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Massachusetts Institute of Technology Department of Economics Working Paper Series

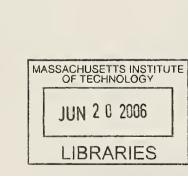
WHY HAS CEO PAY INCREASED SO MUCH?

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Working Paper 06-13 January 26, 2006 Revised: May 8, 2006

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Why Has CEO Pay Increased So Much?

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May 8, 2006. First version: January 26, 2006*

Abstract

This paper develops a simple competitive model of CEO pay. A large part of the rise in CEO compensation in the US economy is explained without assuming managerial entrenchment, mishandling of options, or theft. CEOs have observable managerial talent and are matched to assets in a competitive assignment model. Under very general assumptions, using results from extreme value theory, the model determines the level of CEO pay across firms and over time, and the pay-sensitivity relations. The model predicts a cross-sectional constant-elasticity relation between pay and firm size. It also predicts that the level of CEO compensation should increase one for one with the average market capitalization of large firms in the economy. Therefore, the six-fold increase of CEO pay between 1980 and 2003 can be fully attributed to the six-fold increase in market capitalization of large US companies. The model can also be used to study other large changes at the top of the income distribution, and offers a benchmark for calibratable corporate finance. We find a minuscule dispersion of CEO talent, which nonetheless justifies large pay levels and differences. The empirical evidence is broadly supportive of our model. The size of large firms explains many of the patterns in CEO pay, in the time series, across industries and across countries. (JEL D2, D3, G34, J3)

Keywords: Executive compensation, wage distribution, pay performance sensitivity, extreme value theory, superstars, calibratable corporate finance.

^{*}xgabaix@mit.edu, alandier@stern.nyu.edu. We thank Hae Jin Chung, Sean Klein and Chen Zhao for excellent research assistance. For helpful comments, we thank Daron Acemoglu, Olivier Blanchard, Alex Edmans, Bengt Holmstrom, Hongyi Li, Kevin J. Murphy, Andrei Shleifer, David Yermack and seminar participants at Chicago, MIT and the University of Southern California. We thank Carola Frydman and Kevin J. Murphy for their data.

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1 Introduction

This paper proposes a neoclassical model of equilibrium CEO compensation. It is simple, tractable and calibratable. CEOs have observable managerial talent and are matched to firms competitively. The marginal impact of a CEO's talent is assumed to increase with the value of the assets under his control. The model generates testable predictions about CEO pay across firms, across countries, and across time. In particular, it also explains, quantitatively, much of the rise in CEO compensation since the 1980s. In the model's view, this increase in pay is due to the rise in the market value of firms.

Our talent market is neoclassical and frictionless. The best CEOs go to the bigger firms, which maximizes their impact. In the benchmark case, incentive considerations do not matter. The paper extends earlier work (e.g., Lucas 1978, Rosen 1981, 1982, 1992, Tervio 2003), by drawing from extreme value theory to obtain general functional forms for the spacings in the distribution of talents. This allows to solve for the variables of interest in closed form without loss of generality, and generate concrete predictions. In equilibrium, under very general conditions, we establish that the compensation of a CEO in firm i is:

CEO compensation
$$(n) = D(n_*) \cdot S(n_*)^{1-\kappa} \cdot S(n)^{\kappa}$$
 (1)

where κ and $D(n_*)$ are positive constants, S(n) is the size of firm *i*, and $S(n_*)$ is the size of a reference firm – for instance, the median market capitalization amongst the largest 500 firms. Hence, the model generates the well-established relationship between compensation and size (with $\kappa \simeq 1/3$ empirically).

The model also predicts that average compensation should move one for one with typical market capitalization $S(n_*)$ of firms.¹ Figure 1 offers evidence for this effect. Historically, in the U.S. at least, the rise of CEO compensation coincided with an increase in market capitalization of the largest firms. Between 1980 and 2000, the average asset value of the largest 500 firms has increased by a factor of 6 (i.e. a 500% increase). The model predicts that CEO pay should increase by a factor of 6. The result is driven by the scarcity of CEOs, competitive forces, and the six-fold increase in stock market valuations. Incentive concerns or managerial entrenchment play strictly no role in this model of CEO compensation. In our view, the rise in CEO compensation is a simple mirror of the rise in the value of large US companies since the 1980s. Our model also predicts that countries experiencing a lower rise in firm value than the US should also have experienced lower executive compensation growth, which is consistent with European evidence (e.g. Conyon and Murphy 2000). We show that a large fraction in cross-country differences in the level of CEO compensation is explained by differences in firm size. We also show that within the US, the distribution of firm size within industries determines the level of compensation as our model predicts: both firm size, S, and the benchmark firm size, $S(n_*)$, are significant predictors of CEO compensation.

Finally, we offer a calibration of the model, which could be useful to guide future quantitative models of corporate finance. The main surprise is that the dispersion of CEO talent distribution appeared to be extremely small at the top. If we rank CEOs by talent, and, at the head of a firm, replace CEO number 1 by CEO number 1000, the value of that firm will decrease by only 0.04%. However, these very small talent differences translate into considerable compensation differentials, as they are magnified by the size of very large firms.

¹Plugging $S = S(n_*)$ in Eq. 1, the compensation in the reference firm is $D(n_*) S(n_*)$.

The rise in executive compensation has triggered a large amount of public controversy and academic research. Our theory is to be compared with the three main types of economic arguments that have been proposed to explain this phenomenon.

The first explanation attributes the increase in CEO compensation to the widespread adoption of compensation packages with high-powered incentives since the late 1980s. Holmstrom and Kaplan (2001, 2003) link the rise of compensation value to the rise in stock-based compensation following the "LBO revolution" of the 1980s. Both academics and shareholder activists have been pushing throughout the 1990s for stronger and more market-based managerial incentives (e.g. Jensen and Murphy 1990). According to Inderst and Mueller (2005) and Dow and Raposo (2005), higher incentives have become optimal due to increased volatility in the business environment faced by firms. Cuñat and Guadalupe (2005) document a causal link between increased competition and higher pay-to-performance sensitivity in US CEO compensation. In the presence of limited liability and/or risk-aversion, increasing the performance sensitivity of a CEO's compensation requires a rise in the dollar value of compensation to maintain his participation. However, this link between the level and the "slope" of compensation has not been extensively calibrated. An exception is Gayle and Miller (2005) who estimate a structural model of executive compensation under moral hazard. CEOs of large companies are typically very wealthy individuals. One can doubt that their level of risk-aversion and the limited liability constraint represent quantitatively important economic frictions. For this reason, it remains unclear that whether increased incentives can explain the very large increase in CEO pay.

Following the wave of corporate scandals and the public focus on the limits of the US corporate governance system, a "skimming view" of CEO compensation has gained momentum. The tenants of the "skimming view" (e.g. Bebchuk et al. 2002) explain the rise of CEO compensation simply by an increase in managerial entrenchment. "When changing circumstances create an opportunity to extract additional rents—either by changing outrage costs and constraints or by giving rise to a new means of camouflage—managers will seek to take full advantage of it and will push firms toward an equilibrium in which they can do so" (Bebchuk et al. 2002). Stock-option plans are viewed by these authors as a way to increase CEO compensation without attracting too much notice from the shareholders. According to them, "high-powered incentives" is just an excuse used by management to justify higher "rent-extraction". A milder form of the skimming view is expressed in Hall and Murphy (2003) and Jensen, Murphy and Wruck (2004). They attribute the explosion in the level of stock-option pay to an inability of boards to evaluate the true costs of this form of compensation. "Why has option compensation increased? Why has it increased with the market? (...) We believe the reason is that option grant decisions are made by board members and executives who believe (incorrectly) that options are a low-cost way to pay people and do not know or care that the value (and cost) of an option rises as the firm's share price rises" (Jensen, Murphy and Wruck 2004). These forces have almost certainly been at work, but it is unclear how important they are for the typical firm. For instance, Rajan and Wulf (2006) challenge the view that perks are pure managerial excess by showing that companies offer high perks precisely when those are likely to be productivityenhancing. In that spirit, the present paper offers a purely competitive benchmark that explains the rise in US CEO compensation without assuming changes in the extent of rent extraction. In our model, this rise is an equilibrium consequence of the substantial increase in firm size. We also show in an extension how an underestimation by some firms of the real cost of stock-options can affect the wage other firms have to pay.

A third type of explanation, perhaps more related to our paper, attributes the increase in CEO compensation to changes in the nature of the CEO job. Hermalin (2004) argues that the rise in CEO compensation reflects tighter corporate governance. To compensate CEOs for the increased likelihood of being fired, their pay must increase. Frydman (2005) and Murphy and Zabojnik (2005) provide evidence that CEO jobs have increasingly placed a greater emphasis on general rather than firm-specific skills. Such a trend increases CEOs' outside options, putting upward pressure on pay. However, a main difficulty with their proposal is quantitative. Changes in the skill set of CEOs appear small to moderate (Frydman 2005), while the level of CEO compensation increase by a factor of 5 to 10. It is hard to envision a calibrated model where moderate changes could explain very large changes in levels of compensation. Moreover, given the rise in the number of MBAs among executives and the spread of executive education, it is doubtful that the scarcity of general skills is a major factor explaining the rise in CEO compensation. By contrast, our model explains this increase readily by the demand for top-talent. When stock market valuations are 6 times larger, CEO "productivity" is multiplied by 6, and pay increases by 6 as firms compete to attract talent. Effort considerations determine, in a second and subordinate step, the relative mix of average pay and incentives. This way, we derive a simple benchmark for the pay-sensitivity estimates that have caused much academic discussion (Jensen and Murphy 1990, Hall and Liebman 1998, Murphy 1999, Bebchuk and Fried 2003).

Perhaps closest in spirit to our paper is Himmelberg and Hubbard (2000). They derive a market equilibrium model of CEO compensation, that joins market equilibrium forces, and incentive considerations. By focusing primarily on market equilibrium forces, we get a very tractable model, and insights about the effect of firm size in the time series and the cross-section. We address incentives later, in section 6.

Our paper connects with several other literatures. One recent strand of research studies the evolution of top incomes in many countries and over long periods (e.g. Piketty and Saez 2006). Our theory offers one way to make predictions about the largest incomes. It could be enriched by studying the impact of the fat-tailed distributions of returns on the ex post value of options, which we suspect is key to understanding the very large increase in income inequality at the top recently observed in several countries.

Recent papers in asset pricing explore between labor income risk and asset prices (e.g. Lustig and van Nieuwerburgh 2005, Santos and Veronesi 2006). Our model generates a perfect correlation of the human capital of executives and the stock market. As the human capital of entrepreneurs is also likely to be correlated with equity prices, we see that the high human capital agents in the economy are likely to have a human capital very correlated with stock market risk. This may help with the equity premium puzzle.

