Essays on Macroeconomic Risks and Stock Prices

by

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Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Economics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2011

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Abstract

In this thesis, I study the relationship between macroeconomic risks and asset prices. In the first chapter, I establish that inflation risk is priced in the cross-section of stock returns: stocks that have low returns during inflationary times command a risk premium. I estimate a market price of inflation risk that is comparable in magnitude to the price of risk for the aggregate market. Inflation is therefore a key determinant of risk in the cross-section of stocks. The inflation premium cannot be explained by either the Fama-French factors or industry effects. Instead, I argue the premium arises because high inflation lowers expectations of future real consumption growth. To formalize and test this hypothesis, I develop a consumption-based general equilibrium model. The model generates a price of inflation risk consistent with my empirical estimates, while simultaneously matching the joint dynamics of consumption and inflation, the aggregate equity premium, and the level and slope of the yield curve.

In the second chapter, with L. Kogan and Dmitry Livdan, we study the relation between returns on the aggregate stock market and aggregate real investment. While it is well known that aggregate investment rate is negatively correlated with subsequent excess stock market returns, we find that it is positively correlated with future stock market volatility. Thus, conditionally on past aggregate investment, the mean-variance tradeoff in aggregate stock returns is negative. We interpret these patterns within a general equilibrium production economy. In our model, investment is determined endogenously in response to two types of shocks: shocks to productivity and preference shocks affecting discount rates. Preference shocks affect expected stock returns, aggregate investment rate, and stock return volatility in equilibrium, helping model reproduce the empirical relations between these variables. Thus, our results emphasize that the time-varying price of aggregate risk plays and important role in shaping the aggregate investment dynamics.

In the third chapter, with S. Parsa, we show a novel relation between the institutional investors' intrinsic trading frequency—a commonly used proxy for the investors's investment horizon—and the cross-section of stock returns. We show that the 20%
of stocks with the lowest trading frequency earn mean returns that are 6 percentage points per year higher than the 20% of stocks that have the highest trading frequency. The magnitude and predictability of these returns persist or even increase when risk-adjusted by common indicators of systematic risks such as the Fama-French, liquidity or momentum factors. Our results show that the characteristics of stockholders affect expected returns of the very securities they hold, supporting the view that heterogeneity among investors is an important dimension of asset prices.

JEL classification: E31, E44, G12

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Acknowledgments

First and foremost, I would like to acknowledge the pivotal role of my advisors, Ricardo Caballero and Leonid Kogan, in the developing of this thesis. Ricardo has been a continuously present force, inspiring me since my days as an undergraduate at MIT and helping me transition into graduate school. His capacity to grasp the fundamental qualities of a given problem, his ability to tackle large, ambitious projects, his relentless work ethic and his high research standards have been a role model worth of admiration for me. I know I will carry his strive for excellence and love of Economics throughout my entire career. Leonid has been the most generous, nurturing advisor one could hope for. His sharp intellect is coupled with a remarkable ability to clearly explain his ideas. Adding his unquestionable patience to the mix makes him a superb person to interact with. When my funding needed a boost, when my equations did not seem to work, when ideas were running dry, Leonid was always there and willing to help.

Many other friends and colleagues have substantially shaped this thesis. Xavier Gabaix, Adrien Verdelhan, Guido Lorenzoni and Ivan Werning provided extremely valuable feedback and comments. I am thankful to Matt Notowididgo, Sahar Parsa, Jennifer La'O, Alp Simsek, Jonathan Goldberg, Maya Eden, Jenny Simon, Michael Powell, Luigi Iovino, Ivo Welch, Hui Chen, Gustavo Manso, Roberto Rigobon, Dmitry Livdan and Stavros Panageas for their comments and helpful discussions. A special mention goes to Christine Breiner, Ryan Kabir, Nicholas Hoff, Matt Notowididgo and Stephen Steiner for their enormous help and support in the many dimensions of creating this thesis. Individual chapters of this thesis have further acknowledgments specific to each of them.

Last but not least, I want to thank my parents and sister. Without their constant direction, encouragement and presence, their all-embracing generosity, humanity and magnanimity, I would have not been able to be where I am today.
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Chapter 1

Inflation Risk and the Cross-Section of Stock Returns

1.1 Introduction

In this paper, I document how and why inflation risk is priced in the cross-section of stock returns. Using a Fama-MacBeth (1973) procedure, I find that stocks whose returns are negatively correlated to inflation shocks command a risk premium. I estimate the market price of inflation risk to be \(-0.33\) when measured as the Sharpe ratio of an inflation-mimicking portfolio. The price of inflation risk is therefore comparable in magnitude to the price of risk for the aggregate market. The negative price of risk means that inflationary periods correspond to bad states of nature: investors are willing to accept lower unconditional returns when holding securities that are good hedges against inflation. I argue the premium arises because high inflation today predicts low real consumption growth over many subsequent periods. I develop a model that uses

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\(^1\)I am grateful to Ricardo Caballero and Leonid Kogan for invaluable inspiration and support throughout my PhD studies. I thank Christine Breiner, David Cesarini, Hui Chen, Maya Eden, Jonathan Goldberg, Farah Kabir, Jennifer La’O, Guido Lorenzoni, Gustavo Manso, Marti Mestieri, Matt Notowididgo, Sahar Parsa, Michael Powell, Jenny Simon, Alp Simsek, Ivo Welch, participants of the MIT Macroeconomics Seminar and particularly Xavier Gabaix and Adrien Verdelhan for helpful comments and discussion.
the relationship between inflation and consumption to generate a price of inflation risk consistent with my empirical estimates.

By studying the cross-section of stock returns, I not only uncover a new source of information about the inflation premium in the economy, but also provide insights about the distribution and pricing of inflation risk of individual firms. Measures of the inflation risk premium have had a natural starting point in the yield curve. With the development of sophisticated no-arbitrage models of the term structure and the emergence of Treasury Protected Inflation Securities (TIPS), estimates of the inflation risk in the bond market have become more reliable and widely available. Another conventional way to estimate the inflation premium is to study the joint time-series behavior of inflation and aggregate market returns. A landmark example is Modigliani and Cohn (1979), who find a negative correlation between inflation and the S&P500 over the 1970’s and propose an explanation based on inflation illusion. Other recent economic explanations of the inflation premium in the aggregate market are based on important contributions by Wachter (2006) using habit formation, Gabaix (2008) using rare disasters and Bansal and Shaliastovich (2010), who use long-run risk. If the fundamental mechanisms of the real effects of inflation originate at the level of individual households or firms, studying the cross-section of stocks can provide valuable additional information that is masked in the aggregate market and the yield curve.

The variation cross-sectional returns associated with inflation is not well described by any of the risk factors most commonly used to price assets. For example, the Fama-French factors have a pricing error of 2.8% per year when confronted with portfolios sorted on exposure to inflation. Industry effects also fail to explain a significant fraction of the spread in returns of inflation-sorted portfolios. Consequently, firm characteristics that differ across sectors of the economy—like menu costs, leverage, tax liabilities or labor relations—although important, should be supplemented by further factors to

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fully understand stocks’ cross-sectional heterogeneity in inflation risk.

I propose an explanation of the cross-sectional inflation premium by arguing that high inflation is a bad state of nature because it predicts low future real consumption growth. I formalize and test this hypothesis by developing a consumption-based equilibrium model. The model takes the stochastic processes for consumption and inflation as given and asset prices are then determined endogenously through the representative agent’s Euler equation. After estimating parameters using generalized method of moments (GMM), I show that the model can quantitatively replicate the observed inflation premium while simultaneously matching key empirical moments of consumption, inflation, bond yields and the aggregate stock market.

To generate an inflation premium consistent with the data, my model has three key ingredients, all of which are necessary. The first ingredient—as already mentioned—is that high inflation predicts low future real consumption growth. I estimate that an increase of one percentage point in inflation this month is associated with a decrease of 2.3 percentage points in real consumption growth over the next two years. Additionally, I show that several lags of inflation are useful in predicting consumption, even after controlling for current inflation and current consumption growth. Piazzesi and Schneider (2005) also find that inflation is a leading indicator for consumption and use this relationship to rationalize the inflation premium in the yield curve.

The second ingredient is that inflation is persistent. Inflation persistence is widely documented in the literature, for example in Fuhrer and Moore (1995), Stock and Watson (2005), Campbell and Viceira (2001) and Ang, Bekaert and Wei (2007). Ang et al. show that the first-order autocorrelations of inflation at the monthly and quarterly frequencies are 0.92 and 0.77 respectively. Furthermore, inflation persistence decays slowly over the business cycle, with a first-order autocorrelation of 0.35 at 10 quarters. That inflation is quite persistent will be important in my model to quantitatively match the large inflation premium: more persistent inflation induces a larger market price of inflation risk because it affects consumption growth negatively for a longer period of
time.

The third ingredient is a representative agent with recursive Epstein-Zin-Weil (EZ) utility. With EZ preferences, shocks to expectations about future consumption growth are priced in addition to shocks to consumption growth itself. Since inflation predicts consumption growth, inflation shocks are priced in my model. This property of EZ utility is explored by many authors in the macro-finance literature.

To give brief intuition of the model, consider what happens when a positive inflation shock hits a two period economy. Inflation unexpectedly jumps up and remains above its initial value in the second period. Consumption is unchanged in the first period and predictably decreases in the second period. The price of the wealth portfolio—which is simply a claim to future consumption—will change due to income and substitution effects. The income effect makes the price of period-2 consumption go down since the representative agent’s wealth has decreased and therefore demands less consumption. The substitution effect makes the price of period-2 consumption increase because the representative agent would like to smooth her consumption path by shifting consumption away from period 1 and into period 2. When the elasticity of intertemporal substitution (EIS) is greater than one, which is the relevant case in my model, the representative agent is not willing to pay a high price to smooth consumption and the income effect dominates the substitution effect. The wealth portfolio now has lower returns. An inflation-mimicking portfolio, since it co-varies negatively with the return on the wealth portfolio, reduces the volatility of expected consumption growth. If the representative agent is averse to risk in expected consumption growth, inflation shocks have a negative market price of risk.

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4Aversion to risk in expected consumption growth is equivalent to having a preference for early resolution of uncertainty. For EZ preferences, this happens when the product of the coefficient of relative risk aversion and the EIS is greater than one.
Conceptually, my model builds on Parker and Julliard’s (2005) idea that ultimate consumption risk depends on an asset’s correlation not only with present consumption but also with consumption growth over many subsequent periods. The particular mechanism I consider is most closely related to Bansal and Yaron’s (2004) long-run risk model. In their model, asset prices are driven by a small, persistent long-run predictable component of consumption. Long-run risk is priced in their model for the same reasons that inflation is priced in mine. However, there are four important differences. First, predictability of consumption using inflation, and inflation persistence itself, operate at business cycle frequencies. Bansal and Yaron’s long-run risk operates at substantially lower frequencies, of 10 years or more. Second, inflation shocks have a higher variance than long-run risk shocks. The combination of higher volatility and lower persistence of inflation shocks makes their inflation premium comparable in magnitude to the premium earned by the low volatility and high persistence long-run risk shocks. Third, inflation is directly observable while long-run risk must be inferred from asset prices using the model’s assumptions. The observability of inflation provides key additional moments to test my model. In particular, a successful model must match, as I do, the correlation between asset returns and inflation, while having a realistic process for inflation with several lags and heteroskedasticity. Fourth, while theoretical connections between long-run risk and consumption growth have not been thoroughly explored, there is a vast theoretical literature proposing mechanisms for why inflation has real effects.

**Related literature** Almost all studies of the inflation premium look at the time series of aggregate stock returns and the yield curve instead of the cross-section of stock returns. Bansal and Shaliastovich (2010) attach to the canonical long-run risk model a

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process for inflation and use it to price nominal bonds. The inflation premium in their model arises not because inflation feeds back into consumption, but because inflation is exposed to the same real shocks that drive consumption and long-run risk. Gabaix (2008) explains the inflation premium in a model with rare disasters. Inflation is priced because when a disaster occurs, inflation tends to increase. Wachter (2006) explains the inflation premium using i.i.d. consumption and habit-formation. All three models, unlike the one I propose, were originally designed to explain classic pricing puzzles such as the equity premium puzzle and the failure of the expectations hypothesis.

A notable exception to using time-series estimates is Chen, Roll and Ross (1986). They postulate a model with many macroeconomic and aggregate factors, including inflation innovations. The emphasis is not in estimating risk premia precisely but to find plausible state variables for asset prices. They find that inflation is priced only for the 1968-1977 subsample and, in contrast to my results, that stocks are weak hedges against inflation. Their study differs from mine in several respects. They use 20 portfolios sorted on size as their test assets, while I use individual stocks and portfolios sorted on inflation risk, which is the relevant variable. Their sample ends in 1984 and contains many fewer securities than mine. Finally, they use yearly instead of monthly data and a different Fama-MacBeth procedure.

Piazzesi and Schneider (2005) analyze how the fact that inflation predicts future consumption growth affects the pricing of nominal bonds. Their paper can be viewed as the counterpart of my paper in the bond market. They argue, consistent with my findings, that inflation is bad news for future consumption, producing an upward sloping yield curve. While I use rational expectations throughout, they study the impact of changing investors’ beliefs. They find that learning is important in an environment in which investors cannot easily distinguish permanent and transitory movements in inflation. While I do not allow for learning or endogenously changing beliefs, I do analyze exogenous structural changes in the relationship between inflation and consumption over my sample. Another important difference is that I allow for heteroskedasticity in
the inflation process.

1.2 Measuring Inflation Risk

In this section, I estimate that the price of risk of inflation shocks is -0.33 using a two-step Fama-MacBeth procedure. The differences in stock returns arising from inflation risk are captured neither by the Fama-French factors nor by other standard pricing models. Industry effects play a limited role in explaining the large heterogeneity in inflation exposures present in the cross-section of stocks.

1.2.1 Data

I use monthly data for the period 1959-2009. For inflation, I use the consumer price index (CPI) from the Bureau of Labor Statistics. For consumption growth, I use real personal consumption expenditures (PCE) in non-durables and services from the Bureau of Economic Analysis. Individual stock returns are from the Center for Research in Security Prices (CRSP) and I use the CRSP value weighted index for aggregate market returns. I use the entire universe of CRSP, which for my sample has 27,688 companies represented and 3,262,429 total month-company observations. Yields for bonds are obtained from the Fama-Bliss bond files, and the risk-free rate is from the Fama risk-free rate files, both available in CRSP. Fama-French and momentum factors, industry portfolios, short and long-term reversal factors are all obtained from Professor French’s website. The Cochrane-Piazzesi factor is from Professor Piazzesi’s website. Oil prices are from the International Monetary Fund. I chose 1959 as the start of my sample to coincide with availability of PCE data.

\footnote{In the following section, I will eliminate 1% of the sample due to outliers.}
1.2.2 Inflation betas

In the first step of the Fama-MacBeth procedure, I measure firms’ exposure to inflation by estimating their “beta”, just as one would do in a CAPM setting. Instead of market returns, I use inflation innovations as the risk factor. I only use past information when estimating risk exposures to eliminate look-ahead bias when I later form inflation portfolios—an investor living in any period of my sample could have replicated my results in real time. For each stock \( n = 1, \ldots, N \) and each time period \( t = 1, \ldots, T \), I find an estimate of the inflation beta \( \hat{\beta}_{n,t} \) by running a weighted least-squares regression of excess returns \( R_{n,t} \) on inflation innovations \( \Delta \pi_t = \pi_t - \pi_{t-1} \), using all observations in the interval \([1, t-1]\). Since the dependent variables are excess returns, I am considering the exposure of real returns to inflation\(^7\). I use weights that decay exponentially with the distance between observations and have a half-life of five years. This estimator efficiently captures time variation in betas by using all available past information. I choose to have decaying weights because recent observations are more likely to contain information about inflation exposure going forward. The weighted-least squares estimator resembles the original 5-year rolling window estimator used in Fama and French (1992), with the advantage of using more information to produce smoother estimates\(^9\). The estimator is given by

\[
\left( \hat{\alpha}_{n,t}, \hat{\beta}_{n,t} \right) = \arg \min_{\alpha, \beta} \sum_{\tau=1}^{t-1} K(t - \tau) \left( R_{i,\tau} - R^f_{i,\tau} - \alpha - \beta \Delta \pi_{\tau} \right)^2
\]

with weights

\[
K(t - \tau) = \frac{\exp(-|t - \tau - 1| h)}{\sum_{\tau=1}^{t-1} \exp(-|t - \tau - 1| h)}.
\]

---

\(^7\)Throughout the paper, I will use a superscript “e” to denote excess returns, so for example, if the risk free rate is \( R^f_t \), excess returns for stock \( n \) at time \( t \) is \( R^e_{n,t} = R_{n,t} - R^f_t \).

\(^8\) \( R^\text{nominal}_{n,s} - R^\text{nominal}_{f,s} = (R^\text{nominal}_{n,s} - \pi_s) - (R^\text{nominal}_{f,s} - \pi_s) = R^\text{real}_{n,s} - R^\text{real}_{f,s} \)

\(^9\) I later report that estimates for the market price of inflation risk are similar when using the weighted-least squares and the 5-year rolling window. The main difference is in the standard error. The weighted least squares estimator also performs better when predicting ex-post exposures, which I attribute to the reduction in noise coming from using more observations.
I use \( h = \log(2)/60 \) to get a half-life of 5-years. The least squares estimator in (1.1) can also be thought of as a kernel estimator with exponential kernel given by (1.2) and bandwidth\(^{10} h \). The 5-year rolling window estimator also satisfies (1.1) but uses a flat kernel that becomes zero after 5 years. The interpretation of the estimates is straightforward: A value of \( \hat{\beta}_{n,t} = -2 \), for example, means that a change in inflation of one percentage point is associated with a decrease in excess returns of two percentage points over the same time period. Ex-ante (backward-looking) betas are useful as a measure of risk insofar as they also capture risk exposure going forward. Elton, Gruber, and Thomas (1978) show that making a simple Vasicek adjustment to the ex-ante betas can make ex-ante exposures better predictors of ex-post exposures. The Vasicek adjustment is a Bayesian updating procedure in which the prior distribution is given by the beta \( \hat{\beta}_{n,t} \) estimated from the time-series and the posterior distribution is obtained by incorporating information about the cross-sectional distribution of \( \hat{\beta}_{n,t} \) for fixed \( t \). The formula is:

\[
\hat{\beta}_{n,t}^{\text{adj}} = w_{n,t} \hat{\beta}_{n,t} + (1 - w_{n,t}) EXS \left[ \hat{\beta}_{n,t} \right] \quad (1.3)
\]

\[
w_{n,t} = 1 - \frac{\text{var}_{TS}(\hat{\beta}_{n,t})}{\text{var}_{TS}(\hat{\beta}_{n,t}) + \text{var}_{XS}(\hat{\beta}_{n,t})}, \quad (1.4)
\]

where the subscripts \( TS \) and \( XS \) denote means and standard deviations taken over the time series (over the variable \( t \)) and the cross section (over the variable \( n \)) respectively. Vasicek betas are a weighted sum of each stock’s beta and the mean beta in the cross-section. The adjustment places higher weight on individual betas that are more precisely estimated and higher weight on the cross-sectional mean when the cross-section has less dispersion. From this point forward, inflation betas refer to estimated Vasicek-adjusted betas and I will drop the superscript \( \text{adj} \) and the hat.

Figure 1 depicts the histogram of inflation betas for four different time periods. I have selected January of 1979, 1983, 1994 and 2009 to portray the shape of the distri-

\(^{10}\)See Ang and Christensen (2010).
bution of betas in different macroeconomic conditions and inflationary regimes. Betas have significant dispersion in all four time periods, with values\textsuperscript{11} ranging from -25 to +15. The mean of the distribution moves considerably through time. Strikingly, during the downturn of 2009 the mean inflation beta is positive. Campbell, Sunderam and Viceira (2010) document, consistent with Figure 1, that the “nominal-real” covariance of inflation with the real economy is positive on average but has been negative since the downturn of 2001.

Figure 2 shows the time series of inflation beta for the aggregate market, a five-year zero-coupon nominal Treasury bond and four well-known firms representative of different sectors of the economy. The market’s inflation beta is a good proxy for the mean of the distribution of betas shown in Figure 1. Figure 2 displays mostly negative betas for the aggregate market in the 1980’s and 1990’s, with positive values at the beginning and end of the time series. Compared to the market, the excess returns on the 5-year bond have a small and almost constant beta. This means that the spread of the 5-year real risk-less rate over the 1-month real risk-less rate has little exposure to inflation. Figure 2 also shows that individual companies’ betas tend to move together with the market, especially at lower frequencies, yet still exhibit considerable cross-sectional heterogeneity. The correlation between firms’ betas is also time varying. For example, Coca-Cola and General Electric move in lockstep in the 1970’s but move in opposite directions in the 1980’s. I focus exactly on exploiting this type of cross-sectional variation to identify the inflation premium. In this respect, my research differs substantially from Campbell, Sunderam and Viceira (2010) and from most other studies of the inflation premium, as explained in the introduction.

\textsuperscript{11}I cut from the sample stocks with betas in the top and bottom 0.5% of the distribution because their betas are extreme. Results are robust to windorizing with a threshold for betas of ±25.
1.2.3 Inflation-sorted portfolios

Figure 2 makes clear that individual estimates of inflation betas have substantial high frequency variation. If some of that variation is due to noise (e.g. measurement error), statistical inference of the inflation premium or the distribution of betas can be challenging. Following Fama and MacBeth (1973), and Black, Jensen and Scholes (1972), I address this problem by forming portfolios, in the hope that idiosyncratic variation will average out within each portfolio. To form portfolios, I perform a double-sort based on inflation betas and size. Size is measured by market equity (price multiplied by shares outstanding) in June of the previous year, just as in the construction of the Fama-French factors. At time $t$, each firm belongs to one of 10 deciles of the inflation beta distribution and one of 10 deciles of the size distribution. I create 100 value-weighted portfolios by grouping stocks that belong to the same beta and size deciles. This procedure implies rebalancing portfolios every month. In practice, however, around 80% of the firms remain in the same portfolio after one year and about 30% after 5 years, which is not surprising given that estimates in betas for two consecutive months only differ by one observation. As when creating size and book-to-market factors in Fama-French, I reduce the 100 inflation-and-size portfolios to 10 inflation-only portfolios by collapsing the size dimension. Specifically, at time $t$, each stock $n$ has an exposure to inflation given by $\hat{\beta}_{n,t}$ and a size of $size_{n,t}$. I pick cutoffs $\beta^0, \beta^1, ..., \beta^{10}$ such that the sets

$$N^{i,t} = \{n : \beta^i \leq \hat{\beta}_{n,t} < \beta^{i+1} \} \text{ for } i = 0, ..., 9,$$

have the same number of elements. Similarly for size, I pick cutoffs $size^0, size^1, ..., size^{10}$ such that the sets

$$M^{j,t} = \{n : size^j \leq size_{n,t} < size^{j+1} \} \text{ for } j = 0, ..., 9,$$

(1.5)

(1.6)
have the same number of elements\textsuperscript{12}. I then form 100 value-weighted portfolios $P_{1,t}, ..., P_{100,t}$ by grouping stocks that belong to the same inflation exposure and size groups, i.e.

$$
\bar{P}_{q,t} = \{n : n \in N^{i,t} \text{ and } n \in M^{j,t} \text{ with } q = i + 10j\}
$$

(1.7)

with return

$$
\bar{R}_{q,t} = \sum_{n \in \bar{P}_{q,t}} \omega_{n,t} R_{n,t},
$$

(1.8)

$$
\omega_{n,t} = \frac{\text{size}_{n,t}}{\sum_{n \in \bar{P}_{q,t}} \text{size}_{n,t}}.
$$

(1.9)

Finally, to collapse the 100 double-sorted portfolios into 10 inflation-sorted portfolios $P_{1,t}, ..., P_{10,t}$, I average the returns of portfolios across size groups, so that the resulting ten portfolios have returns given by:

$$
R_{r,t} = \frac{1}{10} \sum_{q=10(r-1)}^{10r} \bar{R}_{q,t} \text{ for } r = 1, ..., 10.
$$

(1.10)

The resulting 10 portfolios have differential ex-ante exposure to inflation innovations but little variation in size. Ideal test assets have identical exposure to every risk factor except for inflation. In that case, any difference in mean returns can be interpreted as compensation for inflation risk. When reducing the original 100 portfolios to the new 10 portfolios, I eliminate most of the differential exposure to size. Conveniently, size smoothing also makes exposures to the market and other risk factors much more homogeneous across portfolios. The resulting portfolios are therefore much closer to ideal test assets and allow me to better isolate the effects of inflation\textsuperscript{13}.

\textsuperscript{12}If the total number of stocks is not divisible by 10, then some of the groups may contain one more stock than others.

