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Understanding Bandwidth Limitations in Robot Force Control

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Abstract

This paper provides an analytical overview of the dynamics involved in force control. Models are developed which demonstrate, for the one-axis explicit force control case, the effects on system closed-loop bandwidth of a) robot system dynamics that are not usually considered in the controller design; b) drive-train and task nonlinearities; and c) actuator and controller dynamics. The merits and limitations of conventional solutions are weighed, and some new solutions are proposed. Conclusions are drawn which give insights into the relative importance of the effects discussed.

Acknowledgments

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This research was originally presented in April 1987 at the IEEE International Conference on Robotics and Automation in Raleigh, North Carolina [4].

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Introduction

Force control originated from the need to allow machines to interact in a controlled manner with uncertain environments [7]. In order to control force with purely position-based systems, a precise model of the mechanism and knowledge of the exact location (and stiffness) of the environment are required. With the use of force feedback from the interaction which takes place, very little model information is required in order to close a loop around contact force.

A useful example, shown in Figure 1, is the task of following a specified contact force "trajectory". We will give desired contact force commands $F_d(t)$ to the robot arm controller. We would like the robot to contact its workpiece with the specified interaction force. The measurement from the wrist force sensor, $F_s(t)$, is used by the controller to calculate the force error, $F_e(t)$. Note that the workpiece position may be changing, and the controller will still attempt to track the desired force trajectory. This paper discusses some of the practical limitations on the bandwidth of such systems.

There are many implementations of force controllers. The scheme described above, which uses the endpoint force information and makes no use of position feedback information, is termed *explicit force control* [8]. Other methods of accommodating environmental constraints include *stiffness control* [11], *damping control* [14], and *impedance control* [6]. In contrast, these methods all use joint position and/or velocity in addition to endpoint force feedback to achieve the desired response. They implement various control schemes which use sensed forces to alter inner position and velocity loop setpoints. In the case of *hybrid control* [9], directions can be specified for either pure position control or force control. All of these schemes are reviewed in [15]. Since these methods close servo loops around the joint position and/or velocity, their effective impedance then becomes the "open-loop" plant for the endpoint force control loop. From this viewpoint, the importance of understanding the stability of the explicit force loop is clear.

The major obstacle in the development of active robot force control is the performance limitation inherent in all of the above implementations. In order to track faster force trajectories, the closed-loop bandwidth must be increased. To accomplish this, the control gains are raised. However, the gains cannot be increased beyond some limits, as the system will become violently unstable. This tradeoff of stability versus performance results in sluggish closed-loop systems. It is not clear, however just what gives rise to this performance limitation. After all, the limit is generally not predicted by the model used in the controller design. Researchers have proposed many possible sources of the stability problem:

- environment stiffness
- sensor dynamics
- workpiece dynamics
- arm flexibility
- actuator bandwidth
- digital sampling rate
- control saturation
- low-pass filtering
- impact forces upon workpiece contact
- drive train backlash
- bearing friction

In fact, many of these suspected causes may actually be able to drive some mechanical systems unstable. They are all worth considering; however, careful analysis shows that not all of them are important in robot systems. This paper addresses these issues by developing dynamic models intended to describe the behavior of robot systems under the influences that these effects may have. The discussion presented here extends the work described in [3].

This analysis begins with simple dynamic models of the robot arm, sensor, grip, and workpiece. The difficulty presented by non-colocated sensors and actuators is discussed. Next, limitations in the actuator dynamics are investigated to show their importance. The analysis continues with the consideration of drive-train and task nonlinearities, and various linear controller designs. Finally, the relative importance of the effects presented is assessed, and promising solution ideas are proposed. Throughout the analysis, the case of one-axis explicit force control is discussed, although the principles apply to the other force control schemes as well. Numerical values are not given for the model parameters, so the results are appropriate to a broad class of robot systems. The plots also do not contain numerical markings on the axes. Only their general shapes are significant.

Linear Plant Dynamics

To begin the analysis, we will first consider the dynamics of a single-axis robot arm, coupled to its workpiece through the force sensor. In this linear model, shown in Figure 2, the robot endpoint is always attached to the workpiece by the sensor. The robot is described by the single mass m_r, and is coupled to the workpiece mass m_w through the sensor stiffness k_s. The actuator is represented by the input force F. The equations of motion describing the response of the model to its input can be derived and are given in [3]. Implementing the explicit force control law $F = k_f[F_d - F_s]$, and plotting the root locus as the feedback gain k_f is varied, shows that for **all gains**, the system is stable, Figure 3.

