The Hyperbolic Lattice: morphology, kinematics, and potential applications

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This thesis is a study of the kinetics and morphology of the hyperbolic lattice. Experiments began with simple models of sticks and string and progressed to the development of a surgical retractor made from biocompatible materials and the design for a lightweight deployable emergency shelter. These applications share many criteria while spanning a wide range of scales and manufacturing methods. The mechanics of soft bodied organisms such as the worm and the sea anemone were observed to better understand the kinetic models being made. A unifying theme of this study has been an interest in the correlation of form, mobility, and structural behavior.

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Figure 0.1 Skeletal construction of the human hand illustrating the compressive components. A system of tensile elements (muscles, tendons, and ligaments) are required to make a highly tuned kinetic tensegrity system capable of precise control.
0.1 minimum weight
In order to survive, nature has learned to produce structures of extreme efficiency with the least amount of material resource. Efficiency, in this case, is a developed knowledge of the interdependent relationship of the structure, the form, and the purpose of the organism. The need for minimum weight varies according to the function and environment of the organism. Airborne structures, out of necessity, have minimized the weight of their structural system; water born organisms, in contrast, are only marginally effected by gravity. A whale, for instance, is far larger than any land animal and can attain this magnitude only because its body density is similar to its surrounding medium of sea water. Once on land and subject to the full force of gravity, the whale is in danger of collapsing under its own weight. In nature, one condition is sure, whenever weight can be minimized it will be metabolically advantageous.

It is quite obvious, but none the less intriguing, that to obtain a minimum weight to strength ratio, the designer necessarily minimizes the density of an assembly. A decrease in density correspondingly causes an increase in porosity. Highly porous three dimensional lattices are used by both nature and humans’ to reduce the weight of a given structure. An example is the interior of the bird bone [Fig. 0.2] and the lattice construction the space frames of Alexander Graham Bell [Fig. 0.3]. The compressive struts of each are minimized to a fine network, visually depicting the flow of forces through the system.

The notion of minimal density led Robert Le Ricolais to comment that “the art of designing structures is knowing where to put the holes.” [Ref. 1] This notion led him in his search for a tensile material of infinite length (and load carrying capacity) with zero cross section. Nature has provided the ‘solution’ in the development of planetary systems. Gravity, with zero cross section and density and an arguably infinite porosity, tensely holds relatively massive bodies in orbit. Gravity, in a sense, is the ‘material’ of Le Ricolais’s search. A similar argument could be made, on a far smaller scale, to
describe atomic stability. The atom is composed of ‘compressive islands’ held in orbit by tensile forces. Differences in both atoms and solar systems are fundamentally a result of their various spectrums of material density (distribution).

The distribution of material in space is a significant concern of many architects, engineers, and designers. Engineers in particular, often strive for higher structural and functional efficiency at lower rates of material consumption. An example is the geodesic dome or sphere popularized by Buckminster Fuller [Fig. 0.6]. This system encloses the greatest possible volume per unit weight of material. Fuller was interested in creating the maximum benefit with the minimum use of energy and materials. He invented the contraction ‘dymaxion’ from the words dynamic, maximum, and ion to describe his pursuit [Ref. 0.2].

Kenneth Snelson developed a method of construction termed ‘tensegrity’ [Fig. 0.6] by Fuller. Tensegrity is a contraction of ‘tensional integrity’, defined by Pugh as “a set of discontinuous compressive components interacting with a set of continuous tensile components to define a stable volume in space.” [Ref. 0.3] Both the atomic and planetary models are precedents of this system. Vertebrate skeletons are also composed of this structural type; the compressive bones are held together by a continuous prestressed tensile network. The relevance of tensegrity to this thesis is two fold:

1. Tensegrities, such as the human musculo/skeletal system, have a wide range of motion and still maintain stability. Vertebrates are in continual kinetic response to the external stimuli of a constantly fluctuating environment. In this perpetual flux, a dynamic equilibrium is maintained, known to biologists as homeostasis [Ref. 04]. Amidst the kinetic chaos its surroundings the musculoskeletal system is finely tuned and capable of precise control.

2. Tensegrities do not concentrate forces locally. They distribute external load to a large number of members in the structural system. They “are mechanically
stable not because of the strength of individual members but because of the way the entire structure distributes and balances mechanical stresses." [Ref. 0.5] This sharing of load allows each member to be thinner than if it had to bear the entire load itself thus creating structures with high strength to weight ratios. This manner of load dispersal is also indicative of all lattice structures and may be one reason they are prevalent in nature’s engineering. D’arcy Thompson noticed that the form of an organism is a diagram of the forces that have acted upon it. [Ref. 0.6] Thompson was aware that biological growth is stimulated by pressure, illustrated in his diagram of the correlation of trabecular bone growth and the lines of stress in the femur [Fig. 0.8]. Fuller, after seeing Snelson’s tensegrity constructions, postulated that “nature always used a balance of tension and compression.” [Ref. 0.7]

This thesis is concerned directly with material distribution in space. The presumption has been made that all materials are arranged into lattice types in which each exhibits unique structural, kinetic, and mechanical properties. Central to this work is the ability of hyperbolic lattices to change their form. Examples of nature’s engineering have been used to inform the author’s designs and to illustrate principles of kinetic structures. The following work is primarily an inquiry into the kinetic possibilities of the hyperbolic lattice and its specific applications in the fields of medicine and construction.

“Future design of engineering composite materials may be based on the structures and functions of biological soft and hard tissues. These biological materials include soft tissues such as mucus, cartilage, tendon, and skin; and hard tissues such as skeletal units, teeth, mollusk shells, and scales. The uniqueness of biological materials from several facts including their complex and intricate, ordered structures and their multifunctionality.”

Stephen Wainwright

Figure 0.8 Head of the human femur in section. Concentrations in the trabecular network illustrate patterns of growth in response to stress.
The hyperboloid, along with the conoid and the hyperbolic paraboloid, is part of a small group of structures known as ruled or warped surfaces. These are surfaces that are of double curvature and can be constructed from straight lines or straight structural members [Fig. 1.1]. The simplicity of their construction has led to their employment in numerous engineering and architectural applications.

Inspiring uses of the hyperboloid include the work of Russian engineer Vladimere Suchov (1853 -1939) and Russian born, American engineer Lev Zetlin (b. 1918). Both stacked individual hyperboloids into columns. Suchov creatively utilized the hyperbolic lattice to support water tanks, light houses and power lines as early as the 1890's. These projects began as simple singular units and culminated when he stacked these 'blocks' into lightweight towers that were simple to construct, beautiful, and incredibly strong. Zetlin applied the same method of stacking for the design of a 1000' foot office tower proposed for Milwaukee in 1972 [Fig. 1.2]. He incorporated a system of active cables to control the oscillation of the tower. Catalan Antonio Gaudi (1852 - 1929) made use of the hyperboloid in the column capitals of El Temple Expiatori de la Sagrada Familia (begun in 1883) and the conoid in the roof surface of the Sagrada Familia Parish School (1909). The Spanish born Mexican designer Felix Candella divided the surface of the hyperbolic paraboloid into sections and reassembled them into structurally ef-
Figure 1.3  Schematic drawing of an octagonal groined vault made of four intersecting hypars. It was constructed in concrete as a restaurant in Xochimilco, Mexico, 1958. (Candella)

Figure 1.4  Given the foci F1 and F2, and the tranverse axis AB. Using F1 and F2 as centers, and any radius R1 greater than F1B, strike arcs. With these same centers and a radius equal to R1-AB, strike arcs intersecting the first arcs. These intersections are points of the hyperbola.

Figure 1.5  A vertical cross section through a cone reveals a hyperbola.

Figure 1.6  Asymmetrical hyperboloid.

Figure 1.7  Diagram of a symmetrical hyperboloid generated from straight lines and its circular cross section.

Figure 1.8  Illustration of sections taken through a hyperbolic surface. The sections describe a hyperbola.
ficient, undulating structures [Fig. 1.3]. These geometric forms are easily generated with a scale and straight edge, two common tools of the carpenter, architect and engineer.

1.1 generating a hyperboloid
Figure 1.4 describes the construction of a hyperbola. The revolution of the hyperbola about the y-axis describes a hyperboloid of revolution of one sheet. Revolution about the x-axis describes a hyperboloid of revolution of two sheets [Fig. 1.9]. The surface of the hyperboloid is doubly curved and may be defined by straight lines (rulings) [Fig. 1.6]. Its surface is anticlastic, also referred to as having negative curvature. A cross section through a vertical plane of a hyperboloid yields hyperbolas; a cross section through a horizontal plane yields a circle [Fig. 1.7], and a cross section taken through any other plane reveals an ellipse.

