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***Abstract*—Fueled by the promises of high spectral efficiency, adaptive OFDMA has attracted enormous research interests over the last decade. The significant capacity gain of adaptive OFDMA comes from fast adaptation of resource allocation in response to instantaneous channel conditions. Despite years of efforts to improve the practicality of adaptive OFDMA, such promising technique is still far from real implementation due to the prohibitively high computational complexity and excessive control overhead. This paper is an endeavor to address the problem by proposing a slow adaptation scheme, where resource allocation is adapted on a much slower time scale than the fluctuation of wireless channel fading. Specifically, the slow adaptive OFDMA is formulated into a stochastic programming problem, which adapts resource allocation according to the channel statistics within an adaptation window rather than according to instantaneous channel conditions. By tuning the length of the adaptation window, we could engineer a desirable tradeoff between spectral efficiency and computational complexity. Furthermore, the proposed scheme can be modified to accommodate inelastic traffics. The modification, referred to as "safe" slow adaptation, ensures worst-case data rates to all users. In this work, safe slow adaptation is formulated into a conic linear program, which is efficiently solved via interior-point methods. Through extensive simulations, we show that the proposed schemes drastically reduce the computational complexity and control overheads, while achieving satisfactorily high spectral efficiency and QoS provisioning as their fast-adaptation counterpart does with a much higher cost.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been identified as one of the leading candidates for supporting broadband and multimedia services in future wireless systems. The inherent multi-carrier nature of OFDM allows flexible use of adaptive resource allocation to significantly enhance system capacity and resource utilization. Since the pioneer work in [1], there has been enormous research interests to design algorithms that adaptively allocate subsets of subcarriers to different subscribers according to the instantaneous channel conditions (see [2] and [3] for a survey). Such technique, recently referred to as adaptive orthogonal frequency division multiple access (OFDMA), has been considered as a primary alternative in the developing standardization, such as IEEE802.16 WiMAX and 3GPP-LTE, and thus holds great promise in future wireless communication systems.

Despite the potential huge increase in spectral efficiency, adaptive OFDMA has not yet made it to real implementation due to a number of hurdles that need to be addressed seriously. First, existing schemes (e.g. [1]–[6]) adapt subcarrier allocation according to instantaneous channel conditions. In other words, the optimal subcarrier allocation problem has to be solved by the base station (BS) for each channel realization. Considering the fact that the wireless channel fading varies at the order of milliseconds, the on-the-fly optimization quickly becomes computationally infeasible even with a small number of users and subcarriers. Second, fast adaptive resource allocation unavoidably requires highly frequent signaling between the BS and mobile users to keep the users informed of the latest allocation decision. The overhead therefore incurred may significantly decrease the potential gain in system performance [7]. By the same token, the overhead due to frequent channel-state-information (CSI) feedback can also be overwhelming, as the instantaneous CSI is needed at the transmitter side for fast adaptive OFDMA.

To overcome the above hurdles, we propose a slow adaptive OFDMA scheme in this paper. In particular, subcarrier allocation is adapted on a slower time scale than that of channel fading variation. By doing so, both the computational complexity and signaling overhead can be considerably reduced. The important problem that remains is how to obtain a subcarrier allocation that remains optimal for a period of time before the next adaptation epoch. This paper, to the best of our knowledge, is the first attempt in the literature to address the above problems. The contributions of this paper are summarized as follows.

- We formulate slow adaptive OFDMA into a stochastic programming (SP) problem, where subcarrier allocation is optimized in response to the channel statistics within an adaptation window.
- By tuning the time scale of adaptation, the proposed scheme provides a flexible tradeoff between throughput gain and computational complexity as well as overhead. Noticeably, the proposed scheme may simultaneously

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achieve higher throughput and lower computational complexity than fast adaptive OFDMA, when the inband signaling overhead is taken into account.

- For inelastic applications, we propose a safe slow adaptive OFDMA scheme to guarantee the worst-case throughputs of users in each time window. The safe slow adaptation is formulated into a second order cone program (SOCP), which can be solved efficiently via interior point methods [8].

The rest of the paper is organized as follows. System model is described in Section II. In Section III, slow adaptive OFDMA is formulated into a SP. We show how to calculate the statistic mean of data rate within an adaptation window in the same section. The safe slow adaptive OFDMA scheme is presented in Section IV. In Section V, the performance of the proposed schemes are investigated through extensive simulations. Finally, the paper is concluded in Section VI.

