = \bar{R}^2, \tag{21}$$

To see how this index corrects for finite firm size, suppose that a proportion γ of the population matches with other workers of exactly the same skill, and that a proportion $1-\gamma$ matches randomly. Let σ_T^2 denote the variance of the underlying stochastic process generating workers' skills. As noted above, s_b^2 for the set of firms whose workers are drawn randomly will converge to σ_T^2/K as $N\rightarrow\infty$, where K is the number of workers per firm. Since the variance of wages within firms plus the variance of mean wages between firms must equal the total variance of wages, the variance of wages within the set of firms with randomly drawn workers converges to $(1-1/K)\sigma_T^2$. Since the variance of wages within the firms consisting of perfectly-matched workers is zero, s_w^2 for the set of all firms converges to $(1-\gamma)(1-1/K)\sigma_T^2$. Note that in a sample of N workers, the expectation of s_T^2 is $(N-1)\sigma_T^2/N$. Substituting these expressions for s_w^2 and s_T^2 into (21) and using the fact that $K = N/J$, we see that ρ_c converges to γ, which, as desired, is the proportion of the population that matches with others of the same skill. Simulations suggest that the expectation of ρ_c is γ even when N is small, and even when firms vary in size.

Two properties of ρ_c are worth noting. First, holding N and J constant, increases in ρ correspond to increases in ρ_c. In Section III, we demonstrate how changes in the distribution of skill affect ρ. Given that we hold the number of workers per firm constant, changes in ρ_c will always be of the same sign as changes in ρ. Second, the adjusted R-squared may be negative if the correlation between the skills of workers in the same firm is less than that expected from random assignment of workers to firms.

II.B. Trends in Segregation

Wages, experience, and worker classification are all imperfect indicators of skill, but together they paint a consistent picture of increasing segregation. Positive correlation of wages among workers in the same firm may be due to segregation by skill or to compensating differentials, efficiency wages, or pressures for equity within the firm. However, there is little reason to think these latter factors increased in importance over the period, especially given the decline of unions in the United States and Britain. Worker classification (for example, into production and non-production worker categories) is also somewhat problematic as a measure of skill, since it reflects characteristics of the job, as well as of the worker. Nonetheless, non-production worker status is correlated with wages, and thus is presumably correlated with skill. Berman, Bound, and Machin [1994] show that in the United States, non-production workers are more highly educated than production workers and much more likely to be managers or professionals.

Where possible, we measure indicators of skill at plant level rather than firm level. Plant data are likely to be more economically meaningful than firm data, since it is not clear that anything of economic importance changes if a holding company acquires both Microsoft and McDonald's. However, even the plant data may be misleading since they include only workers employed by the firm owning the plant and therefore indicate increased segregation when firms contract out work.[3] The American and British data are at the plant level, whereas the French data are at the level of the business unit, which is conceptually closer to the American firm than to the plant.

The best available data come from France. The Wage Structure Surveys (ESS), performed by INSEE in 1986 and 1992, examined a sample of 10,719 business units and 318,332 workers in 1986, and 3,803 business units and 42,783 workers in 1992.[4] The sample was drawn from all business units with at least ten employees, excluding agriculture, transportation, telecommunications, and services to individuals. Table I, drawn from Kramarz, Lollivier, and Pele [1996], shows that between 1986 and 1992, the correlation of log wages among workers in the same business unit rose from 0.36 to 0.44, the correlation of experience rose from 0.11 to 0.16 and the correlation of seniority rose from 0.24 to 0.31. Segregation according to each of the six categories of worker classification rose as well. For example, unskilled blue-collar workers were more likely to be grouped together in firms, as were engineers, professionals, and managers. Worker classification is likely to be a good

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[3] In the U.S., increases in the importance of pensions, combined with pension laws requiring all workers to be treated equally, may have provided a small impetus toward such contracting out over the period studied, but any increase in correlation from this source is likely to have been more than counter-balanced by declines in wage correlation within plants due to the weakening of unions.

[4] The ratio of workers to business units is smaller in 1992 because fewer workers were sampled per firm.

measure of skill in France, because it is governed by fairly precise rules. We should also mention that Kramarz, Lollivier, and Pele found that besides increasing segregation, the period from 1986 to 1992 was also a period of rising inequality.

Table I: Segregation Indexes for Various Indicators of Skill in France
(95% confidence interval in brackets.)

	1986	1992
Log Wages	0.36 [0.3554; 0.3663]	0.44 [0.4316; 0.4518]
Experience (Age)	0.11 [0.1081; 0.1127]	0.16 [0.1554; 0.1665]
Seniority	0.24 [0.2364; 0.2450]	0.31 [0.3027; 0.3203]
Unskilled Blue-Collar Worker dummy	0.31 [0.3058; 0.3159]	0.34 [0.3324; 0.3508]
Skilled Blue-Collar Worker dummy	0.23 [0.2265; 0.2349]	0.26 [0.2535; 0.2693]
Foreman dummy	0.08 [0.0785; 0.0820]	0.10 [0.0970; 0.1044]
Clerk dummy	0.08 [0.0785; 0.0820]	0.15 [0.1457; 0.1568]
Technician dummy	0.14 [0.1376; 0.1433]	0.17 [0.1652; 0.1768]
Engineer, Professional, or Manager dummy	0.14 [0.1376; 0.1433]	0.21 [0.2044; 0.2180]

Source: Kramarz, Lollivier, and Pele [1996]

In the United States, data linking workers and their plants are hard to come by. However, segregation of workers by wage can be calculated using the Davis and Haltiwanger [1991] estimates of the variance of wages among all workers from the CPS and of the variance of average wages between plants from the Census of Manufactures. Unfortunately, taking the ratio of estimates from two different data sources makes the measure of segregation noisier. Moreover, data are available only for manufacturing workers and reliable data only for production workers within manufacturing. However, since services are a growing share of the economy, and services such as finance, restaurants, and law are highly segregated by skill, their exclusion is likely to lead to underestimation of any increase in sorting.

Table II reports between- and within-plant variances of production worker wages in manufacturing,[5] taken from Davis and Haltiwanger [1991]. The final line reports the correlation between wages of production workers within the same plant.[6] Both within-plant and between-plant variances of wages have increased, reflecting the widely documented rise in inequality in recent years. However, if there had been no realignment of the way relative skills were grouped within firms, both variances would have increased in the same ratio. In fact, the between-plant variance increased proportionately more than the within-plant variance. Thus, although the data are imperfect, and estimates for 1984 seem particularly suspect, the correlation between wages of production workers within the same firm increased from 1975 to 1986, as shown in the final row.[7]

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[5]It is not possible to obtain reliable data on non-production workers because of differences in the sampling frames between the two surveys used by Davis and Haltiwanger. However, the available data show no trend in segregation among non-production workers. The segregation index was .447 in 1975 and .443 in 1986.

[6] Note that these correlations are higher than those reported in France, because these data include only production workers, whereas the French data include all workers. The variance of production workers' wages within plants is much smaller than that of all workers.

[7]Given the large sample sizes, sampling error is likely to be small, and noise in these estimates is probably due to other factors, such as business-cycle effects.

Table II: Sorting of Manufacturing Production Workers by Wages: U.S. Plants

	1975	**1977**	**1979**	**1982**	**1984**	**1986**
s_T^2	9.00	9.77	10.78	11.81	11.96	13.84
s_W^2	2.13	2.37	2.19	1.82	1.00	2.76
s_b^2	6.86	7.40	8.58	9.99	11.0	11.19
ρ	0.76	0.76	0.80	0.85	0.92	0.80

Source: Davis and Haltiwanger [1991], ρ calculated as s_b^2/s_T^2.