1.2 specific study
This thesis is an investigation into the kinematics and morphology of the hyperbolic lattice constructed from joints and struts. Rotation of the joints (illustrated in Figure 1.11 as 'points and nodes) allows these structures to have a high range of motion. They are capable of a 'folding' sequence from a compact, substantially cylindrical bundle, to an expanded volume Fig. 1.13. The surface remains hyperbolic, although though the degree of curvature changes, during all stages of expansion and contraction. The system opens in a smooth and synchronized manner expanding into a lattice that is strong, flexible, and resilient. The combination of

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]

Fig. 1.10 Equation describing a hyperboloid of one sheet.

Fig. 1.9 Illustration of a hyperboloid of one sheet (left) and a hyperboloid of two sheets (right).

Fig. 1.11 Drawing illustrating the components of the kinetic hyperboloid.
Figure 1.12 Stents are used as venal or arterial scaffolding, they provide support to a weak or damaged biological conduit. The stent above is deployed via a balloon. The device undergoes plastic deformation to retain its expanded shape and therefore is not retractable.

US Patent 5,562,725

structural integrity and smooth deployment make a wide variety of applications possible.

Applications could range in scale from space station components to deployable emergency shelters to surgical instrumentation and implants. The mechanics and geometry studied in this thesis apply to the design of medical instruments and emergency shelters, both fields share concerns, requirements, and design criteria. Minimally invasive surgical procedures, such as stenting (vessel scaffolding) [Fig. 1.12] require devices that are expandable, mechanically reliable, strong, and resilient. Emergency relief shelters have similar mechanical design requirements with the addition of being lightweight and easily transportable.

The intent of this research has been to adapt the static hyperboloid into a kinetic system such that it could be folded into a compact bundle and expand into a structure with a substantial and useful volume [1.13]. The research progressed from the design of a spectrum of models to the prototyping of components to construct a kinetic hyperboloid. The mechanics of the system have been analyzed and a range of forms have been tested to determine their behavior under load.

Figure 1.13 Partial expansion sequence of a hyperboloid with flexible joints.
This chapter introduces a brief survey of lattice types and examples of their biological, architectural, and engineering applications. Lattice structures are light and porous frameworks and appear to be the predominant means of assembly of biological systems. An intent of this work is to identify parallels between nature and human engineered structures and apply the knowledge nature has developed to the engineering of everyday life. The lattice method of construction, specifically applied to the hyperboloid, will be investigated.

2.0 lattice types

The definition of 'lattice' implies a two dimensional surface structure according to the first definition given by the Encyclopedia Brittanica [Fig. 2.1]. The second definition, familiar to the crystallographer clearly associates the lattice with the third dimension, the world of space. These definitions are quite useful in an approach to understanding structural and spatial principles at every scale.

Lattices can be generally separated into three categories according to their dimension and surface curvature [Fig. 2.3]. The following is a description of each class. It should be noted that planar and curvilinear lattices are topologically similar.

planar

A two-dimensional lattice in its simplest form is planar. The struts [Fig. 2.2] intersect vertices in a flat plane. This group includes a wide variety of structures including flat fabrics and many types of spider webs. The form of the lattice is dependant on the elastic modulus of the material. A cotton fabric for example, could take the form of a plane or a complex wrinkling pattern, while remaining topologically the same structure.

curvilinear

The planar lattice may be wrapped into a cylinder, or distorted to form surfaces of double curvature such as the dome and the hyperboloid. The struts intersect vertices (joints) on a curved surface such as a sphere or hyperboloid. The lattice shells, constructed by Jorg Schlaich, as well as the geodesic domes of
dimensions

planar  curvilinear  spatial

Figure 2.3
R. Buckminster Fuller and the hyperbolic structures of this study fit into this category. The geometric patterning of each is quite similar to early basket designs, which may have been the first human use of the lattice.

The Mongolian yurt [Fig. 2.3e] is an example of a curvilinear lattice and an excellent model of lightweight, portable building technology. The yurt is similar in concept to the kinetic hyperboloid studied in this thesis. The yurt is cylindrical and constructed from simple pin joints, causing bending in the slats as the structure unfolds. Yurts may be folded into a compact bundle and then later expanded into a large volume.

Three dimensional lattice structures comprise the world of crystals and space frames. The octet truss, first designed by Alexander Graham Bell (1907) for lightweight observation towers, is probably the most prolific architectural use of the lattice. It has an extremely high strength to weight ratio, and not surpris-
ingly, is geometrically similar to the interior trabecular network of bone [Fig. 2.5] and the spiders’ web [Fig. 2.7].

2.1 the spiral lattice: nature

Nature, in the search for the most effective spatial distribution of material at the lowest metabolic cost, makes abundant use of the spiral lattice at all scales of construction. The results of a study by M. M. Giraud-Guille have determined that collagen fibrils are distributed in spiral lattice patterns in human bone. [Ref. 2.1] The spiral lattice pattern [Fig. 2.4] is referred to by Giraud-Guille as “twisted plywood.” The silica exoskeleton of the Radiolarian is also constructed of a spiral array wrapped into a spherical form [Fig. 2.8c]. These skeletons are similar in geometric arrangement to architectural geodesic spheres, which, in turn, mimic the construction of flies’ eyes, pollen grains and one of the smallest units of life, the mammalian cells [Fig. 2.8a and b].
2.1.1 worms
The use of the opposing spiral lattices for structural support is prolific in organic systems. The body walls of the nematode *Ascaris* (a small, parasitic, active worm with high internal pressure) are composed of multiple layers of spirally wound collagen fibers. Each spiral layer is opposed (alternating left and right handed) to the next, forming a cross helical reinforcement system. The spirally wound fibers act in tension, resisting the organisms internal hydrostatic pressure. In effect, the form and volume of the organism are dependent on the stiffness and geometry of the tensile fibers. The patterning and stiffness of the fiber play an important role in maintaining internal pressure.

The spiral lattice pattern is an efficient method of construction and can be made kinetic to serve as a means of locomotion. The spiral layers overlap and describe an interstitial pattern of rhombi quite similar to the hyperboloid structures studied in this project. The volume of the worm body (Ascaris) is maximum at a fibre angle of 54°44' [Fig. 2.9]. A decrease in fiber angle is accompanied by an extension (a stride) and a increase in fiber angle causes a contraction. The combination of cross helical (opposing spiral lattice) tensile reinforcing and internal pressure contributes to the strength and versatility of the organism.

2.1.1 sea anemones
The anemone [2.10 and 2.12] is an hydrostatic ani-

Figure 2.9  Diagram illustrating the pattern of tensile reinforcing of the worm Ascaris. The form of the worm is dependent upon the pattern and stiffness of the fibers as they resist internal pressure preventing the worm from exploding.

Figure 2.10  Forest of sea anemone.

Figure 2.11  The caterpillar locomotes in a similar manner as the Ascaris and is capable of substantial cantelevering with fluid compressive elements.
The organism inhabits shallow water where tidal currents are strong and swift. They respond to this stressful environment by remaining supple and limber. Most anemones have rooted themselves to the sea floor and have developed highly flexible bodies. The anemone yields to forces in its environment, swaying with the current rather than offering rigid resistance like the hard corals (and most buildings). The anemone can open its mouth, contract its muscles and reduce its volume to a minimum. A reverse operation will enable a substantial change in volume bringing the anemone into a column four times higher than it is wide. The organism may also sway from side to side sweeping the ocean floor with its tentacular crown. The tentacles have kinetic form and volume characteristics similar to the trunk, although on a smaller scale [Fig. 2.13]. These movements are generated by a very low hydrostatic pressure and accordingly are relatively slow. Anemone body walls are composed of a layered system of collagen fibrils. "There is a cross-he-
lical array in the outer layer and a denser array of circumferential and radially oriented fibers in the inner layer." [Ref. 2.2] The spiral lattice of reinforcing fibrils, as in the worm, regulate the form, the kinetic ability, and the internal pressure of the organism.

There may be industrial or medical applications for a structural material with this ability. A structure with these properties could be valvelike to augment flow within a vessel. The entire conduit might be kinetic as in the tubular body of the anemone. A familiar example is the blood vessels in the eyes, which expand and constrict to control blood flow. A mechanical counterpart might allow precise flow control with a minimal loss of pressure.

2.1.3 cactus skeletons
Cactus skeletons [Figs. 2.14 and 15] are constructed of a similar geometrical arrangement of fibres comprising a lightweight and porous spiraling lattice. The mechanics and material properties of the cactus skel-
The seemingly paradoxical situation of constructing a surface of double curvature with straight lines may have inspired interest in the hyperboloid. It has been used in a variety of occupations by architects and engineers in sections, as whole elements, and in multiple layers [Fig. 2.16]. The form is well known to architects working with thin shell concrete and designers of contemporary fabric roofs. Since hyperboloids can be constructed from uniform standardized joints and components they can be fabricated quite efficiently.