II. System Model

This paper considers a multiuser OFDM system with K users and N subcarriers. In traditional fast adaptive OFDMA systems, BS adapts subcarrier-to-user allocation according to instantaneous channel conditions to maximize the system throughput. As shown in Fig. 1a, subcarrier allocation is performed in each time slot, where a slot is the maximum time unit within which the channel fading does not change. Let $r_{n,k}^{(t)} = W \log_2\left(1 + \frac{p_t \cdot (g_{n,k}^{(t)})^2}{\Gamma N_0}\right)$ denote the transmission rate of user k on subcarrier n at time t, where p_t is the transmission power on a subcarrier, $g_{n,k}^{(t)}$ is the channel gain of user k on subcarrier n at time t, W is the bandwidth of a subcarrier, N_0 is the power spectrum density of Gaussian noise, and Γ is the capacity gap that is related to the target bit error rate (BER) and coding-modulation schemes. At each time t, fast adaptive OFDMA solves the following problem:

$$\begin{aligned} \max_{c_{n,k}^{(t)}} \quad & b^{(t)} = \sum_{k=1}^{K}\sum_{n=1}^{N} c_{n,k}^{(t)} r_{n,k}^{(t)} \\ s.t. \quad & b_k^{(t)} = \sum_{n=1}^{N} c_{n,k}^{(t)} r_{n,k}^{(t)} \geq q_k, \forall k \\ & \sum_{k=1}^{K} c_{n,k}^{(t)} \leq 1, \forall n \\ & c_{n,k}^{(t)} \geq 0, \forall n, k \end{aligned} \tag{1}$$

where $c_{n,k}^{(t)}$ denotes the fraction of airtime assigned to user k on subcarrier n, $b^{(t)}$ is the total system throughput at time t, $b_k^{(t)}$ is data rate of user k at time t, and q_k is the minimum data rate requirement of user k. The above fast-adaptive resource allocation is extremely costly in practice, because $g_{n,k}^{(t)}$ (and hence $r_{n,k}^{(t)}$) varies at a time scale of milliseconds. As a result, Problem (1) has to be solved very frequently.

In this paper, we propose a slow adaptation scheme as shown in Fig. 1b. Instead of allocating subcarriers in each slot, subcarrier allocation is updated once every *adaptation window* of length T. More precisely, the BS allocates subcarriers to users at the beginning of each adaptation window, and the allocation remains unchanged till the beginning of the next window. Unlike fast-adaptive OFDMA systems, the BS in slow-adaptive OFDMA systems does not have the luxury of knowing the channel realizations $g_{n,k}^{(t)}$ over the entire window when it makes the subcarrier-allocation decision. However, it is reasonable to assume that the BS knows $g_{n,k}^{(t_0)}$ (or equivalently $r_{n,k}^{(t_0)}$) when it makes subcarrier-allocation decision for adaptation window $[t_0, t_0+T]$, although $g_{n,k}^{(t)}$ (or $r_{n,k}^{(t)}$) for the remaining time is unaccessible. Therefore, we need to rely on the channel statistics estimated at t_0 to make the optimal decision.

Let $c_{n,k}$ denote the subcarrier allocation for a given application window. Then, the time-average throughput of user k during the window becomes

$$\begin{aligned} \bar{b}_k &= \frac{1}{T}\int_T \sum_{n=1}^{N} c_{n,k} r_{n,k}^{(t)} dt \\ &= \frac{1}{T}\sum_{n=1}^{N} c_{n,k} \int_T r_{n,k}^{(t)} dt \\ &= \sum_{n=1}^{N} c_{n,k}\bar{r}_{n,k} \end{aligned} \tag{2}$$

where

$$\bar{r}_{n,k} = \frac{1}{T}\int_T r_{n,k}^{(t)} dt \tag{3}$$

is the time-average data rate of user k on subcarrier n during the given adaptation window. The time-average system throughput is given as

$$\bar{b} = \sum_{k=1}^{K} \bar{b}_k = \sum_{k=1}^{K}\sum_{n=1}^{N} c_{n,k}\bar{r}_{n,k}. \tag{4}$$

Note that the key difference from (1) is that $c_{n,k}$ remains constant within the window of length T, regardless of the variation of $r_{n,k}^{(t)}$.

When T approaches infinity (or much larger than the channel coherence time in practice), the time-average data rate $\bar{r}_{n,k}$ converges to the ensemble average of the random process $r_{n,k}^{(t)}$, if the underlying channel is an ergodic process. That is, $\bar{r}_{n,k} = E[r_{n,k}]$, where the expectation is taken over all possible channel states. In a general case where T is not sufficiently long, $\bar{r}_{n,k}$ differs from its ensemble mean.

Before leaving this section, note that we have assumed that the data transmission rate $r_{n,k}^{(t)}$ is a function of the instantaneous channel gain $g_{n,k}^{(t)}$. In other words, fast rate adaptation is adopted, although subcarrier allocation is performed on a slower time scale. The reason is that the computational complexity and control overhead needed for fast rate adaptation is very low compared with adaptive subcarrier allocation. As a matter of fact, fast rate adaptation has already been implemented in many practical systems.