The core model is in section 2. Section 3 presents empirical evidence, and is broadly supportive of the model. Section 4 proposes a calibration of the quantities used in the model. Section 5 presents various theoretical extensions of the basic model. Section 6 presents the model's predictions for the size of incentives, and the pay-performance sensitivities. Section 7 concludes.

2 Basic model

2.1 A simple assignment framework

There is a continuum of firms and potential managers. Firm $n \in [0, N]$ has size S(n) and manager $m \in [0, N]$ has talent T(m). As explained later, size can be interpreted as earnings or market capitalization. Low n denotes a larger firm and low m a more talented manager: S'(n) < 0, T'(m) < 0. In equilibrium, a manager of talent T receives a compensation $\omega(T)$. There is a mass n of managers and firms in interval [0, n].

We consider a firm's problem. The firm starts with earnings a_0 . At t = 0, it hires a manager of talent T for one period. The manager's talent increases the firm's earnings according to:

$$a_1 = a_0 \left(1 + CT \right) \tag{2}$$

for some C > 0. C represents how much CEO talent affects this year's earnings.

First, suppose that the CEO's actions at date 0 impact earnings only in period 1. The firm's earnings are $(a_1, a_0, a_0, ...)$. The firm is a price-taker vis-a-vis CEO compensation: It chooses the optimal talent for its CEO, T, by maximizing current earnings, net of the CEO wage $\omega(T)$.

$$\max_{T} \frac{a_0}{1+r} \left(1 + C \times T\right) - \omega \left(T\right)$$

Alternatively, suppose that the CEO's actions at date 0 impact earnings permanently. The firm's earnings are $(a_1, a_1, a_1, ...)$. The firm chooses the optimal talent CEO T to maximize the present value of earnings, discounted at the discount rate r, net of the CEO wage $\omega(T)$:

$$\max_{T} \frac{a_0}{r} \left(1 + C \times T \right) - \omega \left(T \right) = M$$

Up to a constant, the two programs above are equivalent to:

$$\max_{T} S \times C \times T - \omega \left(T \right) \tag{3}$$

If CEO actions have a temporary impact, $S = a_0/(1+r)$, while if the impact is permanent, $S = a_0/r^2$. If CEO talent has a limited impact, (CT close to 0), then $a_0/(1+r)$ is close to the earning of the firm (the realized earnings are a_1), while a_0/r is close to the market capitalization M of the firm.³ Below, we interpret "size" as "market capitalization", but "earnings" are a second plausible interpretation.

Specification (2) can be generalized. For instance, the CEO impact could be modeled as $a_1 =$

² If the impact last for T periods, the formula is $S = a_0 \left(1 - (1+r)^{-T} \right) / r$.

 $^{{}^{3}}$ Eq. 13 rationalizes a potential way to ascertain if ČEO impact is temporary (affecting current earnings) or permanent (affecting market capitalization). One would run a regression of wages on earnings, sales, and market capitalization, and see which variables dominate. Technological change, or fashions, may change the relative strength of earnings or market capitalization in setting CEO pay. This leaves a free parameter that may be useful in some cases. For instance, there was a stock market increase in the 1950s, but, in Frydman and Saks (2005)'s sample, CEO pay did not move much. It could be that this was because firms thought stock market prices were too noisy to be a sound guide to corporate decisions, and firms prefer to use revenues and earnings. Another possibility is that the phenomenon is confined to Frydman and Saks' particular sample.

 $a_0 + Ca_0^{\gamma}T$ +independent factors, with $\gamma > 0$. If large firms are more difficult to change than small firms, then $\gamma < 1$. Decision problem (3) becomes:

$$\max_{T} S^{\gamma} \times C \times T - \omega(T)$$

If $\gamma = 1$, the model is constant return to scale with respect to firm size. On a priori grounds, one can view the benchmark of constant return to scale as the most appealing one – the constant to scale benchmark is generally the most successful empirically, for firm-level or country-level production functions. The time-series prediction in Proposition 2 and section 3.1 yields an empirical estimate consistent with $\gamma = 1$. We believe $\gamma = 1$ is the natural benchmark, and it appears to be consistent with the long term stylized facts, but we keep a general γ factor in the paper.

We now turn to determination of equilibrium wages, which requires us to allocate one CEO to each firm. Given the absence of asymmetric information, the equilibrium allocation will match the CEO indexed by n with the firm indexed by n. We call w(n) the equilibrium compensation of a CEO with index n. Firm n, taking the compensation of each CEO as given, picks the potential manager m that maximizes performance net of salary:

$$\max CS(n)^{\gamma} T(m) - w(m) \tag{4}$$

which gives: $CS(n)^{\gamma} T'(m) = w'(m)$. At the optimum, there is associative matching: m = n, the best firms go with the best managers. Hence:

$$w'(n) = CS(n)^{\gamma} T'(n)$$

We normalize to 0 the reservation wage of the least talented CEO (n = N).⁴ Hence:

$$w(n) = -\int_{n}^{N} CS(u)^{\gamma} T'(u) du$$
(5)

Specific functional forms are required to proceed further. We assume a Pareto firm size distribution with exponent $1/\alpha$:

$$S\left(n\right) = An^{-\alpha} \tag{6}$$

This fits the data reasonably well with $\alpha \simeq 1$, a Zipf's law. See section 4 and Axtell (2001), Luttmer (2005) and Gabaix (1999, 2006) for evidence and theory on Zipf's law for firms.

Using Eq. (5) requires to know T'(u), the spacings of the talent distribution.⁵ As it seems hard to have any it confidence about the nature, and distribution of talent, one might think that the situation is hopeless. Fortunately, section 2.2 will show that extreme value theory gives a definite prediction about the functional form of T'(u).

⁴If the outside opportunity wage of the worse executive is w_0 , all the wages are increased by w_0 . This does not change the conclusions at the top of the distribution, as w_0 is likely to be very small compared to the expressions derived in this paper.

⁵We call T'(n) the spacing of the talent distribution because the difference of talent between CEO of rank n + dn and CEO of rank n is T(n + dn) - T(n) = T'(n) dn.

2.2 The surprisingly universal functional form of the talent spacings at the top, T'(n): an insight from extreme value theory

Extreme value theory shows that, for all "regular" continuous distributions, a large class that includes all usual distributions (including uniform, Gaussian, exponential, lognormal, Weibull, Gumbel, Fréchet, Pareto), there are some constants β and B such that the following equation holds for the spacings in the talent distribution in the upper tail (i.e., for small n):

$$T'(n) = -Bn^{\beta-1},\tag{7}$$

perhaps up to a "slowly varying" function, a notion defined below.

The rest of this subsection is devoted to explaining (7), but the reader willing to accept it can just go to section 2.3, which derives the implications for CEO pay.

We adapt the presentation from Gabaix, Laibson and Li (2005), Appendix A, and recommend Embrechts et al. (1997) and Resnick (1987) for a textbook treatment. The following two definitions specifies the key concepts:

Definition 1 A function L defined in a right neighborhood of 0 is slowly varying if: $\forall u > 0$, $\lim_{x\to 0^+} L(ux)/L(x) = 1$.

Prototypical examples is L(x) = a or $L(x) = a \ln 1/x$ for a constant a. If L is slowly varying, it varies more slowly than any power law x^{ε} , for any non-zero ε .

Definition 2 The cumulative distribution function F is regular if f is differentiable in a neighborhood of the upper bound of its support, $M \in \mathbb{R} \cup \{+\infty\}$, and the following tail index ξ of distribution F exists and is finite:

$$\xi = \lim_{t \to M} \frac{d}{dt} \frac{1 - F(t)}{f(t)}.$$
(8)

We refer the reader to Embrechts et al. (1997, p.153-7) for the following Fact.

Fact 1 The following distributions are regular in the sense of Definition 2: uniform $(\xi = -1)$, Weibull ($\xi < 0$), Pareto, Fréchet ($\xi > 0$), and Gaussian, lognormal, Gumbell, lognormal, exponential, stretched exponential, and loggamma ($\xi = 0$).

Fact 1 means that essentially all continuous distributions usually used in economics are regular.

In what follows, we note $\overline{F} = 1 - F$. The index ξ is an index of fatness of the distribution. A higher ξ means a fatter tail.

 $\xi > 0$ means that the distribution is in the domain of attraction of the Fréchet, i.e. roughly behaves like a Pareto: $\overline{F}(t) \sim t^{-1/\xi} L(1/t)$ for $t \to \infty$.

 $\xi < 0$ means that the distribution's support has a finite upper bound M, and for t in a left neighborhood of M, the distribution behaves as $\overline{F}(t) \sim (M-t)^{-1/\xi} L(M-t)$. This is the case that will turn out to be relevant for CEO distributions.

Finally $\xi = 0$ means that the distribution is "in the domain of attraction of the Gumbel". This includes the Gaussian, exponential, lognormal and Gumbel distributions.

Call \tilde{T} the random talent, and \overline{F} its countercumulative distribution: $P\left(\tilde{T} > t\right) = \overline{F}(t)$, and $f(t) = -\overline{F}'(t)$ its density. Call x the corresponding upper quantile, i.e. $x = P\left(\tilde{T} > t\right) = \overline{F}(t)$. The talent of CEO in the top x-th upper quantile of the talent distribution is our function T(x):

$$T\left(x\right) = \overline{F}^{-1}\left(x\right)$$

So the derivative is:

$$T'(x) = -1/f\left(\overline{F}^{-1}(x)\right)$$
(9)

Eq. 7 is the simplified expression of the following Proposition.

Proposition 1 (Universal functional form of the spacings between talents). For any regular distribution with tail index ξ , there is a slowly varying function L such that:

$$T'(x) = -x^{-\xi - 1}L(x)$$
(10)

Proof. The first step for the proof was to observe (9). The expression for $f(\overline{F}^{-1}(x))$ is easy to get, e.g. from the first Lemma of Appendix B of Gabaix, Laibson and Li (2005), which itself comes straightforwardly from standard facts in extreme value theory. For completeness, we transpose the arguments in Gabaix, Laibson and Li (2005). Call $t = \overline{F}^{-1}(x), j(x) = 1/f(\overline{F}^{-1}(x))$:

$$xj'(x)/j(x) = -x\frac{d}{dx}\ln f(\overline{F}^{-1}(x)) = -x\frac{f'(\overline{F}^{-1}(x))}{f(\overline{F}^{-1}(x))}\frac{d}{dx}\overline{F}^{-1}(x)$$
$$= xf'(\overline{F}^{-1}(x))/f(\overline{F}^{-1}(x))^{2}$$
$$= \overline{F}(t)f'(t)/f(t)^{2} = -(\overline{F}/f)'(t) - 1$$

so $\lim_{x\to 0} xj'(x)/j(x) = \lim_{t\to M} -(\overline{F}/f)'(t) - 1 = -\xi - 1$. Because of Resnick (1987, Prop. 0.7.a, p. 21 and Prop. 1.18, p.66), that implies that j has regular variation with index $-\xi - 1$, i.e. (10).⁶

If x is not the quantile, but a linear transform of it: $\hat{x} = \lambda x$, for a positive constant λ then Proposition 1 still applies: the new talent function is $T(\hat{x}) = \overline{F}^{-1}(\hat{x}/\lambda)$, and $T'(\hat{x}) = -\left[\lambda f\left(\overline{F}^{-1}(\hat{x}/\lambda)\right)\right]^{-1}$.