### 2.2.1 sections of the hyperboloid
Sections of the hyperboloid or hyperbolic paraboloid are commonly known as saddle shapes. They are the predominate shape of cable nets and architectural membranes. If one of the two opposing lines of curvature (one concave and one convex) is circular, the saddle can be mapped on to the surface of a hyperboloid. Under the direction of Frei Otto, the Institute for Lightweight Structures at the University of Stuttgart has experimented with tensile surfaces of minimum curvature formed from soap film [Figure 2.19]. The film is of uniform stiffness and is unable to locally concentrate load. The resulting surface is
Figure 2.17  Ilesia de San Jose, Monterrey, Mexico, 1959

Figure 2.18  San Vicente de Paul Chapel, Coyoacan, D.F., 1960
Figure 2.19 Soap bubble experiment by Frei Otto illustrating a hyperbolic paraboloid of minimum surface. The surface tension of the soap film is equal at every point along the surface.

Figure 2.20 Minimum surface cable net covered with rip stop nylon patterned from flat panels. The structure was scaled from the soap film experiment illustrated above in Figure 2.19.

therefore the minimum area for the boundary conditions. Figure 2.20 is a cable net covered with fabric that was scaled from Otto’s soap bubble experiment. The tensile minimum surface taken naturally by the soap film is often used compressively by designers of concrete shells.

Felix Candella believed that strength should come from the form of a structure and not its mass. He worked extensively with hyperbolic surfaces, dividing them into sections and arranging them into a structurally balanced assemblies. Figures 2.17 and 18 show the tiling of two and three sections respectively. The surfaces of concrete shells are at once tensile and compressive; the concave ‘arch’ of the form relies on the compressive capacity of concrete and the convex ‘arch’ resists forces through tensile reinforcing.

2.2.2 whole hyperboloids
Figure 2.21 shows developments in cooling tower design and constructive methods. The surface of each
Illustration of the progression of cooling tower design from early wooden structures to highly integrated structural concepts. 2.19a is a wooden shell structure reinforced by an exoskeletal wooden lattice. 2.19b relies solely on its lattice for support and is sheathed with non structural panels. 2.19c is similar in concept to the bicycle wheel. A compressive hoop is suspended from a compressive mast enveloped in a structurally integral cable net. A non structural skin is later applied.
design is hyperbolic and relies on its curvature to maintain its shape under load. In recent evolutions, the surface is formed from tensile cables that transfer their load to a compressive hoop, a central compressive mast and eventually to the earth.

Frei Otto has experimented with the physical construction and the many forms of the hyperbola. The minimum surface forms are models of cooling towers [Fig. 2.23]. The hyperboloid is often used for natural draft cooling towers because its form encourages air to flow over its surface with little inertial loss from friction. Otto is presenting a lightweight alternative to the common construction of comparatively massive concrete shell towers. The surfaces of Otto's hyperboloids are purely in tension, confining compressive forces to the rings (or splines) and the central mast.

Models were made with compressive hoops and soap film [Fig. 2.22]. The film, similar to a cable, is only capable of withstanding tension and cannot locally concentrate load. Soap film therefore always pro-
duces a minimum surface that is relative to its boundary conditions. In the models, the distance between the hoops was varied to generate a number of hyperbolic forms, each of minimum surface. The compressive hoops and tensile skin are supported by a compressive mast. The morphological method of finding various forms of the hyperboloid in chapter 3 is similar to Otto's incremental approach with soap film.

The cooling tower engineered by Jorg Schlaich [Figs. 2.24 to 26] is conceptually similar to Otto's. The surface is formed by two sets of opposing spiraling cables combined with longitudinal cables and clad with an internal aluminum skin. The tensile forces of the cables are balanced by the compressive hoops and the central mast. The prestress in the cable net is greater than the compressive force generated by live loading, in this case, the wind, assuring that the surface is always in tension. The tower could be viewed
as a rapidly assembled structure. The cable skin was prefabricated and raised from the mast much like the sail on a ship is hoisted. The opposing spiral lattice pattern of the cables are similar to the strut patterns of the hyperbolic lattice studied in this thesis.

2.2.3 multiple layers:
Vladimere Suchov connected hyperboloid 'blocks' into columnar assemblies. Six blocks are stacked to form the tower in Figure 2.29. Suchov's constructions were designed for numerous applications including the support of water tanks and to carry high tension power lines across the Soviet Union [2.27 to 29]. The spiral weave is incredibly light and has the strength to support massive volumes of water. He was able produce these structures with little variation in material or connections. The struts are made of angle iron and riveted where their flat surfaces meet. The angles are arranged to provide a cross section large enough to resist out of plane buckling.

This method of construction (hyperbolic lattice) does not concentrate load locally but instead disperses it
Figure 2.29
Lattice structure supporting high tension power lines.

Figure 2.30
Buckling of a tower constructed on the USS Michigan in 1918. The tower collapsed during a storm. Maximum buckling occurred between the circumferential hoops.
to a multitude of elements. Since the load is shared by a large number of struts, each strut can therefore be made out of relatively thin, light 'fibers'. Through this method of load distribution, the hyperbolic lattice frees itself from the proverb 'the chain is only as strong as its weakest link.'

Lev Zetlin and Associates engineered a 1000' tower proposed for Milwaukee in the early seventies [Fig. 2.31]. The tower was to be erected of five stacked hyperboloid blocks, ranging in scale from sixty feet to ninety feet in height and made of straight steel pipe. As in Suchov's structures, struts are designed to respond either in 'pure' tension or 'pure' compression, i.e., bending is eliminated from the system. Circumferential tensile rings resist the outward thrust of the struts. The tower was designed to resist 150 m.p.h. winds, though the top of the tower deflected two feet and six inches. Zetlin corrected the deflection with an active control system of cables in the lower two blocks where bending was the greatest. Zetlin designed a sensor control system of gyroscopes to gauge accel-
eration, velocity, and position which sent signals to control hydraulic jacks that tightened or slackened cables as needed. The signals would be alternated to limit the building oscillation to a minimum. [Ref. 2.3]

Le Ricolais's column 'pseudo sphere' [Fig. 2.32 e] is similar in construction to Schlaich's cooling tower [Fig. 2.24]. The outer surfaces of both are formed from spiraling tensile cables balanced by an interior central mast. One major difference is that the prestress of Le Ricolais's column is internally self-contained and balanced. It does not rely on the ground to resolve the prestress and significantly reduces the load transferred to the footings. This self-containment allows it to become a discreet component (beam or column) of a building system. In contrast, the ground balances
the tensile and compressive forces of Schlaich's structure.

Le Ricolais often looked to nature for inspiration and structural understanding. He recognized that vertebrate systems [Fig. 2.30 a and d] relied on both tensile and compressive elements for mobility and structural integrity. The compressive elements (bones) in vertebrates are isolated within a continuous network of tensile components (muscles, tendons and ligaments). Le Ricolais's 'psuedosphere' simulates this type of construction as the central compressive post is isolated from the compressive hoops and both are held in place by a continuous tensile weave. This method of construction results in an assembly that mimics our own musculoskeletal system. [Ref. 2.4]
the virtual
A significant portion of this project has been devoted to developing a family of hyperbolic lattices. Matrices [Fig. 3.4] have been used to methodically explore the possible structural arrangements of the hyperboloid. All forms within the matrix have a degree of relation that stems from the chosen input variables, in this case, points and nodes [Fig. 3.1]. Diverse combinations and permutations were revealed as hyperboloids grouped themselves according to incremental formal variations. It was found that the kinetic mechanics of the initial physical model can be applied to an extensive, if not infinite, family of hyperboloids.

3.1 topological characteristics
There are two variables that topologically define a kinetic hyperboloid, referred to in this thesis as a block [Fig. 3.2]; the number of 'points' and the number of 'nodes'. The term 'point' refers to the 'v' joints at the extremities of the structure. They are the points where two coplanar struts terminate and therefore are of valency two [Fig. 3.1]. The term 'node' refers to the 'x' crossing of struts located on the surface of the hyperboloid, four struts emanate from each node and

morphism- pref. 1. shape; form; structure
matrix- n. 1. A situation, substance, object, etc., within which something is contained, originates, or develops.
(Encyclopedia Britannica, online ed., 1999)
therefore has a valency of four.

The top and bottom points of all kinetic hyperboloids describe a polygon, the simplest case of these is a triangle. A series of anti-prismatic layers are described as one moves along the axis of the structure. The rotation at each nodal latitude is proportional to the polygon that is described by the number of points. The formula \( (360^\circ)/2x \) (where \( x \) is the number of points) holds true for any hyperboloid. For example, a structure with five points describes a pentagon and will have a rotation of \( (360)/2(5) = 36^\circ \) between each nodal latitude.

A structure with an odd number of nodes will describe a prism if its outermost points are connected. An anti-prism is described by a block composed of an even number of nodes. The importance of these geometrical forms is shown in the following sections concerned with tilings and space filling arrangements.

The use of matrices [Fig. 3.7] allows visual interaction and grouping of similar and opposing properties of individual kinetic hyperboloids. For this thesis, the number of points and nodes have been varied and their effects on the form of the hyperboloid observed. The height of the hyperboloids is held constant and all structures are symmetrical. For further study the matrix can be extended to contain asymmetries, columns, tilings and packing arrangements.