\textsuperscript{13}There are other advantages to averaging along size. The model I develop does not have any size or book-to-market effects, so using assets that have size and book-to-market exposure will only complicate estimation and make results difficult to interpret. Having 10 instead of 100 portfolios also makes it feasible to estimate non-linear standard errors with GMM when I test factor models.
One main feature of the 10 inflation portfolios is that they exhibit a spread in returns: the highest beta portfolio has a mean return of 5.16% per year, compared to 6.91% for the lowest beta portfolio. To put the spread of 1.75% in perspective, the analogous spread induced by size and book-to-market differences are 2.6% and 4.8% respectively. Columns 1 and 3 of Table 1 show the mean ex-ante inflation betas and the mean excess returns for all portfolios. Column 2 shows the mean post-ranking betas. To construct the post-ranking beta of a portfolio at time $t$, I freeze the time-$t$ portfolio weights and regress the excess returns of the fixed-weights portfolio on inflation innovations, using the five years of data starting at $t + 1$. The post-ranking betas can be thought of as an out of sample test for the estimates of the ex-ante betas. Table 1 shows that ex-ante betas align neatly with post-ranking betas, showing that portfolios do capture ex-post exposure to inflation. Post-ranking betas are squeezed together compared to ex-ante betas, which is a well-known effect in this set-up\(^{14}\). The range of post-ranking betas is also more reasonable than for the noisy ex-ante betas; it is difficult to imagine a stock whose returns have a systematic 10-fold reaction to inflation.

A first pass “long-short” estimator of the inflation premium can be found by looking at the last row of Table 1, which computes the spread in betas and returns between the highest and lowest beta portfolios. Dividing the 1.75% spread in returns by the difference in their ex-post betas, I find a slope of $\lambda_{Long-Short} = -0.74$. I divide this crude non-linear slope estimator by the standard deviation of returns of the long-short portfolio to find a market price of inflation risk of -0.23. Higher inflation beta is associated with lower mean returns, which implies a negative market price of risk. The price of risk obtained in this way is not statistically significant. However, combined with how well returns align with ex-ante and post-ranking betas, the obtained value for $\lambda_{Long-Short}$ provides further motivation to more deeply analyze how inflation is priced in the cross-section of stock returns. The estimator $\lambda_{Long-Short}$ is inefficient because it discards changes in the cross-section of stocks that occur every period – it simply

\(^{14}\)See Elton and Gruber (1995).
averages across time first and then ignores all but the two corner portfolios. In the next section, I will formally statistically test whether the spread in returns can be attributed to the differences in betas by using the entire cross-sectional variation of the 10 portfolios over time. The test will confirm that inflation is priced in the cross-section of stock returns in a statistically significant manner, with a market price of risk of -0.33.

Table 2 analyzes other characteristics of the portfolios. The first four columns show that portfolios are not systematically different in terms of their exposure to market, size, book-to-market or momentum. The last three columns show that portfolios are only slightly different in terms of their industry composition. Industries are defined by the first two digits of the Standard Industry Classification code (SICCD). Column 5 is a Herfindahl industry concentration index obtained by summing the squares of the shares of firms in each industry within a portfolio. A value of 1 for this index means that all companies in the portfolio belong to the same industry, and the closer the index is to 0, the more diversified the portfolio is across industries. Column 6 provides a measure of distance in the distribution of industries between a given portfolio and the remaining 9 portfolios. The index is normalized so that a value of 1 means that the portfolio in question has the same exact distribution of industries as the remaining 9 portfolios, and the measure decreases toward 0 when there is no intersection between industries in the portfolios. Column 7 uses the same measure as Column 6 to compare each portfolio’s industry distribution with the distribution of the same portfolio five years later. Low beta, high return portfolios have slightly higher industry concentration and persistence but the message of Columns 5-7 is that portfolios are by and large well-diversified, similar to each other and not very persistent in their industry composition. Hence, there are no large industry differences within portfolios, across portfolios or along different time periods. The observed pattern implies that the bulk of the heterogeneity of inflation risk in the cross-section of stock returns cannot be ascribed to industry effects. Table 2 has important implications for any theory that attempts to explain the variation of stock returns induced by their differential exposures to inflation. For example, menu
costs, taxes, leverage, or labor relations between a firm and its employees, although important, cannot provide a complete explanation of why different firms react differently to inflation, as these characteristics vary strongly by industry while inflation-sorted portfolios don’t.

To further confirm that the spread in returns of inflation portfolios are not driven by market exposure, size, industry effects or other standard factors that are commonly used to explain returns, I run time-series regressions of inflation portfolios’ returns on different risk factors. I consider the Fama-French factors, momentum, short and long-term reversal factors, liquidity, oil, industry portfolios and the Cochrane-Piazzesi factor. Table 3 shows results for different combinations of the factors. The mean absolute pricing errors—the average of the absolute value of the intercepts or “alphas”—are on the order of 1.88% to 4.28% per year, which is of the same order of magnitude as the difference in returns between the lowest and highest inflation beta portfolios. In addition to being economically sizable, I show in Table 3 that the pricing errors are also statistically different from zero by performing a Gibbons-Ross-Shanken (GRS) test (1989). The GRS test is an F-test adjusted for finite sample bias for the hypothesis that the pricing errors for the 10 portfolios are jointly zero. Oil and industry portfolios perform better than other factors under the GRS measure but still have large pricing errors that are statistically different from zero at the 1% level.

1.2.4 Market price of inflation risk

In this section, I use the inflation portfolios of the last section to perform the second step of the Fama-MacBeth procedure. The goal of this section is to produce an estimate for the market price of inflation risk implied by the cross-section of stock returns.

To do so, I start by running one cross-sectional regression for each time period $t$. The dependent variables are the time $t$ returns for the 10 inflation portfolios and the independent variables are the estimated time $t$ post-ranking inflation betas obtained in
the first stage of Fama-MacBeth \(^\text{15}\):

\[
R_{p,t}^e = a_t + \lambda_t \beta_{p,t} + \varepsilon_t \tag{1.11}
\]

\[p = 1, ..., 10.\]

The estimated coefficient \(\hat{\lambda}_t\) measures the average extra returns earned by assuming one extra unit of inflation beta at time \(t\). Table 4 reports the average annualized price of inflation risk in my sample. I show both the average \(\bar{\lambda}\) of \(\lambda_t\), which gives the risk premium per unit of inflation beta, and \(\bar{\lambda}/\text{std} (\Delta\pi_t)\), which gives the risk premium associated with a one standard deviation shock in inflation innovations. Since \(\lambda_t\) is persistent over time, I use Newey-West standard errors with 12 lags to construct standard errors and verify that both estimates for the price of risk are statistically different from zero.

The negative value of \(\bar{\lambda}\) implies that inflation shocks correspond, on average, to bad states of nature. Holding assets that have low excess returns when inflation is increasing must offer higher mean returns as compensation for bearing inflation risk. Another way to understand the inflation premium is to imagine that each period we move from the first to the last decile in the distribution of ex-post betas. In this case, the associated expected increase in returns is 8.79%. This exercise is not the same as moving from portfolio 1 to 10 every period. When moving from decile to decile in the distribution of individual betas, we do it unconditionally, while portfolios are conditioned on size because of the initial double-sort. Stocks with lower inflation beta are also smaller on average. When moving unconditionally across the distribution, both effects are captured in the extra returns. The 8.79% can then be thought of as a total derivative, while the 1.75% spread in returns is closer to a partial derivative.

To compare the market price of inflation risk to the aggregate market’s price of risk, it is more useful to look at the normalized \(\bar{\lambda}/\text{std} (\Delta\pi_t)\). Table 4 shows that under this

\(^{15}\)Thus, the independent variables in this regression are themselves regression coefficients.
measure, the price of risk of inflation is comparable to the market's, for which 0.3 is a good approximation. This means that an inflation-mimicking portfolio has about the same Sharpe ratio as the market. Inflation is therefore an important component of risk in the cross-section of stock returns.

An alternative to forming portfolios when measuring prices of risk, advocated by Ang, Liu and Schwarz (2010), is to use individual betas in the second stage regression (1.11). The rationale is that while betas may be more precisely estimated when forming portfolios, efficiency in the estimate for $\lambda$ is increased when no information about the cross-section is destroyed. Column 2 of Table 4 reports the estimates for the inflation premium using individual stocks. As a robustness check, Column 3 reports the estimate for $\lambda$ obtained when creating portfolios using a 5-year rolling window instead of an exponential kernel. All three measures are similar, especially the ones in Columns 2 and 3. The estimator using individual stocks does have a smaller variance, confirming the message of Ang, Liu and Schwarz (2010).

1.3 Model

In this section, I present a consumption-based model that can explain and quantitatively match the inflation premium I estimated in the last section. I first consider a simple version to illustrate how assets are priced. This version is similar in its mechanics to Bansal and Yaron's (2004) long-run risk model. I then present a version of the model that has richer processes for inflation, consumption and dividends, that is more suitable for quantitatively estimating and testing of the model.

1.3.1 Set-up

Environment The model is an exchange economy with a single representative agent. Time is discrete and indexed by $t \in \{0, 1, \ldots\}$. For each period $t$, there is one consumption good denoted by $C_t$ which represents the economy's real aggregate consumption.
I will use lower-case letters to denote the logarithm of the corresponding variable, so for example $c_t = \ln C_t$.

The joint process for consumption growth $\Delta c_{t+1} = \ln C_{t+1} - \ln C_t$ and inflation $\pi_t$ is exogenous and given by

$$
\pi_{t+1} = \mu_\pi + \rho_\pi (\pi_t - \mu_\pi) + \sigma_\pi \varepsilon_{t+1} \quad (1.12)
$$
$$
\Delta c_{t+1} = \mu_c + \rho_c (\pi_t - \mu_\pi) + \sigma_c \eta_{t+1} \quad (1.13)
$$

The stochastic disturbances $\varepsilon_s, \eta_r$ are i.i.d. standard normal for all $s, r \in \{0, 1, \ldots\}$. Eq. (1.12) shows that inflation follows an $AR(1)$ process with constant auto-regressive coefficient $\rho_\pi \in (-1, 1)$ and unconditional moments controlled by the constants $\mu_\pi$ and $\sigma_\pi^2$:

$$
E\pi_{t+1} = \mu_\pi \quad (1.14)
$$
$$
Var(\pi_{t+1}) = \frac{\sigma_\pi^2}{1 - \rho_\pi^2} \quad (1.15)
$$

The $AR(1)$ specification for inflation captures, in a stylized way, the persistent nature of inflation. Eq. (1.13) models real consumption growth as being a predictable function of past inflation. When an inflation shock $\varepsilon_t$ hits the economy, inflation reacts contemporaneously and the effect extends into subsequent periods. However, consumption growth only starts reacting to the inflation shock the next period. Thus, inflation leads consumption growth and inflation shocks translate not into changes in current consumption, but into shocks to consumption expectations. The sign and magnitude of the predictability is given by the constant $\rho_c$. When $|\rho_c|$ is large and when $\rho_\pi$ is close to 1, inflation shocks have a large, persistent effect on future consumption growth. When $\sigma_\pi^2$ is large, inflation is very volatile and small inflation shocks also have a large effect on future consumption and consumption expectations.

Table 5 justifies my choice for process (1.12). Column 1 of Table 5 shows that
eq. (1.12) is not unreasonable as a first approximation, although it does mask many features of inflation dynamics. Column 2 shows that inflation has more inertia than hinted by its first autoregressive coefficient, with several lags significant and comparable in magnitude to the first lag. Ang, Bekaert and Wei (2007) show that at the ten quarter horizon, the first order autocorrelation of inflation is still 0.35. Including multiple lags in the monthly process, or looking at longer horizons, makes clear that inflation shocks are active throughout the entire business cycle, spanning a window of 2 to 4 years. A potentially restrictive assumption in (1.12) is that the parameters for inflation are not time-varying. Taylor (1998) argues for a break in inflation regimes before and after the Volcker era. Without trying to produce sophisticated econometric analysis of breaks and switching regimes, Panel A of Table 6 shows basic evidence that inflation was more volatile and persistent before 1980. Under the interpretation in Taylor (1998), the reason is that the Federal Open Market Committee accommodated inflation before Volcker but started leaning against it in the early 1980’s. For both the simple model and my main specification, I will keep the inflation parameters constant. However, I will later exploit the time-variation in inflation parameters to test my model. This will be an important validation of my model, because it focuses on the key mechanism generating the inflation premium. I will show that the model successfully replicates the change in inflation premium observed in the data when inflation persistence and volatility change.

Table 7 addresses the empirical evidence of consumption predictability using inflation. Column 1 shows that an increase in one percentage point in inflation this month is associated with an expected decrease of 1.5 percentage points in real consumption growth over the next year. Column 2 shows that up to three lags of inflation contribute in predicting consumption. Column 3 shows that if enough lags of inflation are included, past consumption need not be included in (1.13). The last three columns of the

---

16 On the other side of the argument, Orphanides (2004) uses information available to the FOMC in real time to argue there was no change in regimes. I consider both cases.

17 The two subsequent lags are not significant and get smaller in magnitude.
table show the same regressions for a two-year horizon.

I take the process for consumption as exogenous and therefore do not attempt to explain why inflation predicts consumption. There is already a large and sophisticated body of literature in Macroeconomics that puts forward several mechanisms that generate real effects of inflation and real consumption predictability. For example, Clarida, Gali and Gertler (1999) provide a reduced-form model that captures a large class of dynamic equilibrium models of nominal rigidities. After computing expectations that ultimately come from agents' optimization, their process for inflation and consumption can be mapped to eqs. (1.12) and (1.13) if I allow for contemporaneous correlation between inflation and consumption (which I do below). Other explanations for the real effects of inflation include monetary policy, menu costs, rational inattention and informational frictions. I take no position as to which explanations are correct or quantitatively important, but instead empirically estimate a joint process for consumption and inflation that is flexible enough to accommodate any of these models and capture their main dynamic characteristics.

If I substitute inflation for long-run risk in eqs. (1.12) and (1.13), I obtain the same basic specification as Bansal and Yaron (2004). However, my model differs conceptually from theirs in significant ways. First, inflation is observable, so its process and its relation to consumption can be estimated directly. In the long-run risk model, the variable predicting expected consumption growth is inferred from asset prices and assumptions about preference parameters. In the next sections, I will use the observable properties of consumption and inflation, such as the presence of multiple lags, to depart from Bansal and Yaron's (2004) specification. This departure will provide me with additional moment restrictions to test my model in a way that would not be possible in the long-run risk model. Second, even though inflation and long-run risk have a similar functional form, their stochastic properties are drastically different. Long-run risk has an extremely long half-life, operating at frequencies of 10 to 30 years instead of the 1

\[18 \text{See the introduction for references}\]
to 2 years for inflation. The counterpart for a long half-life of long-run risk is that its volatility is about 200 times smaller than inflation's. The lower persistence and higher volatility of inflation generates a market price of risk similar to the one for the higher persistence, lower volatility long-run risk.

**Representative Agent** The representative agent has recursive Epstein-Zin-Weil (EZ) preferences. If she enters period $t$ with wealth $W_t$, then her utility is defined by

$$ U_t(W_t) = \left( (1 - \delta) C_t^{1-1/\psi} + \delta E_t [U_{t+1}(W_{t+1})^{1-\gamma}]^{1-1/\psi} \right)^{1-\gamma} $$  \hspace{1cm} (1.16)

The constant $\delta \in (0, 1)$ is the discount rate, $\gamma > 0$ is the coefficient or relative risk aversion and $\psi > 0$ is the elasticity of intertemporal substitution (EIS). It is convenient to define the constant $\theta = \frac{1-\gamma}{1-1/\psi}$, which measures the relative magnitude of risk aversion against the EIS. The EZ utility function is a generalization of the familiar constant relative risk-aversion (CRRA) utility function, which is obtained when $\theta = 1$. With CRRA utility, the coefficient of relative risk aversion is the inverse of the elasticity of intertemporal substitution. EZ disentangles these two conceptually different parameters —there is no reason to assume that the desire to smooth over time is the same as the desire to smooth over different states of nature. In fact, the EIS is important in a dynamic deterministic economy while the lack of uncertainty makes risk-aversion irrelevant. Similarly, $\gamma$ is relevant in a static economy with uncertainty, while the EIS is not. Because inflation operates through the predictable component of consumption, the EIS will play a crucial role in determining the asset pricing implications of inflation.

Another important trait of the EZ utility is that it is not time-separable (it cannot be written as a sum of period utilities). To understand why non-separability is important, consider the following example from Duffie and Epstein (1992). An agent picks between two consumption plans for a long number of periods before any consumption is realized. Consumption plan A is obtained by tossing a fair coin every period and giving the agent high or low consumption in each period depending on the outcome of the toss in that
period. Consumption plan B is obtained by a single coin toss before all consumption takes place and gives the agent high consumption in every period if heads and low consumption in every period if tails. In plan B, uncertainty about consumption is resolved early, while for plan A uncertainty is resolved gradually. When \( \theta < 1 \), which is the case I consider in my model, the representative agent prefers early resolution of uncertainty —i.e. likes to plan ahead— and prefers plan B. This case occurs when \( \gamma \psi > 1 \), requiring high risk aversion or high EIS.

Early resolution of uncertainty can also be understood in terms of aversion to risk in consumption growth. Expected consumption growth is mean-reverting in plan A and constant for plan B. When the agent prefers early resolution of uncertainty, she also has a preference for less risk in expected consumption growth and plan B is more desirable than A. In my model, positive inflation shocks will command a premium because inflation induces this type of risk.

The representative agent’s budget constraint is given by

\[
W_{t+1} = R_{c_{t+1}} (W_t - C_t),
\]

where \( R_{c_{t+1}} \) is the return on the wealth portfolio. The wealth portfolio is the asset that pays consumption \( C_t \) each period as its dividends. Therefore, the agent consumes \( C_t \) out of wealth \( W_t \) and invests the remainder in the economy’s aggregate endowment (consumption) technology.

**Assets** There are \( 1 + N \) assets in the economy indexed by \( n \). The first asset is the wealth portfolio described in the last paragraph. The other \( N \) assets are defined to be levered claims to consumption as in Abel (1999). They pay exogenous dividends given by

\[
\Delta d_{n,t+1} = \mu_{n,d} + l_n \rho_c (\pi_t - \mu_\pi) + \varphi_n \omega_{n,t+1} \text{ for } n = 1, \ldots, N,
\]

with \( \omega_{n,t+1} \) i.i.d. standard normal across time, across assets and with respect to all other shocks in the economy. The process (1.18) for dividend growth has the same form as the
process (1.13) for consumption growth. These processes can have different mean growth rates given by $\mu_{nd}$, different volatilities given by $\varphi_n$ and, more importantly, different exposures to inflation given by $l_n\beta$, where $l_n$ is an asset-specific leverage parameter. The difference between the sum of dividends and aggregate consumption is assumed to come from other sources of income such as human wealth, which I do not explicitly model. When I estimate the parameters of the model with GMM, the $N$ assets will be mapped to the 10 inflation portfolios that I constructed in the empirical section of this paper.

1.3.2 Asset pricing

**Representative agent’s problem** The representative agent’s problem in period $t$ is to pick a consumption path $\{C_s\}_{s=t}^{\infty}$ to maximize utility (1.16) subject to the budget constraint (1.17) and the exogenous processes for consumption (1.13) and inflation (1.12).

**Stochastic discount factor and inflation-CCAPM** The first order condition for the representative agent’s problem implies that the return $R_{n,t+1}$ of any tradable asset $n$ satisfies the Euler equation

$$1 = E_t[SDF_{t+1}R_{n,t+1}]$$

(1.19)

with a stochastic discount factor given by

$$\log SDF_{t+1} = sdf_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}.$$  

(1.20)

Rearranging equations (1.19) – (1.20) and using the log-normal structure of the
set-up, I find that expected excess returns $E_t^e [r_{n,t+1}]$ follow a two factor model

$$E_t^e [r_{n,t+1}] = -Cov_t (sdt_{t+1}, r_{n,t+1})$$

$$= \frac{\theta}{\psi} Cov_t (\Delta c_{t+1}, r_{n,t+1}) + (1 - \theta) Cov_t (r_{n,t+1}, r_{c,t+1})$$

Equation (1.22) states that the risk premium of asset $n$ depends on the covariance of its returns $r_{n,t+1}$ with two factors. The first one is consumption growth $\Delta c_{t+1}$, just as in the consumption-CAPM. The second one is the return on the wealth portfolio $r_{c,t+1}$. The wealth portfolio arises with non-separable utility because, as explained above, the shape of the entire path of consumption matters when computing utility, rather than just the sum of expected utilities across periods. Because the return on the wealth portfolio $r_{c,t+1}$ is the price of the stream of all future consumption, it incorporates information about future consumption that is not included in $\Delta c_{t+1}$. For an agent with non-separable utility, this additional information should be useful when computing marginal utilities and asset prices. For the EZ specification, it turns out that the return on the wealth portfolio $r_{c,t+1}$ is a sufficient statistic for the entire future path of expected consumption growth and hence the only other pricing factor beyond contemporaneous consumption. When $\theta < 1$, the representative agent is averse to risk in expected consumption growth. Assets that covary positively with the return on the wealth portfolio induce more expected consumption growth risk and have a positive risk premium.

Linearizing $r_{c,t+1}$ around the mean wealth-consumption ratio (Campbell 1991), the pricing equation (1.22) can be re-written as an inflation-consumption-CAPM:

$$E_t^e [r_{n,t+1}] = \gamma Cov_t (\Delta c_{t+1}, r_{n,t+1}) + \frac{(\gamma - 1/\psi) (\rho_c - 1/\psi)}{(\kappa_1 - \rho) (1 - 1/\psi)} Cov_t (\pi_{t+1}, r_{n,t+1})$$

where $\kappa_1$ is a linearization constant that depends on the mean wealth-consumption ratio$^{19}$. In a broader model, other state variables that determine returns on the wealth

$^{19}$In practice, the constant is close to 1 for most parameter values.
portfolio should be included. The prediction that inflation is priced because it predicts consumption should be robust to the inclusion of any other factors as long as inflation does not cease to have predictive power. The inflation-CCAPM (1.22) summarizes all the asset pricing content of the model. The market price of risk of consumption shocks is positive and given by $\gamma \sigma_c$ as in the CCAPM.

The magnitude and sign of the market price of risk for inflation shocks depend on the model's parameters. The larger the variance of inflation shocks $\sigma_\pi$, the larger the premium. In addition, if

(i). inflation predicts consumption growth negatively ($\rho_c < 0$) and inflation is persistent ($\rho_\pi > 0$),

(ii). the substitution effect dominates the income effect ($\psi > 1$), and

(iii). the representative agent dislikes uncertainty in expected consumption growth ($\gamma - 1/\psi > 0$),

then assets that have low returns when inflation shocks are positive will command a risk premium. The larger any of the three effects, the larger the premium.

I now give intuition for these components. The product of components (i) and (ii), given by $\frac{1}{1-1/\psi} \frac{\rho_c-1/\psi}{\kappa_1-\rho_r}$, captures how inflation shocks affect returns to the wealth portfolio. It "translates" eq. (1.22) to eq. (1.23). When $\rho_c < 0$, positive inflation shocks are bad news for future consumption growth. If $\psi > 1$, the substitution effect is larger than the income effect, and the adverse shock to expected consumption growth leads to smaller returns of the wealth portfolio. To see this, I use the budget constraint to write the log consumption-wealth ratio as

$$c_t - w_t + a = (1 - \psi) E_t \left[ \sum_j b^j r_{c,t+j} \right] = E_t \left[ \sum_j b^j (r_{c,t+j} - \Delta c_{t+j}) \right]$$

(1.24)

for some constants $a$ and $b$. The first equality shows that when $\psi > 1$, today's consumption decreases relative to wealth when expected returns rise – the substitution effect
dominates. The second equality shows that a fall in expected returns is associated with a fall in expected consumption growth.

Component (iii) determines how shocks to the wealth portfolio are compensated in equilibrium. If the representative agent prefers earlier resolution of uncertainty and is therefore averse to risk in expected consumption growth, holding assets that covary with the returns on the wealth portfolio must have a positive risk premium, since they increase the volatility of expected consumption growth.

Combining ingredients (i), (ii) and (iii) implies that when an inflation shock hits the economy, expected consumption growth decreases, the returns on the wealth portfolio decrease and assets that pay off poorly in those bad states of nature command a risk premium.

**Inflation Betas and Unconditional Returns** To compare the model to the data, it is useful to understand the Fama-MacBeth procedure in the model. Inflation betas are endogenous in the model and given by the coefficient of the univariate regression of excess returns on inflation innovations

\[
\beta_{n,t} = \frac{\text{Cov}_t (\Delta \pi_{t+1}, r_{n,t+1}^e)}{\text{Var}_t (\Delta \pi_{t+1})} = \frac{l_n \rho_c - 1/\psi}{\kappa_{n,1} - \rho_{\pi}}. \tag{1.25}
\]

The betas are not time-varying, but will be when I introduce stochastic volatility. Given that \( \rho_c < 0 \) and \( \kappa_{n,1} - \rho_{\pi} > 0 \), the sign of \( \beta_{n,t} \) depends on the relative magnitudes of firms' leverage and the EIS. Positive leverage makes betas negative because they inherit the consumption risk induced by inflation. When the EIS is small, the agent is reluctant to change her consumption path after an inflationary shock. In that case, the adjustment to the new economic conditions requires large changes in prices.

Combining the inflation-CCAPM eq. (1.23) and the inflation betas, I obtain the cross-sectional regression in the second stage of the Fama-MacBeth procedure

\[
r_{n,t}^e = \gamma \sigma_c^2 + \frac{(\gamma - 1/\psi) (\rho_c - 1/\psi) \sigma_{\pi}^2}{(\kappa_1 - \rho_{\pi}) (1 - 1/\psi)} \beta_{n,t} + \xi_t, \tag{1.26}
\]
where \( \xi_t \) is a random disturbance. Comparing this equation to its empirical counterpart (1.11), we can identify the coefficient

\[
\lambda_t = \frac{(\gamma - 1/\psi) (\rho_c - 1/\psi) \sigma^2}{(\kappa_1 - \rho_\pi) (1 - 1/\psi)}
\]  

(1.27)

Fama-MacBeth is therefore the right procedure to estimate inflation risk in my model. This fact not only helps justify the empirical methodology I employ, but also makes straightforward the comparison of the model and the data.