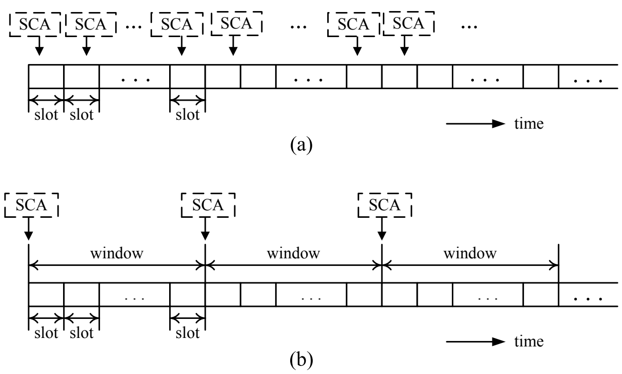

Fig. 1. Adaptation timescales of fast and slow adaptive OFDMA system (SCA = SubCarrier Allocation).

III. Slow Adaptive OFDMA

A. Problem Formulation

At the beginning of an adaptation window $[t_0, t_0+T]$, $\bar{r}_{n,k}$ appears random to the BS, as the BS cannot estimate the channel gains for the entire window except $r_{n,k}^{(t_0)}$. Therefore, the slow adaptive subcarrier allocation is mathematically formulated as

$$\begin{aligned}
\max_{c_{n,k}} \quad & E_{\mathbf{g}}[\bar{b}|r_{n,k}^{(t_0)}] = \sum_{k=1}^{K}\sum_{n=1}^{N} c_{n,k} E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}] \qquad (5)\\
s.t. \quad & E_{\mathbf{g}}[\bar{b}_k|r_{n,k}^{(t_0)}] = \sum_{n=1}^{N} c_{n,k} E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}] \geq q_k, \forall k\\
& \sum_{k=1}^{K} c_{n,k} \leq 1, \forall n\\
& c_{n,k} \geq 0, \forall n,k
\end{aligned}$$

where the expectation is taken over the random channel process $\mathbf{g} = \{g_{n,k}^{(t)}\}$ for $t \in [t_0, t_0+T]$. The above problem can be easily solved using linear programming (LP), if $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$ is known.

B. Computation of $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$

To solve (5), the major difficulty is nothing but to acquire the expected values $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$. In the following, we consider Rayleigh fading channel as one of the typical channel models. The calculation method we adopt here can also be extended to other distribution but with more complicated expressions.

When T is very large, we have $\bar{r}_{n,k} = E[r_{n,k}]$ due to ergodicity, and hence $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}] = E[r_{n,k}]$, which can be easily calculated from the distribution of $g_{n,k}$. For a Rayleigh fading channel, the probability density function (PDF) of $g_{n,k}$ is

$$f_{g_{n,k}}(x) = \frac{2x}{\sigma_k}\exp\left(-\frac{x^2}{\sigma_k}\right) \qquad (6)$$

with σ_k being the long-term average channel gain of user k. Then, we have

$$E[r_{n,k}] = \int_0^{\infty} W\log_2\left(1+\frac{p_t x^2}{\Gamma N_0}\right)\frac{2x}{\sigma_k}\exp\left(-\frac{x^2}{\sigma_k}\right)dx. \qquad (7)$$

In a general case where T is not necessarily long enough, the calculation of $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$ is much more involved. As will be proved in Theorem 1, $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$ can be calculated from Eqn. (8).

Theorem 1. *Given a window* $[t_0, t_0+T]$ *and the initial data rate* $r_{n,k}^{(t_0)} = W\log_2\left(1+\frac{p_t\cdot(g_{n,k}^{(t_0)})^2}{\Gamma N_0}\right)$ *where* $g_{n,k}^{(t_0)}$ *follows the Rayleigh distribution in* (6)*, the expected average data rate of user* k *on subcarrier* n *can be computed as*

$$\begin{aligned}
E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}] = \int_0^{\infty}\int_0^{T}\Big\{&\alpha\cdot r\cdot 2^{r/W}\cdot\exp(-\beta\cdot 2^{r/W})\\
&\cdot I_0\left(\gamma\cdot R(\tau)\sqrt{2^{r/W}-1}\right)\Big\}d\tau dr
\end{aligned} \qquad (8)$$

where $R(\tau) = J_0(2\pi f_d\tau)$ *denotes the time correlation of the channel, where* $J_0(\cdot)$ *is the zero-order Bessel function and* f_d *is the Doppler frequency of the channel. Likewise,* $I_0(\cdot)$ *is zero-order modified Bessel function, i.e.* $I_0(x) = J_0(jx)$*.* α*,* β *and* γ *are all functions of* $\tau = t - t_0$ *given as*

