

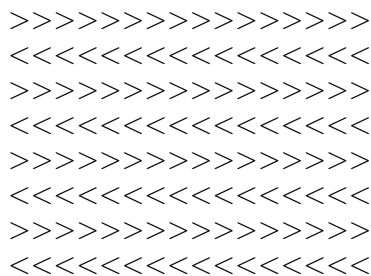
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18.701 Algebra I  
Fall 2007

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## 18.701 Practice Quiz 2

1. Let  $V$  be the real vector space whose elements are the polynomials of degree  $\leq 4$ , and let  $W = \mathbb{R}^2$ . Let  $T : V \rightarrow W$  be the linear transformation defined by  $T(f) = (f(2), f'(2))^t$ , where  $f'$  denotes the derivative. Determine the dimension of the kernel (the nullspace) of  $T$ .
2. As usual,  $\rho_\theta$  stands for the operator of rotation of the plane through the angle  $\theta$  about the origin, and  $r$  is reflection about the horizontal axis.
  - (a) Determine the matrix of the composed linear operator  $m = r\rho_\theta$ .
  - (b) Geometrically,  $m$  is reflection about a line. Determine this line.
  - (c) What are the eigenvalues of  $m$ ?
  - (d) Is  $m$  a diagonalizable operator?
3. The rotation through the angle  $\frac{\pi}{2}$  about the point  $(1, 2)^t$  can be written in the form  $t_v\rho_\theta$ , where  $t_v$  is translation by the vector  $v$ . Determine  $v$  and  $\theta$ .
4. The figure below depicts part of a pattern  $F$  that covers the plane  $\mathbb{R}^2$ . Let  $G$  be the group of symmetries of  $F$ .
  - (a) Determine the point group of  $G$ .
  - (b) Let  $T_G = T \cap G$  be the subgroup of translations in  $G$ . Determine the index of  $T_G$  in  $G$ .



5. Let  $G$  be the group of symmetries of a regular tetrahedron  $T$ , including the orientation-reversing symmetries.
  - (a) Decompose the set of faces of  $T$  into orbits, and describe the stabilizer of a face.
  - (b) Determine the order of  $G$ .
6. Let  $G$  be a group of order 20 whose center is the trivial group  $\{1\}$ . Let  $x$  be an element of  $G$  of order 4. What can you say about the order of the conjugacy class of  $x$ ?