### 1.3.3 A general version

The model presented in the last section, although simple, can quantitatively generate an inflation premium as large as the one estimated in Table 4. I do a back of the envelope calculation of the premium with the following reasonable parameters: \( \gamma = 3, \ \psi = 1.5, \ \rho_\pi = 0.6, \ \rho_c = -0.1 \) and \( Var(\pi_t) = (1 - \rho^2_\pi)\sigma^2_\pi = 1.5% \) per year. The market price of risk in this case is \( \tilde{\lambda} = -0.316 \) which is very much in line with my empirical estimates.

However, as discussed in the previous section, the dynamics for consumption and inflation are more elaborate than \( AR(1) \). A more compelling model should generate the empirically observed price of inflation risk using processes for inflation and consumption that more closely adjust to the data. I therefore consider a richer version of my model and show that it can indeed price inflation in the cross-section of stock returns in accordance to my empirical estimates. The generalized processes for inflation, consumption and dividends are given by

\[
\pi_{t+1} = \mu_\pi + \sum_{s=0}^{2} \rho_{\pi,s} (\pi_{t-s} - \mu_\pi) + \sigma_{\pi,t+1} \xi_{t+1} + \varphi_{\pi} c_{t+1} \eta_{t+1} \tag{1.28}
\]

\[
\sigma^2_{\pi,t+1} = \sigma^2_\pi + \sum_{s=0}^{1} \nu_{\pi,s} (\sigma^2_{\pi,t-s} - \sigma^2_\pi) + \sigma_{\pi,w} u_{t+1} \tag{1.29}
\]

37
\[
\Delta c_{t+1} = \mu_c + \sum_{s=0}^{2} \rho_{c,s} (\pi_{t-s} - \mu_\pi) + \sigma_{c,t+1} \eta_{t+1} \\
\sigma_{c,t+1}^2 = \sigma_c^2 + \nu_c (\sigma_{c,t-s}^2 - \sigma_c^2) + \sigma_{cw} w_{t+1} \\
\Delta d_{n,t+1} = \mu_{n,d} + l_n \sum_{s=0}^{2} \rho_{s,c} (\pi_{t-s} - \mu_\pi) + \varphi_{nc} \sigma_{c,t+1} \omega_{n,t+1}
\]

where the shocks \( \varepsilon_t, \eta_t, u_t, w_t \) and \( \omega_{n,t} \) are i.i.d. normal across time and across processes.

Consumption and inflation now have three lags of inflation and stochastic volatility. Dividends are still levered consumption with parameter \( l_n \). The inclusion of lags is justified by Tables 5 and 7, and the discussion following eqs. (1.12) and (1.13). Stochastic volatility for inflation follows an AR(2) process, creating GARCH-like effects, including heteroskedasticity and persistence. GARCH effects in inflation are documented in Bollerslev, Russell and Watson (2009) and Bollerslev (1986). Stochastic volatility for consumption and dividends are also AR(2), which allows the model to match the widely documented time-varying volatility of stock returns.

The inclusion of stochastic volatility plays a dual role. As explained by Campbell and Beeler (2009) in the context of long-run risk, stochastic volatility in dividends and consumption help generate a realistic aggregate equity premium. In my model, it will also help match the average level of returns in inflation portfolios. Stochastic volatility in consumption and dividends plays no role in generating a spread in returns across stocks with different exposures to inflation. Stochastic volatility of inflation, on the other hand, does increase the mean inflation premium. This is most easily seen in the context of the simple model. Using Jensen's inequality and replacing \( \sigma_\pi \) by a time-varying process \( \sigma_{t,\pi} \) with mean \( \bar{\sigma}_\pi \), we have

\[
\bar{\lambda} = E[\lambda_t] = \frac{(\gamma - 1/\psi) (\rho_c - 1/\psi)}{(\kappa_1 - \rho_\pi) (1 - 1/\psi)} E[\sigma_{t,\pi}^2] \geq \frac{(\gamma - 1/\psi) (\rho_c - 1/\psi)}{(\kappa_1 - \rho_\pi) (1 - 1/\psi)} \bar{\sigma}_\pi^2.
\]

As a last modification from the simple model, I allow correlation between contemporaneous inflation and consumption, parametrized by \( \varphi_{xc} \). In many models, supply (e.g.
productivity) shocks will tend to increase consumption and lower inflation. A demand shock, on the other hand, will tend to increase both. In those models, the correlation between consumption and inflation captures the relative strength of these two competing effects. Apart from theoretical considerations, there is a negative contemporaneous correlation between inflation and consumption at the monthly frequency that could not be otherwise captured. After estimating the parameters of the model, I find that this correlation does not contribute significantly to the inflation premium.

1.4 GMM Estimation

In this section, I estimate the parameters of the general model by using generalized method of moments. I find that the model can reproduce the observed market price of risk for inflation, the aggregate market's risk premium and its volatility, the level and volatility of the risk free rate and the level and slope of the yield curve, while simultaneously matching the processes for consumption and inflation. I use a standard 2-step feasible GMM. In the first step, I use the identity matrix as the weighting matrix. In the second step, I use as weighting matrix the inverse of the variance-covariance matrix estimated using the parameters found in the first step.

1.4.1 Moments

I classify the 59 moment conditions that I use into four groups:

1. **Consumption and inflation** (11 moments). I estimate all the OLS moments of eqs. (1.28) and (1.30), together with the variance-covariance matrix of same-period inflation and consumption growth. These are the natural moments to estimate for the linear exogenous processes for inflation and consumption.

2. **Inflation portfolios** (30 moments). I include the mean and variance of returns of the 10 assets of the model, together with their inflation betas. The empirical
moments corresponding to these assets naturally come from the inflation portfolios constructed in the empirical section.

3. **Aggregate market** (6 moments). I match the mean of aggregate dividend growth and the mean, variance and inflation beta of the market's return. In addition, I include the mean and variance of the price-dividend ratio as a moment condition to highlight the model's ability to match a property of the aggregate market that proves difficult to match in other models. The volatility of the price-consumption ratio also helps identify consumption and inflation's stochastic volatility.

4. **Bonds** (12 moments). I incorporate as moment conditions the means and variances of nominal bond yields for 1, 2, 3, 4 and 5 year maturities together with the mean and variance of the risk-free rate.

1.4.2 **Parameters**

I divide the 53 parameters to be estimated into five categories:

1. **Consumption and inflation** (11 parameters). The vector of parameters is \( \Theta_{C\pi} = (\mu_c, \mu_\pi, \varphi_{C\pi}, \rho_{C\pi}, \rho_{C\pi}, \sigma_c, \sigma_\pi) \) with \( s = 0, 1, 2 \). If I were estimating just the process for consumption and inflation (without stochastic volatility), the 11 parameters would be exactly identified from their corresponding 11 moment conditions and could be estimated by OLS.

2. **Inflation portfolios** (3 \( \times \) 10 = 30 parameters). The vector of parameters is \( \Theta_p = (\mu_{n,d}, l_n, \varphi_{nc}) \) for \( n = 1, ..., 10 \). It includes the mean \( \mu_{n,d} \), leverage parameter \( l_n \) and volatility \( \sigma_n \) of dividend growth for each stock. If these were the only parameters to be estimated, they would be exactly identified (through a non-linear transformation) by the 30 moments for inflation portfolios discussed above.
3. **Aggregate market** (3 parameters). The vector of parameters is \( \Theta_m = (\mu_m, l_m, \sigma_m) \), which are the mean, leverage parameter and volatility of the market’s dividend process.

4. **Stochastic volatility** (6 parameters). The vector of parameters is \( \Theta_{vol} = (\sigma_{\pi w}, \sigma_{cw}, \nu_{\pi, s}, \nu_{c, s}) \) with \( s = 0, 1 \). These are the variances and auto-regressive coefficients for volatility.

5. **Preferences** (3 parameters). The three preference parameters \( \Theta_u = (\delta, \gamma, \psi) \) are the discount rate, the coefficient of risk aversion and the EIS. The small number of preference parameters and the functional form of the stochastic discount factor consistent with equilibrium is one of the main reasons why the GMM system is overidentified.

### 1.4.3 Estimation results

The preference parameters obtained from the calibration are \( \gamma = 8.46 \) and \( \psi = 1.44 \). The EIS is the key preference parameter to match the slope of returns with respect to inflation betas. The EIS that I estimate is very close to the one used in the LRR literature, which is generally calibrated to be \( \psi = 1.5 \). A relatively high level of risk aversion \( \gamma \) contributes to the large inflation premium, but is more important in determining the overall level of returns than their sensitivity to inflation. Because dividends and consumption are exposed to the same underlying volatility shocks, GMM faces a tradeoff between matching the high returns of stocks and the low volatility of consumption. Increasing gamma reduces this tradeoff. If my model included long-run risk, disaster risk, or some other additional source of consumption risk, it would be able to match the equity premium with a lower \( \gamma \). Table 8 reports the other parameters.

The main goal of the parameter estimation of the model is to test whether the model can generate an inflation premium consistent with the data using realistic processes for inflation and consumption. Column 4 of Table 4 shows the model’s results when per-
forming the same Fama-MacBeth procedure that I used in the data. Comparing to the empirical estimates in columns 1, 2 and 3, we see that the model can reproduce all of the inflation premium. Table 10 goes deeper into the model's predictions for inflation portfolios. The table shows that the model can closely match the individual average betas and returns of the inflation portfolios. Table 9 shows that the source of heterogeneity in the model's betas and returns comes mostly from having a different exposure to consumption and inflation and not from their difference in volatility loadings.

Table 11 shows that the model successfully matches basic moments for inflation and consumption, while Table 12 shows the degree of consumption predictability and inflation persistence in the model. The standard deviation for inflation and consumption are 1.34% and 2.25% in the model and 1.14% and 2.14% in the data. The slightly higher volatility of inflation is important to match the observed price of risk for inflation. The other moments for consumption and inflation are accurately aligned to their empirical counterparts.

Table 11 also shows that the model replicates the Sharpe ratio and the mean and standard deviation of the price-dividend ratio for the aggregate market. The level of the nominal risk-free rate is also closely matched, although its volatility in the model is less than half of what we observe in the data. I find the same pattern for the yield curve: the model produces an upward sloping yield curve, but the volatility of yields is too small and decays with horizon faster than in the data.

Table 14 shows interesting moments that were not targets of my GMM calibration. The table verifies that consumption is as persistent in the data as in the model. Koijen, Lustig, Van Nieuwerburgh and Verdelhan (2010) emphasize the moments of the wealth-consumption ratio as a means to differentiate between asset pricing models. Table 14 corroborates that my model performs well in this dimension.

Tables 11-14 compare my results to Bansal and Shaliastovich (2010). Their model is a standard long-run risk model with an exogenous process for inflation attached to it. Unlike my model, inflation does not feed back into consumption or any other real
variables and acts just as a conversion factor between nominal and real prices. In their model, an inflation premium arises because inflation itself is exposed to consumption and long-run risk shocks.

Before I compare their results to mine, two caveats are in order. First, I estimate my model with GMM, while they resort to calibration to choose parameters. Picking parameters using GMM may give their model a better fit and provide a more even comparison between the two. Second, their calibration is for a slightly different period and done at the quarterly frequency after time-aggregating monthly series from their model.

I also emphasize that Bansal and Shaliastovich's (2010) model was not designed to match the inflation premium in the cross-section of stock returns. It is therefore not surprising to find that the inflation premium in their model is about half the size of the one I find in the data, as can be seen in Table 4. Their model is designed to explain predictability puzzles in bond and currency markets while matching the level and volatility of nominal yields and the market’s return. As can be seen from Tables 11-14, they succeed at matching the means and variances of aggregate market returns, the risk-free rate, bond yields of all maturities, consumption and inflation. My model, on the other hand, is designed to explain neither the equity premium puzzle nor the predictability puzzles of bond and currency markets. In this respect, the principal objective of our models is different and they can be regarded as complementary.

Another concern that I address is the possibility that the inflation process has changed throughout my sample, perhaps due to a change in monetary policy. I re-estimate the model in two separate subsamples, one before and one after 1980. Panel B of Table 2 shows that in both the model and the data, the higher persistency of inflation before 1980 is associated with a higher inflation premium. This is an important validation for the model: the change in key parameters determining the inflation premium in the data and the model imply the same reaction of the inflation premium. This exercise is perhaps the closest we can get to a “natural experiment” in models of
1.5 Conclusion

A stock's inflation risk can be written as the product of the market price of inflation risk and the stock's quantity of risk. In this paper, I estimate both by using a two-step Fama-MacBeth procedure. Inflation betas measure the quantity of risk. The coefficient in a cross-sectional regression of excess returns on betas measures the market price of risk. I find that stocks whose returns covary negatively with inflation shocks have unconditionally higher returns. This implies that the average market price of risk of inflation shocks is negative: periods with positive inflation shocks tend to be bad states of nature and investors are willing to pay insurance in the form of lower mean returns when holding an inflation-mimicking portfolio. I estimate that holding such a portfolio gives the agent a Sharpe ratio of -0.33.

I argue that the negative price of inflation risk arises because high inflation today predicts low growth in future real consumption. I develop a model that is able to match the observed inflation market price of risk when estimated by GMM to have the same level of consumption predictability and inflation persistence as in the data.

A limitation of the model is that it takes the distribution of betas—the distribution of the quantity of risk—as given. Full understanding of inflation risk in the cross-section of stocks requires also explaining why the quantity of risk varies from firm to firm. There are four classic explanations in the literature: (i) Summers (1981) and Feldstein (1980) argue that taxes are responsible for firms' inflation risk; (ii) Fama (1981) points out that positive supply shocks increase future expected dividends but lowers the current price level, inducing a spurious correlation between stock returns and inflation; (iii) Cohen, Polk and Vouleeehah (2005), based on Modigliani and Cohn (1979), propose inflation illusion; and (iv) Mundell and Tobin put forward expected inflation and shoe leather costs. Other macroeconomic models, although not specifically
designed to address the stock market’s heterogeneity in inflation risk, can also provide important insights. For example, the work of Nakamura and Steinsson (2008) implies that firms’ inflation risk is heterogeneous due to differences in menu costs and the variance of idiosyncratic productivity shocks. In this paper, I begin the exploration of why firms have different inflation betas and find that a sizable amount of heterogeneity in firm’s inflation riskiness does not depend on what industry the firm belongs to, its size, book-to-market or exposure to fluctuations in oil prices. Theories that rely solely on the aforementioned effects will most likely need additional ingredients to provide a comprehensive explanation of inflation risk.
1.6 References


Ang, Andrew, and Dennis Kristensen, 2009, Testing Conditional Factor Models, CREATES Research Papers, School of Economics and Management, University of Aarhus.


1.7 Tables and Figures

Figure 1-1: Histogram of inflation betas for different time periods.
Figure 1-2: Time series of inflation betas for the aggregate stock market, the five year nominal bond, and four representative companies.
Table 1: Inflation-sorted portfolios have returns well aligned with inflation betas

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Ex-ante $\beta$</th>
<th>Post-ranking $\beta$</th>
<th>$E[R^*_t]$</th>
<th>$\sigma(R_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = 1$</td>
<td>-5.61</td>
<td>-2.32</td>
<td>6.91</td>
<td>15.8</td>
</tr>
<tr>
<td>2</td>
<td>-3.52</td>
<td>-1.71</td>
<td>6.87</td>
<td>13.3</td>
</tr>
<tr>
<td>3</td>
<td>-2.56</td>
<td>-1.51</td>
<td>6.44</td>
<td>16.9</td>
</tr>
<tr>
<td>4</td>
<td>-1.80</td>
<td>-1.10</td>
<td>6.29</td>
<td>14.7</td>
</tr>
<tr>
<td>5</td>
<td>-1.17</td>
<td>-0.874</td>
<td>6.36</td>
<td>14.0</td>
</tr>
<tr>
<td>6</td>
<td>-0.555</td>
<td>-0.762</td>
<td>6.17</td>
<td>12.8</td>
</tr>
<tr>
<td>7</td>
<td>0.104</td>
<td>-0.597</td>
<td>5.58</td>
<td>14.8</td>
</tr>
<tr>
<td>8</td>
<td>0.853</td>
<td>-0.365</td>
<td>5.53</td>
<td>15.0</td>
</tr>
<tr>
<td>9</td>
<td>1.86</td>
<td>-0.007</td>
<td>5.56</td>
<td>15.6</td>
</tr>
<tr>
<td>$p = 10$</td>
<td>4.10</td>
<td>0.064</td>
<td>5.16</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Spread (1 minus 10) | -9.71 | -2.38 | 1.75 | 2.40 |

Notes: To construct portfolios, I first find stock $n$'s inflation beta at time $t$, $\beta_{n,t}$, by regressing its excess returns on inflation innovations, only using observations that occurred before $t$. I give smaller weight to more distant observations by using an exponential kernel with a half-life of five years. I construct 10 inflation portfolios by initially double-sorting stocks on 10 groups according to size (market equity) and 10 groups according to $\beta_{n,t}$, and then averaging across size. The ex-ante betas are the averages across time of $\beta_{p,t}$ for each portfolio $p$. I find post-ranking betas by freezing portfolio weights at time $t$ and regressing the excess returns of this fixed-weights portfolio on inflation innovations, using the five years of data starting at $t + 1$. The second column shows the average across time of portfolios' post-ranking beta. The third and fourth columns report mean and standard deviation of excess returns in percentage points per year. I use all stocks in CRSP. Observations are monthly from February 1959 to December 2009. Even though returns align well with inflation betas, the spread in returns between portfolios is not statistically significant. Table 4 shows that using the more efficient Fama-MacBeth procedure leads to an inflation premium that is similar in magnitude but also statistically significant.
<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Risk-factor exposures</th>
<th>Industry properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market</td>
<td>Size</td>
</tr>
<tr>
<td>p = 1</td>
<td>0.992</td>
<td>0.990</td>
</tr>
<tr>
<td>2</td>
<td>0.997</td>
<td>0.979</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.985</td>
</tr>
<tr>
<td>4</td>
<td>0.999</td>
<td>0.983</td>
</tr>
<tr>
<td>5</td>
<td>0.987</td>
<td>0.964</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.981</td>
</tr>
<tr>
<td>7</td>
<td>1.01</td>
<td>0.985</td>
</tr>
<tr>
<td>8</td>
<td>1.03</td>
<td>0.995</td>
</tr>
<tr>
<td>9</td>
<td>1.03</td>
<td>0.992</td>
</tr>
<tr>
<td>p = 10</td>
<td>1.01</td>
<td>0.985</td>
</tr>
<tr>
<td>Spread</td>
<td>-0.018</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Notes: Risk-factor exposures are the coefficients from a regression of excess returns of inflation portfolios on the Fama-French factors. Industry concentration measures how diversified a portfolio is – a value of 0 means very diversified and a value of 1 means that all firms belong to the same industry. Industry correlation is a measure of distance between portfolio $p$ and the remaining 9 portfolios. A value of 0 means that portfolio $p$ does not share any industries with the other portfolios and a value of 1 means that the industry distribution of $p$ is identical to the distribution of all other portfolios combined. Industry persistence is analogous to industry correlation but compares portfolio $p$ to itself 5 years later.
**Table 3: Standard factor models have large errors when pricing inflation portfolios**

\[
R_{pt}^e = a_p + b_p X_t + e_{p,t}
\]

<table>
<thead>
<tr>
<th>Factors (X_t)</th>
<th>Model number</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>x</td>
<td>Yes</td>
</tr>
<tr>
<td>HML / SMB / Mom</td>
<td>x</td>
<td>Yes</td>
<td>Yes</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
</tr>
<tr>
<td>ST rev + LT rev</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
</tr>
<tr>
<td>Oil</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
</tr>
<tr>
<td>CP factor</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Test \(H_0: all \ a_p = 0\)**

|               | Mean \(|a_p|\) | p-value | \(R^2\) | \(N\) |
|---------------|---------------|---------|---------|-------|
| Mean \(|a_p|\) | 2.08          | 0.005   | 58.3%   | 540   |
| p-value       | 2.80          | 0.000   | 60.9%   | 540   |
|               | 2.68          | 0.000   | 61.2%   | 540   |
|               | 1.88          | 0.027   | 58.6%   | 540   |
|               | 3.11          | 0.000   | 57.2%   | 528   |
|               | 2.31          | 0.043   | 62.3%   | 540   |
|               | 4.28          | 0.000   | 63.8%   | 528   |

Notes: This table reports the results of regressing inflation portfolio’s excess returns \(R_{pt}^e\) on asset pricing factors \(X_t\). Monthly observations, ending in December of 2009 and starting depending on availability of factors \(X_t\). The Fama-French factors, short and long-term reversal factors, and industry portfolios are obtained from Professor French’s website. Oil returns are from the IMF. The Cochrane-Piazzesi factor is from Professor Piazzesi’s website. The mean \(|a_p|\) (mean absolute pricing error) is in percentage points per year. The p-values are for the the null hypothesis \(H_0\) that all pricing errors are zero using a Newey-West variance-covariance matrix with 60 lags and the GRS statistic (which is an F-test adjusted for finite sample bias).
Table 4: Inflation Price of Risk in the Cross-Section of Stocks: Fama-MacBeth Estimates

<table>
<thead>
<tr>
<th></th>
<th>Results from $R_{p,t}^e = a_t + \lambda_t\beta_{p,t} + \epsilon_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 portfolios</td>
</tr>
<tr>
<td>$\bar{\lambda} = \frac{1}{T} \sum \hat{\lambda}_t$</td>
<td>-0.368**</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
</tr>
<tr>
<td>$\bar{\lambda}/\sigma_\pi$</td>
<td>-0.323</td>
</tr>
</tbody>
</table>

Notes: (*, **) Significant at the 5%, 1% level.