$$\alpha = \frac{\sigma_k\Gamma N_0\ln 2}{4TWp_t\delta}\exp\left\{\frac{\Gamma N_0[(\delta-\sigma_k^2/4)2^{r_{n,k}^{(t_0)}/W}-\delta+\sigma_k^2/2]}{\sigma_k p_t\delta}\right\}$$

$$\beta = \frac{\sigma_k\Gamma N_0}{4p_t\delta},\quad \gamma = \frac{\Gamma N_0\sqrt{2^{r_{n,k}^{(t_0)}/W}-1}}{2p_t\delta},\ \delta = \frac{\sigma_k^2 - R^2(\tau)}{4}.$$

Proof: The following proof applies to all n and k. Hence, we omit the subscript n,k for simplicity of notation. From (6), the joint Rayleigh distribution of $g^{(t_1)}$ and $g^{(t_2)}$ can be derived as

$$\begin{aligned}
f(g^{(t_1)}, g^{(t_2)}) =& \frac{g^{(t_1)}g^{(t_2)}}{\delta}\exp\left\{-\frac{\sigma}{4\delta}\left((g^{(t_1)})^2+(g^{(t_1)})^2\right)\right\}\\
&\cdot I_0\left\{\frac{g^{(t_1)}g^{(t_2)}}{2\delta}R(\tau)\right\}.
\end{aligned}$$

Knowing $r^{(t)} = W\log_2\left(1+\frac{p_t\cdot(g^{(t)})^2}{\Gamma N_0}\right)$, we can obtain the following distributions:

$$f(r^{(t)}) = (g^{(t)})' f(g^{(t)})$$

and

$$f(r^{(t_1)}, r^{(t_2)}) = J\cdot f(g^{(t_1)}, g^{(t_2)})$$

where $J = \left|\frac{\partial(g^{(t_1)}, g^{(t_2)})}{\partial(r^{(t_1)}, r^{(t_2)})}\right|$ is the Jacobian determinant.

Hence, the conditional rate distribution is

$$\begin{aligned}
f(r^{(t_2)}|r^{(t_1)}) =& \frac{f(r^{(r_2)}, r^{(t_1)})}{f(r^{(t_1)})}\\
=&\frac{2^{\frac{r^{(t_2)}}{W}}\sigma\Gamma N_0\ln 2}{4Wp_t\delta}\exp\left\{\frac{\Gamma N_0(2^{\frac{r^{(t_1)}}{W}}-1)}{\sigma p_t} - \frac{\sigma\Gamma N_0}{4p_t\delta}(2^{\frac{r^{(t_1)}}{W}}\right.\\
&\left.+2^{\frac{r^{(t_2)}}{W}}-2)\right\}\cdot I_0\left\{\frac{\Gamma N_0\sqrt{(2^{\frac{r^{(t_1)}}{W}}-1)(2^{\frac{r^{(t_2)}}{W}}-1)}}{2p_t\delta}R(\tau)\right\}
\end{aligned}$$

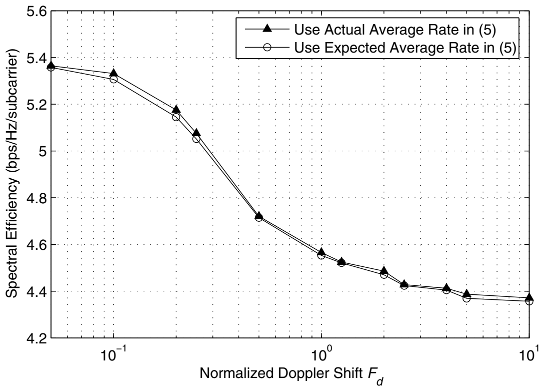

Fig. 2. Performance comparison between systems using the actual average rate $\bar{r}_{n,k}$ and expected average rate $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$ in Problem (5).

Finally, we can calculate the conditional expectation of mean rate as follow,

$$E_{\mathbf{g}}[\bar{r}|r^{(t_0)}] = \frac{1}{T}\int_T E[r^{(t)}|r^{(t_0)}]dt$$
$$= \frac{1}{T}\int_T \left(\int_0^\infty r^{(t)} f(r^{(t)}|r^{(t_0)})dr^{(t)}\right) dt$$
$$= \int_0^T \int_0^\infty \underbrace{\frac{\sigma\Gamma N_0 \ln 2}{4TWp_t\delta} \exp\left\{\frac{\Gamma N_0[(\delta - \frac{\sigma^2}{4})2^{\frac{r^{(t_0)}}{W}} - \delta + \frac{\sigma^2}{2}]}{\sigma p_t \delta}\right\}}_{\alpha}$$
$$\cdot r \cdot 2^{r/W} \exp\left\{-\underbrace{\frac{\sigma\Gamma N_0}{4p_t\delta}}_{\beta} 2^{r/W}\right\} \cdot I_0\left\{\underbrace{\frac{\Gamma N_0\sqrt{2^{\frac{r^{(t_0)}}{W}} - 1}}{2p_t\delta}}_{\gamma}\right.$$
$$\left.\cdot R(\tau)\sqrt{2^{r/W}-1}\right\} dr d\tau. \qquad \square$$

