VALUE SUBTRACTED, NEGATIVE SHADOW PRICES OF FACTORS IN PROJECT EVALUATION, AND IMMISERIZING GROWTH: THREE PARADOXES IN THE PRESENCE OF TRADE DISTORTIONS*

J. N. Bhagwati, T. N. Srinivasan, and Henry Wan, Jr.

Number 204    July 1977
VALUE SUBTRACTED, NEGATIVE SHADOW PRICES OF FACTORS IN PROJECT EVALUATION, AND IMMISERIZING GROWTH: THREE PARADOXES IN THE PRESENCE OF TRADE DISTORTIONS*

J. N. Bhagwati, T. N. Srinivasan, and Henry Wan, Jr.

Number 204 July 1977

*Thanks are due to Lance Taylor and John Black for helpful comments. Research support by the National Science Foundation (Grant No. SOC77-07188) is gratefully acknowledged. The views expressed here do not represent those of the Department of Economics, Massachusetts Institute of Technology, or any organization with which the authors are affiliated.
The literature on trade distortions has now turned up three, seemingly unrelated, paradoxes:

(1) **Value Subtraction**: If inputs and outputs are evaluated at international, rather than (distorted) domestic, prices, the resulting value-added at international prices may show value subtraction — as observed in the early empirical studies in Pakistan by Soligo and Stern (1965) and in India by Bhagwati (1968a) and Bhagwati and Desai (1970).

(2) **Negative Shadow Prices for Primary Factors in Project Evaluation**: Srinivasan and Bhagwati (1976) have shown that, if primary factors are withdrawn for use in a project, from existing activities which are subject to trade distortions, then their shadow prices for evaluating the project may well be negative. Thus, paradoxically, it pays the economy to withdraw factors for use in projects that produce nothing!

(3) **Immiserizing Growth**: Finally, Bhagwati (1958) and Johnson (1967) have produced cases where growth may be immiserizing, rather than welfare-improving, for a country with trade distortions.

These three paradoxes, however, are related in an essential manner in the following way:

**Proposition I**: (i) Value subtraction necessarily implies that some factors will carry negative shadow prices for project evaluation in the presence of the given trade distortions. (ii) The presence of negative shadow factor prices, however, does not necessarily imply value subtraction.
Proposition II: Negative shadow prices for factors in project evaluation are yet another manifestation of immiserizing growth.

These propositions are established in the rest of the paper.¹

II

Consider the following model where intermediates are explicitly introduced, permitting us to analyze the phenomenon of value subtraction: a phenomenon that obviously cannot arise when there are no intermediates. It is then easily shown, by exploring the relevant dualities, that (some) negative shadow factor prices so derived are necessarily implied by value subtraction but that value subtraction is not necessarily implied by negative shadow prices.

Thus, let \( X, x, \mathbf{p} \) and \( \mathbf{p}^* \) be \( n \)-dimensional vectors representing gross outputs, net outputs, (distorted) domestic output prices and world prices, respectively. And let \( \mathbf{v}, \mathbf{w} \) and \( \mathbf{w}^* \) be \( n \)-dimensional vectors representing (primary) factor quantities and factor shadow prices based on domestic and world prices, respectively. Let then the isoquant producing one unit of output \( j \) be:

\[
(1) \quad F_j(a_{1j}, \ldots, a_{nj}, b_{lj}, \ldots, b_{nj}) = 1
\]

where \( a_{ij} \) is the unit usage of primary factor \( i \) for output \( j \) and \( b_{kj} \) is the unit usage of intermediate input \( k \) for output \( j \). Constant returns

¹Note that, in the following analysis, drawing upon the earlier work of Findlay-Wellisz (1976) and Srinivasan-Bhagwati (1976) we consider only the effects of the production distortion implied by the trade distortion, and do not explicitly bring into the analysis the consumption distortion. However, this does not affect the essence of our analysis, as discussed in depth in Appendix II of Bhagwati and Wan (1977).
and cost minimization will make

\[ a_{ij} = a_{ij}(w,p) \quad b_{kj} = b_{kj}(w,p) \quad i = 1, \ldots, n, \quad j, k = 1, \ldots, n \]

under the assumption of strictly convex isoquant surfaces. The matrix equations:

\[ A(w,p)X = v \quad B(w,p)X + x = X; \quad A = [a_{ij}], \quad B = [b_{kj}] \]

now reflect competitive resource allocation, with \( X \) and \( x \) representing respectively the gross and net output vectors. Note that \( X \) is nonnegative, while \( x \) need not be.

