Welfare Analysis of Imperfect Information Equilibria

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P. Diamond*

1. Introduction

Recently there has been considerable analysis of optimal rules for a consumer with imperfect information about prices for a commodity.¹ These models have been used to describe behavior on one side of markets with imperfect information.² Generally, the approach taken has used an explicit and relatively simple description of the marketing technology to permit the derivation of concrete results about equilibrium, sometimes an explicit calculation of the distribution of prices in the market. Often, one condition of equilibrium has been knowledge by consumers of the price distribution in the market. This analysis differs from these two common traits. The focus is to consider a fairly general model of equilibrium (although a static, partial equilibrium model) to relate welfare analysis to characteristics of the market.

*Financial support by NSF gratefully acknowledged.

¹For a recent survey, see Lippman and McCall (1976).

Part of this generality is to consider a reduced form of consumer behavior which need not come from optimization from a known distribution. This permits the possibility that marginal cost pricing is not optimal even when the government controls the entire supply side of the market.\(^1\) The paper begins with analysis of this issue including the case of optimal behavior in the presence of an unknown distribution, following the analysis of Rothschild (1974). By means of a two-price example, it is shown that marginal cost pricing will generally be nonoptimal when the consumer has a subjective Dirichlet prior which is the correct distribution. The assumption that the consumer does not know the distribution to be correct (with certainty) gives rise to this possibility. In the case considered, the adjustment of subjective prior distributions leads consumers to search too little, implying that increasing the spread in prices (relative to the spread in marginal costs of supply) increases consumer welfare. This follows even if the government does not know the correct distribution.

With the reduced form description of consumer behavior described in Section 3, the model is closed by consideration of a Nash equilibrium with expected profit maximizers who do

\(^1\)This result is a necessary part of analysis of government actions in a private market and may not be interesting as a guide to pricing in government stores.
know the demand curves they face with certainty. For this model we will derive the expressions for the derivatives of social welfare with respect to prices and selling effort. These derivatives, in turn, can be used to express the first order conditions for welfare maximization.
2. Consumer Behavior

Consider the problem of a consumer seeking to purchase a commodity being purchased just once. While the quantity purchased may vary with price, the entire purchase is to be made in a single store. We assume that there are \( J \) stores and \( K \) possible prices that might be charged at each store. Thus there are \( K^J \) possible price distributions which might exist in the market. We assume that stores differ only in price and in the cost of travel (i.e., the commodity is homogeneous) and that search costs do not affect demand.

The consumer has some idea (perhaps a subjective prior distribution) about the nature of the price distribution when the effort to purchase is commenced. A consumer strategy is made up of three parts, each step based on prices actually observed up to that time. The first part is the sequence of stores to visit. This might be random with equal probability (as in Rothschild (1974)) or might involve a conscious sequence (as in Salop (1973)) based on differential costs of visiting or differential expectations of the prices to be found in any store. The second part is the

\[ ^1 \text{Attention here is paid to situations where consumers do not engage in "portfolio diversification" in their purchasing behavior. Some repeat purchase situations (e.g., restaurants, gasoline) may be squeezed into the framework analyzed.} \]
decision whether to visit an additional store or cease information gathering activity. The third part is to make the best purchase given the information gathered. We assume that the consumer is always successful in this last element and pay it no further attention.

Now assume that the government is setting prices at all J stores and that marginal costs of supply differ among stores.¹ We take the number of stores as given. (Presumably location considerations can justify the maintenance of stores with different marginal costs of supply.) We can assume many consumers simultaneously engaged in this purchase. For some sets of behavior rules among the consumers, marginal cost pricing will be Pareto optimal. For other sets, marginal cost pricing will not be optimal. It would be desirable to have a characterization of the situations of each kind and, in the latter case, a description of the desirable direction of divergence from marginal cost pricing. This discussion is not so ambitious, merely offering a few examples to illustrate both possibilities and to point up some

¹For example, state liquor stores having varying rent or costs of delivery of merchandise to the store.
elements affecting the direction of divergence in a simple case.¹

Let us start with the case where the consumer knows which one of the $K^J$ price distributions is the one that exists and is maximizing utility. Then we are in the familiar certainty model with distinct locations, and marginal cost pricing is desirable. Given that the consumer knows the distribution and prices equal social marginal costs, the consumer is solving the same social allocation problem as is used to define Pareto optimality. Thus the private solution is necessarily the social optimum.

Considerable attention in the literature has been paid to the symmetric case. Assume that search and shopping costs are the same for all stores.² Assume that whatever price distribution is chosen by the government, the consumer knows the distribution that exists in the market, but not which prices are to be found at which stores. (Thus no more than $J!$ of the $K^J$ possibilities have positive weight in subjective probabilities.) Assume that consumers follow optimal sequential search rules. Then again, given the

¹Both empirical research on consumer behavior in the face of unknown prices and theoretical analysis are needed to develop interesting characterizations. For an empirical analysis, see White and Munger (1971).

²This is a peculiar assumption since locational differences in residence, work, and travel are a pervasive fact.
information structure and given marginal cost pricing, the consumer is solving the social welfare problem, and the resulting allocation is optimal.

Even if consumers are not following optimal sequential search rules, marginal cost pricing will be optimal if the consumer is following the socially best strategy given the set of strategies he can be induced to follow by different pricing rules.¹ For example, the consumer might visit \( n \) randomly selected stores and then purchase at the lowest price (including shopping cost) found. More interestingly, the consumer may select the number of stores to be visited before purchase to optimize expected utility given the known price distribution. Again, given the class of strategies followed, the planner could not induce a better strategy by having prices diverge from marginal costs.

These examples are characterized by the property that consumer beliefs about the price distribution do not change as a result of observed prices and thus strategies can be described in terms of the alternatives at hand and a subjectively unchanging menu of alternatives to be sought. It is easy to construct examples of consumer behavior which would warrant different pricing rules. Only some of these examples

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¹In a different context, this situation has been analyzed by McFadden (1969).
are interesting, but they illustrate the roles that prices play. We shall consider three examples.

