Use of low-density, reticulated, elastomeric foam impregnated with Newtonian and non-Newtonian fluids to design an impact absorption material

by

Yves Matton


Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

Master of Science in Mechanical Engineering

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Abstract

The development of new threats in recent conflicts, such as improvised explosive devices (IEDs), requires the development of improved protection for US soldiers. The development of improved materials for helmets, in particular, is motivated by the social and economic costs of head injury. A versatile liner, adaptable to different types of helmets, with different constraints, would be useful.

In this thesis, we first review the statistics related to head injuries from motor vehicle and recreational accidents and then describe the state of the art of current helmet design. An experimental study of the response of a widely used helmet liner material (polystyrene foam) and a new potential liner material (low-density, reticulated, elastomeric foam impregnated with Newtonian and non-Newtonian fluids) under impact shows some complementarities and leads to the concept of a composite material that would take advantage of the properties of the two materials.

To conduct an extensive design analysis, comprehensive models are developed to model the behavior of each material under a wide range of impact energies. A complete model for the composite bilayer of the two materials is then compared to experimental data; the model gives a good description of the data.

Using these results, three case studies are developed for a motorcycle helmet, a football helmet and a military helmet. The three case studies show a variety of constraints in term of thickness of the liner and impacting energies. Simulations are conducted using the models developed to indentify potential designs that would meet the requirement in term of peak linear acceleration (PLA) and in term of the specific constraints of each type of helmet. Fi-
nally, in an experimental study, some of the proposed designs are tested for repeated loading. The proposed designs enhance the level of protection in terms of peak linear acceleration and show promising behavior under repeated impact testing.

Thesis Supervisor: Lorna J. Gibson
Title: Matoula S. Salapatas Professor of Materials Science and Engineering
Acknowledgments

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Viscosity of the fluid impregnated the polyurethane foam = 1.1 Pa.s

Viscosity of the fluid impregnated the polyurethane foam = 0.5 Pa.s

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Acceleration versus displacement - Experimental results for Design 2: Polystyrene 9.5mm FFF 9.5mm, Impact Energy of 70J

Acceleration versus displacement - Experimental results for Design 3: Polystyrene 6mm FFF 13mm, Impact Energy of 70J
Chapter 1

Introduction

Each year, more than 2 million Americans suffer a head injury and approximately 0.5 million are hospitalized [2]. Estimates of deaths attributable to head injury range from 40,000 to 100,000 per year. The US Department of Health and Human Services estimates for Traumatic Brain Injury in the US for the year 2010 are shown in Figure 1-1 with an estimated cost of over $60 billion dollars.

Figure 1-1: Estimated number of Traumatic Brain Injuries in 2010 in the US [3]
Around 80,000 people are left with a disability after a head injury accident [2]. Evaluation of the total cost of head injuries for 1985 is $37.8 billion: $4.5 billion for direct costs, $20.6 billion for mortality cost and $12.7 billion for morbidity cost [4]. Head injuries accounted for 29% of total injury cost and 25% of the injury linked death rate. However, head injury only represented 13% of the injury incidence rate [4].

In 2006, more recent work by Finkelstein et al. [5] to tried to summarize data on injuries and concluded that in 2000, the U.S. health care system charges for medical lifetime costs added up to $92.4 billion per year. Head and neck injuries accounted for 26% of this bill. The lifetime costs for fatal injuries added up to $143 billion per year, head and neck injuries accounting for 30% of this total. Total costs for head and neck injury, including loss of productivity added up to $95.4 billion per year accounting for 23.5% of the total lifetime costs of injuries in the US for the year 2000 [5].

Over the nonmilitary populations, the head injury rate in the US is reported to be between 21 and 231 cases per 100,000 individuals per year [2]. However this rate can vary considerably depending on geographic regions and type of populations. Head injury incidence in the US military, for different age groups is shown in Figure 1-2.
Figure 1-2: Rates of Head Injury for various age groups in the US military [2]

For instance, head injury rates are much higher in young individuals, and average injury rate is twice as large for males than for females.

In the civilian population, the three major causes of head injury are motor vehicle crashes (50%), falls (25%) and recreational or sport injuries (10-15%) [2].

The mean cost of inpatient head injury cure and rehabilitation programs was $52,000 in 1993 as reported by Dahmer et al. [6]. In a study published in 1996, Ommaya et al. report the mean cost and mean length of stay in hospitals for patients admitted with head injury diagnoses were $4,438 and 8.1 days respectively [2], as shown on Table1.1. Obviously costs of medical care have risen considerably since then but the length of stay is still valid.

More recent data from Finkelstein et al. show that for the year 2000, the number of hospitalized patients for head or neck injury (including traumatic brain injury) in the US was 299,692 and the associated medical cost was 9.04 billion of dollars [5], which represent an average cost of $30,164 for hospital costs only. Traumatic Brain Injuries represented only
<table>
<thead>
<tr>
<th>Diagnosis</th>
<th>Private Hospitals</th>
<th>Average LOS (SE)</th>
<th>Military Hospitals</th>
<th>Average LOS (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trauma and head injury</td>
<td>$5,220</td>
<td>6.8 (0.7)</td>
<td>$4,866</td>
<td>10 (0.6)</td>
</tr>
<tr>
<td>LOC &gt; 1 hour</td>
<td>$10,802</td>
<td>27 (5.1)</td>
<td>$3,912</td>
<td>16.7 (2.0)</td>
</tr>
<tr>
<td>LOC</td>
<td>$3,902</td>
<td>5 (0.4)</td>
<td>$1,092</td>
<td>4.2 (0.3)</td>
</tr>
<tr>
<td>No LOC</td>
<td>$2,521</td>
<td>6.2 (2.2)</td>
<td>$842</td>
<td>2.8 (0.2)</td>
</tr>
<tr>
<td>LOC of unspecified length</td>
<td>$2,874</td>
<td>7.5 (0.8)</td>
<td>$1,378</td>
<td>5.9 (0.3)</td>
</tr>
<tr>
<td>Nontrauma and head injury</td>
<td>$3,721</td>
<td>9.8 (2.5)</td>
<td>$1,884</td>
<td>6.7 (0.9)</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$4,438</td>
<td>8.1 (0.8)</td>
<td>$1,378</td>
<td>5.1 (0.2)</td>
</tr>
</tbody>
</table>

*a Private facilities exclude skilled nursing, psychiatric, and rehabilitation facilities. Private costs are CHAMPUS-allowed costs. Military costs are based on average cost per occupied bed day plus associated private facility costs.

LOC, loss of consciousness; LOS, length of stay; SE, standard error.

Table 1.1: Median cost and mean length of stay in private and military hospitals for patients admitted with head injury diagnoses* [2]

3% of the injury incident for the year 2000 in the US, but represented a lifetime medical costs of more than 11% of the total costs for injury incidents [5].

1.1 General motivation

Suggesting new helmet designs to reduce head related injury and improve head protection requires a good knowledge and understanding of injury mechanisms. At the same time, a review of state of the art protection gives a better understanding on how current designs try to mitigate injuries resulting from an impact or a blast wave. This section gives an overview of types of head injuries and then describes three situations of particular interest. Subsequently, a review of state of the art head protection for three applications is developed and limitations of current analysis and design are described.
1.2 Head injury

Understanding the biomechanics and classifying the types of head injury is necessary to identify the relevant characteristics and parameters to offer the best protection in helmets. Head injury may be broadly defined as temporary or permanent damage to one or more of the head components from a blow to the head such that encountered in a traffic accident. Generally speaking, head injury can be grouped into four categories such as scalp damage, skull fracture, brain injury, neck injury or a combination of these injuries. Brain Injury can be subdivided into focal and diffuse injuries [7, 8] and in an accident most often these injuries overlap to a certain extent. As reported by Shuaeib et al [9], scalp injuries are of a lesser importance than the others, and neck injuries are of low occurrence, compared with brain and skull injuries. These two types of injury will not be developed further on in this study.

1.2.1 Skull fracture

During an accident, skull fracture may be caused by a rigid object - such as road posts, tree branch, motorcycle parts, etc - penetrating the skull. Depending on the extent of helmet coverage (full face helmet, three-quarter shell helmet or half-shell helmet), the outer shell of the helmet may prevent such penetration by spreading out the force applied to the head. Furthermore, some minor skull fracture may not cause brain injury, and it could be argued that the skull breaking is one of the natural mechanisms to absorb energy. [10]. Even if skull fracture might be at first sight traumatic, this mechanism will not be considered in this study as a primary concern because typical values of loads reported to cause brain damage are much lower than those that cause skull fracture [9]. A rough calculation for a direct head impact to the temporal area for instance, gives that skull fracture would happen if the impacted area was less than $5 \, cm^2$ and the localized pressure exceeds 4 MPa (2 kN localized force) [11]. To produce a situation with such characteristics, the impacting object would have to be sharp.
### Table 1.2: Distribution of principal diagnosis for patients with head injuries admitted to a military and private hospital in fiscal year 1992 (CHAMPUS: Civilian Health and Medical Program of the Uniformed Services) [2]

<table>
<thead>
<tr>
<th>Principal Diagnosis</th>
<th>CHAMPUS Admissions (n = 1,360)</th>
<th>Military Admissions (n = 4,160)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull fracture total</td>
<td>21%</td>
<td>13%</td>
</tr>
<tr>
<td>Intracranial injury total</td>
<td>46%</td>
<td>51%</td>
</tr>
<tr>
<td>Concussion total</td>
<td>16%</td>
<td>21%</td>
</tr>
<tr>
<td>Trauma and head injury</td>
<td>8%</td>
<td>7%</td>
</tr>
<tr>
<td>Nontrauma and head injury</td>
<td>9%</td>
<td>8%</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>100%</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

enough to penetrate the helmet entirely. The current standard test associated with such a situation uses a conical steel indenter impacting with an energy of 90J. (12]. However, it is reported in the literature (11], that the proportion of accidents with such localized dangers is extremely low and that standard tests should be changed to use only flat or hemispherical strikers. This is also confirmed by Ommaya et al., as shown in Table 1.2: skull fractures represent only 20% of head injuries [2].

### 1.2.2 Brain injury

The forces acting on the brain during injury produce complex movements and deformation. When a blunt object strikes the freely mobile head, an acceleration injury occurs. If the moving head suddenly strikes a blunt object, a deceleration injury occurs. Injuries to the brain have in recent times been referred to as Traumatic Brain Injury, simply abbreviated as TBI, thus distinguishing them from general head injury. TBI could be defined as any damage affecting the brain function and resulting from non-penetrating mechanical head loading of the contact and non-contact type [9]. Traumatic Brain Injuries are characterized by lesions to both white and gray brain matter and subsequent evolution of secondary pathogenic events.
Traumatic brain injury is a nondegenerative, non-congenital injury to the head arising from blunt or penetrating trauma of from acceleration/deceleration forces [14]. A Traumatic Brain Injury (TBI) is a form of head injury for which the patient suffers a decreased level of consciousness, amnesia, a skull fracture, objective neurological or neuropsychological abnormality, diagnosed intracranial lesion, or when death occurs as a consequence of head injury [14]. Consequences of TBI ranges from death to physical disabilities or long term cognitive, behavioral and social deficits. In 2008 the direct and indirect cost of TBI is estimated at $56 billion a year as Galarneau et al. report [14].

Non-contact TBI-producing mechanical loading is generally an acceleration of the head transmitted through the neck as a result of overall body motion. It can also be induced by a blast: as this is a major topic for improvement of military helmets and other protective equipment, this point will be developed later on. This type of acceleration may be difficult to mitigate in some situations, such as motorcycle accidents, as the motorcyclist does not move effectively as a single, rigid, free body during impact. Indeed the neck works as a joint and allows for relative movements between the head and the rest of the body as described by Newman et al. [15].

Closed head TBI resulting from non-penetrating head impacts can be categorized as diffuse or focal. Diffuse TBI refers to bulk mechanical effects associated with axonal, neural, micro-vascular and brain swelling injuries. This class of injury is usually a consequence of distributed loading conditions that generally induce relatively low energy damage affecting substantial volumes [16]. A typical situation is that encountered on helmeted-head impact where both the shell and liner work to distribute the load over an area of the head as large as possible. Focal TBI occurs in localized regions of the brain subjected to tensile or compressive stresses. R. Bullock [10] gives further details on these mechanisms of brain injuries.
The conclusion of the Schuaeib et al. study [9], which provides a comprehensive review for head injury, is that diffuse brain injury is the major concern in most helmeted head impacts. The primary aim is therefore to reduce the head’s acceleration during an impact.

It has been reported that concussion could be reproduced in the laboratory by delivering controlled blows to the freely movable head by a pendulum (acceleration concussion), but this is much more difficult if the head is fixed (compression concussion) [10]. Following a blow to the head there may be either linear (translational) or angular (rotational) movements of the skull. If the blow is directed eccentrically, the result is a combined translational and rotational acceleration type of injury. Pure translational acceleration creates intercranial pressure gradients, while pure rotational acceleration produces rotational of the skull relative to the brain and is particularly likely to tear para-sagittal bridging veins [16]. However, Mills and Gilchrist [17] concluded that the rotational acceleration is generally insufficient to cause serious diffuse brain injury. As a consequence the major parameter to be taken into account in helmet design is to reduce the linear (translational) acceleration during an impact. Since a cure is not attainable at this time, the only alternative is to develop intervention strategies to prevent or minimize these injuries [18].
1.3 Specificity of three types of people at risk

Head injuries can occur from many causes and result in various trauma. That is why it is necessary to study in depth the specificities of each of the following areas: Military Injury, Motorcycle related Injury and Sport related Injury.

1.3.1 Military injury

Military forces are exposed to very specific threats, and therefore are to be considered as a specific case for this study. Recent conflicts have demonstrated a radical change in the major source of injuries and death among US soldiers. Indeed, the increasing number of Improvised Explosive Devices (IEDs), resulting in major trauma are reported as a major threat in the recent literature [19], [20], [21].

According to the Department of Defence 'Personnel and Procurement Statistics', over 73% of all US Military casualties in operations in Iraq and Afghanistan have been caused by explosive weaponry [13]. As of March 2009, the Department of Defense reported that the Military Health System (MHS) has recorded 43,779 patients diagnosed with a Traumatic Brain Injury from 2003 to 2007. Direct cost associated to TBI patients are estimated to $100 million for care and $10.1 million for prescriptions after the TBI diagnosis. Associated with these casualties, the MHS has identified 39,365 patients diagnosed with post-traumatic stress disorder (PTSD). Cost for direct care and prescriptions add up to $76.9 million. So, finally, the total cost of brain injury adds up to more that $185 million over the 5 year period under study [22].

Progress made in protective equipment for soldiers against bullets and bombs makes the current risk of blast or shock injuries relatively more important for soldiers in the 21st cen-
tury. IEDs, rocket-propelled grenades and land mines are major threats for soldiers serving in Iraq or in Afghanistan. Galarneau et al. [14] even raise the point that brain injury could be considered as the "signature wound" of the Iraqi war, similar to Orange Agent exposure in the Vietnam war.

According to Beekley et al. [23] relating statistics of an American battalion in Afghanistan, blast and motor vehicle crash related injury account for up to 32% of traumatic injury, which is close to the proportion of gunshot wounds (34%), which was historically the highest proportion of war injuries.

Improvised Explosive Devices (IED) and land mines represent a tremendous threat to military convoys traveling in Iraq and Afghanistan. Gondusky et al. [19] investigated battle injuries sustained by a mechanized battalion operating in Iraq. Over the 32 attacks studied, 120 marines were injured, causing 188 injuries. The vast majority of the resulting wounds were affecting the head and upper extremity (70%). Figure 1-3 shows the proportion of injuries for each type of event. The main point of interest is that 97% of the injuries were caused by IEDs (62%) or land mines (35%).
Figure 1-3: Injuries by event type. Redrawn from [19]

Figure 1-4 gives a representation of localization of injuries on the same sample of soldiers. Head injury (16%) represent clearly the highest proportion of injuries after ear injuries (23%). Because of the specific threat of blast for soldiers, a comprehensive description of the blast-related injuries is given. For a more detailed description, the reader can refer to the cited papers.

Type of blast injury

Primary blast injuries occur as a direct result of the change in atmospheric pressure due to the blast wave. On the other hand, secondary blast injuries are wounds resulting from objects put in motion by the blast, which then can hit nearby people. Finally tertiary blast injuries are due to people themselves put into motion by the blast and hitting surrounding obstacles such as the ground or a wall. [20]. A quaternary type of injury is sometimes reported ([13]),
such as burnings due to the explosion itself.

Figures reported by Warden et al. [20] show that TBI is a major issue for the Iraq and Afghanistan wars. Indeed, contrary to previous conflicts, which showed a significant number of penetrating brain injuries, the wars in Iraq and Afghanistan seem to show a different pattern. The rate of closed TBI injury is reported to be 88% compared with penetrating brain injury which account for only 12% [20]. Warden et al. also report that the number of serious brain injuries is 5 times higher than the number of amputees. The explanation given is that progress made in body armors has allowed soldiers who would have died without body amour to survive the blast but that they survive with head injuries.
Bell et al. [21] also conclude that Operation Iraqi Freedom (OIF) resulted in the highest concentration of severe closed and penetration head trauma since the Vietnam conflict. The study points out the fact that Type I and Type III (primary and tertiary head injury) are the predominant mechanisms of closed-head injury.

**Basic mechanisms of explosive injuries**

Immediately after an explosion a pressure wave travels radially outward from the source. This blast wave starts with a single pulse of highly pressured air that lasts a few milliseconds. This over-pressure is the main cause of primary blast injury. The blast wave velocity in the air is extremely high and mainly depends mainly on the type and amount of explosive material used [13]. For instance Owen [24] reports that a 25 kg charge of trinitrotoluene (TNT) can lead to a 2 milliseconds overpressure of 690 kPa (100 psi), whereas a 2000 kg charge would lead to 10 milliseconds overpressure of the same magnitude. Immediately following this high pressure wave, a low pressure wave (called suction) follows. This time varying pressure is described by Phillips et al. [25] and is shown in Figure 1-5.

The time an object or a person in the path of the shock wave will undergo the pressure effects is called the duration of the blast wave. Furthermore, the expanding gases from the blast wave put the surrounding air into motion, resulting in high speed winds following the pressure wave. To give a comparison, during a hurricane, typical wind speed is about 200 km/h and exerts an over-pressure of only 1.72 kPa [13], on the other hand a lethal blast-induced by a 25 kg TNT charge exerts an over-pressure of 690 kPa travelling at about 2,414 km/h [24]. Figure 1-6 illustrates the sequences of injury mechanisms linked to a blast.

Cernak et al.[13] report that the magnitude of damage from the blast wave depends on five factors: (1) the amplitude of the peak of the initial positive-pressure wave (an overpressure of 690 to 1,724 kPa, for example, 100 to 250 psi, is considered potentially lethal) (Champion
Figure 1-5: Illustration of the ideal pressure-time history of an air blast in an undisturbed, free-field environment (a Friedlander waveform). The impulse is the integral of pressure over time [25]).

et al [26]); (2) the duration of overpressure (3) the medium of explosion (water or air) (4) the distance from the incident blast wave and (5) the degree of focusing because of a confined area or walls. Indeed blast wave reflexions result in the fact that explosions near hard solid surfaces become amplified two to nine times because of shockwave reflection.

Some more complicated phenomena can occur. For instance, in confined area reflected blast waves can interact one with another and generate a complex wave. Cernak et al. [13] report that the mechanisms of blast injury suggest that the major physical components of the blast-body interaction are implosion, spalling and inertial effects. Implosion occurs when the high pressure wave compresses a gas bubble in a liquid medium, raising the pressure in the bubble to a much higher pressure than the shock pressure; once the pressure wave have
Figure 1-6: Complex injurious environment because of blast: primary blast effects, that is, effects of the blast wave itself (primary blast injury); secondary blast effects caused by particles propelled by blast-force (secondary, that is, penetrating blast) injury; and tertiary blast effects caused by acceleration and deceleration of the body and its impact with other objects (tertiary blast injury similar to 'coup–counter-coup'[13]).

passed, the bubbles can then re-expand brutally and damage the surrounding tissues [25]. Spallation is "the disruption that occurs at the boundary between two media of different densities" [13]; spallation occurs when a compression wave in a denser medium is reflected at the interface with a less dense medium. Inertial effects occur at the boundary between two medium of different densities: the lighter material will be more accelerated than the heavier one, resulting in a large stress at the interface.

Recent results [13] suggest a frequency dependence of the blast effects: high-frequency
(0.5–1.5 kHz) low-amplitude stress waves target mostly organs that contain abrupt density changes from one medium to another (for example, the air–blood interface in the lungs or the blood–parenchyma interface in the brain), and low frequency (inferior to 0.5 kHz) high-amplitude shear waves disrupt tissue by generating local motions that overcome natural tissue elasticity [27] (for example, at the contact of gray and white brain matter).

Thus, explosions may cause four major patterns of injury: (1) primary blast injury caused by the blast wave itself; (2) Secondary injury caused by the fragments of debris propelled by the explosion; (3) tertiary injury because of the acceleration of the body or part of the body by the blast wind; and (4) flash burns because of the transient, but intense, heat of the explosion.

Relevance of improving protection against head impact threat

McEntire [28] reported that paratrooper injuries for the period of 1985 to 1989 adds up to 277 paratroopers who suffered head injuries that resulted in at least one lost work day and four died as a result of their injuries. More recent studies conducted by Knapik et al. [29] report an injury rate of 6 injuries per 1000 aircraft exits, and a rate of head injury of 13.8% of the total casualties. These figures are in accordance with the study of McEntire et al. which reports that paratrooper injuries that occurred at Fort Bragg, NC, between May 1993 and December 1994 showed an overall military parachute injury rate of 8 injuries per 1000 aircraft exits, with head injuries accounting for 18.4% of the total casualties [28].

McEntire [28] reported that Airborne operations regularly expose paratroopers to head impact risks during flight (unexpected turbulence or evasive maneuvers), during aircraft exit (impact with the door frame or fuselage), descent (collision with other paratroopers once parachute is open), the parachute landing fall (average speed impact is 20 km/h), and after landing (collision in obstacles if paratrooper is dragged on the ground in high winds).
However, the vast majority of injuries occurred during the landing phase (77.8%) and 89.4% involved concussion or brain contusion [28].

McEntire notices that even relatively mild head impacts, even if they do not threaten soldiers lives directly, can cause short-term loss of abilities from dizziness, headaches, memory loss, lack of ability to concentrate etc. Given the necessity for efficiency in communication, aggressiveness, and responsiveness on the battlefield, the aforementioned symptoms can become significant and jeopardize soldier survivability and the success of the unit's mission. Therefore one can conceive that there is a critical need to protect the integrity of soldiers body and ability to think by reducing the injury rate to a minimum, thus preserving the efficiency of the military unit.

1.3.2 Motorcycle injury

Motorcycle accidents are major contributors to the number of motor vehicle fatalities. Offner et al. [30] report that in 1985 over 4,400 motorcyclists were killed, representing 10% of all motor vehicle fatalities. More recent studies by the NHTSA (National Highway Traffic Safety Administration) report an increase to 5,407 fatal crashes in 2008, accounting for 10.7% of all annual fatal crashes. At the same time 400,000 other motorcyclists were injured, requiring medical evaluation. Viano et al. shows that around 40% of serious brain injuries are due to traffic-related causes, as shown in Figure 1-7.
Costs associated with injured motorcyclists are also high. Offner et al. report that over the sample they studied in Arizona hospital in 1985, motorcyclists not wearing helmets incurred medical care expenses on average 28% higher than those wearing a helmet ($17,120 versus $13,368 [30]). Similarly Lacy [32] reported that in 2001, in Florida, helmet use reduces fatalities by 35% and that average charges for injured non-helmeted motorcyclist was on average $26,805 versus $12,736 for helmeted motorcyclist. The NHTSA reports the annual costs for Florida for the two categories of motorcyclists from 1995 to 1999 (Table 1.3).
<table>
<thead>
<tr>
<th>Year</th>
<th>Cost for Helmeted</th>
<th>Cost for Unhelmeted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>$3,316,398</td>
<td>$927,253</td>
<td>$4,243,650</td>
</tr>
<tr>
<td>1996</td>
<td>$3,307,837</td>
<td>$924,859</td>
<td>$4,232,696</td>
</tr>
<tr>
<td>1997</td>
<td>$3,283,867</td>
<td>$918,157</td>
<td>$4,202,024</td>
</tr>
<tr>
<td>1998</td>
<td>$3,641,703</td>
<td>$1,018,207</td>
<td>$4,659,909</td>
</tr>
<tr>
<td>1999</td>
<td>$3,831,749</td>
<td>$1,071,343</td>
<td>$4,903,092</td>
</tr>
<tr>
<td>1995-1999</td>
<td>$17,381,554</td>
<td>$4,859,819</td>
<td>$22,241,371</td>
</tr>
<tr>
<td>Average Annual Cost</td>
<td>$3,476,311</td>
<td>$971,964</td>
<td>$4,448,274</td>
</tr>
</tbody>
</table>

Table 1.3: Annual costs for motorcycle casualties in Florida [32]

Specificity of motorcycle Injury

Richter et al. [33] studied data from accidents in Hannover, Munich and Glasgow from July 1996 to July 1998 to investigate injury mechanisms in helmet protected motorcyclists. The first conclusion is that impact speed of the first collision was on average 55 km/h, which corresponds to 15.3 m/s. Furthermore, in an accident with axial load shift (axis of the load changes during the accident) and a helmet with a weight greater than 1.5 kg, the risk of a basal skull fracture is increased. This study concludes that helmet weight reduction is an important factor in reducing the rate of injury.