Assuming then that the Leontief inverse \( [I - B(w,p)]^{-1} \) exists, (3) may be written as:

\[ A(w,p)[I - B(w,p)]^{-1}x = v, \]

with its distorted and nondistorted duals:

\[ w'^{\prime}A(w,p)[I - B(w,p)]^{-1} = p' \]
\[ w'^{*}\prime A(w,p)[I - B(w,p)]^{-1} = p'^{*} \]

respectively. Denoting \( P \) and \( P^* \) as value-added with and without distortion respectively, we then get:

\[ p'[I - B(w,p)] = P' = w'A(w,p) \geq 0 \]
\[ p'^{*}[I - B(w,p)] = P'^{*} = w'^{*}A(w,p) \]

Now, from (8), we observe that, due to the distortion of intermediate input-usage coefficients (reflecting cost-minimization in response to the distorted domestic prices), \([I - B(w,p)] \) may be such that \( P_j^* \) may be negative for some output \( j \). Now, since
it is clear from (8) that, if the value-added for output $j$, $P_j^*$, is negative, then there must be some $w_i^*$ negative since $P_j^* = \sum_{i=1}^{n} a_{ij}(w,p)w_i^*$ is a non-negatively weighted sum of the $w_i^*$'s. On the other hand, some negative $w_i^*$'s are compatible with value addition rather than subtraction. Hence, the value subtracted phenomenon implies, but is not implied by, (some) negative shadow factor prices. Therefore Proposition I is established.

III

The shadow factor prices were obtained in the analysis above as the solutions to the matrix equations (6). This is the procedure stated in Diamond-Mirrlees (1976) and Srinivasan-Bhagwati (1976) and is equivalent to the Little-Mirrlees (1969) rule for shadow-pricing factors under which the changes in the outputs of traded goods, resulting from the change in factor supplies, should be evaluated at international prices.²

But as soon as this equivalence is appreciated, it is readily seen that the phenomenon of negative shadow factor prices under trade distortions in project analysis is but the mirror image of the phenomenon of the immiserizing growth of a trade-distorted small country, as analyzed by Johnson (1967), Bertrand and Flatters (1971) and Martin (1977). For, a negative shadow factor price implies, as per the Little-Mirrlees (1969) version, that the change of national output of tradeables from the trade-distorted situation, as a factor decumulates, is positive at the given international prices whereas the Johnson (1967) case of immiserizing growth shows that the change of

²However, for problems that arise with either of the two techniques when the numbers of factors and goods are unequal, as also for "stationarity" of the "marginal-variational" shadow factor prices that may be computed, see Bhagwati and Wan (1977).
output of the economy (producing only the tradeables), as resources accumulate or technology improves, is negative. Proposition II is thus established.

IV

For the applied economist, the baring of the underlying relationship among the three paradoxes is of importance since they are not just curiosa but are likely to be encountered in the real world with its heavy incidence of trade distortions. Thus, for example, the phenomenon of value subtraction has been encountered in several empirical studies of protection. Moreover, the phenomenon of immiserizing growth, in the presence of a tariff distortion, is also a matter of some empirical relevance: Little-Scitovsky-Scott (1970) have argued that the growth rates of highly-protected developing countries are seriously overstated by evaluation at domestic, rather than international, prices; and that the latter could show negative rates of growth. Finally, while we are not aware of any project analysts actually having calculated negative shadow prices, it is not at all heroic to imagine that, if the shadow prices were calculated with enough sophistication and accuracy in the real world of highly-protected developing countries, the project analyst would find some negative shadow factor prices.

3 Note, however, two things. (i) There may be alternative, statistical and economic, explanations of why value subtraction may be found in practice. These are examined in depth in Bhagwati and Desai (1970, Ch. 17, Appendix I). (ii) Moreover, even if the explanation in the text above is the correct one, as is certainly the case frequently, the reader should not infer that the situation is necessarily welfare-reducing and the activity with value subtraction may well be worth maintaining for dynamic reasons. Thus, there may be dynamic advantages, as for example in the analyses of learning effects in Clemhout-Wan (1970), Bardhan (1971) and Kemp (1976, Chapter 17); and of a putty-clay model in Findlay (1973, Chapter 8); and corresponding advantages owing to uncertainty endogenous to first-period trade levels, as with the case of an oil-embargo depending on current import dependence, as analyzed in Bhagwati-Srinivasan (1976).

4 Again, the question as to whether growth rates should be measured at international prices is rather more complex. This issue has been explored in depth in Bhagwati and Hansen (1972).
References