Sorting by production worker/non-production worker status also increased. Table III shows that the correlation of a dummy variable for non-production worker status among American manufacturing workers in the same plant increased from 0.195 to 0.228 between 1976 and 1987.[8]

Table III: Sorting by Production Worker/Non-Production Worker Status: U.S. Plants

Year	1976	1977	1980	1981	1983	1984	1985	1986	1987
NPW/Emp	0.261	0.260	0.282	0.287	0.302	0.291	0.304	0.314	0.310
ρ	0.195	0.192	0.196	0.199	0.215	0.218	0.225	0.231	0.228

Source: Census of Manufactures

British evidence also shows increased segregation by worker classification.[9] The 1984 and 1990 Workplace Industrial Relations Surveys (WIRS) divide employment in each of 402 British establishments into manual and non-manual components. Manual employment is then further subdivided into unskilled, semi-skilled, and skilled; and non-manual employment is

[8]We thank Andrew Bernard, Brad Jensen, and the U.S. Bureau of the Census for supplying the information in Tables II and III. Data for 1976 and 1977 may not be strictly comparable with that from other years, since a different sampling methodology was used by the census in those years.

[9]We thank Stephen Machin for providing the information in Table IV.

subdivided into (i) clerical, secretarial, and administrative staff; (ii) supervisors and foremen; (iii) junior technical/professional; (iv) senior technical/professional, and (v) middle/senior management. Table IV shows that sorting increased in most worker classifications. For example, the correlation of a dummy variable for middle/senior managers between workers in the same firm increased from 0.058 in 1984 to 0.077 in 1990. Sorting of senior professional/technical staff also increased, contributing to an increase in sorting among all upper-grade, non-manual workers. There was a rise in segregation among clerical workers, and among all non-manual workers taken as a whole. Within the category of manual workers, sorting declined among skilled and semi-skilled manual workers, but increased among unskilled manual workers. Note that the British evidence, unlike that from the United States and France, indicates that segregation increased among a fixed sample of plants rather than just in the economy as a whole.

Section III explores how segregation and wages are jointly determined given the technology and the distribution of skill.

Table IV: Sorting in 402 British Establishments, 1984-1990

	Mean 1984	Mean 1990	ρ 1984	ρ 1990
Managers	0.074	0.080	0.058	0.077
Senior. Prof/Tech	0.072	0.083	0.230	0.276
Junior Prof/Tech	0.095	0.090	0.294.	0.301
Ttl.Upper Gr. Non-Manual	0.241	0.253	0.299	0.323
Clerical	0.233	0.222	0.273	0.322
Total Non-Manual	0.511	0.525	0.446	0.469
Skilled Manual	0.162	0.163	0.341	0.323
Semi-skilled Manual	0.143	0.132	0.391	0.367
Unskilled Manual	0.185	0.180	0.415	0.444
Total Manual	0.489	0.475	0.446	0.469

III. Theory

Let us extend the analysis of Section I -- which was conducted for two- and three-point skill-distributions -- to more general distributions. We maintain the hypotheses of competitive markets and production function (6).

We consider a pair of distributions of types on the integers between $\underline{n}$ and $\bar{n}$, parametrized by $\theta \in \{0,1\}$. For given θ, let $p(n;\theta)$ be the number of workers of skill-level n. The values $\theta = 0$ and $\theta = 1$ correspond to the "pre-shift" and "post-shift" distributions, respectively. Let $\mu(\theta)$ be the mean of $p(\cdot;\theta)$. That is, $\mu(\theta) = \sum_{n=\underline{n}}^{\bar{n}} np(n;\theta) / \sum_{n=\underline{n}}^{\bar{n}} p(n;\theta)$. We assume that $\mu(\theta)$ also equals $n^*(\theta)$, the mode of the distribution.

Recall from Section I that, if the ratio of two workers' skill-levels was greater than $A = 2/(1+5^{1/2})$, then it was more efficient for each of them to match with another of his or her own skill-level (self-matching), rather than match with one another (cross-matching). Let n_1 be the smallest integer no less than $An^*(1)$; let n_2 be the smallest integer no less than $(3/2)^{1/2}n^*(1)$; and let n_3 be the smallest integer no less than $\sqrt{2}\,n^*(1)$. We assume that, for $\theta = 0, 1$, $p(n;\theta)$ is "sufficiently peaked" near the mode $n^*(\theta)$. More precisely, "sufficiently peaked" means

$$p(n^*(\theta);\theta) > \sum_{n'=An^*(\theta)}^{n^*(\theta)-1} p(n';\theta) + \sum_{n'=n^*(\theta)+1}^{n^*(\theta)/A} p(n';\theta), \tag{22}$$

$$\sum_{n'=An}^{An^*(\theta)} p(n';\theta) < p(n;\theta), \text{ for all } n \in [n_1, n^*(\theta)]\,, \tag{23}$$

and

$$p(n;\theta) > \sum_{n'=\sqrt{2}n^*(\theta)}^{n/A} p(n';\theta), \text{ for all } n \in [n^*(\theta), n_3]\,. \tag{24}$$

As will become clear below, condition (22), which is imposed mainly to simplify the computations,[10] ensures that at least some workers of modal skill will have to self-match, i.e., match with others of the same skill in competitive equilibrium. Given the zero-profit condition, this will serve to tie down their wage (which is why (22) is so convenient to impose). Conditions (23) and (24) imply that in equilibrium, some workers of every skill level between n_1 and $n^*(\theta)$ and between $n^*(\theta)$ and n_3 will also match with modal workers.

Both the mean level and the dispersion of skill have increased in recent decades. We consider a shift in the distribution of skill in which, as θ increases from 0 to 1, (i) the

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[10]Condition (22) can be relaxed considerably. However, the argument becomes correspondingly more complicated.

mean rises, i.e.,

$$n^*(1) = n^*(0) + 1; \tag{25}$$

and (ii) the lower and upper tails get fatter symmetrically but retain their "relative shapes," i.e., there exists $\eta > 1$ such that[11]

$$\frac{p(n;1)}{p(n;0)} = \alpha, \text{ for all } n \le n_1 \text{ and } n \ge n_2. \tag{26}$$

III.A. Wages

We first show that if the distributions $p(\cdot\ ;\ \cdot)$ are "tight" enough -- i.e., if $\bar{n}$ is sufficiently close to $\underline{n}$ -- then an increase in θ promotes equality in the sense that wages in the lower tail of the distribution rise and wages in the upper tail of the distribution fall.

<u>Proposition 1</u> (The Trickle-Down Principle): Suppose that $p(\cdot;\cdot)$ satisfies (22)-(25). Given

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[11]We adopt (26) not because it is necessarily realistic, but because it facilitates the presentation of our results. In particular, we wish to argue that, if the skill-distribution is sufficiently dispersed, then a typical worker in the lower end of the distribution will see his wage fall and one in the upper end of the distribution will experience a wage increase when the mean skill-level rises. We demonstrate this (Proposition 2) by showing that the average wages in the lower and upper tails, respectively, fall and increase. But a change in the average wage would not be a good measure of what happens to a given worker if the relative distribution of skills also changed. Hence, (26) makes comparisons easier, although our qualitative results do not depend on it.

$\underline{n}$, if $\bar{n}$ is near enough $\underline{n}$ so that $\underline{n} > (2/3)^{1/2}\,\bar{n}$, then

(27) $$w(n;0) < w(n;1), \text{ for all } n<n^*(0),$$

and

(28) $$w(n;0) > w(n;1), \text{ for all } n > n^*(1).$$

We establish Proposition 1 through a series of Lemmas.