### 3.1.1 points

There is a distinct relationship between the number of points and the surface curvature of the resulting lattice. As the number of points is increased, while the number of nodes is held constant, the form approximates a cylinder and the angle of inclination [Fig. 3.1] of the struts increases. Measurements taken from physical models have shown that the expansion ratio (cross section of compact bundle as compared to the cross section of the structure at full volume) increases with the addition of points. The graph [Fig. 3.13] illustrates the plotting of computer simulated expansion ratios for a structure with 12 points and 7 nodes.
3.1.2 nodes
The nodal placement is kept symmetrical along each strut for this exercise [Figure 3.6]. An increase in the number of nodes (with points remaining constant) has the opposite effect of an increase in points. The form departs from the cylinder as nodes are increased and the curvature of the hyperbolic surface increases. The focus of the hyperbola approaches the vertical axis of the hyperboloid with a nodal increase and the expansion ratio diminishes.

3.1.3 limits
A limit was found when the number of points exceeds the number of nodes by one ($p = n + 1$) [Fig. 3.7]. In this arrangement the path of a strut is 180 degrees from bottom polygon to top polygon. If the struts are assumed to be straight, they would meet at the center of the hyperboloid. Since the struts cannot pass through each other, a degree of curvature is required in order for them to complete their path. This denotes the limit to the construction of a hyperboloid with straight struts.

3.1.4 asymmetrical nodal placement
The placement of nodes directly affects the hyperbolic curvature. The 'waist' (located at the dashed line in Figure 3.6) of the hyperboloid constrains where the nodes are collected. For example, if the nodes are grouped towards the 'top' of the structure, the top polygon shrinks and the bottom polygon expands — forming structures that approach cones.

Figure 3.6 Nodes are held symmetrical about the horizontal axis for the structures in the matrix.

Figure 3.7 Plan view of limits for lattices with 6, 8, and 10 points.
Figure 3.8a

matrix, points and nodes, elevation

3n  5n  7n  9n

6p

8p

10p

12p
matrix-points and nodes-plan

Figure 3.8b
3.2 expansion of the matrix
The matrix of Figure 3.6 can be extended into further dimensions to generate columns, tilings and all space filling arrangements.

3.2.1 linear assemblies (columns)
The unit blocks can be stacked together to form columns. These have the same kinetic characteristics as the single block. They are able to compress into a compact planar arrangement and may be extended by actuating a single block. Furthermore, variations in the nodal spacing produces arches, spirals, and knots. It is important to note, however, that the knot will not have the same folding characteristics as it is a closed figure.

3.2.2 tiling
Only three prisms will fill space by themselves: the square, the triangular and the hexagonal [Fig. 3.11]. A wide variety of dissimilar prisms can be mixed to tile the plane [Fig. 3.9c]. The interstitial spaces may be filled or left open [Fig. 3.8 b and c]. Possible appli-
Figure 3.12 Drawing of the structure 12p7n fully closed and 50 percent open (measured as a function of strut length).

Figure 3.13 Graph comparing the expansion of the diameter of a circle inscribed in the polygons at the extremities of the structure 12p7n in relation to the diameter of a circle inscribed in the polygon at the equator.

Figure 3.11 Only the triangular, the cubic, and the hexagonal prisms fill space by themselves. The hyperboloid blocks describe prisms and therefore a hyperboloid block could be substituted for each prismatic cell.
Figure 3.14

Figure 3.15
able porosity. Figure 3.10 illustrates a pattern of four tiled kinetic hyperboloids. The entire assembly can fold into a compact bundle [3.14, 3.15, and 3.16].

3.2.3 packing (space filling)
Planar tilings can be stacked to form multilayered structures. As stated in the previous subsection, only the rectangular, the triangular, and the hexagonal prisms tile plane, analogously they are the only prisms which fill space by themselves. Packings of kinetic lattices can form structures of high porosity. The porosity can be varied according to the degree to which the hyperboloid is expanded. A singular hyperboloid could be expanded locally, while others are restrained, causing the entire packing arrangement to curve and wrinkle.

3.3 computational generation
The morphological method of finding form described above is particularly suited to mathematical algorithms. The topological characteristics and kinetic mechanisms of the hyperboloid were discussed with
Ryota Matsuura of Molecular Geodesics, Inc., who developed programs to automatically generate hyperboloid 'blocks' and the 'hyper-polyhedra'. The input arguments are strut height, number of points and the number of nodes. The program assumes a distance between upper and lower points as seen in Fig. 3.2. This distance can be adjusted to simulate the kinematics of the system. It can be programmed to move through its full expansion/contraction sequence - from bundle, to largest volume, to plane. Figures 3.14 - 17 illustrate examples of hyperboloid blocks generated with the program.

An interesting morphological extension has been to map the polygonal face of any hyperboloid to the corresponding face of a polyhedron. As with the flat tilings, there are many possible arrangements. A polyhedron may be 'fully mapped' (every face of the polyhedron is tiled) or 'partially' mapped. The truncated tetrahedron is composed of triangular and hexagonal faces. In the model [Fig. 3.17] only the hexagonal faces have been mapped. The structures fold in a synchronized manner similar to the singular blocks.

A 'closed' structure, (a polyhedron whose primary faces have been fully mapped) as in the case of the truncated tetrahedron [Fig. 3.17], will fold into a compact bundle whose length is twice the strut length. This is true of any closed structure. If at least one primary face is not tiled, as to make a structure with a hole, the assembly will fold to a length equal to one strut. Figures 3.18a and 3.19a depict the development of the program to include the additional input argument of 'polyhedron.' This variable facilitates the mapping of the hyperboloids onto any polyhedron.

Physical models were constructed to verify the digital output from the program. The formal and kinetic results of each process, digital and physical, matched closely. This close correlation expedited the design process as the digital models are more quickly generated than the physical.
truncated tetrahedron constructed from $6p3n$ blocks

Figure 3.17
truncated octahedron constructed from $6p5n$ blocks

virtual model

physical model

Figure 3.18
truncated pentagonal dodecahedron constructed of 10p5n blocks

virtual model

physical model

Figure 3.19
Structural and kinetic analyses were carried out on a series of hyperbolic lattices. The structural analysis was performed under static vertical loading conditions in order to explore the relationships between the form of the hyperboloids and their load carrying capacity. Three different cases were considered with the SAP2000 Nonlinear Structural Analysis Package. The kinetic analyses were carried out in Pro Engineer to determine the specific rotation of the joints. Two different structures were considered and compared.

4.1 structural
4.1.1 uniform loading on a selection of individual hyperboloid blocks: case 1
A selection of forms, shown in bold [Table 4.1], was selected from this matrix upon which to perform full nonlinear analyses.

The foreseeable application of these lattices as rapidly deployable emergency shelters was kept in mind during the selection of scale, geometry, materials and loads for the analysis. Cables were placed around the tops of all models representing either an actual cable or to simulate a membrane canopy.

The dimensions of all structures are held at a constant height of 3.5m and cable length (polygon edge length) of 1.5m. This was assumed to be the position of maximum deployment for all lattices. Loads were applied to the top of each point, calculated using a unit load per area and the tributary area of the enclosed 'polygon wedge,' as shown in Fig. 4.1.

Due to the use of a constant polygon edge length, the lattice blocks with differing numbers of points will have differing tributary areas. The change of

Figure 4.1 Demonstration of the calculation of the tributary area for applied point loads.
surface curvature of the hyperboloids may be more unusual than those created if the tributary area had been used as the constant parameter. To evaluate this effect, an additional case, 12p7nA, was considered. This block was arranged so as to have an equivalent tributary area to the 10p7n block. To achieve this a cable length of 1.244m was used. The height remained unchanged.

All the models analyzed had constant cross sectional strut properties. The material properties adopted were those of high strength steel cable and pultruded fibre reinforced composite sections. The material and section properties used are given in Table 2.

All points at the base of the blocks were assumed to have pinned supports (1 degree of freedom DOF). Although, all the other joints are actually pinned, they were modeled as fixed joints (0 DOF) for simplificity. Preliminary analysis proved that this was acceptable as the forces were found to be transmitted axially through the struts and caused negligible bending moments. The use of a large cable section helped to

<table>
<thead>
<tr>
<th>Material</th>
<th>Struts</th>
<th>Cables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber Reinforced Composite</td>
<td></td>
<td>High Strength Steel</td>
</tr>
<tr>
<td>E</td>
<td>17.2 GPa</td>
<td>200 GPa</td>
</tr>
<tr>
<td>σ_u</td>
<td>277 MPa</td>
<td>1800 MPa</td>
</tr>
<tr>
<td>ρ</td>
<td>1700 kg/m³</td>
<td>7850 kg/m³</td>
</tr>
<tr>
<td>Section Type</td>
<td>Tube</td>
<td>Circular</td>
</tr>
<tr>
<td>Dimensions</td>
<td>51 mm o.dia.</td>
<td>50 mm dia.</td>
</tr>
<tr>
<td></td>
<td>3 mm thick.</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Matrix of lattice blocks.