To illustrate Proposition 1, we can give a few examples. For $\xi > 0$, the prototype is a Pareto distribution: $\overline{F}(t) = kt^{-1/\xi}$. T(x) satisfies: $x = \overline{F}(T(x)) = kT(x)^{-1/\xi}$, which yields $T(x) = (k/x)^{\xi}$, and $T'(x) = -\xi k^{\xi} x^{-\xi-1}$. Eq 10 is satisfied, with function L(x) that is simply a constant: $L(x) = \xi k^{\xi}$.

For $\xi < 0$, the prototypical example is a power law distribution with finite support: $\overline{F}(t) = k (M-t)^{-1/\xi}$, for $t < M < \infty$. A uniform distribution corresponds to $\xi = -1$. It is easy to verify that L(x) is an exact constant, $L = -\xi k^{\xi}$.

Another simple case is that of an exponential distribution: $\overline{F}(t) = e^{-(t-t_0)/k}$, for k > 0, which has tail exponent $\xi = 0$. Then, $T(x) = t_0 - k \ln x$, T'(x) = -k/x. Eq.10 is verified, again with a constant L = k.

⁶One can check that the result makes sense, in the following way: If $j(x) = Bx^{-\xi-1}$, for some constant B, then $\lim_{x\to 0} xj'(x)/j(x) = -\xi - 1$.

A last case of interest is that of a Gaussian distribution of talent $\tilde{T} \sim N(\mu, \sigma^2)$, which has tail exponent $\xi = 0$. Then, with ϕ and Φ respectively the density and the cumulative of a standard Gaussian, $T(x) = \mu + \sigma \Phi^{-1}(x)$, $T'(x) = \sigma/\phi (\Phi^{-1}(x))$, and standard calculations (using $1 - \Phi(y) \sim \phi(y)/y$ for $y \to \infty$) show $T'(x) = -x^{-1}L(x)$ with $L(x) \sim \sigma/\sqrt{2\ln(1/x)}$. Again, L(x) is slowly varying.

From section 2.3 on, we will consider the case where Eq. 7 holds exactly, i.e. L(x) is a constant. Comparing with Proposition 1, that means that we set $\beta = -\xi$, and use the benchmark where L(x) is a constant *B*. When L(x) is simply a slowly varying function, the Propositions below should be understood "up to slowly varying function", e.g. the right-hand size should be multiplied by slowly varying functions of the inverse of the size of a firm. Such corrections typically do not make a material difference to the predictions, but cause great notational burden.

We conclude that (7) should be considered a very general functional form, satisfied, to a first degree of approximation, by any reasonable distribution of talent. ⁷. Eq. 7 allows us to be specific about the functional form of T'(x), at very low cost in generality, and go beyond the previous literature.

2.3 Implications for CEO pay

Armed with functional form (7), we can now solve for CEO wages. Equations 5, 6 and 7 imply:

$$w(n) = -\int_{n}^{N} A^{\gamma} B C u^{-\alpha\gamma+\beta-1} du = \frac{A^{\gamma} B C}{\alpha\gamma-\beta} \left[n^{-(\alpha\gamma-\beta)} - N^{-(\alpha\gamma-\beta)} \right]$$
(11)

In what follows, we focus on the case $\alpha \gamma > \beta$.⁸

We consider the domain of very large firms, i.e. take the limit $n/N \rightarrow 0$, which gives:

$$w(n) = \frac{A^{\gamma}BC}{\alpha\gamma - \beta}n^{-(\alpha\gamma - \beta)}$$
(12)

To interpret Eq. 12, we consider a reference firm, for instance firm number 250 – the median firm in the universe of the top 500 firms. Call its index n_* , and its size $S(n_*) = S(n_*)$. $S(n_*)$ is the size of a "reference" large firm, and can be used for time-series prediction. We obtain the following:

Proposition 2 Let n_* denote the index of a reference firm – for instance, the 250-th largest firm. In equilibrium, for large firms (small n), the manager of index n runs a firm of size S(n), and is paid:

$$w(n) = D(n_*) S(n_*)^{\beta/\alpha} S(n)^{\gamma-\beta/\alpha}$$
(13)

where $S(n_*)$ is the size of the reference firm and

$$D(n_*) = \frac{-Cn_*T'(n_*)}{\alpha - \beta} \tag{14}$$

⁷In the language of extreme value theory, $-\beta$ is the tail index of the distribution of talents, while α is the tail index of the distribution of firm sizes. Gabaix, Laibson and Li (2005, Table 1) contains a tabulation of the tail indices of many usual distributions.

⁸ If $\alpha \gamma < \beta$, Eq. 11 shows that CEO compensation has a zero elasticity with respect to x for small x, so that it has a zero elasticity with respect to firm size. Given that empirical elasticities are significantly positive, we view the relevant case to be $\alpha \gamma > \beta$.

is independent of the firm's size. In particular, the compensation in the reference firm is

$$w(n_*) = D(n_*) S(n_*)^{\gamma}$$
 (15)

- Cross-sectional prediction: compensation varies with firm size according to $S^{\gamma-\beta/\alpha}$.
- Time-series prediction: compensation changes over time with the size of the reference firm $S(n_*)^{\gamma}$.
- Cross-country prediction: for a given firm size S, CEO compensation will vary across countries, as the market capitalization of the reference firm, $S(n_*)^{\beta/\alpha}$, using the same rank of the reference firm across countries.

Proof. If the talent distribution and the population do not change, $\frac{-Cn_*T'(n_*)}{\alpha-\beta}$ is just a positive constant.

As $S = An^{-\alpha}$, $S(n_*) = An_*^{-\alpha}$, $n_*T'(n_*) = Bn_*^{-\beta}$, we can rewrite Eq. 12,

$$(\alpha \gamma - \beta) w (n) = A^{\gamma} B C n^{-(\alpha \gamma - \beta)} = C B n_*^{\beta} \cdot (A n_*^{-\alpha})^{\beta/\alpha} \cdot (A n^{-\alpha})^{(\gamma - \beta/\alpha)}$$
$$= -C n_* T' (n_*) S(n_*)^{\beta/\alpha} S(n)^{\gamma - \beta/\alpha}$$

Ξ

The first prediction is cross-sectional. Starting with Roberts (1956), many empirical studies (e.g. Barro 1990, Frydman and Saks 2005, Kostiuk 1990, Rosen 1992) have documented that CEO compensation increases as a power function of firm size $w \sim S^{\kappa}$, in the cross-section. A typical empirical exponent is $\kappa \simeq 1/3$. ⁹We propose to name this regularity "Roberts' law", and display it for future reference:

Roberts' law for the cross-section: CEO Compensation ~ Firm size^{$$\kappa$$} (16)

Eq. 13 predicts a Roberts' law, with an exponent $\kappa = \gamma - \beta/\alpha$.¹⁰ Section 4 will conclude that the evidence points to $\alpha \simeq 1$, $\gamma \simeq 1$ and $\beta \simeq 2/3$.

The second prediction concerns the time-series. Eq. 13 predicts that the average wage depends on the size of the reference firm to the power γ , $S(n_*)^{\gamma}$. For instance, in the U.S., between 1980 and 2000, the average market capitalization of the top 500 firms has increased by a factor of 6 (i.e. a 500% increase). In the benchmark of constant return to scale, $\gamma = 1$, the model predicts that CEO pay should increase by a factor 6. This effect is very robust. Suppose all firm sizes S are multiplied by a factor λ . In Eq. 5, the right-hand side is multiplied by λ^{γ} . Hence, the wages, in the left-hand side, are multiplied by λ^{γ} . The reason is the shift in the willingness of top firms to pay for top talent.

The contrast between the cross-sectional and time-series prediction is interesting. Based purely on an empirical knowledge of the cross-sectional link between compensation and size (16), with an

⁹As the empirical measures of size may be different from the true measure of size, the empirical κ may be biased downwards, though it is unclear how large the biase is. In the extension in section 5.3, there is no downwards biased. Suppose that the effective size is $S'_i = C_i S_i$, so that $\ln w_i = \kappa (\ln C_i + \ln S_i) + a$ for a constant a. If C_i and S_i are independent, regressing $\ln w_i = \hat{\kappa} \ln S_i + A$ will still yield an unbiased estimate of κ .

¹⁰Sattinger (1993, p.849) presents a model with a lognormal distribution of capital and talents, that predicts a Roberts' law with $\kappa = 1$.

empirical $\kappa \simeq 1/3$, one might be tempted to conclude that, if all top firm sizes are multiplied by 6, then the average compensation should be multiplied by $6^{\kappa} \simeq 1.8$. However, and perhaps surprisingly, in equilibrium, the effect is actually an increase in compensation of 6.¹¹

Third, the model predicts that CEOs heading similar firms in different countries will earn different salaries.¹² Suppose that the size $S(n_*)$ of the 250th U.K. firm is λ times smaller than the size of the 250th U.S. firm ($\lambda = S(n_*)^{\text{US}}/S(n_*)^{\text{UK}}$), and, to simplify, that the distribution of talents at the top is the same. Consider two firms of equal size, one British, one American. The salaries of their CEOs should not be equal. Indeed, according to Eq. 13, the salary of the US CEO should be $\lambda^{\beta/\alpha}$ higher than the British CEO.¹³

A few additional remarks are in order. A Rosen (1981) "superstar" effect holds. If $\beta > 0$, the talent distribution has an upper bound, but wages are unbounded as the best managers are paired with the largest firms, which allows them to command a high compensation. It is easy to generalize the model to other superstars, e.g. entertainers and star athletes. One could interpret S(n) as various forums (e.g., tournaments, TV shows) in which superstars can perform. The same universal functional form for talent (7) applies, and the decision problem remains similar.