Fig. 2 investigates the spectral efficiency of the slow adaptive OFDMA, given by $\bar{b}/N$, where $\bar{b}$ is defined in (4). Assume there are 3 users and 16 subcarriers. The average received signal to noise ratio (SNR) $p_t\sigma_k/N_0$ is set to be 22dB for all users and the target BER is set to 10^{-4}. We fix the Doppler shift f_d to 50Hz and vary the adaptation window size T, so that the normalized Doppler shift, defined as $F_d = f_d T$, varies from 0.05 to 10. The line with triangles corresponds to the ideal case where the actual realization of $\bar{r}_{n,k}$ is used instead of $E_{\mathbf{g}}[\bar{r}_{n,k}|r_{n,k}^{(t_0)}]$ in Problem (5). The line with circles corresponds to the case where the expected average data rate calculated by (8) is used in Problem (5). The figure shows that the gap between the two curves is negligible. This implies that the simple mean-rate estimation given in Theorem 1 performs almost as well as the ideal case even when the adaptation window is large.

Remark 1. *Problem* (5) *is formulated into an LP which can be efficiently solved by standard methods, e.g. the path-following algorithm. The complexity involved depends on the number of variables (NK) and constraints ($NK+N+K$). We note that there is already plenty of research on fast adaptive OFDMA. The work focusing on LP formulation (e.g. [4], [5]) can also be applied to the slow adaptation here, including their efficient heuristic algorithms, whereas our scheme only affords $1/T$ computational complexity of fast schemes within each window. Moreover, computing the expectation in* (8) *requires $\mathcal{O}(M^2)\cdot NK$ computations where M is the number of function evaluations required for one-dimensional integral. In practice, however, we may pre-compute the data and store them in a lookup table, and then only a table lookup is needed at the beginning of each window. Hence, the computation of the expectation will not be the major burden of the computational complexity in slow adaptation.*

IV. "SAFE" SLOW ADAPTIVE OFDMA

The above formulated slow adaptive OFDMA guarantees the *expected* average throughput of users to be higher than q_k per unit time during each window. The *actual realized* throughput $\bar{b}_k$ could be higher or lower than q_k in different application windows. While this is good enough for elastic traffics where users value their long-term average data rates, it may not work for inelastic traffics that require the *actual* throughput to be higher than a certain threshold during each period of interest. To address the need of inelastic traffics, we modify the formulation (5) to propose a safe slow adaptive OFDMA scheme that guarantees the actual throughput $\bar{b}_k$ to be higher than q_k for each and every application window.

Let vector

$$\mathbf{r_0} = \left[E_{\mathbf{g}}[\bar{r}_{1,1}|r_{1,1}^{(t_0)}], \cdots, E_{\mathbf{g}}[\bar{r}_{N,1}|r_{N,1}^{(t_0)}], \cdots, \right.$$
$$\left. E_{\mathbf{g}}[\bar{r}_{1,K}|r_{1,K}^{(t_0)}], \cdots, E_{\mathbf{g}}[\bar{r}_{N,K}|r_{N,K}^{(t_0)}]\right]^T \in \mathbb{R}^{NK}$$

denote the statistical mean of time-average data rate estimated at the beginning of an application window. Likewise, let

$$\mathbf{r} = \left[\bar{r}_{1,1}, \cdots, \bar{r}_{N,1}, \cdots, \bar{r}_{1,K}, \cdots, \bar{r}_{N,K}\right]^T \in \mathbb{R}^{NK}$$

denote the actual realization of the time-average data rate. Typically, $\bar{r}_{n,k}$ varies around its statistical mean. Therefore, at time t_0, it is reasonable to assume that $\mathbf{r}$ will lie in an ellipsoid centered around $\mathbf{r_0}$ during the application window that follows [9]. That is, $\mathbf{r} \in \mathcal{U}$ where

$$\mathcal{U} = \left\{\mathbf{r} \in \mathbb{R}^{NK} : \mathbf{r} = \mathbf{r}_0 + \mathbf{R}\mathbf{v},\ \|\mathbf{v}\|_2 \le \rho_0\right\}. \qquad (9)$$

In the above, $\mathbf{R}$ is some $NK \times NK$ symmetric positive semidefinite matrix, $\|\cdot\|_2$ is the Euclidean norm of a vector, and ρ_0 corresponds to the maximum deviation of $\mathbf{r}$ from $\mathbf{r_0}$.