Table 4.2 Material and section properties adopted.
ensure that the struts did not bend and was shown not to effect the actual load path of the structures.

4.1.2 single point load on a selection of individual blocks: case 2
To explore the localized behavior of the lattices the structures from Case 1 were also analyzed with a single vertical point load applied at one point on their tops. All other parameters remained unchanged.

4.1.3 uniform loading on one block at various stages of expansion: case 3
Case 3 was an exploratory study into the loads acting on one of the structures at its various stages of deployment. The 10p3n form was selected for this. It was analyzed through three stages of expansion with polygon edge lengths of 0.5m, 1m and the original 1.5m of case 1. The height of the structure was changed to correspond with the polygon edge lengths and the unchanged strut lengths. All other parameters from Case 1 were unchanged and the uniform point loads of 10p3n Case 1 were used.

results and observations
4.1.4 case 1
The typical axial force diagram for all the structures in Case 1 is shown in Fig. 4.2. As expected, the variation between the tensile force, (T), in the top cable and compressive force, (C), in the struts was seen to differ among the structures. To allow comparison, these two values were considered as a ratio, C/T, and plotted against various variables for the structures.

The first of the two sets of results presented here is shown in Fig. 4.4. This figure displays the ratio C/T against the number of nodes, n, for each model. From this plot, it can be seen that there is a small decrease in the C/T ratio for an increased number of points and a constant number of nodes. If the number of nodes is increased and points are kept constant, however, a far more dramatic decrease is seen in the ratio C/T. The second observation is to be expected, as an increase in n increases the curvature, in turn decreasing the angle between the cable and the strut and hence
Figure 4.3  Diagram displaying the definition of measurement of curvature, the lever arm vs. height ratio.

Figure 4.4  Ratio of strut to cable load vs. number of nodes for case 1.

Figure 4.5  Ratio of lever arm vs. height vs. number of nodes for all models.

Figure 4.6  Ratio of strut to cable force vs. lever to height ratio for cases 1 and 3.

Figure 4.7  Ratio of horizontal to vertical reaction vs. lever to height ratio for cases 1 and 3.
requiring a greater tensile force, $T$, for a given load, $C$, in the strut.

The initial observation of a marginal decrease in $C/T$ with an increase in the number of points, $p$, is a little surprising. As previously stated, an increase in $p$ causes the lattices to approach the cylindrical form by both decreasing the curvature and moving the struts' angle of inclination closer to the vertical. Both of these would be expected to increase $C/T$. An initial explanation for this contrary behavior could perhaps be the use of the constant cable length as a constraint rather than the constant tributary. Considering the forces as ratios means that the variation in loaded areas has been accounted for. However, the effect on the curvature has not. The much closer results between $12p7nA$ and $10p7n$ where the tributary areas are equal, than between $12p7n$ and $10p7n$, would appear to support this.

Therefore, to clarify this observation further, a deeper consideration of the results was undertaken. Firstly, although the parameters $p$ and $n$, together, can describe the curvature, it is recognized that a clearer quantification was desired and an additional variable was defined. This measurement of curvature - the lever arm versus height ratio, $l/h$, is defined in Fig. 4.6. To show the curvatures of the structures, Fig. 4.6 has been plotted with $l/h$ versus $n$. The results of this plot support both of the initial expectations that increasing $n$ will strongly increase curvature (i.e., $l/h$) and increasing $p$ will decrease curvature, though weakly.

The second set of results, therefore, plots the $C/T$ ratios against the $l/h$ ratios in Fig. 4.5. These show a definite and strong trend between the two ratios. The results of Case 3, to be discussed in Section 4.2.3, have also been included and further support this relationship. For example, examining the $10p$ results for Cases 1 and 3, a smooth line can be drawn.

Finally, from Fig. 4.7 it is interesting to observe that a linear relationship would appear to exist between the ratio of the horizontal reaction to the
vertical and the lever on height ratio.

4.1.5 case 2
The typical axial force diagram for this case is shown in Fig. 4.8 (dark grey = compression). From this diagram some very interesting general observations can be made. Primarily, a single point load is dispersed back through the action of the cable. It appears that the load is not only resisted by the struts immediately below the point of loading but that it is transferred to the adjacent struts as well. This develops a clear compressive zone in the structure.

The load path to the adjacent struts was typically back through two cable bays, T1 and T2 as indicated on Fig 4.11. The loads in the other struts and cable bays were substantially less. The dispersive nature was found to increase with curvature as demonstrated by

![Figure 4.9](image_url)  
**Figure 4.9** Ratio of force in strut 1 over cable 1 vs. number of nodes for case 2.

![Figure 4.10](image_url)  
**Figure 4.10** Ratio of force in strut 1 over that in struts 2 and 3 vs. number of nodes for case 2.
the Figs. 4.9 and 10. The cable force ratios (T1 and T2) have been plotted against the number of nodes (n) as a measure of the distribution mode. (A sufficient relationship was observed with this variable. Hence, in the interests of brevity the results versus the lever to height ratio are not included.) The struts and cables considered are labeled on Fig. 4.8.

Fig. 4.10 plots C1/T1 versus n - a similar ratio to that considered in Case 1. It is interesting to note from this, that for n = 3, the ratio is greater than 1 and relatively constant for all values of p. For greater values of n, the behavior is similar to that of Case 1 with both p and n decreasing the ratio. Figure 10 best displays the effect of an increase in the number of nodes on the distributing capabilities of the structures. Quite clearly it can be seen that the contribution of the C3 struts increases greatly with an increase in n, to the point that it almost equals that of the C2 strut when n = 7. It is also important to note that the adjacent struts C2 and C3, which are in compression, have partner struts to which they connect at the v-joints at the top. These partners carry tensile forces of a similar magnitude. Finally, Fig. 4.11 further corroborates our observations with the cable force in the second bay increasing greatly with increased n, actually becoming equal to or greater than the force in the first bay when n = 7.

A full explanation for this behavior will not yet be
ventured, with the hope that further investigations will assist in its clarification. These studies of Case 2 questions whether the measurement of curvature is not the control variable of interest as it appeared for Case 1; perhaps the strut angle of inclination should be the variable under scrutiny.

The final consequences of this behavior are that the load bearing characteristics are determined by the geometry of the structure. The arrangement is such that a single load is transferred out to many supports much more evenly than in a traditional structure. Furthermore, as can be seen by the force diagram in Fig. 4.8, the force in the support directly below the load does not receive the maximum downward reaction (reflected by the small axial forces in the connecting struts at this support) as do traditional structures. Surprisingly, the reaction at this location might even be an uplift load! This would suggest that the structure is self-righting or stabilizing under uneven loads.

4.1.6 case 3
The results of this case study proved the expectations that increasing the angle of strut inclination closer to vertical allows a more direct path for the vertical loads, hence increasing the strut load, C, and decreasing the cable force, T, such that the ratio is greater than 1.0 for the most closed form. These results predict that the force required for expansion decreases as the hyperboloid expands. The angle of strut inclination approaches the horizontal plane as the hyperboloid expands, therefore increasing the lever arm of the structure.

4.1.7 conclusions
This analysis has initiated an understanding of the structural behavior of lattice hyperboloid under vertical loading. The results were not always intuitive and have hence left some unanswered questions. The two variables, the number of nodes, n, and the number of points, p, were used to describe a selection of lattice block units. An effect on the load carrying capacity was seen by these variables. Increasing n typically increased the dispersive capability of the form
and increased the cable forces. A change in $p$ had less obvious effects. However, it too, unexpectedly caused an increase in the form's dispersive capabilities, though only slightly. It is also worth noting that an increase in the number of nodes decreases the buckling length of a given strut.

To further explain the behavior, the effects of the angle of inclination and curvature as variables were questioned and as yet are still not fully understood. The result of these studies, therefore, is that rather than answering all questions, many more have been raised. Future explorations are anticipated to address some of these issues through considering more models from the matrix of the previous chapter, and examining other features such as different support conditions, cable locations, stacking of the units, asymmetrical structures and lateral loading.

4.2 kinetic analysis

The reliability of kinetic systems depends largely on the method of joinery. Before design began, the structures 8p3n and 10p5n were modeled in Pro Engineer to better understand the specific location and motion of each joint [Fig. 4.14]. The joints were modeled with 5 degrees of freedom (DOF). A sleeve was allowed to translate and rotate along the strut (2 DOF), and then pinned (1 DOF). The analysis showed that there is minimal
translation during expansion. Out of plane rotation during expansion from bundle to final position is limited to 10°, measured from an axial view of the strut [Fig. 6.10b].

4.2.1 joints
The frame has three joint types, classified according to their degree of freedom and number of connecting struts. Type 1 requires three DOF and is located at the extremities of the structure, referred to as a v-joint in chapter 3. Types 2 and 3 are located on the interior surface of the hyperboloid, referred to as nodes in chapter 3. The motion of the x-joints vary in complexity, from a simple pin (1DOF) to joints with five degrees of freedom. The center x-joint, of a symmetrical structure, can be constructed from a simple pin (type 3). The x-joints near the extremities require out of plane rotation for deployment, therefore the closer the x-joint is to the extremity, the more freedom it requires.