Consider a consumer who samples n stores, calculates a statistic based on these n observations (e.g. variance) and samples more stores for some values of the statistic and not for others. Depending on the social desirability of more sampling, it may be worthwhile to have prices diverge from marginal costs to affect the probability of further search. For example, the differences in marginal cost may not be large enough to justify any search, yet the consumer does more searching if the first two prices observed differ at all. Then uniform pricing may be optimal to avoid wasteful search, even though the quantities actually purchased at stores may not be optimal given purchase at particular stores (since price differs from marginal cost). Alternatively consumers may not search enough because of beliefs on the structure of prices.¹ A few outrageously priced stores may alter beliefs and so induce a superior quantity of search.

Consider a consumer who makes an optimal selection of

¹There is some evidence that consumer beliefs about grocery prices reflect the nature of the store as well as actual prices and are often quite inaccurate. See Brown and Oxenfeldt (1972).
the number of stores to be sampled solely on the basis of his subjective prior on the distribution of prices. Assume that his subjective prior is wrong. Assume that changing the actual distribution of prices will change his prior. Since there is no welfare loss from wrong quantities purchased for the first (infinitesimal) move of prices away from marginal costs, a change that results in a better selection of the number of stores to be sampled would raise welfare.

As a third example, consider a consumer optimally searching, but from an unknown distribution. Assume he has a Dirichlet prior. (That is, he has subjective beliefs on the number of firms at each price. When he observes a firm charging a particular price, he increases the number of firms he believes are charging the observed price and decreases the numbers charging each other price in proportion). Assume that his prior distribution is the correct distribution. Even so it may be socially advantageous to diverge from marginal cost pricing. To illustrate this, let us make the example very simple. Assume that the consumer is minimizing the expected cost of purchasing one unit and that search costs are constant over stores and length of search. Assume that costs are either $c_1$ or $c_2$ ($c_1 < c_2$) at each location. The government chooses the prices $p_1$ and $p_2$ at the two locations ($p_1 \leq p_2$). Assume that the consumer
know the two prices, but doesn't know either which prices are at which locations, or the fraction of stores which have the lower price. He has subjective beliefs that the fraction of the stores with the low price is \( \mu \). If he observes a store with the high price, his belief about the fraction of stores with the low price will shift to \( \frac{\mu}{\rho+1} \). Thus \( \rho \) measures the lack of confidence of the consumer in his subjective prior.\(^1\) (\( \rho = 0 \) corresponds to acting as if he knows the fraction to be \( \mu \) with certainty.) This corresponds to the optimal probability adjustments for someone with a Dirichlet prior.\(^2\) Given his beliefs (fully described by the pair \( (\mu, \rho) \)), his costs per search and the difference in the two prices, the consumer has one of two plans--to purchase at the next store visited, or only if the next store visited is a low-price store. If he doesn't purchase, the consumer will revise \( (\mu, \rho) \) by the rule above and then reconsider his decision. The larger the fraction of stores believed to have low prices, the less advantageous appears the policy of buying at the next store regardless of its price. The less the confidence of the consumer in his beliefs, the more

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\(^1\)With an additional observation, confidence will shift to \( (\rho^{-1}+1)^{-1} \).

\(^2\)For details see Rothschild (1974). A formal proof of the discussion here appears in the Appendix.
appealing the purchase at whatever price is sampled.\(^1\) This follows from the fact that observation of a high-price store will give a larger decrease in the believed fraction of low price stores the larger the lack of confidence. Thus we can show the behavior rule of the consumer in a simple diagram\(^2\)

\[
\begin{array}{c|c}
\text{fraction of low-price stores (µ)} & 1 \\
\hline
\text{purchase if next store is low-price} & \text{purchase at the next store regardless of price} \\
\hline
0 & 1 \text{ lack of confidence (ρ)}
\end{array}
\]

Figure 1

The socially correct rule (assuming that the number of firms is large enough that visits can be viewed as sampling with replacement) is the rule which would follow if the consumer optimized, taking the price distribution as given--that is, the rule he would follow if \(ρ\) were zero. In this setting of

\(^1\)This result is contrary to the view that lack of confidence tends to increase the value of search. A result of this sort in Kohn and Shavell (1974) (comparing known and unknown distributions) is based on the same posterior distribution, not the same prior, as they point out.

\(^2\)The curve dividing regions need not pass to the left of \((1,1)\), but might.
only two prices, consumers always buy when they find a low price. However, when they find a high price they review their beliefs in the structure of the market, decreasing their expectations of finding a low price. Thus they are more likely to accept a high price than if they were certain about the fraction of low-price stores in the market. Now consider a market with many consumers, all having the correct subjective probability $\mu$ before beginning shopping, but varying costs of search and confidence. All consumers finding a low-price store in their first search will purchase. Of those finding a high-price store, those with sufficient confidence ($\rho$ near zero) or search costs so they are far from the margin of indifference between the two strategies will follow the socially correct rule. However, some with low confidence and search costs so that they are close to the margin will buy when social efficiency would call for them to search again. All those who do search again have lower values of $\mu$ and $\rho$. Some of them will now purchase at a high-price store rather than continuing search despite the fact that for this simple case efficiency calls for no change in reservation price. If the government increases the spread in prices relative to the spread in marginal costs, it increases the incentives for search. There are three effects of such a move—when purchasing, consumers now buy at the wrong price (although in the model presented,
with inelastic demand this has no efficiency cost), and some
individuals with sufficient confidence are induced to search
when they shouldn't. Some individuals without sufficient
confidence (or who have had a series of observations of high
prices) are induced to undertake more searching, which is
socially advantageous.¹ For sufficiently small movements
from marginal cost pricing, the first two effects become
vanishingly small, while the third effect does not. Thus
some movement from marginal cost pricing is desirable, with
the optimal size depending on the mix in the population of
the critical consumer parameters (confidence, search costs).²

¹This picture would be further complicated if we inclu-
ded consumers with incorrect priors.

²I leave for future analysis the question of whether
the more commonly found behavior of consumers tends to
justify increases or decreases in price spreads, since this
depends on pricing behavior as well as consumer behavior.
3. Modeling Consumer Behavior

The discussion above makes clear that the asymmetry between government knowledge of price distributions and consumer subjective beliefs (even if correct) can result in a situation where marginal cost pricing is not optimal.\(^1\) However, the situation is further complicated by the fact that imperfect consumer information is part of the market environment in which firms operate. This can create incentives for pricing that differ from marginal cost pricing and incentives for selling efforts to affect the stores at which consumers purchase. To proceed with the analysis, we need a formal model of consumer behavior. We shall present that now and then briefly relate it to the discussion of the previous section.