Higher-Order Dynamics

When the higher-order dynamics of the arm itself are included in the model, the effect of *non-colocation* is illustrated. Notice that in the model of Figure 4, the actuator and sensor are not attached to the same point on the arm, as they are in the model of Figure 2. The actuator and sensor in the model of Figure 4 are said to be non-colocated [5]. The root locus plot, Figure 5, now shows that for **low** gains the system is stable, and for **high** gains the system is unstable. This phenomenon is discussed in more detail in [3].

It is now interesting to observe the effect of significant dynamics (at frequencies within the desired closed-loop bandwidth) attributable to motion of the robot base or of the sensor and grip. The robot base mass is included in the next model, shown in Figure 6, as well as the sensor mass and grip compliance. The root locus plot for this model, using the simple force control law, is given in Figure 7. It shows that while these new effects did change the dynamics of the system, there is still only one mode which goes unstable. If the net stiffness of the sensor, grip, and workpiece is lower than that of the arm itself, stability can be maintained, but still only for limited bandwidth. Soft grip and/or sensor have been suggested [1, 10] even though large steady-state errors may result if uncompensated.

In fact, if we remove all the dynamics except for those of the arm itself, and represent the workpiece as a rigid wall, we have the model of Figure 8. The root locus plot, Figure 9, for this simpler system still shows the same basic shape. These models show that any dynamics to the left of the actuator or to the

right of the sensor that are significant, add poles and zeros in equal numbers. The dynamics between the actuator and sensor add more poles than zeros. These dynamics cause modes to go unstable as the loop is closed by the force control law. Therefore, much of the remaining analysis will refer to the two-mass robot model of Figure 8, since it is the simplest model with the non-colocated actuator/sensor pair.

Limited Actuator Bandwidth

The explicit force control law $F = k_f[F_d - F_s]$ assumes the actuator to respond perfectly. Suppose that the actuator cannot respond well to components of the input signal above some cutoff frequency. This effect can also drive a stable system to instability. The very simple model of Figure 10 shows the actuator modeled with a first-order lag. For $a=\infty$, the system is stable, even at very high gains. For finite a, however, the lag introduced by the actuator at frequencies above its first-order cutoff at a, drives this **colocated** actuator/sensor pair unstable, as shown by the root locus plot in Figure 11.

Unlike the workpiece, sensor, and base dynamics, which each add poles and zeros equal in number, the actuator bandwidth cutoff adds more poles than zeros, which degrades the performance more seriously, just as did the higher-order arm dynamics.

Summary

The important conclusions to draw from the robot system model analysis are:

- If arm vibratory modes are significant, then the closed-loop bandwidth limitation will be maintenance of stability for these modes.
- If workpiece, sensor, or base dynamics are significant, then these open-loop cutoffs will also limit the closed-loop bandwidth.
- If grip compliance is significant, it degrades performance by lowering the effective loop gain, $k_f k_s k_g$.
- If actuator bandwidth is significant, then the closed-loop bandwidth is also restrained by this crossover frequency.

Note that since sensed contact force F_s is proportional to endpoint position, the issues in force control of a flexible arm become analogous to those in *endpoint control* of a flexible space structure. Research in that area can be applied directly to robotics [2].

Plant Nonlinearities

There are many nonlinear dynamic aspects of robot systems to be considered. Some of these include: dynamic coupling between axes, joint friction, transmission backlash, and actuator saturation. For the most part, these effects should be neglected in control system design. The higher-order dynamics and cross coupling terms are often small [12]. However, friction, backlash, and saturation can indeed give rise to undesirable behavior, in particular when coupled with integral control. Coulomb friction and stiction can change the stability bounds found by linear analysis [13].

A useful generalization to guide initial controller design is the following: Nonlinearities may decrease the allowable loop gain (and thus decrease the closed-loop bandwidth) **only** for linear systems which already have stability limitations, like that shown in Figure 8. For systems with significant

nonlinearities, the choice of controller configuration must be verified by simulation, where the inclusion of the nonlinear effects is relatively easy.

The example presented below discusses the workpiece contact discontinuity, which represents one of the more difficult nonlinearities to be dealt with. Other examples include actuator saturation [12] and stiction [13]. This area is a topic of ongoing research.

Discontinuity at the Workpiece Contact

Perhaps the most severe mechanical nonlinearity for the closed-loop system to deal with is the discontinuity at the workpiece contact. When the robot endpoint **is** in contact with the workpiece, the system behaves like the linear system analyzed above. When the endpoint **is not** in contact with the workpiece, then not only do the dynamics change (the first mode becomes a rigid-body mode, etc.), but also the closed-loop system becomes open loop (but not unforced) since the feedback from the sensor goes away.

Figure 12 shows a nonlinear version of the two-mass model of Figure 8. In this model, the robot and workpiece can separate when the contact force becomes zero. When the linear control law $F = k_f[F_d - F_s]$ is implemented as above, nonlinear simulations have shown time responses resembling Figure 13.