Another specificity of motorcycle injury is the fact that motorcyclists can be subject to multiple impacts in one accident, due to bouncing on the ground or on different obstacles (other vehicles, trees...)

Curnow et al. [34] report that in the early days of helmets for road users, all deaths from head injury and severe effects such as coma were attributed to lesions to the brain that are obvious at examination after death. These include so-called focal injuries which comprise contusions, lacerations and the subdural haematoma (SDH) that may follow. They occur at the site of impact when an external object which penetrates the skull or bone of a damaged skull strikes the brain. Back in 1943, Cairns and Holbourn [35] hypothesised that a helmet with a hard-shell could spread the force of a blow over a larger area, reducing the local pressure and so limiting the risk of such injury. The conclusion of their study is that it
is indeed the case. However, most of the recent studies do not show any knowledge of the mechanisms of brain injury, specify how many helmets had hard shells or relate brain lesions to skull fractures.

However, as previously reported, most injuries to the brain, including contusion, haematoma and concussion, commonly occur without damage to the skull, lesions often being reported both at the site of impact and opposite it (Richardson,[36]). Richardson also report that initially a theory of coup and contre-coup was proposed. By the 1940s, it was expressed in terms of linear acceleration: the skull undergoes a rapid acceleration as a result of a blow to the head and then strikes the loosely attached brain near that site: this is the coup injury. Then the brain moves back in the other direction to strike the skull at the opposite side: this is the contre-coup. Concussion was initially attributed to haemorrhage, but Denny-Brown and Russell showed that its cause is physical stress on neurons, which they attributed to linear acceleration. Several measures of severity of an impact and injuries have been defined after that and will be discussed later on.

As a logical consequence, helmets have been developed to absorb some of the energy of impact and reduce the rate of deceleration, thus protecting the brain. However, recent studies ([37] and others) have raised the point that linear acceleration is not the only cause of brain injury, and that angular acceleration is another important factor. Linear acceleration is more linked to the dynamic of the whole body in a motorcycle accident, and helmets have not been proved so far to be able to mitigate this phenomenon effectively. That is why focussing on reducing linear acceleration still makes sense in the idea of improving protection thanks to the use of helmets.
1.3.3 Sport related injury

General considerations

Annually around 300,000 sport-related concussions happen in the United States [38]. Indeed sports involving impact (with other players, with obstacles...), high speed or moving objects (balls, hockey puck...) are associated with a risk of head and spinal cord injury (SCI). Given the primary purpose of sport practice (leisure, recreational), catastrophic accidents are even less acceptable. However personal and social costs of severe head and neck injury can be tremendously high. For instance, 497 players have died while playing American football in the Unites States between 1945 and 2005 [39]; of these, 69% died from fatal brain injuries and 16% from SCI [40]. Similarly, a study of deaths in football in Victoria, Australia, identified nine cases of intracranial injury resulting from head impacts in the period 1968–1999 [41]. Independent studies report numerous statistics on sport-related casualties in rugby [42, 43, 44], horse racing [39], wrestling [42], hockey and soccer [45].

Catastrophic injury risks also exist in a lot of other sports such as boxing and diving, motor, and snow sports as well. McIntosh et al. [39] give a summary of head and Neck injury in several sports, as shown in Table 1.4.
<table>
<thead>
<tr>
<th>Sport</th>
<th>Injury incidence, proportion of head and neck injury, and special issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rugby</td>
<td>19.4 lost time injuries per 1000 player hours (under 15 level to international combined); head injury 14–25%, concussion 5–15%; neck injury 2.6–7.5%, risk of catastrophic injury</td>
</tr>
<tr>
<td>Ice hockey</td>
<td>96 per 1000 player hours (junior), 53 per 1000 player game hours; 4–18% at the professional level, 1995–2001 &amp; concussion 2–20%; intracranial injury and blinding</td>
</tr>
<tr>
<td>American football</td>
<td>40 per 1000 athletic exposures; 6.1% concussion &amp; 0.41 concussions per National Football League game; neck injury 5%; severe and fatal injury</td>
</tr>
<tr>
<td>Baseball</td>
<td>Facial injury 28%, head injury 11%, oral injury 5% in 5–14 year olds and emergency visits; leading cause of sports related eye injury in the USA</td>
</tr>
<tr>
<td>Youth soccer/football</td>
<td>0.6–29 injuries per 1000 player hours, depending on level of play; head injury 4–20%; fatal injuries in children caused by falling goal posts; debate over concussion and heading</td>
</tr>
<tr>
<td>Boxing</td>
<td>16% of injuries in professional boxing are concussion; risk of severe acute and long term brain injury</td>
</tr>
<tr>
<td>Cricket</td>
<td>2 injuries per 1000 hours in first class ; head injury 3–25%; orofacial and eye injuries</td>
</tr>
<tr>
<td>Professional horse racing</td>
<td>606 injuries per 1000 jockey years (1993–1996, USA); head, neck, and facial injury 19%; severe spectrum of injuries from falls and collisions</td>
</tr>
<tr>
<td>Skiing &amp; snowboarding</td>
<td>2–6 injuries per 1000 skier days; brain injury accounted for 29% of all injuries admitted to hospital, 50–88% of all fatalities, and in general 3–15% of all injuries ; head injury 7% of all injuries 19; collision with rigid objects, such as trees, may be fatal</td>
</tr>
</tbody>
</table>

Table 1.4: Head and Neck Injury in selected sport [39]
Specificity of sport related head injury

Even if of less importance in terms of number of casualties than motorcycle-related head injuries, sport-related concussion is now widely recognized as an important public health issue in the United States and worldwide [46]. Furthermore, the incidence rate of concussions in contact and collision sports continues to be relatively high in spite of rule changes and advances in protective equipment [47]. Among the various types of head injury, concussion is reported as one of the most common injuries in many collegiate sports [46]. Recent data (2002-2003 season) from the National Collegiate Athletic Association (NCAA) Injury Surveillance System reveal that concussion accounted for a significant percentage of total injuries among athletes participating in various collegiate sports such as ice hockey (12.2%), football (8%), and soccer (4.8%) [45].

In particular, football players are highly exposed to repeated head injury, sometimes within the same season. Indeed every team of the National Football League NFL plays sixteen regular games and four preseason games during each season. Zhang et al. [18] report that each year, approximately 150 professional athletes are diagnosed as having sustained an apparent or suspected TBI (during regular season NFL is composed of 32 teams of 53 players, which adds up to 1696 players). So more than 8.8 % of the players are likely to undergo a TBI each year. This is also the case for collegiate football players. Indeed Guskiewicz et al. [38] report that among the 4251 players in NCAA Football league under study, 184 players (6.3%) had a concussion, and 12 (6.5%) of these players accumulated a repeat concussion within the same season. Furthermore, it seems that there is a correlation between reported number of previous concussions and current likelihood of concussion. Guskiewicz et al. showed that players who endured of 3 or more previous concussions are 3 times more likely to be diagnosed with a concussion than players with no concussion history. Studies seem to show that there is a period following a concussion during which the player is more sensitive to impacts: among the repeated concussion case within the same season, more than 75% occurred within 7 days of the first injury [38]. Additionally, a history of repeated concussions slows down the process
of recovery: 30.0% of those with three or more previous concussions had a symptom duration of 1 week compared with 14.6% of those with a single previous concussion.

The conclusions drawn by Guskiewicz et al. [38] are that (i) previous concussions might increase the risk of future concussive injuries and that (ii) this history of concussions could induce a slower recovery of an hypothetical following concussion, (iii) after a concussion, there may be a 7- to 10-day period of time of increased sensitivity to head impacts, that can increase the probability of recurrent concussive injury. Given the frequency of football matches for a collegiate or National Football League player, reducing the risk of a first concussion seems to be of high importance.
1.4 Current state of the art for brain protection

As it has been reviewed with the previous examples, head injuries are a major concern, in various type of activities and involving tremendous social and economical costs. Use of helmets is widespread in the three areas of interest (military personnel, sports and more specifically football players, and motorcycle users). Before attempting to develop a better protection it is necessary to review the state of the art for existing helmets and the current standards.

1.4.1 Measuring head impact severity

Helmets are used to protect against blows, impacts or blast wave, so defining criteria to compare the performance of various models and designs of helmets is of primary importance. Curnow et al. [34] report that current standards for head injury protection are the Gadd Severity Index GSI and the Head Injury Criterion (also designed as HIC). These criteria are based on the Wayne State University tolerance curve (see Figure 1-8), which has been developed based on head acceleration results from animal concussion tests and cadaveric skull fractures. The peak linear acceleration (PLA) is another useful measure of an impact severity. Even if they cannot fully account for the complex motion of the brain within a deformable skull and neglect the angular acceleration of the head to injury production, they give a good basis to compare helmet performance.

The Peak Linear Acceleration is simply the maximum of the linear acceleration during the impact. It is used in many standards as a threshold which should not be exceeded during an impact.

The Head Injury Criterion (HIC) is an empirical integral criterion that evaluates the possible severity of human brain injury induced by an impact in terms of kinematic parameters of this impact [48]. Formally, this criterion is defined by equation 1.1:
\[
HIC = \max_{t_1, t_2, t_2-t_1 \leq \Delta} \left( \left[ \frac{1}{t_2-t_1} \int_{t_1}^{t_2} a(t) dt \right]^{2.5} (t_2-t_1) \right) \quad [HIC] \equiv L^{2.5} \cdot T^{-4} \tag{1.1}
\]

where \(a(t)\) is the magnitude of the acceleration of the center of mass of the brain, \(\Delta\) is a constant parameter having a dimension of time [48]. \(L\) is a unit of length and \(T\) is a unit of time. The Head Injury Criterion reflects the dependence of the severity of the injury on both the mean magnitude of the head acceleration induced by the impact pulse and the duration of this pulse. This means that one can sustain an impact with a higher acceleration if the total duration of the pulse is shorter and vice versa.

Figure 1-8: HIC Wayne University Curve
1.4.2 Military helmets

This section focuses on a few helmets currently used by the US Army. However, data on this type of equipment is difficult to obtain so all the results in this section are from the report by McEntire and Whitley [28]. The two types of helmets under study are the Advanced Combat Helmet (ACH) and the Paratrooper and Infantry Personnel Armor System for Ground Troops Helmet (PASGT helmet).

In this report the performance of each is characterized by the Peak Linear Acceleration transmitted within a standard head form and compared against the recommended threshold for mean and maximum acceleration. This study focuses on the risks linked to head impact.

Standardized tests on military helmets

The report by McEntire et al. [28] tested and evaluated different helmet configurations. These included the ACH (shown in Figure 1-9, and two PASGT helmets, the infantry and paratrooper configurations (shown in Figure 1-10).

Figure 1-9: External views of the ACH helmet [28]
Figure 1-10: Assembly illustration of the paratrooper PASGT helmet configuration with the Parachutist Impact Liner (PIL), parachutist nape pad, and nape strap. [28]
Total weight of the ACH helmet is 1.36 kg.

The test procedure was performed in accordance with the Federal Motor Vehicles Safety Standard (FMVSS) 218 (U.S. Department of Transportation), that is to say, using a hemispherical anvil impacting a head-form wearing the helmet. For more information on the standards the reader can refer to the aforementioned document.

Two impact velocities, 3 and 4.4 meters per second (m/s) were used to determine the energy attenuation of the helmets at the various combinations of helmet type and impact site.

**Combat helmet impact acceleration threshold**

Protective helmets used by Army aviators and civilian motorcycle riders have well defined blunt impact performance thresholds. This acceleration threshold is fundamentally based the aforementioned Wayne State University Curve. The result of this research was a head acceleration tolerance curve, shown in Figure 1-8, which suggested an acceleration and time dependency relationship.

The FMVSS 218 incorporates time dependency into their standard. The acceleration threshold for different standards are shown in Table 1.5

<table>
<thead>
<tr>
<th>Reference Standard</th>
<th>Acceleration Level (G)</th>
<th>Time Limit</th>
<th>Impact Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMVSS 571.218</td>
<td>400</td>
<td>Peak</td>
<td>All</td>
</tr>
<tr>
<td>FMVSS 571.218</td>
<td>300</td>
<td>4 ms</td>
<td>All</td>
</tr>
<tr>
<td>FMVSS 571.218</td>
<td>150</td>
<td>2 ms</td>
<td>All</td>
</tr>
<tr>
<td>ANSI-Z90.1</td>
<td>300</td>
<td>Peak</td>
<td>All</td>
</tr>
<tr>
<td>Snell 2000</td>
<td>300</td>
<td>Peak</td>
<td>All</td>
</tr>
<tr>
<td>U.S. Army aviator helmet (HGU-56/P)</td>
<td>175</td>
<td>Peak</td>
<td>Headband</td>
</tr>
</tbody>
</table>

Table 1.5: Standard Threshold for Military Helmet [28]
As it will be described later on, the motorcycle helmet standards with peak acceleration limits of 300 g are intended to prevent serious head injury. However, protection from concussive head injury would be more appropriate in Army operational environments as emergency medical treatment is not always readily available. By following Slobodnik's [49] recommendation for aircrew helmets, the report advises that the blunt impact performance goal for infantry helmets should also be a peak acceleration limit of 150 g. However, this requirement may be difficult to achieve due to the limited standoff distance available between the scalp and inside surface of the PASGT helmet shell. Indeed as the McEntire et al. notice, ballistic protection requirements often drive the design of combat helmets, and blunt impact protection has previously received minimal design consideration.

More recently the goals have been stated as [50] a threshold of 150g for a 10 ft/s (3 m/s) impact on the 5.5 kg helmeted head-form, and with an ideal goal of the same threshold up to a 17 ft/s (5.18 m/s) impact. To comply with the constraints of the ACH helmet, the pads should also be less than 3/4 of inch (19 mm) thick.
1.4.3 Motorcycle

Motorcycle helmets are very widespread, and present generally similar structures. Standards can vary from one area to another (example ECE 22.05 in Europe versus FMVSS 218 in the U.S), so we will focus on one situation, giving to the reader the opportunity to adapt all the results to other standards, by modifying the threshold, the other aspects of the test associated with each standard being very similar.

Motorcycle helmet design

Current motorcycle helmet design appears to be very standardized. The helmet is composed of three layers. As noted by Schuaeib et al. [51] the main helmet components are the shell and the foam liner. The comfort foam, in contact with the head, is only of little use in protecting the head.

The function of the shell is to resist the penetration of any object, preventing it from reaching the head. This is the main protection against direct skull damage. This rigid layer distributes the load over a larger area, thus improving the energy absorption capabilities of the foam liner. The foam liner is a deformable material, softer than the shell, which primarily use is to absorb energy. A picture of a modern motorcycle helmet, and a schematic from Gilchrist et al. [52], showing the different layers of a typical motorcycle helmet, are given in Figure 1-12. Average weight of a motorcycle helmet is 1.5 kg.
Current research focuses on improving the energy absorption capacity of helmets and new designs and materials are currently being investigated [51]. However currently polystyrene is commonly selected, due to its good impact energy absorption capacities, its low density, and the ease of manufacturing.

**Standardized tests on motorcycle helmets**

Standard tests for brain injury are now well established, and following the example of the motorcycle helmet industry, some sport manufacturers have developed similar standards (see next section). Schuaeib et al. [9] describes a review of the primary acceleration-based standards for preventing brain injury. Two major standards are today used: the Peak Linear Acceleration (PLA) which reports the highest acceleration recorded during the impact, and the Head Injury Criteria (HIC) (see previous section). Most international standards require the PLA of the head not to exceed a certain threshold, under a direct impact of a mass of 5kg traveling between 6 and 7.5 m/s. This corresponds to impact energies between 90J and 140J. For instance the U.S pass/fail limit in national standards is a maximum Peak Linear Acceleration of 300g [53]. This is equivalent to a 15kN load if the headform mass is 5kg.
1.4.4 Sport

Equipment and rules have evolved over the years to try to reduce fatalities and serious injury rates resulting from falls or impacts between football players. However, as Zhang et al. [18] note, current football helmets have an effective padding system which can prevent severe head injuries but do not effectively prevent concussion; this is also supported by the data in the previous section relative to sport injuries.

Football helmet design

Various brands offer helmets for the practice of football, and even if each model has its own characteristics all the helmets present the same structure: a metallic face guard protects the face of the players from being struck by the ground or other players, while allowing the player to have a good vision of the field and his teammates. This element is very standardized and has undergone very few design or technological changes since its introduction. A rigid outer shell complete the external shell to wrap the head of the player in a rigid shell composed of this hard-composite shell plus the face guard. Inside the rigid shell is some padding, or a liner foam.
This is the part which is the object of most of the research to try to offer better protection. Finally, webbing is attached to the outer-shell to maintain the helmet on the player's head even during an impact. Two examples of football helmets are given here. Figure 1-13 shows a schematic of a typical football helmet. Figure ?? shows pictures from a Riddell Revolution Helmet, which is the model currently used by NFL players. The thickness of the padding ranges between 25 and 35 mm depending on the location in the shell. The total weight of the helmet is 1.8 kg.
Standardized tests on football helmets

Football helmets are tested and approved by the National Operating Committee on Standards for Athletic Equipment (NOCSAE). Football helmets (as well as Hockey, Lacrosse...helmets) are defined in the official document as well as Standard test methods and equipment used in evaluating the performance characteristics of protective headgear/equipment [55]. The standardized test configuration is shown in Figure 1-14; components of the testing setup are
described in Table 1.6.

Figure 1-14: Standardized Impact test schematics for football helmets as defined by the NOCSAE standard [55]
<table>
<thead>
<tr>
<th>CODE</th>
<th>DESCRIPTION</th>
<th>AVAILABILITY</th>
<th>DRAWINGS AVAILABLE</th>
<th>SIRC PART NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Drop Carriage</td>
<td>SIRC</td>
<td>Yes</td>
<td>1001</td>
</tr>
<tr>
<td>2</td>
<td>½&quot; MEP Testing Pad</td>
<td>SIRC</td>
<td>No</td>
<td>1006</td>
</tr>
<tr>
<td>2</td>
<td>⅛&quot; MEP Faceguard Testing Pad</td>
<td>SIRC</td>
<td>No</td>
<td>1007</td>
</tr>
<tr>
<td>2</td>
<td>3&quot; MEP Calibration Pad</td>
<td>SIRC</td>
<td>No</td>
<td>1005</td>
</tr>
<tr>
<td>3</td>
<td>Hook-eye Tumbuckle, Forged Steel, 36&quot; with a 6&quot; take-up</td>
<td>SIRC/H</td>
<td>N</td>
<td>1043</td>
</tr>
<tr>
<td>4</td>
<td>⅛&quot; Wire Rope Thimble</td>
<td>SIRC/M</td>
<td>N</td>
<td>1044</td>
</tr>
<tr>
<td>5</td>
<td>1/8&quot; Spring Music Wire</td>
<td>SIRC/M</td>
<td>N</td>
<td>1045</td>
</tr>
<tr>
<td>6</td>
<td>⅛&quot; Wire Rope, Tiller Rope Clamp, Bronze</td>
<td>SIRC/M</td>
<td>N</td>
<td>1046</td>
</tr>
<tr>
<td>7</td>
<td>36&quot; 16 x 3&quot; Eye Bolt</td>
<td>SIRC/H</td>
<td>N</td>
<td>1041</td>
</tr>
<tr>
<td>8</td>
<td>36&quot; Forged Eye Bolt</td>
<td>SIRC/H</td>
<td>N</td>
<td>1040</td>
</tr>
<tr>
<td>9</td>
<td>Right Angle DC Hoist Motor</td>
<td>SIRC/G</td>
<td>N</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>DC Motor Speed Controller (Reversible)</td>
<td>SIRC/G</td>
<td>N</td>
<td>2001</td>
</tr>
<tr>
<td>10</td>
<td>Single Groove Sheave (Pulley), 3 ¾&quot;</td>
<td>SIRC/G</td>
<td>N</td>
<td>2002</td>
</tr>
<tr>
<td>11</td>
<td>Top Mount Plate</td>
<td>SIRC</td>
<td>Y</td>
<td>2003</td>
</tr>
<tr>
<td>12</td>
<td>18&quot; Top Channel Bracket</td>
<td>SIRC/H</td>
<td>N</td>
<td>2004</td>
</tr>
<tr>
<td>13</td>
<td>Wall Mount Channel Bracket, 4' x 1 56&quot;</td>
<td>SIRC/H</td>
<td>N</td>
<td>2005</td>
</tr>
<tr>
<td>14</td>
<td>Mechanical Release System</td>
<td>SIRC</td>
<td>Y</td>
<td>2006</td>
</tr>
<tr>
<td>15</td>
<td>Lift Cable, Wire Rope, 20' Coil</td>
<td>SIRC/H</td>
<td>N</td>
<td>2007</td>
</tr>
<tr>
<td>16</td>
<td>Anvil Base Plate</td>
<td>SIRC</td>
<td>Y</td>
<td>2010</td>
</tr>
<tr>
<td>17</td>
<td>Anvil</td>
<td>SIRC</td>
<td>Y</td>
<td>2011</td>
</tr>
<tr>
<td>18</td>
<td>Headform Adjuster</td>
<td>SIRC</td>
<td>Y</td>
<td>2012</td>
</tr>
<tr>
<td>19</td>
<td>Headform Rotator Stem</td>
<td>SIRC</td>
<td>Y</td>
<td>2013</td>
</tr>
<tr>
<td>20</td>
<td>Headform Threaded Lockring</td>
<td>SIRC</td>
<td>Y</td>
<td>2016</td>
</tr>
<tr>
<td>21</td>
<td>Headform Collar</td>
<td>SIRC</td>
<td>Y</td>
<td>2014</td>
</tr>
<tr>
<td>22</td>
<td>Nylon Bushing</td>
<td>SIRC</td>
<td>Y</td>
<td>1803</td>
</tr>
<tr>
<td>23</td>
<td>Small Headform</td>
<td>SIRC</td>
<td>N</td>
<td>1100</td>
</tr>
<tr>
<td>23</td>
<td>Medium Headform</td>
<td>SIRC</td>
<td>N</td>
<td>1101</td>
</tr>
<tr>
<td>23</td>
<td>Large Headform</td>
<td>SIRC</td>
<td>N</td>
<td>1102</td>
</tr>
</tbody>
</table>

Table 1.6: Standardized Impact test components for football helmets as defined by the NOCSSAE standard [55]

Impacting parameters are defined in term of height of drop as shown on Table 1.7

The Severity Index is very similar to HIC and is defined as [55]

\[ SI = \int_0^T [A(t)] dt \]  \hspace{1cm} (1.2)

where \( A(t) \) is the acceleration of the headform measured in term of g (9.81 m/s), and \( t=0 \) corresponds to the point in time where the measured acceleration exceeds 4g and \( t=T \).
<table>
<thead>
<tr>
<th></th>
<th>FRONT</th>
<th>SIDE</th>
<th>F. BOSS</th>
<th>R. BOSS</th>
<th>REAR</th>
<th>TOP</th>
<th>RANDOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Temperature</td>
<td>36 (91)</td>
<td>36 (91)</td>
<td>48 (122)</td>
<td>48 (122)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
</tr>
<tr>
<td>High Temperature</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
<td>60 (152)</td>
</tr>
</tbody>
</table>

Table 1.7: Impact height drop [inches (cm)] for Football helmet testing as defined by the NOCSAE [56]. These heights correspond to the parameter $H$ in Figure 1-14. Ambient temperature is 72°F and high temperature is 115°F.

corresponds to the point in time where the measured acceleration falls for the first time below 4g [55]. The limit threshold is given as a limit in Severity Index of 1200.

So for a constant acceleration, a SI of 1200 corresponds to a deceleration of 150g over a period of 4.35 milliseconds.
1.5 Limitation of current analysis and designs

Based on this review, several of limitations are apparent in the current helmet design. Military helmets need a better liner material, to be able to fulfill the new standards for impacts, which have evolved from impacts at 10 ft/s to impacts at 17ft/s. Moreover, absorbing multiple impacts is of interest since soldiers can sustain several blows in a single mission. Current football helmets do not effectively protect players from concussion, and do not have a multiple loading capacity. This is an issue since it has been identified that multiple Traumatic Brain Injury history tends to favor future injuries of the same type. Furthermore, reducing the weight of the helmet could help prevent injuries during falls. Motorcycle helmets are currently using only high density polytstyrene, which presents very good mechanical properties for impact absorption. However, the plastic deformation induced during an impact crushes the liner, so that protection on following impacts (bouncing on the ground, against trees, fences, sidewalk...) is very poor. Mills [57] showed that event if multiple impacts usually does not occur exactly at the same place on the helmet because of the rotation of the victim, impacts are usually located very close on the helmet, so a crushed liner has dramatic effects on a second impact. So there are also some possibilities of improvement here. Mills [57] developed two interesting case study on a bicycle helmet and a motorcycle helmet that give a good overview on methods currently used to design a helmet.

1.5.1 Goals of this study

Based on the previous considerations, this study aims at suggesting a better design for the military helmet liner, developing a method that is also applicable for football and motorcycle helmets.
1.5.2 Outline of the thesis

To reach this goal we will first review the basics mechanical properties of materials of interest for helmet design. Then we will introduce our experimental testing methods and apparatus. The following chapter will introduce modeling of the various materials, and a comparison with the experimental data. Then we will describe our optimization method using the modeling to design a better helmet. Finally we will conclude our study and suggest potential topics of interest for future research in this area.