For simplification, we make the following assumptions. Consumer preferences can be described by expected utility. Utility is linear in income to spend on other goods. Efforts in purchasing are calculated in financial terms. (Thus, when purchasing, demand depends only on the price at which the purchase is made.) The distribution of prices and selling efforts affect the likelihoods that consumers will purchase from different suppliers and the buying effort needed to

\(^1\) If the government knows that consumers have the correct prior, the government need not know the actual distribution. It would be interesting to try to weaken this condition by having a distribution of priors in the population.
complete the transaction. Introducing the notation we have

\[ p = \langle p_1, \ldots, p_J \rangle \]

prices charged by the J sellers

\[ y = \langle y_1, \ldots, y_J \rangle \]

selling efforts by the J sellers

\[ x^h(p, y) \]

expected value of buying effort by consumer h

\[ \pi(p, y) = \langle \pi^h_j(p, y) \rangle \]

probability that consumer h purchases from store j

\[ \pi^h_0(p, y) \] is the probability that the consumer is unable to complete a transaction.\(^1\)

\[ q^h(p_j) \]

quantity purchased if purchase is completed at price \( p_j \).

\[ u^h(q^h(p_j)) \]

utility derived from the commodity\(^2\) (gross of purchase price and buying effort)

\[ v^h(p_j) = u^h(q^h(p_j)) - p_j q^h(p_j) \]

utility derived from the commodity net of purchase cost but gross of buying effort\(^3\)

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\(^1\)This needs to be distinguished from the case that the prices are such that the consumer doesn't want to complete a transaction even in the absence of the cost of buying effort.

\(^2\)Since \( q^h \) is assumed to be optimally chosen \( u^h(q^h(p_j)) = p_j \).

\(^3\)By the standard properties of an indirect utility function we have \( v^h(p_j) = -q^h(p_j) \).
w^h(p,y) is the expected utility for the consumer in a market with the vectors p and y.

From the definitions we have:

\[ w^h(p,y) = \sum_{j=0}^{J} \pi^h_j(p,y)(u^h(q^h(p_j)) - p_jq^h(p_j)) - x(p,y) \]

\[ = \sum_{j=0}^{J} \pi^h_j(p,y)v^h(p_j) - x(p,y) \tag{1} \]

For welfare analysis we need to know the partial derivatives of \( w^h \) with respect to prices and selling efforts. Let us differentiate first with respect to the price charged by the k-th supplier.

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1. We ignore the important issue that much consumer information comes from other consumers. As long as the set of consumers is held fixed, these functions can be viewed as reduced forms.

2. Note that most models that have been analyzed are of this general form.

3. We take \( p_0 \) to be \( \infty \).

4. For later reference we can also consider the value to the consumer of an increase in selling effort.

\[ \frac{\partial w^h}{\partial y_k} = \frac{\partial x^h}{\partial y_k} + \sum_{j=0}^{J} \frac{\partial \pi^h_j}{\partial y_k} v^h(p_j) \]

This term will be useful later in the analysis of social welfare.
\[ \frac{\partial w^h}{\partial p_k} = -\pi_k^h q^h(p_k) + \left\{ \sum_{j=0}^{J} \frac{\partial \pi_j^h}{\partial p_k} v^h(p_j) - \frac{\partial x^h(p,y)}{\partial p_k} \right\} \]  

(2)

When the term in tall brackets is zero, we will say that the classical condition of consumer demand holds, for this condition is the basis for marginal cost pricing, as can be seen by differentiating social welfare, which is the sum of expected utilities plus the expected profits of suppliers. If we denote the costs per unit of supply by the j-th firm by \( c_j \), we can write social welfare as

\[ S(p,y) = \sum_{h=1}^{H} w^h(p,y) + \sum_{j=1}^{J} (p_j - c_j) \sum_{h=1}^{H} \pi_j^h(p,y) q^h(p_j) - \sum_{j=1}^{J} y_j \]  

(3)

Differentiating this expression with respect to \( p_k \) we obtain

\[ \frac{\partial S}{\partial p_k} = -\sum_{h=1}^{H} \pi_k^h q^h(p_k) + \sum_{h=1}^{H} \left\{ \sum_{j=0}^{J} \frac{\partial \pi_j^h}{\partial p_k} v^h(p_j) - \frac{\partial x^h(p,y)}{\partial p_k} \right\} \]

\[ + \sum_{h=1}^{H} \pi_k^h q^h(p_k) + (p_k - c_k) \sum_{h=1}^{H} \pi_k^h q^h(p_k) \]  

(4)

\[ + \sum_{j=1}^{J} (p_j - c_j) \sum_{h=1}^{H} \frac{\partial \pi_j^h}{\partial p_k} q^h(p_j). \]

If the classical condition holds (the expression in tall brackets is zero for each consumer) then marginal cost
pricing is a solution of the first order conditions obtained by setting $\frac{\partial S}{\partial p_k}$ equal to zero. Conversely, if the classical condition isn't satisfied, unless divergences across consumers just balance (the sum of terms in tall brackets equals zero), marginal cost pricing cannot be a solution to these first order conditions. The issue raised in the previous section was that of examining the determinants of the sign of the difference from the classical condition, that is, the sign of

$$\frac{\partial w^h}{\partial p_k} + \eta^h_k q^h(p_k). \tag{5}$$

While we considered a few examples where this term was and wasn't zero, a systematic study of the sign and size of this expression remains to be made.
4. **Equilibrium**

Given the general formulation of consumer behavior, and the assumption of perfect information expected profit maximization by suppliers we can now express the conditions for equilibrium and for welfare maximization for different sets of public controls. For the definition of equilibrium we use the Nash equilibrium concept, with firms taking both the selling efforts and prices of other firms as given. We assume that demand curves are known and that firms are expected profit maximizers. These last two assumptions are not fully satisfactory, but some limit on the complexity of the analysis is needed. Thus the equilibrium conditions are the $J$ conditions.¹

\[
p_j, y_j \text{ maximize } \sum_{h=1}^{H} h_j(p, y)(p_j - c_j)q^h(p_j) - y_j \quad (10)
\]

When it is not necessary to consider the shutdown condition (or other candidates for multiple solutions) we can describe equilibrium by the $2J$ first order conditions.