Lemma 1: For any n', if n is such that

(29) $$n < An',$$

then workers of skills n and n' cannot be matched in equilibrium.

Proof: As demonstrated in Section I, total output for two workers of skill n and two workers of skill n' when they are cross-matched (i.e., $2nn'^2$) is be less than that when they are self-matched (i.e., $n^3+n'^3$) if $n/n'<A$. *Q.E.D.*

Lemma 2: $w(n^*(\theta);\theta) = (n^*(\theta))^3/2.$

Proof: From (22), even if all workers of all skills $n'\neq n^*(\theta)$ between $An^*(\theta)$ and $n^*(\theta)/A$ were matched with workers of skill $n^*(\theta)$, there would still be some $n^*(\theta)$-workers left over.

Moreover, from Lemma 1, no skill outside the interval $[An^*(\theta), n^*(\theta)/A]$ can be matched with $n^*(\theta)$ in equilibrium. Therefore, at least some $n^*(\theta)$-workers must be self-matched in equilibrium. But this means that they earn 1/2 the output of a firm with two workers of skill $n^*(\theta)$. *Q.E.D.*

Lemma 3: If $n < n^*(\theta)$ and $n' > n^*(\theta)$, then workers of type n and n' cannot be matched in equilibrium.

Proof:[12] Total output with one (n,n') firm and one (n^*,n^*) firm is $Y = n'^2n + n^{*3}$, where $n^*=n^*(\theta)$. Total output with the assignments (n,n^*) and (n^*,n') is $Y^* = n^{*2}n + n'^2n^*$. To prove the lemma, we need to show that $Y^* > Y$. Note that

(30) $$Y^*-Y = n^{*2}n + n'^2n^* - n'^2n - n^{*3}.$$

Defining $z = n/n^*$, and $z' = n'/n^*$, we obtain

(31) $$\Delta \equiv \frac{Y^*-Y}{n^{*3}} = z + z'^2 - z'^2z - 1 = (z'^2-1)(1-z).$$

Since $n' > n^*$, $z' > 1$, and since $n < n^*$, $z < 1$. Thus the result follows. *Q.E.D.*

Lemma 4: If $n, n' \in (An^*(\theta), n^*(\theta))$, then n and n' cannot be matched in equilibrium.

Proof: Assume without loss of generality that $n' \geq n$. As in the proof of Lemma 3, we

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[12]We thank Charles Morcom for the proofs of Lemmas 3, 4, and 6, which are simpler than the ones we originally developed.

show that total production by the firms (n,n') and (n^*,n^*) (i.e., Y) is less than that by the firms (n ,n^*) and (n',n^*)(i.e., Y'). Here, $Y = n'^2n + n^{*3}$, and $Y' = n^{*2}n + n^{*2}n'$. This implies that

$$Y'-Y = n^{*2}n + n^{*2}n' - n'^2n - n^{*3}. \tag{32}$$

Dividing the right-hand side of (32) by n^{*3}, we obtain

$$z + z' - z'^2z - 1 = z(1-z'^2) + z'-1 = (1-z')(z(1+z')-1). \tag{33}$$

Now, $1-z'$ is positive, since $n'<n^*$. As for $z(1+z')-1$, it is increasing in both z and z', so we need only check that it is non-negative at the bottom of the range: $z = z' = A$. But at this point it vanishes. *Q.E.D.*

<u>Lemma 5</u>: If $n \in (An^*(\theta), n^*(\theta))$, then some n-workers are matched with $n^*(\theta)$ in equilibrium and so $w(n;\theta) = n(n^*(\theta))^2 - (n^*(\theta))^3/2$.

<u>Proof</u>: If $n \in (An^*(\theta), n^*(\theta))$, then from (23) and Lemma 1, the n-workers cannot be matched exclusively with skills no greater than $An^*(\theta)$. But from Lemma 3, they cannot be matched with skills greater than $n^*(\theta)$ and, from Lemma 4, they cannot be matched with skills greater than $An^*(\theta)$ and less than $n^*(\theta)$. Thus by process of elimination, some n-workers must be matched with $n^*(\theta)$-workers in equilibrium. From Lemma 2, we conclude that $w(n;\theta) = n(n^*(\theta))^2 - (n^*(\theta))^3/ 2$. *Q.E.D.*

Lemma 6: If $n,\ n' \in (n^*(\theta),\ \sqrt{2}\,n^*(\theta))$, then n and n' cannot be matched in equilibrium.

Proof: Assume without loss of generality that $n' \geq n$. Let Y' denote total production with the assignments (n^*,n) and (n^*,n'), where $n^*=n^*(\theta)$. Then $Y' = n^2n^* + n'^2n^*$. Let Y denote production with the assignments (n,n') and (n^*,n^*). Then $Y = n'^2n+n^{*3}$. Dividing $Y' - Y$ by n^{*3} we obtain

(34) $$z^2 + z'^2 - z'^2 z - 1 = z'^2 (1-z) + z^2-1 = (z-1)(z+1-z'^2).$$

Now, z-1 is positive. As for z+1 - z'^2, it is increasing in z and decreasing in z'. Hence it attains a minimum when z = 1 and $z' = \sqrt{2}$, at which point it vanishes. *Q.E.D.*

Lemma 7: If $n \in (n^*(\theta),\ \sqrt{2}\,n^*(\theta))$, some n-workers are matched with $n^*(\theta)$ in equilibrium and so

(35) $$w(n;\theta) = n^*(\theta)n^2 - \frac{(n^*(\theta))^3}{2}.$$

Proof: If $n \in (n^*(\theta),\ \sqrt{2}\,n^*(\theta))$, then from (24) and Lemma 1, the n-workers cannot be matched exclusively with skills greater than $\sqrt{2}\,n^*(\theta)$. From Lemma 3, they cannot be matched with skills less than $n^*(\theta)$. And, from Lemma 6, they cannot be matched with skills greater than $n^*(\theta)$ and less than or equal to $\sqrt{2}\,n^*(\theta)$. Thus some n-workers must be matched with $n^*(\theta)$-workers in equilibrium. From Lemma 2, we deduce (35).

Q.E.D.

We can now establish Proposition 1. Given a sufficiently tight distribution of skills, all workers will have skill greater than $An^*(0)$. Lemma 5 implies, therefore, that, for each skill level below the mean, at least some workers will have managers of mean skill in equilibrium. Hence, an increase in the mean skill implies that low-skill workers will have better managers and therefore that their firms will produce more output. Because the wage that their managers can obtain from self-matching increases by less than this increase in output, the assistants will receive part of the increased value of their matches. That is, the wages of the low-skilled will rise. A tight distribution also means that no worker has skill greater than $\sqrt{2}\,n^*(\theta)$. Lemma 7 implies, therefore, that for each skill level above the mean, at least some workers will have assistants of mean skill in equilibrium. An increase in mean skill produces an increase in output for such matches. However, the wage from self-matching for those of mean skill will increase by more than output, and hence wages for high-skill workers will fall in response to this shift in mean skill. More formally, we have:

<u>Proof of Proposition 1</u>: Suppose $\bar{n}$ is near enough $\underline{n}$ so that $\underline{n} \geq (2/3)^{1/2}\bar{n}$. Because

$(2/3)^{1/2} > 3/4 > A$, we have $\underline{n} \geq (2/3)^{1/2}\bar{n} > 3n^*(\theta)/4 > An^*(\theta)$. Hence, if $n < n^*(\theta)$,

Lemma 5 implies that $w(n;\theta) = n(n^*(\theta))^2 - (n^*(\theta))^3/2$. Thus $dw(n;\theta)/dn^*(\theta) = n^*(\theta)(2n - 3n^*(\theta)/2)$. Since $n > 3n^*(\theta)/4$, this derivative is positive, and so $w(n;\theta)$ is increasing in θ.

