Tipping Points: Referral Homophily and Job Segregation

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November 8, 2007
Abstract

How does referral recruitment contribute to job segregation? Current theory emphasizes the segregated nature of job-seekers’ information and contact networks. The job-seeker perspective characterizing most research on network effects in the labor market leaves little role for organizational influence. But referrals are necessarily initiated within a firm by referrers. This paper focuses on the neglected half of the referring dyad and seeks to explain the segregating effects of referring from the referrer’s perspective. Our main finding is that if a firm can get its under-represented group to refer more, referral recruitment can be made neutral to job segregation, or even integrative. Our analysis reveals a tipping point in referring dynamics – precisely how much more the under-represented group needs to refer to neutralize the segregating effects of referring. We build upon previous research to generate a formal model of referring dynamics as a regular Markov population process. We use this model to build theory regarding the segregating effects of referring, and the role of organizations in this process. In so doing, we show the prevailing wisdom fails to explain how referring contributes to job segregation. We reveal the conditions necessary for referring to segregate and identify policy levers for firms to mitigate this effect.
Introduction

Most organizations recruit new workers via informal referrals, along with other more formal methods (Marsden and Gorman 2001). Reliance on referral recruitment is particularly strong among smaller firms without dedicated recruiting budgets or personnel, firms which constitute about half of the labor market (Bartram et al. 1995; Barber, Wesson and Roberson 1999; Mencken and Winfield 1998). Referral recruitment is also important to job-seekers. Approximately half of all workers in the U.S. used their personal contact networks to find their current jobs (Granovetter 1995; Marsden and Gorman 2001).

Despite being a pervasive labor market practice, there have been numerous calls to reduce or even eliminate referral recruitment in organizations (Braddock and McPartland 1987; LoPresto 1986; Padavic and Reskin 2002; Roos and Reskin 1984). Because referral recruitment has long been theorized as contributing to job segregation1 (Doeringer and Piore 1971; Kanter 1977; Marsden 1994; Marsden and Gorman 2001; Moss and Tilly 2001; Mouv 2002; Reskin, McBrier and Kmec 1999). Job segregation has numerous organizational and societal costs. Perhaps the most prominent of these effects is gender wage inequality – most of which is attributable to the sex segregation of jobs (Baron and Bielby 1986; Bayard et al. 2003; Kmec 2003; Petersen and Morgan 1995; Tomaskovic-Devey 1993). In addition, segregation introduces harmful labor market rigidities – where fluctuations in demand can result in shortages (e.g., recent shortages in engineers and nurses) or gluts for particular jobs rather than being absorbed by job mobility (Anker 1997; Kahn 2000; Padavic & Reskin 2002:58-9). From an organizational and legal perspective, job segregation is often taken as evidence of violations in U.S. Equal Employment opportunity (EEO) laws (Gutman 2000), and is associated with the successful prosecution of EEO

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1 Although referral recruitment is viewed to have associations with both the race and sex segregation of jobs, in this paper we focus on sex segregation. The simplifying choice to focus on sex segregation was a pragmatic one: modeling the dynamics of a dichotomous variable such as sex is more straightforward than a multi-valued (and potentially multi-dimensional) variable as race. We recognize this choice as a limitation. Sex and race are both important and omnipresent social markers of individuals in organizational contexts (Ashforth and Humphrey 1995; Heilman 1995; Hirschfield 1999), and these particular markers interact in complex ways (Chafetz 1997), especially in the context of the labor market (Browne and Misra 2003; Robinson et. al. 2005). This simplification neither makes any assumptions of the relative importance of one type of segregation over another, nor reflects a view that the homogenous treatment of groups by race or sex, and not both simultaneously, is unproblematic. We plan to expand the model in the future to include race and sex simultaneously.
complaints (Hirsch 2006). Thus, organizations face a dilemma: referral recruitment is economically efficient (e.g., Fernandez, Castilla and Moore 2000), but also places the organization at risk for contributing to job segregation. Our analysis shows that it is possible for organizations to manage referral recruitment to preserve the practice but eliminate its segregating effects.

The current literature on referral processes and their relation to job segregation suffers from two major deficits. The first is its almost exclusive focus on job-seekers and their networks. The referral process requires a dyad.\(^2\) The referral is the job-seeker using her networks to identify job opportunities. The other half of the dyad is the referrer – necessarily an organizational member aware of a job opportunity who shares that information with the referral. From the perspective of organizations, there is little the organization can do to influence either the network structure or job-search behaviors of the job-seeker. For this reason, organizations have largely been absolved from any responsibility for the segregating outcomes of referral recruitment (e.g., National Research Council, 2004: 43). But the referrer is an organizational member and subject to organizational influence; a fact demonstrated by practices such as the referral bonus. Despite this potential for managing at least half of the referrer-referral dyad, organizations have been given few tools to mitigate the segregating effects of referring.

The second deficit characterizing this literature is the lack of a process-based understanding of the segregating effects of referring. Although some empirical work has established a link between referral processes and job segregation (Braddock and McPartland 1997; Fernandez and Fernandez-Mateo 2006; Fernandez and Sosa 2005; Mouw 2002; Petersen, Saporta and Seidel 2001), there has been no formal mechanism-based theory of how one is associated one with the other. Mechanism-based theories can serve as invaluable aids in designing organizational interventions and moving research forward (Hedström 2005; Reskin 2003; Schelling 1998).

\(^2\) In this paper, we define referral applicants as those applicants who can identify an in-firm referrer by name. Indeed, referral bonus policies are predicated on precisely this type of dyadic relationship. We recognize that some definitions of referral applicants are more broad and could include contacts from non-employees e.g., those who learned of a job advertisement through a friend not employed by the hiring company.
This paper addresses both of these deficits. We operationalize a mechanism-based theory of referral processes as a formal mathematical model. Our model is designed from the perspective of the referrer, focusing on the networking yields of referring employees. Our analysis leads to some important and novel insights. Notably, we uncover an implicit confusion in the literature regarding the nature of referral homophily: is referral homophily *absolute* – that is, set in the labor market and independent of job-specific factors, or is it *relative* – that is, to some extent varying from job to job? We are able to resolve this confusion in a manner that challenges the received wisdom concerning how referring contributes to job segregation. In so doing, we are able to specify the conditions under which referring can segregate. This finding leads us to our major revelation: organizations *can* manage the referral process to render it neutral to job segregation, or even turn it into an *integrating* mechanism. We identify that sex differences in referring rates yield a critical tipping point at which referring will have no segregating effect and beyond which referring will actually serve to integrate a job.

Our paper is organized as follows: in the first section, we review the existing literature on referral networks and job segregation to identify the elements of a mechanism-based theory. The second section identifies an implicit disagreement in the literature regarding the structuring of referrer-referral ties, formalizes these two distinct views, and presents empirical findings addressing the conflict. The third section introduces our regular Markovian population process model of the segregating effects of referring, and uses an analysis of this model to resolve the conflicting views from before by exposing their necessary consequences. The fourth section expands upon the model to reveal an important policy lever in asymmetries in referring rates. This analysis uncovers the titular tipping points of referral processes. The simplicity of the tipping point result presents immediate propositions and potential organizational policy recommendations. The final section summarizes our findings and lays out a research agenda to expand upon this referrer-based perspective of networks and labor market outcomes.
Background
Granovetter’s groundbreaking work (1973) was arguably the harbinger of the large stream of research into the labor market role of social networks. Possibly inheriting Granovetter’s perspective, this subsequent scholarship has largely taken the form of seeking to explain labor market outcomes as a function of the characteristics of job seekers' networks. When investigating gendered aspects of referring from the perspective of job seekers, researchers have sought to reveal the proportion of referrers that are the same sex as the job seeker (e.g., Drentea 1998; Hanson and Pratt 1991; Mencken and Winfield 2000). This formulation prioritizes job-seekers' behaviors and network characteristics at the cost of neglecting any effects deriving from the behaviors of those people who initiate the referral process – referrers.