Energy levels for testing

Based on the standardized test reported in the previous part, three types of impact tests are conducted in this study. The way tests are defined vary from one area of application to another (speed of impact for military purposes, height of drop for sports helmets, energy of impact for motorcycle helmets). However, given the fact that the weight of the helmeted-head is approximately the same for all the applications, and that one can find an equivalence between height of drop, impacting speed and impacting energy, from now on in this study, the tests are designated in function of their impacting energy only, the weight of the impactor being constant, and close to the reference value for a helmeted head (between 5 and 6 kg [28, 53, 50]). Three levels of energy have been selected.

30 J impact corresponds to the current level of impact test for military purposes [28], this is equivalent to the impact of a 5.5 kg helmeted head at a speed of 10 ft/s (3.05 m/s).

70 J impacts corresponds to the standardized tests reported by the NOCSAE for a football helmet. This is equivalent to a fall of a 5 kg helmeted head from 1.5 m. This is also equivalent to an impact of a 5.5 kg helmeted head at a speed of 17 ft/s (5.18 m/s), which is the new target for military helmets [50].
Finally 130J impacts corresponds to the standardized tests for motorcycle helmets [53], which is the equivalent to the impact of a 5.5 kg helmeted head at a speed of 7 m/s.
Chapter 2

Literature review

2.1 Foam behavior

When asking for the definition of a foam, answers can be very different. Indeed, this type of structure is present in numerous materials of everyday life. According to Wikipedia, foams are defined as a substance that is formed by trapping many gaseous bubbles in a liquid or a solid. A common characteristic of foams is that they are of relative density (defined as the ratio of the density of the foam over that of the material that composes it) very small compared with unity. Many types of foams exist and a classification is possible based on several criteria. First comes the relative density (or equivalently the volume fraction of solid), then the type of cells. Open-cell foams are solid only at the edges of the polyhedra, while closed-cell foams have solid membranes over the faces of the polyhedra [58].

2.1.1 Conventional foams

A characteristic stress-strain curve for a cellular solid in compression is characterized by 3 regimes [59]. As depicted on Figure 2-1 the foam first exhibits a linear elastic regime, corresponding to cell edge bending or face stretching. This regime is valid for strains smaller than the elastic buckling strain $\varepsilon_{el}^*$. Then the foam shows a stress plateau, over which stress
is approximately constant as the strain increases. During this phase the cells progressively collapse by elastic buckling for flexible foams. Finally when all the cells are collapsed, a densification regime corresponding to the loading of cells and faces against each other, occurs in this phase, the stress increases rapidly as the strain is increased.

According to Gibson and Ashby (ref), one can find 3 regimes for compression of a flexible foam.

\[
\text{For } 0 < \epsilon < \epsilon_{\text{elastic}} \text{ then } \sigma^* = \epsilon E^* \tag{2.1}
\]
For $\epsilon_{\text{elastic}} < \epsilon < \epsilon_D \left(1 - \frac{1}{D}\right) + \epsilon_{\text{elastic}}^*$, then $\sigma^* = \sigma_{\text{elastic}}^*$ \hspace{1cm} (2.2)

For $\epsilon_D \left(1 - \frac{1}{D}\right) + \epsilon_{\text{elastic}}^* < \epsilon$, then $\sigma^* = \frac{\sigma_{\text{elastic}}^*}{D} \left(\frac{\epsilon_D}{\epsilon_D - \epsilon}\right)^m$ \hspace{1cm} (2.3)

Where

$$\epsilon_D = 1 - 1.4 \left(\frac{\rho^*}{\rho_s}\right) \hspace{1cm} (2.4)$$

For open-cell reticulated flexible foams with a relative density around 3%, Dawson [60] reports a densification regime beginning at a strain called $\epsilon_d$ (densified strain), which is of value $\epsilon_d \approx 0.60$. So the transition from the plateau regime to the densified regime is considered occurring for $\epsilon > \epsilon_d$.

$D$ is defined such as

$$D = \frac{\epsilon_D}{\epsilon_D - \epsilon} \hspace{1cm} (2.5)$$

where the strain $\epsilon$ is the strain at which the stress at the end of the plateau region begins to exceed the elastic buckling stress.

### 2.1.2 Negative Poisson’s ratio foams

Most materials contract laterally when stretched, and expand when compressed. Poisson’s ratio ($\nu$), which is defined as the negative transverse strain divided by the axial strain in the direction of stretching (or compressing) force, is positive for those materials.

$$\nu = -\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{axial}}} \hspace{1cm} (2.6)$$

For rubbers and biological tissues $\nu \approx 0.5$, for aluminum $\nu \approx 0.33$, and $\nu \approx 0.1$ to 0.4 for
typical polymer foams [61]

In an isotropic material, the allowable range of Poisson’s ratio is from -1.0 to +0.5, based on thermodynamic consideration of strain energy [61]. Love [62] presented an example of cubic ”single crystal” with a Poisson’s ratio of -0.14.

Lakes [61] produced foams with negative Poisson’s ratio from low-density open-cell polymer foams, by causing the ribs of each cell to protrude inward, resulting in a permanent reentrant structure such as the idealized cell shown on Figure 2-2.

Figure 2-2: Idealized reentrant unit cell produced by a symmetrical collapse of a 24-sided polyhedron with cubic symmetry [61]

Lakes used a polyester foam with the following characteristics: density of 0.03 g/cm³, Young’s modulus of 71 kPa, cell size of 1.2 mm, and a Poisson’s ratio of 0.4.

The method used by Lakes to create the reentrant structure is as follows. Specimen of conventional foam were compressed triaxially (three orthogonal directions), and placed in a mold. The mold was heated to a temperature slightly above the softening temperature of the foam material (163 C to 171 C in this case). The mold was then cooled to room temperature and the foam was extracted. Specimens that were given a permanent volumetric compression
factor of between 1.4 to 4 during this transformation were found to exhibit negative Poisson’s ratios. For example, a sample of this foam compressed with a volumetric compression factor of 2 exhibited a Poisson’s ratio of -0.7. Figure 2-3 shows an example of a foam sample before and after preparation. Polymer foams exhibited negative Poisson’s ratio as small as -0.7 and values to -0.8 have been observed in metal foams.

Figure 2-3: (top) Stereo photograph of a conventional open-cell polymer foam. Scale mark 2 mm. (bottom) Stereo photograph of a reentrant foam. Permanent volumetric compression factor is 2.7. Poisson’s ratio is -0.6. Scale mark 2 mm. [61]

Properties of foam with negative Poisson’s ratio

Many phenomena in the deformation of elastic materials depend on Poisson’s ratio. The simplest is that a material with a negative Poisson’s ratio will be fatter in cross-section when stretched, and thinner when compressed.
R. Lakes [61] observed that foams with negative Poisson’s ratio were found to be more resilient than conventional foams. Foams with a typical structure of tetrakaidecahedral cells exhibit a linear compressive regime in stress-strain curve up to about 5% strain. When strain is increased passed this threshold, the cell ribs buckle and the foam collapses at roughly constant stress. On the contrary, reentrant foams behave following a nearly linear stress-strain relationship up to more than 40% strain, with no sudden collapse.

It is notable that the theory of elasticity contains no characteristic length scale. So the phenomenon of negative Poisson’s ratio does not depend on the cell-size. Based on energy conservation, one can deduce that the range of possible Poisson’s ratio for isotropic materials is from -1.0 to 0.5 [61]. The general theory of elasticity predicts some unusual phenomena in solids with negative Poisson’s ratio. For instance, the indentation of a block of material, for a given pressure, is proportional to \((1 - \nu^2)/E\), in which \(E\) is Young’s modulus. Thus, a material exhibiting a negative Poisson’s ratio close to the thermodynamic limit \(\nu = -1.0\) will be extremely difficult to indent even if the material is compliant (indeed negative Poisson’s material can exhibit greater value of \(\nu^2\) than conventional materials).

The origin of this phenomenon can be explained by studying the relation between the shear modulus \(G\), the bulk modulus \(B\) and Poisson’s ratio:

\[
B = \frac{2G(1 + \nu)}{3(1 - 2\nu)}
\]  \hspace{1cm} (2.7)

When the Poisson’s ratio approaches 0.5 (as in rubbery solids), the bulk modulus exceeds greatly the shear modulus and the material is referred to as incompressible. When Poisson’s ratio approaches -1.0, the material becomes highly compressible \((B \rightarrow 0)\); its bulk modulus is much less than its shear modulus.

In most two-dimensional situations, the stress concentrations have no dependence at all.
on Poisson’s ratio [62]. In three dimensions, there may be a significant dependence of the stress concentration factor upon Poisson’s ratio value [62].

**Tailoring the Negative Poisson’s ratio**

The four basics elastic constants are: Tensile (Young’s) Modulus (E) - Measure of resistance to tensile load-, the Poisson ratio (ν) - Measure of change in cross section under tension, the shear modulus (G) -measure of resistance to applied shear load- and the bulk modulus (B) - measure of resistance to an applied hydrostatic pressure.

\[
G = \frac{E}{2(1+\nu)} \quad B = \frac{E}{3(1-2\nu)}
\]  
\[
E = \frac{9BG}{3B+G} \quad \nu = 1/2 \left( \frac{3B-2G}{3B+G} \right)
\]  

Evans [63] noted that in most structural applications materials are used in sheet or beam form. So it is more important to have high shear modulus rather than a high bulk modulus. So as the above equations indicate, tailoring the Poisson’s ratio allows enhancement of material properties significantly. Also other benefits are expected, such as better indentation resistance (as explained previously), shock absorption or fracture toughness [63].

**Example of application of negative Poisson’s ratio foam**

**Wrestling Mat**

Lakes et al. [62] examine the case of a wrestling mat or a knee pad. To choose the material they consider the penetration rigidity \( F/u \) where \( F \) is the force of indentation and \( u \) is the maximum displacement recorded. Given the two applications under study, it is necessary that the padding system works as well for large or small impactors.

If the impactor is sufficiently small compared to the mat/pad a good approximation is to consider the mat as an elastic half space loaded with a circular pressure distribution of
radius \( a \),

\[
[F/u]_{\text{narrow}} = G a_n / (1 - \nu) \quad \text{(2.10)}
\]

where \( a_n \) refers to the radius of a narrow impactor. On the other hand, for impactor much larger that the protective device (mat or pad) thickness, it can be approximated that the compression is uniform over a layer of thickness \( H \), and radius \( a_w \), and in this layer the the lateral Poisson effect is restrained. Lakes et al. [62] make the assumption that the force \( F \) is uniformly distributed over the layer. This gives that

\[
[F/u]_{\text{wide}} = \frac{G a_w^2}{2H(1+\nu)} \quad \frac{1-2\nu}{1-2\nu} \quad \text{(2.11)}
\]

where \( a_w \) refers to the radius of a wide impactor and \( H \) is the thickness of the protective device. To be useable in both situations (ratio \( \text{Area}_{\text{Impactor}} / \text{Area}_{\text{Mat}} \gg 1 \) and \( \text{Area}_{\text{Impactor}} / \text{Area}_{\text{Mat}} \ll 1 \)), the following ratio must be close of the order of unity:

\[
\frac{[F/u]_{\text{wide}}}{[F/u]_{\text{narrow}}} = \frac{2a_w^2 (1 - \nu^2)}{a_n H (1 - 2\nu)} \quad \text{(2.12)}
\]

Plotting this ratio shows that material exhibiting a negative Poisson’s ratio will offer the best protection ( the material is compliant enough so that it absorbs distributed forces, but on the other hand it is rigid enough so that a very localized force will not crush it completely), and rubbery materials are the worst in this application. As an example, Lakes et al. [62] show that for \( a_w = 10H \), \( a_n = 0.1H \), then the optimal Poisson’s ratio is \( \nu = -0.9993 \).

**Deformation mechanisms in negative Poisson’s ratio materials: structural aspects**

Lakes identified the microstructural features associated with negative Poisson’s ratio [64]. A material’s Poisson’s ratio is determined by several aspects of its microstructure: the presence of rotational degrees of freedom, non affine deformation kinematics, or anisotropic structure. The early development of elasticity theory summarized in the ”Cauchy relations” predicted a
Figure 2-4: (a) Conventional Honeycomb as a Poisson’s ratio of +1. (b) Re-entrant structure can exhibit a Poisson’s ratio of -1 providing an appropriate choice of angle and cell ribs dimensions [65].

Poisson’s ratio of 1/4 for all materials described. However, materials can exhibit a Poisson’s ratio different from 1/4 under certain conditions. Specifically (i) non-central forces between particles in the solid, (ii) forces which do not depend on distance alone or (iii) anisotropy, including non-centrosymmetry [64]. The range for Poisson’s ratio for isotropic materials is 
\[-1 < \nu < 1/2\] in 3D, and \[-1 < \nu < 1\] in 2D [64].

Specifically, a honeycomb composed of regular hexagonal cells has a Poisson’s ratio of +1 [59] Figure 2-4. The deformation is not affine because some pairs of nodal points move apart during stretching while other do not. Re-entrant structures as shown in Figure 2-2, exhibit a negative Poisson’s ratio, since compressive strain leads to a decrease in transverse section.

In 2D, as shown on Figure 2-4(b) by choosing correctly angles and ribs dimension a Poisson’s ratio of -1 can be achieved. For instance, for a 2D cell shown on Figure 2-5, choosing \(h/L = 2\) and \(\theta = \pi/6\)
Figure 2-5: Example of 2D cell exhibiting a lateral Poisson's ratio of -1 if $h/L = 2$ and $\theta = -\pi/6$

The non-affine kinematics are seen to be essential for the production of negative Poisson's ratios for isotropic materials containing central force linkages of positive stiffness. Non-central forces combined with pre-load can also give rise to a negative Poisson’s ratio in isotropic materials. Finally a chiral microstructure with non-central force interaction or non-affine deformation can exhibit a negative Poisson’s ratio [65].

2.2 Shear thickening fluid

Introduction

Shear thickening fluids are of interest because of the versatility of behavior they demonstrate depending on the conditions in the sample. Like Magneto-Rheological fluids, or Electrorheological fluids, they undergo a sharp transition in viscosity when a set of parameters is reached. They are part of the so-called "field-activated" fluids. The main advantage of this type of fluid is that its change in viscosity is directly triggered by the phenomena which is
to be mitigated: high rate of deformation.

This type of fluid has given rise to a great number of studies attempting to account for their behavior ([66], [67], [68], [69] and others), but so far explanations for many of the phenomena are still under dispute.

A shear thickening fluid is composed of highly concentrated suspensions in a carrier fluid. The shear-thickening behavior can be continuous, showing an increasing viscosity when shear rate increases, or can be discontinuous, showing a viscosity jump when a certain shear rate is reached as shown on Figure 2-6.

![Figure 2-6: Viscosity discontinuity in concentrated monodisperse suspensions [69]. VF: volume fraction of solid, Reduced viscosity = suspension viscosity/ suspending fluid viscosity](image)

General behavior of shear is described and discussed extensively by Barnes [68]. The main points are that first the suspension exhibits a decrease in viscosity, exhibiting a behavior which can be described by a power-law type relationship between viscosity and shear rate.
Then, when a critical shear rate $\dot{\gamma}_c$ is reached, the suspension’s viscosity begins to increase. After this shear thickening region a decrease in viscosity is generally observed. This typical behavior is represented on Figure 2-7.

![Figure 2-7: Schematic representation of viscosity versus shear rate for shear thickening systems, with each curve representing a different phase volume. $\dot{\gamma}_c$ and $\dot{\gamma}_m$ are the shear rates at the beginning and the end of the shear-thickening region][68].

Given the large literature on the subject, the reader can refer to some comprehensive reviews for more detailed discussions on the parameters of importance (Barnes [68], Hoffman [69], Stickel and Powell [70]).

**Shear thickening mechanism**

Before going to the parameters that allow one to build a fluid that will have the desired properties in term of critical shear rate, viscosity jump and reversibility of the phenomena for instance, it is interesting to study the shear thickening phenomena and the proposed mechanisms.
The two main theories suggested so far are the Order-Disorder theory, and the "Hydro-cluster" theory.

Hoffman [69] was the first to suggest a mechanism for the shear thickening behavior of highly concentrated colloidal dispersions. He observed that monodisperse suspensions under shear rate generate diffraction patterns under white light. Famous examples of such patterns are given in Figure 2-8.

According to the order-disorder theory, when the suspension is sheared, particles initially collide randomly one with another, align in hexagonally packed layers, in plane parallel to the plane of shear. After a critical stress is reached, flow instabilities grow and induce particle motions out of the ordered layers. The particles then collide into each other and produce a rise in viscosity.

Analysis of the order–disorder mechanism predicts that the instability should be controlled by dimensionless groups scaling as

\[ \mu \dot{\gamma} a^2 / \varepsilon \Psi_0^2 \]  
(2.13)

when particles are stabilized by charge, and

\[ \mu \dot{\gamma} a^2 V_s / (0.5 - \chi) \Delta^2 kT \]  
(2.14)

when particles are stabilized by steric stabilization[71]. Here \( a \) is the particle radius, \( \dot{\gamma} \) the shear rate \( \mu \) is the carrier fluid viscosity, \( \varepsilon \) is the dielectric constant of the fluid, \( \Psi_0 \) is the surface potential of the particles, \( V_s \) is the volume of the solvent molecule, \( \chi \) is the Flory-Huggins parameter, \( \Delta \) is the thickness of the layer yielding steric repulsion, \( k \) is Boltzmann’s constant and \( T \) is the absolute temperature. So the onset of shear thickening should be controlled by \( \dot{\gamma} \sim 1/a^2 \).
The "Hydrocluster" theory explains the shear thickening behavior of highly concentrated suspension by the force balance between the hydrodynamic forces imposed by the shearing flow, and the forces arising from particle interactions. As shown on Figure 2-9, in equilibrium there are random collisions between the particles and there is no particular structure, so they resist flow.

But as the shear rate increases, particles get organized in the flow, so the resulting fluid viscosity lowers. At even higher shear rates (or similarly shear stress), hydrodynamic interactions between particles dominate over stochastic ones, so layers "jam" one into another, forming what is called "hydroclusters", resulting in a much higher rate of energy dissipation and an abrupt increase in viscosity.

As Wagner et al. [72] state, the dynamics of colloidal dispersions is by nature a multi-body, multiphase fluid-mechanics problem. But if one consider the case of a single particle, the fluid drag results in the Stokes-Einstein-Sutherland fluctuation-dissipation formula:

\[ D_0 = \frac{kT}{6\pi\mu a} \]
Where $D_0$ is the diffusivity, $kT$ the thermal energy of a particle, $\mu$ the carrier fluid viscosity and $a$ the hydrodynamic radius of the particle. In the case of a Brownian interaction among the particles [73], the characteristic time scale for a particle to diffuse over a distance equal to its radius is then $a^2/D_0$.

The Peclet number $P_e$ gives a ratio between the shear rate of the flow $\dot{\gamma}$ to the particle’s diffusion rate so that:

$$P_e = \frac{\dot{\gamma}}{D_0} = \frac{\dot{\gamma}a^2}{D_0}$$

Using the shear stress, shear rate relationship $\tau = \mu \dot{\gamma}$, it is also possible to express this dimensionless number as a function of the shear stress $\tau$:

$$P_e = \frac{\dot{\gamma}a^2}{D_0} = \frac{\dot{\gamma}a^2}{kT \frac{6\pi \mu a}{kT}} = \frac{6\pi \tau a^3}{kT}$$

(2.17)
According to this theory, the shear thickening onset should then be varying as

$$\tau_{\text{crit}} \sim 1/a^3$$

(2.18)

in the case of Brownian interactions.

This theory was first suggested by Bossis and Brady [74] and then developed by Wagner, based on Stokesian Dynamics Simulations [66], [75]. This has been supported by following rheo-optical experiments (Ultra Low Angle Neutron Scattering Technique) by Wagner and Kalman [76].

So finally, both models report a dependence on the size of the particle, which seems to show that one can set the critical shear rate by choosing an appropriate size of particles, all other parameters being equal. Both advocates give some experimental data supporting their theory, but many other parameters than the particle size can affect the behavior of the solution so no clear consensus has been reached so far. This study will not dwell on the specific size dependency but is developed with the idea that shear-thickening results can be adapted to various situations, just by changing the radius of the particles suspended.

Shear Thickening Parameters of importance

The parameters identified as being of importance for the characteristics of the suspension are the volume fraction of particles, the size of the particles, their shape, the viscosity of the carrier fluid and the particle-particle interactions.

Numerous literature reviews (Laun [77], Barnes [68] and Stickel [70]) give good correlations between various experimental studies to give an empirical understanding of each of these parameters. For further details on the influence of these parameters, the reader can refer to the aforementioned articles; here we give only the main results.
Volume fraction dependence

As explained previously, shear thickening has to do with particles colliding into each other, so the volume concentration of particles ($\Phi$) is a key parameter in determining onset of shear thickening and severity of the increase or jump in viscosity. As shown in Figure 2-7, increasing the particle concentration results in 3 effects. First, as $\Phi$ increases, the critical shear rate $\dot{\gamma}_c$ decreases. Furthermore, the jump in viscosity is greater, and finally there is an overall increase in the suspension's viscosity. This has been reproduced experimentally by Dawson [60], as shown in Figure 2-10.

![Figure 2-10: Viscosity versus shear rate for various concentrations of silica-based non-Newtonian fluid ~ 48% (■), ~ 50% (▲), ~ 52% (○) [60].](image)

Batchelor [73] extended the Einstein viscosity relation for hard sphere suspensions to get the following asymptotical development:

$$\eta = \eta_s \left(1 + 2.5\Phi + 6.2\Phi^2 + O(\Phi^3)\right)$$  \hspace{1cm} (2.19)
where the first power of $\Phi$ represents the single sphere viscous dissipations in the fluid, the second order is the contribution from the two-particles interactions and so on.

**Particle size dependence**

Particle size dependence is of major interest, since both theoretical models predict a strong correlation between the particle size and the onset of the shear thickening phenomenon. Several studies synthesize a number of experimental studies on particles of different size. Among those, Barnes [68] report values of shear thickening onset values for a number of different particles of various sizes in Table 2.1.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Particles/fluid</th>
<th>Repulsion</th>
<th>Temp. (K)</th>
<th>$\phi$ (Pa s)</th>
<th>$\dot{\gamma}_c$ (1/s)</th>
<th>$\sigma_c$ (Pa)</th>
<th>$a$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laun et al. (1992)</td>
<td>PSEA/EtGly</td>
<td>Charge</td>
<td>291</td>
<td>0.587</td>
<td>0.021</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Bender and Wagner (1996)</td>
<td>Silica/THFA</td>
<td>...</td>
<td>298</td>
<td>0.59</td>
<td>0.005</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>D'Haeene thesis (1992)</td>
<td>PMMA/decalin</td>
<td>Steric</td>
<td>298?</td>
<td>0.59</td>
<td>0.00255</td>
<td>84</td>
<td>30</td>
</tr>
<tr>
<td>Hoffman (1987)</td>
<td>PVC/DOP</td>
<td>Steric</td>
<td>298</td>
<td>0.57</td>
<td>0.054</td>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>Boersma thesis (1990)</td>
<td>PVC/DOP</td>
<td>Steric</td>
<td>298</td>
<td>0.60</td>
<td>0.054</td>
<td>5</td>
<td>18.9</td>
</tr>
<tr>
<td>Boersma et al. (1990)</td>
<td>PS/water</td>
<td>Charge</td>
<td>293</td>
<td>0.575</td>
<td>0.001</td>
<td>400</td>
<td>80</td>
</tr>
<tr>
<td>Boersma thesis (1990)</td>
<td>PS/water</td>
<td>Charge</td>
<td>293</td>
<td>0.584</td>
<td>0.001</td>
<td>107</td>
<td>11.8</td>
</tr>
</tbody>
</table>

Table 2.1: Critical shear rate ($\dot{\gamma}_c$) for various types of particles. $\mu$ is the solvent viscosity, PSEA is styrene ethylacrylate copolymer, PVC is polyvinyl chloride, PMMA is polymethylmethacrylate, PS is polystyrene, EtGly is ethylene glycol, DOP is dioctyl phthalate or 2-ethylhexyl phthalate, and THFA is tetrahydrofurfural alcohol. Repulsion indicates the type of stabilization, M or P stands for mono-disperse of poly-disperse [71].

Barnes also compiled data from previous studies and plotted the critical shear rate versus
the size of the particles (see Figure 2-11). The data show a dependency of the type $\dot{\gamma}_c \sim a^{-2}$.

His collection of data is from studies with different "controlled" conditions such as solvent viscosity, chemistry of the particle, charge, all of which can affect the critical shear stress.

![Critical shear rate for various sizes of particles.](image)

This supports the claim that it is possible to "tune" the shear thickening onset of a fluid, by choosing the right size of particles.

**Particle shape dependence**

Particle shape is reported to have a major impact on shear thickening, but as Barnes [68] notes it is not clear whether it is really the shape of the particles or its effect on effective phase volume, particle size or other parameters. Effective phase volume (or effective volume fraction) is defined as

$$\Phi_c = \Phi_c (1 + R_g A_g \rho)$$  \hspace{1cm} (2.20)
where \( A_p \) is the particle specific area, \( R_g \) is the radius of gyration of the particle and \( \rho \) is the particle density [78]. Clarke’s study of shape [79] suggests that increasing anisotropy results in much more shear thickening. Thus a suspension of non-spherical particles can exhibit shear-thickening behavior at a volume concentration much lower than the value usually reported for spherical particles (\( \Phi > 30 \% \), [80]). Barnes [68] gives an example of viscosity versus shear rate for different particle shapes, at a phase volume \( \Phi = 20\% \) (see Figure 2-12).

![Figure 2-12: The effect of shape on shear thickening for \( \Phi = 0.2 \) [68].]