The estimates $\hat{\lambda}_t$ are the annualized coefficients of a cross-sectional regression of excess returns at time $t$ on the estimated inflation betas for the same period. The estimate $\bar{\lambda}$ is the average over time of the cross-sectional estimates $\hat{\lambda}_t$. The second row normalizes $\bar{\lambda}$ by the standard deviation of inflation innovations. Column 1 corresponds to my main specification with 10 inflation-sorted portfolios. Column 2 uses individual stocks as advocated in Ang, Liu and Schwarz (2010). Column 3 is the same as column 1 but uses a simple 5-year rolling window to estimate inflation betas. Columns 4 and 5 show the price of inflation risk implied by my model and Bansal and Shaliastovich (2010). The first three columns show in parenthesis Newey-West standard errors with 12 lags and Shanken's adjustment. Column 4 reports GMM asymptotic standard errors. Observations are monthly from February 1959 to December 2009.
### Table 5: Inflation is Persistent

<table>
<thead>
<tr>
<th></th>
<th>( \pi_{t-1} )</th>
<th>( \pi_{t-2} )</th>
<th>( \pi_{t-3} )</th>
<th>( R^2 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{t-1} )</td>
<td>0.629**</td>
<td>×</td>
<td>×</td>
<td>39.6%</td>
<td>611</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_{t-2} )</td>
<td>×</td>
<td></td>
<td></td>
<td>0.091**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>( \pi_{t-3} )</td>
<td>×</td>
<td></td>
<td></td>
<td>0.107**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.040)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td></td>
<td>41.6%</td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>611</td>
<td></td>
<td></td>
<td>609</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (***) Significant at the 1% level.
Inflation is seasonally adjusted CPI. Monthly observations, February 1959 to December 2009. OLS standard errors are in parenthesis.
TABLE 6: INFLATION REGIMES
BEFORE AND AFTER 1980

Panel A: Regression of $\pi_t$ on its lag

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Pre-1980</th>
<th>Post-1980</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.629**</td>
<td>0.679**</td>
<td>0.547**</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.047)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>39.6%</td>
<td>45.2%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$N$</td>
<td>611</td>
<td>251</td>
<td>360</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.14</td>
<td>1.14</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Panel B: Inflation premium $\lambda$

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Pre-1980</th>
<th>Post-1980</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>-0.368</td>
<td>-0.371</td>
<td>-0.317</td>
</tr>
<tr>
<td>Model</td>
<td>-0.377</td>
<td>-0.401</td>
<td>-0.324</td>
</tr>
</tbody>
</table>

Notes: (***) Significant at the 1% level.
Inflation is seasonally adjusted CPI. Monthly observations, February 1959 to December 2009. OLS standard errors in parenthesis.
### Table 7: Inflation predicts consumption growth

<table>
<thead>
<tr>
<th>Inflation lags</th>
<th>$\Delta c_{t-t+k}$ on lags of inflation and consumption growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 12$</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>-1.15*</td>
</tr>
<tr>
<td></td>
<td>(0.507)</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
<td>$\times$</td>
</tr>
<tr>
<td></td>
<td>(0.250)</td>
</tr>
<tr>
<td>$\pi_{t-3}$</td>
<td>$\times$</td>
</tr>
<tr>
<td></td>
<td>(0.298)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumption lags</th>
<th>$\Delta c_{t-1}$ on lags of consumption growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 12$</td>
</tr>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>$\times$</td>
</tr>
<tr>
<td></td>
<td>(0.186)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>6.12%</td>
</tr>
<tr>
<td>$N$</td>
<td>599</td>
</tr>
</tbody>
</table>

Notes: (*) Significant at the 5%, 1% level. Monthly observations, February 1959 to December 2009. Inflation is seasonally adjusted CPI. Consumption is non-durables and services components of PCE. Newey-West standard errors with $2k$ lags are in parenthesis.
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\delta$</td>
<td>0.989</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution</td>
<td>$\psi$</td>
<td>1.44</td>
<td>(0.18)</td>
<td></td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\gamma$</td>
<td>8.46</td>
<td>(0.93)</td>
<td></td>
</tr>
<tr>
<td><strong>Consumption growth parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean of consumption growth</td>
<td>$\mu_c$</td>
<td>0.0026</td>
<td>(0.0012)</td>
<td></td>
</tr>
<tr>
<td>Consumption loadings on inflation</td>
<td>$\rho_{c,0}$</td>
<td>-0.140</td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho_{c,1}$</td>
<td>-0.083</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho_{c,2}$</td>
<td>-0.052</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>Consumption volatility level</td>
<td>$\sigma_c$</td>
<td>0.0065</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Consumption volatility persistence</td>
<td>$\nu_{c,0}$</td>
<td>0.79</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_{c,1}$</td>
<td>0.31</td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>Consumption volatility of volatility</td>
<td>$\sigma_{cw}$</td>
<td>$1.1 \times 10^{-5}$</td>
<td>(2.4 $\times 10^{-5}$)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: GMM asymptotic standard errors in parenthesis.
### Table 8B: GMM Estimates of Parameters for Inflation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean of inflation rate</td>
<td>$\mu_\pi$</td>
<td>0.0038</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00064)</td>
</tr>
<tr>
<td>Inflation auto-regressive coefficients</td>
<td>$\rho_{\pi,0}$</td>
<td>0.621</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.05)</td>
</tr>
<tr>
<td></td>
<td>$\rho_{\pi,1}$</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.08)</td>
</tr>
<tr>
<td></td>
<td>$\rho_{\pi,2}$</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td>Inflation volatility level</td>
<td>$\sigma_\pi$</td>
<td>0.0039</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>Inflation volatility persistence</td>
<td>$\nu_{\pi,0}$</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.21)</td>
</tr>
<tr>
<td></td>
<td>$\nu_{\pi,1}$</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>Inflation volatility of volatility</td>
<td>$\sigma_{\pi w}$</td>
<td>$3.8 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.6 $\times 10^{-5}$)</td>
</tr>
<tr>
<td>Volatility loading on consumption shocks</td>
<td>$\varphi_{\pi c}$</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.84)</td>
</tr>
</tbody>
</table>

Notes: GMM asymptotic standard errors in parenthesis.
<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Mean of dividend growth</th>
<th>Dividend leverage on consumption</th>
<th>Volatility loading of dividend growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>p = 1</td>
<td>0.0028</td>
<td>-1.12</td>
<td>0.896</td>
</tr>
<tr>
<td>2</td>
<td>0.0026</td>
<td>-1.06</td>
<td>0.874</td>
</tr>
<tr>
<td>3</td>
<td>0.0022</td>
<td>-1.01</td>
<td>0.830</td>
</tr>
<tr>
<td>4</td>
<td>0.0031</td>
<td>-0.997</td>
<td>0.858</td>
</tr>
<tr>
<td>5</td>
<td>0.0022</td>
<td>-0.909</td>
<td>0.916</td>
</tr>
<tr>
<td>6</td>
<td>0.0026</td>
<td>-0.838</td>
<td>0.860</td>
</tr>
<tr>
<td>7</td>
<td>0.0021</td>
<td>-0.944</td>
<td>0.929</td>
</tr>
<tr>
<td>8</td>
<td>0.0023</td>
<td>-0.891</td>
<td>0.881</td>
</tr>
<tr>
<td>9</td>
<td>0.0026</td>
<td>-0.900</td>
<td>0.862</td>
</tr>
<tr>
<td>p = 10</td>
<td>0.0028</td>
<td>-0.773</td>
<td>0.812</td>
</tr>
</tbody>
</table>
### Table 10: Estimates for Inflation Portfolios

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>$E[R^e_t]$</td>
</tr>
<tr>
<td>$p = 1$</td>
<td>-2.32</td>
<td>6.91</td>
</tr>
<tr>
<td>2</td>
<td>-1.71</td>
<td>6.87</td>
</tr>
<tr>
<td>3</td>
<td>-1.51</td>
<td>6.44</td>
</tr>
<tr>
<td>4</td>
<td>-1.10</td>
<td>6.29</td>
</tr>
<tr>
<td>5</td>
<td>-0.874</td>
<td>6.36</td>
</tr>
<tr>
<td>6</td>
<td>-0.762</td>
<td>6.17</td>
</tr>
<tr>
<td>7</td>
<td>-0.597</td>
<td>5.58</td>
</tr>
<tr>
<td>8</td>
<td>-0.365</td>
<td>5.53</td>
</tr>
<tr>
<td>9</td>
<td>-0.007</td>
<td>5.56</td>
</tr>
<tr>
<td>$p = 10$</td>
<td>0.064</td>
<td>5.16</td>
</tr>
</tbody>
</table>

**Spread**

(1 minus 10)

-2.38 | 1.67 | -1.78 | 1.83 |

Notes: The data section reproduces the ex-post betas and returns of Table 1. I compute Columns 3 and 4 using model parameters estimated via GMM.
**Table 11: Moments for inflation, consumption, the aggregate market and the risk free rate**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>B.S. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[\pi_t]$</td>
<td>4.47</td>
<td>4.52</td>
<td>3.30</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.14</td>
<td>1.34</td>
<td>1.82</td>
</tr>
<tr>
<td>$\mathbb{E}[\Delta c_t]$</td>
<td>3.14</td>
<td>3.14</td>
<td>1.92</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t)$</td>
<td>2.14</td>
<td>2.25</td>
<td>1.35</td>
</tr>
<tr>
<td>$\text{corr}(\pi_t, \Delta c_t)$</td>
<td>-0.26</td>
<td>-0.26</td>
<td>-0.34</td>
</tr>
<tr>
<td>$\mathbb{E}[R^m_{t,e}]$</td>
<td>6.65</td>
<td>7.25</td>
<td>5.01</td>
</tr>
<tr>
<td>$\sigma(R^m_{t,e})$</td>
<td>15.5</td>
<td>16.8</td>
<td>15.2</td>
</tr>
<tr>
<td>$\mathbb{E}[P_t/D_t]$</td>
<td>26.97</td>
<td>25.42</td>
<td>21.71</td>
</tr>
<tr>
<td>$\sigma(P_t/D_t)$</td>
<td>7.32</td>
<td>8.32</td>
<td>12.17</td>
</tr>
<tr>
<td>$\mathbb{E}[R^f_t]$</td>
<td>1.18</td>
<td>1.26</td>
<td>1.19</td>
</tr>
<tr>
<td>$\sigma(R^f_t)$</td>
<td>0.97</td>
<td>0.45</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: For the data column, I report annualized estimates from monthly observations for February 1959 to December 2009. I compute moments for the model using parameters estimated via GMM for the same period. Column 3 reports the results in Bansal and Shaliastovich (2010).
Table 12: Estimates of Consumption Growth
Predictability and Inflation Persistence

Panel A: Regression of consumption growth $\Delta c_t$ on lags of inflation

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>B.S. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.02</td>
<td>-0.14</td>
<td>-0.30</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
<td>-0.11</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\pi_{t-3}$</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Panel B: Regression of inflation $\pi_t$ on its lags

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>B.S. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.52</td>
<td>0.62</td>
<td>0.65</td>
</tr>
<tr>
<td>$\pi_{t-2}$</td>
<td>0.09</td>
<td>0.12</td>
<td>0.63</td>
</tr>
<tr>
<td>$\pi_{t-3}$</td>
<td>0.11</td>
<td>0.10</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Notes: I compute Columns 2 using the GMM estimates of my model. Column 3 reports results from Bansal and Shaliastovich's (2010) model.
TABLE 13: ESTIMATES OF THE NOMINAL YIELD CURVE

<table>
<thead>
<tr>
<th>Bond Maturity</th>
<th>Data</th>
<th>Model</th>
<th>B.S. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E[y_t^{(n)}]$</td>
<td>$\sigma(y_t^{(n)})$</td>
<td>$E[y_t^{(n)}]$</td>
</tr>
<tr>
<td>1 year</td>
<td>6.40</td>
<td>0.86</td>
<td>6.25</td>
</tr>
<tr>
<td>2 years</td>
<td>6.63</td>
<td>0.87</td>
<td>6.26</td>
</tr>
<tr>
<td>3 years</td>
<td>6.81</td>
<td>0.87</td>
<td>6.27</td>
</tr>
<tr>
<td>4 years</td>
<td>6.95</td>
<td>0.87</td>
<td>6.29</td>
</tr>
<tr>
<td>5 years</td>
<td>7.03</td>
<td>0.88</td>
<td>6.35</td>
</tr>
</tbody>
</table>

Notes: Bond yield data are from the Fama-Bliss bond files. I compute Column 2 using the GMM estimates of my model. Column 3 reports the results in Bansal and Shaliastovich (2010).
### Table 14: Moments not targeted in GMM estimation

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>B.S. (2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{corr}(\Delta c_t, \Delta c_{t-1})$</td>
<td>-0.26</td>
<td>-0.26</td>
<td>0.35</td>
</tr>
<tr>
<td>$\mathbb{E}[W_t/C_t]$</td>
<td>88.59</td>
<td>26.42</td>
<td>48.97</td>
</tr>
<tr>
<td>$\sigma(W_t/C_t)$</td>
<td>14.11</td>
<td>17.23</td>
<td>12.59</td>
</tr>
</tbody>
</table>

Notes: Columns 1 and 3 for the wealth-consumption ratio $W_t/C_t$ are from Koijen, Lustig, Van Nieuwerburgh and Verdelhan (2010). I compute Column 2 using the GMM estimates of my model.
Chapter 2

Aggregate Investment and Stock Returns

with Leonid Kogan and Dmitry Livdan

2.1 Introduction

In this paper\(^1\) we explore the relation between aggregate real investment and stock market volatility, an important aspect of the broader relation between financial markets and the real economy. In a recent influential paper, Bloom (2009) analyzes the impact of large transient volatility shocks on aggregate investment. We focus on a different aspect of the joint dynamics of volatility and investment, relating aggregate investment to persistent changes in return volatility. We document a new empirical pattern: high aggregate investment rate forecasts persistently high subsequent market volatility. It is well known (e.g., Abel (1983), Caballero (1991)) that the sign of the investment-volatility relation depends on the structure of the economic environment.

---

\(^1\)We thank the participants of the brown bag finance seminar at MIT for helpful discussions and comments. Leonid Kogan acknowledges financial support for this project from JPMorgan Chase.
To help narrow down the range of possible structural explanations for the observed positive correlation between aggregate real investment and expected future stock market volatility, we rely on the additional empirical patterns in the joint dynamics of stock returns, investment, and output.

An important feature of the stock return-investment dynamics is negative correlation between aggregate investment rate and subsequent excess stock market returns, studied in Cochrane (1991).²

In our model, time-varying discount rates generate both the negative relation between investment and future excess returns and the positive relation between aggregate investment and future stock market volatility. The first relation is well understood. It is consistent with the basic partial-equilibrium intuition that, ceteris paribus, an exogenous decline in discount rates should increase the net present value of potential investment projects, and thus should raise the aggregate investment rate.

To see the intuition behind the second relation, consider the classic Gordon model for stock valuation. The price of the stock is proportional to the expected future dividend, and inversely proportional to the difference between the expected growth rate of dividends, and the discount rate, both assumed constant:

\[
P_0 = \frac{E_0[D_1]}{r - g},
\]

where time is discrete, \(P_0\) is the stock price at time 0, \(E_0[D_1]\) is the expected time-1 dividend, \(r\) denotes the cost of capital, and \(g\) denotes the expected dividend growth rate. Assume, furthermore, that dividend growth is homoscedastic, so heteroscedastic stock return volatility is not generated mechanically by a similar pattern in cash flows.

² The relation between investment and subsequent excess stock market returns has also been studied in the cross-section of firms, e.g., Titman, Wei, and Xie (2004), Chen, Novy-Marx, and Zhang (2010), Kogan and Papanikolaou (2010).
Consider a comparative-statics experiment: holding the expected future dividends fixed, reduce the discount rate by a small amount. This has an effect of increasing the stock price at time 0, which is a well-known effect of time-varying discount rates on the volatility in stock returns. Note that the magnitude of the impact of a discount rate change on the stock price depends on the initial difference between the discount rate and the expected growth rate: if \( r - g \) is relatively low, the same change in the discount rate has larger impact on the stock price than it would at higher levels of \( r - g \). This simple observation prompts a conjecture: if discount rates experience homoscedastic shocks, an exogenous decline in discount rates should give rise to higher future return volatility. Since a decline in discount rates also naturally leads to an increase in the aggregate investment rate, we thus conclude that time-varying discount rates may give rise to a positive correlation between real investment and future stock market volatility.

The above conjecture is based on ad hoc arguments ignoring the general equilibrium considerations and liberally using comparative statics in lieu of rigorous dynamic analysis. We formalize these arguments using a general-equilibrium production economy model. The economy in our model is affected by two types of shocks: productivity shocks and preference shocks. Our framework is very similar to canonical real business cycle models in its treatment of production. The only deviation from the standard setting is in our assumption that the representative household is subject to preference shocks. Effectively, preference shocks generate exogenous variation in risk aversion of the representative household, and with it variation in the market prices of risk. We calibrate our model to match the key unconditional moments of consumption growth and financial asset returns. We then verify that our model generates the same qualitative predictive relations as we document empirically and comes close in replicating the magnitude of the observed effects.

Our analysis further supports the idea that accounting for the time-varying price of
risk in financial markets is important for understanding the dynamics of real economic activity. Modern asset pricing literature has emphasized the significance of time-varying price of risk, or return predictability, for understanding the key properties of asset return behavior, such as excess volatility of asset returns and high equity premium (e.g., Campbell and Cochrane (1999), Cochrane (1999)). Our paper adds to this body of work by arguing that time-varying price of risk may also be the cause of persistent changes in return volatility that we document. Thus, we tie together the core asset pricing results on return predictability and the growing literature on the connections between real economic activity and time-varying uncertainty (e.g., Bernanke (1983), Leahy and Whited (1996), Bloom, Bond, and Reenen (2007), Bloom (2009)). As shown in Bloom (2009), stock market volatility is a key indicator of economic uncertainty. Our analysis in this paper offers an economic interpretation of the empirical relations between market volatility and real investment.

The rest of the paper is organized as follows. Section 2.2 describes the data and empirical results. Section 2.3 presents the theoretical model. Section 2.4 presents calibration results and robustness checks. Section 2.5 concludes.

2.2 Empirical Results

2.2.1 Data and procedures

Our sample starts 1947Q1 and ends 2009Q4 for a total of 252 quarters. We use lowercase letters for logs of all variables throughout this section and the rest of the paper.

3 We find that excluding the 2008-2009 period from our sample has no effect on the qualitative results, and has only minor effect on the point estimates. Thus, our conclusions are robust to excluding the financial crises period. We also find that our conclusions are unchanged if we exclude the immediate post-war period of 1947-1952.
As a measure of aggregate stock returns, we use returns on the CRSP value weighted portfolio, available from Kenneth French’s website. We construct quarterly returns, $r_t$, from the daily returns. To construct excess returns, we subtract the three-month T-bill rate, $r_f^t$, available in Kenneth French’s website.

We also use quarterly data on realized volatility. Specifically, we construct a quarterly series $\text{vol}_t$, defined as the log of the standard deviation of daily returns within quarter $t$. We find that our results are robust to Winsorizing the volatility series or using alternative measures of realized volatility, such as absolute values of quarterly returns.

Quarterly data for the macroeconomic variables is from U.S. National Income and Product Accounts (NIPA) available directly from the Bureau of Economic Analysis (BEA). For GDP, $Y_t$, we use real business gross domestic product (Table 1.3.6, line 2). For investment, $I_t$, we use real nonresidential fixed private domestic investment (we multiply the real index Table 1.5.3, line 28, by the nominal investment in 2005—the base year for the real index—in Table 1.5.5, line 28 and then we divide by 100). Quarterly capital stock values, $K_t$, are interpolated from annual values using the quarterly investment series $I_t$. The annual series for real capital stock is private nonresidential fixed assets (we multiply the real index in NIPA’s fixed assets table 4.2, line 1, by the nominal capital in 2005—the base year for the real index—in table 4.1, line 2 and then we divide by 100), with year-end 1946 as the starting point. For each quarter, a fraction of the annual capital increment is added to the current end-of-year stock, with the fraction given as the year’s investment to date, divided by total

---

4 $r_t$ is cumulative log return on the value-weighted portfolio of NYSE, NASDAQ, and AMEX stocks.
5 For example, in order to obtain the standard deviation of the market return for 1952Q1, we use the standard deviation of the time series of daily market returns from January 1, 1952 to March 31, 1952.
6 In 2003, the BEA undertook comprehensive revisions of all NIPA data series. Our data incorporates these revisions.
7 In billions of chained (2005) dollars.
investment for that year. Unlike the standard inventory-based method of constructing capital stock (e.g., Cochrane (1991)), the above method does not rely on any particular model of capital accumulation.

2.2.2 Empirical findings

Summary statistics

We use two variables in predictive regressions. The first is the natural log of the investment rate, $i_t - k_t = \ln(I_t/K_t)$, where investment rate is measured as the ratio of the quarterly GPDI to the end-of-quarter capital stock (our timing convention is analogous to the one used in defining trailing dividend yield). The second variable, $y_t - k_t = \ln(Y_t/K_t)$, is the log of the ratio of quarterly output to the end-of-quarter capital stock. We view this variable as a proxy for average profitability in the economy.\(^8\)

We start by summarizing the key moments of investment, profitability, and financial asset returns in our sample.

[Table 2.1 ]

In addition to the first two moments of the key variables, we estimate their autocorrelation. We find that profitability is highly persistent in our sample, with an eight-quarter autocorrelation coefficient of 0.56. Aggregate investment rate shows less persistence, with autocorrelations of 0.48 and below beyond the eight-quarter horizon. Consistent with commonly reported results, consumption and output growth rates exhibit very little autocorrelation. Investment growth is also close to being uncorrelated over time. Stock returns are virtually uncorrelated over time, but stock return volatility is persistent. Autocorrelations of volatility decline at a relatively slow rate, starting at

\(^8\)Our interpretation is justified as long as the output share of capital is approximately constant. Alternatively, one may simply view the two variables as jointly approximating the state vector in the economy.
0.58 at a one-quarter horizon and declining to 0.34 and 0.10 at four and eight-quarter horizons respectively. This pattern of decline suggests that market volatility possesses more persistence than what could be generated by a simple first-order autoregressive specification. Our results in Table 2.4 reinforce this observation.

**Predictability of excess stock returns and return volatility**

We first analyze predictability of stock returns. Tables 2.2 and 2.3 report predictive regressions of single-quarter and multi-quarter excess stock returns on lagged values of investment rate and profitability. Our regressions extend the results in Cochrane (1999) to our longer sample and to a more general specification. We run two predictive regressions:

\[ r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \epsilon_{t,t+h} \quad (2.2) \]

and

\[ r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \epsilon_{t,t+h} \quad (2.3) \]

Before running these regressions, we de-trend all right-hand-side variables. We do this so that low-frequency movements in the variables under consideration do not drive our results, since we cannot evaluate statistical significance of such effects in our sample. De-trending has little effect on the predictive regressions for returns, but is potentially important for predictive regressions of return volatility below, since return volatility exhibits some low-frequency persistence. As a robustness check, we perform the same regression on the original series, and find qualitatively similar results.

In the second specification, we include profitability as a second predictive variable in addition to the investment rate. The predictive relation between the investment rate and future excess stock returns indicates time-variation in expected stock returns. According to conventional intuition, the aggregate investment rate is negatively affected
by the discount rates on future projects because higher discount rates imply lower net present value of cash flows produced by new investments. Therefore, as long as discount rates on cash flows from new investments and on those produced by existing assets are not too different, it is natural that the investment rate is negatively correlated with future excess stock returns. This argument can be refined by observing that investment decisions are affected by profitability of new investment projects in addition to their discount rates. Persistence of aggregate profitability (see Table 2.1) suggests that lagged aggregate profitability may be a useful predictor of future profitability of new investments, as long as profitability of new projects is not too different from profitability of existing physical assets. Therefore, aggregate profitability is a potentially useful control in predictive regression of excess stock returns on the lagged aggregate investment rate.

It is worth noting at this point that, while our regressions are inspired by the common intuition, our interpretation of the empirical results relies on a fully specified general equilibrium model that we develop in the following sections. In the model, some of the vague statements used in the previous intuitive argument are not necessary. All of the relevant variables are derived endogenously in equilibrium, and relations between them can be quantified. For instance, discount rates on cash flows from new investments are equal to those on cash flows from existing assets, and thus the aggregate investment rate is a useful predictor of future excess stock returns. In anticipation of the formal equilibrium analysis below, we note that in our model economy there are two structural shocks: shocks to productivity and preference shocks. Discount rates, as well as profitability, are affected by both shocks in equilibrium. Without arguing further that investment rate and profitability are each primarily affected by a single structural shock, the two-shock structure of the model implies that the two-variable predictive regression emerges as a natural agnostic linear approximation to the model’s
equilibrium relation between the conditional moments of financial asset returns and equilibrium state variables.

[Table 2.2]

As in Cochrane (1999), we find that the investment rate predicts future excess stock returns negatively. Adding lagged profitability as a control does not seem to affect the results substantially, and coefficients on lagged profitability are not statistically significant.

In Table 2.5, we report analogous predictive regressions for a cumulative sum of excess stock returns over multiple quarters. These regressions highlight the joint significance of predictability in excess returns at multiple horizons. The explanatory power of these regressions is quite low, but there is evidence of predictability in returns at horizons up to 16 quarters.

[Table 2.3]

We next summarize the results on predictability of stock return volatility in Tables 2.4 and 2.5. In Table 2.4, we report the results of a univariate predictive regression of volatility on lagged investment rate,

\[ \text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h} \] (2.4)

and of multivariate regressions, adding lagged profitability and lagged realized volatility to the forecasting equation:

\[ \text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1} \] (2.5)

and

\[ \text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3 \text{vol}_{t-1} + \varepsilon_{t,t+h+1} \] (2.6)
As in Tables 2.3, we use both the investment rate and profitability in predictive regressions.

[Table 2.4]

The investment rate predicts future return volatility with a positive sign at all horizons up to 16 quarters. This pattern is stronger and has higher statistical significance in a two-variable regression, which also includes aggregate profitability.

In the third panel of Table 2.4, we add lagged realized volatility to the regression. We do this for two reasons. First, as shown in Bloom (2009), market volatility has a negative impact on future investment at short horizons. Our model is not flexible enough to capture this pattern, but such a relation may affect our empirical findings. In particular, at relatively short horizons, it may weaken the positive relation between investment rate and future market volatility produced by our current specification.

We find that adding lagged volatility leaves the results qualitatively unchanged, while significance and point estimates of coefficients on investment rate increase at short horizons.

Second, realized volatility is a noisy proxy for the true conditional volatility, and it is useful to see how well realized volatility can predict its own future values compared to the two macro-economic state variables we use in our regressions. Looking at the first panel of Table 2.4, we find that investment rate and profitability jointly explain 22 percent of variation in realized return volatility at an eight-quarter horizon, while lagged realized volatility adds only 2 percent of explanatory power when added to the regression, percent of its own future variation. The pattern of $R^2$ is consistent with this observation. Adding lagged volatility to the forecasting regression significantly boosts its explanatory power at short horizons, for instance, raising the $R^2$ from 0.22 to 0.40 at a one-quarter horizon, but has negligible effect at horizons longer than six quarters. This indicates that there are sources of short-horizon predictability in return volatility.
not captured by our predictive variables.

[Table 2.5]

In Table 2.5, we report analogous predictive regressions for a cumulative sum of realized intra-quarter volatility over multiple quarters. These regressions highlight the joint significance of predictability in volatility at multiple horizons. We find that the investment rate is a highly statistically significant predictor of future stock return volatility at horizons up to 16 quarters. Profitability enters negatively in a multi-variate forecasting regression, and is marginally statistically significant. Lagged realized volatility enters positively, and is highly statistically significant. The pattern of coefficients and t-statistics on lagged volatility is consistent with its predictive ability being relatively short-lived.

2.3 The Model

2.3.1 Formulation

Technology

We assume that there exists a competitive representative firm. This firm uses capital and labor to produce a single consumption good. We denote the capital stock by $K_t$, the input of labor by $L_t$, and the flow of output by $Y_t$. We assume the standard Cobb-Douglas production function

$$Y_t = e^{x_t} K_t^\alpha L_t^{1-\alpha},$$ (2.7)
where the productivity shock $x_t$ follows

$$dx_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma_x dW_t.$$  

We assume that capital depreciates at the constant rate $\delta$ and can be replenished through investment. Denoting the investment rate by $i_t$,

$$dK_t = (i_t - \delta)K_t dt. \quad (2.8)$$

We assume that new capital can be created from the consumption good subject to convex adjustment costs, so that the flow cost of creating new capital at the investment rate $i_t$ is given by

$$I_t = \frac{a}{\lambda} i_t^\lambda K_t,$$  

where $\lambda > 1$. Thus, the marginal cost of capital creation, measured in units of the consumption good, is positively related to the investment rate.