Referrers are necessarily firm employees, and potentially subject to organizational influence. In one simple example of firm influence on referring behavior, many firms offer referral bonuses to encourage referring behavior among their employees (e.g., Fernandez and Castilla 2001). However, when investigating the relationship between referring and the segregation of jobs, consideration for the role of referrers has largely been absent.

The existing literature positing associations between referring and job segregation outcomes is voluminous (Bielby 2000:123; Braddock and McPartland 1987; Doeringer and Piore 1971; Drentea 1998; Fernandez and Sosa 2005; Granovetter 1995; Hanson and Pratt 1991; Kanter 1977; Kmec 2005; Marsden 1994; Marsden and Gorman 2001; Moss and Tilly 2001; Mouw 2002; Padavic and Reskin 2002; Petersen, Saporta and Seidel 2000; Reskin 1993; Reskin, McBrier and Kmec 1999; Roos and Reskin 1984; Straits 1998). Studies about the associations between referring and job segregation tend to emphasize the role of homophily in job seekers’ networks – the degree to which job seekers receive their job information from socially similar, here same-sex, others. (Drentea 1998; Hanson and Pratt 1991; Huffman and Torres 2001, 2002; Mencken and Winfield 2000; Rankin 2003; Straits 1998).

The job-seeker perspective regarding the segregating effects of referring posits that job seekers differentially seek out job opportunities from similar others, or naturally exist in segregated contact networks and therefore are dependent upon socially similar referrers for job referrals. The salient point is
that homophily in job seekers’ networks does not involve the organizations that are offering the jobs. From the perspective of organizations, there is nothing the organization can do to influence the level of this type of homophily in the labor market.

We take a referrer-based perspective, and concern ourselves with the structure of referrers’ networks. These networks are composed of dyads consisting of the referrer and her referred job applicant – the referral.³ We consider the role of homophily in these networks by attending to same-sex referring (SSR) behaviors of referrers. For both male and female referrers in an organization, we define their SSR probability as the probability that a referral applicant generated by the referrer is the same sex as the referrer. We will use these probabilities in constructing our model of the segregating effects of referring. We denote the SSR probability for male referrers as $P$, and the SSR probability for female referrers as $Q$. Male referrers then generate referral applicants that are male with a probability $P$ and female with a probability $1-P$. Similarly, the referral applicants generated by female referrers are female with a probability $Q$ and male with a probability $1-Q$.

We have not yet defined the meaning of homophily from the referrers’ perspective. The reason is that the referrer-based perspective on referring dynamics reveals the literature to be equivocal on how referral homophily should be defined. The fundamental disagreement rests in whether referral homophily as a general social phenomenon, independent of job-specific factors, and a property of a labor market, or is referral homophily in part contingent on job-specific factors. Through its silence on the role of organizations, the dominant job-seeker perspective implicitly supports the former proposition. The latter job-specific view has been voiced briefly in an empirical case study (Fernandez and Sosa 2005), where it received empirical support.

Examples in the literature where homophily among social ties has been defined explicitly (e.g., Heckathorn 2002; Mollica, Gray and Trevino 2003; Skvoretz, Fararo and Agneessens 2004) all

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³ Our emphasis is on the referrer-referral dyad. Referrers who share a job opportunity with a potential referral who does not in turn actually apply for the job is not a successful generation of a referral applicant. The dyad is formed both by the actions of the referrer in sharing job opportunity information, and by the actions of the referral in applying for that particular job.
emphasize defining homophily as a biasing deviation away from a referent of unbiased or random ties. That is, if ties were formed randomly, there would still be some ties to socially similar people just by chance (usually reflective of their representation within the population of potential alters). Homophily is the adjustment away from this random network towards one that is more socially similar than what would occur by chance.

The literature on biased networks addresses these concepts directly. This literature presents the tie formation process as being composed of both a random component and a biased component (Skvoretz, Fararo and Agneessens 2004; Heckathorn 2002:20). The biased component in this case is homophily – the promotion of same-sex ties above and beyond the random component. When homophily is perfect, referrers generate exclusively same-sex referral applicants. Homophily can also be negative, as in heterophily. Under perfect heterophily (perfect negative homophily), referrers generate exclusively opposite-sex referral applicants. When homophily is wholly absent, referrers generate referral applicants in a random and unbiased manner. The result of unbiased referring is the counterfactual against which referral homophily is defined. Defining this counterfactual composition of referral applicants absent the influences of homophily is critical in structuring referral dynamics. The two positions on defining referral homophily have fundamentally different views on the determination of the counterfactual composition produced by unbiased referring.

We introduce a parameter $c$, for the counterfactual composition (as percent female) of referral applicants generated by unbiased or random referring, and a parameter $h$, for the biasing component of referrer-referral ties resulting from the tendency towards homophilous (same-sex) ties. As $c$ gives the percent female, $1 - c$ gives the percent male of the referral applicants generated by unbiased referring. The $c$ parameter is identical for male and female referrers by definition. These two new parameters ($c$ and $h$) serve as the building blocks for the same-sex referring probabilities, $P$ and $Q$. We can write both probabilities as the deviation from the counterfactual composition resulting from unbiased referring ($c$ for

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4 We make no claims regarding the antecedents of homophily as a phenomenon. Whether due to agentic choices of the referrer, extant social or cultural norms, imposed structural constraints, or combinations of these and other forces, contact networks tend toward homophily (McPherson, Smith-Lovin and Cook 2001).
women and \( l - c \) for men), and the bias term \( (h) \) from homophily. The probability that referral applicants generated by female referrers are also female, \( Q \), can be written as: \( Q = c + h \). Similarly, the probability that referral applicants generated by male referrers are male, \( P \), can be written as: \( P = l - c + h \). Whether and to what extent the homophily adjustment parameter, \( h \) varies by sex will be discussed in more detail below. In terms of these new parameters, the disagreement in the literature is in how the counterfactual composition of women among the referrals produced without bias \( (c) \) should be defined.

**Referral homophily: Two views**

The random component of referring must reflect the outcome if a referrer were to refer without regard to the ascriptive category (here, sex) of her potential referral. Would this composition be singular and invariant across different jobs in the labor market, or might job-specific factors also influence this counterfactual composition of referral applicants resulting in many different values for \( c \) in a given labor market? We found most of the theoretical statements about referral processes silent regarding any job-specific dependencies for \( c \), and consistent with the job-invariant view. An alternative view along with some empirical evidence explicitly supports a job-specific definition of \( c \). These two distinct perspectives concerning the counterfactual each entail different dynamics for the segregating effects of referring.

**Dominant view**

**Theory**

The dominant view of the association between referral hiring and job segregation posits that the fact of homophily in referral networks entails that male referrers will generate a predominantly male set of referral applicants, and that female referrers will generate a predominantly female set of referral applicants. Given this tendency, referral recruitment can keep male dominated jobs male, and female dominated jobs female. We use the following quotes to illustrate this point:

- “Current employees and others who distribute information about [a job] opening through interpersonal channels will tend to pass it along to socially similar persons” (Marsden 1994:981);
- “[M]ore often than not, members of workers’ networks are their same sex and race” (Reskin and Padavic 2002:81);
• “Occupational sex segregation persists in white-collar jobs in part because information networks are sex segregated” (Roos and Reskin 1984:245).