For instance, one could model the decision of the choice of a movie star in the following way. Given a movie of expected revenues S, a movie star with talent T will, on average, increase revenues by CT per cent, for some constant T. So the studio's decision problem is: $\max_T S^{\gamma}CT - w(T)$, a problem isomorphic to our firms'. Similar conclusions follows; for instance, as movie revenues increase by a factor λ , the salaries of top movie stars increase by a factor λ^{γ} .

It would be interesting to estimate the parameters α and β

for those other markets. It is tempting to imagine that the empirical analysis would yield fairly universal values for those parameters. One can anticipate a fat-tailed distribution, perhaps with $\alpha \simeq 1$ for the size of scales of operation (TV audiences, movie audiences, and the like) on which stars operate, as the Zipf's law is so often found in many domains of activity, such as cities, firms, and banks (see the references in Gabaix 1999, 2005). It would be striking if the absolute value of the tail index of the talent distribution, β , was consistent across domains – for movie stars, singers, athletes, and the like.¹⁴ One can certainly expect it to be in the range (0, 1), which indicates a bounded talent ($\beta > 0$), that nonetheless gets unbounded rents ($\beta < \alpha$, as per Eq. 11). There are now detailed studies of the talent markets for bank CEOs (Barro and Barro 1990), lawyers (Garicano and Hubbard 2005), software programmers (Andersson et al. 2006), rock and roll stars (Krueger 2005), movies and actors (de Vany 2004). It would be interesting to apply the analytics of the present paper to these markets, and see to what extent variations in the sizes of stakes (size of banks, size of contested amounts in lawsuits, concert revenues, movie revenues) explain the evolution in top pay in these markets.

It would be also interesting to understand better possible variation in C. One might expect C

¹¹Sattinger (1993) illustrate qualitatively this effect in assignment models.

¹²In the present analysis, we assume identical distribution of top talents across the countries compared in the thought experiment, e.g. identically-sized countries. Section 5.4 discusses the potential impact of country size on the talent distribution at the top.

¹³This is qualitatively consistent with the findings of Conyon and Murphy (2000).

¹⁴One could even link this to the production of idea, a theme that has received considerable attention recently. Small differences in talents in the creation of ideas, matched in equilibrium with fat-tailed distribution of resources (e.g., a large laboratory), create a Pareto-tailed distribution of ideas, a mechanism that can complement those of Kortum (1997) and Jones (2005).

to be higher in more competitive industries, because in more competitive industries, a small cost advantage has a bigger difference. As discussed by Shleifer (2004), another interpretation of CEO talent is that the CEO might be good affecting the share value of the firm, i.e. the price earning ratio, even if he is not good at affecting earnings per se. Hence, in moment of stock market booms, if investors are too optimistic and gullible in the aggregate, C can be higher. One might expect C, perhaps estimated as the residual of a regression of log CEO wage on log firm size, to increase at the beginning of a stock market bubble.

The model predicts the spacing of CEO salaries between firms of similar size. Then Proposition 2 implies the following log-linear interpolation rule. If firm size satisfies $S = \sqrt{S_1S_2}$, then CEO pay satisfies $w = \sqrt{w_1w_2}$. This relation suggest an extension of the present work, with a dynamic process whereby a number of firms, spread throughout the distribution, increase their compensation (believing C is high); this causes other firms to follow suit by the log-linear interpolation rule. We do not pursue this idea in the present paper, and instead assume that the technological parameters are all common knowledge.

3 Empirical Evidence

We motivate our paper by the large increase in CEO compensation observed in the US since the 1980s. We show that changes in firm size explain the bulk of this phenomenon. This section provides two further empirical tests of the relevance of our theory. We first document whether (and to what extent) cross-country variation in the level of CEO compensation can be explained by differences in firm-size. Second, within the US, we look at whether our model can shed light on the cross-section and the dynamics of the distribution of CEO compensation across sectors.

3.1 Time-Series Evidence for the USA

The most basic prediction of our theory with constant returns to scale (Proposition 2, with $\gamma = 1$) is that the average CEO compensation (in a group of K top firms) should be proportional to the average size of the firms in that group. This section shows that the USA evidence is quite supportive of this prediction, with $\gamma = 1$.

In the USA, between 1980 and 2000, the average asset market value of the largest 500 firms (including debt and equity) has increased (in real terms) by a factor of 6 (i.e. a 500% increase). This rise can be decomposed as follows: both the asset price to earnings ratio and earnings have increased by a factor of around 2.5 during that period. The model predicts that CEO pay should increase by a factor of 6.

To check whether this is a reasonable order of magnitude, we use two different indices of CEO compensation. The first one (JMW_compensation_index) is based on the data of Jensen Murphy and Wruck (2004)¹⁵. Their sample runs from 1970 onwards and is based on all CEOs included in the S&P 500, using data from Forbes and ExecuComp. CEO total pay includes cash pay, restricted stock, payouts from long-term pay programs and the value of stock options granted using ExecuComp's modified Black-Scholes approach. A shortcoming of these data is that total pay prior to 1978 excludes option grants, and total pay between 1978 and 1991 is computed using the amounts realized from

¹⁵We are grateful to Kevin J. Murphy for sharing his data with us.

exercising stock options, rather than grant-date values. The latter can create a mechanical positive correlation between stock-market valuations and pay in the short-run.

Our second compensation index (FS_compensation_index), based on the data from Frydman and Saks (2005) does not have this bias: It reflects solely the ex-ante value of compensation rather than its ex-post realization. FS_compensation_index sums cash compensation, bonuses, and the ex ante (Black-Scholes value at date granted) of the indirect compensation, such as options. The data of Frydman and Saks (2005), however, include less companies and are not restricted to CEOs. The data are based on the three highest-paid officers in the largest 50 firms in 1940, 1960 and 1990. The Size data for year t are based on the closing price of the previous fiscal year as (1) we need a proxy of the reference size at the time compensation is set and (2) we want to avoid any mechanical link between increased performance and increased compensation.

The correlation of the mean asset value of the top500 companies in compustat is 0.93 with FS_compensation_index and 0.97 with JMW_compensation_index. Apart from the years 1978-1991 for JMW_compensation_index, there is no clear mechanical relation that produces the rather striking similar evolution of firm sizes observed in Figure 1, as the indices reflect ex-ante values of compensation at time granted (not realized values).

We can estimate γ , via the relation $w_* = D(n_*) S(n_*)^{\gamma}$, by the following regression, reported in Table 1.

$$\Delta_t(\ln w_t) = \widehat{\gamma} \times \Delta_t \ln S_{*,t} + b \times \ln w_{t-1} + c \times \ln S_{*,t-1}$$

which gives a consistent estimate of γ . The regression is consistent with $\gamma = 1$.

It would be highly desirable to study the deeper past of the US historical evidence. The main sources are a book by Lewellen (1968), and the recent ambitious working paper by Frydman and Saks (2005). The two studies are in some conflict.¹⁶ In particular, Lewellen (1968, p.147) finds very high increase in before-tax compensation in the 1950s, while Frydman and Saks find essentially no change during that period. It appears that a key difference is in the treatment of indirect compensation, particularly options and pensions. Pensions are are not included in Frydman and Saks (2005) study, but, by the end of the 1950s are very high in the Lewellen study.¹⁷ In the end, we think it best to await the resolution of these methodological and data issues (in particular the final version of the Frydman-Saks project) to examine the past of US compensation. We now turn to the cross-country evidence.

3.2 Cross-Country Evidence

In most countries, public disclosure about executive compensation is either non-existent or much less complete than in the US. This makes the collection of an international dataset on CEO compensation a highly difficult and country-specific endeavour. For instance Kaplan (1994) collects firm-level information on the sum of the compensation of all directors using official filings of large Japanese companies at the beginning of the 1980s. When it comes to comparing levels (rather than performance sensitivities, which is the main goal of Kaplan (1994)), an additional difficulty is the variation in tax-systems, pension-benefits, perquisites and cost of life across countries. We rely on on the data collected by Abowd and Bognanno (1995) and updated in Abowd and Kaplan (1999). international

¹⁶We thank Carola Frydman for helpful conversations on this topic.

¹⁷Lewellen attributes the increased importance of indirect compensation to a reaction to taxes. The marginal tax rates on direct compensation were very high, and the tax rates on indirect compensation were much lower.

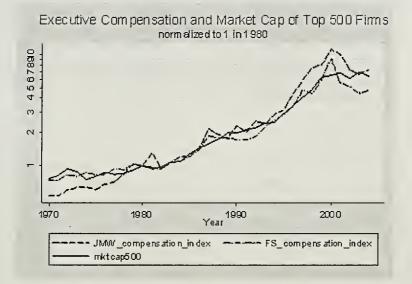


Figure 1: FS_compensation_index is based on Frydman and Saks (2005). Total Compensation is the sum of salaries, bonuses, long-term incentive payments, and the Black-Sholes value of options granted. The data are based on the three highest-paid officers in the largest 50 firms in 1940, 1960 and 1990. JMW_compensation_index is based on the data of Jensen Murphy and Wruck (2004). Their sample encompasses all CEOs included in the S&P 500, using data from Forbes and ExecuComp. CEO total pay includes cash pay, restricted stock, payouts from long-term pay programs and the value of stock options granted using ExecuComp's modified Black-Scholes approach.Compensation prior to 1978 excludes option grants, and is computed between 1978 and 1991 using the amounts realized from exercising stock options. Size data for year t are based on the closing price of the previous fiscal year. The firm size variable is the mean of the biggest 500 firm asset market values in Compustat (the market value of equity plus the book value of debt). The formula we use is mktcap=(data199*abs(data25)+data6-data60-data74). Quantities are deflated using the Bureau of Economic Analysis GDP deflator.

compensation data are constructed using information from statistical agencies and compensation consulting companies (such as Towers Perrin). The data are available on John Abowd's website and their construction is explained in the above-mentioned papers. We use the 1996 CEO compensation variables (i.e. the most recent year available). As our theory predicts the costs paid by firms for talent, we construct the before-tax and purchasing-power adjusted compensation of the CEO of "a company incorporated in the indicated country with \$200-500 million in annual sales (1990 dollars)"¹⁸. This variable includes an estimate of country-specific perquisites attached to the compensation. To get information on a country's typical firm's, we use Compustat global data for 1995. We compute the median size of the top 50 firms along two different proxies of size: net income (DATA32) and book value of assets (DATA89), which gives us two proxies for country-specific firm size. These values are in local-currency and to compare them we use the same purchase-power adjustment as for the compensation variable. We then regress the log of the country CEO compensation on the log of our measure of country benchmark firm size.