Furthermore Dawson [60] gives some experimental evidence that spherical particles show minimal hysteresis compared with non spherical precipitated calcium carbonate (PCC) which tends to show irreversible behavior as shown on Figure 2-13.
Figure 2-13: Viscosity versus shear stress (a) PCC particles (▲) ascending and (○) descending sweep stress, (b) Silica spherical particles (●) ascending and (◇) descending sweep stress [60]
Particle size distribution dependence

As Bettin reports [80], if the particles are of different sizes, they can pack more efficiently. This is due to a geometric configuration in which the smaller particles can fit in the gap between the bigger ones. As a result, one can achieve greater volume fractions. So comparing two solutions - presenting the same volume fraction $\Phi$, one composed of monodisperse particles, and the other one composed of polydisperse particles, the monodisperse one will exhibit a stronger shear thickening, at a lower critical shear rate [68]. This result is also confirmed by Dawson using PCC polydisperse particles and silica-based monodisperse particles. Barnes gives an example of this phenomena for $CaCO_3$ particles Figure 2-14.

![Figure 2-14: Viscosity of calcium carbonate blends at $\Phi = 0.48$, as a function of shear rate. Clay A 12 $\mu m$, Clay B 0.65 $\mu m$ [68].](image)
Particle-particle interaction dependence

There can be shear thickening in a suspension, only when there is no overall attraction between the particles, that is when they are deflocculated [68]. Freundlich [81] states that "dilatancy is reduced or annihilated if the particles show the least tendency to adhere to each other". Thus to be able to observe a shear thickening behavior, particles must be either neutral or repel each other due to interactions, either electrostatic, entropic or steric. Barnes [68] summarized the literature on this subject concluding that deflocculated suspensions generally have a low viscosity at low shear rate and can exhibit a shear thickening. On the other hand flocculated suspensions have a high viscosity at low shear rate and exhibit a shear thinning as shear rate increases, but do not exhibit a shear thickening behavior. These results are summarized in Figure 2-15.

Figure 2-15: Effect of flocculation on shear thickening behavior [68].
Shear thickening current applications

Shear Thickening suspensions are of high interest for human protection, since they can be incorporated into flexible matrices, and do not affect the flexibility of the matrices at low deformation rates, since their viscosity remains low. However, if a sharp rise in shear rate is induced by a blow, an impact or a blast wave, they exhibit a sharp jump in viscosity, being able to stiffen the material and absorb a large amount of energy.

For instance Hayes and Robinovitch [82] patented a hip-padding protection belt, that is composed of pouches of shear-thickening fluid. This is aimed to be worn and be flexible under normal circumstances, and to be activated by any fall or impact to protect the hips of the person wearing it. A schematic of one of the configuration is shown on Figure 2-16(a) and performance are related on Figure 2-16(b).

More recent work by Wagner et al. [83], [84] and Lee et al.[85] have been focusing on ballistic applications. Impregnating Kevlar® fabric with shear thickening fluid seems to give very promising results for personal protection. Both the size of the nanoparticles used [85], and the effect of the particles in the fluid have been studied by Kalman et al. Kalman20091. Figure 2-17 gives an overview of the performances of this type of material versus conventional ones.
Figure 2-16: (a) Perspective view of the protective garment (b) Maximum peak load for a hip-form undergoing a 120J impact. Ensolite Horse-shoe and Dilatant Horse-shoe are two configurations of the patented applications, other products are pre-existing products from concurrence[82]
Figure 2-17: (a) Penetration depth of the projectile in the clay witness (orange material on the picture) for the STF unimpregnated (A-D) and Impregnated (E,F) Kevlar® samples (b),(c) Comparison of front kevlar layer for a non-impregnated (target D) and an impregnated (target F) target after ballistic impact [83].
2.3 Fluid filled foam

2.3.1 Newtonian fluid

Recent studies [60], have raised interest in the notion of intrinsic permeability of open-cell foams and fluid-flow through the foams. Indeed, this parameter was first studied by Gent and Rusch in 1966 [86] who demonstrated that the average cell diameter is a function of the applied compressive strain. Gent and Rusch suggested a relation between strain and average cell diameter and developed a simple model based on flow through an array of smooth tubes. Although the transition from fully laminar (where Darcy’s law is applicable) to the turbulent regime is usually reported to occur for $Re > 2000$ in a smooth tube, they experimentally showed that this transition was occurring at $Re \approx 1$ for open-cell foams.

However, in a foam the path of the fluid is actually more tortuous than in the case of flow through a smooth pipe with uniform cylindrical section. Comiti et al. [87] developed a theoretical model for this transition, finding a value of $Re = 0.83$ for flow through porous media.

A complete review of this simple model will be described in the next chapter and adapted to the modeling of the response of the foam.

2.3.2 Permeability model

Assuming the foam is isotropic, the relative density under uniaxial compression is given by [60]:

$$\frac{\rho^*}{\rho_s} = \frac{\rho_0^*}{\rho_s} \frac{1}{(1 - \epsilon)(1 + \nu \epsilon)^2} \tag{2.21}$$

where $\epsilon$ is the strain, which is taken positive in compression, $\rho^*$ is the density of the foam at strain $\epsilon$, $\rho_0^*$ is the density of the foam at strain $\epsilon = 0$, $\rho_s$ is the density of the solid material
of which the foam is made and $\nu$ is the Poisson’s ratio of the foam.

Poisson’s ratios of open-cell, reticulated foam are usually between 0 and 0.3 in the linear elastic regime of the compression. However, for strains greater that the buckling strain (about 0.075), the cells collapse and buckle without expanding much laterally, so that their Poisson’s ratio in this regime is close to zero. Since this study leads to compressive strains far higher than the elastic bulking strain, $\nu \epsilon$ is taken equal to zero in the rest of this study for low-density, open-cell, reticulated, flexible foams. Using equation 2.21, expanding the relative density term in Brace’s equation (after Brace [88]) given by equation 2.22

$$k = A d^2 \left( 1 - \frac{\rho^*}{\rho_s} \right)^3$$

(2.22)

using the relation between relative density and strain with the approximation $\nu \epsilon \approx 0$

$$\frac{\rho^*}{\rho_s} = \frac{\rho^*}{\rho_s (1 - \epsilon)}$$

(2.23)

and replacing in equation 2.22, gives the intrinsic permeability as a function of strain (Eq 2.24).

$$k = A d^2 \left( 1 - \frac{\rho^*}{\rho_s} \frac{1}{(1 - \epsilon)} \right)^3$$

(2.24)

Where $A$ is an empirical constant and $d$ is the average diameter of the cell. $A$ is given by Brace as 0.025 for a porous microstructure composed of tubes of circular sections.

The model suggested by Gent and Rusch [86] considers the foam as composed of an array of circular tubes. The average diameter of a cell is found to be proportional to the diameter of the cross-section of the tube. In the case of uni-directional compressive strain, for low-density foams, and for strains smaller than the elastic buckling strain, the model makes the assumption that each tube deforms in the same proportion as the bulk material. That means that the average cross-section diameter can be expressed as:
\[ d_{el} \approx d_0 (1 - \epsilon)^{1/2} \quad \text{for } 0 < \epsilon < \epsilon_{el}^{*} \quad (2.25) \]

This equation is valid in the elastic regime, and \( d_0 \) is the average cell size at strain equal zero (\( \epsilon = 0\% \)).

The model developed by Dawson, Gibson and McKinley [89], [90], suggests a similar form for the average size of cell for the densified regime:

\[ d_d = d_0 (1 - \epsilon)^{a} \quad \text{for } \epsilon = \epsilon_d \quad (2.26) \]

where \( a \) is an empirical constant, which has been determined for the type of foam under study. So finally, the permeability of the foam can be determined as:

\[ k_{el} = Ad_0^2 (1 - \epsilon) \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon} \right)^{3} \quad \text{for } 0 < \epsilon < \epsilon_{el}^{*} \quad (2.27) \]

\[ k_{el}^{*} = Ad_0^2 (1 - \epsilon_{el}^{*}) \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon_{el}^{*}} \right)^{3} \quad \text{for } \epsilon = \epsilon_{el}^{*} \quad (2.28) \]

\[ k_{d} = Ad_0^2 (1 - \epsilon_d)^{2a} \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon_d} \right)^{3} \quad \text{for } \epsilon = \epsilon_d \quad (2.29) \]

And the corresponding volume fractions of the cells remaining in the linear elastic regime \( \chi_{el}^{*} \) and densified regime \( \chi_d \), for strain greater that the elastic buckling strain are

\[ \chi_{el}^{*} = \frac{(\epsilon_d - \epsilon)(1 + \epsilon_{el}^{*})}{(1 + \epsilon)(\epsilon_d - \epsilon_{el}^{*})} \quad \text{for } \epsilon_{el}^{*} < \epsilon < \epsilon_d \quad (2.30) \]

\[ \chi_d = \frac{(\epsilon - \epsilon_{el}^{*})(1 + \epsilon_d)}{(1 + \epsilon)(\epsilon_d - \epsilon_{el}^{*})} \quad \text{for } \epsilon_{el}^{*} < \epsilon < \epsilon_d \quad (2.31) \]

In the case of a flow in the direction of compression, and using Gent and Rusch model [86], in the case of viscous dominated flow, the pressure drop across the specimen can be
related to the permeability of the foam by the following relation:

\[ \frac{\Delta p_i}{h_i} = \frac{\mu}{k_i} U \]  

(2.32)

where \( h_i \) is the length of each regime in the direction of the flow, \( k_i \) is the intrinsic permeability of each regime and \( U \) is the flow velocity. \( U \) is assumed uniform and constant through each regime because of continuity. Using equation 4.106, the total pressure drop over the specimen is the sum of the pressure drop in the different parts as shown on Figure 2-18, Figure 2-19. If the specimen is of constant cross section, the length of each regime is proportional to the volume fraction of each regime, so that as Dawson, Gibson and McKinley [90] deduced:

\[ k_T = k_{ei} \quad \text{for} \quad 0 < \epsilon < \epsilon_{e1}^* \]  

(2.33)

\[ k_T = \frac{k_d k_{ei}^*}{\chi_{ei} k_d + \chi_d k_{ei}^*} \quad \text{for} \quad \epsilon_{e1}^* < \epsilon < \epsilon_d \]  

(2.34)

Figure 2-18: Magnified compression labs photos of densified region for a saturated, 90ppi, polyurethane foam specimen. A) 0.20 strain; B) 0.40 strain; C) 0.60 strain; D) 0.80 strain. [60]
Results obtained by Dawson et al [90], [89], [60], show very good agreement between experimental data and model predictions as shown on Figure 2-20.
<table>
<thead>
<tr>
<th>Foam Type (ppi)</th>
<th>Avg. Cell Dia. (μm)</th>
<th>Before Pre-Compression</th>
<th>After Pre-Compression</th>
<th>$k_p/k_o$</th>
<th>$k_d/k_o$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>235</td>
<td>4.85</td>
<td>5.62</td>
<td>0.893</td>
<td>0.221</td>
<td>0.75</td>
</tr>
<tr>
<td>70</td>
<td>235</td>
<td>4.67</td>
<td>5.02</td>
<td>0.902</td>
<td>0.214</td>
<td>0.76</td>
</tr>
<tr>
<td>80</td>
<td>210</td>
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<td>3.68</td>
<td>0.897</td>
<td>0.198</td>
<td>0.80</td>
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<td>90</td>
<td>175</td>
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</tbody>
</table>

Figure 2-20: (a) Data for each grade of foam. The permeability at 0% strain for each grade of foam before and after precompression. (i) and (||) correspond to the direction of flow being perpendicular and parallel to the rise direction, respectively. (b) The normalized permeability $k/k_o$ plotted vs. strain $\epsilon$ for 70 ppi polyurethane foam. Gent and Rusch regression (...), Hilyard and Collier regression (---), Dawson, Germaine, and Gibson Model (- - -). Experimental data from Dawson et al. study (●). [60]
2.3.3 Shear thickening fluid

Considering the case of a power-law viscosity fluid squeezed between two parallel plates, Dawson et al. [91] developed an analytical model to describe the compressive response of an open-cell foam impregnated with non-Newtonian fluid.

The fluid is assumed to have a viscosity given by

\[ \eta = m(\dot{\gamma}^{n-1}) \]  

(2.35)

where \( \dot{\gamma} \) is the shear rate in the fluid, and \( m \) is a constant depending on the fluid itself.

In the case of a flow in a rectangular channel, using the Navier-Stokes equations of motion in the axial direction, the shear stress in the fluid and the pressure drop across the channel are linked by

\[ \frac{\partial \tau_{yz}}{\partial y} = -\frac{\partial P}{\partial z} \]  

(2.36)

where \( \tau_{yz} \) is the shear stress in the fluid, which is linked to the viscosity and speed gradient by

\[ \tau_{yz} = -\eta \frac{\partial V_z}{\partial y} \]  

(2.37)

So combining these three equations and using the appropriate boundary conditions (free surface on the outer part of the parallelepiped, no slip condition on the base of the sample) Dawson et al. conclude that the velocity profile is:

\[ V_z = \left( -\frac{1}{m} \frac{\partial P}{\partial z} \right)^{1/n} \frac{1}{\frac{1}{n} + 1} \left( B^{1/n+1} - (B - y)^{1/n+1} \right) \]  

(2.38)

Similarly, in a cylindrical sample (see Figure 2-21) they derive the following expression for the volumetric flow rate and the average stress in the fluid:
Figure 2-21: Lubrication fluid flow model assuming the absence of foam. (a) At 0% strain (b) At any given strain $\epsilon$ [60]

\[ Q(r) = \pi rh^2 \left( \frac{-h}{2m} \frac{\partial P}{\partial r} \right)^{1/n} \frac{1}{1/n + 2} \]  

(2.39)

\[ \sigma_f = 2 \left( \frac{2n + 1}{n} \right)^n \frac{m}{n + 3} \left( \frac{R_0}{h_0} \right)^{n+1} \left( \frac{-\dot{h}}{h} \right)^n \left( \frac{1}{1 - \epsilon} \right)^{\frac{3(n+1)}{2}} \]  

(2.40)

where $R_0$ is the initial radius of the sample, $h_0$ is its initial height, $\epsilon$ is the strain and $\dot{h}/h$ is the instantaneous strain rate of the foam. To account for the tortuosity of the fluid path through the open-cell foam a constant is added to the equations. So finally the average stress in the fluid (which is the total force divided by the sample cross section area) is
\[ \sigma_{Avg} = 2C_R \left( \frac{2n + 1}{n} \right)^n \frac{m}{n + 3} \left( \frac{R_0}{h_0} \right)^{n+1} \left( \frac{-\dot{h}}{h} \right)^n \left( \frac{1}{1 - \epsilon} \right)^{\frac{3(n+1)}{2}} \] (2.41)

Tests done on cylindrical samples give good agreement in term of average stress for various strains. Test were conducted with an Instron 1321 or an Instron Droptower (see in Chapter on Experimentation) for a description of these apparatus). The results are summarized in Figure 2-22.
Figure 2-22: (a) True stress plotted against instantaneous strain rate for 70 ppi foam filled with 61% volume fraction silica based non-Newtonian fluid. Regimes R1- R4 correspond to fluid behavior regimes (b) True stress plotted against instantaneous strain rate for 70 ppi foam filled with 61% volume fraction silica based non-Newtonian fluid, ranging from 0.10 to 0.40 strain. Model corresponds to regimes R3 and R4 of the fluid given by 2.41 [60]
Chapter 3

Experiments

In this chapter, the experimental methods and the testing apparatuses used to obtain the experimental data are described. The results are then presented and analysed.

3.1 Shear thickening fluid preparation and characterization

3.1.1 Materials and method

Materials

For the suspension of the silica nanoparticles, we have used ethylene glycol (VWR, West Chester, PA). The density of ethylene glycol is $1.113g/cm^3$. The viscosity was measured with an ARG2 rheometer (TA Instruments, New Castle, DE), with a 40 mm aluminum cone. The viscosity at $22.5^\circ C$ of the solution is 16.5 mPa.s.

A similar test at $25^\circ C$ shows a viscosity of 13.5 mPa.s. The effect of exposure of ethylene glycol to ambient air on viscosity has been studied and no significant changes were observed. By comparison, glycerol incorporates moisture in air leading to a dramatic decrease in vis-
This can be confirmed by the fact that water and ethylene glycol have comparable viscosity and density $\mu_{\text{water}} = 1.002 mPa.s$ so incorporation should not change dramatically the rheological properties of the mixture, contrary to the effect of air moisture on glycerol for instance ($\mu_{\text{glycerol}} \approx 1000 \mu_{\text{water}}$).

The silica nanoparticles are produced by Fiber Optics Center Inc (New Bedford, MA). The description of the product is the following: AngstromSphere 0.25um Silica Spheres with Particle Size Standard Deviation 10 %, dry form.

According to the data given by the manufacturer the properties of the particles are the following:

- Diameter of the particle is $250 \pm 25\mu m$.

- The density, determined by pycnometry, is $\rho = 1.72 \pm 0.02 g/cm^3$.

- Purity > 99.9%

- Dielectric constant < 3.8

- The shape of the particle is spherical as we can see in Figure 3-1:
Figure 3-1: SEM image of the Nanoparticles given by the manufacturer

Observations made with the SEM tend to confirm the properties in term of size, distribution and shape (see Figure 3-2).
Suspension of the particles

Previous studies assume that the volume fraction at which shear thickening begins is about 50%. So the aim is to suspend the particles at about this volume fraction.

To achieve this goal a vortex mixer was used to mix the particles with the solvent (VWR Digital Vortex mixer, VWR, West Chester, PA), and an ultrasonic bath was used to ensure proper mixing (Branson Model 2200 Sonifier). The dilution process requires adding dry silica particles to ethylene glycol and then mixing the suspension with a digital vortex mixer. Sonication for one hour in an ultrasonic bath is supposed to finish the process [60]
It is observed that for low volume fraction solutions (below 30%), obtaining a very homogeneous solution is straightforward with this process. However, for volume fractions around 50% and higher, even if the solution seemed homogeneous after the dilution process, some small aggregates were noticed when putting the suspension in the rheometer. The observed aggregates, size around 0.5mm, which persist after vortex mixing, are broken into pieces after sonication.

After several tests using various suspension methods, the conclusion is that the best results have been obtained with a combination of progressive dilution, sonication plus heating. By gently heating the ethylene glycol during the mixing it gives a very good dispersion. The procedure used is the following.

- Prepare in a sealable tube the appropriate quantity of ethylene glycol (take into account that about 2% of the mass of ethylene glycol is going to evaporate during the preparation).
- Prepare in another tube the appropriate quantity of silica nanoparticles
- Preheat the ethylene glycol in a water-bath at 60 degrees Celsius.
- Add the silica particles to the ethylene glycol, in increments of about 10% of the total mass of silica
- After each step, use the vortex mixer to produce a visually homogeneous suspension. Then put the tube containing the mix in ultrasonic bath for 5 minutes, with the heating set at 60 degree Celsius.
- Finish the process by sonicating the suspension for one hour, at 60 degree Celsius.
- Before testing, put the mix in a vacuum oven (Vacuum Oven, VWR, West Chester, PA) for 15 minutes, at 60 degree Celsius, with a pressure vacuum of 25 inches of mercury. This will allow small bubbles to escape from the solution.
3.1.2 Rheological characterization of the shear thickening suspension

To measure the viscosity of the sample a classic parallel disk geometry was used, in an ARG2 rheometer (TA Instruments, New Castle, DE), with a 40 mm diameter aluminium plate equipped with a solvent trap. The main limitations of the rheometer are the following:

- Torque (L) for steady state flow: $0.01 \, \mu \text{N.m} < L < 200 \, \text{mN.m}$

- Angular velocity for steady state flow: $0 \, \text{rad/s} < \dot{\theta} < 300 \, \text{rad/s}$ for stress controlled measurement and $1.4 \times 10^{-9} \, \text{rad/s} < \dot{\theta} < 300 \, \text{rad/s}$ for strain controlled measurement.

Previous studies [80, 60] have shown that the plates must be coated with a rough surface to prevent slip at the boundary. The ideal case is to coat the geometry and the bottom plate with particles similar to the one suspended in the solution, however this option is difficult to
realize and sandpaper tends to give similar results.

Based on Dawson et al.'s [60] previous work on very similar measurements, first some 1000 grit sandpaper was used. (6" PSA Abrasive Discs 1000 grit, 4S Premium Stearated Aluminum Oxide. Brownell Industrial Supply, Attleboro, MA). Waterproofness of the sandpaper did not seemed to be optimal. Furthermore, the size of the particles used for 1000 Grit sandpaper is around 18.3μm, much larger than the silica nanoparticles, so it was decided to use a roughness as close as possible to the particle size. Finally a 2000 Grit sandpaper was used (particle size around 10.3 μm Disc PSA 5×0 2000 Film BAC TAB McMaster-Carr (PT No 809775-53082)). To coat the geometry a disk was cut to fit best the geometry, and then adjusted it once it is adhered to the plate. A similar technique was used to coat the bottom plate.
Figure 3-5 shows a scanning electron micrograph of the sandpaper. A elemental analysis tends to show that the abrasive particles are $Al_2O_3$.

During all the measurements, to prevent evaporation, the solvent trap of the rheometer was used.

**Testing procedure**

As stated before, the device used is an ARG2 rheometer, with a 40 mm diameter aluminium plate. Both the bottom plate and the geometry are coated with the aforementioned sandpaper. A gap of 165 $\mu m$ was set between the bottom and the top plates, which is more than 15 times the average size particle of the sandpaper used, and allows tests in all of the samples with the same gap. Indeed for lower volume fraction samples, the centrifugal force tends to be important at higher shear rates (fluid expulsion observed for velocity around 60 rad/s).
After loading the sample, the geometry is lowered, by steps of 100 μm from a 1500 μm gap to 300 μm, followed by steps of 50 μm to reach 200 μm. The final step lowers the geometry to a 165 μm gap.

• **1- Preshear:** After elimination of the excess of sample, we proceed to a short preshear. This preshear is to "erase" the memory of the fluid. Basing our choice on previous work (Wagner, 2001, Matthew Dawson 2008), we apply a stress control steady state, with ten points per decade, from 0.5 to 4700 Pa with maximum duration point of 30 s.

• **2 - Rest 1:** Let the fluid recover during 2 minutes.

• **3 - Steady state flow 1:** Apply a stress sweep from 1 Pa to the maximum stress that can be imposed by the rheometer (namely 15940 Pa with the 40 mm plate), with 10 points per decade of stress. The sample was allowed to equilibrate for 10 s before each stress measurement. The viscosity was then averaging on a 20 s period. The maximum stress for dilute suspension was dictated by centrifugal force.

• Then operations 2 and 3 are repeated.

• **4- Rest 2**

• **5 - Steady state flow 2**
3.1.3 Results

Reproducibility

One of our major goals was to obtain reproducible results. Thanks to the preparation protocol and the preshear, results obtained are highly reproducible. Figure gives an illustration of two viscosity measurements for the same concentration, but with different samples (same preparation, but changing the samples between the two measurements).

![Viscosity vs shear stress graph]

For high volume fractions ($\Phi > 0.59$), we observe a strong shear thickening. This shear thickening can result in two different values of viscosity for a same shear rate, given that we are working at an imposed shear stress.
Summary of the results

Figure 3-7: Viscosity vs shear rate for a volume fraction ($\Phi$) of silica of 0.615

Figure 3-8: Viscosity vs shear rate for various volume fractions ($\Phi$) of silica
Concentrations of the suspension were determined by drying a portion of the sample and weighing the sample before and after drying. The samples were dried at 80 degrees Celsius, under a vacuum of 30 inches of mercury until no change in mass had been recorded for 6 hours.

**Limit of High Strain rate**

Given the limitations of the rheometer ARG2 in the maximum applied torque, a smaller geometry has been used to study the shear thickening suspension at higher shear rate. The geometry used is a 8 mm diameter steel cone. The preshear and testing procedure are the same as previously, the only difference is that the gap between the geometry and the bottom plate was not conserved from one experiment to another. Indeed, given the extremely small volume that is needed to do the measurement, and the high viscosity at low shear rate, the technique used to introduce the sample is the following:

- Apply a drop of the sample on the geometry with a spatula.
Figure 3-10: (a) Sample on the geometry, (b) Sample ready to be tested

- Lower the geometry to the point were the sample presents a straight meniscus

Consequently the gap set for the viscosity measurement was between 165 \( \mu m \) and 300 \( \mu m \). The influence of the gap on the viscosity measurements was studied on the same sample and did not lead to any discrepancy in the measurements.

As we can see in Figure 3-11, viscosity reaches a maximum before another decrease. The maximum shear rate or shear stress attained is dictated by centrifugal effects which tends to eject the sample out of the geometry.
Figure 3-11: Viscosity vs shear stress for various volume fractions (Φ) of silica

Figure 3-12: Viscosity vs shear rate for various volume fractions (Φ) of silica

The hydroelastic model plotted is based on the work of Kalman and al. [1].
At high stresses, two particles elastically deform when they come into close contact, with a thin fluid layer between them. This phenomenon is treated as an elastohydrodynamic deformation with Hertzian contact, similar to previous models of pastes that treat the deformation of a single particle near a wall [92]. Meeker et al. [92] argues that the stress resulting from the lubrication forces acting between elastically deformable particles (with Hertzian contacts) scales as:

$$\sigma \sim \left( \frac{\eta_s V G_0}{R} \right)^{1/2} \left( \frac{G_0}{G_p} \right)^{1/6}$$  \hspace{1cm} (3.1)$$

where $\sigma$ is the stress, $\eta_s$ is the solvent viscosity, $V$ is the plate velocity, $R$ is the plate radius, $G_0$ is the particle shear modulus, and $G_p$ is the particle Young's modulus. Assuming that $(G_0/G_p)^{1/6} \approx O(1)$, the scaling of the stress as a function of shear rate becomes:

$$\sigma \sim (V/R)^{1/2}(\eta_s G_0)^{1/2} = (\dot{\gamma})^{1/2}(\eta_s G_0)^{1/2}$$  \hspace{1cm} (3.2)$$

this can also be rewritten as:

$$\eta \sim (\dot{\gamma})^{-1/2}(\eta_s G_0)^{1/2}$$  \hspace{1cm} (3.3)$$

For more details on the hydroelastic model see the Appendix section at the end of this chapter. For our model we have chosen $\eta_s = 16.5mPa.s$, which is the viscosity of ethylene glycol at 22.5 degrees Celsius. The particle shear modulus $G_0 = 31$ GPa has been chosen following the data given by the manufacturer.