**Households**

We model households as a representative consumer. The representative agent owns the representative firm and supplies labor competitively in the labor market. We assume that the representative household is endowed with a constant flow of labor, normalized to one, which it supplies inelastically.

We describe preferences of the representative household by a time-separable isoelastic utility function subject to preference shocks. In particular, the representative
household evaluates consumption streams \(\{C_t\}\) according to

\[
\mathbb{E}_0 \left[ \int_0^\infty e^{-\delta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} \, dt \right].
\]

We assume that the preference shock \(\xi_t\) evolves according to

\[
d\xi_t = (b_t - \frac{\sigma_t^2}{2}) \, dt + \sigma_t \, dW_t,
\]

\[
d\sigma_t = -\theta \sigma_t \, dt + \nu \, dW_{\sigma t}, \quad W_{\sigma t} = \rho W_t + \sqrt{1 - \rho^2} W_t'.
\]

In our specification, stochastic \(\sigma_t\) implies that the representative household has a state-dependent marginal utility with respect to consumption. Specifically, state-dependence is driven by the same shocks as the productivity process. This specification can be viewed as a reduced-form description of time-varying aversion to risk or time-varying beliefs.\(^9\)**10** The exact interpretation is not critical for our analysis. Our interpretation of the empirical patterns is based on the time-varying discount rates, and does not hinge on the exact source of discount rate variation. The process \(b_t\) plays the role of time-varying subjective rate of time preferences. This process does not affect the

\(^9\)If we set \(b_t = 0\) in the definition of \(\xi_t\), our preference specification is isomorphic to a model of a household with the same isoelastic preferences but distorted beliefs. In particular, under the distorted beliefs, the Brownian motion \((W_t)\) that drives the productivity process acquires a drift \(\sigma_t\). Thus, the representative household exhibits the time-varying degree of optimism or pessimism, and perceives the productivity process as

\[
dx_t = \left( \mu - \frac{\sigma_X^2}{2} + \sigma_t \right) \, dt + \sigma_X \, d\tilde{W}_t,
\]

where \(\tilde{W}_t\) is a Brownian motion under the subjective distorted beliefs of the representative household. Clearly, such a distortion in beliefs affects the equilibrium discount rates.

\(^{10}\)Our reduced-form description may reflect a variety of economic phenomena. The most immediate connection is with the models emphasizing habit formation, e.g., Campbell and Cochrane (1999). Gallmeyer, Hollifield, and Zin (2005) use a similar description of preferences to capture time-varying risk aversion. Time-varying discount rates also arise naturally as a result of the dynamic wealth redistribution across a heterogeneous population of market participants, e.g., Chan and Kogan (2002), Gårleanu and Panageas (2007), Guvenen (2009).
qualitative implications of our model and is introduced for purely technical reasons, helping stabilize the risk-free interest rate in equilibrium.

Financial markets and asset prices

We assume that there exists a complete set of zero-net-supply state-contingent claims, prices of which are summarized by the state-price density process \( \pi > 0 \) and that the time-\( t \) price of any long-lived asset with cash flow \( X \) is given by the bubble-free pricing equation

\[
E_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} X_s \right] dt.
\]

We denote the equilibrium short-term risk-free rate by \( r_t \).

In addition to the state-contingent claims, we assume that the representative household is endowed with a single stock share, which is a claim on the dividends of the representative firm. Thus, the representative firm is all equity financed.\(^{11}\) The dividends are equal to output net of investment costs and labor costs. Denoting the wages paid by the representative firm by \( w \), the aggregate dividend flow rate is

\[
D_t = Y_t - \frac{a}{A} i_t^A K_t - w_t L_t.
\] (2.13)

2.3.2 Equilibrium

We adopt the standard definition of competitive equilibrium. In equilibrium, the representative household and the representative firm take prices of state-contingent claims and the wage rate as given. The representative household maximizes its expected utility, while the representative firm maximizes its market value. All markets clear.

\(^{11}\)The assumption of equity financing for the representative firm is without loss of generality, since the assumptions of the Modigliani-Miller theorem hold in our setting, and therefore the choice of capital structure does not affect equilibrium policies.
Definition 1. The competitive equilibrium is described by a collection of stochastic processes $\pi^*, w^*_t, L^*, C^*, Y^*, K^*, i^*$, and $D^*$, such that

1. $Y^*, K^*, L^*$, and $i^*$ satisfy the technological constraints (2.7) and (2.8).

2. $C^*$ and $L^*$ maximize the representative household’s objective, taking the state-price density, dividends, and wages as given,

$$\max_{\{C^*, L^*\}} E_0 \left[ \int_0^\infty e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right],$$

subject to

$$E_0 \left[ \int_0^\infty \frac{\pi^*_t}{\pi^*_0} (C_t - D_t^* - w_t^* L_t^*) dt \right] = 0;$$

3. $i^*, L^*$, and $D^*_t$ maximize the representative firm’s value, taking the state-price density as given,

$$\max_{\{i^*, L^*, D^*\}} E_0 \left[ \int_0^\infty \frac{\pi^*_t}{\pi^*_0} D_t dt, \right],$$

subject to (2.7), (2.8), and (2.13).

4. Labor market clears,

$$L^*_t = 1,$$

and consumption market clears,

$$C^*_t = D^*_t + w_t L^*_t.$$

2.3.3 Solution

Since financial markets in our model are frictionless and there are no externalities, equilibrium consumption and investment policies can be determined by solving the
central planner’s problem. The central planner maximizes the expected utility of the representative household

$$E_0 \left[ e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1 - \gamma} dt \right].$$

subject to the aggregate resource constraint

$$C_t + I_t = Y_t$$

and to (2.8), (2.9), (2.10), (2.11).

Equilibrium prices can be recovered from the central planner’s solution for equilibrium quantities using individual optimality conditions:

$$\pi_t = e_t^{-\beta t + \xi_t} (C_t^*)^{-\gamma},$$

and

$$w_t^* = (1 - \alpha)Y_t^*.$$

We solve for equilibrium numerically using finite-difference approximations. We first solve the dynamic program of the central planner and determine equilibrium consumption and investment policies, and the state-price density. We then compute the price of the aggregate stock market as the expected value of future dividends discounted with the equilibrium state-price density. We derive the risk-free interest rate as the negative of the drift of the state-price density.
2.4 Calibration and Simulation Results

2.4.1 Parameter calibration

The starting point of our calibration is the canonical real business-cycle model. Indeed, if our model had no preference shocks, \( v = 0, \theta = 0, \rho = 0 \), we could pick parameters that are standard in the literature, simplifying calibration. We find that if we set the parameters not relating to preference shocks in that manner, we can successfully match all the unconditional moments in Table 2.1 that do not relate to asset prices. We then pick the values of \( v, \theta, \) and \( \rho \) to reproduce the empirically observed moments for asset prices and the conditional moments of Tables 2.2-2.5. This strategy is possible because most of the moments relating to quantities are decoupled from preference shock parameters. For example, the steady-state level of the investment rate is

\[
E [i_t - k_t] = \frac{\mu - \sigma_X^2/2}{(1 - \alpha)} + \delta, \tag{2.18}
\]

while the mean and volatility of output growth are given by

\[
E [d_{yt}] = \frac{\mu - \sigma_X^2/2}{(1 - \alpha)} \tag{2.19}
\]

\[
\text{st.dev.} (d_{yt}) = \sigma_X, \tag{2.20}
\]

which do not depend on any preference parameters.

We set the values for the model's production technology to \( \alpha = 0.33, \mu = 0.015, \sigma_X = 0.03, a = 10 \) and \( \lambda = 5 \), which are similar to those find in the literature. For the standard preference parameters, we pick a discount parameter of \( \beta = 0.02 \) and a reasonable coefficient of relative risk aversion \( \gamma = 10 \).
We pick an auto-regressive coefficient for the preference shock of $\theta = 0.4$, which represents a half-life of about 7 quarters. Tables 2.4 and 2.5 show that this is exactly the horizon for which investment has its strongest predictive force. While this persistence parameter helps control the timing of predictability, we set the volatility of preference shocks to $\nu = 0.3$ to match the magnitude of predictability of stock volatility and returns. Finally, we set the correlation between shocks to productivity and preference shocks to $\rho = -0.9$. The negative correlation implies that times of low productivity coincide with times of high volatility.

To understand the mechanics of the model and gain further intuition into how different parameters affect our results, we analyze how macroeconomic variables and asset prices respond to preference shocks.

[Figure 2-1]

Figure 2-1 shows the steady-state probability distribution function of the two state variables, the preference shock $\sigma$ and profitability $y_t - k_t$. Since $\rho < 0$, they are negatively correlated. Figure 2-2(a) shows how the investment rate and consumption (normalized by capital) behave as a function of the state variables. Investment is decreasing in the preference shock: a positive shock to marginal utility will, ceteris paribus, reduce investment and increase consumption. On the other hand, investment is increasing in profitability, since the latter is simply a capital-adjusted measure of productivity: $y_t - k_t = x_t + (\alpha - 1)k_t$.

Figure 2-3 shows the impulse-response functions of the key variables to a positive preference shock. Because the capital stock cannot change instantaneously but productivity $x_t$ decreases when a positive preference shock hits the economy (because $\rho < 0$), profitability decreases after the shock. The representative agent’s optimal response to a drop in productivity is to reduce the investment rate. In addition, because the preference shock raises marginal utility, the representative agent has an added incentive to
invest less and consume more. Both effects lead to a sharp decrease in investment. As the preference shock reverts to its mean, investment slowly returns to its steady-state levels, with the speed of adjustment controlled mainly by the convexity of adjustment costs.

Figure (2-4) shows the impulse response function for asset prices. Even though we have a general equilibrium model and cash flows are not constant, the partial equilibrium intuition of the Gordon formula given in the introduction still holds. In this case, an increase in the cost of capital is associated with persistently high expected stock returns and low volatility. Tables 2.9-2.12 confirm the result by replicating our empirical regressions using 2,500 sample paths generated by the model and then computing the averages of regression coefficients, t-statistics, and $R^2$ across the simulated samples replications.

Simulation results suggest that our equilibrium model captures, at least qualitatively, the key empirical patterns: the negative predictive relation between the investment rate and future excess stock returns, and the positive relation between the investment rate and future return volatility. Comparing the empirical numbers to simulation results, we identify two areas for future improvements.

First, excess stock returns in the model are much more predictable than in the data, as indicated by the high values of $R^2$'s in Tables 2.11, 2.12. This may be partly because too much of stock return volatility in the model is driven by the preference shocks. In addition, since we know that high explanatory power for future excess stock returns can be obtained using financial valuation ratios (e.g., Campbell and Cochrane (1999)),
it appears that the aggregate investment rate is a more precise proxy for the preference shocks in the model than it is in the data.

Second, the hump-shaped pattern of the regression coefficients of stock return volatility on the aggregate investment rate is an interesting feature of the data that is not captured by the model. In our model, even though the effects on returns and volatility are persistent, the largest responses occur contemporaneously with the arrival of shocks. Our model therefore matches the observed empirical patterns at the frequencies of 4 to 6 quarters and onwards, and over-estimates the effects of investment on volatility in the short run. A potential resolution of this discrepancy is to introduce transient components, either in the volatility of productivity or in productivity itself. Indeed, Bloom (2009) considers exactly these type of shocks and obtains a negative relation between investment and volatility of returns at short horizons.

2.5 Conclusion

In this paper we establish a new empirical fact: the aggregate investment rate is strongly positively correlated with future stock market volatility. Together with the well-known negative relationship between the investment rate and subsequent excess stock returns, this implies that, conditionally on the aggregate investment rate, stock market exhibits a negative mean-variance tradeoff. We interpret these empirical patterns using a general-equilibrium production economy model. In our model, the qualitative empirical correlation patterns among the aggregate investment and productivity on one hand, and the conditional moments of the stock market returns on the other hand, arise because of time-varying discount rates. Thus, our paper emphasizes the importance of time-varying discount rates for understanding not only the behavior of financial markets, but also for interpreting the dynamics of the key macroeconomic variables.
We are working on extending our model to incorporate the negative short-term correlation between stock-return volatility and subsequent real investment. Together with the results obtained in this paper, the extended model should further clarify the respective roles played by the technology and preference shocks in shaping the observed joint dynamics of aggregate investment and financial asset returns.
Bibliography


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2.6 Appendix: Solution and Numerical Procedure

2.6.1 Central's Planner's Problem

The value function \( J(\xi, \sigma, x, K) \) of the central planner's problem satisfies the following HJB equation:

\[
\max_i \left[ e^\xi \left[ \frac{e^\alpha K - \frac{\gamma}{\lambda} K^2}{1 - \gamma} + iKJ_K \right] - \delta KJ_K - \theta \sigma J_\sigma + \left( b - \frac{\sigma^2}{2} \right) J_\xi + \frac{\sigma^2}{2} J_{xx} + \rho \sigma \chi vJ_x \sigma + \rho \sigma vJ_\xi \sigma + \sigma \sigma \chi J_{xx} = \beta J. \right. \tag{2.21}
\]

\[
(\mu - \frac{\sigma^2}{2})J_x + \frac{\sigma^2}{2} J_{\xi \xi} + \frac{\sigma^2}{2} J_{xx} + \rho \sigma \chi \nu J_x \sigma + \rho \sigma v J_\xi \sigma + \sigma \sigma \chi J_{xx} = \beta J.
\]

We look for solutions of the form

\[
J(\xi, \sigma, x, K) = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} V(\sigma, z), \tag{2.22}
\]

where

\[
z \equiv x + (\alpha - 1)k.
\]

Substituting (2.22) back into (2.21) and using

\[
J_\xi = J_{\xi \xi} = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} V,
\]

\[
KJ_K = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} [(1 - \gamma)V + (\alpha - 1)V_z],
\]

\[
J_x = J_{\xi \xi} = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} V_z,
\]

\[
J_{xx} = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} V_{zz},
\]

\[
J_{x\sigma} = \frac{e^{\xi+(1-\gamma)k}}{1 - \gamma} V_{z\sigma},
\]

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we obtain

\[ (e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} + \mathcal{L}V = 0 \]  \hspace{1cm} (2.23)

where Dynkin operator \( \mathcal{L} \) has the following form

\[
\mathcal{L} = (\rho v - \theta) \frac{\partial}{\partial \sigma} + \left[ \mu + \sigma X - \frac{\sigma^2_\chi}{2} - (1 - \alpha) (i^* - \delta) \right] \frac{\partial}{\partial z} + \\
+ \frac{v^2}{2} \frac{\partial^2}{\partial \sigma^2} + \frac{\sigma^2_\chi}{2} \frac{\partial^2}{\partial z^2} + \rho \sigma X v \frac{\partial^2}{\partial \sigma \partial z} - [\beta - b - (1 - \gamma) (i^* - \delta)].
\]  \hspace{1cm} (2.24)

The optimal investment rate, \( i^* \), satisfies

\[
a (i^*)^{\lambda-1} = \left[ V - \frac{1 - \alpha}{1 - \gamma} V_x \right] \left[ e^z - \frac{a}{\lambda} (i^*)^\lambda \right]^\gamma. \]  \hspace{1cm} (2.25)

2.6.2 Stock Price

The stock price is given by the discounted sum of dividends:

\[
S_t = e^{-\xi_t} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(s-t) + \xi_s} \left( \frac{C_t^*}{C_s^*} \right)^\gamma D_s ds \right] = e^{-\xi_t} (C_t^*)^\gamma \alpha (1 - \gamma) J(\xi_t, \sigma_t, x_t, K\mathcal{Q})
\]

\[
- (1 - \alpha) e^{-\xi_t} (C_t^*)^\gamma \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(s-t) + \xi_s + (1-\gamma)k_s} \left( \frac{C_s^*}{K_s} \right)^{-\gamma} \frac{a}{\lambda} (i_s^*)^\lambda ds \right] =
\]

\[
= K_t \left( \frac{C_t^*}{K_t} \right)^\gamma \left[ \alpha V(\sigma_t, z_t) - (1 - \alpha) \Phi(\sigma_t, z_t) \right],
\]

where the Feynman-Katz formula implies that \( \Phi(\sigma_t, z_t) \) satisfies the following PDE

\[
\frac{a}{\lambda} (i^*)^\lambda \left( e^z - \frac{a}{\lambda} (i^*)^\lambda \right)^{\gamma-1} + \mathcal{L}\Phi = 0.
\]  \hspace{1cm} (2.27)
2.6.3 Steady state distribution

The joint steady state distribution of $(\sigma, z), p_\infty(z, \sigma)$, satisfies the following Kolmogorov backward equation:

\[
- \left[ \theta + (1 - \alpha) \frac{\partial i^*}{\partial z} \right] p_\infty(z, \sigma) = -\theta \sigma \frac{\partial p_\infty(z, \sigma)}{\partial \sigma} + \mu \frac{\sigma^2}{2} - (1 - \alpha)(i^* - \delta) \frac{\partial p_\infty(z, \sigma)}{\partial z} + \frac{v^2}{2} \frac{\partial^2 p_\infty(z, \sigma)}{\partial \sigma^2} + \frac{\sigma^2}{2} \frac{\partial^2 p_\infty(z, \sigma)}{\partial z^2} + \rho \sigma v \frac{\partial^2 p_\infty(z, \sigma)}{\partial \sigma \partial z}.
\]

This equation is solved so that the probability density function integrates to one:

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_\infty(z, \sigma) dz d\sigma = 1.
\]

2.6.4 Numerical Procedure

We discretize the HJB equation on the $(\sigma, z)$ using the following approximations:

\[
V_z = \frac{V(z + \Delta z, \sigma) - V(z, \sigma)}{\Delta z}, \text{ if } \sigma \geq 0
\]

\[
V_z = \frac{V(z, \sigma) - V(z - \Delta z, \sigma)}{\Delta z}, \text{ if } \sigma < 0
\]

\[
V_\sigma = \frac{V(z, \sigma + \Delta \sigma) - V(z, \sigma)}{\Delta \sigma}, \text{ if } (\rho v - \theta) \sigma \geq 0
\]

\[
V_\sigma = \frac{V(z, \sigma) - V(z, \sigma - \Delta \sigma)}{\Delta \sigma}, \text{ if } (\rho v - \theta) \sigma < 0
\]

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Since \((\mu + (1 - \alpha) \delta) \geq 0, (\sigma^2 X_i / 2 + (1 - \alpha) i) \geq 0\), we use the one-sided approximation for the second derivatives:

\[
V_{\sigma \sigma} = \frac{V(z, \sigma + \Delta \sigma) + V(z, \sigma - \Delta \sigma) - 2V(z, \sigma)}{\Delta \sigma^2}
\]

\[
V_{z z} = \frac{V(z + \Delta z, \sigma) + V(z - \Delta z, \sigma) - 2V(z, \sigma)}{\Delta z^2}.
\]

For the cross-partials, we use

\[
V_{\sigma z} = \left[ \frac{2V(z, \sigma) + V(z + \Delta z, \sigma + \Delta \sigma) + V(z - \Delta z, \sigma - \Delta \sigma)}{2\Delta z \Delta \sigma} - \right. \\
\left. - \left[ \frac{V(z + \Delta z, \sigma) + V(z - \Delta z, \sigma) + V(z, \sigma + \Delta \sigma) + V(z, \sigma - \Delta \sigma)}{2\Delta z \Delta \sigma} \right] \right]^{(2.32)}
\]

if \(\rho \geq 0\) and

\[
V_{\sigma z} = - \left[ \frac{2V(z, \sigma) + V(z + \Delta z, \sigma - \Delta \sigma) + V(z - \Delta z, \sigma + \Delta \sigma)}{2\Delta z \Delta \sigma} - \right. \\
\left. + \left[ \frac{V(z + \Delta z, \sigma) + V(z - \Delta z, \sigma) + V(z, \sigma + \Delta \sigma) + V(z, \sigma - \Delta \sigma)}{2\Delta z \Delta \sigma} \right] \right]^{(2.33)}
\]

if \(\rho < 0\). We define the transition probabilities between the discretized points to be:

\[
p((z, \sigma) \rightarrow (z, \sigma + \Delta \sigma) | i) = \frac{1}{Q^A} \left[ (\rho u - \theta) \sigma^+ \Delta \sigma \Delta z^2 + \frac{\sigma^2 v}{2} \Delta z^2 - \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

\[
p((z, \sigma) \rightarrow (z, \sigma - \Delta \sigma) | i) = \frac{1}{Q^A} \left[ (\rho u - \theta) \sigma^- \Delta \sigma \Delta z^2 + \frac{\sigma^2 v}{2} \Delta z^2 - \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

\[
p((z, \sigma) \rightarrow (z + \Delta z, \sigma) | i) = \frac{1}{Q^A} \left[ (\mu + (1 - \alpha) \delta + [\sigma]^+ \sigma X^+ \Delta \sigma \Delta z^2 + \frac{\sigma^2 x}{2} \Delta \sigma \Delta z^2 - \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

\[
p((z, \sigma) \rightarrow (z - \Delta z, \sigma) | i) = \frac{1}{Q^A} \left[ \frac{\sigma^2 x}{2} + (1 - \alpha) i^+ [\sigma]^+ \sigma X^+ \Delta \sigma \Delta z^2 + \frac{\sigma^2 x}{2} \Delta \sigma \Delta z^2 - \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

\[
p((z, \sigma) \rightarrow (z + \Delta z, \sigma + \Delta \sigma) | i) = p((z, \sigma) \rightarrow (z - \Delta z, \sigma - \Delta \sigma) | i) = \frac{1}{Q^A} \left[ \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

\[
p((z, \sigma) \rightarrow (z - \Delta z, \sigma - \Delta \sigma) | i) = p((z, \sigma) \rightarrow (z + \Delta z, \sigma - \Delta \sigma) | i) = \frac{1}{Q^A} \left[ \frac{\sigma^2 x v}{2} | \rho | \Delta \sigma \Delta z \right]
\]

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where
\[ Q^\Delta((z, \sigma), i) = |(\rho v - \theta) \sigma| \Delta_z \Delta_z^2 + \left[ \mu + \sigma_X |\sigma| + \frac{\sigma_X^2}{2} + (1 - \alpha) (i^* + \delta) \right] \Delta_\sigma \Delta_z + v^2 \Delta_z^2 + \sigma_X^2 \Delta_\sigma^2 - \sigma_X v |\rho| \Delta_\sigma \Delta_z \]

and
\[ \bar{\beta} = \beta - b - (1 - \gamma)(i^* - \delta) \]

The transition probabilities are positive if
\[ \frac{v \Delta_z}{|\rho| \sigma_X} > \Delta_\sigma > \frac{v |\rho| \Delta_z}{\sigma_X} . \]

Plugging in the approximations into the HJB equation and using the definitions above, we get:
\[ V(z, \sigma) \left[ 1 + \frac{\Delta_z^2 \Delta_\sigma^2}{Q^\Delta \bar{\beta}} \right] = (e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} \frac{\Delta_z^2 \Delta_\sigma^2}{Q^\Delta} + \sum_{s \in S} V(s) p((z, \sigma) \to s) \]

where \( S \) is the set of nearest neighbors of \((z, \sigma)\). The discretized Bellman equation is:
\[ V(z, \sigma) = \min_i \left\{ (e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} \frac{\Delta t}{1 + \bar{\beta} \Delta t} + \frac{1}{1 + \bar{\beta} \Delta t} E_t [V(z', \sigma')|(z, \sigma)] \right\} \]

with
\[ \Delta t = \frac{\Delta_\sigma \Delta_z^2}{Q^\Delta} \]

and
\[ \bar{\beta} = [\beta - b - (1 - \gamma)(i^* - \delta)]. \]
We solve the Bellman equation using policy iteration.