The regression results, reported in Table 2, show that the variation in typical firm size explains about a third of the variance in CEO compensation across countries. The coefficient on firm size is more significant when using income, possiblie due to heterogeneity in depreciation rules across countries¹⁹. The use of book values instead of market values biases the exercise against us: The US price-earnings ratios were notably larger than those of Europe during that period (see e.g. IMF (1997)). Therefore, an even larger part of the gap in compensation between the US and other countries would be explained if we were using market values. The results are robust to controlling for GDP per capita, which interestingly appears not to be a significant predictor of the level of CEO compensation in a country. We also try to control for social norms, as societal tolerance for inequality is often put forth as a possible explanation for differences at the top of the income distribution across countries. Our social norm variable is based on the World Value Survey's E035 question in wave 2000, which gives the mean country sentiment toward the statement "We need larger income differences as incentives for individual effort." We find no significant predictive power of this country-level variable on the level of CEO compensation.

3.3 Cross-Industry Evidence

CEO labor markets are likely to be partially segmented by industry: The best outside options of Carlos Gohsn, CEO of Renault and Nissan are certainly in the automobile industry. We exploit this insight to test whether our theory is compatible with the joint distribution of compensation and firm size across industries. The advantage of this approach is that we are able to rely on US data, where compensation data is largely available and defined in a homogeneous manner across companies.

Using ExecuComp data (1992-2003) merged with Compustat to retrieve firm size information, we select each year the top n compensation packages for each of the 48 Fama-French (1997) sectors. We show results with n = 20 and n = 10 successively. We drop sectors that do not have more than n companies in ExecuComp. CEO Compensation packages are sorted using ExecuComp's total compensation variable, TDC1, which includes salary, bonus, restricted stock granted and Black-

¹⁸The formula we use to construct this variable based on Abowd's website data is (basebon+volben+perqs+longterm)/ppp_oecd, i.e. the sum of base salary, bonuses, perquisites and long-term compensation adjusted by the OECD purchase-power parity variable.

¹⁹For example, in many countries, contrary to the US, firms can not have different values for tax-purpose depreciation and economic depreciation.

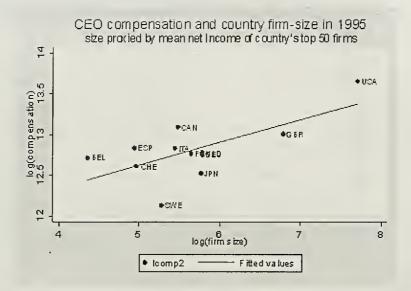


Figure 2: Compensation data are from Abowd and Kaplan (1999). We use the 1996 pretax and purchasing power parity adjusted compensation of CEOs. The exact formula is (basebon+volben+perqs+longterm)/ppp_oecd)), which sums the base salary, bonuses, voluntary benefits, perquisits and long-term compensation of the CEO of "a company incorporated in the indicated country with \$200-500 million in annual sales (1990 dollars)". Firm size is the 1995 mean net income of a country's top 50 firms in Compustat Global.

Scholes value of stock-options granted. Similarly, using the Compustat annual data, we compute for every year the mean and median of the top n market capitalizations (book value of debt plus market value of equity) for each of the 48 Fama-French sectors. We therefore end-up with the upper-tail of the compensation and a measure of firm size for each industry-year, namely 5844 CEO compensation observations, spread over 12 years. We convert all nominal quantities in 2000 dollars, using the GDP deflator from the Bureau of Economic Analysis. Using this sample, Figure 3 plots the 10th compensation, for 1996, in each sector as a function of the sector's reference firm size (i.e. the 10th market cap) in 1995.²⁰

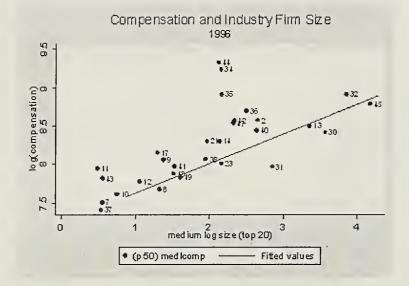


Figure 3: Median Compensation in 1996, and Median Firm size in 1995, by Industry (Fama-French 48 sectors), in the top 20 firms of each industry.

Figure 3 gives us a sense of the very strong positive relation between CEO compensation and industry size. However, it could just come from the general relation between CEO pay and firm size, Roberts' law. Our sample however allows us to test much finer predictions of our model. Consider the *j*th compensation package in sector *i* at year *t*. Calling $S_{i,j,t}$ the size of the firm and $S_{i,t}^*$ its corresponding industry-year reference size, our model predicts:

$$\ln(w_{i,j,t}) = \ln D_i^* + \frac{\beta}{\alpha} \ln(S_{i,t}^*) + (1 - \frac{\beta}{\alpha}) \ln(S_{i,j,t})$$

We perform three estimations of this equation. First, if one is willing to assume that the sensitivity of performance to talent (C) does not vary much across industries, so that $D_i^* = D$, one can run a simple cross-sectional regression:

 $^{^{20}}$ There appears to be three outliers to the relation in Figure 3, for sectors 34 (business services), 35 (computers) and 44 (banking). In terms of our model, this could mean that those industries have particularly high C's. The impact of a CEO would be particularly important for these three industries.

$$\ln(w_{i,j,t}) = d + e \times \ln(S_{i,t}^*) + f \times \ln(S_{i,j,t}) + \epsilon_t$$

We provide estimates of the coefficients of this OLS regression with t-stats clustered at the industry level and at the firm level, as a same firm might appear several years. We consider two industry-year sample sizes, n = 10 and n = 20. $S_{i,t}^*$ is the median market capitalization of the n firms in the industry-year (i, t).

Second, we allow for the sensitivity of performance to talent to vary across industry and therefore include industry fixed-effects. In such a regression, the identification relies on the time-variations of sizes and compensations across industries between 1992 and 2003.

$$\ln(w_{i,j,t}) = d_i + e \times \ln(S_{i,t}^*) + f \times \ln(S_{i,j,t}) + \epsilon_t$$

Third, we allow for firm fixed-effects, allowing for the performance to talent sensitivity to be firm-specific.

The results, reported in Table 3, are highly supportive of our theory. In particular, the industry fixed-effect and firm fixed-effect specifications give an estimate of β/α quite compatible with the back-of-the envelope calibration of section 1, which was suggestive of $\beta/\alpha \approx 2/3$. Note that we do not impose in these regressions that e + f = 1.

4 A calibration

We propose a calibration for the model. We hope it is a useful step in the long-run goal of calibratable corporate finance, and for the macroeconomics of the top of the wage distribution. We present here indicative numbers, that will be made more exact in a future iteration of this paper.

The empirical evidence and the theory on Zipf's law for the size of firms makes one expect $\alpha = 1$ (Axtell 2001, Gabaix 1999, 2006, Ijiri and Simon 1977, Luttmer 2005). However, the existing evidence covers firms as measured by number of employees, and assets, but not total firm value (debt plus equity). We therefore estimate α for the market value of the large firms.

It is well established that Compustat suffers from a retrospective bias before 1978 (e.g. Kothari, Shanken and Sloan 1995). This implies that companies present in the data set prior to 1978 were in reality included after 1978. So, to avoid the retrospective bias, we study the years 1978-2004.

For each year, we rank firms by total firm value (market value of debt + equity²¹), and order $S_{(1)} \geq S_{(2)} \geq \dots$ We study the best Pareto fit for the top n = 500 firms. We estimate the Pareto exponent α for each year by two methods the Hill estimator, $\alpha^{Hill} = (n-1)^{-1} \sum_{i=1}^{n} \ln S_{(i)} - \ln S_{(n)}$, and OLS regression, which is the regression coefficient of: $\ln(S) = -\alpha^{OLS} \ln(\operatorname{rank} - 1/2) + \operatorname{constant}(Gabaix and Ibragimov 2006 show that the <math>-1/2$ term is optimal and removes a small sample bias). If we want to estimate $\zeta = \alpha$, the Pareto exponent, we take $\zeta^{Hill} = 1/\alpha^{Hill}$, and for the OLS procedure, ζ is the regression coefficient of: $\ln(\operatorname{rank} - 1/2) = -\zeta^{OLS} \ln(S) + \operatorname{constant}$. The mean and cross-year standard deviations are respectively: α^{Hill} : 1.095 (s.d. 0.063), α^{OLS} : 0.869 (s.d. 0.071), ζ^{Hill} : 0.916 (s.d. 0.056), ζ^{OLS} : 1.115 (s.d. 0.095). Figure plots the log log plot for the year 2004.

²¹We define the total firm value as (data199*abs(data25)+data6-data60-data74.

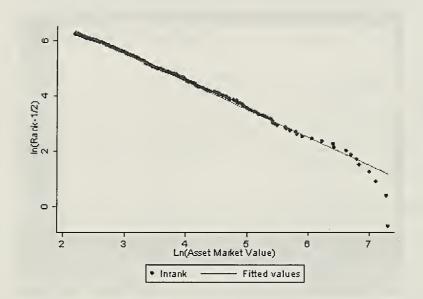


Figure 4: Size distribution of the top 500 firms in 2004. In 2004, we take the top 500 firms by total firm value (debt + equity), order them by size, $S_{(1)} \ge S_{(2)} \ge ... \ge S_{(500)}$, and plot $\ln S$ on the horizontal axis, and $\ln (\text{Rank} - 1/2)$ on the vertical axis. Regressing: $\ln (\text{Rank} - 1/2) = -\zeta^{OLS} \ln (S)$ +constant, yields: $\zeta^{OLS} = 1.007$ (s.e. 0.063), $R^2 = 0.988$. The standard error on ζ^{OLS} is $\zeta^{OLS} \cdot (2/500)^{1/2}$ (Gabaix and Ibragimov 2006).

All in all, the results for the market capitalization of firms are consistent with the Zipf's law benchmark found for other measures of firm size, namely

 $\alpha\simeq 1$

The time-series evidence of section 3.1 suggests:

$$\gamma \simeq 1.$$

The evidence on the firm-size elasticity suggests $w \sim S^{1/3}$, which by Eq. 13 implies

$$\beta \simeq 2/3$$

A value $\beta > 0$ implies that the distribution has an upper bound T_{max} , and that in the upper tail, talent density is (up to a slowly varying function of $T_{\text{max}} - T$):

$$P\left(T>t
ight)=B'\left(T_{\max}-t
ight)^{1/eta}$$
 for t close to T_{\max}

It would be interesting to compare it to talent distributions that can directly be observed, such as sprinters'. Even more interesting would be to endogenize the distribution T of talent, perhaps as the outcome of a screening process or a random growth process.

We index firms by rank, the largest firm having rank n = 1. Formally, if there are N firms, the fraction of firms larger than S(n) is n/N: $P\left(\tilde{S} > S(n)\right) = n/N$. The reference firm is the median firm in the universe of the top 500 firms. Its rank is $n_* = 250$.