Results are similar to the one obtain by Kalman et al. on figure 3-13
3.1.4 Discussion

The prepared suspension shows the characteristic behavior of a shear-thickening fluid: as the stress is increased the viscosity first decreases. When a critical shear stress is reached (which depends on the volume fraction of silica nanoparticles) the viscosity increases very rapidly. Limitations due to the testing apparatus: e.g. a limit of a maximum torque of 200mN/m, and in the centrifugal forces in the solution in the geometry used, limit the analysis to a range of shear stress. So it is hard to predict the behavior of the solution after this phase of increase in viscosity. The theoretical upper limit described by Meeker et al. [92] gives us several ideas of what can really happen. One hypothesis is that the viscosity in the solution keeps increasing as stress is increased until it actually reaches the limit plotted in Figure 3-12. Indeed the apparent decrease in viscosity observed in Figure 3-12 and in Figure 3-13 could be due to slip of the extremely viscous suspension along the plates of the rheometer.

3.1.5 Conclusion

A method to prepare a shear thickening suspension has been tested with success and the rheological characterization shows the desired behavior. This shear thickening suspension can be a good candidate to impregnate foams in order to provide a rate-dependent responsive
Figure 3-14: (a) Conventional NDI 90PPI sample, (b) Conventional NDI 90PPI sample

material. This aspect will be developed further in the section on the impact test, and in the chapter on modeling of the system.

3.2 Negative-Poisson's ratio foam

3.2.1 Materials and method

General fabrication procedure

For the tests, four types of materials were used. First of all, specimens were open-cell, polyurethane-based polyester foams (New Dimensions Industries, Moonarchie, NJ), with average cell diameters of 175 μm (90 PPI type), 210 μm (80 PPI) and 235 μm (70 PPI). Relative density of the foams was calculated using the manufacturer's value for the density of the polyurethane (ρₖ = 1.078g/cm³), corresponding to a relative density ρₖ/ρₛ ≈ 0.03. The dimensions of each sample were measured with a digital caliper, capable of an accuracy of 0.01mm. SEM images of foam samples are given in Figure 3-14.

The foam specimens used in our study were cut with a hot wire cutter, enabling accurate rectangular blocks to be cut with very smooth faces. The flexible foam, with a low Young's
modulus, can easily be squeezed into a mold to achieve tri-axial compression. The method used is based on the technique published by Friis et al. [93] and developed by Chan et al. [94], and described below.

The VWR oven is preheated to 210 degree C. The square section mold (with inner dimensions: 35.07 mm x 35.07 mm x 96.45 mm) is used for the foam, which is cut oversize (example 47.34 mm x 47.34 mm x 102.98 mm). The inner walls of the mold are then lubricated with some general purpose lubricant (WD40), and the foam is then placed into the mold, which is then progressively set up to its final position, and then held clamped. The compressed foam with the buckled cell edges is then placed in the oven at 210 degree C for 46 minutes to ”set” the new configuration. The heating time is very critical because the transformation temperature of the foam does not reach the oven temperature. The establishment of the correct heating time is discussed below. The mold is then removed from the oven and cooled at room temperature for 40 minutes. Finally the foam is taken out of the mold and gently stretched in each of the three orthogonal directions to overcome any adhesion of the cell ribs.

**Tri-axial compression mold**

The tri-axial compression mold is designed in accordance with Chan et al. [94] The mold is composed of 6 parts, as represented in Figure 3-15, and a picture is shown in Figure 3-16.
Figure 3-15: Schematic of the mold used to compress tri-axially the foam

Figure 3-16: Triaxial compression Mold
Determination of softening temperature - DSC measurement

Determining the softening temperature of the foam is critical, because it will determine the temperature at which the foam can be deformed permanently without melting the sample.

Lakes [61] did not report how he determined the foam temperature, and Chan et al. [94] used a small specimen of foam, placed inside a test tube, and heated it using a Bunsen burner. A thermocouple was inserted in the middle of the sample to measure the temperature. The foam softening temperature was recorded when the cell ribs began to collapse. However, this method is not very accurate and is valid only to give estimate of the softening temperature.

The softening temperature given by the manufacturer for the NDI foam is 193 degree Celsius. This temperature has been checked thanks to a Perkin Elmer Diamond Differential Scanning Calorimeter (DSC), which is a power compensated differential calorimeter, that can be used to find heat flow, melting temperature and glass transition temperature.

The DSC measurement was performed with a ramp of 20 degree Celsius per minute between 90 and 250 degree Celsius.

The first change in slope of heat flow vs temperature in the sample was recorded around 188°C. Given the low density of the foam, samples used to determine softening temperature weighed around 6 mg, so a precision of 5 degree Celsius is reasonable.
As suggested by Chan et al., the conversion temperature of the foam should be 5 to 20 degree Celsius lower than the softening temperature, in order to maximize stress relaxation and minimize cell-rib adhesion. Therefore a temperature in the foam around 180 degree Celsius is a suitable target for our experiment.

**Determination of softening temperature - Experimental procedure**

Under static loading conditions, heating allows permanent deformation of the strained foam. At the softening temperature, stress in the foam relaxes to zero; so that a conventional foam with an outward cell structure can be converted into a re-entrant cell structure.

As noticed by Chan et al., if the heating time is too short, the foam cannot be "set", with the result that, after it comes out of the mould, the foam soon expands to its original size as all the internal stress has not been relaxed.

On the other hand, if the heating time is too long, the foam will be either melt, or can even decompose. In order to maximize the stress relaxation process and minimize sticking and
structural collapse it is very important to determine the correct heating time.

The process to study the heating time is the following: the oven is set to a temperature of 200 degree Celsius, and the sample is compressed in the mold with a thermocouple at the foam/mold interface and another one inserted in the middle of the sample. Time-temperature profiles were studied to determine a suitable heating time.

In Figure 3-18, we can see that for a heating time of 46 minutes, the temperature in the middle of the sample reaches 178 degree Celsius and the temperature of the interface foam/sample (185 degree Celsius) does not exceed the softening temperature, so that the whole sample is in a range of temperature 5 to 20 degree Celsius lower than the softening temperature.
Characterization of negative Poisson’s ratio foam

Samples were tested in an Intron 4201, with a 500N load cell. The crosshead speed was chosen to be 3mm/min which corresponds to 0.05 mm/s. Given the size of the samples, this corresponds to a strain rate less than 0.002/s which is consistent with the choice made by Friis et al. [93].

A set of 4 points was drawn on the samples, in order to measure the deformation both in the axial and radial directions, as shown in Figure 3-19.

Load and displacement were recorded with a Labview interface, using the analogical outputs of the Instron 4201 and a 16 bit analog to digital converter. Data to determine Poisson’s ratio were acquired with a digital camera 9 MegaPixels (Panasonic TZ5).
Figure 3-19: **Left** Sample preparation for testing **Right** Example of Sample during a tensile test

**Tensile tests**

To perform tensile tests, tensile fixtures were used and samples were cemented onto the surface of the testing device with a cyanoacrylate adhesive (Mc Master Carr, Loctite 454, Prism Surface Insensitive Gel). As suggested by Friis et al. [93], special care was taken to ensure that specimens were centered directly with the line of action of the ram of the machine. For these tests, the entire sample was used. Samples were tested up to 20% strain, to avoid ripping them apart.

**Compression tests**

Uniaxial compression test were performed with compression fixtures. To limit friction, fixtures were coated with TEFLON back-adhesive sheets. To avoid buckling of the whole sample, axial extremities of specimens were cut to obtain samples of 40 mm long. Samples
Figure 3-20: Tensile test with a conventional foam sample were compressed up to 50% strain. Figure 3-21 shows the configuration for the compression tests.
3.2.2 Results

Fabrication of re-entrant structure

Images of the foam, taken with a Scanning Electron Microscope (see Figure 3-23) were taken to ensure the re-entrant structure of the foam after the process of conversion of the foam. Foam samples dimension were measured just after the preparation process, and were recorded as stable over a time period of 3 weeks, so the process seems to be a durable modification of the foam.
Figure 3-22: (a) Re-entrant foam-volumic compression factor = 2.3 - foam type: NDI 90PPI sample-SEM image, (b) Re-entrant foam-volumic compression factor = 2.3 - foam type: NDI 90PPI sample-SEM image

Figure 3-23: (a) Re-entrant foam-volumic compression factor = 3 - foam type: NDI 90PPI sample-SEM image, (b) Re-entrant foam-volumic compression factor = 3 - foam type: NDI 90PPI sample-SEM image
Characterization of the Poisson’s ratio of the re-entrant structure

Finally samples with a volumic compression factor of 1.5, 1.7, 2.3 and 3.5 were fabricated and tested. Results are shown in Figure 3-24.

Figure 3-24: Poisson Ratio vs Strain for 90PPI foam

As can be seen in Figure 3-25, the optimal compression factor is around 2.3. We can now compare Poisson’s ratios of conventional and modified foams.

Similar results were found with the 70 PPI foam, as shown in Figure 3-27.
Figure 3-25: Poisson Ratio vs Volumetric compression factor for 90PPI foam
Figure 3-26: Poisson Ratio vs Strain for 90PPI foam modified and conventional samples
Figure 3-27: Poisson Ratio vs Strain for 70PPI foam modified and conventional samples
3.2.3 Discussion

In this section we transformed a conventional foam into a re-entrant structure. The characterization of the Poisson’s ratio over a wide range of strain, for tensile and compressive tests showed the desired behavior. After the process to transform a conventional foam into a re-entrant structure, a negative Poisson’s ratio was observed for the 70PPi and and the 90PPi open-cell reticulated flexible polyurethane based foam, both in compression and in tensile tests. The size of the sample realized is smaller than the size actually needed for a helmet pad, but the method (determination of the softening temperature, characterization method of the Poisson’s ratio) is validated so samples of larger size could be made with appropriate tools (larger size oven, precision bandsaw to cut the foam, etc).

3.2.4 Conclusion

This type of re-entrant structure can be of interest for numerous applications [62, 63]. Preparing a negative Poisson’s ratio foam and characterizing it was a great achievement, but given the small value of this Poisson’s ratio, the chapter on modelling will show that the pore size modification when compressed, compared to a conventional foam, will not play a significant role in the impact response for a fluid filled foam.
3.3 Fluid filled foam and polystyrene impact tests

3.3.1 Materials and method

Materials

For the tests, four types of materials were used. First of all, specimens of open-cell, polyurethane-based polyester foams (New Dimensions Industries, Moonarchie, NJ), with average cell diameters of 175 μm (90 PPI type), 210 μm (80 PPI) and 235 μm (70 PPI). Relative density of the foams was calculated using the manufacturer’s value for the density of the polyurethane (ρs = 1.078g/cm³), corresponding to a relative density ρf/ρs ≈ 0.03. The foam was then cut using a hot-wire cutter, into uniform parallelepipeds with a square basis measuring 100 × 100 mm, and variable height. The dimensions of each sample were measured with a digital caliper, capable of an accuracy of 0.01mm.

The Newtonian fluid used to impregnate the polyurethane samples is glycerol (VWR, West Chester, PA), where the density is reported to be ρ = 1260kg/m³ and viscosity μ = 1.1 Pa.s at 23 °C. The polystyrene used is a high-density polystyrene (Cordek, West Sussex, U.K.) with density 0.055g/cm³. Samples were also cut using a band-saw in parallelepipeds of 100 × 100 mm, with a height of 25mm. The ABS used (McMaster Carr, supply Co., Atlanta, GA) has a density of 1.08g/cm³.

To prepare the shear thickening fluid, the same materials as those reported in the section on the shear thickening fluid have been used. The shear thickening fluid is prepared according to the process identified earlier, at a volumetric concentration of 61% of silica particles.

Open-cell polyurethane foam samples impregnated with a fluid are manually filled with the appropriate fluid, and then allowed to rest for two hours, in a air-sealed container, as recommended by previous studies (M.Dawson, PhD thesis [60]).
Experimental testing apparatuses

In order to test various materials under reproducible conditions, an Dynatup Drop-Tower (Dynatup 9250 HVSeries, Instron Corp., Canton, MA) was used. Schematic of the apparatus is given in Figure 3-29 The load cell used has a maximum load of 45kN, the settings of the systems are the following: sampling period of 75ms, sampling rate of 5MHz. The mass of the impactor used is 7.3 kg, and for energies up to 55 J, the impactor is raised above the sample (up to a height of 77 cm), and then is dropped, falling under gravity’s action. For greater energies, the impactor is raised to compress two springs (spring constant 3.5 kN/m), to be able to achieve impacting speed up to 20 m/s and energies up to 1603 J. The impactor is a cylinder of diameter 60 mm, and all the fixtures are those provided by Instron.
Figure 3-29: (a) Instron Dynatup Droptower 9250HV Schematics (From Instron User’s Manual)
Experimental procedure

Samples are loaded in the Dynatup Drop-Tower a zero gap is performed using a sheet of plastic paper, to determine the point of contact between the sample and the impactor. This point is then referenced as 0 strain. During multiple loading tests, samples are simply re-centered in case the various rebounds of the impactor may have moved them. Figure 3-30 gives a representation of the configuration for an impact test. Temperature in the testing room is maintained at 23°C during all the experiments.

Figure 3-30: Schematic of an impact test realized with a sample in the Instron Droptower
3.3.2 Results

Impact tests data

Results are presented for glycerol-impregnated foam and for the polystyrene foam layer. The Instron Droptower gives displacement, velocity of the impactor, as well as the load recorded by the load cell, as a function of time. Given the fact that the impactor is first impacting the aluminum sheet, some noise is introduced in the first milliseconds of the data, so raw data as well as smoothed data (time step of 0.5 ms) are given. Figure 3-32 gives displacement,
velocity and load as a function of time (t=0 is the instant the impactor reaches the sample), for a 30 J impact on a 16 mm-thick glycerol filled foam layer. The Figure also shows load versus displacement.

Figure 3-32: Example of a 16 mm thick Glycerol Impregnated foam impact response to a 30J impact

Figure 3-33 gives a summary of the results for 3 different energies for glycerol-filled foam tests, showing load versus displacement.
Figure 3-33: 16 mm thick Glycerol Impregnated foam impact response for 3 different energies
Figure 3-34: 16 mm thick Glycerol Impregnated foam impact response for 3 different energies
Figure 3-35: 16 mm thick Glycerol Impregnated foam impact response for 3 different energies
Figure 3-36: 16 mm thick Glycerol Impregnated foam impact response for 3 different energies

Figure 3-37 gives displacement, velocity and load as a function of time (t=0 is the instant the impactor reaches the sample), for a 70J impact on a 25 mm-thick polystyrene layer. The Figure also shows load versus displacement.
Figure 3-37: Example of a 25 mm thick polystyrene layer impact response to a 70J impact.

Figure 3-38 gives a summary of the results for 3 different energies for polystyrene tests, showing load versus displacement.
Figure 3-38: 25 mm thick polystyrene layer impact response for 3 different energies
Figure 3-39: 25 mm thick polystyrene layer impact response for 3 different energies
Figure 3-40: 25 mm thick polystyrene layer impact response for 3 different energies
Multiple impact tests have been conducted to study the response of the polystyrene and of the glycerol filled foam to successive impacts. Tests have been performed with an impact energy of 70J and samples were simply re-centered between two impacts. Tests have been performed on a 12mm thick polystyrene sheet, a 18 mm thick polystyrene sheet and a 13 mm thick polyurethane open cell foam impregnated with glycerol. The goal was to perform
5 successive impacts. This was achieved for all the sample tested except for the 12 mm thick polystyrene one, for which the fourth impact reached a peak load close to the maximum value the load cell can record (45kN), so no additional impact was performed on this sample to avoid risk of damage to the load cell. Results are shown in Figure 3-42.

![Figure 3-42: Peak Load for 5 successive 70J impacts on three types of sample.](image)

3.3.3 Discussion

The Instron Droptower allowed a complete characterization of the materials under study, for the 3 impact energies chosen. There are clearly some shortcomings exhibited by each of the materials alone. Indeed the glycerol filled foam sample ends up in acceleration (or load) higher than the defined threshold for the 130J energy test (150g, which is equivalent to 10,600 N for the impactor used). Furthermore the density of the glycerol (1.260 g/cm³) adds weight compared to a layer of polystyrene of the same thickness. On the other hand, multiple
impacts on the same glycerol-filled foam sample shows little variation in absorbing abilities. On the contrary, the polystyrene layer has excellent impact absorption characteristics, but after one impact the plastic deformation is such that subsequent impacts induce transmitted loads way higher than the first one, exceeding the acceptable threshold.

### 3.3.4 Conclusion

In this section the polystyrene and glycerol-filled foam response to 3 energies of impact have been characterized. Each type of material seems to exhibit interesting characteristics. Experimental data allows the development of models to simulate the response of composite materials to try to identify an optimal design.
**Appendix: Elastohydrodynamic model**

In this section, we present the case of an elastic particle, in the situation of elastohydrodynamic slip. We consider a spherical particle of radius $R$, and elastic modulus $G_p$ pressed against a wall moving at velocity $V$.

Figure 3-43: Schematic view of the facet of the deformed particle and the lubricated film formed, when the particle is squeezed against a wall.

Considering the case presented in Figure 3-43, equations for the elastohydrodynamic interaction between the particle and the wall are given by [95]:

\[
\nabla \cdot [\delta^3(x,y) \nabla p(x,y)] = -6\eta_s V \frac{\partial \delta(x,y)}{\partial x}
\]

(3.4)

\[
\delta(x,y) = -h_0 + \frac{r^2}{2R} + w(x,y)
\]

(3.5)

\[
w(x,y) = \frac{1}{G_p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p(\xi,\eta)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} \, d\eta d\xi
\]

(3.6)

In this system, $p$ is the pressure, $x$ and $y$ and $\xi$ and $\eta$ are the Cartesian coordinates in the...
plane of motion of the particle, \( r = (x^2 + y^2)^{1/2} \) is the radial coordinate parallel to the wall, \( w \) is the elastic deformation, and \( \nabla \) is the two-dimensional gradient operator. We assume the deformation to be linear, as Meeker does [92], so \( G_p = 2E/(1 - \nu^2) \), \( E \) is the Young's modulus of the particle, and \( \nu \) is the Poisson ratio.

Equation 3.4 represents the hydrodynamic flow in the lubricated layer between the particle and the wall. Equation 3.6 represents the elastic deformation due to the hydrodynamic pressure field. Equation 3.5 couples the two aforementioned equations with a geometrical representation of the deformation.

At rest, the contact between the elastic particle and the wall can be treated as Hertzian contact. The radius of the flat area \( r_0 \) and the pressure in this zone are given by (Johnson, 1985 [96]).

\[
\begin{align*}
\xi_0 &= h_0/R, \\
r_0 &= R\xi_0^{1/2} \quad \text{and} \quad p_0 = \frac{2G_p}{\pi^2} \xi_0^{1/2}
\end{align*}
\]  

(3.7)

\( \xi_0 = h_0/R, \) and is defined as the compression ratio. We can deduce \( \xi_0 \) from the balance between osmotic pressure and contact stress of the particle. Osmotic pressure is expected to be such that \( P_{osmotique} \sim G_0 [97] \). \( G_0 \) is the shear modulus. The normal stress acting on a particle is \( p_0(r_0/R)^2 \sim G_p\xi_0^{1/2} \). So finally

\[
\xi_0 \sim \left(\frac{G_0}{G_p}\right)^{2/3}
\]  

(3.8)

During the flow, if we consider that the lubricated layer dimension is small compared with the compression of the particle at rest \( \delta \ll h_0 \), then the particle compression in the flow is not very different to that at rest \( oh \approx h_0; r_0 \) is not changed. The hydrodynamic pressure \( p \) is of order \( p_0 \).

Using (1), we can deduce that:
\[ \frac{\delta^3 P_0}{r_0^2} \sim \frac{\eta_s V \delta}{r_0} \]  

so that the film thickness \( \delta \) is

\[ \delta \sim (\eta_s V R/G_p)^{1/2} \]  

The viscous drag on a particle scales as

\[ F_d \sim \frac{\eta_s V}{\delta^2 r_0^2} \quad \text{or} \quad F_d \sim \xi_0 (\eta_s) V R^3 G_p 1/2 \]  

Finally, we can express the shear stress

\[ \sigma \sim \frac{F_d}{R^2} \sim \left( \frac{\eta_s V G_0}{R} \right)^{1/2} \left( \frac{G_0}{G_p} \right)^{1/6} \]
Chapter 4

Modeling

Designing helmet liners using only experiments is a time-consuming and very expensive process. Furthermore, the results obtained can not be used to design helmets with with a different set of constraints. The aim of this chapter is to suggest physical models which predict accurately the impact response of the materials of interest (polystyrene and fluid filled foam) and to compare these predictions with the experimental data. The model will then be used in the following chapter to suggest an optimal design, based on an optimization under constraints. Parameters of interest will be identified throughout the physical analysis in this chapter and will be varied in the optimization algorithm.

4.1 Polystyrene models

Polystyrene foam is a very lightweight material, renowned in the motorcycle helmet industry for its very good impact absorption features. In this section we suggest two models to account for the impact response of a polystyrene foam sheet and find the relevant system of mechanical equations that will then be solved numerically.
4.1.1 SLS model

Introduction

The standard linear solid (SLS) model, also known as the Zener model, is a method of modeling the behavior of a viscoelastic material using a combination of springs and dashpots in series and in parallel to represent elastic and viscous components, respectively. Previously, the Maxwell and the Kelvin–Voigt models have been studied. But these models proved insufficient, as the Maxwell model does not describe creep well, and the Kelvin–Voigt model does not describe stress relaxation well. SLS is the simplest model that describes both phenomena. Furthermore, with this model it becomes possible to ensure initial conditions corresponding to what is experimentally observed, namely an initial displacement and stress equal to zero, at t=0.

Solving the model

The SLS model is composed of a spring in parallel with a Maxwell element (spring and dashpot in series), as shown in Figure 4-1. Springs, which represent the elastic component of a viscoelastic material, obey Hooke’s Law:

\[ \sigma_{Spring} = E \epsilon \]  \hspace{1cm} (4.1)

where \( \sigma \) is the applied stress, \( E \) is the Young’s Modulus of the material, and \( \epsilon \) is the strain.

On the other hand, the dashpot represents the viscous component of a viscoelastic material. That means that the applied stress depends on the time rate of change of the strain:

\[ \sigma_{Dashpot} = \eta \frac{d\epsilon}{dt} \]  \hspace{1cm} (4.2)

where \( \eta \) is the viscosity of the dashpot component.

Figure 4-1 shows how the three elements of the model are connected.
The following physical relations have to be satisfied.

For parallel components $\sigma_{\text{total}} = \sigma_1 + \sigma_2$ and $\epsilon_{\text{total}} = \epsilon_1 = \epsilon_2 = \epsilon$

For series components $\sigma_2 = \sigma_{S2} = \sigma_D$ and $\epsilon_2 = \epsilon_{S2} + \epsilon_D$

Where indices 1 refers to stress and strain in branch 1 of the model, indices 2 refers to same quantities in branch 2, and S2 refers to the spring in branch 2, and D to the dashpot in branch 2.

Then we can deduce

$$\sigma_1 = E_1 \epsilon$$ (4.3)
\[ \dot{\epsilon}_2 = \frac{\dot{\sigma}_2}{E_2} + \frac{\sigma_2}{\eta} \quad (4.4) \]

\[ \sigma = \sigma_1 + \sigma_2 = E_1 \epsilon + \eta \dot{\epsilon} - \frac{\dot{\sigma}_2}{E_2} \eta \quad (4.5) \]

\[ \dot{\sigma}_2 = \dot{\sigma} - \dot{\sigma}_1 = \dot{\sigma} - E_1 \dot{\epsilon} \quad (4.6) \]

So that finally

\[ \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E_2} = \left( \frac{E_1 + E_2}{E_2} \right) \dot{\epsilon} + \frac{E_1 \epsilon}{\eta} \quad (4.7) \]

Using the definition of stress and strain, \( \sigma = F/A \) and \( \epsilon = x/h \), with \( A \) the area of the sample impacted, and \( h \) its height, we introduce \( k = AE/h \) and \( \mu = A\eta/h \), to end up with the following force displacement relationship:

\[ \frac{F}{\mu} + \frac{\dot{F}}{k_2} = \left( \frac{k_1 + k_2}{k_2} \right) \ddot{x} + \frac{k_1 x}{\mu} \quad (4.8) \]

Using dynamics’ fundamental principle for the mass \( M \) we can write

\[ M \ddot{x} = Mg - F + F_i \quad (4.9) \]

We assume that as soon as the impactor is in contact with the sample, then \( \dot{x}_{Impactor} = \dot{x}_{Sample} = \dot{x} \), so that using dynamics’ fundamental principle for the impactor:

\[ m \ddot{x} = mg - F_i \quad (4.10) \]

So finally

\[ M_t \ddot{x} = M_t g - F \quad (4.11) \]
with \( M_t = M + m \).