2.6.5 Steady state distribution

After solving the HJB equation, the steady state is given by the eigenvector of the transpose of the transition probability matrix associated with the eigenvalue of 1, and then normalized so that it integrates to unity.
Table 2.1: Summary Statistics
This table reports the summary statistics of data and model output. $I_t$ is the real nonresidential fixed private domestic investment, $Y_t$ is the real business gross domestic product, $K_t$ is the real capital stock, $\Delta y_t$ is the growth of $Y_t$, $\Delta i_t$ is the growth of $Y_t$, $r_t$ is the market return (value-weighted CRSP index), $r_f^t$ is the risk-free rate (3-month T-Bill yield), and $\text{vol}_t$ is the time series for the natural log of the realized market volatility. Lowercase letters represent natural logarithms. The sample is quarterly and spans the period from 1947 to 2009. Model is simulated 2,000 times using parameters reported in Table 2.8 and the averages across simulations are reported.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[i_t - k_t]$</td>
<td>-1.3965</td>
<td>-2.6562</td>
</tr>
<tr>
<td>std[$i_t - k_t$]</td>
<td>0.1365</td>
<td>0.0798</td>
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<tr>
<td>$E[y_t - k_t]$</td>
<td>-0.4871</td>
<td>-7.3794</td>
</tr>
<tr>
<td>std[$y_t - k_t$]</td>
<td>0.0677</td>
<td>0.0758</td>
</tr>
<tr>
<td>$E[r_t - r_f^t]$</td>
<td>0.0145</td>
<td>0.0123</td>
</tr>
<tr>
<td>std[$r_t - r_f^t$]</td>
<td>0.1106</td>
<td>0.06831</td>
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<tr>
<td>$E[\text{vol}_t]$</td>
<td>-2.1753</td>
<td>-2.9239</td>
</tr>
<tr>
<td>std[$\text{vol}_t$]</td>
<td>0.4255</td>
<td>0.1736</td>
</tr>
<tr>
<td>$E[r_f^t]$</td>
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<td>0.0061</td>
</tr>
<tr>
<td>std[$r_f^t$]</td>
<td>0.0088</td>
<td>0.0078</td>
</tr>
<tr>
<td>$E[\Delta y_t]$</td>
<td>0.0083</td>
<td>0.0054</td>
</tr>
<tr>
<td>std[$\Delta y_t$]</td>
<td>0.0124</td>
<td>0.0150</td>
</tr>
<tr>
<td>$E[\Delta i_t]$</td>
<td>0.0099</td>
<td>0.0052</td>
</tr>
<tr>
<td>std[$\Delta i_t$]</td>
<td>0.0276</td>
<td>0.0398</td>
</tr>
</tbody>
</table>
Table 2.2: Predictability of Excess Stock Returns: Single Period

This table reports the results of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill, $r_{t+h} - r_{f,t+h}$, on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Regressions are performed for values of the lag $h$ between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, $t$-statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted $R^2$ in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 2.1 for the definition of all relevant variables.

| Variables $r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \epsilon_{t,t+h}$ | Horizon h (in quarters) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $a_1$           | -0.50          | -0.42          | -0.28          | -0.22          | -0.15          | -0.16          | -0.17          | -0.14          | -0.16          | -0.13          | -0.11          | -0.10          | -0.13          | -0.20          | -0.39          | -0.23          | -0.07          | -0.09          | -0.02          | -0.11          | -0.02          | -0.07          | -0.01          | -0.01          | -0.00          | 0.01          | 0.05          | 0.02          | 0.01          | -0.04          |
| $t$-stat        | (-2.42)        | (-1.94)        | (-1.27)        | (-1.02)        | (-0.68)        | (-0.72)        | (-0.54)        | (-0.78)        | (-0.76)        | (-0.62)        | (-0.69)        | (-0.56)        | (-0.47)        | (-0.44)        | (-0.61)        | (-0.96)        | (-1.55)        | (-0.93)        | (-0.28)        | (-0.37)        | (-0.07)        | (-0.41)        | (-0.06)        | (-0.25)        | (-0.05)        | (-0.02)        | (-0.04)        | (0.22)         | (0.09)         | (0.04)         | (-0.18)        |
| $R^2$           | [0.02]         | [0.02]         | [0.01]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.00]         | [0.03]         | [0.02]         | [0.01]         | [0.01]         | [0.00]         | [0.01]         | [0.01]         | [0.00]         | [0.01]         | [0.01]         | [0.00]         | [0.01]         | [0.01]         | [0.01]         | [0.01]         | [0.01]         |
Table 2.3: Predictability of Excess Stock Returns: Multiple Periods

This table reports the results of multi-period regressions of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Multi-period excess returns are defined as a sum of single-period excess returns, $\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s})$. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones, Neuberger, and Nolte (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, t-statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted $R^2$ in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 2.1 for the definition of all relevant variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s})$</td>
<td>$a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.50</td>
<td>-0.92</td>
<td>-1.17</td>
<td>-1.35</td>
<td>-1.51</td>
<td>-1.66</td>
<td>-1.75</td>
<td>-1.89</td>
<td>-1.99</td>
<td>-2.06</td>
<td>-2.14</td>
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<td>-2.26</td>
<td>-2.30</td>
<td>-2.39</td>
<td>-2.53</td>
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<td>(-2.19)</td>
<td>(-1.87)</td>
<td>(-1.64)</td>
<td>(-1.47)</td>
<td>(-1.38)</td>
<td>(-1.29)</td>
<td>(-1.24)</td>
<td>(-1.19)</td>
<td>(-1.12)</td>
<td>(-1.07)</td>
<td>(-1.02)</td>
<td>(-0.96)</td>
<td>(-0.92)</td>
<td>(-0.89)</td>
<td>(-0.90)</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
<td>[0.04]</td>
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<td>[0.05]</td>
</tr>
<tr>
<td>$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s})$</td>
<td>$a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$</td>
<td></td>
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</tr>
<tr>
<td>$a_1$</td>
<td>-0.39</td>
<td>-0.62</td>
<td>-0.66</td>
<td>-0.72</td>
<td>-0.77</td>
<td>-0.87</td>
<td>-0.86</td>
<td>-0.86</td>
<td>-0.86</td>
<td>-0.82</td>
<td>-0.78</td>
<td>-0.74</td>
<td>-0.66</td>
<td>-0.62</td>
<td>-0.60</td>
<td>-0.63</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.55)</td>
<td>(-1.25)</td>
<td>(-0.91)</td>
<td>(-0.75)</td>
<td>(-0.64)</td>
<td>(-0.61)</td>
<td>(-0.52)</td>
<td>(-0.48)</td>
<td>(-0.41)</td>
<td>(-0.36)</td>
<td>(-0.31)</td>
<td>(-0.27)</td>
<td>(-0.23)</td>
<td>(-0.20)</td>
<td>(-0.18)</td>
<td>(-0.19)</td>
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<td>-2.79</td>
<td>-2.98</td>
<td>-3.36</td>
<td>-4.29</td>
<td>-4.72</td>
<td>-5.52</td>
<td>-5.66</td>
<td>-6.16</td>
<td>-6.55</td>
<td>-7.00</td>
<td>-7.51</td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>(-0.74)</td>
<td>(-1.04)</td>
<td>(-1.14)</td>
<td>(-1.04)</td>
<td>(-0.98)</td>
<td>(-0.87)</td>
<td>(-0.85)</td>
<td>(-0.84)</td>
<td>(-0.87)</td>
<td>(-0.88)</td>
<td>(-0.91)</td>
<td>(-0.92)</td>
<td>(-0.95)</td>
<td>(-0.96)</td>
<td>(-0.98)</td>
<td>(-1.01)</td>
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<tr>
<td>$R^2$</td>
<td>[0.03]</td>
<td>[0.04]</td>
<td>[0.05]</td>
<td>[0.05]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.06]</td>
<td>[0.07]</td>
<td>[0.07]</td>
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<td>[0.08]</td>
<td>[0.09]</td>
<td>[0.09]</td>
<td>[0.10]</td>
</tr>
</tbody>
</table>
Table 2.4: Predictability of Volatility of Excess Stock Returns: Single Period

This table reports the results of regressions of log volatility of returns on CRSP value-weighted index, \( \text{vol}_{t+h} \), on the log of investment rate, \( i_t - k_t \), the log of profitability, \( y_t - k_t \), and lagged log volatility \( \text{vol}_{t-1} \). Volatility is calculated using intra-quarter daily returns and has a time trend removed. Regressions are performed for values of the lag \( h \) between 1 and 16. All series have been de-trended. The table reports OLS estimates of the regressors and \( t \)-statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags. Adjusted \( R^2 \) is shown in square brackets. The sample is quarterly and spans the period 1947 to 2009. See the caption to Table 2.1 for the definition of all relevant variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
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<tbody>
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<td>( a_1 )</td>
<td>1.12</td>
<td>1.26</td>
<td>1.33</td>
<td>1.50</td>
<td>1.57</td>
<td>1.65</td>
<td>1.62</td>
<td>1.60</td>
<td>1.54</td>
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<td>(4.53)</td>
<td>(5.14)</td>
<td>(5.52)</td>
<td>(5.95)</td>
<td>(5.91)</td>
<td>(5.75)</td>
<td>(5.51)</td>
<td>(4.83)</td>
<td>(4.14)</td>
<td>(3.28)</td>
<td>(2.67)</td>
<td>(2.04)</td>
<td>(1.58)</td>
<td>(1.23)</td>
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<tr>
<td>( R^2 )</td>
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<td>0.15</td>
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<td>0.17</td>
<td>0.14</td>
<td>0.11</td>
<td>0.08</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Univariate regression: \( \text{vol}_{t+h} = a_0 + a_1 (i_t - k_t) + \varepsilon_{t,t+h} \)

| \( a_1 \)  | 2.02 | 2.08 | 2.07 | 2.19 | 2.16 | 2.17 | 1.97 | 1.81 | 1.61 | 1.47 | 1.28 | 1.08 | 0.92 | 0.67 | 0.48 | 0.35 |
| \( t \)-stat | (6.05) | (6.02) | (5.76) | (6.22) | (6.37) | (6.76) | (6.17) | (5.45) | (4.87) | (4.13) | (3.33) | (2.57) | (2.00) | (1.41) | (1.03) | (0.75) |
| \( a_2 \)  | -3.38 | -3.10 | -2.78 | -2.63 | -2.21 | -1.94 | -1.30 | -0.80 | -0.27 | -0.09 | 0.19 | 0.27 | 0.40 | 0.64 | 0.71 | 0.70 |
| \( t \)-stat | (-3.96) | (-3.67) | (-3.40) | (-3.27) | (-2.84) | (-2.52) | (-1.75) | (-1.08) | (-0.36) | (-0.11) | (0.24) | (0.33) | (0.50) | (0.81) | (0.92) | (0.88) |
| \( R^2 \)   | 0.22 | 0.23 | 0.23 | 0.26 | 0.26 | 0.26 | 0.24 | 0.22 | 0.20 | 0.17 | 0.14 | 0.11 | 0.09 | 0.06 | 0.04 | 0.03 |

Multivariate regression I: \( \text{vol}_{t+h} = a_0 + a_1 (i_t - k_t) + a_2 (y_t - k_t) + \varepsilon_{t,t+h+1} \)

| \( a_1 \)  | 1.09 | 1.47 | 1.69 | 1.98 | 1.99 | 2.07 | 2.09 | 2.12 | 2.13 | 1.90 | 1.64 | 1.44 | 1.36 | 1.11 | 0.77 | 0.72 |
| \( t \)-stat | (5.01) | (5.08) | (5.15) | (6.00) | (5.82) | (6.26) | (5.65) | (5.38) | (5.43) | (5.53) | (3.90) | (3.41) | (2.86) | (2.25) | (1.66) | (1.52) |
| \( a_2 \)  | -1.55 | -1.91 | -2.04 | -2.23 | -1.88 | -1.76 | -1.53 | -1.41 | -0.89 | -0.92 | -0.50 | -0.42 | -0.42 | -0.17 | 0.19 | 0.04 |
| \( t \)-stat | (-2.96) | (-2.74) | (-2.65) | (-2.77) | (-2.38) | (-2.25) | (-1.93) | (-1.75) | (-1.12) | (-1.26) | (-0.64) | (-0.55) | (-0.51) | (-0.20) | (0.25) | (0.04) |
| \( a_3 \)  | 0.48 | 0.31 | 0.19 | 0.11 | 0.09 | 0.05 | -0.06 | -0.16 | -0.17 | -0.23 | -0.19 | -0.23 | -0.23 | -0.15 | -0.19 |
| \( t \)-stat | (8.80) | (4.30) | (2.68) | (1.58) | (1.37) | (0.80) | (-0.78) | (-2.02) | (-2.24) | (-3.05) | (-2.07) | (-2.19) | (-2.53) | (-2.31) | (-1.64) | (-1.82) |
| \( R^2 \)   | 0.40 | 0.31 | 0.26 | 0.26 | 0.26 | 0.27 | 0.24 | 0.24 | 0.22 | 0.21 | 0.17 | 0.13 | 0.12 | 0.10 | 0.06 | 0.05 |

Multivariate regression II: \( \text{vol}_{t+h} = a_0 + a_1 (i_t - k_t) + a_2 (y_t - k_t) + a_3 \text{vol}_{t-1} + \varepsilon_{t,t+h+1} \)
Table 2.5: Predictability of Volatility of Excess Stock Returns: Multiple Periods

This table reports the results of multi-period regressions of log volatility of returns on CRSP value-weighted index, \( \log \text{volt+} \), on the log of investment rate \( i_t - k_t \), the log of profitability \( y_t - k_t \), and lagged log volatility \( \log \text{volt-} \). Volatility is calculated using intra-quarter daily excess returns and has a time trend removed. Multi-period log volatility defined as a sum of single-period log volatility, \( \sum_{s=1}^{h} \log \text{volt+s} \). Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, \( t \)-statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted \( R^2 \) in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 2.1 for the definition of all relevant variables.

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<td>18.14</td>
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<td>(4.42)</td>
<td>(4.74)</td>
<td>(5.04)</td>
<td>(5.32)</td>
<td>(5.55)</td>
<td>(5.75)</td>
<td>(5.93)</td>
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<td>(6.08)</td>
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Univariate regression: \( \sum_{s=1}^{h} \log \text{volt+s} = a_0 + a_1(i_t - k_t) + \epsilon_{t,t+h} \)

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<td>[0.50]</td>
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Multivariate regression I: \( \sum_{s=1}^{h} \log \text{volt+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \epsilon_{t,t+h} \)

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<td>20.87</td>
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<td>(-3.35)</td>
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<td>(-2.72)</td>
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<td>[0.48]</td>
<td>[0.49]</td>
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<td>[0.49]</td>
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<td>[0.47]</td>
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</tr>
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</table>

Multivariate regression II: \( \sum_{s=1}^{h} \log \text{volt+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3\log \text{volt-1} + \epsilon_{t,t+h} \)

<table>
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<tr>
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<td>(5.38)</td>
<td>(4.67)</td>
<td>(3.73)</td>
<td>(2.60)</td>
<td>(1.77)</td>
<td>(1.03)</td>
<td>(0.71)</td>
<td>(0.36)</td>
<td>(0.09)</td>
<td>(-0.22)</td>
<td>(-0.41)</td>
<td>(-0.62)</td>
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<td>[0.44]</td>
<td>[0.45]</td>
<td>[0.46]</td>
<td>[0.48]</td>
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<td>[0.49]</td>
<td>[0.48]</td>
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</tbody>
</table>
Table 2.6: Predictability of S&P 500 Earnings Growth Volatility

This table reports results of regressions of the absolute value of demeaned log growth rate of real one-period S&P 500 earnings, \( |e_{t+h} - e_{t+h-1}| \), on the log of investment rate, \( i_t - k_t \), and the log of profitability, \( y_t - k_t \). Regressions are performed for values of the lag \( h \) between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, Newey and West (1987) corrected t-statistics (in parentheses), and adjusted \( R^2 \) in square brackets. The sample is quarterly and spans the period from 1962 to 2007. See the caption to Table 2.1 for the definition of all other relevant variables.

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<td>-0.004</td>
<td>0.001</td>
<td>0.003</td>
<td>0.016</td>
<td>0.029</td>
<td>0.035</td>
<td>0.039</td>
<td>0.054</td>
<td>0.057</td>
<td>0.057</td>
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<td>(-1.089)</td>
<td>(-0.467)</td>
<td>(-0.302)</td>
<td>(-0.079)</td>
<td>(0.015)</td>
<td>(0.065)</td>
<td>(0.318)</td>
<td>(0.526)</td>
<td>(0.571)</td>
<td>(0.624)</td>
<td>(0.808)</td>
<td>(0.954)</td>
<td>(1.081)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>[0.094]</td>
<td>[0.058]</td>
<td>[0.034]</td>
<td>[0.016]</td>
<td>[0.004]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.002]</td>
<td>[0.006]</td>
<td>[0.008]</td>
<td>[0.010]</td>
<td>[0.019]</td>
<td>[0.022]</td>
<td>[0.021]</td>
</tr>
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</table>

Univariate regression: \( |e_{t+h} - e_{t+h-1}| = a_0 + a_1(i_t - k_t) + \epsilon_{t+h} \)

| a2                      | -0.180 | -0.047 | -0.017 | 0.013 | 0.019 | 0.031 | 0.036 | 0.037 | 0.053 | 0.069 | 0.076 | 0.081 | 0.099 | 0.101 | 0.097 |
| t-stat                  | (-3.696) | (-2.162) | (-1.030) | (-0.320) | (0.227) | (0.339) | (0.562) | (0.666) | (0.680) | (0.974) | (1.150) | (1.133) | (1.156) | (1.306) | (1.500) | (1.795) |
| \( R^2 \)               | [0.090] | [0.057] | [0.037] | [0.024] | [0.016] | [0.011] | [0.010] | [0.010] | [0.014] | [0.020] | [0.024] | [0.027] | [0.040] | [0.041] | [0.036] |

Multivariate regression: \( |e_{t+h} - e_{t+h-1}| = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \epsilon_{t+h} \)
Table 2.7: Predictability of Dividend Growth Volatility

This table reports results of regressions of the absolute value of demeaned log growth rate of real one-period dividends of the CRSP value-weighted index, $|d_{t+h} - d_{t+h-1}|$, on the log of investment rate, $i_t - k_t$, and the log of profitability, $y_t - k_t$. Regressions are performed for values of the lag $h$ between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, $t$-statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted $R^2$ in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 2.1 for the definition of all other relevant variables.

<table>
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<tr>
<td>$</td>
<td>d_{t+h} - d_{t+h-1}</td>
<td>= a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$</td>
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<td>$</td>
<td>d_{t+h} - d_{t+h-1}</td>
<td>= a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$</td>
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Table 2.8: Calibration Parameters
This table reports parameters used to calibrate the model.

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<td>Preference Shock $\xi_t$: Volatility</td>
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<td>Productivity Shock: Variance</td>
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<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Adjustment Costs Scale Parameter</td>
<td>$a$</td>
</tr>
<tr>
<td>Adjustment Costs Elasticity to Investment Rate</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Correlation Between $dW_t$ and $dW_{ot}$</td>
<td>$\rho$</td>
</tr>
</tbody>
</table>
Table 2.9: Predictability of Volatility of One Period Excess Stock Returns: Model Simulation

This table reports the results of regressions of log volatility of returns on CRSP value-weighted index, \( \text{vol}_{t+h} \), on the log of investment rate, \( i_t - k_t \), the log of profitability, \( y_t - k_t \), and lagged log volatility \( \text{vol}_{t-1} \). Volatility is calculated using intra-quarter daily returns and has a time trend removed. Regressions are performed for values of the lag \( h \) between 1 and 16. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 2.8 and the averages across simulations are reported. Volatility series has been adjusted by removing the time trend. The table reports OLS estimates of the regressors and Newey and West (1987) corrected \( t \)-statistics (in parentheses). Adjusted \( R^2 \) is shown in square brackets.

<table>
<thead>
<tr>
<th>Horizon h (in quarters)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( a_1 )</td>
<td>0.837</td>
<td>0.714</td>
<td>0.599</td>
<td>0.450</td>
<td>0.429</td>
<td>0.394</td>
<td>0.282</td>
<td>0.287</td>
<td>0.224</td>
<td>0.180</td>
<td>0.162</td>
<td>0.095</td>
<td>0.107</td>
<td>0.068</td>
<td>0.080</td>
<td>-0.012</td>
</tr>
<tr>
<td>( t )-stat</td>
<td>(6.655)</td>
<td>(5.580)</td>
<td>(4.231)</td>
<td>(2.727)</td>
<td>(2.694)</td>
<td>(2.153)</td>
<td>(1.538)</td>
<td>(1.520)</td>
<td>(1.135)</td>
<td>(0.872)</td>
<td>(0.827)</td>
<td>(0.484)</td>
<td>(0.522)</td>
<td>(0.323)</td>
<td>(0.374)</td>
<td>(-0.068)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>[0.121]</td>
<td>[0.088]</td>
<td>[0.062]</td>
<td>[0.035]</td>
<td>[0.031]</td>
<td>[0.026]</td>
<td>[0.014]</td>
<td>[0.014]</td>
<td>[0.009]</td>
<td>[0.006]</td>
<td>[0.005]</td>
<td>[0.002]</td>
<td>[0.002]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

Univariate regression: \( \text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h} \)

Multivariate regression I: \( \text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1} \)
Table 2.10: Predictability of Volatility of Multi-Period Excess Stock Returns: Model Simulations

This table reports the model results of multi-period regressions of log volatility of returns on CRSP value-weighted index, $\text{vol}_{t+h}$, on the log of investment rate $i_t - k_t$, the log of profitability $y_t - k_t$, and lagged log volatility $\text{vol}_{t-1}$. Volatility is calculated using intra-quarter daily excess returns and has a time trend removed. Multi-period log volatility defined as a sum of single-period log volatility, $\sum_{s=1}^h \text{vol}_{t+s}$. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 2.8 and the averages across simulations are reported. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, $t$-statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted $R^2$ in square brackets.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
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</tr>
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<tbody>
<tr>
<td>Horizon h (in quarters)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Univariate regression: $\sum_{s=1}^h \text{vol}<em>{t+s} = a_0 + a_1 (i_t - k_t) + \varepsilon</em>{t,t+h}$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.121</td>
<td>0.182</td>
<td>0.215</td>
<td>0.211</td>
<td>0.208</td>
<td>0.203</td>
<td>0.194</td>
<td>0.188</td>
<td>0.183</td>
<td>0.177</td>
<td>0.167</td>
<td>0.155</td>
<td>0.151</td>
<td>0.145</td>
<td>0.139</td>
<td>0.127</td>
</tr>
<tr>
<td>Multivariate regression I: $\sum_{s=1}^h \text{vol}<em>{t+s} = a_0 + a_1 (i_t - k_t) + a_2 (y_t - k_t) + a_3 \text{vol}</em>{t-1} + \varepsilon_{t,t+h}$</td>
<td></td>
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</tr>
<tr>
<td>$a_1$</td>
<td>0.829</td>
<td>1.549</td>
<td>2.159</td>
<td>2.636</td>
<td>3.093</td>
<td>3.482</td>
<td>3.757</td>
<td>4.042</td>
<td>4.281</td>
<td>4.491</td>
<td>4.646</td>
<td>4.736</td>
<td>4.876</td>
<td>5.002</td>
<td>5.142</td>
<td>5.156</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.053</td>
<td>0.025</td>
<td>-0.046</td>
<td>-0.165</td>
<td>-0.293</td>
<td>-0.397</td>
<td>-0.442</td>
<td>-0.448</td>
<td>-0.476</td>
<td>-0.504</td>
<td>-0.582</td>
<td>-0.670</td>
<td>-0.715</td>
<td>-0.784</td>
<td>-0.871</td>
<td>-1.024</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(0.377)</td>
<td>(0.087)</td>
<td>(-0.108)</td>
<td>(-0.280)</td>
<td>(-0.386)</td>
<td>(-0.429)</td>
<td>(-0.410)</td>
<td>(-0.361)</td>
<td>(-0.337)</td>
<td>(-0.316)</td>
<td>(-0.326)</td>
<td>(-0.341)</td>
<td>(-0.333)</td>
<td>(-0.337)</td>
<td>(-0.347)</td>
<td>(-0.383)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.117</td>
<td>0.179</td>
<td>0.212</td>
<td>0.209</td>
<td>0.207</td>
<td>0.203</td>
<td>0.193</td>
<td>0.187</td>
<td>0.182</td>
<td>0.176</td>
<td>0.166</td>
<td>0.155</td>
<td>0.150</td>
<td>0.145</td>
<td>0.140</td>
<td>0.129</td>
</tr>
</tbody>
</table>
Table 2.11: Predictability of Single Period Excess Stock Returns: Model Simulations

This table reports the model results of of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill, $r_{t+h} - r_{f,t+h}$, on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Regressions are performed for values of the lag $h$ between 1 and 16. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 2.8 and the averages across simulations are reported. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, $t$-statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted $R^2$ in square brackets.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
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<th>11</th>
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<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.256</td>
<td>-0.239</td>
<td>-0.207</td>
<td>-0.178</td>
<td>-0.148</td>
<td>-0.114</td>
<td>-0.101</td>
<td>-0.083</td>
<td>-0.067</td>
<td>-0.050</td>
<td>-0.022</td>
<td>-0.017</td>
<td>-0.033</td>
<td>-0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>(-7.407)</td>
<td>(-8.196)</td>
<td>(-6.355)</td>
<td>(-5.078)</td>
<td>(-3.857)</td>
<td>(-3.746)</td>
<td>(-3.128)</td>
<td>(-2.874)</td>
<td>(-2.166)</td>
<td>(-1.914)</td>
<td>(-1.384)</td>
<td>(-0.605)</td>
<td>(-0.464)</td>
<td>(-0.845)</td>
<td>(-0.945)</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>[0.172]</td>
<td>[0.152]</td>
<td>[0.113]</td>
<td>[0.083]</td>
<td>[0.057]</td>
<td>[0.033]</td>
<td>[0.044]</td>
<td>[0.027]</td>
<td>[0.027]</td>
<td>[0.018]</td>
<td>[0.012]</td>
<td>[0.007]</td>
<td>[0.001]</td>
<td>[0.003]</td>
<td>[0.000]</td>
<td></td>
</tr>
<tr>
<td>$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>-0.264</td>
<td>-0.250</td>
<td>-0.219</td>
<td>-0.194</td>
<td>-0.168</td>
<td>-0.137</td>
<td>-0.155</td>
<td>-0.129</td>
<td>-0.110</td>
<td>-0.095</td>
<td>-0.079</td>
<td>-0.051</td>
<td>-0.046</td>
<td>-0.062</td>
<td>-0.031</td>
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</tr>
<tr>
<td>t-stat</td>
<td>(-8.251)</td>
<td>(-9.217)</td>
<td>(-7.213)</td>
<td>(-6.683)</td>
<td>(-6.010)</td>
<td>(-5.536)</td>
<td>(-5.872)</td>
<td>(-5.049)</td>
<td>(-4.238)</td>
<td>(-4.164)</td>
<td>(-2.880)</td>
<td>(-2.062)</td>
<td>(-1.574)</td>
<td>(-2.132)</td>
<td>(-1.118)</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.055</td>
<td>0.071</td>
<td>0.081</td>
<td>0.096</td>
<td>0.113</td>
<td>0.118</td>
<td>0.129</td>
<td>0.140</td>
<td>0.136</td>
<td>0.140</td>
<td>0.144</td>
<td>0.145</td>
<td>0.146</td>
<td>0.148</td>
<td>0.150</td>
<td>0.154</td>
</tr>
<tr>
<td>$R^2$</td>
<td>[0.178]</td>
<td>[0.166]</td>
<td>[0.131]</td>
<td>[0.109]</td>
<td>[0.094]</td>
<td>[0.073]</td>
<td>[0.092]</td>
<td>[0.082]</td>
<td>[0.080]</td>
<td>[0.073]</td>
<td>[0.071]</td>
<td>[0.066]</td>
<td>[0.061]</td>
<td>[0.064]</td>
<td>[0.068]</td>
<td>[0.067]</td>
</tr>
</tbody>
</table>
Table 2.12: Predictability of Multi-Period Excess Stock Returns: Model Simulations