Because $n^*(\theta) > \underline{n}$ and $(3/2)^{1/2} < \sqrt{2}$,

(36) $$n < \bar{n} \leq \sqrt{\frac{3}{2}}\, n^*(\theta) < \sqrt{2}\, n^*(\theta)\,,$$

And so, if $n > n^*(\theta)$, Lemma 7 implies that $w(n;\theta) = n^*(\theta)n^2 - (n^*(\theta))^3/2$. Hence

(37) $$dw(n;\theta)/dn^*(\theta) = n^2 - \frac{3}{2}(n^*(\theta))^2\,.$$

From (36), this derivative is negative, and so $w(n;\theta)$ is decreasing in θ.

Q.E.D.

We now turn to the case in which the distributions $p(\cdot;\cdot)$ are more diffuse. In that case, an increase θ works <u>against</u> equality. To see this, let $n_1(\theta)$ be the smallest integer greater than $An^*(\theta)$ (recall that $n_1 = n_1(1)$), and let S_1 consist of all workers with skills no greater than $n_1(1)$. Lemma 8 below will show that any worker in S_1, except possibly for an $n_1(0)$- or $n_1(1)$-worker, is matched in equilibrium with another worker in S_1. Moreover, from Lemmas 3, 4, and 8, any $n_1(0)$- or $n_1(1)$- worker not matched with another worker in S_1 is matched with an $n^*(\theta)$-worker. Hence, the average wage of workers in S_1 equals the sum of

(i) the total output of matches between workers in S_1 and (ii) the wages of $n_1(0)$- and $n_1(1)$-workers matched with $n^*(\theta)$-workers, all divided by the number of workers in S_1. But $w(n_1(1);\theta)$ falls as θ increases because the wage of an $n^*(\theta)$-worker from self-matching increases faster than the output from an $n_1(1)$-$n^*(\theta)$ match. Moreover, Lemma 1 implies that in equilibrium there are no matches between $n_1(0)$- and $n^*(1)$-workers, and so, for the purpose of calculating (ii), we can assume that $w(n_1(0);\theta)$ also falls as θ increases. Furthermore, from (26), the relative distribution of skills in S_1 does not change as θ increases. Hence, the decrease in (ii) cannot be made up for by an increase in (i). We conclude that the average wage of workers in S_1 must fall.

Next consider the set S_2 consisting of workers of skill no less than n_2 (the smallest integer no less than $(3/2)^{1/2}n^*(1)$). The average wage of workers in S_2 equals the sum of (i) the total output from matches between workers in S_2, and (ii) the wages of workers in S_2 matched with workers with skills in the interval $[n^*(\theta),n_2\text{-}1]$, all divided by the number of workers in S_2. Now, n_2 is big enough so that, for $n \geq n_2$, the wage of an $n^*(\theta)$-worker from self-matching increases more slowly than the output from an $n^*(\theta)$-n match. Hence, the wage of an n-worker matched with an $n^*(\theta)$-worker increases with θ. Moreover, the argument of Proposition 1 implies that, for $n' \in (n^*(\theta),n_1)$, $w(n';\theta)$ falls with θ. Hence, the wage of an n-worker ($n \geq n_2$) matched with an n'-worker must rise. We conclude that (ii) must rise with θ and, so from (26), the average wage in S_2 also rises.

Formally, we can state:

Proposition 2: Suppose that the distributions $p(\cdot;\cdot)$ satisfy (22)-(26) and are sufficiently diffuse so that $\underline{n} < n_1$ and $n_2 < \bar{n}$. Then the average wage in the tail below n_l falls as θ increases from 0 to 1. More precisely, we have

$$(38) \qquad \frac{\sum_{n=\underline{n}}^{n_1} w(n;1)\,p(n;1)}{\sum_{n=\underline{n}}^{n_1} p(n;1)} < \frac{\sum_{n=\underline{n}}^{n_1} w(n;0)\,p(n;0)}{\sum_{n=\underline{n}}^{n_1} p(n;0)}.$$

Similarly, the average wage in the tail above n_2 increases with θ:

$$(39) \qquad \frac{\sum_{n=n_2}^{\bar{n}} w(n;1)\,p(n;1)}{\sum_{n=n_2}^{\bar{n}} p(n;1)} > \frac{\sum_{n=n_2}^{\bar{n}} w(n;0)\,p(n;0)}{\sum_{n=n_2}^{\bar{n}} p(n;0)}.$$

The formal proof of Proposition 2 is given in Appendix 2.

<u>Lemma 8</u>: Let $n_1(\theta)$ be the smallest integer no less than $An^*(\theta)$ (so that $n_1(1) = n_1$). If $n < An^*(\theta)$ and an n-worker is matched with a worker of skill $n' \geq An^*(\theta)$ in equilibrium, then $n' = n_1(\theta)$.

The proof, which is in Appendix II, calculates the profit for a firm hiring an assistant of skill $n < An^*(\theta)$. It shows that profit will be higher with a manager of skill $n_1(\theta)$ than with one of greater skill.

Segregation

We now turn to the issue of segregation. We will show that an increase in skill dispersion accompanied by a rise in the mean skill-level leads to an increase in the index of segregation, provided that the shift in the mean is not too large relative to that of dispersion.

To make this last proviso formal, we will no longer work with distributions on the integers alone. Instead fix (rational) numbers $\underline{b}$ and $\bar{b}$ with $0 < \underline{b} < \bar{b}$. For each integer $r = 1, 2, \ldots$, choose integers $\underline{n}^r$ and $\bar{n}^r$, such that $\underline{b} = \underline{n}^r/r$ and $\bar{b} = \bar{n}^r$. For each r, we consider a pair of distributions $(p_r(\cdot;0), p_r(\cdot;1))$, where for all integers $n \in [\underline{n}^r, \bar{n}^r]$, $p_r(n;\theta)$ is the number of workers of skill n/r. From (25), notice that, if r is big, the shift in the mean is small when θ increases from 0 to 1.

Let us suppose that there exist $\alpha > 1$ and $\beta < 1$ such that, for all r, $(p_r(\cdot;0), p_r(\cdot;1))$ satisfies (22)-(26) and

$$\textbf{(40)} \qquad p_r(n;1) < \beta p_r(n;0), \text{ for all } n \in (n_1, n_2).$$

Condition (40) says that the shift from $\theta = 0$ to $\theta = 1$ reduces probability more-or-less uniformly in the middle of the distribution. Henceforth, we will drop the subscript r when this does not create ambiguity.

For n, n' and θ, let $z(n, n'; \theta)$ be the equilibrium number of matches between n- and n'-workers. Let $\mu(n, n') = (n + n')/2$. Then, from (19), the index of segregation can be expressed as

$$(41)\quad \rho(\theta) = \frac{\sum_{n=\underline{n}}^{\bar{n}} \sum_{n'=\underline{n}}^{\bar{n}} (\mu(n,n') - \mu(\theta))^2 z(n,n';\theta)}{\sum_{n=\underline{n}}^{\bar{n}} \sum_{n'=\underline{n}}^{\bar{n}} (\mu(n,n') - \mu(\theta))^2 z(n,n';\theta) + \sum_{n=\underline{n}}^{\bar{n}} \sum_{n'=\underline{n}}^{\bar{n}} (n - \mu(n,n'))^2 z(n,n';\theta)}.$$

Proposition 3: Suppose that, for each $r = 1,2,...$, the pair $(p_r(\cdot;0), p_r(\cdot;1))$satisfies (23)-(26) and (40). Then, for r sufficiently big, $\rho(1)>\rho(0)$.