The sample year is 2004.²² The median compensation amongst the top 500 (by market capitalization) firms is $w_* = \$7.0 \cdot 10^6$. The market capitalization of firm $n_* = 250$ in 2003 is $S(n_*) = \$25 \cdot 10^{9}.^{23}$ Using Proposition 2, we get: $w_* = \frac{BCn_*^9}{\alpha - \beta}S(n_*)$, so $BC = (\alpha - \beta)\frac{w_*n_*^{-\beta}}{S(n_*)}$, i.e.

$$BC = 2.4 \cdot 10^{-6} \tag{17}$$

It means that, the difference in marginal product between the top CEO and the 2nd CEO is CT'(1) = BC. The top CEO increases a firm's market value by only $2.4 \cdot 10^{-4}$ %. This means that the spacings between talent is very small. But this very small difference is big enough, in this neoclassical model, to generate large differences in compensation, as the small differences act on very large firms.

Tervio (2003), backs up talent differences in CEOs over a range. We answer his question in our framework. The difference of talent between the top CEO and the K-th CEO is:

$$T(1) - T(K) = -\int_{1}^{K} T'(n) dn = \int_{1}^{K} B n^{\beta - 1} dn = \frac{B}{\beta} \left[K^{\beta} - 1 \right]$$

For K = 250, this differences yields: BC(T(1) - T(250)) = 0.014%. If firm #1 replaced its CEO #1 with CEO #250, its market capitalization would go down by 0.014% (there is no typo here

²²As elsewhere, the numbers are in constant 2000 dollars, deflated by the GDP deflator.

²³ Proposition 3 indicates: $w(n) = AB\overline{C}n^{-\alpha+\beta}/(\alpha-\beta)$. It confirms that the correct procedure to estimate C is to take firm size number n in the universe of all firms (which yields an estimate of A via $S(n) = An^{-\alpha}$), and salary number n in the universe of all CEO pay.

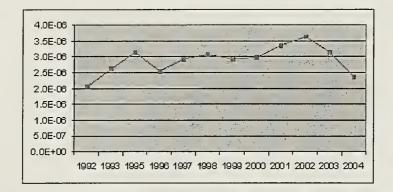


Figure 5: Estimate of *BC* for each year. The estimate is as $BC_t = (\alpha - \beta) \frac{w(n^*, t)n_*^{-\beta}}{S(n^*, t-1)}$, for $n^* = 250$, with $\alpha = 1$, $\beta = 2/3$, $w(n^*, t)$ the total (expected) compensation of the 250-th highest paid CEO, and $S(n^*, t-1)$ the market value of the 250-th largest firm.

- the number is 0.014%), a difference that may seem minuscule at first. However, this very small difference in talent between CEO #1 and CEO #250 still makes the pay of CEO #1 be exceed that of CEO #250 by $(250)^{1-\beta/\alpha} - 1 = 250^{1/3} - 1 = 530\%$.

We see the economics of superstars in action. CEO #1 increases a firm's market capitalization by only 0.014% (= $BC(250^{\beta} - 1)/\beta$) more than CEO #250; however, is paid 530% (= $250^{1-\beta/\alpha} - 1$) more.

In the "temporary impact" interpretation, where CEO affects earnings for just one year, one multiplies the estimate of talent by the price-earnings ratio. Taking an empirical price/earnings ratio of 15, replacing CEO #250 by CEO #1 increases earnings by: $15 \times 0.014\% = 0.2\%$.

Such a small difference might be due to the difficulty of inferring talent. Here the talent is the expected value of the CEO's talent, given noisy signals that include his past performance. The distribution of true (but unknowable) talent is surely greater.

Another way to put the finding (17) is the following. If there is a paradox in CEO pay, it is that firms must think that talent differentials between the top CEOs are surprisingly small. Otherwise, they would pay CEOs much more.

We reproduce the calculation of BC in the years 1992-2004, and find that it is quite stable. The mean value is $2.90 \cdot 10^{-6}$ and the standard deviation $0.44 \cdot 10^{-6}$. Figure 5 plots the estimates.

5 Extensions

5.1 Heterogeneity in Sensitivity to Talent across Assets

The impact of one unit of CEO talent per dollar of asset might vary substantially depending on the nature of the asset. For example, the value of young high-tech companies might be more sensitive to CEO talent that the value of a mature company of similar size. We therefore extend the model to the case where C's differ across firms. Defining the "effective" size of a firm is $\hat{S} = CS$, one can apply the results of the section 2 directly. We assume that any firm *i* has a "CEO sensitivity" C_i

drawn from a distribution independently from its size S_i , so that its effective size is $\hat{S}_i = C_i S_i$. We can now formulate the analog of Proposition 2.

Proposition 3 Call n_* a reference index of talent – e.g., the index of the 250-th most talented manager. In equilibrium, for large firms (small n), the manager of rank n runs a firm whose "effective size" CS is ranked n, and is paid:

$$w = D(n_*) \left(\overline{C}S(n_*)\right)^{\beta/\alpha} (CS)^{1-\beta/\alpha}$$
(18)

with $D(n_*) = \frac{-n_*T'(n_*)}{\alpha-\beta}$, and $S(n_*)$ is the size of the reference firm and \overline{C} , the average impact, is defined as the following non-linear mean over the \widetilde{C} of all firms:

$$\overline{C} = E \left[\widetilde{C}^{1/\alpha} \right]^{\alpha} \tag{19}$$

In particular, the reference compensation is:

$$w(n_*) = D(n_*)\overline{C}S(n_*) \tag{20}$$

Proof. We need to calculate the analog of (6) for the effective sizes $\widehat{S}_i = C_i S_i$. For convenience, we set n to be the upper quantile. Then, by (6), $n = P(S > s) = A^{1/\alpha} s^{-1/\alpha}$. Hence:

$$n = P\left(\widehat{S} > s\right) = P\left(CS > s\right) = P\left(S > s/C\right) = E\left[P\left(S > s/C \mid C\right)\right]$$
$$= E\left[A^{1/\alpha}C^{1/\alpha}s^{-1/\alpha}\right] = A^{1/\alpha}E\left[C^{1/\alpha}\right]s^{-1/\alpha}$$

so $\widehat{S}(n) = \widehat{A}n^{-\alpha}$ with $\widehat{A} = AE[C^{1/\alpha}]^{\alpha} = A\overline{C}$. Then, the proof of Proposition 3 applies.

5.2 Executives below the CEO

The upper-tail of the distribution of managerial talent might not be allocated solely to the top CEO positions in firms. For example, a division manager at General Electric might have a managerial talent index comparable to the CEO of a relatively big company. It is therefore natural to generalize the model to the top H executives of each firm. For that purpose, we assume the following enrichment of Eq. 2: $a_1/a_0 = 1 + \sum_{h=1}^{H} C_h T_h$. The h-th ranked executive improves firm productivity by his talent T_h and a sensitivity C_h , with $C_1 \geq ... \geq C_H$. There are no complementarities between the talents of the various managers in our simple benchmark. On the other hand, in equilibrium, there will be assortative matching, as very good managers work together in large firms, and less good managers work together in smaller firms.

A firm of size S wants to hire H executives with talent $(T_h)_{h=1...H}$, to maximize its value net of costs:

$$\max_{T_1,\dots,T_H} \sum_{h=1}^H S \times C_h \times T_h - \sum_{h=1}^H \omega(T_h).$$
(21)

One can readily see that these are in fact H independent simple optimization problems:

$$\max_{T_{h}} S \times C_{h} \times T_{h} - \omega (T_{h}), \ h = 1, ..., H$$

In other words, each firm S can be considered as vector of "single-manager" firms with effective sizes $(S \times C_h)_{h=1...H}$ to which the last proposition can be applied. The next Proposition describes the equilibrium outcome.

Proposition 4 (Extension of Proposition 2 to the top H executives). In a model where the top H executives increase firm value, the compensation level of the h-th executive h in firm i, is:

$$w_{i,h} = D(n_*) \left(H^{-1} \sum_{k=1}^{H} C_k^{1/\alpha} \right)^{\beta} S(n_*)^{\beta/\alpha} S(n)^{1-\beta/\alpha} C_h^{1-\beta/\alpha}$$
(22)

with $D(n_*) = \frac{-n_*T'(n_*)}{\alpha-\beta}$.

Proof. The proof is very simple, given Proposition 3. As per Eq. 21, each firm behaves as H independent firms, with effective size $C_h S$, h = 1...H. The average productivity (19) is now: $\overline{C} = \left(H^{-1}\sum_{k=1}^{H} C_k^{1/\alpha}\right)^{\alpha}$. So

$$w(n) = \frac{-n_* T'(n_*)}{\alpha - \beta} \left(H^{-1} \sum_{k=1}^H C_k^{1/\alpha} \right)^\beta S(n_*)^{\beta/\alpha} S(n)^{1-\beta/\alpha}$$

and the *h*-th executive in firm i earns (22).

In a given firm, the ratio between the CEO's pay and that of the *h*-th executive is $(C_1/C_h)^{1-\beta/\alpha}$. Hence, one could go within a firm, and infer the marginal productivity of talent, C_h , from the relative wages of the executive, according: $w_1/w_h = (C_1/C_h)^{1-\beta/\alpha}$. It would be interesting to unite this with other ideas in the organization of a firm, e.g. Garicano and Rossi-Hansberg (2006).

Rajan and Wulf (2006) document a flattening of large American firms in the 1990s. More executives report directly to the CEO and their more prominent position in the organization also translates into higher wages. In our framework, the increased role played by managers below the CEO in value creation could be modeled as a smaller C_1/C_h . It could be empirically related to the flattening of compensation (smaller w_1/w_h).

One could extend the impact to the full hierarchy of a firm, which would generate that large firms pay more, because they hire more talented workers. Fox (2006) indeed provides evidence that large firms pay more because they hire more talented workers.

5.3 If other firms pay their CEO more, how much is a firm forced to follow?

It is often casually argued that a large amount of herding drives the dynamics of CEO compensation: If a few firms increase compensation (for example because they believe talent impact has increased) the others might somehow be "forced to follow". It is also argued that the dynamics of compensation in a specific industry, say the finance industry, can have large "contagion" effects to the rest of the economy. Here again, other firms might be forced to align their compensation policy to these new standards. However, these arguments have mostly remained informal statements. We propose to examine them formally using our model.