This relationship is valid as long as the impactor and the sample are in contact.

We can now solve the following system

\[
M_t \ddot{x} = M_t g - F
\]  

(4.12)

\[
\frac{F}{\mu} + \frac{\ddot{F}}{k_2} = \left( \frac{k_1 + k_2}{k_2} \right) \ddot{x} + \frac{k_1 x}{\mu}
\]  

(4.13)

\[
x(t = 0) = 0
\]  

(4.14)

\[
\dot{x}(t = 0) = V_0
\]  

(4.15)

\[
F(t = 0) = 0
\]  

(4.16)

Analytic solutions of this linear system exist, but finding the eigenvalue and eigenvectors of the linear transformation end up being highly demanding for computational resources, so the system has been solved with a standard ODE solver in Matlab (ode23). To solve this system we introduce the following vector:

\[
Y = \begin{bmatrix} x \\ \dot{x} \\ F \end{bmatrix} \quad \text{so that} \quad \dot{Y} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{F} \end{bmatrix}
\]  

(4.17)

So finally
\[
\dot{Y} = MY + g \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{with} \quad M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ k_1 k_2 / \mu & k_1 + k_2 & -k_2 / \mu \end{bmatrix}
\] (4.18)

Comparison of the results predicted by the model and experimental data shows that the Standard Linear Solid model fails to account for the plastic behavior exhibited by polystyrene during the impact. Good agreement is obtained for each of the data set (30J, 70J and 130J) as shown in Figures 4-2,4-3,4-4,4-5, but optimizations of the parameters \(k_1, k_2\) and \(\mu\) shows that different values are found for the three energies. This proves that the model is not able to represent the system, indeed the strain rate dependence of the material is supposed to be taken into account by the dashpot element.

![Displacement versus time](image1)

![Velocity versus time](image2)

![Load versus time](image3)

![Load versus Displacement](image4)

Figure 4-2: SLS Model response and Experimental data 30J impact
Figure 4-3: SLS Model response and Experimental data 70J impact
Figure 4-4: SLS Model response and Experimental data 70J impact-Displacement versus time and Velocity versus time
Best fit values for the parameters have been identified using a minimization of the distance between curves of experimental data and model prediction. Values of $k_1$, $k_2$, and $\mu$ are given in table 4.1.

<table>
<thead>
<tr>
<th>Energy</th>
<th>Parameters</th>
<th>$k_1$ (N/m)</th>
<th>$k_2$ (N/m)</th>
<th>$\mu$ (N.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30J Impact</td>
<td>1 300 000</td>
<td>5 000 000</td>
<td>3 300</td>
<td></td>
</tr>
<tr>
<td>70J Impact</td>
<td>700 000</td>
<td>5 000 000</td>
<td>3 300</td>
<td></td>
</tr>
<tr>
<td>130J Impact</td>
<td>100 000</td>
<td>2 000 000</td>
<td>4 300</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Optimal SLS parameters value for the three types of impact

However, from the computed Load-displacement graphs, one can see that SLS model recovers all the deformation. On the contrary, experiments show that there is an irrecoverable deformation. This behavior cannot be neglected as it is an important way of dissipating
energy for materials such as polystyrene. This leads to the consideration of models including a plastic behavior.

4.1.2 Friction pot

Introduction

To take into account plasticity, which is an important way of absorbing energy in the particular case of polystyrene, a friction-pot is introduced in the model.

A friction pot is a mechanical element with the following properties: below a certain threshold ($\sigma_F$ or $F = K$), the element is equivalent to a rigid element, with no mechanical deformation. When $\sigma_F$ is reached, then the stress in the element is equal to $\sigma_F$, no matter what is the displacement, or the strain rate.

Several models have been implemented and identified. A schematic of the one showing the best concordance with experimental data is shown on Figure 4-6.

Solving the model

The friction-pot model we solve here is shown on Figure 4-6, since it is the one that models the polystyrene most accurately. The friction pot model is composed of two branches, each composed of a spring in series with a friction element, in parallel with a Maxwell element (spring and dashpot in series). As previously, springs, which represent the elastic component of a viscoelastic material, obey Hooke’s Law:

$$\sigma_{Spring} = E\epsilon$$  \hspace{1cm} (4.19)

where $\sigma$ is the applied stress, $E$ is the Young’s Modulus of the material, and $\epsilon$ is the strain.

On the other hand, dashpot represents the viscous component of a viscoelastic material. That means that the applied stress depends on the time rate of change in the strain:
\[ \sigma_{Dashpot} = \eta \frac{de}{dt} \] (4.20)

where \( \eta \) is the viscosity of the dashpot component.

And finally, friction elements represents the plastic behavior of the material under study, so that

If \( \sigma < K \), \( \epsilon_{Friction} = 0 \), otherwise \( \sigma = K \) (4.21)

Figure 4-6 shows how the three elements of the model are connected.

![Figure 4-6: Friction-pot model](image)

The following physical relations have to be satisfied.
For parallel components $\sigma_{total} = \sigma_1 + \sigma_2 + \sigma_3$ and $\epsilon_{total} = \epsilon_1 = \epsilon_2 = \epsilon_3$

For series components $\sigma_1 = \sigma_{S1} = \sigma_{F1}$ and $\sigma_2 = \sigma_{S2} = \sigma_{F2}$ and $\epsilon_3 = \epsilon_{S3} + \epsilon_D$

Where indices 1 refers to stress and strain in branch 1 of the model, indices 2 refers to same quantities in branch 2, indices 3 refers to same quantities in branch 3, and S refers to the springs, and D to the dashpot, and F to the friction elements. In this model we assume $K_1 < K_2$ and $E_1 > E_2$ to ensure the uniqueness of the solution.

Then we can deduce

When $\sigma_1 < K_1$ and $\sigma_2 < K_2$ \hspace{0.5cm} (both friction pots stationary) \hspace{1cm} (4.22)

\[ \sigma_1 = E_1 \epsilon \] (4.23)

\[ \sigma_2 = E_2 \epsilon \] (4.24)

\[ \dot{\epsilon}_3 = \frac{\dot{\sigma}_3}{E_3} + \frac{\sigma_3}{\eta} \] (4.25)

\[ \sigma = \sigma_1 + \sigma_2 + \sigma_3 = E_1 \epsilon + E_2 \epsilon + \eta \dot{\epsilon} - \frac{\dot{\sigma}_3}{E_3} \eta \] (4.26)

\[ \dot{\sigma}_3 = \dot{\sigma} - \dot{\sigma}_1 - \dot{\sigma}_2 = \dot{\sigma} - E_1 \dot{\epsilon} - E_2 \dot{\epsilon} \] (4.27)

So that finally

\[ \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E_3} = \left( \frac{E_1 + E_2 + E_3}{E_3} \right) \dot{\epsilon} + \left( \frac{E_1 + E_2}{\eta} \right) \epsilon \] (4.28)
When $K_1 = \sigma_1$ and $\sigma_2 < K_2$ \hspace{1cm} \textit{(friction pot 1 moves and friction pot 2 stationary)} \hspace{1cm} (4.29)

\[
\sigma_1 = K_1
\] \hspace{1cm} (4.30)

\[
\sigma_2 = E_2 \epsilon
\] \hspace{1cm} (4.31)

\[
\dot{\epsilon}_3 = \frac{\dot{\sigma}_3}{E_3} + \frac{\sigma_3}{\eta}
\] \hspace{1cm} (4.32)

\[
\sigma = \sigma_1 + \sigma_2 + \sigma_3 = K_1 + E_2 \epsilon + \eta \dot{\epsilon} - \frac{\dot{\sigma}_3}{E_3} \eta
\] \hspace{1cm} (4.33)

\[
\dot{\sigma}_3 = \dot{\sigma} - \dot{\sigma}_1 - \dot{\sigma}_2 = \dot{\sigma} - E_2 \dot{\epsilon}
\] \hspace{1cm} (4.34)

So that finally

\[
\frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E_3} = \left( \frac{E_2 + E_3}{E_3} \right) \dot{\epsilon} + \left( \frac{E_2}{\eta} \right) \epsilon + \frac{K_1}{\eta}
\] \hspace{1cm} (4.35)

When $K_1 = \sigma_1$ and $K_2 = \sigma_2$ \hspace{1cm} \textit{(both friction pots move)} \hspace{1cm} (4.36)

\[
\sigma_1 = K_1
\] \hspace{1cm} (4.37)

\[
\sigma_2 = K_2
\] \hspace{1cm} (4.38)
\[
\epsilon_3 = \frac{\sigma_3}{E_3} + \frac{\sigma_3}{\eta} \tag{4.39}
\]

\[
\sigma = \sigma_1 + \sigma_2 + \sigma_3 = K_1 + K_2 + \eta \dot{\epsilon} - \frac{\sigma_3}{E_3} \eta \tag{4.40}
\]

\[
\dot{\sigma}_3 = \dot{\sigma} - \sigma_1 - \sigma_2 = \dot{\sigma} \tag{4.41}
\]

So that finally

\[
\frac{\sigma}{E_3} + \frac{\dot{\sigma}}{\eta} = \dot{\epsilon} + \left( \frac{K_1 + K_2}{\eta} \right) \tag{4.42}
\]

Using the definition of stress and strain, \( \sigma = F/A \) and \( \epsilon = x/h \), with \( A \) the area of the sample impacted, and \( h \) its height, we introduce \( k = AE/h \), \( \mu = Ah/h \), \( K'_1 = AK_1 \) and \( K'_2 = AK_2 \) to end up with the following strain stress relationship:

**If** \( x < K'_1/k_1 \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \left( \frac{k_1 + k_2 + k_3}{k_3} \right) \dot{x} + \left( \frac{k_1 + k_2}{\mu} \right) x \tag{4.44}
\]

**If** \( K'_1/k_1 < x < K'_2/k_2 \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \left( \frac{k_2 + k_3}{k_3} \right) \dot{x} + \left( \frac{k_2}{\mu} \right) x + \frac{K'_1}{\mu} \tag{4.45}
\]

**If** \( x > K'_2/k_2 \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \frac{K'_2}{\mu} \tag{4.47}
\]
Using dynamics’ fundamental principle for the mass \( M \) we can write

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \ddot{x} + \left( \frac{K'_1 + K'_2}{\mu} \right)
\]  

(4.48)

We assume that as soon as the impactor is in contact with the sample, then \( \ddot{x}_{\text{Impactor}} = \ddot{x}_{\text{Sample}} = \ddot{x} \), so that using Dynamics’ fundamental principle for the impactor:

\[
m\ddot{x} = mg - F_i
\]  

(4.50)

So Finally

\[
M_i\ddot{x} = M_tg - F
\]  

(4.51)

with \( M_t = M + m \).

This relationship is valid as long as the impactor and the sample are in contact.

We can now solve the following system

\[
M_i\ddot{x} = M_tg - F
\]  

(4.52)

If \( x < \frac{K'_1}{k_1} \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \left( \frac{k_1 + k_2 + k_3}{k_3} \right) \ddot{x} + \left( \frac{k_1 + k_2}{\mu} \right) x
\]  

(4.54)

If \( \frac{K'_1}{k_1} < x < \frac{K'_2}{k_2} \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \left( \frac{k_1 + k_2 + k_3}{k_3} \right) \ddot{x} + \left( \frac{k_1 + k_2}{\mu} \right) x
\]  

(4.55)
\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \left( \frac{k_2 + k_3}{k_3} \right) \ddot{x} + \left( \frac{k_2}{\mu} \right) x + \frac{K'_1}{\mu}
\]  

(4.56)

If \( x > \frac{K'_2}{k_2} \)

\[
\frac{F}{\mu} + \frac{\dot{F}}{k_3} = \ddot{x} + \left( \frac{K_1 + K_2}{\mu} \right)
\]

(4.58)

\[x(t = 0) = 0\]

(4.59)

\[\dot{x}(t = 0) = V_0\]

(4.60)

\[F(t = 0) = 0\]

(4.61)

This can also be expressed using

\[
Y = \begin{bmatrix} x \\ \dot{x} \\ F \end{bmatrix} \quad \text{and} \quad \dot{Y} = \begin{bmatrix} \ddot{x} \\ \dddot{x} \\ \ddot{F} \end{bmatrix}
\]

(4.62)

So that

\[
\dot{Y} = MY + g \begin{bmatrix} 0 & & 0 \\ 1 & & \text{Cst} \\ 0 & & 1 \end{bmatrix}
\]

(4.63)

Where \( M \) is a 3 by 3 matrix and \( \text{Cst} \) is a constant defined as follows:
If \( x < K'_1/k_1 \) then \( M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ (k_1 + k_2)k_3/\mu & k_1 + k_2 + k_3 & -k_3/\mu \end{bmatrix} \) and \( Cst = 0 \) \hspace{1cm} (4.64)

If \( K'_1/k_1 < x < K'_2/k_2 \) then \( M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ k_2k_3/\mu & k_2 + k_3 & -k_3/\mu \end{bmatrix} \) and \( Cst = K'_1k_3/\mu \) \hspace{1cm} (4.65)

If \( K'_2/k_2 < x \) then \( M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ 0 & k_3 & -k_3/\mu \end{bmatrix} \) and \( Cst = (K'_1 + K'_2)k_3/\mu \) \hspace{1cm} (4.66)

Analytic solutions of this linear system exist, but finding the eigenvalue and eigenvectors of the linear transformation end up being highly demanding for computational resources, so the system has been solved with a standard ODE solver in Matlab (ode23). To solve this system compute use the previously defined system of equation and we solve it using \( Y \) and \( \dot{Y} \) defined in Equation 5.3.

Comparison of the results predicted by the model and experimental data shows that the Frictionpot model succeeds in accounting for the plastic behavior exhibited by polystyrene during the impact.

Good agreement is obtained for each of the data set (30J, 70J and 130J) as shown in Figures 4-7 - 4-30 , and optimizations of the parameters \( k_1, k_2, k_3, K_1, K_2 \) and \( \mu \) shows that a single set of values is valid for the three energies. This proves that the model is able to represent the system accurately, indeed the strain rate dependence of the material is taken into account by the dashpot element, and the plastic behavior is shown.
Figure 4-7: Friction Pot Model response and Experimental data 30J impact - Displacement versus time and Velocity versus time
Figure 4-8: Friction Pot Model response and Experimental data 30J impact - Load versus time and Load versus Displacement
Figure 4-9: Friction Pot Model response and Experimental data 70J impact - Displacement versus time and Velocity versus time.
Figure 4-10: Friction Pot Model response and Experimental data 70J impact - Load versus time and Load versus displacement
Figure 4-11: Friction Pot Model response and Experimental data 130J impact - Displacement versus time and Velocity versus displacement
Optimal values for the parameters have been found using minimization of the distance between curves of experimental data and model prediction. Values of $k_1$, $k_2$, $k_3$, $K_1$, $K_2$ and $\mu$ are given in Table 4.2.
<table>
<thead>
<tr>
<th>Energy Parameters</th>
<th>$k_1$ (N/m)</th>
<th>$k_2$ (N/m)</th>
<th>$k_3$ (N/m)</th>
<th>$K'_1$ (N)</th>
<th>$K'_2$ (N)</th>
<th>$\mu$ (N.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30J Impact</td>
<td>3 300 000</td>
<td>1 300 000</td>
<td>650 000</td>
<td>3 300</td>
<td>4500</td>
<td>2300</td>
</tr>
<tr>
<td>70J Impact</td>
<td>3 300 000</td>
<td>1 300 000</td>
<td>650 000</td>
<td>3 300</td>
<td>4500</td>
<td>2300</td>
</tr>
<tr>
<td>130J Impact</td>
<td>3 300 000</td>
<td>1 300 000</td>
<td>650 000</td>
<td>3 300</td>
<td>4500</td>
<td>2300</td>
</tr>
</tbody>
</table>

Table 4.2: Optimal Frictionpot parameters value for the three energies of impact

### 4.2 Fluid filled foam models

#### 4.2.1 Newtonian fluid

**Foam contribution**

According to Gibson and Ashby (ref), one can find 3 regimes for the foam compression.

For $0 < \epsilon < \epsilon^*_{\text{elastic}}$ then \[ \sigma^* = \epsilon E^* \] (4.67)

For $\epsilon^*_{\text{elastic}} < \epsilon < \epsilon_D \left(1 - \frac{1}{D}\right) + \epsilon^*_{\text{elastic}}$ then \[ \sigma^* = \sigma^*_{\text{elastic}} \] (4.68)

For $\epsilon_D \left(1 - \frac{1}{D}\right) + \epsilon^*_{\text{elastic}} < \epsilon$ then \[ \sigma^* = \frac{\sigma^*_{\text{elastic}}}{D} \left(\frac{\epsilon_D - \epsilon}{\epsilon_D - \epsilon_D^*_D}\right)^m \] (4.69)

Where

\[ \epsilon_D = 1 - 1.4 \left(\frac{\rho^*_s}{\rho_s}\right) \] (4.70)

Given the relative density of the polyurethane open cell reticulated foam used, $\epsilon_D \approx 1 - 1.4 \times 0.03 \approx 0.96$. The densification regime is reported (Matt Dawson Thesis) to begin at a strain called $\epsilon_d$ (densified strain), which is of value $\epsilon_d \approx 0.60$. So transition from the plateau regime to the densified is considered occurring for $\epsilon > \epsilon_d$
$D$ is defined such as

$$D = \frac{\epsilon_D}{\epsilon_D - \epsilon} \quad \text{(4.71)}$$

Where the strain $\epsilon$ is the strain at which the stress at the end of the plateau region begins to exceed the elastic buckling stress.

**Newtonian fluid response**

Our first analysis is the case of a fluid flowing between two plates, the top one moving at speed $V(t)$, toward the bottom one. We assume here that the flow is instantly fully developed. We solve the problem in a 2-D situation, given that we consider an invariance in the 3rd direction of space.

![Figure 4-13: Compression of a fluid layer](image)

Using Mass Conservation (see Figure 4-13), we can write that

$$-V(t)L = 2\int_{y=0}^{y=h(t)} u(y,t) \, dy \quad \text{(4.72)}$$
Using Navier-Stokes equation, in the x-direction, gives that

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

(4.73)

The flow being fully developed, this equation reduces to

\[
0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}
\]

(4.74)

Navier Stokes in the y-direction gives

\[
\frac{\partial p}{\partial y} = 0
\]

(4.75)

so that we can solve the equations

\[
0 = -\frac{dp}{dx} + \frac{\mu}{\partial y^2} 
\]

(4.76)

Integrating twice with respect to y gives

\[
u(y, t) = \frac{1}{2\mu} \frac{dp}{dx} y^2 + \alpha y + \beta
\]

(4.77)

Non-slip boundary conditions are

\[
u(y = 0, t) = 0 \quad \text{and} \quad \nu(y = h(t), t) = 0
\]

(4.78)

So that

\[
\alpha = -\frac{1}{2\mu} \frac{dp}{dx} h \quad \text{and} \quad \beta = 0
\]

(4.79)

\[
u(y) = \frac{1}{2\mu} \frac{dp}{dx} \left[y^2 - yh(t)\right] \Rightarrow Q = -\frac{1}{12\mu} \frac{dp}{dx} h(t)^3
\]

(4.80)

So using Mass Conservation
\[ V(t)L = \frac{-2}{12\mu} \frac{dp}{dx} h(t)^3 \Rightarrow \frac{dp}{dx} = \frac{-6\mu V(t)L}{h(t)^3} \quad (4.81) \]

So over-pressure (local pressure minus atmospheric pressure) is

\[ p^*(x) = \frac{6\mu V(t)L}{h(t)^3} 2 \int_{s=L/2}^{x=L} dx = \frac{12\mu LV(t)}{h(t)^3} \left[ \frac{L}{2} - x \right] \quad (4.82) \]

\[ \Rightarrow \sigma_f = 2/L \int_{x=0}^{x=L/2} \frac{12\mu LV(t)}{h(t)^3} \left[ \frac{L}{2} - x \right] dx = \frac{3\mu L^2 V(t)}{h(t)^3} \quad (4.83) \]

Using

\[ h(t) = h_0(1 - \epsilon)^{1/2} \quad \text{and} \quad \dot{\epsilon} = \frac{V(t)}{h(t)} \quad (4.84) \]

\[ \sigma_f = \frac{3\mu \dot{\epsilon}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 \quad (4.85) \]

where \( h_0 \) is the initial cell diameter of the foam.

**Fluid filled foam response**

Previous work suggests that fluid-structure interactions can be neglected, so our first model predicts a response such that

\[ \sigma_{total} = \sigma_{fluid} + \sigma_{foam} \quad (4.86) \]

Analysis of the dry foam impact shows that clearly the foam contribution is negligible up to the densification phenomena, so we consider the following behavior for the foam:

\[ \sigma_{total} = \frac{3\mu \dot{\epsilon}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 \quad (4.87) \]

\[ \sigma_{total} = \frac{3\mu \dot{\epsilon}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 + \frac{\sigma_{ci}^*}{D} \left( \frac{\epsilon_D}{\epsilon_D - \epsilon} \right)^m \quad (4.88) \]
Taking into account the tortuosity of the path, a coefficient $C/3$ should be added to the fluid response, so that finally the following form for the response is chosen:

\[
\begin{align*}
\text{for } 0 < \varepsilon < \varepsilon_d & \quad \sigma_{total} = \frac{C\mu \dot{\varepsilon}(t)}{(1 - \varepsilon)} \left( \frac{L}{h_0} \right)^2 \\
\text{for } \varepsilon > \varepsilon_d & \quad \sigma_{total} = \frac{C\mu \dot{\varepsilon}(t)}{(1 - \varepsilon)} \left( \frac{L}{h_0} \right)^2 + \frac{\sigma_{el}^*}{D} \left( \frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \right)^m
\end{align*}
\] (4.89) (4.90)

Literature suggests that for polyurethane $n \approx 1$, and $D \approx 1.55$. (Gibson and Ashby).

Introducing $n$ number of cell per unit length and $H_0$ initial thickness of the specimen, one can deduce $h_0 = \frac{H_0}{n}$, so that:

\[
\begin{align*}
\text{for } 0 < \varepsilon < \varepsilon_d & \quad \sigma_{total} = \frac{C\mu \dot{\varepsilon}(t)}{(1 - \varepsilon)} \left( \frac{nL}{H_0} \right)^2 \\
\text{for } \varepsilon > \varepsilon_d & \quad \sigma_{total} = \frac{C\mu \dot{\varepsilon}(t)}{(1 - \varepsilon)} \left( \frac{nL}{H_0} \right)^2 + \frac{\sigma_{el}^*}{D} \left( \frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \right)
\end{align*}
\] (4.91) (4.92)

On Figure 4-14 - 4-18, we can see that the model is able to predict accurately the glycerol-filled foam behavior for the 3 different energies.
Figure 4-14: 30J impact - Model versus Experimental data.
Figure 4-15: 70J impact - Model versus Experimental data.
Figure 4-16: 70J impact - Model versus Experimental data.
Figure 4-17: 130J impact - Model versus Experimental data.
Figure 4-18: 130J impact - Model versus Experimental data.
The analytic model developed gives also estimates for the fluid and solid contribution prefactors $\frac{3\mu L^2}{h_0^2}$ and $\frac{\sigma_{\text{Elastic}}}{D}$. Using glycerol viscosity (1.1 Pa.s), the elastic stress ($\sigma_{\text{Elastic}} \approx 4.8$ kPa) and the characteristic dimensions of the sample ($L$ is the sample size so $L = 0.10m$ and the pore size is $h_0 \approx 235 \mu m$), so that

$$\frac{3\mu L^2}{h_0^2} \approx \frac{3 \times 1.1 \times 0.1^2}{0.000235^2} \approx 5.98 \times 10^5 \approx 600000 \text{ Pa.s}$$ (4.93)

$$\frac{\sigma_{\text{Elastic}}}{D} \approx \frac{4800}{1.55} \approx 3100 \text{ Pa}$$ (4.94)

The value fitted from the experiments for $\frac{3\mu L^2}{h_0^2}$ and $\frac{\sigma_{\text{Elastic}}}{D}$ are respectively 150000 Pa.s and 4000 Pa. There is a good agreement for the foam contribution, however for the fluid contribution there is a need to study a more complete model in order to ensure that the phenomenon is accurately described and can be predicted by simulations. This is the aim of the permeability model developed in the following part, which will give another estimate of these parameters.

**Permeability model**

Assuming the foam is isotropic, the relative density under uniaxial compression is given by (Matt Dawson [60]):

$$\frac{\rho^*}{\rho_s} = \frac{\rho_0^*}{\rho_s} \frac{1}{(1 - \epsilon)(1 + \nu \epsilon)^2}$$ (4.95)

where $\epsilon$ is the strain, which is taken positive in compression and $\rho^*$ is the density of the foam at strain $\epsilon$, $\rho_0^*$ is the density of the foam at strain $\epsilon = 0$, $\rho_s$ is the density of the solid material and $\nu$ is the Poisson’s ratio of the foam.

Poisson’s ratio of open-cell, reticulated foams are usually between 0 and 0.3 in the elastic regime of the compression. However, for strain greater that the buckling strain (about 0.075), the cells collapse and buckle without expanding much laterally, so that their Poisson’s ratio
in this regime is close to zero. Since this study leads to compressive strains far higher than the elastic buckling strain, \( \nu \epsilon \) is taken equal to zero in the rest of this study for low-density, open-cell, reticulated, flexible foams. Using equation 4.95, expanding the relative density term in Brace’s equation (after Brace, 1977 [88]) given by equation 4.96, gives the intrinsic permeability as a function of strain (Eq 4.97).

\[
k = A d^2 \left( 1 - \frac{\rho_s}{\rho_s} \right)^3
\]  

\[
(4.96)
\]

\[
k = A d^2 \left( 1 - \frac{\rho_0^*}{\rho_s (1 - \epsilon)} \right)^3
\]  

\[
(4.97)
\]

Where \( A \) is an empirical constant, and \( d \) is the average diameter of the cell. \( A \) is given by Brace as 0.025 for a porous microstructure composed of tubes of circular sections.