The degree of curvature of the hyperbolic surface has a substantial influence on the degrees of rotation and translation required. A structure with a high degree of surface curvature will have a high degree of complex motion. Conversely, a hyperboloid that approximates a cylinder requires less complex motion during expansion. Structures of this type have a large interior volume making them useful for the current applications.

The graph in Figure 4.15a illustrates the relationship between the height of the shelter frame, taken as a percentage of the strut length, and the degree of expansion of the structure, defined as the radius of the top polygon and the outer point of the frame. Figure 4.15b shows the relationship between the axis angle, understood as the actual out of plane rotation required for the structure to expand, and the degree of expansion. The graph shows the structure expanding to approximately 80% with relatively little out of plane rotation. The solid line describes type 1 joints and the dashed line indicates type 2 joints. The graph shows them following each other quite closely, indicating that they have similar degrees of out of plane rotation as
Figure 4.14 Kinetic analysis of the motion in each joint of the structure 8p3n.
Figure 6.12a: AXIS ROTATION VS. PERCENT OPEN

Figure 4.15b: AXIS ROTATION VS. PERCENT OPEN

Figure 4.15c: AXIS ROTATION VS. PERCENT HEIGHT

Joint type 1 - solid line
Joint type 2 (upper) - dashed line

Figure 4.15c
Figure 4.16 Graph showing the translation of the joints during a complete expansion-contraction cycle. Measurements are mm e-3.

Figure 4.17 Graph illustrating the in-plane rotation of each joint type during a complete expansion-contraction cycle.
Figure 4.18 Chart illustrating the correspondence of height, interior diameter, and strut inclination for the kinetic hyperboloid 10p5n.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>ID</th>
<th>NUM. POINTS</th>
<th>STRUCTURE HEIGHT</th>
<th>STRUT ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID5</td>
<td>5</td>
<td>10</td>
<td>98.8</td>
<td>8.9°</td>
</tr>
<tr>
<td>ID10</td>
<td>10</td>
<td>10</td>
<td>95.1</td>
<td>17.9°</td>
</tr>
<tr>
<td>ID15</td>
<td>15</td>
<td>10</td>
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<td>27.5°</td>
</tr>
<tr>
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<td>20</td>
<td>10</td>
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</tr>
<tr>
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</tr>
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</tr>
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</tr>
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<td>10</td>
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</tr>
<tr>
<td>ID50</td>
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<td>10</td>
<td>95.1</td>
<td>17.9°</td>
</tr>
<tr>
<td>ID55</td>
<td>55</td>
<td>10</td>
<td>94.1</td>
<td>19.8°</td>
</tr>
</tbody>
</table>

Figure 4.19 A portion of the expansion sequence for the kinetic hyperboloid 10p5n.

10p5n
the structure expands. Figure 4.15c combines the information of the two previous graphs, plotting the axis angle (out of plane rotation) to the height of the structure. The solid line describes type 1 joints and the dashed line indicates type 2 joints. Translation of the joint is negligible as shown in Figure 6.13.

10p5n
The analysis of the structure 10p5n more closely models reality than the 8p3n analysis. In the previous model the struts were assumed to pass through each other. The 10p5n model is composed of overlapping struts, approximating the method of construction used to make the majority of the physical models in chapter 5 and the proposed design surgical retractor and emergency shelter.

Figure 4.18 shows the ratio of the inner diameter of the hyperboloid to the height of the structure as it expands, the angle of inclination (referred to as strut angle in the chart) is also correlated. The graph in figure 4.20 compares the translation of the nodes (located between the center node and the v - joint) for the structures 8p3n and 10p5n. Translation was not an issue in the previous analysis (8p3n) with coincident struts. The graph shows a flat area after a steep curve. The flat area describes a large range of the hyperboloids' motion with negligible joint translation along the strut. This range of motion can therefore be accommodated with relatively simple joint design.
Figure 5.1 The puffer fish is able to increase its volume in order to appear unappetizing.
The early stages of this study depended on physical models for the exploration of structural morphology and the individual structures' response to loading. Although the models have undergone incremental development toward various ends, the means of modeling were chosen to facilitate quick and easy construction. This chapter is devoted to describing the models, and their method of construction, made during this investigation.

The materials of the models changed to simulate the needs of a particular application. The models were frequently handled and therefore required struts that were strong and resilient and joints that could undergo repeated cycling. The medical applications require biocompatible materials, such as silicone and stainless steel.

The motion of the joints ranges from a simple pin, with only one degree of freedom, to complex joints requiring out of plane rotation. The center joint (in a structure with an odd number of nodes) is limited to that of a simple pin. All other joints require out of plane rotation. The degree of out of plane rotation, discussed in chapter 4, increases as the surface curvature of the hyperbola increases. Conversely, the joints of a structure that approximates a cylinder approximate simple pins.

5.1 model types
In order to accommodate the range of motion that the joints required as the exploration continued, a variety of model types were constructed. The rotation of the joint was provided through an exploration of elastomeric and mechanical means.

5.1.1. wood and rubber
Initial models utilized joints small bands, one quarter of an inch in diameter, cut from a length of surgical tubing [Fig. 5.2]. The tubing offers resistance when it is twisted from its resting state by the wooden dowels during expansion. Surgical tubing is available in a wide variety of diameters and wall thickness. The degree of resistance can be controlled by the degree of elasticity of the rubber joint. The
structure became a stronger 'spring' as the wall thickness of the tubing increased. This variable provided an easily observed visual response to the models' behavior under load.

5.1.2 pvc, bunji cord and shrink wrap
A large scale model was constructed from 2.5 inch pvc tubing, 18 feet in length [Fig. 5.3]. The structure was stabilized with tensile rings at the top and bottom polygons and cables (tensors) connecting the upper and lower points. Shrink wrap was applied around the tensors to form an envelope.

5.1.3 stainless steel, silicone and nylon
Stainless steel and silicone were chosen because they are biocompatible, and are the materials that might be used in an actual medical device [Fig. 5.4]. The end joints are made from nylon cord. This joint is extremely durable and able to undergo repeated cycling. The force exerted by the nylon causes the joint to open and as a result the structure has a resting state that approaches full volume. The nylon cord is strung through the stainless tubing and in some ge-
ometries the cord completes a continuous circuit.

5.1.4 stainless, silicone and nitinol
Small scale models were made with nitinol v-joints. .013 nitinol wire was inserted into .015 ID tubing and crimped. The structure shown has nine points and is able to fold to fit in a 5 mm tube. This particular design was intended to create an operable space during a minimally invasive surgical procedure. The device would be inserted through a 5mm tube (trocar), and expand to 5cm (ID at the waist) once in place. The device would locally move organs aside creating an operative space in which the surgeon can work. At the end of the surgical procedure it would be retracted into a 5 mm bundle and withdrawn though the trocar. The five pointed structure below [Fig. 5.6c] is designed to fit into a 3mm tube.

5.1.5 stainless and pins
The intention of this model [Fig. 5.5] was to replace the silicone bands with pins. The pins are made to snap into predrilled holes in the tubular strut. The pins fit loosely to allow freedom of movement beyond a
5.1.6 brass and nitinol
The intention of this experiment was to eliminate overlapping of the struts at the x-joints, making a structure with smooth walls and virtually no relief. The struts are 1/16 brass rod and are of two different lengths. The left-handed spirals are continuous, the right-handed spirals only span the distance between the joints of the lefts. The ends of the short struts are drilled 1/4 inch to accept a .013 inch Nitinol wire. The wire is affixed to the smaller strut, passes though a hole in the continuous strut, then is affixed to another smaller strut [Fig. 5.8 and 5.9]. This method of assembly eliminates the relief of the joint allows unimpeded delivery through tubular instruments. It also more approaches a purely mathematical model of the hyperboloid, making analysis less complicated, and increases the correspondence of the real and the virtual.

5.1.7 flat bars and rivets
Riveting offers a rapid method of assembly with low relief joints. Constructing the joints with rivets however, limits their motion to a simple pin. All out of plane rotation required for expansion and contraction is resolved by flexure of the bars. Faces of the bars meet at every nodal point and are held tightly by a rivet. The bars do not remain straight, as in previous models, but are curved.

The joints are located at quarter points of the strut—each joint is equidistant from one another, forming a cylinder [Fig. 5.10b]. In Figure 5.10d the distance to the end joints is closer than the mid joints. This difference in length results in an increased curvature and a hyperbolic profile.

The riveted assemblies have the same range of motion as those constructed of straight sticks. The degree of ‘springiness’ for a particular structure is dependent on the elasticity and cross section of the bars. Assemblies of thin gauge aluminum tend to hold their shape at any position while a structure made of spring
Steel would always tend toward a compact bundle. This is the equilibrium state of the structure as it is the state in which the bars are least curved. The degree of force required for expansion increases as the structure moves from its rest state.