5.3.1 Competition from a new sector

First, suppose that a new "fund management" sector emerges and competes for the same pool of managerial talent as the "corporate sector". For simplicity, say that the distribution of funds and firms is the same. The relative size of the new sector is given by the fraction π of fund per firm. We assume that talent affects a fund exactly as in Eq. 3, with a common C. The aggregate demand for talent is therefore multiplied by $(1 + \pi)$. More precisely, manager n will now be matched to an asset of size index $\frac{n}{1+\pi}$. The pay of a given talent is multiplied by $(1 + \pi)$, while the pay at a given corporate firm is multiplied by $(1 + \pi)^{\beta/\alpha}$. We see from this simple exercise that it is plausible that the increase in the demand for talent –due to the rise of new sectors, such as venture capital and the money management industry, might have exerted substantial upward pressure on CEO pay.

5.3.2 Strategic complementarity in compensation setting

Second, suppose that all firms except one believe that CEOs have become more productive, i.e. increase their C by a factor $\lambda > 1$. The remaining firm, F, retains the original C. Hence $\overline{C}' = \lambda C$, while firm F keeps its own C. How much will the pay at firm F change? First, if firm F wishes to retain its CEO, then it needs to increase his pay by a factor λ , i.e. "follow the herd" one for one. The reason is simply that firm F's CEO outside option is determined by the other firms (as per Eq. 5), and has been multiplied by λ .

Alternatively, firm F may want to re-optimize, and hire a new CEO with lower talent. Eq. 18 shows that the salary paid in firm F will still be higher than the previous salary, by a factor $\lambda^{\beta/\alpha}$. With $\beta/\alpha = 2/3$, if all firms increase their willingness to pay by 100% ($\lambda = 2$), firm F chooses to increase its CEO pay by $2^{2/3} - 1 = 60\%$. Such a high degree of "strategic complementarity" may make the market for CEO quite reactive to shocks, as initial shocks are not dampened very much.

We believe that the "microstructure" of CEO compensation setting is a promising avenue for empirical research. How is compensation fixed? Some firms might fix compensation by relying on compensation consulting firms that use formulas where size is an explicit determinant. Those formulas might be in turn determined by cross-sectional regressions. When they hire a new CEO, firms have to decide what level in the talent distribution they want to target. Conversely, firms who have a CEO targeted by another firm have to decide whether they are willing to match his outside offer or not. This implies that hiring wages are likely to have particularly high informational content about the market forces that our model describes.

5.3.3 Misperception of the cost of compensation

Hall and Murphy (2003) and Jensen, Murphy and Wruck (2004) have persuasively argued that at least some boards incorrectly perceived stock options to be inexpensive because options create no accounting charge and require no cash outlay. What is the impact on compensation?

To model this, consider if a firm believes that pay costs w/M rather than w, where M > 1 is a misperception of the cost of compensation. Hence Eq. 4 for firm i becomes $\max_m CS_iT(m) - w(m)/M_i$ i.e.

$$\max CM_i S_i T(m) - w(m)$$

Thus, if the firm's willingness to pay is multiplied by M_i . The effective C is now $C'_i = CM_i$. The analysis of section 5 applies: if all firms underestimate the cost of compensation by $\lambda = M$, then total

compensation increases by λ . Even a "rational" firm that does not underestimate compensation will increase its pay by $\lambda^{\beta/\alpha}$ if it is willing to change CEOs, and λ if it wishes to retain its CEO. Hence, other firms' misperceptions considerably affect a rational firm.

5.4 Country size and talent at the top

How does Proposition 2 when the population size varies? We examine two polar benchmarks. We call N the number managers and firms. If the average firm size is independent of population size, N is proportional to the population size.

One benchmark is that the top K CEOs are draws from the whole population of N managers, without preliminary sorting. Then, manager number k is in the upper quantile n = k/N. Hence, the term -nT'(n) can be written: $-nT'(n) = Bn^{\beta} = BN^{-\beta}k^{\beta}$. The compensation of the CEO of firm number k is:

$$w(k) = \frac{BC}{\alpha - \beta} N^{-\beta} k^{\beta} \cdot S(k)$$

Hence, if one uses an absolute rank index (i.e., firm number k_*), then the prefactor $D(n_*)$ is predicted to scale as $N^{-\beta}$, where N is the number of potential managers. If the country is larger, and $\beta > 0$, then the talent distribution in the top say 1000 managers is more concentrated.

In another benchmark, the talent distribution in the, say, top 1000 firms, is independent of country size. This is the case, for instance, if managers have been selected in two steps. First, potential CEOs have to have served in one of the top five positions at one of the top 10,000 firms, where those numbers are simply illustrative. This creates the initial pool of 50,000 potential managers for the top 1000 firms. Then, their new talent is drawn. This way, the effective pool from which the top 1000 CEOs are drawn does not scale with the general of the population, but is simply a fixed number, here 50,000. Then, the $D(n_*)$ term is independent of N.

The two above benchmarks suggest to define the "population pass-through" π in the following way: when the true population is N, the effective number of potential CEOs the top K firms consider, is N^{π} . In the first benchmark, $\pi = 1$, while in the second benchmark, $\pi = 0$. The above analysis means B scales as $N^{-\pi\beta}$.

In conclusion, the scaling of $D(n_*)$ with country size depends on the details of the managerial selection process, and in general scales as $D(n_*) \sim N^{-\pi\beta}$, for $\pi \in [0, 1]$. A dynamic extension of the model is necessary to study further this issue, and we leave it to further research.

6 Extension with pay-performance sensitivity

6.1 Extension of the model with incentives

The previous section determined total compensation without assuming any incentive problem. This section extends the model to allow for effort, thus creating a role for stock options. Its core objective is to propose quantitative, calibratable predictions for the pay-performance sensitivities in CEO pay.

The CEO's objective function is:

$$U = E\left[\frac{c}{1+\Lambda e}\right]$$

where c is the compensation, $e \in \{-1, 0\}$ is effort and $\Lambda \in [0, 1)$ represents the disutility of effort. The CEO is risk neutral, subject to limited liability, $c \ge 0$. e = 0 is the high effort level, and e = -1 is "shirking". As always, the "effort" decision should be interpreted broadly (Holmstrom and Milgrom 1990).

We normalize prices, so that the end-of-period stock-price is:

$$\tilde{P} = 1 + Le + \eta$$

where η is stochastic noise with mean 0. A shirking manager (e = -1) lowers the firm value by L percent.

The contract should elicit high effort.²⁴ We study a particular compensation package, that will turn out to be first best.²⁵ The CEO's compensation c is the sum of fixed pay f, and v options:

$$c = f + v \left(\widetilde{P} - K\right)^+ \tag{23}$$

where $Z^+ \equiv \max(Z, 0)$, K is the strike price. K = 0 corresponds to a payment in shares.

At the optimum, effort is high (e = 0), and the CEO's utility is E[c] = w, the expected compensation. Hence, the market equilibrium is determined in two, independent steps. First, the market equilibrium of the previous section determines w(n), the average compensation of a manager. Then, an incentive scheme elicits high effort, while keeping the total salary equal to w(n).

As in the equilibrium described in our first section, a manager of talent T(n) is paired with a firm of size S(n), and receives a compensation of expected value w(n) determined in Proposition 2. We now determine the minimum amount of options the compensation package should include. The unique optimal contract minimizing the quantity of stock-options is described as follows:

Proposition 5 The following compensation package is optimal. The manager with index n receives a fixed base pay, f^* , and v^* options, with:

$$v^* = w(n) \cdot \frac{\Lambda}{\psi} \tag{24}$$

$$f^* = w(n) \cdot \left(1 - \frac{\Lambda}{\psi} E\left[(1 + \eta - K)^+\right]\right)$$
(25)

where

$$\psi = E\left[(1+\eta-K)^{+} - (1+\eta-K-L)^{+}\right]$$
(26)

is the decrease in option value if the manager shirks: η is the noise around stock returns and L is the percentage decrease in firm value if the manager shirks, and K is the strike price. The expected compensation is equal to w(n), derived in Proposition 2.

Proof. The manager should get his market wage: E[c | e = 0] = w(n). We calculate:

$$E[c | e = 0] = f + vE[(1 + \eta - K)^{+}] = w(n)$$

$$E[c | e = -1] = f + vE[(1 + \eta - K - L)^{+}] = w(n) - v\psi$$

²⁴[Insert short proof for this]

²⁵Given that the manager is risk neutral (for $c \ge 0$), many compensation packages are optimal. The proposition focuses on the contract minimizing the number of stock-options.

The manager exerts a high effort e = 0 if:

$$E\left[\frac{c}{1+\Lambda e} \mid e=0\right] \ge E\left[\frac{c}{1+\Lambda e} \mid e=-1\right] \Leftrightarrow$$
$$w\left(n\right) \ge \frac{w\left(n\right) - v\psi}{1-\Lambda} \Leftrightarrow v \ge v^* := w\left(n\right)\frac{\Lambda}{\psi}$$

If $b = w(n) - v^* E[\eta^+] \ge 0$, this is the solution to the problem.

In the world described by Proposition 5, options are not indexed to the market.²⁶ Hence, an economist should not use the lack of option indexing as evidence of inefficiency. Also, CEOs will be rewarded for luck (Bertrand and Mullainathan 2001), but again, this does not violate efficiency. Both those features are consistent with a first best compensation scheme.²⁷

Murphy (1985), Jensen and Murphy (1990) and Hall and Liebman (2000), estimate empirical pay-performance measures. Those measures, b^{I} and b^{II} , are obtained by regressions of the type:

$$\Delta \text{\$Compensation} = b^{I} \cdot \Delta \text{Value of the firm + controls}$$

$$\Delta \ln \text{Compensation} = b^{II} \cdot \Delta \ln \text{Value of the firm + controls}$$

If there is no debt, Δ lnValue of the firm the stock market return of the firm. We define the theoretical counterparts of those sensitivity measures: $b^{I} = \frac{1}{S}E\left[\frac{dc}{d\eta}\right]$, and $b^{II} = E\left[\frac{dc}{d\eta}\right]/E[c]$.²⁸

The next Proposition derives predictions for these quantities. We start with a lemma useful to interpret q/ψ .