The nonlinear system exhibits limit cycles for some values of gain. For **all** gains higher than the critical gain predicted by the linear analysis, the system will limit cycle. For **some** gains lower than the critical gain predicted by the linear analysis, the system will also limit cycle. For **other** low gains, however, the system is always stable. The limit cycle performance depends upon all the system parameters, as well as the initial conditions, and of course the feedback gain. (Coulomb friction, on the other hand, can extend the stability range found by linear analysis [13].) Most significant, however, is the fact that the discontinuity only brings about limit cycles in systems which do have a critical gain limit. With a rigid-body robot model, no limit cycle response is displayed.

Controller Dynamics

The explicit force control law $F = k_f[F_d - F_s]$ may only be the starting point in an actual controller design. The control may in fact be implemented digitally, and may include other dynamics. Note that the closing of inner position and velocity loops alters the effective impedance. This merely changes $G(s)$, which in turn moves the "open-loop" poles for the force loop.

Digital Control Implementation

Historically, digital effects have predominated the force-controlled instabilities observed in the laboratory. Slow sampling can certainly miss the important dynamics as they take place. Whitney presents criteria for choosing the digital sampling interval for various force control schemes [15]. With modern computer-based controllers, sample rate can be sufficiently fast to avoid this cause of instability.

Filtering

A common thought, upon the observance of the violently unstable behavior of a force-controlled robot in the laboratory, is to "filter out the components of the signal which are exciting the unstable dynamics". This scheme is depicted in the block diagram of Figure 14.

Suppose that the plant $G(s)$ is the two-mass model of Figure 8, which displays unstable behavior

that results from the non-colocation. Let us see what is the effect of adding a low-pass filter in the feedforward loop. The new root locus plot is drawn in Figure 15. The system is still unstable at high gains. In fact, comparing Figure 15 with Figure 9 shows that the filter can make the system go unstable even faster (at lower gain and lower closed-loop bandwidth). This is not surprising, since filtering adds poles, and so there are now four asymptotes. Low-pass filtering is a part of almost every controller implementation, and both the order and cutoff frequency of the filter must be chosen carefully. Filtering the sensed force rather than the force error signal yields the same closed-loop characteristic equation, and so it has the same root locus plot. Filtering in the feedforward loop is usually preferred for better disturbance rejection.

PI Control

To eliminate steady-state error to step changes in desired contact force, integral control is often added. The PI force controller includes the integral of the force error and takes the form of Figure 16. To demonstrate the effect of adding the integral term to the simple force controller, we will let $G(s)$ represent the two-mass model of Figure 8. For $k_i=0$, we have the root locus plot of Figure 9. For $k_i>0$, we have the root locus plot shape (for varying k_f) drawn in Figure 17. The PI controller adds a pole at the origin and adds a zero at $s=-k_i$. Clearly this scenario is detrimental to performance. This is analogous to adding a lag compensator, which "destabilizes" the system.

PD Control

Faster response can often be achieved by allowing the actuator to respond to the changes in the error in addition to the error itself. The PD force controller would be constructed as in Figure 18. Again, let $G(s)$ represent the two-mass model of Figure 8 which displayed an instability at high k_f. For $k_d=0$, we have the root locus plot of Figure 9. For $k_d>0$, we have the root locus plot shape (for varying k_f) that is shown in Figure 19. The derivative term adds a zero at $s=-1/k_d$, which contributes phase **lead**. This zero tends to "stabilize" the closed-loop system. Compare the root locus plot of Figure 19 with that of Figure 9.

Lead Compensation

Since the PD force controller looked promising, let us now investigate the effect of adding a lead compensator to the simple force control law. The lead compensator makes the controller take the form of Figure 20. To demonstrate the effect of the lead compensator, consider again the two-mass robot model of Figure 8 as $G(s)$. The root locus plot is drawn in Figure 21. The lead compensator adds a zero at $s=-a$ and a pole at $s=-b$. With the zero at a lower frequency, phase lead is introduced over a finite frequency range. This effect can **increase** the closed-loop bandwidth. In the root locus plot shown in Figure 21, the loci which cross the imaginary axis do so at a higher frequency with the lead compensator. Judicious choice of the frequencies a and b can result in superior closed-loop performance. Higher-order compensators may be able to increase the bandwidth even further, but may also be more susceptible to modeling errors.

Summary

The important conclusions to draw from the controller implementation analysis are:

- Digital effects are significant, but should not present a problem in modern computer-based systems.
- Low-pass filtering and PI control add destabilizing poles, which introduce phase lag and limit closed-loop bandwidth.

- PD control and lead compensation add zeros which provide phase lead at low frequency. This effect is able to increase closed-loop bandwidth.