To ensure that the minimum data rate constraints are satisfied for all possible realizations $\mathbf{r}$, the slow adaptation problem (5) is reformulated as follows.

$$\begin{aligned} \max_{\mathbf{c}} \quad & \min_{\mathbf{r}\in\mathcal{U}} \sum_{k=1}^{K}\sum_{n=1}^{N} c_{n,k}\bar{r}_{n,k} & (10)\\ s.t. \quad & \sum_{n=1}^{N} c_{n,k}\bar{r}_{n,k} \ge q_k, \forall k, \forall \mathbf{r} \in \mathcal{U} \\ & \sum_{k=1}^{K} c_{n,k} \le 1, \forall n \\ & c_{n,k} \ge 0, \forall n, k \end{aligned}$$

Writing it into a more condensed form, we have

$$\begin{aligned} \min_{\mathbf{c}} \quad & t \\ s.t. \quad & \mathbf{r}^T\mathbf{c} \geq -t,\ \forall \mathbf{r} \in \mathcal{U} \\ & (\mathbf{B}_k\mathbf{r})^T\mathbf{c} \geq q_k,\ \forall k,\ \forall \mathbf{r} \in \mathcal{U} \\ & \mathbf{D}\cdot\mathbf{c} \geq \mathbf{d} \end{aligned} \tag{11}$$

where $\mathbf{c} = [c_{1,1}, \cdots, c_{N,1}, \cdots, c_{1,K}, \cdots, c_{N,K}]^T \in \mathbb{R}^{NK}$; $\mathbf{B}_k = \begin{pmatrix} \mathbf{b}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{b}_K \end{pmatrix} \in \mathbb{R}^{NK\times NK}$, $\mathbf{b}_i = \mathbf{I}_N$ if $i = k$, otherwise $\mathbf{b}_i = \mathbf{0} \in \mathbb{R}^{N\times N}$; $\mathbf{D} = \begin{pmatrix} -\mathbf{I}_N & \cdots & -\mathbf{I}_N \\ & \mathbf{I}_{NK} & \end{pmatrix} \in \mathbb{R}^{(N+NK)\times NK}$, and $\mathbf{d} = [\underbrace{-1, \ldots, -1}_{N}, \underbrace{0, \ldots, 0}_{NK}]^T \in \mathbb{R}^{N+NK}$; $\mathbf{I}_M$ is the M-by-M identity matrix.

In the sequel, we show that (11) is equivalent to a second order cone program (SOCP) that can be efficiently solved.

Theorem 2. (11) *is equivalent to the following SOCP:*

$$\begin{aligned} \min_{\boldsymbol{c}} \quad & t \\ s.t. \quad & \left(\frac{\boldsymbol{r}_0^T\boldsymbol{c} + t}{\rho_0}, \boldsymbol{R}^T\boldsymbol{c}\right) \succeq_{\mathcal{Q}} 0 \\ & \left(\frac{\boldsymbol{r}_0^T\boldsymbol{B}_k^T\boldsymbol{c} - q_k}{\rho_0}, \boldsymbol{R}^T\boldsymbol{B}_k^T\boldsymbol{c}\right) \succeq_{\mathcal{Q}} 0,\ \forall k = 1, \cdots, K \\ & \boldsymbol{D}\cdot\boldsymbol{c} \geq \boldsymbol{d} \end{aligned} \tag{12}$$

where $\succeq_{\mathcal{Q}}$ is the partial order defined on the second order cone $\mathcal{Q} = \{(t, x) \in \mathbb{R}\times\mathbb{R}^n : t \geq \|x\|_2\}$, i.e. we have $(t, x) \succeq_{\mathcal{Q}} 0$ iff $(t, x) \in \mathcal{Q}$.

Proof: The first constraint in (11)

$$\mathbf{r}^T\mathbf{c} + t = (\mathbf{r}_0 + \mathbf{R}\mathbf{v})^T\mathbf{c} + t \geq 0, \quad \forall \mathbf{r} \in \mathcal{U}$$

can be expressed as

$$\begin{aligned} & \min_{\mathbf{v}\in\mathbb{R}^{NK}, \|\mathbf{v}\|_2\leq\rho_0} (\mathbf{r}_0^T\mathbf{c} + t) + \mathbf{v}^T(\mathbf{R}^T\mathbf{c}) \\ & = (\mathbf{r}_0^T\mathbf{c} + t) - \rho_0\|\mathbf{R}^T\mathbf{c}\|_2 \geq 0. \end{aligned} \tag{13}$$