Proposition 6 The pay-performance sensitivities for a manager in firm i are:

$$b_i^I = \frac{\Delta \$Compensation}{\Delta Value \ of \ the \ firm} = \frac{-n_* T'(n_*)}{\alpha - \beta} S(n_*)^{\beta/\alpha} S^{\gamma - \beta/\alpha - 1} \frac{q\Lambda}{\psi}$$
(28)

$$b_i^{II} = \frac{\Delta \ln Compensation}{\Delta \ln Value \ of \ the \ firm} = \frac{q\Lambda}{\psi}$$
(29)

where q is the probability that the option will be exerted. q = 1 if the option is simply a stock grant (K = 0), and q = 1/2 if the strike is equal to the initial stock price, and the noise symmetrically distributed around 0. In the limit of small L, where L is the decrease in firm value caused by manager shirking:

$$\frac{q}{\psi} = \frac{1}{L} + o\left(\frac{1}{L}\right). \tag{30}$$

²⁶ Calling σ the volatility of the firm's stock price. In the simple case of Gaussian noise η and a strike price equal to the initial price, K = 1 the expected value from the option is: $E\left[\eta^+\right] = \sigma/\sqrt{2\pi}$. We use that fact that if $\eta \sim N\left(0, \sigma^2\right)$, $E\left[\eta^+\right] = \sigma/\sqrt{2\pi}$. So the ratio of average compensation coming from options to total compensation is:

Option share :=
$$\frac{\text{Compensation coming from options}}{\text{Total compensation}} = \frac{\Lambda}{\psi} \frac{\sigma}{\sqrt{2\pi}}$$
 (27)

The share of compensation that is given in options increases with the volatility of the firm, but is independent of firm size.

²⁷ The reason is that the manager is locally risk neutral, which may reasonably approximate reality.

²⁸One can define the b's in terms of covariances, and one gets the same expressions if R has a symmetrical around 0.

which implies, for small L's: $b_i^{II} \sim \Lambda/L$.

Proof. We observe that $d\left(\left(\eta-y\right)^{+}\right)/d\eta = 1\left\{\eta > y\right\}$, so

$$q = E \frac{d((1+\eta-K)^+)}{d\eta} = E[1\{1+\eta-K\}] = P(1+\eta-K>0)$$

is the probability that the option will be exerted.

So $E[dc/d\eta] = qv^*$. If the manager has v^* options, compensation moves, on average, by a factor qv^* times the changes in returns. So in the incentive package of Proposition 5, the change in compensation is $E\frac{dc}{d\eta} = w \cdot \frac{\Lambda}{\psi}q$, so

$$b^{II} = \frac{1}{w} E \frac{dc}{d\eta} = \frac{1}{w(n)} w(n) \cdot \frac{q\Lambda}{\psi} = \frac{q\Lambda}{\psi}$$

Eq. 13 is: $w = D(n_*) S(n_*)^{\beta/\alpha} S^{\gamma-\beta/\alpha}$, so we get:

$$b^{I} = \frac{1}{S} E \frac{dc}{d\eta} = \frac{1}{S} D\left(n_{*}\right) S(n_{*})^{\beta/\alpha} S^{1-\beta/\alpha} \cdot \frac{q\Lambda}{\psi} = D\left(n_{*}\right) S(n_{*})^{\beta/\alpha} S^{\gamma-\beta/\alpha-1} \frac{q\Lambda}{\psi}$$

Finally, to study the limit $L \to 0$, Eq. 26 gives: $\frac{d\psi}{dL}(L=0) = P(1+\eta-K>0) = q$, hence $\lim_{L\to 0} \psi(L)/L = \psi'(0) = q$.

 b^{I} is the Jensen-Murphy (1990) measure. The model predicts that it decreases with the size of a firm, with an elasticity equal to β/α , which the previous calibration assessed to be around 1/3. Indeed, Jensen and Murphy (1990), and the subsequent literature (Schaeffer 1998), have found that b^{I} decreases with firm size. It would be interesting to test the specific scaling $b_{i}^{I} \sim S_{i}^{\gamma-\beta/\alpha-1}$, which is $b_{i}^{I} \sim S_{i}^{-2/3}$ with the calibrated values of section 4.

 b^{II} is predicted not to scale with firm size, at least in the baseline model, where Λ and ψ do not depend on firm size. The empirical literature (see the survey by Murphy 1999) is qualitatively consistent with this prediction.

6.2 Calibrating effort and incentives

We use Eq. 29 and 30, which gives $b^{II} \simeq \Lambda/L$. In the model, there is no way to know Λ or L separately, but Λ/L can be identified. What is a good value for L, which is the amount by which a firm value decreases if the CEO slacks? We think that 30% is a plausible upper bound: $L \leq 0.3$. It means that the CEO can willfully decrease the firm value by up to 30% before the board takes steps to remove him.

The empirical literature (Murphy 1999) finds $b^{II} \simeq 1/3$. So we conclude: $\Lambda = L \times b^{II} \lesssim 0.30 \times 1/3 = 0.10$. This means that utility that the CEO gets from shirking is around 10% of total pay, at most. The calibration indicates that firms must consider the shirking problem to be relatively minor.

7 Conclusion

We provide a simple, calibratable competitive model of CEO compensation. The principal contribution is that it can explain the recent rise in CEO pay as an equilibrium outcome of the substantial growth in firm size. Our model differs from other explanations that rely on managerial rent extraction, greater power in the managerial labor market, or increased incentive-based compensation. In addition, it suggests why the recent growth in CEO pay has been concentrated in the US. The model can be generalized to the top executives within a firm and extended to analyze stock options, the impact of outside opportunities for CEO talent (such as the money management industry), and the impact of misperception of the price of options on the average compensation. Finally, the model allows us to propose a calibration of various quantities of interest in corporate finance and macroeconomics, the dispersion and impact of CEO talent, and the cost and impact of CEO effort.

8 Appendix A

The following table documents the increase, in ratios, of mean and median market capitalizations and price-earning ratios of the largest n firms of the Compustat universe (n = 100, 500, 1000) between 1980 and 2003, as ranked by market capitalization. For instance, the median Price/Earnings ratio of the top 100 firms was 2.4 times greater in 2003 than it was in 1980. The market price of equity, E, is computed by multiplying the number of shares outstanding by the sock price at the end of the fiscal year, and the market value of debt is proxied by its book value, D. We report total market capitalization, E + D, and price-earnings ratios defined as (E + D)/Operating Income. All quantities are real, using the GDP deflator. Between 1980 and 2003, US GDP increased by 100% (source: Bureau of Economic Analysis).

Increase in:	Market Capitalization		Price/Earning ratio		
	Median	Mean	Median	Mean	
Top 100	630%	700%	140%	230%	
Top 500	400%	540%	100%	100%	
Top 1000	440%	510%	100%	120%	

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	$\Delta \ln (\text{Compensation})$				
	Murphy index	Frydman-Saks index			
$\Delta \ln$ Market	1.344	1.029			
	$(4.12)^{***}$	$(3.14)^{***}$			
In Compensation(-1)	-0.579	-0.898			
	$(3.70)^{***}$	$(5.27)^{***}$			
ln Market(-1)	0.797	0.775			
	$(3.52)^{***}$	$(4.89)^{***}$			
Constant	-0.301	0.164			
	$(1.75)^{*}$	-1.15			
Observations	44	44			
R-Squared	0.37	0.43			

Table 1: CEO pay and the size of large firms, 1970-2003

Explanation: OLS estimates, absolute value of t statistics in parentheses. We estimate:

 $\Delta_t(\ln w_t) = \widehat{\gamma} \times \Delta_t \ln S_{*,t} + b \times \ln w_{t-1} + c \times \ln S_{*,t-1}$

which gives a consistent estimate of γ .

log(total compensation)						
log(mean net income)	0.28 (2.91)	0.23 (2.07)	0.27 (2.47)			
log(mean book assets)	~ /	. ,	~ /	0.21 (2.14)	0.15^{+} (1.44)	0.19 (1.90)
$\log(gdp/capita)$		0.51 (0.79)		()	0.74 (1.09)	~ /
"Social Norm"		()	0.001 (0.09)		()	0.005 (0.74)
Constant	11.23 (20.50)	9.91 (5.64)	11.22 (19.12)	10.95 (12.52)	9.14 (4.85)	10.81 (11.79)
Observations	12	12	12	12	12	12
R-squared	0.46	0.49	0.46	0.31	0.39	0.35

Table 2: CEO pay and typical firm size across countries

Explanation: OLS estimates, absolute value of t statistics in parentheses. Compensation information comes from Abowd and Kaplan (1999) data (available on John Abowd's website). We regress the log of CEO total compensation before tax in 1996 on the log of a country specific firm size measure. Total compensation is the sum of base salary, bonuses, voluntary benefits and long-term compensation (i.e. basebon+volben+perqs+longterm). The firm size measure is based on 1995 Compustat Global data: We use the mean size for each country top 50 firms where size is proxied as assets (data89) or net income (data32). Both the size and compensation variable are purchase-power adjusted, using the OECD adjustment provided in Abowd's data. The Social Norm variable is based on the World Value Survey's E035 question in wave 2000, which gives the mean country sentiment toward the statement "We need larger income differences as incentives for individual effort".

log(total compensation)							
	Top 20 in Industry			Top	Top 10 in Industry		
log(market cap)	0.239 (9.54) (19.25)	0.235 (10.10) (20.49)	0.355 (8.21) (9.31)	0.158 (6.68) (10.18)	$0.167 \\ (7.48) \\ (11.81)$	$\begin{array}{c} 0.315 \ (5.37) \ (6.39) \end{array}$	
log(median industry mkt cap)	$0.259 \\ (5.68) \\ (15.12)$	$0.764 \\ (14.15) \\ (27.65)$	0.581 (8.72) (14.07)	$0.335 \ (7.21) \ (16.27)$	$0.643 \\ (7.91) \\ (17.92)$	0.468 (5.32) (8.84)	
Constant	5.848 (29.69)	4.805 (29.53)	4.186 (14.77)	6.502 (32.74)	5.642 (28.73)	4.81 (11.00)	
Industry Fixed Effect Firm Fixed Effect	NO NO	YES NO	NO YES	NO NO	YES NO	NO YES	
Observations R-squared	5844 0.37	$\begin{array}{c} 5844 \\ 0.5 \end{array}$	$5844 \\ 0.68$	$\begin{array}{c} 3538\\ 0.4 \end{array}$	3538 0.56	3538 0.69	

Table 3: Cross-Industry Evidence: CEO Pay, Firm Size and Industry Benchmark Firm Size.

Explanation: OLS estimates, absolute value of t-statistics in parentheses. The first row of reported t-stats are clustered at the industry level and the second one at the firm level. Using ExecuComp data (1992-2003) merged with Compustat to retrieve firm size information, we select each year the top n (n = 10, 20) compensation packages for each of the 48 Fama-French sectors. We drop sectors that do not have more than n companies in ExecuComp. We use ExecuComp's total compensation variable, TDC1, which includes salary, bonus, restricted stock granted and Black-Scholes value of stock-options granted. Our industry reference size is the nth market capitalization (book value of debt plus market value of equity) computed by year-industry using Compustat annual for n = 10, 20. All nominal quantities are converted in 2000 dollars using the GDP deflator of the Bureau of Economic Analysis.

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