5.1.8 Wound Nitinol
The intent of this model was to make a kinetic structure from a single piece of .013 inch Nitinol wire [Fig. 5.11]. The wire was wound around a brass mandrel then the shape was programmed into the Nitinol by heating at 1300°F for ten minutes then cooling rapidly. A geometry was chosen that could be made from a continuous circuit and the tail of the wire was crimped to its head. The pins in the mandrel were made to be removable so as to be able to release the Nitinol lattice. Once off the mandrel the structure could be physically deformed, and when heated, it returned to its preset shape. Structures produced on this mandrel allowed shape memory lattice hyperboloids to be constructed with expanded diameters of 1.25 cm.

5.2 Actuation
Many methods of actuating the lattice are possible. The lattice is can be viewed as an assembly of rhombi (the interstitial spaces between the struts). The vertices of all of these rhombi are pin jointed and are therefore mobile. The structure may be made kinetic by applying force in such a manner so as to change the shape of the rhombus. When force is applied locally to one rhombus, the rest of the structure moves in concert. The more joints that are actuated, the higher the force of deployment. A graduated actuator can offer precise control of radial expansion and contraction.

5.2.1 Nitinol
Recognizing that an open or closed ‘rest state’ (as well as general kinetic control) could be beneficial for specific applications, research in remotely controlling the volume of the structure began. The shape memory alloy Nitinol is available in a variety of profiles including rods similar in diameter to the nylon cord with which previous modes had been made (Nitinol is currently considered biocompatible).
Nitinol is an alloy of nickel and titanium (about 50/50) which undergoes a “phase transformation in its crystalline structure when cooled from the stronger, high temperature form (Ausinite) to the more malleable, low temperature form (Martensite).” [Ref. 5.1] This inherent phase transformation is the basis for the unique properties of these alloys- in particular Shape Memory and Superelasticity. The Nitinol wire works is a strong and flexible joint able to undergo repeated cycling with minimal fatigue when used in its superelastic phase.

Another model was constructed with the nitinol wire ordered with a preprogrammed ‘hot’ shape of 60° set to trigger at 50°C. The material was malleable at temperatures below 50°C, and sprung to the programmed shape at the target temperature. The wire was crimped into the tube ends, making the v-joints of the structure [Fig 5.12]. Heat, supplied via water or an electrical charge, triggered the structure to full expansion. An eight pointed structure with sixteen v-joints was modeled. Each joint additively increased the force of expansion.

Figure 5.13 shows a model with a nitinol spring attached to an upper and lower point of one of the kinetic hyperboloids. Low voltage from a battery heats the spring causing it to contract, decreasing the distance of upper and lower points, expanding the assembly. The entire structure expands from the actuation of two points. Multiple springs could be applied to the structure increasing the force of expansion and resistance to lateral loading.

Figure 5.13 a b Kinetic hyperboloid actuated via a shape memory piston.
5.2.2 hydraulic
The hydraulic model was constructed of stainless steel tubing with silicone joints [Fig. 5.14]. Flexible plastic tubing was fed through the hollow struts in a continuous circuit. The end of the tubing was connected to a syringe and the entire conduit filled with water. It was anticipated that an increase in pressure, as a result of pressing the syringe, would cause the tubing at the v-joints to straighten. This action is similar to the promiscus of the butterfly. The structure responded only minimally. It is speculated that the wall thickness of the inner tubing was too large compared to the pressure applied.

5.2.3 pneumatic
The pneumatic model [Fig. 5.15] expands as a surrounding bladder is pressurized. The bladder is attached to the upper and lower extremities of the structure. As the bladder inflates, the distance between the points decreases, causing expansion. Inflating the bladder expanded the structure and then began to exert lateral pressure, thereby resisting further expansion. A bladder could be designed to control the
final height and shape of the structure. If expansion beyond full volume is required, then the inner wall of the bladder could be made of a material with a lower elastic modulus than the outer layer. The inner membrane would expand with less curvature than the outer allowing free expansion of the frame.

Figure 5.16 is a fluid driven tensile actuator constructed of inextensible cables anchored to end rings with an inner elastic membrane. The tensile strands define the ruled surface of a hyperboloid when deflated. When pressurized the actuator assumes a spherical form, shortening the overall length of the mechanism. This device could actuate the structure in controllable increments in proportion to the pressure applied.

Figure 5.16 is proposed to actuate pneumatically via the mechanism illustrated in Figure 5.17. The actuators are intended to be placed along cables connecting the outer points of the hyperboloid, offering precise height and volume control in response to pressurization.
There are many applications in architecture, industry, and medicine that call for expandable or retractable structures. The two applications investigated in this thesis apply to the fields of medicine and construction: a surgical retractor and an emergency shelter. The mechanical parameters of these applications are surprisingly similar. In each case, specific morphologies of the structure could be designed to meet requirements of size, shape, and loading capacity.

The structure developed can be accurately described within Bulson's categorization of rapidly assembled structures. He classifies hinged structures as assemblies where "the elements are permanently connected to each other by pins or joints that allow articulation. Structural integrity after unfolding is achieved by the cunning use of linkages or sliding connections. Simple examples are the umbrella, the deck chair, or the foldable music stand. The unfolding can be operated manually for small scale structures or hydraulically for large scale structures. In all cases the joint design is a key feature." [Ref. 6.1]

6.1 medical device
In the medical field, an application that requires a strong, resilient, and expandable structure is tissue retracting. An open tissue retractor is used in open surgeries to separate the walls of an incision in order to allow the surgeon space to operate. Standard retractors are similar to hooks and require one or two assistants to spread the wound. Self locking retractors [Fig. 6.1] exist, however the long handles of the device can impede access to the wound site. These types of retractors exert linear force to open the wound causing trauma to the ends of the incision. The end of any 'crack' is a point of stress concentration whether the material is skin or steel.

The hyperbolic retractor in its compact state would be inserted into an incision and expanded. The device would be remotely actuated and self locking allowing the surgeon unobstructed access to the incision. Due to the circular cross section, the device would impart load evenly to the walls of the incision reducing trauma at the incision ends. Cur-
Currently, designs are being developed in which the ends of the structure are based on ovals. This arrangement yields a structure that has an elliptical cross section rather than circular (when taken horizontal to the main axis). An elliptical section will more closely approximate an expanded incision and therefore more equally distribute forces. The nodes of the structure can be arranged to make a retractor that creates a small proximal aperture with a larger distal aperture. Due to the foldable nature of the system it may be inserted through a small opening [Fig. 6.2], and can reduce the overall size of incision necessary. The lattice pattern of the device may also be varied generating 'side ports' allowing the surgeon to gain access to the field from the sides of the structure as well as from the ends.

A number of the construction techniques investigated in the preceding chapter could be employed to build working prototypes and eventually a manufacturable device. A possibility that lends itself to easy manufacture is a one-piece molding process. The material would be a biocompatible polyethylene and cast around a hyperbolic mandrel.

Another class of medical devices that requires an expandable structure is endoluminal prostheses.

Figure 6.3 Model of the prototype retractor with latex sheath. The sheath helps to evenly disperse the force the retractor exerts on the skin and reduces the possibility of pinching from the scissoring of the joints.
such as catheters, dilators or stents, which serve as scaffolding for a vessel or organ. Common devices are made of braided or woven filaments, which are extended in the axial direction in a collapsed state and shortened in the axial direction in the expanded state [Fig. 6.4]. A disadvantage of these structures is that they are limited in their range of strength and expansion. They undergo substantial axial shortening in order to obtain their expanded state, as seen in the angle of inclination of the fibers in Figure 6.4. A benefit of the proposed device is that it could be locked incrementally at various stages of expansion allowing precise control of dilation. There seems to be extensive opportunity in the medical instrument field for kinetic devices that offer compactability and mechanical strength combined with precise control.

**Figure 6.4** Design for a stent comprised from fibers which are woven into a tubular and expanding network. The stent shortens axially during expansion. The fibers are of rectangular cross section and attached at their ends.

Patent # 5,503,636
6.2 emergency shelter
Portable architecture has a long history ranging from nomadic housing to prefabricated housing transported to a site [Fig. 6.7] to the temporary encampments of the circus. War, genocide, and natural disasters displace people from their homes and as a result temporary shelters are necessary. The properties of the kinetic hyperboloid make it an excellent candidate for an emergency shelter. It is compact in its folded state and expands into a resilient framework that encloses a large volume. The components are modular and are suited for low cost mass production. The design facilitates rapid assembly without tools producing an easily deployable autonomous emergency shelter.

The vast majority of shelters available are time consuming to construct and often require training. The shelter in Fig. 6.5 is a multi part system. The roof unfolds then legs and side panels are added. The Deployable Rapid Assembly Surgical Hospital (DRASH) [Figs. 6.6 and 6.7] arrives in a compact bundle and is quickly deployed. A single unit covers 1100 square feet and multiple units can be connected together to form larger floor areas. The structures are lightweight and require no site preparation. Floors (ground level) are added to the enclosure usually in the form of a thick pad. This type of flooring adds substantially to the weight and volume of the assembly. The system consists of an expandable frame, two (inner and outer) pre-attached canopies, and a ground cover. Twelve inches of air is sealed between the fabric layers to help maintain a constant interior.
temperature.