Conclusions

Closed-loop endpoint force control is difficult because it involves non-colocated sensors and actuators. For robot structures that can be modeled as rigid, simply closing the force loop always yields a stsble system. However, significant dynamics of the robot arm **between** the wrist force sensor and the joint actuators are very important. When the control loop is closed to give desirable performance to the rigid-body mode, the higher modes can become unstable. Gevarter provides a detailed discussion of this effect [5]. The dynamics of the robot system that are **not between** the actuator and sensor can also limit the overall system performance, but they do not affect the system stability in the same manner.

Nonlinearities also can change the system performance. The discontinuity at the workpiece contact can decrease the feedback gain allowable to one below the critical gain found by linear analysis. On the other hand, joint friction can extend the stability region [13].

Controller dynamics are equally as important as the plant characteristics. Digital effects should not pose a problem with the computational speed of today's microprocessors. Limited actuator bandwidth, low-pass filtering, and integral control all add low-frequency poles. These poles introduce phase lag, which can even cause rigid robots to have stability limits on the force feedback gain. Derivative control and lead compensation, on the other hand, add low-frequency zeros, which provide phase lead. These control schemes have the potential to raise the closed-loop bandwidth, improving active force control performance.

It may be possible to further improve performance by the use of some more creative control strategies. A sophisticated model of the robot vibratory modes and their coupling of input forces to output motions could be used along with more sensors to give reliable information about the response of these modes. A nonlinear control strategy could be developed, which is matched to the nonlinear dynamic characteristics of the task to be performed. Active nonlinear elements in the mechanical system itself could be included, perhaps at the interface between the sensor and the workpiece. Finally, we could use knowledge about the dynamics involved in the execution of typical tasks in order to change the control strategy as the procedure actually takes place.

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Historical Perspective and State of the Art in Robot Force Control.
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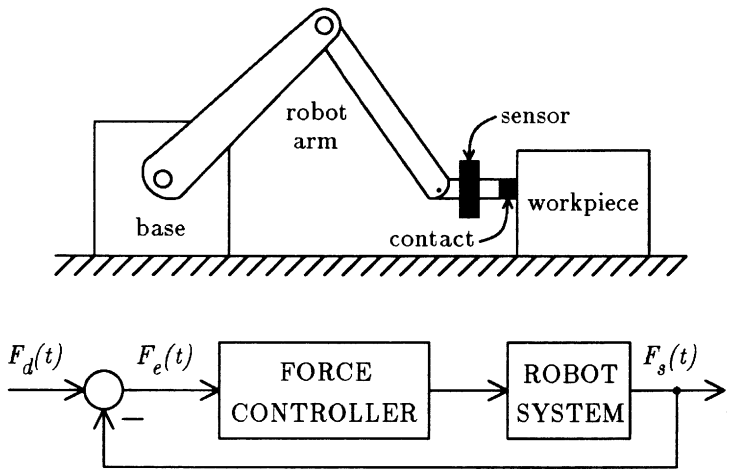
Figures



The force controller is designed to minimize force errors F_e calculated as the difference between the desired force trajectory F_d and the sensed contact force F_s. The endpoint position may even be unimportant.