This table reports the model results of multi-period regressions of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill on the log of investment rate \( i_t - k_t \) and the log of profitability \( y_t - k_t \). Multi-period excess returns are defined as a sum of single-period excess returns, \( \sum_{s=1}^{h} (r_{t+s} - r_{f,t+s}) \). The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 2.8 and the averages across simulations are reported. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, \( t \)-statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted \( R^2 \) in square brackets.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Horizon h (in quarters)</th>
<th>( \sum_{s=1}^{h} (r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>-0.256</td>
<td>-0.495</td>
</tr>
<tr>
<td>( t )-stat</td>
<td>(-7.40)</td>
<td>(-9.43)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>[0.17]</td>
<td>[0.37]</td>
</tr>
</tbody>
</table>

|           | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| \( a_2 \) | -0.264 | -0.514 | -0.732 | -0.926 | -1.087 | -1.216 | -1.368 | -1.495 | -1.621 | -1.731 | -1.823 | -1.898 | -1.950 | -2.000 | -2.066 | -2.095 |
| \( t \)-stat | (-8.25) | (-11.0) | (-12.9) | (-15.3) | (-17.4) | (-18.6) | (-19.6) | (-19.2) | (-17.9) | (-17.2) | (-16.8) | (-16.2) | (-15.8) | (-15.6) | (-15.1) | (-14.8) |
| \( R^2 \) | [0.178] | [0.400] | [0.540] | [0.604] | [0.670] | [0.694] | [0.732] | [0.761] | [0.784] | [0.795] | [0.797] | [0.800] | [0.789] | [0.790] | [0.795] | [0.792] |
Figure 2-1: Steady-state distribution of the state variables $\sigma$ and $y-k$
Figure 2-2: Investment and consumption as a function of the state variables $\sigma$ and $y - k$
Figure 2-3: Model's impulse response function of the investment rate, profitability and consumption after a preference shock.
Figure 2-4: Model's impulse response function of the risk-free rate, expected stock returns and volatility of stock returns after a preference shock.
Chapter 3

Institutional Investors’ Intrinsic Trading Frequency and the Cross-Section of Stock Returns

with Sahar Parsa

3.1 Introduction

Heterogeneity among investors is a prevalent feature of financial markets. Investors differ in many dimensions such as their preferences, their types, their constraints, their information, the markets they participate in and their investment horizon. However, depending on the environment, heterogeneity may play little or no role in equilibrium asset prices. For example, in a world with complete markets, diversity in investors’ characteristics is irrelevant. In particular, all financial claims can be priced through a

\footnote{We are particularly grateful to George-Marios Angeletos, Christine Breiner, Ricardo Caballero, Victor Chernozhukov, Ryan Kabir, Leonid Kogan, Guido Lorenzoni and Pablo Querubin for their help and support.}
representative agent’s stochastic discount factor that is uniquely determined by prices and not by the underlying heterogeneity of investors. Rubinstein (1974), Constantinides (1982), Grossman and Shiller (1981), Krusell and Smith (1998) and many others provide conditions under which aggregation, or at least approximate aggregation, obtains even in the presence of heterogeneous agents and incomplete markets. Nevertheless, there are many theoretical models in which heterogeneity of investors is a key determinant of asset prices. Examples include heterogeneity of beliefs (Geneakoplos (2010), Scheinkman and Xiong (2003)), information (Allen, Morris and Shin (2006), Angeletos, Lorenzoni and Pavan (2010)) and preferences (Constantinides and Duffie (1996), Chan and Kogan (2002)).

On the empirical side, the literature has downplayed the importance of heterogeneity in investors’ characteristics as a source of information to understand stock prices. Most leading asset pricing models\(^2\) ignore heterogeneity, yet successfully match the observed patterns of a wide range of macroeconomic and financial variables. When studying the cross-section of stock returns, a standard approach\(^3\) is to use variables that are inherent to the underlying firm —such as size or book-to-market ratio— and not to the type of investor holding the stock.

This paper exploits institutional investors’ intrinsic trading frequency as a source of heterogeneity to empirically answer the following question: Do the returns of a given security differ in a systematic way when held by investors with different trading frequency? We find that the answer is yes. We show that, even after controlling for security fixed-effects, time fixed-effects, market volatility, trading volume, liquidity, momentum and exposure to the Fama-French factors, the returns of portfolios held by

\(^2\)Both consumption-based models such as Bansal and Yaron (2004)’s long-run risk model, Barro (2005) and Gabaix (2008)’s rare disasters and Campbell and Cochrane (1999)’s habit-formation, and factor models like the Capital Asset Pricing Model (CAPM).

\(^3\)Popularized by Fama and French (1993)
investors with different intrinsic trading frequency differ significantly. Moving from the first to the last quintile in the distribution of trading frequency —that is, moving from stocks held by investors who trade more frequently to those held by investors who trade less frequently— is associated with an expected gain in returns of 6 percentage points over the next year.

Our results allow us to make two contributions. First, we show that stock holders’ characteristics provide information about the cross-sectional distribution of stock returns that is not contained in firm-specific characteristics or aggregate market variables. This is an important finding because it challenges two widely used paradigms in finance: the existence of a representative agent (or in general, of aggregation) and the irrelevance of the identity of stock holders. To understand these two paradigms, consider the net-present value formula for a stock’s price:

\[ P_t = \sum_{s=t+1}^{\infty} \Lambda_s d_s. \]  

(3.1)

According to equation (3.1), if two different investors are not large enough to directly affect the aggregate discount factor \( \Lambda_t \), and do not have a controlling stake in the firm so that they can not influence the cash flow \( d_t \), then the fact that one of them owns the stock —and not the other— makes no difference in the stock’s price. In contrast, we find that stock prices do depend on at least one intrinsic characteristic of its holder, the trading frequency. Because we control for aggregate and firm-specific variables, and because we can study the subset of institutional investors that are small enough so that they can not affect the aggregate discount factor and do not hold a large enough proportion of stocks to control any firms, we provide evidence that investors’ trading horizon are not acting on prices through \( \Lambda_t \) or \( d_t \). We conclude that heterogeneity across investors is an important dimension of asset prices.
The second contribution is to introduce a new variable, the trading frequency of a stock, that helps predict the cross-sectional distribution of returns. We find that our results are a "pricing anomaly" in the sense that common indicators of systematic risks such as the Fama-French factors do not explain the spread in returns between stocks held by high and low-frequency traders.

To obtain our results, we use the Thomson-Reuters Institutional Holdings dataset to get stock positions for large US financial institutions at a quarterly frequency for the period 1980-2005. Following Parsa (2010) we construct a security-specific trading frequency index by taking the weighted average of the intrinsic trading frequencies of the institutional investors who hold the security, with weights given by the size of the position of each investor. We construct the intrinsic trading frequency of investors by using a fixed-effects model. Concretely, we measure an investor's change in his position as the absolute value of the percentage change of number of shares in a given security. We estimate a regression of institutions' turnover of securities on a time fixed effect, a security fixed effect, their interaction, and an institution fixed effect. The institution fixed effect captures the institutions' intrinsic trading frequencies by controlling for any security and market characteristics which could influence the investor's change in his position across time and across securities. In this way, changes in institutional holdings due to events like an increase in market-wide volatility or a flow of information at the security level do not in themselves affect our measure of investors' intrinsic trading frequencies.

To identify systematic differences in stocks with different trading frequencies, we form portfolios by sorting stocks based on their trading frequency on the previous year. We find that the relation between expected mean returns and trading frequency is monotonically decreasing. This pattern holds within subgroups of securities that are independently sorted on size, book-to-market, liquidity and past performance. In addi-
tion, the relationship between trading frequency and returns does not disappear when considering returns that are risk-adjusted by the Fama-French factors, two different measures of liquidity introduced by Sadka (2006) and Pastor and Stambaugh (2003), and the momentum factor of Jegadeesh and Titman (1993).

The remainder of the paper is organized as follows. Section 2 provides a brief description of the literature. Section 3 describes the data as well as the methodology. Section 4 provides the results. Section 5 concludes.

3.2 Literature Review

This paper uses the trading frequency measure developed in Parsa (2010), which is the first paper to suggest the importance of the intrinsic trading frequency to understand properties of asset prices. Parsa (2010) interprets the trading frequency as a measure for short-termism and then studies whether short-termism is associated with excess volatility and a disconnect between prices and fundamentals. In contrast, the present paper studies whether trading frequency, not necessarily interpreted as short-termism, can be used to predict the cross-section of stock returns. The focus of the present paper is on the heterogeneity of investors and stocks — on the cross-sectional aspects — rather than the evolution and relation between volatility, cash flows and fundamentals — the time-series aspects — studied in Parsa (2010). These two papers highlight the usefulness of the measure developed in Parsa (2010) to learn about different aspects of financial markets and the economy.

More generally, this paper connects and contributes to three different strands of the existing literature. First, this paper adds to the vast literature on the relationship between the institutional investors and stock prices. This literature has documented a positive, contemporaneous relation between institutional investors’ buying and stock re-
turns; Lakonishok, Shleifer and Vishny (1992), Grinblatt, Titman and Wermers (1995), Wermers (1999, 2000), Nofsinger and Sias (1999). It has also been highlighted that institutional buying is positively related to short-term expected return, where the expected returns are higher (lower) for stocks experiencing significant institutional buying (selling); see Daniel, Grinblatt, Titman and Wermers (1997), Gompers and Metrick (2001). Most of this literature considers the group of institutional investors to be a homogeneous group. In line with Parsa (2010), this paper contributes to the previous literature by considering the group of institutional investors as a heterogeneous group and by exploiting the heterogeneity among the institutional investors in order to understand stock prices. Thus, this paper contributes to a subset of the literature which explores the heterogeneity of investors. Grinblatt and Keloharju (2001) explores a dataset of the shareholdings in FSCD stocks and documents differences in the buy and sell behavior as well as the performance of different types of investors, such as households, foreign investors, financial institutions and insurance companies. Wermers (1999) focuses on the mutual fund industry and provides evidence on the “herding” behavior of mutual funds as well as their impact on stock prices. Cohen, Polk and Vuolteenaho (2002) study the difference between the trading behavior of institutional investors as opposed to individual investors in their reaction to cash flow news using a VAR-return decomposition at the firm level. In general, the approach in these studies consists of exploring a source of heterogeneity in the type of investors, i.e. mutual funds, retail investors, institutional investors, and so on. In this paper, the heterogeneity is the intrinsic investor trading behavior measured by the trading frequency fixed effect.

Second, this paper is connected to previous studies that have examined the portfolio turnover rate of institutional investors and its interaction with financial markets motivated by the effect of the investment horizon of institutional investors; see Gaspar, Massa and Matos (2005), Ke, Ramalingegowda and Yu (2006), Jin and Kogan
Khan, Kogan and Serafeim (2010), Parsa (2010), Yan and Zhang (2009). Gaspar, Massa and Matos (2005) look at the corporate controls market and show that firms with shareholders having a higher portfolio turnover are more likely to get an acquisition bid, but at a lower premium. Yan and Zhang (2009) find that the trading of institutional investors with a high portfolio turnover rate forecasts future stock returns. This paper is related and adds to the previous studies, as it uses the institution’s equity portfolio churning information. However, following Parsa (2010), it focuses on the institutions’ intrinsic trading characteristic as opposed to its equilibrium trading behavior to find evidence on the relation between the institution’s investment horizon and stock prices. We exploit the variation in the trading behavior intrinsic to the institution by using the fixed-effect trading frequency of the investors. Furthermore, in contrast to earlier studies, the main focus of this study is on the differential response of the stock prices to the interaction of the trading frequency fixed effect rather than on the effects of the demand by institutional investors on stock prices. In this manner, the study is related to Jin and Kogan (2007) as well as Parsa (2010). Jin and Kogan (2007) use the variation in the portfolio turnover rate of the mutual fund managers and its interaction with a measure of investor impatience, defined as the sensitivity of money flows into and out of the fund in response to the short-term performance of the fund. They find that mutual fund managers tend to focus on short-horizon investments due to the short horizon of their investors (and not the other way around). Their evidence suggests that this behavior may result in abnormal returns as it leads to an inflated demand of short horizon investment opportunities at the expense of longer horizon alternatives. However, Jin and Kogan (2007) differs on several points with respect to this study. Similar to Parsa (2010), the measure we construct for the institutional investors’ trading frequency is a “black box”, which captures the component of the institution’s turnover, which is explained by the institution’s intrinsic characteristics as opposed
to the market and/or characteristics of the securities in which they invest. Thus, we do not focus exclusively on one particular channel through which the higher trading frequency of the institutions may affect stock prices. Institutional investors can have different horizons for many reasons: different levels of patience (subjective discount factor), liquidity needs, administrative costs, legal restrictions, competitive pressures related to performance-based pay; see Dow and Gorton (1997), Shleifer and Vishny (1997), Bolton, Scheinkman and Xiong (2006). Instead, the measure used in this study allows us to focus on the whole set of institutional investors and the interaction of their trading frequency with stock prices, as the only information required is the holdings of the investors. Similar to the findings in Jin and Kogan (2007), we provide evidence that the institution’s trading frequency matters for the behavior of stock prices. Finally, this paper complements Parsa (2010), which focuses on the source of the volatility in stock prices between its cash flow and discount factor component as a function of the trading frequency index. Parsa (2010) highlights that the movements of the prices of the securities held by investors trading more frequently is traced back by the long run cash flow of the securities. In line with the results in Parsa (2010), we demonstrate that the portfolio of the securities held by investors trading more frequently is closer to their risk adjusted return.

Finally, this paper connects to the literature on the cross sectional behavior of stock returns. This literature has documented a number of empirical patterns unsupported by a standard Capital Asset Pricing Model.\footnote{The Capital Asset Pricing Model, introduced by Sharpe (1964), Lintner (1965), Mossin (1966), Treynor (1961), implies that the expected stock returns are determined by their level of beta risk through a positive and linear relation.} The firm size, the book-to-market ratio (Basu (1983), Fama and French (1993)), the firm’s prior performance (Jegadeesh and Titman (1993)) and the liquidity (Pastor and Stambaugh (2003), Sadka (2006)) have each been established as an important dimension in order to understand stock prices. This paper
contributes to the previous literature as it underlines a new variable that brings forth information about stock prices, the trading frequency index. However, in contrast to previous work, the role of the trading frequency index in understanding stock prices suggests a new way of looking at asset pricing as it exploits the heterogeneity of the investors characteristics. Not only do we show that the cross-sectional return of the trading frequency portfolio is not explained by their respective market risk or the usual variables (Fama-French factor, liquidity factor, momentum factor), but the dimension of interest is related to a characteristic of the securities, which is embedded in their ownership.

3.3 Data Description and Methodology

In order to study the relationship between the investors’ trading frequency and the cross-section of stock returns, (i) We construct an investor-specific measure of the intrinsic frequency of trading; then (ii) we construct a security-specific measure of the composition of the intrinsic trading frequency of the investors holding the security at a given moment in time. Finally, (iii) we use the security level measure constructed in (ii) to study the relationship between the aforementioned security-specific characteristic and the cross-section of stock returns. In what follows, we begin with a brief description of the different data sources. We then describe, step by step, each of the three former points as well as the results on the relationship between the investors’ trading frequency and the cross-section stock returns.

3.3.1 Data Description

The information used in this study comes mainly from three sources: The Thomson Reuters Ownership Data, the Fama-French factors and the Center for Research in
Security Prices (CRSP). In addition, the one-month Treasury Bill Rate at monthly frequency gives the risk-free interest rate from Ibbotson Associates.

In order to study the institutional investors’ trading frequencies, we use information about the quarterly equity holdings of all the institutions provided by the Thomson Reuters Ownership dataset. The dataset results from the 1978 amendment to the Securities and Exchange Act of 1934 which requires all institutions with greater than $100 million worth of securities under discretionary management to disclose their holdings on all their common-stock positions more than 10,000 shares or $200,000 on the SEC’s form 13F. The institutions included are divided into 5 categories: Banks, Insurance Companies, Investment Companies and Their Managers (e.g. Mutual Funds), Investment Advisors, which includes the large brokerage firms, and all Others (Pension Funds, University Endowments, Foundations). It reports a total of 4382 managers. The data coverage increased in both the securities’ and managers’ dimensions from a total of 573 managers and 4451 securities in 1980 to 2617 managers and 13125 securities in 2005. The institutional investors represented initially 16% of the market they invested in ($954 million) in 1980 but this number increased to about 44% ($17,500 million) in 2005.

The Fama-French and momentum factors are taken from Kenneth French’s website at Dartmouth. The Sadka liquidity measures are described in Sadka (2006). The measure captures non-traded, market-wide, undiversifiable liquidity risk. Finally, the Pastor and Stambaugh (2003) liquidity factor is based on the turnover of the securities.

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5 The dataset was previously known as the CDA/Spectrum 34 database. The institutions in the sample are also referred to as the 13F institutions in reference to the form they are required to file on a quarterly basis.

6 Some of this growth is due to an increase in the value of the equity market throughout the sample period, which forced more institutions to file the 13-F forms, as the rising market pushed their portfolios across the nominal threshold level of $100 million. For more details about the dataset, see Gompers and Metrick (2001).

7 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
The monthly market information—i.e. return, price, shares outstanding—about each security is taken from the Center for Research in Security Prices (CRSP). The set of securities included corresponds to the intersection of our two main data sources, i.e. the securities that belong to the portfolio of the 13-F financial institutions and the market information available in the CRSP. We restrict our attention to securities traded in the NYSE, the AMEX and the NASDAQ, provided they are held by more than 25 institutions, or that the institutions hold at least 10% of the shares outstanding. Our sample has 12455 securities represented and a total of 288760 data points.

3.3.2 Methodology

After briefly introducing the dataset used, the remainder of this section describes each step of the methodology. We start with the institution-specific trading frequency measure. Then we construct the security-specific trading frequency measure as the composition of the trading frequency of the institutions holding the security. Finally, we explain the methodology used to study the relationship between the security-specific measure and the cross section of stock returns. The trading frequency measures closely follow Parsa (2011) where more detailed information about the respective measures can be found.\footnote{More details are provided in the appendix found at http://econ-www.mit.edu/grad/sparsa/research.}

**Institutional Investors Intrinsic Trading Frequency**

Define \( s_{ijt} \) as the number of shares institution \( i \) is holding in security \( j \) at quarter \( t \). We capture the trading frequency of institution \( i \) in each security \( j \) at quarter \( t \) as the absolute value of the percentage change in the position of institution \( i \) in security \( j \) at
quarter $t$: \(^9\)

\[
y_{ijt} = \text{abs} \left( \frac{s_{ijt} - s_{ijt-1}}{1/2(s_{ijt} + s_{ijt-1})} \right).
\] (3.2)

If an institution $i$ is holding the same number of securities at quarter $t$ and $t-1$, then $y_{ijt} = 0$. If on average $y_{ijt}$ is bigger for institution $i$ than institution $i'$, then the institution $i$ is rebalancing its portfolio more frequently than institution $i'$ during a given period of time.

In order to construct a measure that captures an investor's idiosyncratic tendency to change his or her position, once any security or market effects have been partialled out, we exploit $y_{ijt}$'s three dimensions in a three-way, fixed-effect model. In particular, we estimate by ordinary least squares, for each year $T=1980,...,2005$ a regression of the form:

\[
y_{ijt}^T = a_t^T + h_i^T + g_{jt}^T + \beta X_{ijt}^T + \epsilon_{ijt}^T
\] (3.3)

where $y_{ijt}^T$ is the absolute value of the change in the holdings of institution $i$ in security $j$ in quarter $t$ of year $T$, $h_i^T$ is the institution fixed effect; $g_{jt}^T$ is the time-security interaction fixed effect and $X_{ijt}^T$ controls for the size of the portfolio of investor $i$ as well as the size of each security in the portfolio of investor $i$.\(^{10}\) The estimates of $h_i^T$ in equation (3.3) provide an annual measure of the investor's trading frequency that does not confound any security or time effects. The two latter effects are fully absorbed by the term $g_{jt}^T$.

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\(^9\)We are using in the denominator the average number of shares in quarter $t$ and quarter $t-1$ instead of the number of shares in quarter $t-1$. The main reason is to keep $y_{ijt}$ from being forced to be a missing value when the number of shares moves from 0 to a positive number. However, notice that as the number of shares increases from 0 to a positive number $y_{ijt}$ will be equal to 2. Hence, part of the information is clearly missing as a change of an institution's position is treated differently whether it was holding a positive number or 0 at $t-1$.

\(^{10}\)Concretely, the fixed effect measures are computed with respect to the following normalization: $\sum_t \sum_j \delta_{jht}g_{jt} = \sum_i \delta_{jht}h_i = 0$ where $\delta_{jht} = 1$ if $y_{ijt}$ is non missing and 0 otherwise.
We allow the measure of the institution's trading frequency \((h_i^T)\) to change annually in order to capture changes across time that could be driven by investor characteristics, such as the investment horizon associated with changes in its corporate governance, its objective, its CEO, the regulation or its preferences. An investor's intrinsic trading frequency is defined by the fixed effect \(h_i^T\) in regression (3.3). A larger institution's fixed effect \(h_i^T\) is associated with investors who change their positions more often and hence have a higher idiosyncratic trading frequency. Ultimately, \(h_i^T\) provides a measure comparable to a portfolio turnover rate. However, by exploiting the three dimensions of the data (institutions, security and quarter), it combines the changes to an institution's security holdings in one churning rate, which summarizes only the trading behavior that results from the institution.

### 3.3.3 Security Specific Trading Frequency

For each 13-F institution, the Thomson Reuters ownership data reports the securities the investor is holding in his or her portfolio and their respective position in the securities. For each year \(T\), quarter \(t\) and security \(j\) held by a group of institutions \(I_j\), the security \(j\)'s trading frequency index at year \(T\) and end of quarter \(t\) is defined as the weighted average of the fixed effects of the institutions in \(I_j\):

\[
H_{jT} = \sum_{i \in I_j} \omega_{ijT} h_i^T
\]

where the weights are \(\omega_{ijT} = \frac{s_{ijT}}{\sum_{i \in I_j} s_{ijT}}\), and \(s_{ijT}\) is the number of shares outstanding of security \(j\) held by institution \(i\) at year \(T\) quarter \(t\), and \(h_i^T\) is the fixed effect of institution \(i\) at year \(T\). The weight \(\omega_{ijT}\) captures the relative importance of investor \(i\) for security \(j\) at year \(T\) and quarter \(t\), in terms of the number of shares investor \(i\) holds relative to the total number of shares the group of institutional investors is holding.
This implies that the trading frequency of an investor holding 90% of the shares of a security should have a greater effect than the trading frequency of an investor holding only 10% of the shares of a security. The security’s trading frequency index will give more weight to the former investor’s fixed effect than to that of the latter.

$H_{jT}$ maps the institutional investors’ trading frequency, $h^T_i$, to the security. $H_{jT}$ is interpreted as the average trading frequency of the population of institutional investors holding the security $j$ at year $T$. A security $j$ will have a high trading frequency index if, on average, the institutional investors holding the security are characterized by a short investment horizon, proxied by a large $h$. Overall, the institutions are weighted by their relative size with respect to the institutions holding the security. As a consequence, the variation in $H$ can be traced back to one of two sources: (i) For a given pool of investors, the investors with a lower value of the fixed effect are holding a higher share of the security. In other words, the high trading frequency investors represent a higher share of the security, i.e. higher weight $\omega_{ijT}$ on the high $h_{jT}$ (the high trading frequency investors). (ii) For a given weight, the institutions holding the security have a higher institution’s trading frequency. Both sources of variation, translate into a security having higher trading frequency investors than another security or having a higher trading frequency across time.\footnote{The variation of $H_{jT}$ through time is either the result of: (i) investors selling or buying the security characterized by a different horizon, (ii) the investors experiencing a change in their characteristics to trade (which could come from a change in the CEO or a merger), or (iii) both. The variation of $H_{jT}$ across security mainly comes from different securities being held by a population of investors characterized by different horizons at a given moment in time.} Finally, it is important to note that even though the trading frequency fixed effects at the institution level are orthogonal to any security and market characteristics by construction, there is a correlation between the securities characteristics and the trading frequency index. This dependence arises from the portfolio selection of the investors, which ultimately defines the weights $\omega_{ijT}$.
3.3.4 Cross-Section of Expected Stock Return

In order to analyze the effects of trading frequency on the cross-section of expected stock return, we first sort all the securities for each time period into 5 or 10 portfolios based on their measure of trading frequency. For the 5-portfolio case, the portfolios are assembled based on the quintiles in the following way: the first portfolio is the value-weighted portfolio of the 20 percent of the stocks with the lowest trading frequency index the previous year, the second portfolio is the value-weighted portfolio of the 20 percent of the stocks with the next highest trading frequency index the past year, and so on. For the 10-portfolio case, the quintiles are simply replaced by deciles. The main exercise will consist of comparing the average excess return along the trading frequency dimension. Given that the trading frequency fixed effects use all the information for the whole year in which it was estimated, we consider only the trading frequency measure lagged by one year. This assures that we are using exclusively past information in our cross-sectional regression in order to predict the cross section of stock returns. All of the remaining sorting exercises follow the same precept so that an investor could have reproduced our study in real time.