To understand Proposition 3, consider a shift in the skill-distribution satisfying (26) and (40). The effect of such a shift is to move probability weight to the tails. To simplify matters, let us assume for the moment that this shift does not affect the mean. Now, because of the complementarity between tasks, a firm will not hire workers that are too different in skill-level. Thus, if the shift pushes more workers into the tails of the distribution, it will push more firms' means $\mu(n,n')$ into the tails. This means that the dispersion of firms' means -- the numerator and also the first term of the denominator of (41) -- rises with the shift. (This argument assumes that the overall mean $\mu(\theta)$ does not change too much.) In contrast, the dispersion of skill within a firm, $(n-\mu(n,n'))^2 + (n'-\mu(n,n'))^2$ does not depend very much on whether the skill-levels are in the tails or the middle of the distribution -- and thus is not affected very much by the shift -- since it is determined mainly by the trade-off between task-complementarity (promoting matching of similar skill-levels) and differential sensitivity to skill (enhancing matching of different skill-levels). Hence the second term in the denominator of (41) remains about the same. The over-all effect of the shift, therefore, is an increase in ρ. The formal proof can be found in Appendix II.

Several limitations of our model are worth noting. First, we treat all skills as if they can be reduced to a single dimension. A fuller model would allow for multi-dimensional skill, as in Roy [1951]. Second, the model assumes that all workers are producing a single good, so that there is no interaction between workers through the product market. To the extent that high- and low-skill workers specialize in production of different goods, increases in mean-skill will tend to reduce the price of skill-intensive goods, and thus will be more likely to increase relative wages of low-skill workers. This would partially offset the effect described in Proposition 2.

Second, the production function allows no substitutability between quantity and quality of workers. The number of workers in a firm is taken as fixed, rather than as a choice variable. However, many of the results of this model would presumably hold as partial equilibrium results in models that allowed for choice of quantity of workers, as long as quality and quantity were not perfect substitutes. To the extent that high and low-skill workers specialize in producing different goods, it becomes more likely that an increase in the mean-skill level of the economy will increase relative wages of low-skill workers.

There is some gap between the theoretical finding that increased skill-dispersion leads to increased segregation of workers by <u>skill</u> and the empirical finding of increased segregation of workers by <u>wages</u>. Under the model, the relationship between wages and skill is non-linear and endogenous to the skill-distribution. For a high dispersion of skill, there will be a strong correlation between the wages of workers in the same firm, and, for a low dispersion of skill, there will be a weak correlation. Thus, on average, the correlation between the wages of workers in the same firm will increase with skill-dispersion. However, the correlation need not increase monotonically, because over some ranges, small increases in dispersion may not change the pattern of matching by skill, but may change the wage schedule so as to reduce the correlation of wages. Thus there may be special cases in which segregation by wages could increase without a concomitant rise in segregation by skill. This contrasts with conventional efficiency unit models in which an increase in correlation of wages within firms will always correspond to an increase in the correlation of skills within firms.

IV. Skill Dispersion and Segregation across U.S. States

We next argue that, as proposition 3 implies, U.S. states with greater variance of education are more segregated by education. Casual empiricism suggests the same is true for countries.

Kremer and Troske [1995] create segregation indices by state based on several indicators of skill using the Worker-Establishment Characteristics Database (WECD). The WECD matches 199,557 manufacturing workers to 16,144 manufacturing establishments [Troske, 1994]. It was created by linking individual data from the 1990 census with establishment data from the Longitudinal Research Database, using detailed location and industry information. It should not be taken as representative of American firms, since the match rate varies by industry, portion of the country, and plant size. The segregation indices are computed as the adjusted R^2 from regressing worker characteristics on a set of firm dummies in a state. The sample was restricted to firms for which data on more than two workers were available, which led to the exclusion of four states.

States with greater variance of education[13] tend to be more highly segregated by education, as shown in Table V. A regression of the correlation index on the variance of education, with observations heteroscedasticity weighted by the square root of the sample size, yielded a t-statistic of 5.47. Although the model may be a good description of how workers sort into firms by education, it is not a good description of how they sort by age. A regression of segregation by age within plants on the variance of age among workers in the state yielded a t-statistic of -1.69. There was a positive, but not statistically significant, relationship between segregation and the variance of skill when the skill indicator was either the log wages or the predicted wage given age, education, sex, marital status, and race.[14]

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[13] The variance of worker characteristics in each state was calculated using the Sample Detail File from the 1-in-6 census sample, so it is based on a large number of observations and should be representative of manufacturing workers in the state as a whole.

[14]The predicted wage is calculated from a country-wide regression of log wages on age, age^2, and dummies for female, married, black, female*married, female*black, and various education levels.

Table V: Effect of Variance of Skill on Segregation by Skill: U.S. States
Regression of ρ_c for skill indicator on the variance of the indicator in the state.
Heteroscedasticity-weighted using the number of firms per state. (t-statistics in parentheses)

Dependent Variable	Education	Age	Predicted Wage	Log Wage
Constant	0.0014 (0.056)	0.3037 (2.51)	0.117 (1.36)	0.218 (1.802)
Variance of Variable	0.0257 (5.471)	-0.0017 (-1.687)	0.444 (0.591)	0.431 (0.948)
N	46	46	46	46
R^2	0.3995	0.0595	0.0077	0.0196

Source: Kremer and Troske [1995]

Although comparable data are not available for rigorous comparisons across countries, the model is consistent with the widespread view that developing countries, with wide skill-dispersion, are prone to "dualism." The model suggests that if skill-differentials are comparatively small, as in highly traditional societies[15] or modern societies, workers of high- and low-skill will mix together within firms. However, in developing economies with a wide range of skills, such as India or Brazil, low- and high-skill people will work in separate firms, and relatively low-paid workers in the advanced sector will earn more than relatively high-paid workers in the traditional sector. In fact, even the less skill-intensive tasks in the formal sectors of developing countries' economies are often performed by workers who are highly skilled and highly paid in comparison to workers in the informal sector. For example, jobs as clerks in international hotels are highly prized by college graduates in China, and jobs as flight attendants on Thai Air are sought after by the educated in Thailand.

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[15] It seems likely that in pre-modern agricultural societies, such as medieval Europe, Russia under serfdom, or colonial India, many of the better-educated workers helped landlords manage their estates, and thus worked in the same "firms" with serfs or peasants.

V. Interpreting Recent Changes in Wages and Segregation

Under our model, the recent simultaneous increases in inequality and segregation could be due either to skill-biased technological change or a shift in the observable skill-distribution. Both hypotheses are consistent with the data, although the latter provides a more parsimonious explanation.

On the technological front, the model predicts that both segregation and inequality should increase following change in which the effective skill of high-skill workers improves. There is some evidence that recent technological progress has been of this type. For example, Kahn and Lim [1994] argue that total factor productivity growth across American industries has been proportional to the fraction of skilled workers in the industry. Moreover, Berman, Bound, and Griliches [1994] find evidence that adoption of new technologies may have placed a premium on high-skill workers. This means that although the old technologies in which high- and low-skill workers mix are still available, high-skill workers now find it more attractive to leave the firms and form firms with other high-skill workers. This reduces the relative wage of low-skill workers because firms hiring them must pay more to attract complementary high-skill workers.

Our model, however, does not require appeal to largely unobserved skill-biased technological change to account for changes in wages and segregation; the data can be explained through observed changes in the distribution of skill in the United States and the United Kingdom.

There has been a substantial increase in mean-skill in the U.S. over recent decades.