The model suggested by Gent and Rusch [86] considers the foam as composed of an array of circular tubes. The average diameter of a cell is found to be proportional to the diameter of the cross-section of the tube. In the case of uni-directional compressive strain, for low-density foams, and for strains smaller than the elastic buckling strain, the model makes the assumption that each tube deforms in the same proportion as the bulk material. That means that the average cross-section diameter can be expressed as:

\[
d_{el} \equiv d_0 (1 - \epsilon)^{1/2} \quad \text{for } 0 < \epsilon < \epsilon^*_c
\]  

\[
(4.98)
\]

This equation is valid in the elastic regime, and \( d_0 \) is the average cell size at strain equal zero (\( \epsilon = 0\% \)).

The model developed by M.A Dawson, L.J Gibson and G.H McKinley (2009) [91], suggest a similar form for the average size of cell for the densified regime:

\[
d_{d} = d_0 (1 - \epsilon)^\alpha \quad \text{for } \epsilon = \epsilon_d
\]  

\[
(4.99)
\]
where $a$ is an empirical constant, which has been determined for the type of foam under study. So finally, the permeability of the foam can be determined as:

\[ k_{el} = Ad_0^2(1 - \epsilon) \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon} \right)^3 \quad \text{for } 0 < \epsilon < \epsilon_{el}^* \]  

(4.100)

\[ k_{el}^* = Ad_0^2(1 - \epsilon_{el}^*) \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon_{el}^*} \right)^3 \quad \text{for } \epsilon = \epsilon_{el}^* \]  

(4.101)

\[ k_d = Ad_0^2(1 - \epsilon_d)^{2a} \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon_d} \right)^3 \quad \text{for } \epsilon = \epsilon_d \]  

(4.102)

For strain greater than the densified strain $\epsilon_d$ a linear variation with strain is suggested, so that

\[ k_d = Ad_0^2(1 - \epsilon_d)^{2a} \left( 1 - \frac{\rho_0^*}{\rho_s} \frac{1}{1 - \epsilon_d} \right)^3 \left( 1 + (\epsilon_d - \epsilon)^2 \right)^{2a} \quad \text{for } \epsilon > \epsilon_d \]  

(4.103)

And the corresponding volume fractions of the cells remaining in the linear elastic regime $\chi_{el}^*$ and densified regime $\chi_d$, for strain greater that the elastic buckling strain are

\[ \chi_{el}^* = \frac{(\epsilon_d - \epsilon)(1 + \epsilon_{el}^*)}{(1 + \epsilon)(\epsilon_d - \epsilon_{el}^*)} \quad \text{for } \epsilon_{el}^* < \epsilon < \epsilon_d \]  

(4.104)

\[ \chi_d = \frac{(\epsilon - \epsilon_{el}^*)(1 + \epsilon_d)}{(1 + \epsilon)(\epsilon_d - \epsilon_{el}^*)} \quad \text{for } \epsilon_{el}^* < \epsilon < \epsilon_d \]  

(4.105)

In the case of a flow in the direction of compression, and using Gent and Rusch (1966) model, in the case of viscous dominated flow, the pressure drop across the specimen can be related to the permeability of the foam by the following relation:

\[ \frac{\Delta p_i}{h_i} = \frac{\mu}{k_i} U \]  

(4.106)

where $h_i$ is the length of each regime in the direction of the flow, $k_i$ is the intrinsic
permeability of each regime and \( U \) is the flow velocity. \( U \) is assumed uniform and constant through each regime because of continuity. Using equation 4.106, the total pressure drop over the specimen is the sum of the pressure drop in the different parts. If the specimen is of constant cross section, the length of each regime is proportional to the volume fraction of each regime, so that as Dawson, Gibson and McKinley (2007) deduced:

\[
k_T = k_{el} \quad \text{for } 0 < \epsilon < \epsilon_{el}^* \tag{4.107}
\]

\[
k_T = \frac{k_d k_{el}^*}{X_{el}^* k_d + \chi_d k_{el}^*} \quad \text{for } \epsilon_{el}^* < \epsilon < \epsilon_d \tag{4.108}
\]

\[
k_T = k_d \quad \text{for } \epsilon > \epsilon_d \tag{4.109}
\]

The resulting intrinsic permeability has been computed and is shown on Figure 4-19. Where \( A = 0.025 \) as suggested by Brace, and \( d_0 = 235 \ \mu m \), relative density at zero strain \( \frac{\rho_a}{\rho_s} = 0.03 \) according to the properties of the 70 PPI foam (New Dimension Industries). The value of \( a = 0.76 \) (Dawson, PhD Thesis), is the one experimentally determined by Dawson, Gibson and McKinley on the very same foam.
Figure 4-19: Intrinsic Permeability of a 70 PPI foam, computed after the permeability model.
Flow parallel to the direction of compression.

From that one can use mass conservation and Darcy’s law to obtain an estimate of the average stress during the compression of the sample, studying a flow perpendicular to the direction of compression.

**Elastic regime** $0 < \varepsilon < \varepsilon_{\text{el}}^*$ and **densified regime** $\varepsilon > \varepsilon_d$

In these two cases, the sample structure is the same over the thickness of the sample. Schematics of the flow is given on Figure 4-20. Mass conservation gives that
where $\phi$ is the porosity of the foam, $L$ is the size of the sample edge, $h$ is the thickness of the sample, and $U$ is the horizontal velocity, uniform in the vertical direction. Using Darcy’s law, the pressure gradient in the horizontal direction for a viscous Newtonian fluid is such as (Darcy, 1856):

$$
\frac{dP}{dx} = \frac{\mu U}{k_T}
$$

(4.111)

where the permeability $k_T$ is taken to be isotropic. Combining equations 4.110 and 4.111 gives:

$$
\frac{dP}{dx} = \frac{\mu h L}{2h \phi k_T}
$$

(4.112)

Integrating gives

$$
P(x) - P_{atm} = P^*(x) = \frac{\mu h L}{2h \phi k_T} (x - L/2)
$$

(4.113)
where $P^*$ is the pressure minus the atmospheric pressure, namely the over-pressure in the foam.

So finally the average stress $\sigma_f$ in the foam is:

$$
\sigma_f = -\frac{\mu hL^2}{8h\phi k_T} \tag{4.114}
$$

**Bimodal regime** $\epsilon_{el}^* < \epsilon < \epsilon_d$

For strains greater than the elastic buckling strain, but less than the densified strain, the foam is composed of both a fraction of densified cells, and a fraction of elastic buckling cells. Using the assumption of no horizontal variations of the pressure field in the densified and in the elastic region, and using Darcy's law in the two regions gives:

$$
\frac{dP}{dx} = -\frac{\bar{U}_{\text{elastic}}}{k_{el}^*} = -\frac{\bar{U}_{\text{densified}}}{k_d} \tag{4.115}
$$
So that using mass conservation

\[
\frac{\dot{h}L}{2} = \phi h \left( \tilde{U}_{\text{elastic}} x_{\text{elastic}} + \tilde{U}_{\text{densified}} x_{\text{densified}} \right)
\]

(4.116)

\[
= \phi h \left( \tilde{U}_{\text{elastic}} x_{\text{elastic}} + \tilde{U}_{\text{elastic}} \frac{k_d}{k_{\text{el}}} x_{\text{densified}} \right)
\]

(4.117)

\[
= \frac{\dot{h} \phi \tilde{U}_{\text{elastic}}}{k_{\text{el}}} \left( k_{\text{el}} x_{\text{elastic}} + k_d x_{\text{densified}} \right)
\]

(4.118)

\[
\tilde{U}_{\text{elastic}} = -\frac{\dot{h} k_{\text{el}}^* L}{2 \phi \left( k_{\text{el}}^* x_{\text{elastic}} + k_d x_{\text{densified}} \right)}
\]

(4.119)

Using Darcy’s law, it is possible to deduce the pressure gradient across the specimen:
\[
\frac{dP}{dx} = \frac{\mu \tilde{U}}{k^*} \quad (4.120)
\]
\[
= \frac{\mu \dot{h} L}{2 h \phi \left( k^* x^* + k_d x_d \right)} \quad (4.121)
\]

As previously, integrating twice and taking the average over the sample gives the average stress:

\[
\sigma_f = \frac{\mu \dot{h} L^2}{8 h \phi \left( k^* x^* + k_d x_d \right)} \quad (4.122)
\]
\[
= C_1 \frac{\dot{\epsilon}(t)}{1 - \epsilon} \quad (4.123)
\]

So that \(C_1 = \frac{\mu L^2}{8 \phi \left( k^* x^* + k_d x_d \right)}\)

So defining the intrinsic permeability for the three regimes:

\[
k_T = k^* \quad \text{for } 0 < \epsilon < \epsilon_{el} \quad (4.124)
\]
\[
= \left( k^* x^* + k_d x_d \right) \quad \text{for } \epsilon_{el} < \epsilon < \epsilon_d \quad (4.125)
\]
\[
= k_d \quad \text{for } \epsilon_d < \epsilon \quad (4.126)
\]

With that, and using experimental data determined by Matt Dawson in his PhD thesis, one can plot the intrinsic permeability of the foam (Figure 4-22, and the prefactor \(C_1\) for the fluid contribution (Figure 4-23).
Figure 4-22: Intrinsic Permeability of a 70 PPI foam, computed using the permeability model
Figure 4-23: Prefactor $C_1$ for the viscous contribution of a 70 PPI foam, computed using the permeability model.
Permeability Model - Discussion

The permeability shown in Figure 4-22 and the associated value for the prefactor $C_1$ shown in Figure 4-23 show that the model developed in the previous section representing the foam with a simplistic model gives a good estimate of the physical importance of the viscous forces. Indeed the value found previously with the simplistic model gives for the prefactor a value of $5.98 \times 10^5$ Pa.s, and the values reported in Figure 4-23 are between $0.7 \times 10^5$ to $7 \times 10^5$ Pa.s depending on the strain. So this supports the validity of the model. The permeability model is more precise since it gives a permeability as a function of the strain, but to predict the foam response to compressive strain it appears that both models give very close value for the permeability of the foam. The value for the prefactor found from the experiment is $1.5 \times 10^5$ Pa.s which is in agreement with the value found previously, so the modeling of the foam is successful with respect to the permeability of the foam.

4.2.2 Non-Newtonian fluid

Results from the previous section give the response of a fluid filled foam, as a function of the viscosity of the fluid impregnated the foam (see Equations 4.127, 4.128. Experimental results of Chapter 3 give the viscosity of the shear thickening solution as a function of shear rate. So using the model developed with the Newtonian fluid, and replacing the glycerol viscosity, by a viscosity as function of the shear rate (see Equation 4.129), gives a model that combines the analysis conducted on the permeability model and also takes into account the fact that the fluid is non-Newtonian.

\[
\begin{align*}
\sigma_{total} &= \frac{C\mu\dot{\varepsilon}(t)}{(1-\varepsilon)} \left( \frac{L}{h_0} \right)^2 \quad \text{for} \quad 0 < \varepsilon < \varepsilon_d \\
\sigma_{total} &= \frac{C\mu\dot{\varepsilon}(t)}{(1-\varepsilon)} \left( \frac{L}{h_0} \right)^2 + \frac{\sigma_{el}^*}{D} \left( \frac{\varepsilon_D - \varepsilon}{\varepsilon_D - \varepsilon_d} \right)^m \quad \text{for} \quad \varepsilon > \varepsilon_d
\end{align*}
\]
Local shear rate can be computed as follow:

\[ \mu = f(\dot{\gamma}) \]  

(4.129)

\[ \sigma_{\text{fluid}} = \frac{C \mu \dot{c}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 = \dot{\gamma} \mu \]  

(4.130)

So that

\[ \dot{\gamma} = \frac{C \mu \dot{c}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 \]  

(4.131)

According to the experimental results given in Chapter 3 and the limit developed at high shear rate, the viscosity is given in Figure 4-24.

For the viscosity, the relationship for high shear rate (\( \dot{\gamma} > 100/\text{s} \)) is given by the hydroelastic limit (see Chapter 3):

\[ \mu = \dot{\gamma}^{-1/2}(\eta_s G_p)^{1/2} \]  

(4.132)

where \( \eta_s \) is the solvent viscosity (\( \eta = 16.5 \text{ mPa.s} \) for ethylene glycol) and \( G_p \) is the particle Young’s modulus (\( G_p = 31 \text{ GPa} \) for the particle used).

So finally for high shear rate (\( \dot{\gamma} > 100/\text{s} \)),

\[ \mu = \left( \frac{C \mu \dot{c}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 \right)^{-1/2} (\eta_s G_p)^{1/2} \]  

(4.133)

Local shear rate when impactor reaches the sample for a 70J impact test on a 13 mm thick fluid filled foam layer is around

\[ \dot{\gamma} = \frac{5.98 \times 10^5 \times 4.38}{0.013} = 2 \times 10^8 [1/\text{s}] \]  

(4.134)

Similarly, during the test, assuming a strain of 80 % and a velocity of the impactor of 0.5 m/s the local shear rate is
Shear Rate [1/s]  

**Figure 4-24: Viscosity versus shear rate - Model, Experimental result for a volume fraction of silica around 60% and Hydroelastic limit**

\[ \dot{\gamma} = \frac{5.98 \times 10^3 \times 0.5}{0.013(1 - 0.8)} \approx 1.2 \times 10^8 \text{ [1/s]} \]  \hspace{1cm} (4.135)

So the assumption that local shear rate is greater than 100 /s is verified during the test, and the use of formula given in Equation 4.132 is correct.

Experimental results and model predictions are given in Figure 4-25. It appears that experimental results are quite different from the model prediction. This is mainly due to two factors. First of all, the viscosity given by the chosen model is an upper-limit viscosity. So predicted results tend to show a much sharper rise in load versus displacement, as the viscosity of the fluid used for the computation is higher than in reality.
Figure 4-25: Load versus displacement - Model including varying viscosity defined by Equation 4.133 and Experimental results for a 70PPI foam impregnated with a 61% STF fluid, Impact Energy of 70J

Furthermore, experimentally the foam is extremely difficult to impregnate completely, due to the high viscosity of the shear thickening suspension. By weighting the sample after impregnation, it was determined that the maximum impregnation percentage achieved is around 50%. So when impactor hit the sample, there is a first phase during which the impactor only compresses a layer of foam which is not filled, so stress is extremely low. When the sample is compressed such that all the foam is filled with fluid, then stress raises sharply. Taking into account this two phenomena, experimental results and model prediction are plotted in Figure 4-26. On this graph, a constant viscosity of 5 Pa.s has been chosen, as a best fit to the experimental data. This is obviously not a realistic model to predict the behavior of the shear-thickening fluid impregnated foam, but it illustrates the idea that the
The hydroelastic limit is indeed a model and tends to overestimate by several orders of magnitude the viscosity of the shear-thickening suspension. So more experimental work would be needed to be able to model more accurately the viscosity of the shear thickening fluid at high strain rates.

![Load versus displacement](image)

Figure 4-26: Load versus displacement - Model with a constant viscosity of 5 Pa.s, taking into account the partial filling, and Experimental results for a 70PPI foam impregnated with a 61% STF fluid, Impact Energy of 70J

### 4.3 Comparison of models vs experiments

#### 4.3.1 Polystyrene modeling

Using the friction-pot model, a very good agreement between experimental data and model prediction was found for the three impact energies under study. This is shown in Figure 4-27,
exhibiting a very good correlation between experimental results and the model. Interesting points are that, the influence of the various elements of the friction-pot model are tractable on the stress-strain curves. So initially the curve is almost linear, up to the point where the first friction pot element begins to slide.

![Graph showing comparison of experimental data and model response](image)

Figure 4-27: Comparison Model versus experience for a 25 mm thick polystyrene sheet - 3 Impact energies: 30J 70J and 130J

Then the slope of the stress-strain (or equivalently the Load - displacement) curve shows a decrease, indeed the stress in one of the branch of the model has now reached a plateau. Similar behavior is observed when the second friction-pot begins sliding. In the remaining part, the model shows a Maxwell-like behavior, because this is the only active element remaining.
At the end of the test, a plastic deformation is noticeable, characteristic of the behavior of the friction-pot model. Another interesting point is that the maximum load reached over the test (or similarly the PLA) is not very different for the three types of energy (only a 30% increase from the 30J test to the 130J test). This is also characteristic of the behavior of the polystyrene: since energy absorption is due mainly to plastic deformation, in this regime stress does not increase rapidly when strain increases. Complete comparisons is given for 70J impact and 130J impact in Figures 4-28-4-30.

Figure 4-28: 25 mm thick polystyrene layer impact response for a 70J impact. Model versus Experimental data.
Figure 4-29: 25 mm thick polystyrene layer impact response for a 130J impact. Model versus Experimental data
Figure 4-30: 25 mm thick polystyrene layer impact response for a 130J impact. Model versus Experimental data
Parameters used for the simulations are listed in Table 4.3

<table>
<thead>
<tr>
<th>$k_1$ [N/m]</th>
<th>$k_2$ [N/m]</th>
<th>$k_3$ [N/m]</th>
<th>$K'_1$ [N]</th>
<th>$K'_2$ [N]</th>
<th>$\mu$ [Pa.s/m]</th>
<th>Impactor Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.3 \times 10^6$</td>
<td>$1.3 \times 10^6$</td>
<td>$6.5 \times 10^5$</td>
<td>3 300</td>
<td>4 500</td>
<td>2 300</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters used for the friction-pot model

And the equations for the motion of the system are

This can also be expressed using

$$Y = \begin{bmatrix} x \\ \dot{x} \\ F \end{bmatrix} \quad \text{and} \quad \dot{Y} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{F} \end{bmatrix}$$  \hspace{1cm} (4.136)

So that

$$\dot{Y} = MY + g \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + Cst \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$  \hspace{1cm} (4.137)

Where $M$ is a 3 by 3 matrix and $Cst$ is a constant defined as follows:

If $x < K'_1/k_1$ then $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \end{bmatrix}$ and $Cst = 0$  \hspace{1cm} (4.138)

where $M_t$ is the mass of the system composed of the impactor plus the layer of polystyrene.

If $K'_1/k_1 < x < K'_2/k_2$ then $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \end{bmatrix}$ and $Cst = K'_1k_3/\mu$  \hspace{1cm} (4.139)

$$\begin{bmatrix} (k_1 + k_2)k_3/\mu & k_1 + k_2 + k_3 & -k_3/\mu \end{bmatrix}$$
If \( K_2' / k_2 < x \) then

\[
M = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & -1/M_t \\
0 & k_3 & -k_3/\mu
\end{bmatrix}
\]

and

\[
Cst = (K_1' + K_2')k_3/\mu
\]  

(4.140)

### 4.3.2 Glycerol filled foam

Using the model developed in the previous section, a good prediction of the fluid filled foam response is achieved for the three energies under study. This is summed up in Figure 4-31. An interesting point is that contrary to polystyrene, PLA or maximum load reached over an impact is very different depending on the energy of the impactor when reaching the sample. Between the maximum load of a 30J impact and a 130J impact, there is an increase of more than 350%.

Furthermore, the stress-strain curve for the glycerol impregnated foam has a shape very different from the one of the polystyrene. This is mainly due to the fact that two components take part in the stress while compressed. The viscous forces due to the fluid are proportional to the velocity of the fluid, but inversely proportional to the strain to which the sample is submitted. Furthermore the foam contribution is also inversely proportional to the strain, so stress increases more and more rapidly as strain increases, the stress in the sample being reached close to the maximum displacement recorded.
Figure 4.31: 16 mm thick polystyrene layer impact response for 3 different energies. Model versus Experimental data.

Parameters used for the simulations are listed in Table 4.4

<table>
<thead>
<tr>
<th>$\mu(L/h_0)^2$ [Pa.s]</th>
<th>$\sigma_{el}^*$ [Pa]</th>
<th>$\epsilon_D$</th>
<th>$D$</th>
<th>$\epsilon_{el}^*$</th>
<th>$\epsilon_d$</th>
<th>Impactor Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.98 \times 10^5$</td>
<td>4000</td>
<td>0.958</td>
<td>1.55</td>
<td>0.05</td>
<td>0.60</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 4.4: Parameters used for the glycerol filled foam model
with the stress-strain relations defined as follows:

\[
\text{for } 0 < \epsilon < \epsilon_d \quad \sigma_{\text{total}} = \frac{\mu \dot{\epsilon}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2
\]  
\[\text{(4.141)}\]

\[
\text{for } \epsilon > \epsilon_d \quad \sigma_{\text{total}} = \frac{\mu \dot{\epsilon}(t)}{(1 - \epsilon)} \left( \frac{L}{h_0} \right)^2 + \frac{\sigma_{\text{el}}^*}{D} \frac{\epsilon D}{\epsilon D - \epsilon}
\]  
\[\text{(4.142)}\]

Because viscosity is one of the parameter that one might want to vary for a helmet design purpose we can re-write the equations 4.141, 4.142 as follow

\[
\text{for } 0 < \epsilon < \epsilon_d \quad \sigma_{\text{total}} = C \frac{\mu \dot{\epsilon}(t)}{(1 - \epsilon)}
\]  
\[\text{(4.143)}\]

\[
\text{for } \epsilon > \epsilon_d \quad \sigma_{\text{total}} = C \frac{\mu \dot{\epsilon}(t)}{(1 - \epsilon)} + \frac{\sigma_{\text{el}}^*}{D} \frac{\epsilon D}{\epsilon D - \epsilon}
\]  
\[\text{(4.144)}\]

Where \( C \) is a dimensionless constant. Using the values given in Table 4.4 and the viscosity of the fluid used for the experiment (glycerol \( \mu = 1.1 \) Pa.s), we can compute \( C = 5.44 \times 10^4 \).

**Temperature influence**

Viscosity of pure glycerol at 20 degree Celsius is 1.410 Pa.s. However glycerol viscosity depends strongly on the temperature: at 0 C it is 12.070 Pa.s and at 50 C it is 0.142 Pa.s. This strong dependence of viscosity on temperature could be cause some variations in the performance of a liner composed of a glycerol filled foam in extreme conditions. However, since the glycerol filled foam layer is supposed to be the layer in contact with the head of the user, we can imagine that temperature would remain in a relatively restricted interval. This should be checked on complete models of helmet, but for future work this is an area of investigation. Simulations given in the next chapter for higher and lower viscosity than the glycerol can give some guidance on how to take into account the viscosity dependence on temperature in the design process of a helmet.
4.3.3 Complete system modeling

The aim of the next chapter is to propose a design using two layers: a layer of polystyrene and one of glycerol filled foam. Having developed successfully models that predict accurately the response of each type of material under dynamic loading, we can now use the two models in series and predict the behavior of a system composed of the two layers. Results for various thicknesses of layers and various impacting energies are given in Figure 4-32.

![Figure 4-32: Model versus Experimental data for 3 designs and 3 impacting energy](image)

Comparisons between experimental results and modeling show a good agreement. Deviation of the model from experimental data is slightly bigger than for each of the layer
separately. This was predictable but the main point is that prediction of the overall deformation and the PLA acceleration is very accurate. The model as well as the experimental data show first a sharp rise in acceleration as the displacement increases. Then the slope of the curve decreases significantly, allowing to the system to absorb a great amount of energy without reaching high levels of acceleration. This is particularly true in the case of the 70J impact, where a plateau-like region is noticeable.

4.4 Conclusions

In this chapter models to describe and predict the response of a polystyrene layer and a fluid filled foam layer have been developed and compared with experimental data. The use of these model also allowed to predict behavior of systems composed of layers of the two materials, thus enabling to conduct computerized simulations on a very large number of design. Parameters for each layer have been identified so that simulations can be run on various designs, using equations of motion defined previously, stress strain relations and parameters value given in Table 4.3 and 4.4. This will allow us to suggest an optimal design for helmet application as it will be shown in the next chapter.
Chapter 5

Helmet design

In this chapter, using all the results and simulations developed throughout the previous chapter, a method to optimize a liner material for helmet design will be developed and applied to the examples of the football, motorcycle and military helmets. The helmet is assumed to have a rigid external shell, with two inner layers of foam: one, a glycerol-filled open-cell polyurethane foam and the other, a polystyrene foam. Criteria to be met are in terms of maximum peak linear acceleration (PLA) and maximum thicknesses. Other parameters such as the weight of the design are investigated. Parameters of interest are the thickness of each layer and the viscosity of the impregnating fluid.

5.1 Simulation algorithm

To perform simulations over the various designs the two parameters varied in the simulation loop are the thickness of the layer of polystyrene and the thickness of the layer of glycerol filled foam.

Materials parameters are determined at the beginning of the program, before entering the loop and a double "for loop" solve the equation of motion for the system (as in the previous chapter) for each combination of \((h_{\text{poly}}, h_{\text{FFF}})\), where \(h_{\text{poly}}\) is the thickness of the polystyrene layer and \(h_{\text{FFF}}\) is the thickness of the fluid filled foam layer. For each step in the simulation,
the physical value of interest (weight or PLA) is determined and the results are then plotted on a 3D graph.

For instance, to plot Figure 5-1, for each set of thickness \((h_{\text{poly}}, h_{\text{FFF}})\), a simulation is run, using the parameters identified in the previous chapter for each layer, and the characteristic of the impactor (weight and velocity when reaching the sample). From this simulation, a set of data comparable to the one showed in Figure 4-28 is obtained. From that physical parameters of interest (in the case of Figure 5-1, the weight per unit area) are extracted. For each pair of thicknesses of polystyrene and fluid-filled foam, the parameter of interest is calculated and a 3D graph is plotted.