• “Social networks tend toward closure along most dimensions; hence, persons hired through networks tend to be socially similar to those referring them” (Marsden and Gorman 2001:471).

The first two quotes simply emphasize the segregated nature of job referral networks. The third quote explicitly asserts an association between segregated referring networks and the perpetuation of job segregation. The final quote is the most specific regarding the generation of referral applicants. If referral hires tend to be socially similar to their referrers (net other individual characteristics such as human capital), then it follows that referrers tend to generate socially similar referral applicants. In other words, male referrers tend to generate male referral applicants and female referrers tend to generate female referral applicants. The dominant view is silent with regard to any changes in these tendencies contingent on job-specific factors.

Translating this theory into our model parameters, we see the dominant view entailing that:

1. $P$ and $Q$ are both larger than 0.5 (i.e., male referrers generate majority male referral applicants and female referrers generate majority female referral applicants), and

2. $P$ and $Q$ are independent of the particular type of job using referral recruitment.

What does the dominant view entail for the counterfactual parameter, $c$ and the homophily adjustment, $h$?

As the dominant view comes from literature largely emphasizing the job-seeker perspective, inferring an operationalization for $c$ (percent female among the referrals produced by unbiased referring) and $h$ (the homophily adjustment away from $c$) from this body of work is necessarily an interpretive endeavor. We work backwards from the two consequences above to determine the implications of the dominant view for the referrer-based perspective of referring.

One way to view the effect of random or unbiased referring would be to consider the result if a referrer were able to refer every individual in the labor market to apply for the job – who would apply? The dominant view might suggest that a representative subset of referrals would apply, and thus unbiased referring would generate referral applicants who are representative of the labor market as a whole. One
variation on this idea might suggest unbiased referral applicants would be representative of the unemployed (thus presumably job-seeking) labor pool, rather than the whole workforce. U.S. Census Bureau data (2002) suggests that in either case the gender composition of the counterfactual, \( c \), would be close to 0.47 (and with standard deviations of less than 0.05 when looking across the 280 MSAs available in the Census sample file). In this case, both \( c \) and \( 1 - c \) are close to 0.5. Consequently, in order for the referral homophily parameters \( P \) and \( Q \) both to be greater than 0.5, \( h \) can be any positive number between about 0.04 and 0.47. The dominant view of referring dynamics does not make any explicit claims affecting the structure of \( h \), or whether it should be the same or different by sex. With this operationalization, we can reproduce the characteristics of the dominant view: men tend to refer men and women tend to refer women in a manner that is independent from job-specific variation.

Although we presented one possibility for how the dominant view would construct \( c \) and \( h \), we cannot be certain of our interpretation. We do know that the dominant view’s construction of referral homophily as independent from particular jobs entails that \( c \) and \( h \) are properties of the labor market itself and not of the jobs from which the referrers engage in their referring behaviors.

Another way to consider the referral-based implications of the dominant view is to review related empirical research. Empirical cases finding that men tend to refer women or that women tend to refer men would present a challenge to the dominant view.

**Evidence**

An empirical test of the implicit propositions from the dominant view would require identifying the sex of both parts of the referrer-referral applicant dyad for a representative sample of referral applicants and job titles within a labor market. Although we found no empirical studies with this specific design, there are two\(^5\) that are informative. Based on original data surveying job-seekers on the sex of up to three referrers who alerted them to job opportunities while the respondents were still employed, Torres

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\(^5\) A third empirical paper did report same-sex and cross-sex referrals in sex-specific percentages of referrals (Leicht and Marx 1997), but did so in a manner such that the same-sex referral probabilities for men and women were not calculable.
and Huffman (2004) identified the sex-composition of a quasi-representative\textsuperscript{6} set of referrer-referral dyads from a cross-section of white collar jobs in California. Their survey asked only about job leads, with no information as to whether the referral actually applied for the job about which they were alerted by the referrer. Calculations on the data presented by Torres and Huffman (2004) provide estimates of the referral homophily parameters $P$ (SSR probability for men) and $Q$ (SSR probability for women) as 0.79 and 0.53, respectively. That is, of the dyads where men were the referrers, 79% referred other men; and of the dyads with a female referrer, 53% had a female referral. This finding supports the idea that men tend to refer men and women tend to refer women when looking at referrer-referral dyads across many different jobs. However, this data cannot assess whether this tendency is robust within job categories.

A second empirical study provides additional insights. Using data from the 1982 survey of the National Longitudinal Survey of Youth (NLSY), Jacqueline Berger (1995) reported the gender-similarity of hired referrals and their referrers for the ten most common job titles across the four possibilities of male-female dyads. Here, the referrals were those who had already been hired into the job responding with the sex of their referrer.\textsuperscript{7} The data presented in the paper allows bounded estimates of both referral homophily parameters, $P$ and $Q$, for 5 job titles.\textsuperscript{8} The dominant view suggests that $P$ and $Q$ should both be above 0.5 for all jobs. In addition, the dominant view is consistent with the proposition that the various estimates of $P$ should be similar in magnitude across jobs, as well as the same similarity for the various estimates of $Q$. Table 1 presents the estimates for $P$ and $Q$ derived from Berger’s (1995) work.

The $P$’s and $Q$’s shown in Table 1 challenge both of the propositions from the dominant view. First, there are several SSR probabilities that are near or less than 0.5. Across the five jobs, $P$ (SSR

\textsuperscript{6}The data from this paper have important limitations regarding their generalizability as they were generated by a group of white-collar workers in California who elected to join a specific job-search club following a layoff or other involuntary unemployment event. These limitations were noted in the paper (Torres & Huffman, 2004:22-23).

\textsuperscript{7}The risk of selection bias from using hired referral applicants instead of all referral applicants is precisely the risk that hiring decisions have some sex biases. For a discussion of the pitfalls of using post-hire data to investigate a pre-hire phenomenon, see Fernandez and Sosa (2005).

\textsuperscript{8}We consider only those jobs for which we could derive an estimate for the number of entries in all four of the following categories: women referring women, women referring men, men referring men, and men referring women. In some cases, we used the absence of a job from one of the categories as evidence that there were fewer occurrences than the least number in the top ten.
probability for men) has a range from below 0.41 (secretary) to greater than 0.89 (miscellaneous machine operator). The range for Q (SSR probability for women) is from 0.49 (janitor, cleaner) to greater than 0.92 (secretaries). Further, the magnitudes of \( P \) and \( Q \) appear to vary with the degree to which the corresponding jobs are male-dominated or female-dominated. Notably, for female-dominated jobs (cashier, secretary), men appear to tend to generate a majority female referrals. Given this empirical challenge to the implications of the dominant view, we sought an alternative operationalization of referring dynamics from the referrer’s perspective.

**Alternative view**

**Theory**

The dominant view’s formulation of referral hiring and job segregation emphasizes the importance of homophilous networks and sets aside the fact that job specific factors also affect gendered application rates. Would male referrers naturally generate predominantly male referral applicants for a job in a heavily female field such as nursing? Would female referrers naturally generate predominantly female referral applicants for a disproportionately male job such as computer programming? The evidence presented in Table 1 from Berger (1995) suggests otherwise.