The system proposed in this thesis is entirely self contained, has a raised floor and weather proof skin. It can be brought to the site and deployed easily with a simple unidirectional force and without the aid of tools or additional parts. The structure opens in a smooth and synchronized manner when actuated by a downward pull from a top joint. Joints may be designed to assure stability throughout the deployment sequence. The total weight of the shelter is estimated to be 40 pounds (dependent on the type of fabric used) and could be easily deployed by one person.

Figure 6.7 Prefabricated housing unit in transit.
Figure 6.9  Drawing illustrating joint types and locations. Locations are correlated on the model to the right. Images of each joint type are shown on page 87. The notation $Q$ demarks the floor level of the shelter.
6.3 design

Type 1  86  
Floor hub  89
Skin  91

Type 2+3  87  

Type 4  88  
Suspended strut  91
Plans  92
Packaging  94
6.3.1 upper v joint - type 1
The joint requires an in-plane range of motion from $0^\circ - 60^\circ$ [Fig. 6.10] and an out of plane rotation of $10^\circ$ in order for the frame to expand to its final volume. The final height is $70\%$ of the original strut length. The joint is intended to be cast from nylon with an interior angle larger than $60^\circ$ (its angle at full deployment) [Fig. 6.21]. The memory of the nylon would serve as a spring, minimizing the input force required to expand the frame. It is proposed that it could be designed to be self expanding and lock at a desired angle. The joint can be fitted snugly within the tube without the use of a tool. The cylindrical rings plastically deform to make a friction connection.

6.3.2 snap fit spline - type 2 and 3
Figures 6.12 to 6.14 illustrate a snap fit joint that can be easily molded or machined. The slots allow the material to be deformed in order to plunge through the hole in the tube (strut). The material expands once inside, locking the joint in place. The beveled walls of the center disk and the tolerance between the inner shaft and the hole (in the tube) allow for out of plane rotation. This joint can be inserted into the tube by hand without a tool, making onsite repair feasible. This design suffices for joint types 1 and 2. It is intended that the spline would be injection molded nylon.
Figure 6.12 Autocad drawing of the snap fit joint.

Figure 6.13 Joint printed in nylon

Figure 6.14 Drawing of the joint fitted within the struts. Struts are shown in cross section.
6.3.3 lower v joint - type 4

This connection [Figs. 6.15 and 6.16] accepts two main frame members and one floor strut. The floor strut is attached to a sliding track allowing the floor to be levelled independently from the frame. The strut end is pinned to a hinged plate providing the degree of freedom necessary for expansion of the frame.

Figure 6.15 Elevation of the lower v-joint and floor frame assembly.

Figure 6.16 Section (AA) through the hinged plate and track.
Figure 6.18 Plan of the floor frame as it inserts into the track. The floor frame can slide vertically in the track allowing independent leveling of the floor.

Figure 6.17 Initial sketch of the floor assembly with insulated panels.

Figure 6.19 Floor frame to lower v-joint. The floor frame connects to a vertically sliding track allowing the frame to be leveled independently of the structure.
6.3.4 frame
The frame is anticipated to be constructed from fiber reinforced composite tubes. The same material, length, and cross section used in the analysis in Chapter 4.

6.3.5 floor
The floor assembly [Figs. 6.17 - 19] consists of a simply pinned frame that is contained within the bundle and unfolds into place as the structure is deployed [Figs. 6.20]. The frame unfolds into a plane with struts radiating from the center to each point of the structure [Figs. 6.21]. A leg supports the frame in the center, reducing the span of each strut to 1/2 the diameter of the floor polygon. The floor struts are pinned to a central hub [Fig. 6.22].

The frame provides a compressive and tensile ring, assuring stability when the shelter is deployed. The joints of the floor assembly could be designed to add resistance such that the structure would be substantially stable at all stages of unfolding. A strong fabric
could be fitted to the frame – stretching tightly as the structure expands, making a floor similar to the surface of a trampoline. A double layer of fabric could be stretched in the same manner, then inflated once in place. The fabric would provide a vapor barrier, and if inflated, insulation.

6.36 roof strut
This connection [Fig 6.27 and 6.28] is designed to accept cables or straps from the outermost top points of the shelter. The cap is designed to increase the surface area of the strut end, preventing the membrane from puncturing. The straps support the strut, giving pitch to the membrane for drainage. The material is compliant, friction-fitting into the tube.

6.3.6 skin
The skin could be constructed out of a lightweight waterproof PVC fabric and arrive on site preattached to the expandable frame. The fabric to anchor point (footing pad) would be attached within the bundle, and could be tightened quickly and easily. The fabric itself would provide the primary tensile triangulation,
Figure 6.30 Plan and elevation of the shelter.
stabilizing the structure.

A two-piece fabric system [Fig. 6.31] would allow for an extremely simple fabrication process. The two shapes necessary would be a simple circle and a rectangle. A fabric with a slight degree of elasticity would be desirable to form to the double curvature of the hyperbolic frame. In cold climates a layered or insulating fabric could be used.

6.3.7 stakes
Footing pads could be drilled to accept stakes to avoid toppling or uplifting of the structure. The aerodynamic form of the hyperboloid will help to minimize wind loads and help facilitate natural cooling of the shelter. Guy wires could be used to tether the structure to the earth in extreme conditions.

6.3.8 leveling
The floor frame of the expanded shelter could be leveled via an adjustable track and pin system incorporated into the footing, allowing the shelter to adapt to uneven terrain.

6.3.9 sizes
The shelter presented in this chapter is made of fiberglass struts that are twelve feet in length skinned with a lightweight pvc fabric. The geometry of the lattice hyperboloid lends itself to many variations as seen in chapter 3. Accordingly the volume and surface area of a shelter can be varied to make a flexible system suited to the needs of diverse situations.

6.3.10 emergency system
The folded shelter 8p3n [Fig. 6.32 and 6.33] fits

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Figure 6.31 Simple cutting pattern using fabric with a small degree of elasticity.
within a sixteen inch diameter tube. Footing plates are stacked at the end of the tube, providing ballast if dropped with a parachute. The proposed system would pack into a standard shipping container (8' x 8' x 20') which would contain the shelters and support facilities, including food, clothing, generators, medical supplies and telecommunication systems. The container could be transported over land by truck or dropped from the air. The time of assistance could be drastically reduced if the stocked containers were on standing reserve near areas prone to natural disaster. The container, once emptied of the emergency structures, could serve as a medical and communications base.
This thesis has investigated the kinetic and morphological potential of the hyperbolic lattice. The construction of the lattice with flexible joints has revealed that the structure is capable of a wide range of motion. The insertion of the form into matrices has brought into focus the numerous morphological possibilities the hyperboloid, expanding the vocabulary of kinetic lattices.

Physical models were constructed in order gain insight into the relationship of form and motion. Algorithms were developed to computationally derive the hyperboloids in an effort to augment the design process. The program evolved to include a kinetic analysis which was able to track specific joint rotation of the hyperboloid throughout its range of motion. The digital models and computational analyses continually intertwined with the physical models. The digital often revealed what was hard to observe in the physical.

The structural analyses has brought to light properties that are unique to hyperbolic lattice and supported the initial assumption that kinetic hyperbolic lattice is structurally sound and can be stabilized throughout its range of expansion. The kinetic analyses illustrated that substantial expansion ratios can be attained with relatively simple joints and that the structure requires complex joints to expand and contract through its full range of motion. These analyses have initiated an understanding of the structural and kinetic behavior of lattice hyperboloid and have instructed the design of the surgical retractor and the emergency shelter. The kinetic and structural requirements proved to be similar relative to the large difference in scale.

The result of these studies, rather than answer all questions asked, have revealed a wellspring of possibilities. Future explorations are anticipated to address the potential of the kinetic hyperbolic lattice. The matrix used in this thesis had a limited number of input arguments, namely points and nodes. Joint location, various combinations and permutations could be added to the input to produce asymmetries, linear assemblies, tilings, and packing arrange-
ments. These patterning possibilities have not been fully explored and warrant further investigation. Hydraulic, pneumatic, and shape memory alloy methods of actuation have been touched upon but require continued experimentation to develop finely controlled and articulate kinetic systems.

The methodology applied in this thesis to the hyperboloid could be applied to the hyperbolic paraboloid. This form shares the distinction of being a surface of double curvature that is able to be generated from straight lines. It seems reasonable that if the hyperbolic paraboloid were assembled with flexible joints that it would exhibit kinetic mechanics similar to the hyperboloid.

There are numerous application of kinetic structures in the fields of architecture, industry, and medicine. This thesis has been an attempt to increase the vocabulary of kinetic forms and to identify specific applications that utilize the unique properties of the hyperbolic lattice.
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