18.701 Algebra I
Fall 2007

November 3, 2007

18.701 Practice Quiz 2

1. Let V be the real vector space whose elements are the polynomials of degree ≤ 4, and let $W = \mathbb{R}^2$. Let $T : V \to W$ be the linear transformation defined by $T(f) = (f(2), f'(2))^t$, where f' denotes the derivative. Determine the dimension of the kernel (the nullspace) of T.

2. As usual, ρ_θ stands for the operator of rotation of the plane through the angle θ about the origin, and r is reflection about the horizonal axis.
(a) Determine the matrix of the composed linear operator $m = r\rho_\theta$.
(b) Geometrically, m is reflection about a line. Determine this line.
(c) What are the eigenvalues of m?
(d) Is m a diagonalizable operator?

3. The rotation through the angle $\frac{\pi}{2}$ about the point $(1,2)^t$ can be written in the form $t_v\rho_\theta$, where t_v is translation by the vector v. Determine v and θ.

4. The figure below depicts part of a pattern F that covers the plane $\mathbb{R}^2$. Let G be the group of symmetries of F.
(a) Determine the point group of G.
(b) Let $T_G = T \cap G$ be the subgroup of translations in G. Determine the index of T_G in G.

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5. Let G be the group of symmetries of a regular tetrahedron T, including the orientation-reversing symmetries.

(a) Decompose the set of faces of T into orbits, and describe the stabilizer of a face.

(b) Determine the order of G.

6. Let G be a group of order 20 whose center is the trivial group $\{1\}$. Let x be an element of G of order 4. What can you say about the order of the conjugacy class of x?