Hence, we obtain a SOC constraint as

$$\left(\frac{\mathbf{r}_0^T\mathbf{c} + t}{\rho_0}, \mathbf{R}^T\mathbf{c}\right) \succeq_{\mathcal{Q}} 0. \tag{14}$$

Similarly, the second constraint in (11) yields

$$(\mathbf{B}_k\mathbf{r})^T\mathbf{c} = (\mathbf{r}_0 + \mathbf{R}\mathbf{v})^T(\mathbf{B}_k^T\mathbf{c}) \geq q_k,\ \forall k = 1, \cdots, K, \forall \mathbf{r} \in \mathcal{U}.$$

It is equivalent to

$$\begin{aligned} & \min_{\mathbf{v}\in\mathbb{R}^{NK}, \|\mathbf{v}\|_2\leq\rho_0} \mathbf{r}_0^T(\mathbf{B}_k^T\mathbf{c}) + \mathbf{v}^T(\mathbf{R}^T\mathbf{B}_k^T\mathbf{c}) \\ & = \mathbf{r}_0^T\mathbf{B}_k^T\mathbf{c} - \rho_0\|\mathbf{R}^T\mathbf{B}_k^T\mathbf{c}\|_2 \geq q_k, \quad \forall k = 1, \cdots, K. \end{aligned}$$

Hence, we have the SOC constraints

$$\left(\frac{\mathbf{r}_0^T\mathbf{B}_k^T\mathbf{c} - q_i}{\rho_0}, \mathbf{R}^T\mathbf{B}_k^T\mathbf{c}\right) \succeq_{\mathcal{Q}} 0, \forall k. \tag{15}$$

□

Remark 2. *SOCP is a convex problem, and hence the optimal solution of* (12) *can be efficiently solved via interior point method [8], which is a polynomial-time algorithm and can be applied into real-time adaptation.*

V. Numerical Results

In this section, we study the performance of the slow adaptive OFDMA schemes proposed in Sections III and IV. We consider an OFDMA system with 4 users and 64 subcarriers, and each user has a minimum rate constraint of 64 bps. The average received SNR is 22dB and the channel fading follows Rayleigh distribution. The target BER is set to 10^{-4}. In the following, all the plots are averaged over independent 4000 simulation runs.

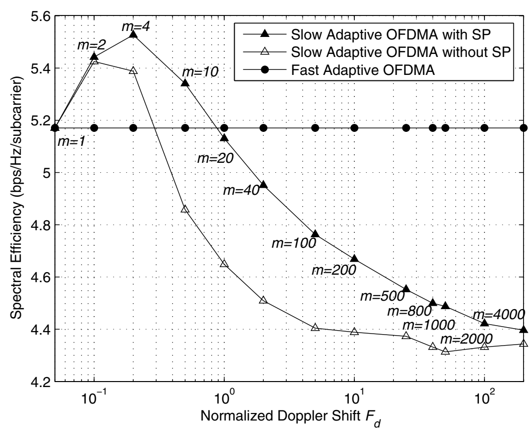

Fig. 3. Spectral efficiency comparison among fast adaptation, slow adaptation with and without SP. The control overhead is considered here.

In Fig. 3, we compare the system spectral efficiency of three adaptive OFDMA systems: fast adaptive OFDMA in (1), the proposed slow adaptive OFDMA with SP in (5), and an intuitive slow adaptation scheme without SP, i.e., to solve (1) at time t_0 and then apply $c_{n,k}^{(t_0)}$ to the whole window. We fix f_d to 50Hz and vary T, so that the normalized Doppler shift F_d varies from 0.05 to 200. Presumably, the larger T, the less frequently the subcarrier allocation is updated. To quantify this, we assume that time unit per fast adaptation $T_0 = 1000\mu s$ is the maximum time unit within which the wireless channel does not change. Then, $m = T/T_0$ quantifies how much times slower the slow adaptation is compared with the fast one. Here, we assume that the control overhead for subcarrier allocation consumes a bandwidth equivalent to 10% of T_0 every time the subcarrier allocation is updated [7]. From the figure, we can see that by tuning T, we can engineer a tradeoff between spectral efficiency and computational complexity as well as control overhead. Due to high control overheads, the traditional fast adaptive OFDMA suffers from throughput degradation. Hence, when T is relatively small (F_d smaller than 1), slow adaptation achieves an even higher spectral efficiency than the fast one, while enjoying a much lower computational cost. It is worth noting that when $m = 100$, the proposed slow adaptation scheme achieves 92% throughput of the fast scheme with only $\frac{1}{100}$ computational complexity. Comparing with the intuitive