Figure 1: A Sample Robot Task and Force Controller

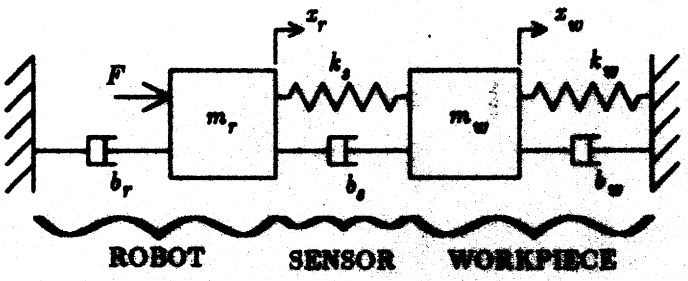


Figure 2: Robot Model Including Workpiece Dynamics

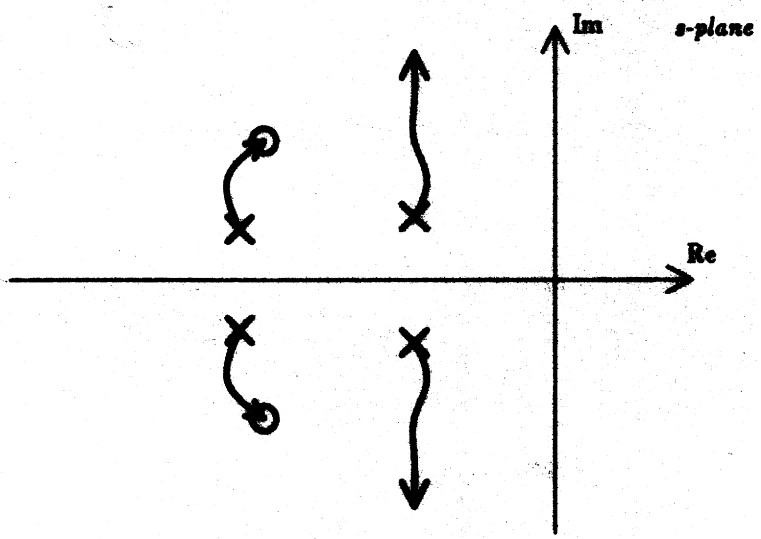


Figure 3: Root Locus Plot for the Model of Figure 2

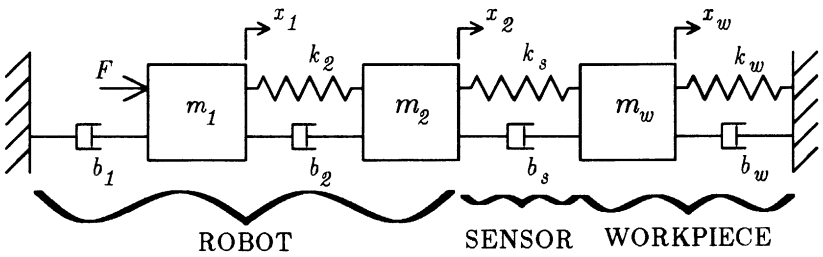


Figure 4: Robot Model Including Higher-Order Arm Dynamics

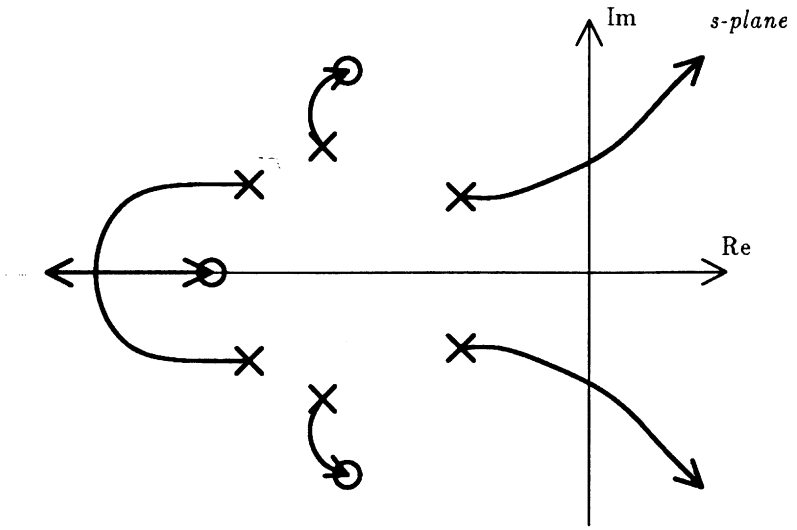


Figure 5: Root Locus Plot for the Model of Figure 4

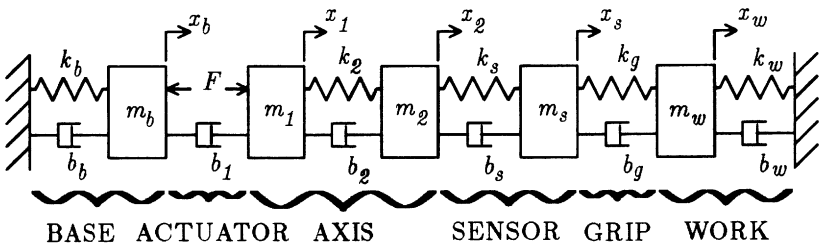


Figure 6: Robot Model Including Base, Sensor, and Grip

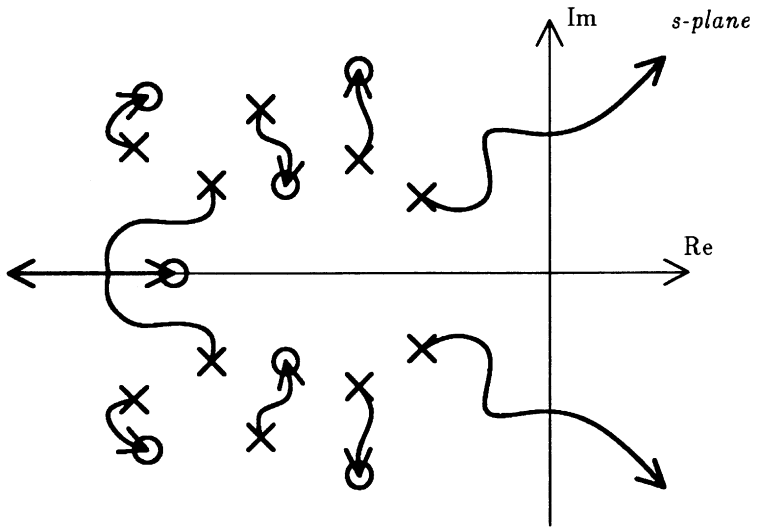


Figure 7: Root Locus Plot for the Model of Figure 6

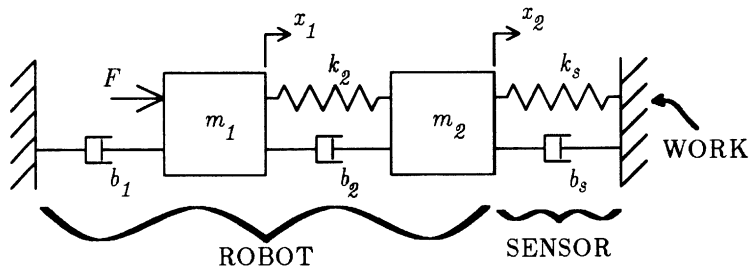