From 1960-1 to 1987-88, the flow of bachelor's degrees as a percentage of the labor force increased 1.58 times. The flow of professional degrees increased 1.65 times, masters' degrees 2.14 times, and doctorates 1.87 times. [U.S. Department of Education, 1990, Table 220, cited in Ehrenberg, 1992; Bureau of the Census, 1992]. Most of the increase in the flow took place during the earlier part of the period, but the stock increased throughout the period, as less educated people left the labor force.

Evidence that the increase in the average skill in the United States has been accompanied by a moderate rise in dispersion of skill is provided by Juhn, Murphy, and Pierce [1993]. They partition increased wage inequality from 1964 to 1988 into increases in the dispersion of observable indicators of skill (education and experience), increases in returns to those observable skills, and increases in inequality due to unobservable factors.[16] Somewhat more than 20% of the growth in inequality due to observable factors can be attributed directly to increased dispersion of skill, holding the wage schedule constant .[17]

We do not have data on the skill-distribution in France, but British data also indicate an increased mean and dispersion of skill. Schmitt [1992] reports the distribution of the

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[16]They measure wage inequality by the gap between log wages of workers at the 10th and 90th percentiles of the wage distribution.

[17]The single biggest source of increased inequality is the increase in dispersion of wages within education-experience groups. This could be due to increased variance in skills that are unobservable to the econometrician or to increased returns to these unobservable skills. If one assumes that the decomposition into quantity and price changes is the same for unobserved as for observed variables, then slightly more than 20 percent of the total increase in inequality is accounted for by the direct impact of widening of the skill-distribution. Under the assumption that the entire change in the residual is due to changes in prices, rather than quantities, of unobserved skill, widening of the skill-distribution explains just under 10 percent of the increased inequality.

British workforce by educational qualifications in 1978-80 and 1986-88, based on data from the General Household Survey. In 1974-76 the median worker had no qualifications beyond primary education. By 1986-88, the median worker had some O-levels or lower-middle vocational education. We calculated the variance of expected log wages conditional on education in both periods, holding constant the wage schedule, as a measure of the variance of the observable skill distribution.[18] The variance of skill, as measured by the variance of expected log wages conditional on education, increased 20% from 0.0410 in 1978-80 to 0.0488 in 1986-88.

Thus, at least in the U.S and Britain, there have been significant changes in the mean and dispersion of the skill-distribution. These changes have not been as profound as the shift in the wage-distribution, but such a discrepancy is to be expected from our theory (see proposition 2 and 3). Of course, to assess the predictions of our model quantitatively would require a more detailed and richer formulation of the production function than that of section III.

One issue that arises in interpreting the empirical results is the timing of the relationship between changes in the skill-distribution and changes in worker-matching. If there are search costs, changes in the skill-distribution may primarily affect matching of new entrants to the labor force and of the unemployed, rather than causing workers who were already matched to switch jobs. Matching patterns among workers may thus respond to changes in the skill-distribution only with a lag. Juhn, Murphy, and Pierce [1993] find the

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[18] The wage schedule is based on Schmitt's estimates of differences in wages by education group for male workers with twenty years of experience in 1986-88.

greatest increase in dispersion of skill in the U.S. occurred from 1964 to 1979. There was a smaller increase in skill dispersion from 1979 to 1988, with about ten percent of the total increase in the wage differential between workers at the 10th and 90th percentiles of the wage distribution accounted for by increasing dispersion of observable skills. Overall, only three percent of the increase in wage dispersion between 1979 and 1988 can be attributed directly to measured increases in skill dispersion. The data on segregation of workers by skill cover the period 1975-86, with the increase in segregation coming between 1977 and 1984. It is possible that changes in sorting during the later period reflect either the small contemporaneous widening of the skill-distribution, or the larger earlier widening of this distribution.

In sum, the data are consistent either with the hypothesis that technological change raised the effective skill of high-skill workers, or with the more parsimonious story that there were simply observed increases the mean and dispersion of skill. In either case, the model can explain the simultaneous rise in segregation and inequality.

Our model is also consistent with the absolute decline in wages for low-skill workers found by Katz and Murphy [1992]. The simplest models of skill-biased technological change do not generate such a decline. Technological change that opens up new opportunities for high-skill workers does not necessarily make low-skill workers worse off, since the older technologies remain available. Technological change that increases the productivity of low-skill workers could reduce their wages by reducing the prices of products they produce. There is, however, mixed evidence about whether there has been a fall in prices of goods produced by low-skill workers and, if so, sufficient to explain the reduction in wages for low-skill workers [Lawrence and Slaughter, 1993; Krugman and Lawrence, 1993; Sachs and Shatz, 1994; Krueger, 1996].

VI. Policy Implications and Directions for Future Research

To conclude, let us consider two prominent social policy issues that have rather different implications in our model than in more standard frameworks, and speculate on the dynamic interaction between inequality and segregation.

In many countries, there are strong social norms promoting equality across workers within a firm. Freeman [1980], for example, provides evidence that unions typically reduce intra-firm wage differentials. Our model implies that social norms for equality within the firm may have perverse effects on the distribution of income in the economy as a whole.[19] If low- and high-skill workers were perfect substitutes, pressures for equality within the firm would lead to increased segregation by skill, but would not affect wages or output. However, in our model, pressures for equality within the firm will reduce output. Moreover, they may perversely reduce wages for low-skill workers by causing high-skill workers to sort into separate firms, thus depriving low-skill workers of the benefit of cross-matching.[20]

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[19]We are grateful to Daron Acemoglu for suggestions on this point.

[20]For example, suppose that some workers had skill 1, and that slightly more workers had skill 1.5, so that rents from cooperation would accrue to the scarce low-skill workers. Some firms would match together the two types of worker, and produce output 2.25. Firms with only high-skill workers would produce output 3.375, so high-skill workers would earn half of this, or 1.6875. Low-skill workers would earn 2.25 - 1.6875, or .5625. If pressures for intra-firm equity made it impossible to have a wage gap this great, then .11 high-skill workers would self-match together, and earn a wage of 1.6875 -- exactly what they earned otherwise.

The model also bears on tax policies, such as those in the Clinton health care proposal, that encourage increased segregation of high-wage workers [Cutler, 1994; Elmendorf and Hamilton, 1994; Sheiner, 1994]. Under standard efficiency unit models, workplace composition would be perfectly elastic to such incentives, and so revenue calculations based on the current workplace composition would seriously overestimate revenue. However, such policies would have no harmful effect on efficiency. Under our model, by contrast, workforce composition would be only imperfectly elastic to incentives for segregation (and so naive revenue estimates might not perform badly). Still, changes in workforce composition would create deadweight losses.

We have taken the distribution of skill as exogenous. Future researchers may wish to endogenize the distribution of skill, and in particular to examine the effect of matching patterns on skill acquisition. In industries in which people learn from high-skill colleagues, a worker often serves as assistant to a high-skill manager when young, and as the manager of low-skill assistants when old.[21] For example, young lawyers and doctors start out assisting more senior colleagues, performing tasks that are only moderately sensitive to skill. In the process, they profit from the wisdom of their elders. Later, when human capital accumulation is less important, most take positions of higher rank and supervise less-skilled subordinates. It is relatively cheap to provide training in this way if there are tasks that are suited to

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Low-skill workers would also self-match and earn a wage of .5 -- less than what they would have earned in the absence of pressures for equity within the firm.

[21] This is not to deny that there are also learning environments in which people maximize learning by performing a high-skill job, even if their co-workers are of lower skill.

workers of medium skill, as in law or medicine.[22]

These examples suggest that increases in segregation will spur future inequality of skill by reducing opportunities for low-skill workers to learn from high-skill co-workers. This raises the possibility that exogenous increases in either segregation or skill inequality may spur a vicious cycle of ever-rising inequality and segregation.