The referrer-referral dyad is not merely the referrer and any other individual with whom she shares job opportunity information, but the referrer and those individuals who act upon that information and become a referral applicant. This insight is reflected in the following passage where the authors react to an earlier statement about the gendered effects of networks in the labor market:

“For gender, Roos and Reskin (1984, p. 245) provide, ‘With respect to sex segregation, the questions of interest are whether the sexes have equal access to personal networks, whether they are equally likely to use them, and whether networks are equally effective for women and men.’ It is not only a question of having access, but also one of utilizing it and of its eventual success.” (Petersen, Saporta and Seidel 2000:768, emphasis added)

Once the referrer-referral dyad is defined as being formed by the application of a referral applicant (as is necessary when looking from the perspective of the organization and its referrers), the
possibility of job-specific variations in the same-sex referring probabilities $P$ and $Q$ emerges. Notably, gendered differences in application rates become relevant when trying to establish the counterfactual baseline necessary for defining homophily.

While there are a multitude of demand-side or organization-based reasons that could affect the demographic composition of applicants (Braddock and McPartland 1987; Kmec 2005; O’Farrell and Harlan 1984), there are unquestionably supply-side or other organizationally exogenous factors also affecting the demographic composition of applicants for different jobs. Job typing (Kmec 2005; Reskin 1993), job queuing (Moss and Tilly 2001; Reskin and Roos 1990), socialization to gendered career aspirations (Marini and Brinton 1984), socio-cultural norms regarding the distribution of domestic versus wage labor (Fuwa 2004), rational choice (Rosen 1997), and individual preferences (Okamoto and England 1999) are only a few examples of organizationally exogenous factors that might affect the gender composition of applicant pools for specific jobs. Some factors are complicated interactions of exogenous and endogenous forces. For example, the labor market as a whole influences the wage for any particular job, although the individual organization can play a role in wage-setting. The wages of a job, in turn, influence the demographic composition of applicants. For our purposes, we need not know precisely why men and women do not apply for the same jobs at the same rates, only that this phenomenon exists, and that these mechanisms could very well affect the composition of referral applicants in ways similar to their effects on non-referral applicants.

Differential application rates have important implications for the definitions of the same-sex referring probabilities $P$ and $Q$ in terms of referral homophily. The counterfactual composition of referral applicants produced by unbiased referring would not be some property of the labor market, but those potential referral applicants who survive the filter of the host of supply and demand side mechanisms affecting application rates. Although this composition may be difficult to estimate directly, an easy-to-measure proxy exists for most organizations: the population of non-referral applicants.
Evidence

Researchers Fernandez and Sosa (2005) first offered this insight that non-referral applicants could serve as the appropriate baseline for comparison when evaluating referral homophily. The idea is that formal recruitment methods are likely to be less biased than referral recruitment (Braddock & McPartland, 1987; Reskin, McBrier & Kmec, 1999), and provide an indicator of the likely composition of the counterfactual applicant pool produced by unbiased referral recruitment. The composition of non-referral applicants then provides an indicator of the net effect of all the forces except referring that result in gendered application rates.

The empirical findings presented (Fernandez and Sosa 2005:878, Table 3) were consistent with their argument. We reproduce the relevant elements of their table here as Table 2a, and indicate how their empirical case corresponds to a specification of referral homophily parameters $P$ and $Q$, as well as their constructors, $c$ and $h$. We note that the empirically-observed values for both $P$ and $Q$ (43.7% and 75.1%, respectively) may be generated using the percent female among non-referral applicants (65%) as the value of $c$, and about 10 percentage points (9.4±0.7) as the homophily adjustment $h$. We also note that this finding presents another empirical case challenging the dominant view in that the male referrers did not generate a majority male group of referral applicants.

Seeking to replicate this finding, we constructed a similar table using a dataset taken from a different empirical setting. (For a detailed discussion of this setting, see Fernandez and Fernandez-Mateo 2006:47-51).

We present the findings here as Table 3b. Again the empirically-observed values for both $P$ and $Q$ (men’s SSR probability: 55.7% and women’s SSR probability: 68.2%, respectively) may be generated using the percent female among non-referral applicants (57.4%) as the counterfactual composition $c$ parameter, and a homophily adjustment $h$ of about 12 percentage points (11.95±1.15).

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Note that this additional dataset is not restricted to a single job-title, but rather represents all of the applicants to the multiple entry-level jobs in this firm. Applicants did not apply to specific jobs, but rather just applied to work at the firm. So even from the referrer’s perspective, referrers would generate referral applicants for an entry-level job application, and not for a specific job.
In the referral homophily data from Fernandez and Sosa (2005), we can see that both men and women generated referral applicants that were more similar by sex relative to the baseline of non-referral applicants. In that case, the homophily adjustment was approximately 10 percentage points for both men and women. Our attempt to replicate this finding in Table 3b reveals a similar positive homophily adjustment for men and women, this time of approximately 12 percentage points (13 points for women, 11 points for men). In both cases, we find support for referral homophily, but this homophily must be viewed from the appropriate baseline: in the ideal case, applicants from unbiased referring, but in the practical case, non-referral applicants.

In casting the referral process in terms of the referrer and from the perspective of the organization, we have identified this conflict in defining referral homophily. The dominant view claims that male referrers generate more male referrals and female referrers generate more female referrals in a manner independent of the particular job. This view is supported by a variety of theoretical claims in the literature, but is challenged by some empirical findings. An alternative view, using non-referral applicants as a proxy for the counterfactual baseline to assess referral homophily, claims homophily acts as an adjusting factor to job-specific application rates. These distinctions have crucial implications for the segregating effects of referring, as we will show in the next section.

**The model**

We conceptualize referring as an example of a population process – that is, a process by which the current population generates new members of the population. Here, the population is job holders, and the reproductive process is referring. Indeed this analogy to population dynamics prompted Rosabeth Moss Kanter to coin the term “homosocial reproduction” (1977: 63) in referring to the informal processes by which male managers maintained the gender composition of their jobs.

Our model makes use of a common analytical tool applied to population processes, namely a Markov process (Keyfitz and Caswell 2005; Kingman, 1969). Although the sociological literature has seen a wide application of Markov models for social processes particularly in the area of social mobility
(cf. Singer and Spilerman 1976:1), there are a number of examples of modeling labor market processes as Markov processes (Lieberson and Fugitt 1967; Montgomery 1994). There is even a recent paper specifically using a Markov model to investigate the association between referring and job segregation (Tassier 2005).\textsuperscript{10}

**Basic functioning and Dynamics**

First, consider a one-job organization composed of men and women who generate the next generation of organizational members through referring. The percent male among new organizational members generated by men is $P$, and $1 - P$ of the men-generated members are female. The percent female among new organizational members generated by women is $Q$, and $1 - Q$ of the women-generated new members are male. These proportions allow us to define a transition matrix $M$ giving the male and female composition of referrals entering the job as generated by the referring behaviors of the employees already in the job. The transition matrix $M$ is defined in equation (1) below, and the corresponding Markov process model is presented in Figure 1a.\textsuperscript{11}

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M = \begin{pmatrix}
    P & 1 - Q \\ \\
    1 - P & Q
\end{pmatrix}
$$

The simplest application of this transition matrix assumes that referring is the only route for recruitment, every job holder refers, that all referral applicants are hired, and each new generation of hired applicants completely replaces the previous generation. We explore this highly restricted model to cultivate an intuition for the dynamics. Then, we will relax many of these unrealistic assumptions.