Figure 8: Robot Model Including Arm Dynamics Only

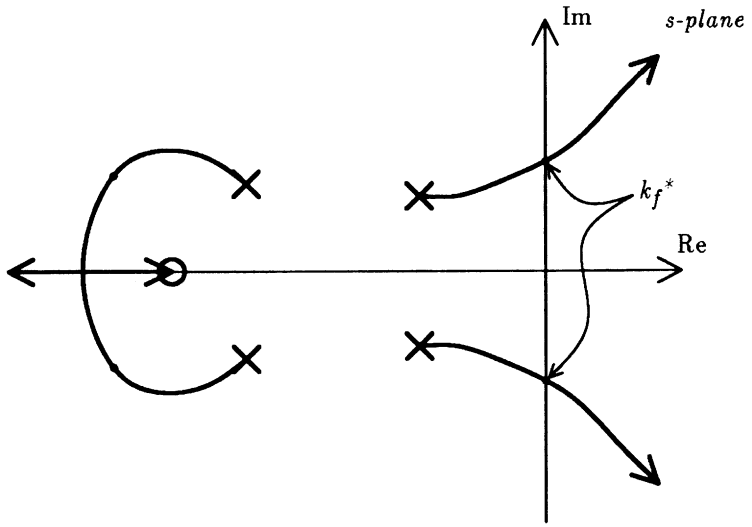


Figure 9: Root Locus Plot for the Model of Figure 8

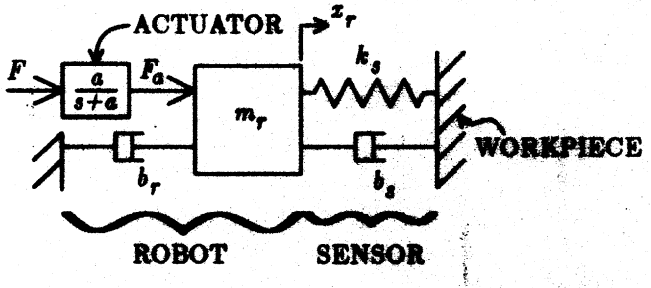


Figure 10: Rigid-Body Robot Model With Actuator Bandwidth Limitation

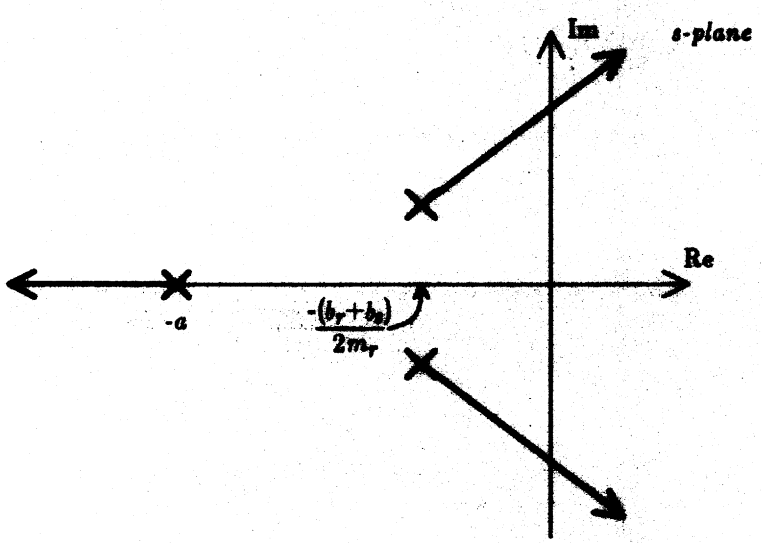


Figure 11: Root Locus Plot for the Model of Figure 10

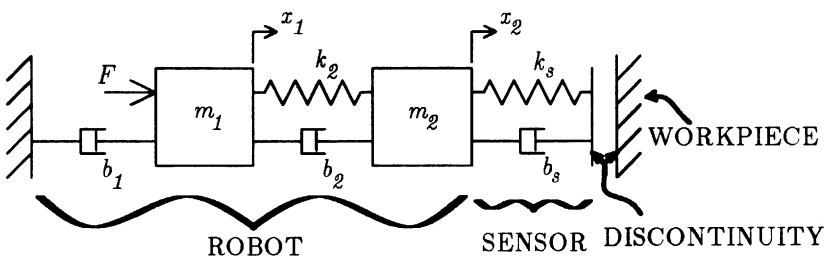


Figure 12: Robot Model with Discontinuity at Workpiece Contact

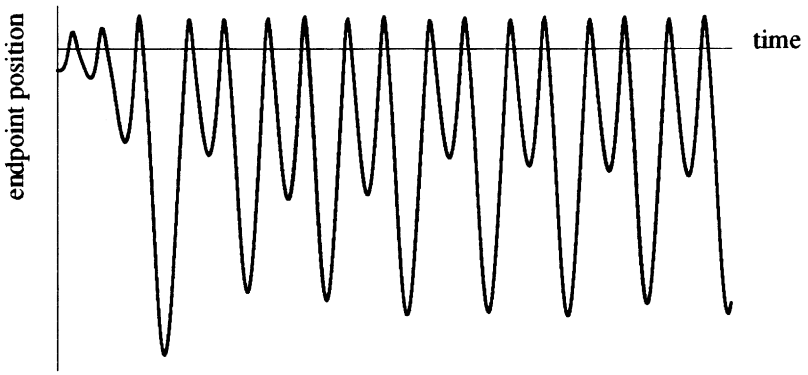


Figure 13: Simulation Time Response for the Model of Figure 12

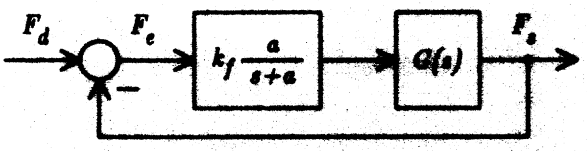


Figure 14: Filtered Force Control Law