Besides its implications for income distribution and the evolution of the skill distribution, the pattern of matching may have political and social implications. A society in which high- and low-skill workers match together in firms is likely to have a more homogenous culture than one in which people match with others of their own skill. Language, customs, and fashions are likely to be more similar if there is more interaction between groups. Societies with wide distributions of skill, such as the contemporary United States, Brazil, and India, are more likely to develop separate subcultures and perhaps to experience political and social conflict between these subcultures. On the other hand, a pattern of matching in which groups of people always play the same role (manager or assistant) may develop into a caste-like system, in which participants work together, but in a rigid hierarchy. There may be advantages to matching patterns in which over wide ranges people are indifferent between being the assistant to a higher level manager or the manager of a lower level assistant, so that they play different roles in different situations.

[22] Of course, this condition does not always obtain. For example, young baseball players would presumably learn a great deal from playing in the major leagues, but the production function makes it important to match the best players together, and so young players train in the minor leagues, probably less effectively.

Appendix I

It is possible to construct a confidence interval for the segregation index under the assumption that the sampling errors in the estimates of the variances of q within and between firms are independent. This will be the case if the variance of q within the firm is independent of the average q within a firm or is a linear function of the average q within the firm.

Given that $s^2_w N/\sigma^2_w$ and $s^2_b J/\sigma^2_b$ are distributed as independent χ^2 variables with N-J and J-1 degrees of freedom respectively,

(A.1)
$$\frac{1}{1 + \dfrac{\dfrac{s^2_w}{\sigma^2_w}}{\dfrac{s^2_b}{\sigma^2_b}}} \sim \frac{1}{1 + b_f},$$

where b_f is an F-distributed random variable with (N-J,J-1) degrees of freedom.

Thus a 95-percent-confidence interval for the index of segregation is

$$\text{(A.2)} \qquad \frac{F(N-J,J-1)_{0.025}}{F(N-J,J-1)_{0.025} + \dfrac{s_w^2}{s_b^2}} \le \rho \le \frac{F(N-J,J-1)_{0.975}}{F(N-J,J-1)_{0.975} + \dfrac{s_w^2}{s_b^2}}$$

where $F(N\text{-}J,\ J\text{-}1)_{0.975}$, and $F(N\text{-}J,\ J\text{-}1)_{0.025}$ are numbers at which the cumulative probability distribution function $F(N\text{-}J,J\text{-}1)$ takes the values 0.975 and 0.025 respectively.

This derivation assumed that the variance of q within a firm was independent of, or linear in, the firm's average q. If the variance of q within firms is convex in the average q in a firm, then when a sample of firms is picked that has a high variance of average q, it will also tend to have a high variance of q within firms, which will make the estimate more precise than the calculations below would indicate. The variance of q within firms will be convex in average q, for example, if the standard deviation of q within the firm is proportional to the average q within the firm. On the other hand, if the variance of q within firms is concave in average q, then the formulas above will understate the width of the confidence interval. Note that this will be the case for dichotomous variables, such as race or production worker status, because the variance of the characteristic within the firm equals $\mu(1 - \mu)$, where μ is the share of people with the characteristic. However, when this share is close to one or zero, the variance within firms will be approximately linear in average q, and the estimated confidence intervals should not be that far off.

Appendix II

Proof of Lemma 8: Consider $n < An^*(\theta)$. Suppose an n-worker is matched with some skill $n' \geq An^*(\theta)$. From Lemma 1, an n worker cannot be matched with $n' \geq n^*(\theta)$. Hence consider $n' \in [n_1(\theta), n^*(\theta))$. Given that, from Lemma 5, $w(n';\theta) = n'(n^*(\theta))^2 - (n^*(\theta))^3/2$, we have

$$\text{(A.3)} \quad v(n,n';\theta) = n(n')^2 - w(n';\theta) = n(n')^2 - n'(n^*(\theta))^2 + \frac{(n^*(\theta))^3}{2} \equiv H(n').$$

It suffices to show that, on the interval $[n_1(\theta), n^*(\theta)]$, $H(n')$ is maximized at $n' = n_1(\theta)$. To establish this, note that the derivative of $H(\cdot)$ is $2nn' - (n^*(\theta))^2$. Now at $n' = n_1(\theta)$, $2nn' \leq 2A^2(n^*(\theta))^2 < (n^*(\theta))^2$, and so the derivative is negative at this point. Suppose there exists $p > n_1(\theta)$ (not necessarily an integer) such that $H(p) = H(n_1(\theta))$. In fact, let p be the smallest such number. Then $dH(p)/dn' \geq 0$. Moreover, because $2nn' - (n^*(\theta))^2$ is increasing in n', $dH(n')/dn' > 0$, for all $n' > p$. Since $H(p) = H(n_1(\theta))$,

$$\text{(A.4)} \quad H(n^*(\theta)) > H(n') \text{ for all } n' \in [n_1(\theta), n^*(\theta)) .$$

But if n is matched with n' in equilibrium, then $H(n') \geq H(n^*(\theta))$, a contradiction of (A.4). We conclude that there is no p such that $H(p) = H(n_1(\theta))$, and so $H(n')$ is maximized at $n' = n_1(\theta)$ as claimed.

Q.E.D.

Proof of Proposition 2: Let $S_1=\{n \mid n \leq n_1\}$. Competitive equilibrium maximizes total product, and, for any match, the sum of the equilibrium wages equals the match's product. Moreover, for any $n < n_1(0)$, Lemma 8 implies that an n-worker is matched with another worker in S_1. Finally from Lemmas 1, 3, and 4, an $n_1(0)$- or n_1-worker is matched either with another worker in S_1 or with an $n^*(\theta)$-worker. Hence, the sum of wages of workers in S_1

$\sum_{n=\underline{n}}^{n_1} w(n;\theta)p(n;\theta)(\equiv R_1(\theta))$ equals

$$\text{(A.5)} \qquad \max_{z(\cdot,\cdot)} \sum_{n'=\underline{n}}^{n_1} \sum_{n=\underline{n}}^{n'} z(n,n')n(n')^2 + \sum_{n=n_1(0)}^{n_1} z(n,n^*(\theta))v(n,n^*(\theta);\theta)$$

subject to

$$\text{(A.6)} \qquad \sum_{n'=\underline{n}}^{n_1} z(n,n') \leq p(n;\theta), \quad n = \underline{n},\ldots,n_1(0)-1$$

and

$$\text{(A.7)} \qquad \sum_{n'=\underline{n}}^{n_1} z(n,n') + z(n,n^*(\theta)) \leq p(n;\theta), \quad n = n_1(0),\, n_1\,,$$

where, for all n, n', $z(n,n')$ is the number of matches between an n-worker and an n'-worker, and $v(n, n';\theta)$ is the wage for an n-worker if matched with an n'-worker (which may or may not occur in equilibrium), assuming the latter is paid his equilibrium wage $w(n';\theta)$.

Let P_1' be the variant of program (A.5)-(A.7) in which $\theta = 1$ in (A.5) but $\theta = 0$ in (A.6)-(A.7). Denote by R_1' the maximized value of the objective in P_1'. Because $A < 3/4$, $n^*(\theta)(2n-3n^*(\theta)/2)$ is negative when $n = n_1(i)$ for i=0,1. Hence, $v(n_1(i),n^*(\theta);\theta)= n_1(i)(n^*(\theta))^2-\frac{1}{2}(n^*(\theta))^3$ is decreasing in θ. We conclude that $R_1' < R_1(0)$. But from (26), the constraints in program P_1' are proportional to (A.6)-(A.7) with $\theta = 1$. Hence,

$$\text{(A.8)} \qquad \frac{R_1'}{\sum_{n=\underline{n}}^{n_1} p(n;0)} = \frac{\sum_{n=\underline{n}}^{n_1} w(n;1)p(n;1)}{\sum_{n=\underline{n}}^{n_1} p(n;1)},$$

and so from (A.8) and the fact that $R_1' < R_1(0)$ we conclude that (38) holds.