\textsuperscript{10} That particular model makes the explicit assumption that contact networks are perfectly segregated, thus $P$ and $Q$ would both equal 1. The resulting analysis finds associations between referring and segregation only when looking across firms within industries. Research has shown that segregation increases with the level of specificity up to the level of the job (Bayard et al. 2003; Jacobs 2001; Kmec 2003), and our outcome of concern is job segregation. As a result, this previous analysis and its associated assumptions are not directly germane to our study.

\textsuperscript{11} We note that we are modeling this population process as a first-order Markov process; that is, a *memoryless* process where the sex of the referral is not influenced by the sex of the referrer's referrer. This implementation choice is largely one of erring on the side of simplicity. As a testament to the under-theorized and under-researched nature of referring dynamics, we could find neither theoretical nor empirical statements addressing whether or not such second-order influences might exist. Further, the temporal dynamics of our model are tantamount to a job where prior to time zero, the job has recruited applicants via unbiased referring only, and at time zero, the types of homophily biases observed in referral recruiting switches on. The change in percent female from time zero on is the necessary the result of the modeled homophily in referring processes and no others.
We define \( j_t \) to be a 2-element column vector (men, women) consisting of the count of men and women in a single job at time \( t \). The initial count of men and women within the job is given by \( j_0 \). The matrix product \( Mj_t \) gives the count of men and women in the job's next generation, \( j_{t+1} \). Each application of the transition matrix to the job's latest generation returns the next generation. If there is some job composition \( j^* \) such that \( Mj^* = j^* \), then the job is at an equilibrium defined by the transition matrix. This equilibrium composition, \( j^* \) is called a fixed column vector of \( M \). The fixed column vector is by definition the eigenvector of \( M \) when \( M \) has an eigenvalue equal to one (Grinstead and Snell 1997).

Our definition of the transition matrix mathematically entails that one is an eigenvalue, for any possible values of \( P \) and \( Q \) (proof in appendix). Therefore, the transition matrix \( M \), defined by the proportion of same-sex referral applicants generated by male and female referrers, determines the equilibrium composition of the job. Thus, there will always be an equilibrium job composition, and that composition is completely determined by the parameters \( P \) and \( Q \).

As a result, we can write formulas for the equilibrium percent female of the job, denoted by \( f^* \) and for the equilibrium percent male, denoted by \( m^* \) in equations (2) and (3), respectively.

\[
f^* = \frac{1 - P}{2 - P - Q}
\]

\[
m^* = \frac{1 - Q}{2 - P - Q}
\]

**Equivalence under less stringent assumptions**

This initial representation of the referral process from the perspective of an organization makes a number of drastic assumptions. First, referring is the sole source of new organizational members. Second, all job holders engage in referring. All referral applicants are hired, and each new generation of job holders completely replaces the previous generation.

Does the convenient analytical solution to our model hold when we relax some of our constraining assumptions? We modify the transition matrix to relax all assumptions but the first. Because we are interested in elucidating referral processes, modeling a system where referrals are the sole source
of applicants helps to highlight referring dynamics. We introduce three new parameters: \( r \) is the proportion of organizational members engaging in referring, \( s \) is the selection-for-hire rate of referral applicants, and \( x \) is the exit rate of current organizational members. Each of these three new parameters may take values from 0 to 1, inclusive. With the addition of these parameters, the transition matrix can account for less-than-universal referring, less-than-universal hiring of applicants, and employee retention as well as turnover. The modified transition matrix is defined in equation (4) below, and the corresponding Markov process model is presented in Figure 1b.

\[
M = \begin{bmatrix}
rsP - x & rs(1 - Q) \\
rs(1 - P) & rsQ - x
\end{bmatrix}
\] (4)

Although the introduction of these parameters can change \( M \)’s eigenvalues, it does not change the equilibrium eigenvector, \( j^* \) (proof in appendix). With the possibility of an eigenvalue other than 1, the equilibrium relation becomes \( Mj^* = \lambda j^* \), where \( \lambda \) is the largest eigenvalue of \( M \). Consider the meaning of \( \lambda \) being other than one. When the composition of the job is \( j^* \), the composition of the job at the next time step will be \( \lambda j^* \). When \( \lambda \) is greater than 1, the job will still have the same proportion of male and female employees, but the total population will increase by the scaling factor \( \lambda \). Similarly, when \( \lambda < 1 \), the total population of the job decreases as determined by \( \lambda \), but the proportion male and female remains the same.

Thus, we see that this new set of parameters does not change the equilibrium percent men and women of the job, only the job’s size (which either grows to infinity or shrinks to zero depending on the value of \( \lambda \)). When \( rs = x \), the job is stable, neither growing nor shrinking, so when \( rs = x \), 1 will always be an eigenvalue of \( M \). So even with this more inclusive transition matrix, there is an equilibrium in the percent female and male in the job, completely determined by referral homophily parameters \( P \) and \( Q \), and given by the same formulas (2) and (3) from above. Rather than affecting the final equilibrium percent female and male in the job, the three parameters \( r \), \( s \), and \( x \) simply affect how quickly the job reaches that equilibrium.

\footnote{In a separate paper, we use an agent-based model to compare the segregating effects of referring with those of screening bias. We find that for a large swath of the labor market referring can have segregating effects comparable to or even exceeding those of empirically documented screening bias levels (Rubineau and Fernandez 2006).}
Identifying the equilibrium composition of the job resulting from referral processes reveals the direction of the effect of referring on the composition of a particular job. For example, if a job's initial (male, female) composition is (100, 100) and the equilibrium composition determined by the transition matrix is (150, 50), we can say referring acts to make the job more male. Similarly, if the equilibrium composition is (80, 120), we can say referring acts to make the job more female. We do not claim that referral processes will bring the job to that equilibrium position. Rather, we use the equilibrium position relative to the original composition of the job to reveal the direction of the effects of referring.\textsuperscript{13}

It is important to note that the new parameters assume unbiased selection and turnover as well as the same rates of referring for men and for women. If selection or turnover rates were biased such that there were sex differences net all other individual characteristics, this would be a clear example of discrimination. Our model begins with the ideal job where these biases have been eliminated. As a result, any and all segregation observed may be attributable to referring processes – the focus of our study. Later in this paper, we will relax the assumption regarding equal rates of referring for male and female job holders. Because the different definitions of $M$ do not affect our outcome of interest, job segregation, we proceed using the simpler version of the transition matrix defined in equation (1).

\textbf{Analysis}

\textbf{Analysis of the Dominant View of Referring Dynamics}

As discussed above, the dominant view holds that $P$ and $Q$ are both above 0.5 and independent of job-specific factors. We now explore the consequences of this view in terms of our model.

If $P$ and $Q$ are set in the labor market and not influenced by job-specific factors, then this yields a single transition matrix $M$ for the entire labor market. Given this single transition matrix $M$ applying to all jobs in the labor market, and the fact that the equilibrium composition of the job $j^*$ is independent of the initial composition of the job, it follows that the dominant view entails that referring processes push all jobs in the labor market to the same final job composition $j^*$.