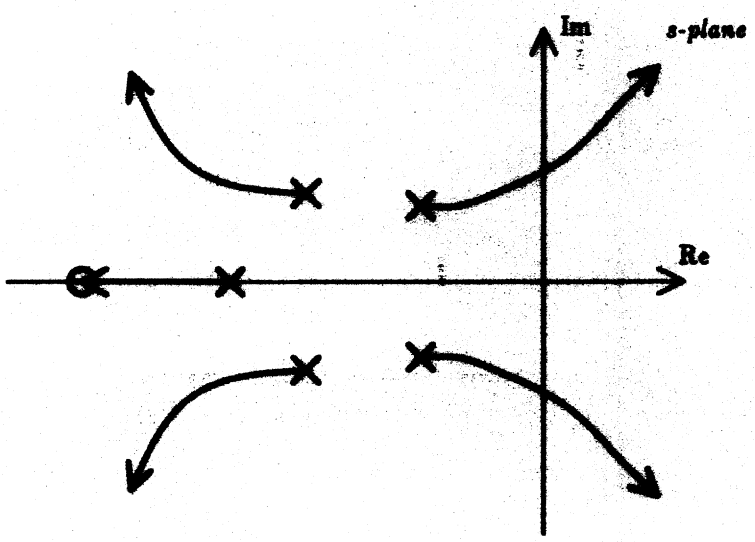


Figure 15: Root Locus Plot for the Controller of Figure 14

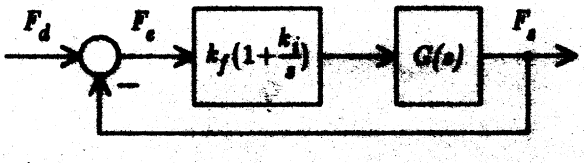


Figure 16: PI Force Control Law

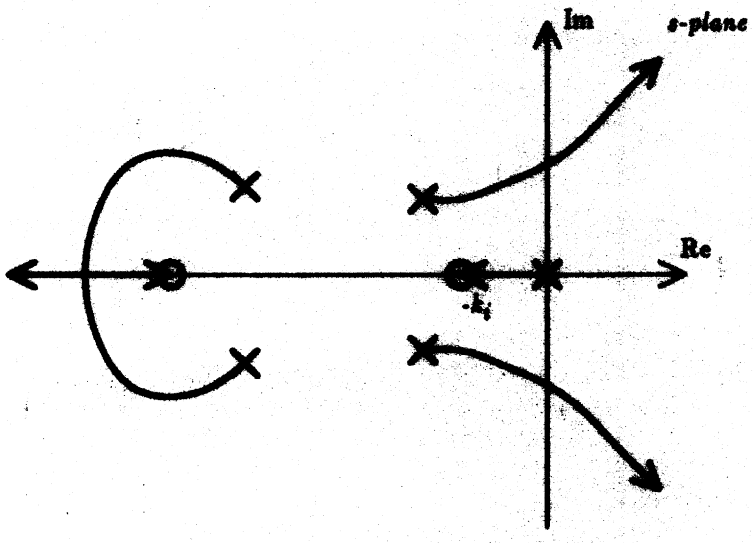


Figure 17: Root Locus Plot for the Controller of Figure 16

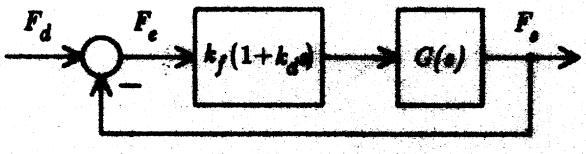


Figure 18: PD Force Control Law

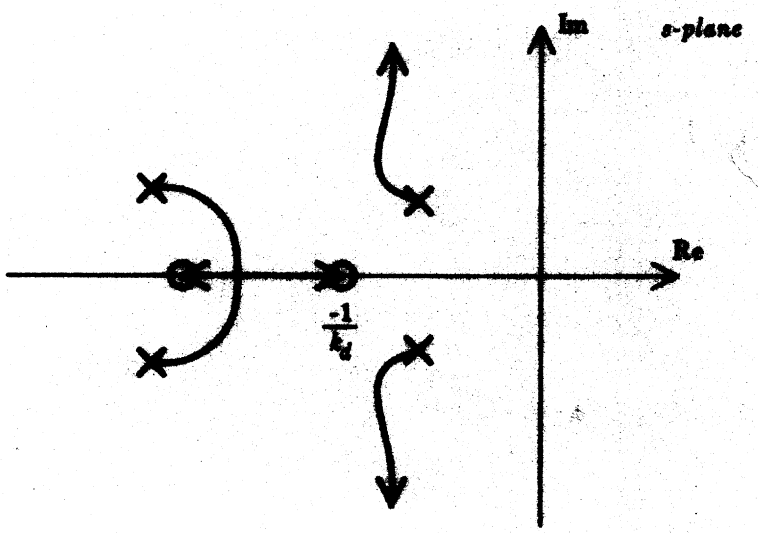


Figure 19: Root Locus Plot for the Controller of Figure 18

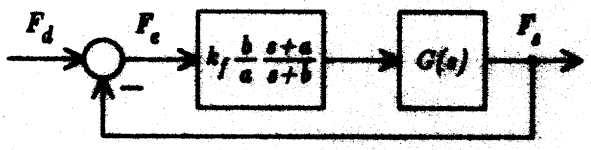


Figure 20: Lead Compensator Force Control Law

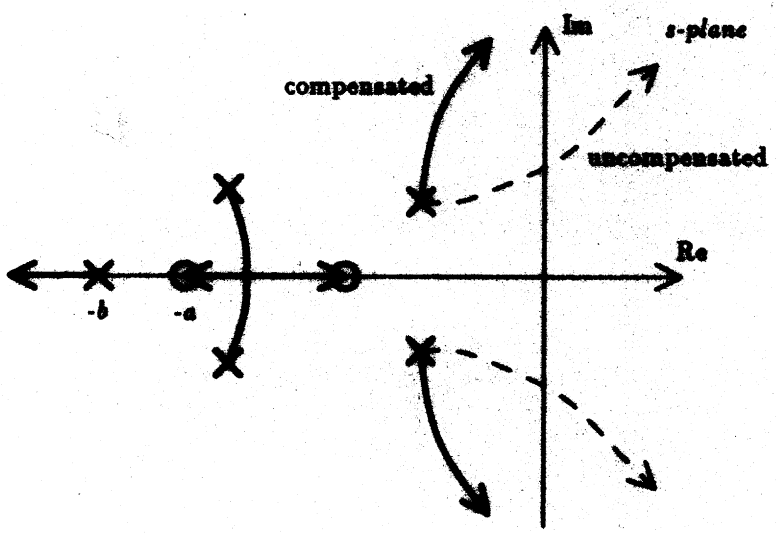


Figure 21: Root Locus Plot for the Controller of Figure 20

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This paper provides an analytical overview of the dynamics involved in force control. Models are developed which demonstrate, for the one-axis explicit force control case, the effects on system closed-loop bandwidth of a) robot system dynamics that are not usually considered in the controller design; b) drive-train and task nonlinearities; and c) actuator and controller dynamics. The merits and limitations of conventional solutions are weighed, and some new solutions are proposed. Conclusions are drawn which give insights into the relative importance of the effects discussed.

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