From Lemma 3, a worker in the set consisting of S_2 of workers of skill no less than n_2 must be matched either with another worker in S_2 or with a worker of type $n' \in [n^*(\theta),n_2-1]$.

Hence the sum of wages in of workers in S_2, i.e., $R_2(\theta) = \sum_{n'=n_2}^{\bar{n}} w(n';\theta)p(n';\theta)$, equals

$$\text{(A.9)} \qquad \max_{z(\cdot,\cdot)} \sum_{n'=n_2}^{\bar{n}} \sum_{n=n_2}^{n'} z(n,n')n(n')^2 + \sum_{n=n_2}^{\bar{n}} \sum_{n'=n^*(\theta)}^{n_2-1} z(n',n)v(n,n';\theta)$$

subject to

(A.10)
$$\sum_{n'=n^*(\theta)}^{\bar{n}} z(n',n) \le p(n;\theta),\ n=n_2,\ldots,\bar{n}.$$

From Lemma 2,

(A.11)
$$v(n,n^*(\theta);\theta) = n^*(\theta)n^2 - \frac{(n^*(\theta))^3}{2},\ \text{for}\ \ n \ge n_2.$$

From Lemma 7, (35), and the fact that $n_2 < \sqrt{2}\,n^*(\theta)$,

(A.12)
$$v(n,n';\theta) = n'n^2 - n^*(\theta)(n')^2 + \frac{(n^*(\theta))^3}{2},$$
$$\text{if}\ n' \in (n^*(\theta),n_2)\ \text{and}\ n \ge n_2\,.$$

For $n \ge n_2$, the fact that $n_2 \ge (3/2)^{1/2} n^*(1)$ definition of n_2 implies that the right-hand side of (A.11) is increasing in θ. Similarly, because $n' < n_2$, the right-hand side of (A.12) is also increasing in θ. We conclude that if P_2' is the program in which $\theta = 1$ in (A.9) but $\theta = 0$ in (A.10), the maximized value R_2' of the corresponding objective satisfies $R_2' > R_2(0)$. But from

(26), the constraints (A.10) when $\theta=0$ are proportional to (A.10) when $\theta = 1$. Hence

(A.13) $$\frac{R_2'}{\sum_{n=n_2}^{\bar{n}} p(n,0)} = \frac{\sum_{n=n_2}^{\bar{n}} p(n;1)\, w(n;1)}{\sum_{n=n_2}^{\bar{n}} p(n;1)}.$$

This yields (39).

Q.E.D.

Proof of Proposition 3: We can decompose (41) as follows. Let B_θ and W_θ be the dispersions of wages between and within firms, respectively, for those firms that hire only workers from the interval $[n_1+1, n_2-1]$. Then

(A.14) $$B_\theta = \sum_{n,n' \in [n_1+1,\, n_2-1]} (\mu(n,n') - n^*(\theta))^2 z(n,n';\theta)$$

and

(A.15) $$W_\theta = \sum_{n,n' \in [n_1+1,\, n_2-1]} (n - \mu(n,n'))^2 z(n,n';\theta)\ .$$

Similarly, let B_θ' and W_θ' be the dispersions, respectively, between and within all other firms. We have:

$$
\text{(A.16)} \qquad B_\theta' = \sum_{n'=\underline{n}}^{\bar{n}} \sum_{n \notin [n_1+1, n_2-1]} (\mu(n,n') - n^*(\theta))^2 z(n,n';\theta) + \sum_{n' \notin [n_1+1, n_2-1]} \sum_{n \in [n_1+1, n_2-1]} (\mu(n,n') - n^*(\theta))^2 z(n,n';\theta)
$$

and

$$
\text{(A.17)} \qquad W_\theta' = \sum_{n'=\underline{n}}^{\bar{n}} \sum_{n \notin [n_1+1, n_2-1]} (n - \mu(n,n'))^2 z(n,n';\theta) + \sum_{n' \notin [n_1+1, n_2-1]} \sum_{n \in [n_1+1, n_2-1]} (n - \mu(n,n'))^2 z(n,n';\theta).
$$

We can rewrite (41) as

$$
\text{(A.18)} \qquad \rho(\theta) = \frac{B_\theta + B_\theta'}{B_\theta + B_\theta' + W_\theta + W_\theta'} \quad .
$$

Now, the proofs of Lemmas 5 and 6 imply that if $n,\ n' \in (n_1,n_2)$, then $z(n,n';\theta) > 0$ if and only if either $n = n^*(\theta)$ or $n' = n^*(\theta)$. But then

(A.19) $$|\mu(n,n') - n^*(\theta)| = |n - \mu(n,n')|,$$

and so

(A.20) $$B_\theta = W_\theta.$$

Moreover, from Lemma 3, if $z(n, n';\theta) > 0$ and $n \leq n_1$, or $n \geq n_2$, then

(A.21) $$|n - \mu(n,n')| \leq |\mu(n,n') - n^*(\theta)|,$$

with strict inequality unless $n' = n^*(\theta)$. Hence,

(A.22) $$W_\theta' < B_\theta'$$

From (A.20) and (A.22) we can re-express (A.18) as

(A.23) $$\rho(\theta) = \frac{B_\theta + B_\theta'}{2B_\theta + 2B_\theta' - \Delta}$$

where $\Delta > 0$. Now, below, we will show that, for r sufficiently big, (40) implies that

(A.24) $$B_1 < \beta B_0$$

and (26) implies that

(A.25) $$B_1' \approx \alpha B_0' \text{ and } W_1' \approx \alpha W_0'.$$

Now, the fact that $\Delta > 0$ and (A.23)-(A.25) imply

(A.26) $$\rho(1) > \frac{\beta B_0 + \alpha B_0'}{2\beta B_0 + 2\alpha B_0' - \alpha\Delta}$$

But because $\alpha > 1 > \beta$ and $\Delta > 0$, the right-hand side of (A.26) exceeds that of (A.23) with $\theta=0$, establishing that $\rho(1) > \rho(0)$. It remains to establish (A.24) and (A.25).

To derive (A.24), note that,

$$B_1 = \sum_{n,n' \in [n_1+1,n_2-1]} (\mu(n,n') - n^*(1))^2 z(n,n';1)$$

(A.27)
$$= \frac{1}{2} \sum_{n \in [n_1+1,n_2-1]} (n-n^*(1))^2 z(n,n^*(1);1) \quad \text{from Lemmas 3-7}$$

$$< \frac{\beta}{2} \sum_{n \in [n_1-1,n_2-1]} (n-n^*(0))^2 z(n,n^*(0);0) \quad \text{from (39) for } r \text{ big enough}$$

$$= \beta B_0,$$

establishing (A.24).

To demonstrate (A.25), note that, for r sufficiently big, $v(\mathrm{n}, \mathrm{n}^*(0);0) \approx v(\mathrm{n}, \mathrm{n}^*(1);1)$ and so from (A.5) to (A.6) and (A.8) to (A.8).

(A.28)
$$z(n,n';1) \approx \alpha z(n,n';0), \text{ for } n \le n' \le n_1 \text{ or } n_2 \le n \le n',$$

which implies (A.25).

Q.E.D.

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