\textsuperscript{13} In fact, the dynamics of the model are to monotonically approach the equilibrium composition (Grinstead & Snell, 1997: 449) and do so very quickly – in geometric time (Bremaud 1991:136).
We can see this result mathematically if we consider a labor market composed of $N$ jobs, the composition of each job at time $t$ is given by a column vector $j_{it}$ ($i=1..N$) consisting of the number of men and number of women on that particular job at that particular time. The 2-by-$N$ matrix $J_t$ gives the composition of all jobs in the labor market at time $t$. The transition matrix $M$ is defined as before. Given this modified construction, and because the same transition matrix $M$ applies to all jobs, it is easy to see that $MJ_t = J_{t+1}$. Iterating through the generations through repeated applications of the transition matrix results in an eventual equilibrium $J^*$ where all the column vectors comprising $J^*$ are equal to $j^*$. Thus a single transition matrix for the entire labor market has the result that referral processes push all jobs towards the same equilibrium composition.

We now consider some other specifications of the transition matrix $M$ and their resulting equilibrium compositions. First, if $P=Q=1$ (both men and women generate exclusively same-sex referral applicants), the final percent female (and male) for the labor market are undefined based on equations (2) and (3). In fact, in this case of perfect homophily for both men and women, all jobs exactly reproduce themselves with each generation. In this extreme case, $J^* = J_0$. Although this result allows the dominant view to have a result where referring preserves the segregation of jobs, we reject the explanatory success of this extreme case. There is neither empirical evidence supporting nor strong theoretical claim for the idea that perfect homophily is a reasonable characterization of actual referral dynamics in the labor market. Rather, none of the empirical settings we have examined has exhibited perfect homophily. Note also that if only one of $P$ or $Q$ equals 1, and the other is anything less than 1, then the equilibrium is the corner solution where the job goes to all male or all female, respectively.

Another necessary result from equations (2) and (3) is that whenever referral homophily parameters $P$ and $Q$ are equal to each other (men and women have the same probability for generating same-sex referral applicants), and in the range (0,1), exclusive, the final percent female (and male) is 50%. Even when both $P$ and $Q$ are above 0.5, as the dominant view suggests is a requirement for referring to segregate, if they are equal, even some arbitrarily high value such as 0.95, the result of
referring will be to integrate all jobs in the labor market. This result is directly contrary to the prevailing wisdom regarding the consequences of referring. Still, this is the necessary result of referring dynamics.

This surprising and counterintuitive result may be made clear with an illustration. Consider an all-male job, and same-sex referring probabilities of 0.95 for both men and women. Although these men generate referrals who are 95% men, 5% are women, making the job more female than the initial composition. These men continue to generate referrals who are 95% men and 5% women, so the referrals from men are always a little less male than the composition of the referring men. At the same time, the new women on the job generate referrals who are 95% women and 5% men. At each step, the job becomes more female than before. This process continues, but cannot make the job all-women or even female-dominated. The majority group will continue to generate members of the minority, who themselves increase their relative composition. The equilibrium position for these dynamics is an exactly equal composition of men and women. At this point, for each new female referral generated by women, there is a new male referral generated by men, and vice-versa. This structural solution is very similar to an earlier analysis showing that the prevailing wisdom concerning the reproduction of gendered norms of nurturing could not in fact explain that outcome (Jackson 1989).

When $P$ and $Q$ are not equal to each other, the final percent female may be something other than 50%. Whatever the final composition, all jobs in the labor market – both male-dominated and female-dominated – move towards this final point. Whenever $P$ is greater than $Q$ (men have a higher probability of producing same-sex referral applicants than women), all jobs drawing upon the labor market are moved towards a majority male equilibrium. Necessarily, this implies that referring acts to integrate all female-dominated jobs in the labor market. In addition, any jobs that are more male-dominated than the equilibrium majority male composition will be made less male-dominated through the operation of referral processes. In fact, the only jobs in the labor market that would be further segregated from referring would be those slightly male-dominated jobs that are between 50% male and the equilibrium composition.
We see analogous results when $P$ is less than $Q$, and all jobs in the market are moved towards a majority female equilibrium. In this case, referring acts to integrate all male-dominated jobs and jobs more female-dominated than the equilibrium. This finding suggests that the dominant view entails that although referring can segregate some relatively integrated jobs, referring tends to integrate most jobs in the labor market. That is, a female-dominated equilibrium entails that referring acts as an integrating force for all male-dominated jobs and for those female-dominated jobs that are more segregated than the equilibrium point. Necessarily, the dominant view's explanation of referral processes cannot realistically result in dynamics which simultaneously maintain the segregation of male-dominated and female-dominated jobs. Figure 2 presents graphically this tendency of referral processes as defined by the dominant view to integrate most jobs in the labor market for an arbitrary equilibrium composition (35% female, 65% male). In Figure 2, we see that referring acts as an integrating force for all the female-dominated jobs and for those male-dominated jobs that are more male-dominated than the equilibrium composition (jobs are represented as dots on a line giving the continuum of possible initial job compositions). The only jobs that are segregated through referring processes are those that are slightly male-dominated, but less male-dominated than the equilibrium composition. The grayed portion of the continuum of jobs experience referring as an integrating force, while jobs in the smaller white portion of the continuum experience referring as a segregating mechanism.

Our analysis of the consequences of the dominant view of referring dynamics regarding job segregation leads us to reject this formulation of referring dynamics. Under this framework, unless referral homophily is in all cases perfect, referring not only cannot maintain levels of job segregation, but referring would actually serve as an integrating force for the majority of the jobs in the labor market. We now turn to a similar analysis of the alternative view of referral dynamics, predicated on referral dynamics being contingent upon job-specific factors.
**Analysis of the Alternative View**

Whereas the dominant view was consistent with an explanation of referral homophily that left no role for organizational influence, the alternative view defines referral homophily explicitly as a function of job-specific factors. Whereas the dominant view focused on homophily in the ties between referrers and job-seekers, the alternative view focuses on homophily in the ties between referrers and *successful* referrals (using again the words of Petersen, Saporta and Seidel 2000: 768) – that is those alters who survive the many gendered filters prior to job application and actually become a job applicant. The counterfactual percent female of applicants produced by unbiased referring, \( c \), must be the net gendering effect of all pre-application labor market forces *except* referring. Naturally, some job-specific factors are included in this broad set of mechanisms.

Whereas the invariance of \( P \) and \( Q \) across the labor market in the dominant view allowed the use of a single transmission matrix, \( M \) for the entire labor market, the alternative view does not allow this wholesale treatment. Specifically, each job has its own counterfactual composition of non-referral applicants, the percent female of which is denoted by \( c \). If we have \( N \) jobs, then we indicate the job-specific nature of our model parameters using again the index \( i (i=1..N) \). This alternative view of the labor market suggests that each job has its own transition matrix, \( M_i \), composed of job-specific values of \( P_i \) and \( Q_i \), which are the result of the equations \( P_i = 1 - c_i + h \), and \( Q_i = c_i + h \).

Our knowledge of the structuring of the homophily adjustment parameter, \( h \), is limited by the evidence from our two empirical cases presented in Table 2a and Table 2b.\(^{14}\) Although in neither of the empirical cases were the observed \( h \) for men exactly equal to the \( h \) for women, we do not have the data to test for significant differences in these values. In both cases, the sex-specific deviations (±0.7 and ±1.15) from the mean value of \( h \) (9.4 and 11.95) were small relative to that mean value – less than 10% (7.4% and 9.6%, respectively). Because we lack clear evidence supporting sex asymmetries in the homophily

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\(^{14}\) In the context of our model, there are two ways we could have \( h \) vary: by job or by sex. Because \( c_i \) is intended to capture all of the job-specific sex biasing effects of recruitment except biased (homophilous) referring, we do not have \( h \) vary by job. In fact, we suggest that the arguments used in the dominant view are more appropriately applied solely to the homophily adjustment to referring, rather than referring as a whole.
adjustment, because we wish to avoid attributing a potentially significant causal role to a new and uninvestigated parameter, and because modeling \( h \) as invariant to sex is more parsimonious, we proceed with our analysis setting \( h \) as a constant (and explore the implications of this choice later in the paper).