Descriptive Statistics

Table I summarizes the descriptive statistics of the main characteristics of the securities in our sample, which consists of 12,455 securities and 288,760 data points. Table I reports the means and the standard deviations for the excess return, the size, the book-to-market ratio, the past performance, the liquidity and the trading frequency index. For each statistic, we also report a number for two groups of securities, the securities held in the previous year by the low and high trading frequency investors. Notice from the last line that the trading frequency index ranges from -0.18 (for the
low trading frequency group of securities) to 0.18 (for the high trading group of securities), while it is close to zero for the full sample, giving a relatively easy benchmark to understand the magnitude of the trading frequency measure. There is more variation within the high trading frequency group than low trading frequency group. Looking at the column of the mean, one can notice that the high trading frequency securities have lower excess returns, are substantially more liquid and are larger than the low trading group of securities. Interestingly, from the momentum line, one can observe that the securities held by the low trading group of securities also exhibit lower past performance. However, one should notice the substantial difference in the standard deviation across the two groups of securities for the liquidity as well as the size, highlighting a difference in the heterogeneity within the groups in terms of the characteristics of the securities. We will show in the next section that after we control for heterogeneity in all of these dimensions in several ways, the portfolios still show the spread in returns stemming from their different trading horizons.

3.4 Results

In this section, we explore the extent to which ownership matters in explaining differences in expected returns in the cross-section of stock by exploiting the heterogeneity in investors' trading frequency.

3.4.1 Is there a relation between trading frequency and returns?

Figure I illustrates the empirical relation between the realized return and the trading frequency index by reporting the average annualized return for the different trading
frequency portfolios. The only difference between Figure I (a) and Figure I (b) is that the number of portfolios formed increased\textsuperscript{12} from 5 to 10. Independent of the number of portfolios considered, there is a clear negative relation between the horizon and the realized return. The higher the average trading frequency of the institutional investors holding the security the previous year, the smaller the realized return this year. The spread in the realized returns is economically significant: The low-trading frequency portfolio exhibits an average annualized return of approximately 11 percentage points and the high trading frequency portfolio is exhibiting an average annualized return of approximately 5.4 percentage points.

Table II shows that the relation exists within sub-groups of different types of securities by double-sorting portfolios with respect to their size, book-to-market ratios, liquidity and past performance. The double sorting is accomplished as follows: (i) We sort all the securities into five groups based on their trading frequency. (ii) We independently sort all the securities into three groups based on each of the dimensions mentioned above. (iii) We construct fifteen different portfolios for each trading frequency and characteristic combination.

Table II shows that a strategy that consists of buying low and selling high trading frequency securities generates an annual return close to 5 percentage points. This difference is statistically significant at the 5\% level, as can be noticed from the t-statistic of the last column. The same pattern is revealed when one looks at stocks divided by any of the other characteristics considered. The difference is the smallest for the group of small securities, which is mainly driven by a higher average return for the high trading group of securities. However, in terms of the statistical significance, the relation remains relatively stable even for the small securities.

\textsuperscript{12}Our results still hold when forming 25 portfolios, although the statistical inference becomes more challenging because, especially in the double sorting, some portfolios end up having a small number of firms.
The natural next step consists of exploring these spreads and the extent to which it can be explained by the characteristics or the risk exposures of the portfolios, and not by their institutional ownership.

3.4.2 Can we explain trading frequency returns by systematic risks?

The first step in exploring the relation highlighted in Figure I is to explore the results controlling for the Fama-French factors. Figure II reports the mean, annualized excess return of the different trading frequency portfolios as a function of the mean excess return predicted by the standard Fama-French model. For each portfolio \( p \), we run the following time-series regression:

\[
R_{p,t} - r_f^t = \alpha_p + (R_{m,t} - r_f^t)\beta_p^m + \text{SMB}_t \beta_p^{SMB} + \text{HML}_t \beta_p^{HML} + \varepsilon_t
\]

\[
t = 1, \ldots, T
\]

where \( R_m - r_f \) is the excess return on a broad market portfolio, SMB (small minus big) is the difference between the return on a portfolio of small and large stocks, and HML (high minus low) is the difference between the return on portfolios of high and low book-to-market stocks, and the time variable \( t \) refers to quarters. The OLS estimates are \( \hat{\alpha}_p \) and \( \hat{\beta}_p \). Figure II plots \( E \left[ R_{p,t} - R_f^t \right] = \frac{1}{T} \sum_{t=1}^{T} \left( R_{p,t} - R_f^t \right) \) in the y-axis and \( E \left[ X_t \hat{\beta}_p \right] = \frac{1}{T} \sum_{t=1}^{T} X_t \hat{\beta}_p \) on the x-axis. Each portfolio is represented by a triangle as well as a number that denotes the quintile of the trading frequency index (increasing from 1 (low trading frequency) to 5 (high trading frequency)). Figure III summarizes the pricing error (alpha) of the different portfolios as a function of the trading frequency index. The average trading frequency within each portfolio ranges from -0.15 for the
low trading frequency group to 0.16 for the high trading frequency group.

Figure II shows a discrepancy between realized and predicted returns. This divergence is more pronounced for the low trading group of securities. Overall, the portfolio defined by the low trading group of institutions exhibits a realized return of 12 percentage points, from which approximately 9 percentage points have been accounted for by the model. Figure II suggests that the ownership matters, and it matters specifically for the low trading frequency group of securities. Figure III shows that the pricing error is a linear and monotonically decreasing function of the trading frequency index. As such, the higher the trading frequency of the institutional investors holding a security, the smaller the underlying alphas.

A more econometrically precise picture of Figures II and III is given in Table III. This table reports the characteristics of trading frequency portfolios from the lowest trading frequency portfolio to highest trading frequency portfolio divided into five value-weighted portfolios. The table reports the Fama-French factor sensitivities, i.e. the slope coefficients in the Fama-French, three-factor-model time-series regressions as well as the alphas and the $R^2$ (from the left to the right). From Table III, one can notice that overall it seems that apart from the low trading group of securities, the model seems to do a fair job from the $R^2$ point of view. However, the market risk does not help explain the difference in the return, as the coefficients of the different portfolios are roughly constant. The risk-adjusted return from the first column (alpha) shows that the portfolio that shorts the high trading frequency securities and buys the low trading frequency securities earns approximately 4 percentage points on an annual basis. The bottom line from Table III is that there is a substantial risk-adjusted average return from the trading frequency strategy that can be implemented.

Given the particular nature of our portfolios, there are two other dimensions of portfolios highlighted in the literature that could account for our results: liquidity
and momentum. More liquid securities are naturally associated with a higher trading frequency index. This high correlation is expected as investors trading more frequently might select a more liquid security. Conversely, investors trading more frequently might increase the liquidity of the securities they invest in by the activities they engage in. For these two reasons, it is necessary to control for the liquidity of these portfolio to make sure that the results are not completely driven by liquidity risk. Likewise, for the momentum, one could expect that high-trading frequency securities might be more correlated to the momentum factor as high trading frequency investors could potentially care more about the short-term price movements and engage in momentum strategies. Figures IV and V illustrate the results after accounting for the two factors. Specifically, for each portfolio, we estimate:

\begin{equation}
R_{p,t} - r_f^t = \alpha_p + (R_{m.t} - r_f^t)\beta_p^m + SMB_t\beta_p^{SMB} + HML_t\beta_p^{HML} + MOM_t\beta_p^{MOM} + LIQ_t\beta_p^{LIQ} + \varepsilon_t, \quad t = 1,\ldots,T,
\end{equation}

where in addition to the variables from (3.6), we have added the liquidity factors based on Sadka (2006) and Pastor and Stambaugh (2003) and the momentum factor. Interestingly, from Figure V one can notice that the introduction of the new factors actually increases the pricing errors. As such, the spread highlighted in Figure II is not confounding these two characteristics. As in Figure III, Figure V shows that the pricing error (alpha) decreases monotonically with the trading frequency.

Table IV summarizes the results for all the cases considered. It reports the statistical significance of the figures just discussed. It compares the estimates of the pricing error, \( \alpha \), for the regressions (3.6) and (3.8) as well as the simple CAPM model and a model controlling for the long run and short run reversal. The \( t \)-statistic is computed using a Newey-West estimator with 3 lags, which is robust to correlation of the error
terms across portfolios, within portfolios and across time. Furthermore, we report in
the column labeled GRS, the “GRS test statistic” for the hypothesis that all \( \alpha_p \) are
jointly zero. It is simply an F-test adjusted for finite samples and is F-distributed,
\( F[M, T - M - 1] \), with \( M \) and \( T - M - 1 \) degrees of freedom, where \( M \) is the number of
factors in \( X_t \). From Table IV, even though the alpha of each portfolio is not statistically
significant on its own, the null hypothesis that all the \( \alpha \) are jointly zero is rejected. Our
results show that an investor can earn on average 3.3 percent per year without being
exposed to any source of the common systematic risks considered here.

3.4.3 Can we explain trading frequency returns by a trading
frequency index?

So far, we have highlighted a relationship between trading frequency and stock returns.
We showed that the relationship cannot be accounted for by the usual factors or vari-
ables used in the literature. Can this difference be explained by a trading frequency
“factor”? In the previous section, Figures III and IV suggest a linear and monotone
negative relation between the trading frequency index and the pricing error. In other
words, the higher the trading frequency index, the closer the return from its fundamen-
tals or from the return predicted by a standard cross-sectional model.

In order to explore this further, we build a trading frequency factor as the differ-
ence between the return of the portfolio of the bottom 20% trading frequency group
of securities and the top 20% trading frequency group of securities. We then try to
explain the extent to which adding this extra factor helps us account for the pricing
error. A first answer to this exercise is summarized in Figure VI and VII. Figure VI
illustrates the relation between the realized excess return and the predicted return and
Figure VII illustrates the relationship between the pricing error and the predicted re-
turn after controlling for the trading frequency factor. In particular, for each portfolio, we estimate:

\[
R_{p,t} - r^s_t = \alpha_p + (R_{m,t} - r_{f,t})\beta_p^m + SMB_t\beta_p^{SMB} + HML_t\beta_p^{HML}
\]

\[
+ MOM_t\beta_p^{MOM} + LIQ_t\beta_p^{LIQ} + TF_t\beta_p^{TF} + \epsilon_t, t = 1,...,T.
\]

(3.9) (3.10)

where in addition to the variables from (3.8), we have added the trading frequency factor TF as defined above. Figure VI shows that the realized return aligns more naturally with the 45 degree line. The difference between the realized and the predicted excess return is by and large accounted for by the inclusion of the trading factor. This is also reported in Figure VII, which shows that the new pricing errors from a model that internalizes the trading frequency factor are smaller and do not have a systematic correlation with the trading frequency measure. Table V shows the related statistical information. On one hand, even though the \(\hat{\alpha}\)'s are smaller and they do not exhibit a specific relation with the trading frequency, one can still reject the null of all \(\hat{\alpha}\)'s being jointly zero. On the other hand, from an economic point of view, the return an investor will make exploiting the trading frequency difference is now substantially smaller after accounting for the trading frequency return. Hence, adding the trading frequency factor, even though it adds new information, provides a mixed response to the spread in returns of the different trading frequency portfolios.

3.5 Conclusion

In this paper, we show that stock returns are predicted by the intrinsic frequency of trading of its institutional holders. Moving from the first to the last quintile in the distribution of the security-specific trading frequency is associated with an expected gain
in returns of 6 percentage points over the next year. The magnitude and predictability of these returns persist or even increase when risk-adjusted by measures of systematic risks such as the Fama-French factors.

The result that stock returns depend on who holds them is at odds with two standard views in finance. The first is that a stock’s price is frictionlessly determined by the discounted sum of its dividends. If two institutional investors are not large enough to directly affect the aggregate discount factor and do not have a controlling stake in the firms in which they invest, then the fact that one of them owns the stock — and not the other — should make no difference in the stock’s return. The second standard view that is challenged by our results is that of the representative agent whose stochastic discount factor prices any given cash flow. In such an economy, the identity and heterogeneous characteristics of stockowners should provide no information about the cross-section of stock returns.

Another way to state our findings is to interpret them as a “pricing anomaly” in the sense that neither aggregate risk factors nor firm-specific characteristics can explain the spread in returns between stocks held by high and low-frequency traders. An explanation of our results will most likely be found by analyzing the “demand side” instead of the “supply side” of the market, i.e. how traders who demand stocks behave, instead of how firms who supply stocks behave.

Herein lies a limitation of our study: even though the relationship between trading horizon and stock returns is empirically strong and pervasive among different subgroups of stocks, there is no theoretical explanation for why this is the case. The apparent breakdown of the relation between stock prices and their corresponding discounted sum of dividends and the emphasis on traders’ heterogeneity suggests that behavioral explanations in the spirit of Shleifer and Vishny (1997) and Daniel, Hirshleifer and Subrahmanyam (2001), could provide potential explanations of our results. At a mini-
mum, explanations will most likely deviate from complete market, representative agent, frictionless economies.
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Table I

Descriptive statistics

The table shows the annualized mean and standard deviation of excess returns $R - R^f$, market capitalization (Size), Book-to-Market ratio (B/M), volume per number of shares outstanding (Liquidity), last quarter’s excess returns (Momentum) and Trading Frequency. Excess returns are from CRSP. Market capitalization is measured as price multiplied by the number of shares outstanding reported in CRSP. The trading frequency of a security is constructed following Parsa (2010). The first column shows the statistics for the full sample of securities, while the last two columns show the statistics for stocks in the lowest and highest quintile of the trading frequency distribution, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Low trading frequency</th>
<th>High trading frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>$R - R^f$</td>
<td>0.0310</td>
<td>0.278</td>
<td>0.0348</td>
</tr>
<tr>
<td>Size</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td>B/M</td>
<td>0.723</td>
<td>0.723</td>
<td>0.723</td>
</tr>
<tr>
<td>Liquidity</td>
<td>3.29</td>
<td>3.29</td>
<td>3.29</td>
</tr>
<tr>
<td>Momentum</td>
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<td>0.304</td>
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<td>Trading Frequency</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0212</td>
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</tr>
<tr>
<td></td>
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<tr>
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<td>0.723</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.21</td>
<td>5.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0286</td>
<td>0.310</td>
<td></td>
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<tr>
<td></td>
<td>0.171</td>
<td>0.130</td>
<td></td>
</tr>
</tbody>
</table>
Table II
Mean Returns and \( t \)-Statistics of Sorted Portfolios

The table shows annualized mean excess returns and the corresponding \( t \)-statistics of value-weighted portfolios formed by sorting on the characteristics defined in Table I. The first row is a single sort on quintiles of trading frequency for each quarter \( t \). The next rows perform a double sort by independently placing each stock into one of five trading frequency quintiles and one of three size, book-to-market, liquidity or momentum groups. Portfolios are formed by grouping stocks that belong to the intersection of two groups. The reported mean returns are the time-series averages of the annualized returns of each portfolio. The column High-Low constructs a zero-investment portfolio by buying the portfolio in the High trading frequency group and shorting the portfolio in the Low frequency group.

<table>
<thead>
<tr>
<th>Trading Frequency</th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High</th>
<th>High-Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-sort</td>
<td>0.129</td>
<td>0.112</td>
<td>0.108</td>
<td>0.0939</td>
<td>0.0789</td>
<td>-0.0505</td>
</tr>
<tr>
<td></td>
<td>[3.37]</td>
<td>[2.89]</td>
<td>[2.62]</td>
<td>[2.03]</td>
<td>[1.53]</td>
<td>[2.18]</td>
</tr>
<tr>
<td>Size</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Small</td>
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<td>0.139</td>
<td>0.105</td>
<td>0.107</td>
<td>-0.0366</td>
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<td>0.108</td>
<td>0.101</td>
<td>0.0923</td>
<td>0.0638</td>
<td>-0.0505</td>
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<tr>
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<td>[1.21]</td>
<td>[-1.94]</td>
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<tr>
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<td>0.105</td>
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<td>0.0789</td>
<td>0.0528</td>
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</tr>
<tr>
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<td>[2.52]</td>
<td>[1.85]</td>
<td>[1.07]</td>
<td>[-1.62]</td>
</tr>
<tr>
<td>Book-to-Market</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>High Growth</td>
<td>0.0843</td>
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<td>0.00407</td>
<td>-0.0454</td>
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<tr>
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<td>[-3.42]</td>
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<td>[0.0001]</td>
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<td>0.137</td>
<td>0.0639</td>
<td>0.0353</td>
<td>-0.113</td>
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<td>[3.35]</td>
<td>[2.68]</td>
<td>[1.35]</td>
<td>[2.53]</td>
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<tr>
<td></td>
<td>More illiquid</td>
<td>Medium</td>
<td>More liquid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>--------------</td>
<td>--------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High past returns</td>
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<td>0.114</td>
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<tr>
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<td>0.177</td>
<td>0.101</td>
</tr>
<tr>
<td>Low past returns</td>
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<td>0.0119</td>
<td>0.236</td>
<td>0.0972</td>
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<td>0.0648</td>
<td>0.00597</td>
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<td>0.021</td>
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<tr>
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<td></td>
<td></td>
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<td>0.0338</td>
<td>0.0618</td>
</tr>
<tr>
<td>More liquid</td>
<td></td>
<td></td>
<td></td>
<td>0.125</td>
<td>0.0838</td>
<td>0.0119</td>
</tr>
<tr>
<td>High past returns</td>
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<td></td>
<td></td>
<td>0.169</td>
<td>0.121</td>
<td>0.118</td>
</tr>
<tr>
<td>Medium</td>
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<td></td>
<td></td>
<td>0.105</td>
<td>0.077</td>
<td>0.179</td>
</tr>
<tr>
<td>Low past returns</td>
<td></td>
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<td></td>
<td>0.0119</td>
<td>0.236</td>
<td>0.0972</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td></td>
<td></td>
<td></td>
<td>0.00959</td>
<td>0.0648</td>
<td>0.00597</td>
</tr>
<tr>
<td>More illiquid</td>
<td></td>
<td></td>
<td></td>
<td>0.0225</td>
<td>0.0216</td>
<td>0.021</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td>0.0398</td>
<td>0.0338</td>
<td>0.0618</td>
</tr>
<tr>
<td>More liquid</td>
<td></td>
<td></td>
<td></td>
<td>0.125</td>
<td>0.0838</td>
<td>0.0119</td>
</tr>
</tbody>
</table>
Table III

Time-Series Regressions of Returns of Trading Frequency Portfolios on Fama-French Factors

The table shows estimates of the intercept, coefficients and $R^2$ of time series regressions of excess returns on the Fama-French factors. Each row corresponds to one of the portfolios constructed by sorting on trading frequency as explained in Table II. $t$-statistics are reported in brackets.

<table>
<thead>
<tr>
<th>Trading Frequency</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}_{MKT}$</th>
<th>$\hat{\beta}_{SMB}$</th>
<th>$\hat{\beta}_{HML}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>-0.0131</td>
<td>1.10</td>
<td>-0.312</td>
<td>0.488</td>
<td>0.926</td>
</tr>
<tr>
<td></td>
<td>[-0.86]</td>
<td>[20.9]</td>
<td>[-4.72]</td>
<td>[6.40]</td>
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</tr>
<tr>
<td>2</td>
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<td>0.24</td>
<td>0.965</td>
</tr>
<tr>
<td></td>
<td>[0.0516]</td>
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<td>[-2.89]</td>
<td>[5.30]</td>
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</tr>
<tr>
<td>3</td>
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<td>-0.0226</td>
<td>-0.0587</td>
<td>0.944</td>
</tr>
<tr>
<td></td>
<td>[1.66]</td>
<td>[30.8]</td>
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<td></td>
</tr>
<tr>
<td>4</td>
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<td>1.01</td>
<td>0.181</td>
<td>-0.103</td>
<td>0.918</td>
</tr>
<tr>
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<td>[1.83]</td>
<td>[27.6]</td>
<td>[3.92]</td>
<td>[-1.94]</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>0.0261</td>
<td>0.923</td>
<td>0.302</td>
<td>0.0758</td>
<td>0.796</td>
</tr>
<tr>
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<td>[1.59]</td>
<td>[16.3]</td>
<td>[4.24]</td>
<td>[0.921]</td>
<td></td>
</tr>
<tr>
<td>High-Low</td>
<td>0.0392</td>
<td>-0.007</td>
<td>0.0246</td>
<td>-0.0165</td>
<td>0.513</td>
</tr>
<tr>
<td></td>
<td>[1.51]</td>
<td>[-1.96]</td>
<td>[5.44]</td>
<td>[-3.17]</td>
<td></td>
</tr>
</tbody>
</table>
Table IV
Pricing Errors of Different Models
When Pricing Frequency-Sorted Portfolios

The table shows the performance of different factors when pricing 5, 10 and 25 portfolios constructed by sorting on stock’s trading frequency as described in Table II. As factors, we consider the market excess return (CAPM), the Fama-French factors (FF), Jegadeesh and Titman’s momentum (UMD), long-term return reversal (Rev) and liquidity factors of Sadka and Pastor/Stambaugh (Liq). All the reported statistics are obtained from regressions of the excess return of the 5, 10 or 25 trading frequency portfolios on the different pricing factors. Mean $|\hat{\alpha}|$ is the average across regressions of the absolute value of the estimate of the intercept in annualized percentage points. GRS is the Gibbons-Ross-Shanken test-statistic (an F-statistic adjusted for finite sample bias) of the null hypothesis that the $\alpha$ for all portfolios are jointly zero, for which we also report its $p$-value. The Mean $R^2$ is the average value of the $R^2$ across regressions.

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th>FF</th>
<th>FF+UMD+Liq</th>
<th>FF+UMD+Rev+Liq</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 portfolios</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean $</td>
<td>\hat{\alpha}</td>
<td>$</td>
<td>0.0324</td>
<td>0.015</td>
</tr>
<tr>
<td>GRS</td>
<td>15.9</td>
<td>15.5</td>
<td>14.2</td>
<td>12.1</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean $R^2$</td>
<td>0.885</td>
<td>0.910</td>
<td>0.910</td>
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</tr>
<tr>
<td>10 portfolios</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean $</td>
<td>\hat{\alpha}</td>
<td>$</td>
<td>0.0307</td>
<td>0.0161</td>
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<tr>
<td>GRS</td>
<td>12.3</td>
<td>11.4</td>
<td>8.38</td>
<td>8.1988</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean $R^2$</td>
<td>0.808</td>
<td>0.831</td>
<td>0.877</td>
<td>0.837</td>
</tr>
<tr>
<td>25 portfolios</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mean $</td>
<td>\hat{\alpha}</td>
<td>$</td>
<td>0.0342</td>
<td>0.0215</td>
</tr>
<tr>
<td>GRS</td>
<td>25</td>
<td>25.9</td>
<td>37.5</td>
<td>30.1</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Mean $R^2$</td>
<td>0.635</td>
<td>0.685</td>
<td>0.812</td>
<td>0.707</td>
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</tbody>
</table>
Table V

Time-Series Regressions of Returns of Trading Frequency Portfolios on Fama-French and a Trading Frequency Factor

The table shows estimates of the intercept, coefficients and $R^2$ of time series regressions of excess returns on the Fama-French factors and a the High-Low portfolio. Each row corresponds to one of the five portfolios constructed by sorting on trading frequency as explained in table II. $t$-statistics are reported in brackets.

<table>
<thead>
<tr>
<th>Trading Frequency</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}^{FREQ}$</th>
<th>$\hat{\beta}^{MKT}$</th>
<th>$\hat{\beta}^{SMB}$</th>
<th>$\hat{\beta}^{HML}$</th>
<th>$R^2$</th>
</tr>
</thead>
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<tr>
<td>Low</td>
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<td>1.02</td>
<td>-0.0220</td>
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<td>[13.2]</td>
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<td>[6.14]</td>
<td></td>
</tr>
<tr>
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<td>[4.34]</td>
<td></td>
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<tr>
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<td>0.0131</td>
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</tr>
<tr>
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<td>[-0.651]</td>
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</tr>
<tr>
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<td>0.0092</td>
<td>-26.3</td>
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<td>0.0194</td>
<td>0.0053</td>
<td>0.952</td>
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<td>[0.122]</td>
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<tr>
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<td>0.0054</td>
<td>-52.8</td>
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<td>0.938</td>
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<tr>
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<td>[0.591]</td>
<td>[-14.7]</td>
<td>[31.8]</td>
<td>[-0.488]</td>
<td>[6.14]</td>
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</tr>
</tbody>
</table>
Figure I.a

Figure I.b
Mean Excess Return Predicted by FF (% per year) vs. Trading Frequency (c) Figure II

(d) Figure III
Trading Frequency (f)

Figure V

(e) Figure IV

(f) Figure V
Figure VI: Mean Excess Return Predicted by FF + Freq Factor (% per year) vs. Trading Frequency (g)

Figure VII: Fitting Error (% per year) vs. Trading Frequency