¹One could replace these assumed perceptions by reaction functions to consider different market structures. In addition by relating utility and demand to $j$ and $y_j$ one could introduce product differentiation.
\[
\sum_{h=1}^{H} \frac{\partial \pi^h_j}{\partial y_j} (p_j - c_j) q^h(p_j) = 1
\]  
\[
\sum_{h=1}^{H} \left[ \pi^h_j q^h(p_j) + \pi^h_j (p_j - c_j) q^{-h}(p_j) + \frac{\partial \pi^h_j}{\partial p_j} (p_j - c_j) q^h(p_j) \right] = 0
\]
\[
j = 1, \ldots, J
\]

These conditions are familiar ones\(^2\)--that the expected marginal revenue product of selling effort be one and that prices be increased until marginal profits are zero. To get on with the welfare analysis we will not consider questions of existence of an equilibrium and existence, specifically, of symmetric and nonsymmetric equilibria when the purchase probability functions are symmetric, recognizing that only some specifications of the general model will yield suitable equilibria.

From the model of Butters (1974), which is a special case of the general model (although not of (11) since free entry, or conversely the shutdown condition is an essential element of his analysis), we know that there is no necessary relationship between \( p \) and \( c \) or \( y \) and \( c \) since he considered firms all with the same costs \( c \) but differing \( p \) and \( y \). In his case, high price firms advertise a lot per sale.

\(^2\)The solution corresponds to monopolistic competitive pricing with demand derivative \( \pi q^+ + q \partial \pi / \partial p \).
5. An Example

To give a sense of the variety of questions which might be explored in models of this type, let us consider a special example. By varying the parameters of the example we will have examples of symmetric and nonsymmetric equilibria with firms of identical characteristics and equilibria with differing firms. We assume that individual demand is the same for all individuals and linear

\[ q^h(p) = \text{Max} [d-ep,0] \]  \hspace{1cm} (12)
\[ d > 0, \ e > 0 \]

The probability of purchase is assumed to have a logit form with a piecewise linear structure. The justification for this structure is that probability varies smoothly with price in the range of "normal" prices but that a firm gets a larger share once its price is enough below the typical price to be visible.\(^1\)

\[ \pi_k(p,y) = \frac{e^{a_k(y_k)-b_k(y_k)p_k+r_k(y_k)\text{Max}[\tilde{p}-p_k-g,0]}}{e^{a_0} + \sum_{j=1}^{J} e^{a_j(y_j)-b_j(y_j)p_j+r_j(y_j)\text{Max}[\tilde{p}-p_j-g,0]}} \]  \hspace{1cm} (13)

\(^1\)One might think it plausible to have another slope for sufficiently high prices. More general definitions of \( \tilde{p} \) would also increase plausibility.
\[ \hat{p} = j^{-1} \sum_{j=1}^{J} p_j \]

Having \( e^{a_0} \) in the expression allows for the difficulty in reaching some potential demanders. In addition we assume that firms only perceive the direct effect of \( p_k \) through its role in the numerator. That is, for individual maximizations firms act as if

\[ \frac{\partial \ln \pi_k}{\partial p_k} = -b_k - \left\{ \begin{array}{ll} r_k & \text{as } p_k \leq \hat{p}_g \\ 0 & \text{as } p_k > \hat{p}_g \end{array} \right\} \] (14)

This formulation excludes the common phenomenon of firms competing against neighboring firms (as in local gas price wars), allowing only competition against the general market.\(^1\) Nevertheless the example is rich enough to generate a range of possible outcomes. To examine pricing behavior let us calculate the perceived derivative of the logarithm of profits with respect to price

\[ \frac{\partial \ln (\text{profits})}{\partial p_k} = \frac{1}{p_k - c_k} - \frac{e_k}{d - ep_k} - b_k \left( \begin{array}{ll} r_k & \text{as } p_k \leq \hat{p}_g \\ 0 & \text{as } p_k > \hat{p}_g \end{array} \right\} \] (15)

\(^1\)Note that with \( r_j = 0 \) for all \( j \), \( \frac{\partial \ln \pi_i}{\partial p_k} = \frac{a_k b_k p_k e^{-a_i+b_i p_i}}{\pi_i b_k e^{a_i+b_i p_i} k} \).
To examine the example we can plot the positive and negative terms in this derivative in a diagram. For the present we plot the first two terms, with their intersection being the equilibrium price if the firm were a monopolist.¹

[Insert Figure 2]

The presence of the additional terms gives an upward shift to the curve representing the negative terms, lowering the price charged. That is, the fact that the probability of purchase is seen to decrease in price implies that firms set lower prices. Let us now consider a sequence of specializations of this example.

Symmetric Equilibrium, No Selling Effort

For ease in demonstrating a symmetric equilibrium let us assume that $r_k$ is zero for all $k$ and that $\pi$ is independent of $y$. For symmetry we have $a_k$, $b_k$, and $c_k$ taking the same values. Then we will have a symmetric equilibrium as shown in the diagram. The larger $b$ (the more sensitive purchase probabilities to price) the lower the equilibrium price.

[Insert Figure 3]

¹We only consider the set of firms for which $d/e > c_k$. As the diagram shows, there is a unique solution to this equation in $(c, d/e)$ which is a local maximum for profits.
If $r_k$ is not zero it is possible to have a case where there does not exist a symmetric equilibrium. The intersection shown in the following diagram cannot be a symmetric equilibrium since it occurs in a range where $p_k < \bar{p}$, violating the condition that $\bar{p} = p_k$ in a symmetric equilibrium.

[Insert Figure 4]

Nonsymmetric Equilibrium, No Selling Effort

If we consider the case where we have two intersections (as shown in the next diagram), then we may have a case

[Insert Figure 5]

where a two-price equilibrium is possible. For this to occur, we must divide the firms into two groups, with a fraction $f$ of the firms charging $p_1$ and $(1-f)$ of the firms charging $p_2$ (with $p_1 < p_2$). We need to satisfy two conditions for this position to be an equilibrium. One is to check that the diagram correctly portraits the local maxima--i.e. that

$$p_1 + g < \bar{p} = fp_1 + (1-f)p_2 \quad (16)$$

or

$$g < (1-f)(p_2-p_1)$$
\[ \frac{\ln \text{profits}}{\text{price}} = \frac{1}{p - c} + \frac{e}{d - ep} \]

\[ c \quad \text{p}_{\text{mon}} \quad d/e \quad \text{price} \]