Evaluating whether this choice is an accurate one requires future empirical research.

Given our operationalizations of \( c_i \) and \( h \), we can rewrite the transition matrix \( M_i \) as in equation (5), with the analogous Markov diagram again provided in Figure 1a (using the initial \( P \) and \( Q \) notation).

\[
M_i = \begin{bmatrix}
1 - c_i + h & 1 - c_i - h \\
c_i - h & c_i + h
\end{bmatrix}
\]  

(5)

The eigenvector of \( M_i \) when the eigenvalue is 1 defines \( j^*_i \), which we can use to determine the equilibrium percent female of the job, \( f^*_i \). The solution is a repeat of equation (2) with a substitution of terms, given as equation (6).

\[
f^*_i = \frac{c_i - h}{1 - 2h} = \frac{1}{1 - 2h} c_i - \frac{h}{1 - 2h}
\]  

(6)

The denominator in equation (6) imposes the constraint that \( h \neq 0.5 \). This result is a carry-over from our definition of \( P \) and \( Q \) in terms of \( c_i \) and \( h \) along with the restriction that \( P \) and \( Q \) are limited to values in the range: (0,1).

One of the first implications of equation (6) is that when there is no homophily adjustment to referring (\( h = 0 \)), the final percent female on the job equals the percent female from the population of non-referral applicants (\( f^*_i = c_i \)). That is, the slope\(^{15} \) associated with the counterfactual \( c_i \) in yielding the final percent female of the job, \( f^*_i \), is exactly one. In a job where all other bias has been eliminated, this is the sensible result.

A second consequence of equation (6) is that when the counterfactual composition of non-referral applicants is 50:50 (\( c_i = 0.5 \)), then the equilibrium composition is 50% female for all possible values of \( h \). That is, any level of homophilous (or heterophilous) referring will still generate a 50:50 equilibrium

\(^{15} \) The final percent female on the job, \( f^*_i \), is linear with \( c_i \), with a slope of \( 1/(1 - 2h) \). The intercept \((-h/(1 - 2h)) \) adjusts such that \( c_i = 0.5 \) is a kind of pivot point, present for all the solution lines relating \( f^*_i \) and \( c_i \). A few solution lines for this formula are presented graphically in Figure 3.
composition for a job that has perfect parity in its counterfactual (where unbiased referring generates a 50:50 composition of applicants).

As Figure 3 illustrates, when $h$ is positive (a positive homophily adjustment), the slope relating $f_i^*$ and $c_i$ is greater than one (and approaches infinity as $h$ approaches 0.5). This means that when $c_i > 0.5$ (the counterfactual composition is female-dominated), $f_i^* > c_i$, or that the job is becoming more female. When $c_i < 0.5$ (male-dominated), $f_i^* < c_i$, that is, the job becomes more male. This behavior is precisely that which is commonly understood in associations between referring and segregation: male-dominated jobs remain or become more male, and female-dominated jobs remain or become more female. We see that the alternative view, with its job-specific formulation of referring dynamics, is able to generate consistently segregating effects from referral processes.

Figure 3 also depicts the effects of heterophily, or when the biasing component of referrer-referral tie formation encourages cross-sex ties.\(^{16}\) We represent heterophily using negative values of $h$. This heterophily has the effect of dampening the slope relating $f_i^*$ and $c_i$. Specifically, for all negative values of $h$, the slope relating $f_i^*$ and $c_i$ remains positive, but approaches zero as $h$ approaches negative infinity. So heterophily alone can serve to lessen the segregation of a job, but cannot change a male-dominated job to a female-dominated one or vice-versa. The increasingly negative values of $h$ shown in Figure 3 reveal this dampening effect on the slope.

Our analysis reveals that this alternative view, with job-specific variations in the structuring of referral processes, is successful in explaining the segregating effects of referring. If the segregating effects of referring depend on job-varying factors, then it is reasonable to consider the job or firm as a primary target for efforts seeking to eliminate these segregating effects. Previous literature has emphasized the importance or desirability of limiting, or to the extent possible, eliminating informal recruitment via referrals to prevent its segregating effects (e.g., Braddock and McPartland 1987, Reskin and McBrier 2000). If these recommendations were based on the false assumption that the segregating effects of

\(^{16}\) See Heckathorn and Jeffri (2001:312) for empirical examples of heterophily in the referral recruitment scenario of sampling the hard-to-measure population of HIV+ individuals.
referring are exogenous to the firm, perhaps there have been overlooked opportunities for intervention. In
the next section, we extend our model to accommodate some empirical findings, and identify an
opportunity for organizations to manage the segregating effects of referring.

**Tipping Points**

We mentioned above that sex biases in hiring, exit, or referring behaviors could change the
equilibrium composition from our analyses. We explained why hiring and exit biases are beyond the
scope of our current analysis, but referring behaviors certainly is germane to our analysis. So far, we
have made an implicit assumption that men and women are equally likely to engage in referring. (We note
that Tassier’s [2005:236] Markov model of referring and labor market segregation makes this identical
assumption.) Is this assumption warranted?

There have been several empirical studies showing that referring behavior can vary significantly
by sex and race (Elliott 2001; Fernandez and Fernandez-Mateo 2006; Fernandez and Sosa 2005). In
addition to the empirical evidence, there is a large body of indirect theoretical support for sex
asymmetries in referring behavior.\(^{17}\) The theoretical claim is not necessarily for any particular direction in
the asymmetry, but that consistent male-female parity in referring would be surprising. In this section we
relax this earlier assumption and explore the implications of sex asymmetries in referring.

We implement sex asymmetries in referring rates by adding a new parameter to our model, \(r_{i}^{\text{asym}}\).
The male referring rate becomes \(r_{i} - r_{i}^{\text{asym}}\), and the female referring rate becomes \(r_{i} + r_{i}^{\text{asym}}\).\(^{18}\) This
formulation allows the referring rates for males and females to be distinct: \(r_{i}\) becomes the central
tendency of referring for job \(i\) (i.e., the unweighted average of the male and female referring rates), while

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\(^{17}\) In the realm of behavior within the organization, examples of sex differences in network structure include
instrumental and social support networks (Ibarra 1992, 1997; Stackman and Pinder 1999) and peer and mentoring
relationships (Ely 1994; McGuire 2000, 2002). Beyond organizations, researchers have found sex-specific
homophily effects in recruitment to Injection Drug Use (IDU) (Heckathorn 2002), in telephone usage (Smoreda and
Licoppe 2000), and in socializing behavior among school-aged children (Hanish et al. 2005; Kiesner, Poulin and
Nicotra 2003; Nangle et al. 2004).