5.2 Constraints for the simulation

Parameters for the simulations are derived from the literature review chapter, and give a set of constraints for each type of helmet. Peak linear acceleration threshold is 150 g for football and military helmets and impacting energy is set at 30 J for current military helmet; the aim is to reach 70 J without reaching the threshold. For motorcycle helmets, the threshold of PLA is 300 g and the impacting energy is 130 J. However, seeing that PLA threshold tends to be lower for the two other applications under study, we will try to propose designs that comply with a PLA threshold of 150 g. Constraints are summarized in Table 5.1. For comparison with the proposed designs, the weight of the polystyrene liner in a motorcycle helmet weighs roughly 400 g (or 0.7 g/cm²).

<table>
<thead>
<tr>
<th>Helmet type</th>
<th>PLA ([m/s^2])</th>
<th>(h_{\text{max}}) [mm]</th>
<th>Energy [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Military - current</td>
<td>150 g</td>
<td>19</td>
<td>30</td>
</tr>
<tr>
<td>Military - new</td>
<td>150 g</td>
<td>19</td>
<td>70</td>
</tr>
<tr>
<td>Football</td>
<td>150 g</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>150 g</td>
<td>25</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 5.1: Design constraints for the three types of helmet
5.3 Results of the simulation

Parameters used in this chapter are those defined in the previous chapter on modeling. Parameters are detailed in Table 4.3 and 4.4.

Here are summarized the parameters used for the simulations.

**Fluid Filled Foam**

Using the parameters detailed in Table 4.4, the equation of motion for a layer of fluid-filled foam is defined by Equations 4.143 and 4.144 and here restated in Equations 5.1 and 5.2

\[
\sigma_{total} = C \frac{\mu \dot{\varepsilon}}{1 - \varepsilon} \quad (5.1)
\]

Equation 5.1 describes the foam response when the strain is less than the densification strain \( \varepsilon_d \). When the strain is greater than the densification strain \( \varepsilon_d \) then the foam response is described by equation 5.2

\[
\sigma_{total} = C \frac{\mu \dot{\varepsilon}}{1 - \varepsilon} + \frac{\sigma_{el}^*}{D} \left( \frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \right) \quad (5.2)
\]

Parameters are detailed in Table 5.2. \( \mu \) is the viscosity of the fluid used to impregnate the foam. When glycerol is used a viscosity of 1.1 Pa.s is used, otherwise viscosity is specified.

<table>
<thead>
<tr>
<th>( C )</th>
<th>( \sigma_{el}^* ) [Pa]</th>
<th>( \varepsilon_D )</th>
<th>( D )</th>
<th>( \varepsilon_{el}^* )</th>
<th>( \varepsilon_d )</th>
<th>Impactor Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5.44 \times 10^5 )</td>
<td>4 000</td>
<td>0.958</td>
<td>1.55</td>
<td>0.05</td>
<td>0.60</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 5.2: Parameters used for the glycerol filled foam simulations

**Polystyrene Foam**

The polystyrene foam behavior is described using the parameters and equation of motion defined in Chapter 4 (see Table 4.3 and Equations 4.137 - 4.140). Equations of motion are restated in Equations 5.4 - 5.7.
To solve numerically the problem, a $3 \times 1$ vector $Y$ is introduced and its derivative with respect to time $\dot{Y}$.

$$Y = \begin{bmatrix} x \\ \dot{x} \\ F \end{bmatrix} \quad \text{and} \quad \dot{Y} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{F} \end{bmatrix} \quad (5.3)$$

The equation of motion of the polystyrene layer can then be synthesized by the following equation:

$$\dot{Y} = MY + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + Cst \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (5.4)$$

Where $M$ is a 3 by 3 matrix and Cst is a constant defined as by Equations 5.5 - 5.7. Initially when the layer of polystyrene is elastically loaded $M$ is described by:

If $x < K'_1/k_1$ then $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ (k_1 + k_2)k_3/\mu & k_1 + k_2 + k_3 & -k_3/\mu \end{bmatrix}$ and $Cst = 0 \quad (5.5)$

where $M_t$ is the mass of the system composed of the impactor plus the layer of polystyrene. When a certain threshold is reached, the plastic behavior begins and $M$ is described by:

If $K'_1/k_1 < x < K'_2/k_2$ then $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/M_t \\ k_2k_3/\mu & k_2 + k_3 & -k_3/\mu \end{bmatrix}$ and $Cst = K'_1k_3/\mu \quad (5.6)$

Finally when a second threshold is reached more plasticity is induced and $M$ is described
by:

\[
M = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & -1/M_s \\
0 & k_3 & -k_3/\mu
\end{bmatrix}
\]

and \( Cst = (K'_1 + K'_2)k_3/\mu \) \quad (5.7)

Using parameters from Table 4.3, parameters used in the simulation are computed and summarized in Table 5.3. Where \( k_i = A \times E_i/h, \mu = A \times \eta/h \) and \( K'_i = A \times K_i, A \) is the area of the sample (100 \( cm^2 \)) and \( h \) is the thickness of the layer (one of the variable of the simulations).

<table>
<thead>
<tr>
<th>( E_1 ) [Pa]</th>
<th>( E_2 ) [Pa]</th>
<th>( k_3 ) [Pa]</th>
<th>( K_1 ) [Pa]</th>
<th>( K_2 ) [Pa]</th>
<th>( \eta ) [Pa.s]</th>
<th>Impactor Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8.25 \times 10^6 )</td>
<td>( 3.25 \times 10^6 )</td>
<td>( 1.625 \times 10^6 )</td>
<td>330000</td>
<td>450000</td>
<td>5750</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 5.3: Parameters used for the friction-pot simulations

Materials

The materials used are those described in Chapter 3.

The flexible foam to be impregnated is an open-cell, polyurethane-based polyester foam (New Dimensions Industries, Moonarchie, NJ), with average cell diameter of 235 \( \mu m \) (70 PPI). Relative density of the foam was calculated using the manufacturer’s value for the density of the polyurethane (\( \rho_s = 1.078 g/cm^3 \)), corresponding to a relative density \( \rho_s^* / \rho_s \approx 0.03 \).

The Newtonian fluid used to impregnate the polyurethane samples is glycerol (VWR, West Chester, PA), where the density is reported to be \( \rho = 1260 kg/m^3 \) and viscosity \( \mu = 1.1 Pa.s \) at 23 \( ^\circ C \). Glycerol is the fluid used by default in most of the simulations. If a different viscosity is used for a simulation, this is mentioned and the viscosity is given. All the Newtonian fluid simulations and experiments are performed with fully impregnated foam.
The polystyrene foam used is a high-density polystyrene (Cordek, West Sussex, U.K.) with density 0.055g/cm³.

To prepare the shear thickening fluid, the same materials as those reported in the section on the shear thickening fluid have been used. The shear thickening fluid is prepared according to the process identified earlier, at a volumetric concentration of 61% of silica particles.

Finally the mass of the impactor is 7.3 kg.

5.3.1 Football helmet

The thicknesses of the polystyrene and fluid-filled foam layers are plotted on a horizontal plane, and weight per unit area or PLA is plotted on the vertical axis of a 3D plot, with a colormap to enhance the visualization. A 3D view of the results is given in Figure 5-1. The solid black area corresponds to non-admissible designs (PLA > 150 g or Total thickness > 30 mm).
Figure 5-1: Weight per unit area in \( g/cm^2 \) as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a Football helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.

It can be seen on Figure 5-1 that the weight of the design increases with the thickness of each layer, and for a given thickness increases with the proportion of fluid filled foam thickness. This is due to the higher density of the fluid filled foam layer. On the contrary, Figure 5-2 shows that PLA decreases when the total thickness increases.

Figures 5-3 and 5-4 are top views of the 3D graph shown in Figure 5-1 and 5-2. Top views give a good representation of the admissible designs. The black area (inadmissible designs) is divided in two parts on these graph: the upper right corner is the area for which thickness is greater than the upper limit of 30 mm of total thickness. On the other hand the lower left corner is the area for which the PLA is greater than the threshold of 150 g.
Figure 5-2: PLA in g [m/s²] as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a Football helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-3: Weight per unit area in $g/cm^2$ as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a Football helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-4: PLA in g m/s² as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a Football helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.

Figures 5-5 - 5-8 show the results of simulations of PLA as a function of thickness of each layer for four viscosities. Figure 5-5 gives results for a fluid with viscosity of 20 Pa.s. Figure 5-6 gives result for a fluid with viscosity 5 Pa.s. Figure 5-7 gives results for a fluid with viscosity 1.1 Pa.s, which is the viscosity of glycerol and Figure 5-8 gives results for a fluid with viscosity 0.5 Pa.s.
Figure 5-5: Admissible designs for a football helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 20 Pa.s.
Figure 5-6: Admissible designs for a football helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 5 Pa.s
The influence of the viscosity of the fluid impregnating the polyurethane foam is very important. Fluid with higher viscosity leads to a higher PLA all other things being equal. That is why area of admissible design depends strongly on the fluid viscosity.

Figure 5-7: Admissible designs for a football helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 1.1 Pa.s
Figure 5-8: Admissible designs for a football helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 0.5 Pa.s

It can be seen that indeed the admissible range for \((h_{\text{Poly}}, h_{\text{FF}})\) depends on the viscosity of the fluid used to impregnate the foam. The higher the viscosity, the smaller the admissible area. However, it can be expected that a more viscous fluid will have other benefits such as a better multi-loading ability. Indeed, energy dissipation depends on the viscous forces, which are directly linked to the viscosity of the fluid. So a larger viscosity should give greater energy dissipation by the fluid, affecting the polystyrene layer and the polyurethane foam itself less.

### 5.3.2 Motorcycle helmet

Results for the simulations for a motorcycle helmet design are given in Figures 5-9 and 5-10 with a 3D view. On Figure 5-9, weight per unit area is shown for admissible design (PLA
less than 150g and Total thickness less than 25 mm). Given the higher energy for the impact (130 J) and the smaller maximum thickness (25 mm) compared with the case of the football helmet, the admissible area is significantly smaller.

Figure 5-9: Weight per unit area in g/cm² as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a Motorcycle helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-10: PLA in g [m/s²] as a function of polystyrene layer thickness and glycerol filled foam layer thickness. Admissible designs for a Motorcycle helmet. The Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.

Figures 5-11 and 5-12 give a top view of Figures 5-9 and 5-10. It can be seen that the area of admissible designs is very small, but still allows an important choice in the thickness distribution of each layer. This example shows that it is possible to design a liner which ends up transmitting a PLA less than 150 g when impacted with a 130J impactor energy. So even with a high energy and a reduced thickness several designs can be chosen, based on the preference in term of weight, PLA and also multi-impact ability. Some experimental tests can be conducted in the admissible area to determine the optimal design once constraints on weight and multi-loading capacity have been decided. For an example of multi-loading ability, see section of this chapter on the military helmet.
Figure 5-11: Weight per unit area in $g/cm^2$ as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a motorcycle helmets. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-12: PLA in g m/s² as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a motorcycle helmet. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figures 5-13 - 5-16 show the results of simulations of PLA as a function of thickness of each layer for four viscosities. Figure 5-13 gives results for a fluid with viscosity of 20 Pa.s. Figure 5-14 gives result for a fluid with viscosity 5 Pa.s. Figure 5-15 gives results for a fluid with viscosity 1.1 Pa.s, which is the viscosity of glycerol and Figure 5-16 gives results for a fluid with viscosity 0.5 Pa.s.

Figure 5-13: Admissible designs for a motorcycle helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 20 Pa.s
Figure 5-14: Admissible designs for a motorcycle helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 5 Pa.s
Figure 5-15: Admissible designs for a motorcycle helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 1.1 Pa.s
Figure 5-16: Admissible designs for a motorcycle helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 0.5 Pa.s
It can be seen from Figures 5-13 - 5-16 that viscosity of the fluid impregnating the polyurethane foam influences largely the design of a motorcycle helmet. Because higher viscosity leads to a higher PLA, with the case of fluids of viscosity 20 Pa.s and 5 Pa.s, no admissible design is found. This means that for those viscosities, the union of the inadmissible area (PLA greater than 150 g) and the inadmissible area (Total thickness greater than 25 mm) covers all the design space investigated. For a viscosity of 1.1 Pa.s or a viscosity of 0.5 Pa.s however, admissible designs represent a non-zero area and so several designs comply with the constraints and could be tested experimentally to determine multi-impact ability.
5.3.3 Military helmet

Results for the simulations of a military liner material are given on 3D graphs in Figures 5-17 - 5-18. Given the constraints in term of PLA threshold (150 g) and total thickness of the liner material (less than 19 mm), the admissible area is reduced. It can be seen that weight of the layer increases with the percentage of fluid filled foam thickness. The effect on PLA is not very clear but all the designs in the admissible area ensure a PLA less than 150 g.

Figure 5-17: Weight per unit area in $g/cm^2$ as a function of polystyrene layer thickness and glycerol filled foam layer thickness. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-18: PLA in g m/s² as a function of polystyrene layer thickness and glycerol filled foam layer thickness. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.

Figures 5-19 and 5-20 give top view of Figures 5-17 - 5-18. Admissible design form an area which divides the plane of investigated designs in two. The black upper right corner are designs which does not comply with the constraint of a total thickness less than 19 mm and the black lower left corner show designs that lead to a PLA greater than the threshold of 150 g.
Figure 5-19: Weight per unit area in $g/cm^2$ as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a military helmet. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Figure 5-20: PLA in g m/s² as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Admissible designs for a military helmet. Figure gives admissible thicknesses for each layer. Solid black area shows inadmissible designs.
Simulations have been run with different viscosity for the fluid impregnating fluid. Figures 5-21 - 5-24 show the results of these simulations. Figures 5-21 and 5-22 show the PLA as a function of the thickness for each layer, respectively for a fluid viscosity of 20 Pa.s and 5 Pa.s. The area is completely black showing that for those fluid viscosity, none of the design investigated comply with the two constraints (PLA less than 150 g and Total thickness less than 19 mm). Figures 5-23 and 5-24 show the results respectively for a fluid viscosity of 1.1 Pa.s and a fluid viscosity of 0.5 Pa.s. The results show that for those viscosity an area of admissible designs exists and so some designs can be suggested for testing.

Figure 5-21: Admissible designs for a military helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 20 Pa.s
Figure 5-22: Admissible designs for a military helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 5 Pa.s
Figure 5-23: Admissible designs for a military helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 1.1 Pa.s
Figure 5-24: Admissible designs for a military helmet. Figure gives PLA as a function of thicknesses for each layer. Solid black area shows inadmissible designs. Viscosity of the fluid impregnated the polyurethane foam = 0.5 Pa.s

Using these results, some experimental tests can be conducted in the admissible area to determine multiple-loading performance of some of the admissible designs. Designs located in the admissible design area are compliant with the two constraints in term of PLA and total Thickness. Multi-loading ability can be tested with experimental impact tests to give some guidance on what design to choose in the admissible area. As shown in Figure 5-25, three designs have been chosen. Designs parameters are summarized in Table 5.4.

<table>
<thead>
<tr>
<th>Design</th>
<th>Polystyrene thickness [mm]</th>
<th>Glycerol filled foam thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design 1</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>Design 2</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Design 3</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 5.4: Design parameters for the three tested designs
Figure 5-25: Weight per unit area in g/cm² as a function of polystyrene layer thickness and glycerol filled foam layer thickness - Figure shows the three tested designs. \( h_{Poly} \) is the thickness of the polystyrene layer, \( h_{FFF} \) is the thickness of the glycerol filled foam layer. Solid grey area shows inadmissible designs.

**Weight of proposed designs - Discussion**

No constraint in terms of weight is given for each of the applications, however to give an idea of the current weight of padding and that of the proposed design we give here an estimate for each situation.

As stated previously typical liner mass is currently around 400 g which corresponds to a weight per unit area of 0.7 g/cm² Total area of padding on a ACH military helmet is around 500 cm². Weight per unit area ranges from 0.6 to 1.5 g/cm², so that the total weight of the padding would range between 300 g and 750 g. On a football helmet the area of padding is around 470 cm². Weight per unit area ranges from 0.7 to 2.2 g/cm², so that the total weight of the padding would range between 330 g and 1030g. In a motorcycle helmet the area of
padding is around 570 cm². Weight per unit area ranges from 0.8 to 1.8 g/cm², so that the total weight of the padding would range between 456 g and 1026 g.

The weight of the padding in the proposed designs are acceptable. A heavier mass will lead to a better head protection, and as previous study [60] showed, a better liner material could lead to a decrease of the shell thickness, resulting overall in a decrease of the total weight of the helmet. So increasing the weight of the liner material seems to be a possible option, while getting in the end a lower or equal mass for the complete helmet.

**Multi-loading testing**

For each of the designs in Table 5.4, and for a simple layer of 19 mm thick polystyrene, drop tower tests have been conducted. The tests are conducted under the same experimental conditions as those described in chapter 3. The impact energy is 70J. Each of the samples is submitted to 5 consecutive impacts, with a rest of 1 minute between each impact. (This corresponds to the minimum time between two impact tests with the drop tower, at this given energy). Between two impacts, the sample is re-aligned with the impactor if it has moved during the previous impact.

To give an reference for comparison, the results for the single layer of 19 mm thick polystyrene are given in Figure 5-26. Results for Design 1 are given in Figure 5-27. Results for Design 2 are given in Figure 5-28, results for Design 3 are given in Figure 5-29.
Figure 5-26, shows that a simple layer of polystyrene exhibits a rapidly increasing PLA as the number of impacts increases. We know from chapter 4 that this is due to plastic deformation that alters irrevocably the structure of the polystyrene layer at each impact, thus leading to worse energy absorption ability after each impact.

Figure 5-26: Acceleration versus displacement - Experimental results for a 19mm thick polystyrene sheet, Impact Energy of 70J
Figure 5-27: Acceleration versus displacement - Experimental results for Design 1: Polystyrene 14mm FFF 5mm, Impact Energy of 70J

Figure 5-27 shows the results of the PLA for Design 1. The increase in PLA over the 5 impacts is significantly reduced compared with the single layer of polystyrene, but the 4th and 5th impacts lead to PLA greater than the threshold of 150 g.
Figure 5-28: Acceleration versus displacement - Experimental results for Design 2: Polystyrene 9.5mm FFF 9.5mm, Impact Energy of 70J

Figure 5-28 shows the results of the PLA for Design 2. The increase in PLA over the 5 impacts is greatly reduced compared with the single layer of polystyrene, but the 4th and 5th impacts lead to PLA of 150 g, very close to the threshold. The increase of PLA between the first and the 5th impact is still noticeable.
Figure 5-29: Acceleration versus displacement - Experimental results for Design 3: Polystyrene 6mm FFF 13mm, Impact Energy of 70J

Figure 5-29 shows the results of the PLA for Design 3. Increase in PLA over the 5 impact is almost not noticeable. The PLA for all the impacts is significantly below the threshold of 150 g.

**Multiple-loading conclusion**

From the results it can be seen that increasing the thickness of the glycerol filled foam layer increases dramatically the multiple loading ability. Indeed, the comparison between the PLA of the first and the 5th impact for all the proposed designs indicates that design 3 shows less than 5% increase in PLA whereas a simple layer of polystyrene shows an increase of 66%. Furthermore designs 2 and 3 proposed exhibit a PLA less than 150 g for the five impacts, thus complying with the threshold for military application.
<table>
<thead>
<tr>
<th>Design</th>
<th>PLA 1\textsuperscript{st} Impact in g (m/s\textsuperscript{2})</th>
<th>PLA 5\textsuperscript{th} Impact in g (m/s\textsuperscript{2})</th>
<th>% increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polystyrene 19 mm</td>
<td>150</td>
<td>250</td>
<td>66%</td>
</tr>
<tr>
<td>Design 1</td>
<td>137</td>
<td>185</td>
<td>35%</td>
</tr>
<tr>
<td>Design 2</td>
<td>120</td>
<td>150</td>
<td>25%</td>
</tr>
<tr>
<td>Design 3</td>
<td>119</td>
<td>125</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 5.5: PLA for the 1\textsuperscript{st} and 5\textsuperscript{th} impact and percentage of increase for the three designs proposed and a layer of polystyrene of the same thickness (19 mm)

5.3.4 Shear thickening Fluid simulations

In Chapter 4, we showed that modeling polyurethane foam impregnated with shear-thickening fluid showed the best agreement with experimental data if we assumed the fluid to have a constant viscosity of 5 Pa.s. Simulations run in the previous sections (5-6, 5-14, 5-22) with a fluid viscosity of 5 Pa.s give results that predict the behavior of a shear-thickening fluid impregnated foam, under the assumption that the foam is fully impregnated. However, so far a full impregnation of the foam has not been achieved. The uncertainty about the fluid behavior at very high shear rate makes it difficult to run specific simulations for this type of fluid. However, the simulation tool has shown good performance at predicting the liner material behavior under impact and the design process can be used to assess the opportunity of a liner material including a shear-thickening fluid impregnating a polyurethane foam if better understanding of the fluid behavior is achieved.

5.4 Conclusion

In this chapter we used the modeling developed in the previous chapter to find a range of parameters that would end up with liner complying with the constraints of each application. Simulations have been performed for the three area of interest: motorcycle helmet, military helmet and football helmet. Once a admissible space of parameters ($h_{poly}, h_{FFF}$) has been determined, experimental testing allowed us to investigate on multi-loading ability. Results
obtained for the military applications show that proposed design can enhance dramatically the performance for multiple impact absorption.
Chapter 6

Conclusions and recommendations for Future Research

6.1 Conclusion

Head injury is a concern of importance in the US and throughout the world, as Chapter one points out. Casualties reported are linked to various activities and among the one that can be addressed by the use of a head protection, military personal, motorcyclists and football players are a good sample of the type of environments and constraints a helmet designer could face. Growing costs of injured people urges an effort to offer better protection.

In this study we have studied the current state of the art for head protection in three areas: Football, Military and Motorcycle. Studying these three areas of application gave us a wide range of energy levels for impact testing, as well as different sets of constraints for each type of design. After identifying the short comings of the existing designs we have identified potential materials that could be part of better designs. The modeling and experimental testing of these materials lead us to develop models for a composite liner material that would include layers of the two materials (glycerol filled foam and polystyrene). Using this mod-
eling, simulations were performed to find designs that would meet the specific requirements to each type of application. Finally, some of the proposed designs were tested to show effectiveness compared to existing liner materials.

We have developed and validated a model for dense polystyrene, based on typical mechanical behaviors reported in the literature. Simulations and experimental data showed very good agreement and allowed investigation of several parameters of importance and design options. Similarly for fluid filled foam, using previous work on permeability of open-cell reticulated foams [60] it has been possible to model the behavior of the filled foam under impact. Once again, good agreement between simulations and experimental data saved a lot of time and material costs to be able to study the influence of the various parameters and test a wide range of designs.

Finally simulations of a complete liner materials were performed, optimized and experimentally tested to verify the accuracy of predicted results and study the multi loading ability of the proposed designs. In the case study of the military helmet liner, increase of PLA between the first and the 5th impact was reduced from 66% to 5% increase between a single layer of polystyrene and a design proposed that meet the requirement in term of PLA and maximum thickness, alternative designs showing intermediate performance but reduced weight were also tested.

Additionally, a negative Poisson’s ratio foam was prepared and tested, and a robust experimental method to prepare it was developed. This could be of interest for future work as stated by Evans et al. [63]. Also, based on previous work on this subject [60, 80], a shear thickening suspension of silica nanospheres was prepared and characterized. Modeling and experimental tests with partially filled foam were performed and compared to simulations, giving reasonable agreement.
6.2 Recommendations for future research

For future research in the same area, interesting an point of development would be to model the entire helmet and make some samples to actually test it on a standardized head form. Using accelerometers inside the head form this could validate the design with test in accordance to current standards [56], [28], [12]. This will require building a complete helmet with the liner and the shell. This type of test and modeling would allow study of the importance of the shell material and thickness. This could be of great interest since previous work on the subjects [51, 60] showed that, for instance, on motorcycle helmets the thickness of the shell could be reduced while providing the adequate protection. This will also allow significant reduction in the total weight of the helmet since the shell is the material with the highest density by far.

Studying the effect of temperature on the proposed material could also be a field of investigation, since the standardized tests generally impose the liner to perform in cold, standard and warm temperature. Effects could be investigated on materials alone, or measurements of temperature in a helmet on a human head could be used to reduce the range of temperature to study.

Modeling and experiments could also be conducted to investigate designs presenting not only various layers of materials, but also geometrical pattern within a layer (macro-channel, fluid-storage pouch...). Investigating on the adequate scale and the type of geometry that could enhance one of the aspect of the material (weight, multi loading ability...) could improve the design which is for the moment only composed of uniform layers.

Furthermore, based on the method developed in this study to prepare a shear-thickening fluid, an experimental method to fully impregnate the foam could be developed, and later a method that could be industrialized could be tested. This would allow testing of this fluid when fully impregnated the foam. Modeling and experimental work to try to determine
the fluid behavior at very high shear rate could help to model the impact response of the impregnated foam.

Additionally, the finite-element analysis of the helmet when subjected to an impact could be developed to try to enhance the padding distribution, based on a stress distribution analysis during an impact. This could lead to a better protection and reduce the weight significantly.

Finally, future work could try to investigate the use of fluid filled foam (with Newtonian or Non-Newtonian fluid) for other types of applications.
Bibliography