Figure 2
Figure 3
Figure 4
Figure 5
Secondly, we need to check that profits are the same at both positions for firms— that is

\[ \pi_1(p_1 - c)q(p_1) = \pi_2(p_2 - c)q(p_2) \]  

(17)

or

\[ e^{a-bp_1 + r(l-f)(p_2-p_1)}(p_1 - c)(d-ep_1) = e^{a-bp_2}(p_2 - c)(d-ep_2) \]  

(18)

or

\[ (l-f)(p_2-p_1) = \frac{1}{r} \left[ b(p_1-p_2) + \ln \frac{(p_2-c)(d-ep_2)}{(p_1-c)(d-ep_2)} \right] \]  

(19)

In addition \( p_1 \) and \( p_2 \) must satisfy the first order conditions (15)

\[ \frac{1}{p_2-c} - \frac{e}{d-ep_2} = b \]  

(20)

\[ \frac{1}{p_1-c} - \frac{e}{d-ep_1} = b + r \]

Thus we have three equations to determine \( p_1 \), \( p_2 \), and \( f \). In addition we need \( f \) to be a proper fraction and to satisfy (16). For some values of the parameters there will exist a two-price equilibrium.\(^1\)

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\(^1\) For example, at \( b = c = e = r = 1 \), \( d = 4 \), equilibrium occurs at \( p_1 = 1.4 \), \( p_2 = 1.7 \), \( f = .54 \).
Free Entry

The two examples above took the number of firms as given. As an example of a free entry model, let us assume a selling effort of \( \bar{y} \) for any active firm, and of zero for an inactive firm. Given the structure of the perceived purchase probability function, in a symmetric equilibrium, the equilibrium price is independent of the number of firms. The number would be determined by the condition that total fixed costs equal total profits

\[
(p-c)q(p) = J\bar{y} \tag{21}
\]

Given the assumed probability structure and the large numbers behavioral assumption—that each person views the state of the market \((e^{a_0} + \sum e^{a_i-bp_i})\) as independent of his own behavior—it is not surprising that price is independent of the number of active firms.

Differing Firms

Given the structure of this example, firms can differ in the marginal cost of supply, \( c \), and in the parameters of the purchase probability function. Either of these

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\(^1\)The zero profit condition and a static model do not seem to be an interesting combination where price reputation matters.
differences can give rise to an equilibrium with a distribution of prices. For simplicity we take \( r_k \) to be zero for all firms so that in the absence of differences between firms we would have a symmetric equilibrium. Implicitly differentiating the first order condition (15), we can examine the cross-section pattern of prices relative to marginal costs and purchase probability sensitivity.

\[
\frac{\partial p_k}{\partial c_k} = \frac{(d-ep_k)^2}{(d-ep_k)^2+e^z(p_k-c_k)^2} \quad (22)
\]

\[
\frac{\partial p_k}{\partial b_k} = \frac{-(d-ep_k)^2(p_k-c_k)^2}{(d-ep_k)^2+e^z(p_k-c_k)^2} \quad (23)
\]

Thus, looking across firms, we see that firms with higher marginal costs charge higher prices, but prices increase less than costs. This implies that the social gain of switching consumers to low price firms exceeds the private gain to the consumer. Equally naturally, firms facing more sensitive purchase probabilities charge lower prices.

**Selling Effort**

To see an example of selling effort, let us assume that \( r_k \) is zero and \( a_k \) and \( b_k \) are each functions of selling effort, \( y_k \). Then, the first order condition for selling effort is

\[\text{The pattern of response of } a_k \text{ and } b_k \text{ to selling effort may well depend on the choice of the type of selling effort engaged in.}\]
\[ H(p_k - c_k)q(p_k)\pi_k(a_k^-(y_k) - p_k(b_k^-(y_k)) = 1 \quad (24) \]

In addition, we should remember that the profit maximizing price is independent of \( a_k \) and decreasing in \( b_k \). Selling effort won't be undertaken at all unless it increases expected sales.\(^1\) Thus at some price we need \( (a^-'(0) - pb^-'(0)) \) to be positive. This can be accomplished with \( b^- \) being either positive or negative—that is some selling effort increases price sensitivity of consumers while other efforts decrease sensitivity. The effect of advertising opportunities (\( a^- \) and \( b^- \) being different from zero) on prices will depend critically on which of these occurs. As one illustration of this proposition let us relate a uniform equilibrium position to the effectiveness of selling effort. To denote effectiveness let us write \( a_k \) and \( b_k \) as functions of \( \lambda y_k \) and examine the changes in \( y \) and \( p \) as \( \lambda \) changes. Thus we have two equations (15) and (24) relating \( p \) and \( y \) to \( \lambda \). Since we will examine symmetric equilibria \( \pi_k \) will be \( J^{-1} \) in equilibrium, simplifying the analysis.\(^2\)

\(^1\)The model assumes that sales per customer are independent of selling effort.

\(^2\)To ensure a positive level of advertising, we assume \( H(p-c)(d-ep)J^{-1}(a^-'(0)-pb^-'(0)) > 1 \) at the equilibrium price occurring with \( b \) equal to \( b(0) \), as determined by (15). For profits to be concave in \( y \) we assume \( (a^-'-pb^-')^2 + a^-'' - pb^-'' < 0 \) for \( p \) in the relevant range. For profits to be concave in \( p \) and \( y \) at an optimal \( p \), a sufficient condition is \( (a^-'-pb^-')^2 + a^-'' - pb^-''c - b^-2(ab-b^-a^-b)^2 < 0 \).
Differentiating these two expressions we get

\[
\frac{dp}{d\lambda} = -D^{-1}J^{-1}b
\]  

(25)

where \( D \) is the determinant of the \( p \) and \( y \) derivatives of the two equations and is positive under suitable assumptions.