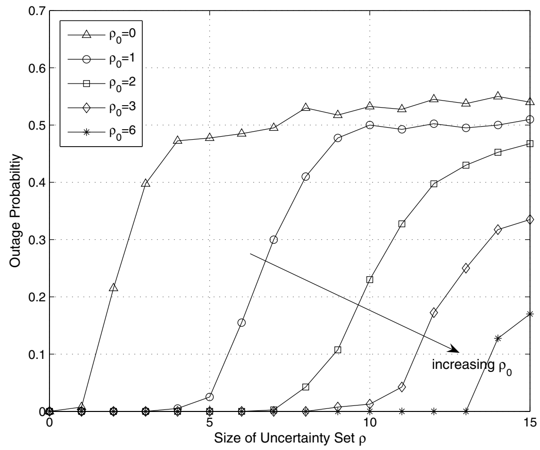

Fig. 4. Outage probability of slow adaptive OFDMA versus the deviation size ρ when $F_d = 2$ and $m = 40$.

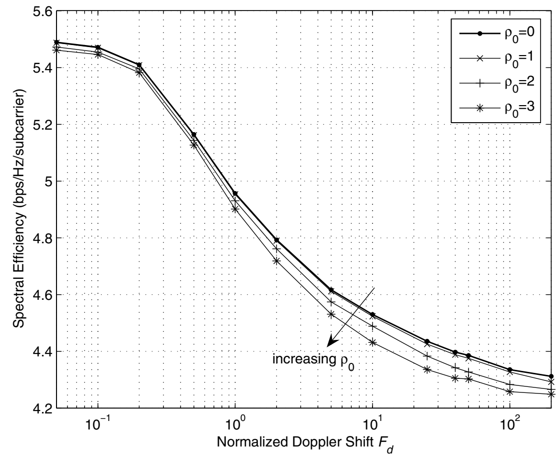

Fig. 5. Spectral efficiency of safe slow adaptive OFDMA with different system parameter ρ_0. We fixed the actual maximum deviation $\rho = 6$.

slow adaptation scheme without SP, the proposed scheme in (5) achieves a much higher throughput (about 10% when $m = 40$) with a similar computational cost.

In Fig. 4 and 5, we investigate the performance of the safe slow adaptive OFDMA scheme proposed in Section IV. We are interested in the occurrence of outage events when the actual data rate enjoyed by user k falls below q_k during some application window. In Fig. 4, we plot the probability a user perceives an outage event[1] versus the deviation of $\mathbf{r}$ from its statistical mean $\mathbf{r}_0$, denoted by ρ. In particular, we vary ρ_0 in (9) and (11) and plot four curves. For comparison, the outage probability of the proposed scheme in (5) is also plotted, which corresponds to $\rho_0 = 0$. From the figure, it can be seen that by safe adaptive OFDMA in (11), outage probability is significantly reduced compared with the scheme in (5). Moreover, it is not surprising that the outage probability is always zero as long as ρ_0 is larger than ρ.

One concern of adopting the safe slow adaptive OFDMA scheme is that the system spectral efficiency may decrease with the increase of safety margin ρ_0. Fortunately, Fig. 5 shows that the throughput degradation with the increase of ρ_0 is negligible. It can be seen that the scheme in (5) (which corresponds to $\rho_0 = 0$) yields an upper bound in the throughput, as there is no safety margin provided. When $\rho_0 = 3$, the throughput gap is as small as 2% compared with the upper bound. Together with Fig. 4, the results imply a tradeoff between the spectral efficiency and outage probability when underlying traffic is inelastic.

VI. Conclusion

Drastically enhancing system spectral efficiency, adaptive OFDMA holds significant promise in next generation wireless system. The major hurdle that renders its real implementation unrealistic is the prohibitively high computational complexity and excessive control overhead. In this paper, we propose a novel slow adaptive OFDMA scheme using stochastic programming, which noticeably reduces the computational cost and control overhead, while achieving satisfactorily high spectral efficiency (e.g. achieving 92% throughput with around $\frac{1}{100}$ computational cost of the fast adaptation scheme). Furthermore, for inelastic traffics caring about worst-case throughput guarantee, we formulate a *safe* slow adaptation scheme that can be efficiently solved via a conic linear program. The numerical results demonstrate two types of tradeoffs: (i) tradeoff between spectral efficiency and computational cost by tuning the length of adaptation window, and (ii) tradeoff between spectral efficiency and outage probability for inelastic traffics by tuning ρ_0 in the safe slow adaptation scheme. How to set the "right" parameters for different applications and how to deal with a formulation with additional constraints on outage probability would be an interesting future research topic.

[1]In the simulation, user k is said to be in outage when $\bar{b}_k \leq q_k(1-\varepsilon)$, where $\varepsilon = 0.001$ is for error tolerance.

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