In this setting we can have a nonuniform equilibrium for suitable values of the parameters. In such an equilibrium one can examine the price-selling effort relationship by examining the optimal price as a function of selling effort since the price is independent of the state of the market (i.e., behavior of other firms). Since the sign of \( \frac{\partial^2 \text{profit}}{\partial p \partial y} \) is the same as that of \( -b' \) firms with larger selling effort could have larger or smaller prices. The equal profit condition is maintained by a suitable distribution across firms of selling effort.
6. Welfare Derivatives

Thus far we have considered an example of the general model. While it might be explored in more detail to obtain a better understanding of the possible workings of the market with particular parameters, we shall push ahead to set up a structure for welfare analysis in the general model. For this purpose we will first examine the impact on social welfare of marginal changes in prices and selling efforts, all other prices and selling efforts held constant. These evaluations will then be used to express in simple form the effect of a policy change on social welfare. Let us start with a price change, for which we define the shadow value $\mu_k$

$$
\mu_k = \frac{\partial S(p,y)}{\partial p_k} = \sum_{h=1}^{H} \left[ \frac{\partial w^h}{\partial p_k} + \pi_k^h q^h(p_k) \right. \\
\left. + \pi_k^h (p_k - c_k) q^h(p_k) + \sum_{j=1}^{J} \frac{\partial \pi_j^h}{\partial p_k} (p_j - c_j) q^h(p_j) \right]
$$

(26)

For the level of selling effort let us call the shadow value $\nu$. 
\[ \nu_k \equiv \frac{\partial S(p, y)}{\partial y_k} = \sum_{h=1}^{H} \left( \frac{\partial w^h}{\partial y_k} + \sum_{j=1}^{J} \frac{\partial \pi^h_j}{\partial y_k} (p_j - c_j) q^h(p_j) \right) - 1 \]

\[ = \sum_{h=1}^{H} \left[ \sum_{j=0}^{J} \frac{\partial \pi^h_j}{\partial y_k} (u^h(q^h(p_j)) - c_j q^h(p_j)) - \frac{\partial x^h}{\partial y_k} \right] - 1 \]  

(27)

Below we will consider some specific policies and express their impact on social welfare in terms of the values of \( \mu \) and \( \nu \). As an introduction to that, we shall examine the full social optimum (defined by \( \mu = \nu = 0 \)) and compare the values of \( \mu \) and \( \nu \) with the effect on expected profits of varying these same controls (as shown in (11) above).

If the government controlled all prices and selling efforts, the first order conditions for social welfare maximization would be the setting of \( \mu_j \) and \( \nu_j \) equal to zero for all \( j \). Let us start by considering pricing. Above we considered the classical condition \( \left( \frac{\partial w^h}{\partial p_k} + \pi_k q^h(p_k) = 0 \right) \). We saw that marginal cost pricing (\( p_j = c_j \)) was a solution to the first order conditions for welfare maximization when this condition was satisfied and that marginal cost pricing did not satisfy the first order conditions when the classical condition was not satisfied. The same results hold here, as is readily seen by inspection of (26). When marginal cost pricing is not optimal the deviation from the classical condition must be weighted along with the change in social
profitability which depends on the downward sloping demand curve and the effect of a price increase on the total pattern of purchase probabilities. As we know from the optimal taxation literature, analysis of optimal prices can be complicated when all the demand crosselasticities are relevant for price determination.

Comparing the social optimality condition with that for profit maximizing price setting (11), we get the familiar differences that a single firm ignores the effect of price increases on consumer welfare \( \left( \frac{\partial w^h}{\partial p_k} \right) \) and on the profits of other firms \( \left\{ \sum_{j \neq k} \frac{\partial \pi^j}{\partial p_k} (p_j - c_j) q_h(p_j) \right\} \). Thus at an equilibrium we know that

\[
\mu_k = \sum_{h=1}^{H} \left( \frac{\partial w^h}{\partial p_k} + \sum_{j \neq k} \frac{\partial \pi^j}{\partial p_k} (p_j - c_j) q_h(p_j) \right)
\]

(28)

Since the two terms tend to be of opposite signs, one cannot generally sign \( \mu_k \) a priori. If demand were relatively inelastic and markups fairly uniform (so that \( (p_j - c_j) q_h(p_j) \) were approximately independent of \( j \)) and if the classical condition held, then \( \mu_k \) would satisfy

\[
\mu_k = \sum_{h=1}^{H} \left( -\pi_k h (p_k) - (p_k - c_k) q_h(p_k) \left( \frac{\partial \pi_k}{\partial p_k} + \frac{\partial \pi_k}{\partial p_k} \right) \right)
\]

\[
= \sum_{h=1}^{H} \left( \pi_k h (p_k - c_k) q_h(p_k) - (p_k - c_k) q_h(p_k) \frac{\partial \pi_k}{\partial p_k} \right)
\]
When everyone who wants to purchase at going prices succeeds \((\pi_0 = 0)\), this expression is negative and private considerations lead to prices which would result in a gain in social welfare from a price decrease, all other prices held constant. This situation doesn't hold with full generality because a price increase might affect consumer search in a favorable way \(\frac{\partial w^h}{\partial p_k} > -h^h(p_k)\) or might shift demand from low markup sellers to high markup sellers, giving a gain in social welfare in excess of the gain to the consumer.

Now let us consider the rules for optimal selling effort \(v_k = 0\). First, if marginal cost pricing holds, then selling effort should be pursued until the sum of gains to consumers equals the marginal cost of effort. The gains to consumers arise from two sources. One is the differences in consumers' surplus from purchases at different prices and the changes in the probabilities of purchase from different suppliers. The second source is the economizing of buying effort. The latter point is clear and needs no elaboration. To pursue the former, let us use the fact that buying probabilities must add to one. Thus we can rewrite these first order conditions as
This rewriting makes clear the role of shifts in demand in the social evaluation of selling effort. If all costs are the same \((c_j = c\) for all \(j\)) and all prices are the same \((p_j = p\) for all \(j\)) then all attention is centered on the gain from purchase rather than nonpurchase and the first order conditions become

\[
\frac{\partial x^h}{\partial y_k} - 1 = 0
\]  

(30)

For some commodities (life insurance, encyclopedias) selling effort makes a large impact on the probability of purchase. The social evaluation of this selling effort depends on the evaluation of consumer surplus.

In contrast to these conditions, one can examine the private incentives for selling effort at given prices. These are shown in the equations for the Nash equilibrium (11). For comparability, we can rewrite the private condition as

\[
- \frac{H}{\sum_{h=1}^{H} \frac{\partial x^h}{\partial y_k}} \left( p_k - c_k \right) q^h(p_k) = 1
\]  

(32)
There are three differences between social and private incentives (given prices and selling efforts of others) that are apparent in this comparison. First, private incentives look to profits while social incentives look to consumer surplus. Second, private incentives ignore the alternative position of consumers and firms while social incentives are based on the difference in consumers' surplus between purchase from the supplier and the realised alternative. Third, social incentives consider the saving of buying effort of consumers.\(^1\) Thus at an equilibrium, we can write \(v_k\) as

\[
v_k = \sum_{h=1}^{H} \left\{ \sum_{j=0}^{J} \frac{\partial n_j^h}{\partial y_k} (u^h(q^h(p_j))) - c_jq^h(p_j) \right. \\
- \left. u^h(q^h(p_k)) + p_kq^h(p_k)) - \frac{\partial x^h}{\partial y_k} \right\}
\]

Note that this expression for \(v_k\) makes use of the profit maximizing conditions for selling efforts, not for prices. Except where selling effort greatly saves buying effort and where selling effort noticeably affects the set of people who don't purchase, although willing to, \(v_k\) will clearly be negative for high price firms and even negative for low

\(^1\)If one were comparing the equilibrium with the full optimum one would also recognize that the various functions are evaluated at different prices and selling efforts.
price firms unless their price is noticeably below prices of other firms. For a firm with a price below the marginal cost of all other firms, $v_k$ will be positive unless such selling effort considerably increases expected buying effort. Thus in Nash equilibrium high price firms tend to spend more on selling than is socially advantageous, given prices and the selling efforts of others.
7. Price Regulation

We have examined market equilibrium in the absence of government intervention and the rules for optimal industry control if all variables are controlled to maximize social welfare. Many markets lie between these limits, with central control of some variables, but not all. In this section we will consider an example of such an intermediate position, examining the rules for social welfare maximization. One such position arises when prices are set to maximize social welfare while selling efforts are still set to maximize individual firm profits. Assuming all prices are set by the government the first order conditions can be described in terms of the welfare multipliers of the previous section and the responses of selling efforts to prices. Writing the solution to the J first order conditions for profit maximizing selling efforts as \( y(p) \), we can write the first order conditions for welfare maximization as

\[
\mu_k + \sum_{j=1}^{J} \nu_j \frac{\partial y_j}{\partial p_k} = 0 \tag{34}
\]

\(^{1}\)One example of this situation is federal government subsidized flood insurance, where pricing by the government has resulted in very little selling effort by insurance companies or agents. See Kunreuther et al. (1976).
Even if the classical condition is satisfied, these conditions will not imply marginal cost pricing when selling effort increases with prices and \( v(c, y(c)) \) is positive (i.e. selling effort raises social welfare when optimal for marginal cost pricing. Given the specific cost structure assumed here, marginal cost pricing will result in zero selling effort, which will not generally be optimal when the probability of no purchase responds to selling effort.

To demonstrate this point, let us consider the special case of equal marginal costs at all outlets. Then, with uniform pricing, we can rewrite the equation for \( v \) in equilibrium as

\[
v_k = \frac{H}{\sum_{h=1}^{H} \left( \frac{\partial \pi^h}{\partial y_k} \right)} \left( u^h(0) - u^h(q^h(p)) + pq^h(p) \right) + (p - c)q^h(p) \sum_{j=1}^{J} \frac{\partial \pi_j^h}{\partial y_k} - \frac{\partial x^h}{\partial y_k}\]

(35)

Making plausible sign assumptions on the various derivatives \( \frac{\partial \pi^h}{\partial y_k} < 0, \frac{\partial \pi_j^h}{\partial y_k} < 0 \) for \( j \neq k \), \( \frac{\partial x^h}{\partial y_k} < 0 \) we have \( v_k \) positive for \( p \) sufficiently close to \( c \). Only with large private selling effort incentives arising from obtaining buyers from other firms will we have a negative value of \( v_k \). For \( p \)
excess of c, this situation becomes plausible. Let us note that uniform pricing and marginal costs result in an expression for \( \mu \) which is

\[
\mu_k = \sum_{h=1}^{H} \left\{ \frac{\partial w^h}{\partial p_k} + \pi_k^h q^h(p) + \pi_k^h (p-c) q^h(p) \right\} - (p-c) q^h(p) \frac{\partial \pi_0^h}{\partial p_k}
\]

Thus optimal pricing depends on the deviations from the classical condition, the marginal deadweight burden from pricing in excess of marginal cost, and the additional social profit from additional purchasers as well as the evaluation of induced selling effort.

Paralleling the method used in this section one could also examine pricing and selling effort for a government firm in an otherwise unregulated private market.
8. **Cost Changes**

Thus far we have examined government regulation or competition with this industry. There are a number of other ways in which public action can affect an industry. For these one would like to know the importance of information imperfection on the return to government action. We shall consider government actions which marginally decrease the cost of producing the good for all firms. Similar analyses could be done where only some firms have cost decreases.\(^1\)

In a classical competitive industry, government should pursue cost reducing activities as long as the cost savings to the industry exceeds the cost of government action. If we denote government expenditures to reduce costs by \(z\), we have the first order condition

\[
\sum_{h=1}^{H} \sum_{j=1}^{J} \pi_j p_j^h \frac{\partial c_j}{\partial z} = -1
\]

By contrast we can consider government actions to reduce costs of a monopolist. Here the return to government action

\(^1\)One could similarly explore the issue of taxation of this industry and policies which directly affect the parameters of the purchase probability functions. The correct shadow price on the output of this industry when used by the government is more complicated, depending on the purchase procedures followed by the government.
involves a response of prices to changed marginal costs. The first order condition is now

\[ \sum_h q^h(p) \frac{\partial c}{\partial z} + \sum_h (p-c)q^h(p) \frac{\partial p}{\partial c} \frac{\partial c}{\partial z} = -1 \]  

(38)

Thus if cost reduction lowers prices, the social gain from cost reduction contains an additional positive term. (Since the monopoly price exceeds the competitive price, the number of units of output on which the cost saving is realized is lower with monopoly than with competition.) Assuming that profits are concave in price and aggregate demand downward sloping, a cost reduction will reduce the monopoly price as can be seen by implicit differentiation of the first order condition for the monopoly price. However, global concavity generally will not hold.

The situation with imperfect information equilibrium is similar to that of monopoly in having to evaluate the price effect, with the additional complication that selling effort generates externalities. Thus the response of selling effort to cost must also be evaluated. Differentiating social welfare less the cost of government expenditures, we get the first order condition (where we have used the reduced form \( p(z), y(z) \))

\[ \sum_h \sum_j h^h(p_j) \frac{\partial c_j}{\partial z} + \sum_j \frac{\partial p_j}{\partial z} + \sum_j \frac{\partial y_j}{\partial z} = -1 \]  

(39)
Again, detailed analysis of the determinants of prices is needed to evaluate this expression. For example, we can examine the particular case presented in Section 5 above. Assuming that selling costs do not affect purchase probabilities, in a uniform equilibrium price will decrease as marginal cost decreases, as can be seen from Figure 3. In the case of a nonsymmetric equilibrium, both the high price and the low price decrease (as can be seen in Figure 5).

Unfortunately, it appears that a general analysis of imperfect information equilibria is too complicated, giving rise to the usefulness of many specific examples.
APPENDIX TO SECTION 5

In Appendix we prove the propositions on optimal search with a two-price distribution which were stated in the text. The notation follows that of Rothschild (1974). We assume that the consumer has a Dirichlet prior and that $p_1 < p_2$.

\[ \mu = (\mu_1, \mu_2) \]

\[ \rho \]

\[ h_i(\mu, \rho) = \left( \frac{\mu_1}{\rho+1}, \ldots, \frac{\mu_1+\rho}{\rho+1}, \ldots, \frac{\mu_n}{\rho+1}, \frac{\rho}{\rho+1} \right) \]

\[ \lambda_i(\mu, \rho) = \mu_i \]

\[ V(\mu, \rho) = \sum_{i} \lambda_i(\mu, \rho) \]

\[ \cdot \min[p_i, V(h_i(\mu, \rho)) + c] \]

\[ A_i \]

From Rothschild we know that

1. $V(\mu, \rho)$ is continuous

2. $A_2 \supset A_1$
3. Search is finite

4. $p_i \geq p_k$ implies $V(h_i(\mu,\rho)) \geq V(h_k(\mu,\rho))$

5. $|p_i - p_k| \geq |V(h_i^S(\mu,\rho)) - V(h_k^S(\mu,\rho))|$

Lemma 1: For $\rho > 0$ $V(h_2(\mu,\rho)) > V(\mu,\rho)$

Proof: $V(\mu,\rho) = \lambda_1(\mu,\rho)p_1 + \lambda_2(\mu,\rho)\text{Min}[p_2,V(h_2(\mu,\rho)) + c]$

$V(h_2(\mu,\rho)) = \lambda_1(h_2(\mu,\rho))p_1 + \lambda_2(h_2(\mu,\rho))$

$\cdot \text{Min}[p_2,V(h_2^2(\mu,\rho)) + c]$

Since $\lambda_1(\mu,\rho) > \lambda_1(h_2(\mu,\rho))$, $p_1 < \text{Min}[p_2,V(\mu^-,\rho^-) + c]$

for all $(\mu^-,\rho^-)$,

$V(h_2(\mu,\rho)) \leq V(\mu,\rho)$ implies $V(h_2^2(\mu,\rho)) < V(h_2(\mu,\rho))$

Repeating the argument we have $V(h_2^{t+1}(\mu,\rho)) < V(h_2^t(\mu,\rho))$ for all $t$. But there exists a $t$ such that $V(h_2^{t+1}(\mu,\rho)) \geq p_2 - c$.

This implies

$V(h_2^t(\mu,\rho)) = \lambda_1(h_2^t(\mu,\rho))p_1 + \lambda_2(h_2^t(\mu,\rho))p_2 > V(\mu,\rho)$

This is a contradiction.

Lemma 2: $\rho^- > \rho > \rho^-$ and $V(h_2(\mu,\rho)) + c = p_2$ imply

$V(h_2(\mu,\rho^-)) + c > p_2$ and $V(h_2(\mu,\rho^-) + c < p_2$

Proof: By Lemma 1 and the definition of $V$,

$V(h_2(\mu,\rho)) = \lambda_1(h_2(\mu,\rho))p_1 + \lambda_2(h_2(\mu,\rho))p_2 = p_2 - c$

$V(h_2(\mu,\rho^-)) = \lambda_1(h_2(\mu,\rho^-))p_1 + \lambda_2(h_2(\mu,\rho^-))$

$\cdot \text{Min}[p_2,V(h_2^2(\mu,\rho^-)) + c]$

Since $\lambda_1(h_2(\mu,\rho)) > \lambda_1(h_2(\mu,\rho^-))$, $V(h_2(\mu,\rho^-)) < V(h_2(\mu,\rho))$

implies $V(h_2^2(\mu,\rho^-)) < p_2 - c$. Repeating the argument
we have $V(h_2^t(\mu,\rho^-)) < p_2 - c$ for all $t$. But there exists a $t$ such that $V(h_2^t(\mu,\rho^-)) \geq p_2 - c$ which is a contradiction.

For $\rho''$ we have $V(h_2(\mu,\rho'')) \leq \lambda_1(h_2(\mu,\rho''))p_1 + \lambda_2(h_2(\mu,\rho''))p_2 < V(h_2(\mu,\rho))$. By a similar argument, $V(h_2(\mu,\rho))$ is nondecreasing in $\rho$.

Theorem: There exists $\rho_i(\mu)$ such that $(\mu,\rho) \begin{cases} \text{is} \\ \text{is not} \end{cases}$ in $A_i$ as $\rho \begin{cases} \geq \\ \leq \end{cases} \rho_i(\mu)$.

Proof: For $i = 1$, $(\mu,\rho)$ are in $A_i$ for all $(\mu,\rho)$. For $i = 2$ the result follows from the continuity of $V$, Lemma 2, and the equivalence between $(\mu,\rho)$ in $A_2$ and $V(h_2(\mu,\rho)) \geq p_2 - c$.

Since $V(\mu_1,1-\mu_1,\rho)$ is strictly decreasing in $\mu_1$, the set $A_2$ has the shape shown in Figure 1 in the text.

Corollary: $V(\mu,\rho)$ is nondecreasing in $\rho$.

Proof: $\rho \geq \rho_2(\mu)$ implies $V(\mu,\rho) = \sum \mu_i p_i$ which is independent of $\rho$. For $\rho < \rho_2(\mu)$

$V(\mu,\rho) = \mu_1 p_1 + \mu_2(V(h_2(\mu,\rho)) + c) < \sum \mu_i p_i$

Since $V(h_2(\mu,\rho))$ is nondecreasing in $\rho$, $V(\mu,\rho)$ is nondecreasing in $\rho$ in this region.
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