\(^{18}\) The choice of which is the result of addition and which of subtraction is unimportant. We chose this convention
because of the empirical case finding that females referred more than males (Fernandez and Sosa 2005).
$r_{i}^{\text{asym}}$ is the difference between the female referring rate and $r_i$. We make $r_{i}^{\text{asym}}$ job-specific because there is empirical evidence of job-specific variation in this asymmetry. Although Fernandez and Sosa (2005) found significant sex differences in referring rates, Fernandez and Fernandez-Mateo (2006) found no such significant differences (although they did find significant difference in the referring rates by racial categories). When $r_{i}^{\text{asym}} > 0$, women tend to refer more than men, and when $r_{i}^{\text{asym}} < 0$, men refer more than women, and when $r_{i}^{\text{asym}}=0$, there is no difference in the referring rates of men and women. Using this new parameter, we define a modified transition matrix, $M_{i}^{\text{asym}}$ below in equation (7); also illustrated as a Markov process diagram in Figure 1c using the $P$ and $Q$ notation from before.

$$M_{i}^{\text{asym}} = \begin{pmatrix}
(r_i - r_{i}^{\text{asym}})(1 - c_i + h) & (r_i + r_{i}^{\text{asym}})(1 - c_i - h) \\
(r_i - r_{i}^{\text{asym}})(c_i - h) & (r_i + r_{i}^{\text{asym}})(c_i + h)
\end{pmatrix}$$

The $f_i^{*}$ equilibrium composition entailed by $M_{i}^{\text{asym}}$ still allows the calculation of a closed-form solution for the final percent female in the job at equilibrium, $f_i^{*}$, albeit a much more complicated one. The equilibrium percent female is now a function of four parameters rather than two. As a shorthand (with the full equation available in the appendix), we denote the function giving $f_i^{*}$ from the four parameters as $g$, written: $f_i^{*} = g(r_i, r_{i}^{\text{asym}}, c_i, h)$. When $r_{i}^{\text{asym}}=0$, the system reduces to that which we have already described, such that $f_i^{*} = g(r_i, r_{i}^{\text{asym}}=0, c_i, h) = (c_i-h)/(1-2h)$. We illustrate the dynamics of $g$ first by holding two parameters constant: $r_i$ – the central tendency for referring, which we set at 0.25, and $c_i$ – the counterfactual percent female among non-referral applicants, set to 0.65 (female-dominated).\(^{19}\) Figure 4 graphs $f_i^{*} = g(r_i =0.25 , r_{i}^{\text{asym}}, c_i =0.65 , h)$, and adds a dotted plane at the 0.65 level to help indicate deviations from the counterfactual.

Figure 4 has several important and notable characteristics. First, when the homophily adjustment is zero (i.e., when the gender distribution of referrals equals the counterfactual gender distribution of non-referral applicants), the final percent female still equals the counterfactual percent female, and sex

\(^{19}\) These settings are approximately what was recorded in the empirical case study presented in Fernandez and Sosa (2005).
asymmetries in referring rates have no effect on the final percent female. The line giving the final percent female when the homophily adjustment is zero is parallel to the $r_{i}^{\text{asym}}$ axis. This finding is logically consistent. When the homophily adjustment equals zero, then referring is completely unbiased with regard to sex, so it should not matter whether men or women are referring more, and any such differences should not have any effect on the final percent female of the job.

As the homophily adjustment increases, the behavior becomes more complex. Most of the surface of Figure 4 is above the dotted plane, indicating that for most values of $r_{i}^{\text{asym}}$ and $h$, referring tends to push this female-dominated job towards an even more female-dominated equilibrium. But $r_{i}^{\text{asym}}$ has a transformative influence on these segregating effects of referring. Note the portion of the surface that is below the dotted plane. When $r_{i}^{\text{asym}}$ is very negative, this female-dominated job becomes less female through the action of referral processes. Increasing $h$ amplifies both the segregating and integrating effects of referring for a given level of $r_{i}^{\text{asym}}$. Importantly, note the intersection of the surface with the dotted plane, indicated with a line on the surface at the 0.65 level. This intersection, indicating the transition point where changes to $r_{i}^{\text{asym}}$ results in either segregation or integration, is a straight line. That is, referring needs some level of homophily adjustment for referring to segregate or integrate, but this critical value for $r_{i}^{\text{asym}}$, which determines whether referring will have a segregating or integrating effect, is the same all values of $h$. We call this critical value the tipping point for the segregating effect of referring. When $r_{i}^{\text{asym}}$ is greater than the tipping point, referring serves to make the job more female, and when $r_{i}^{\text{asym}}$ is less than the tipping point, referring serves to make the job more male. At the critical tipping point value of $r_{i}^{\text{asym}}$ itself, referring is neutral to segregation.

So far, this revelation of the tipping point has been achieved in the context of holding constant two of the four parameters of the function $g$. How does variation in these other two parameters affect the tipping point? Because the tipping point is the value of where $f_i^* = c_i$, we can re-write $g$ to solve for the tipping point: $c_i = g(r_i, r_i^{\text{asym}}, c_i, h)$, and solve for $r_i^{\text{asym}}$. Denoting this critical tipping point value $r_i^{\text{asym}}$ of
as $tp$, it is clear that $tp$ will be some function $k$ such that $tp = k(r_i, c_i, h)$ where $(h>0)$. Solving for this function $k$, we find the formula for the tipping point of referring, given in equation (8) below.

$$tp = r_i (1 - 2c_i)$$

As we observed in Figure 4, the tipping point is in fact independent of $h$. Still, $h$ cannot equal zero for either segregation or integration to occur. An actual firm can easily calculate the tipping point for a job. The tipping point depends only upon the mean rate of referring in the job, $r_i$ (calculable by a firm taking the quotient of total number of job holders generating referral applicants divided by the total number of job holders eligible to generate referral applicants), and the percent female among the counterfactual population of non-referral applicants, $c_i$ (calculable by a firm by documenting the percent female among non-referral applicants to a job). Given that referrer behaviors are within the realm of organizational influence, the identification of this tipping point for the segregating effects of referring presents an important new tool for organizations. If a firm can promote referring behavior among the minority group past the tipping point level, then referral processes can serve to integrate the job, rather than segregate. The policy of managing the behavior of referrers leverages existing homophily dynamics to counter job segregation.

Our analysis first explored the consequences of the dominant view of referring dynamics, and showed that this prevailing wisdom cannot explain how referring contributes to job segregation. When reconceptualizing the referral process from the perspective of the organizationally-based referrers, we identified an alternative view for structuring referral homophily in terms of job-specific factors. Not only was this alternative view more supported by published empirical findings, but it also successfully generates the kind of segregation dynamics that the literature has sought to explain. Resolving this implicit debate in the manner of structuring referral homophily, we expanded our model to incorporate empirical findings that referring rates may differ by sex. Through this additional analysis, we reveal that referring dynamics have a tipping point in referrer behaviors defined as a critical value of the sex

20 We note that this independence renders our decision to model $h$ as not being job-specific somewhat moot; had we modeled $h$ as a job-specific parameter, $h_i$, the results would be the same.
asymmetry in referring. Before reaching the tipping point, referring segregates as is commonly understood. At the tipping point, referring is neutral to segregation, and beyond the tipping point, referring can serve to integrate a job. We identified the formula for the tipping point, and demonstrated how firms could easily calculate the tipping point for their own jobs using data commonly available from hiring departments. In short, we have upended the prevailing wisdom regarding the association between referring and job segregation and introduced a more complete explanation of the phenomenon. We have also identified opportunities for firms to manage referrers so as to preserve referring as a practice but eliminate its segregating effects.

Discussion

Model Robustness

Our goal has been to elucidate the segregating effects of referral processes. We have used a Markov population process model for this purpose, but want to ensure our findings are an outcome of the actual dynamics we model, and not merely a result of our choices for model structure. In this section, we explore the robustness of our model to our modeling assumptions and model structure. Two noted assumptions of our model are that the homophily adjustment parameter $h$ is sex-invariant, and that our percent female in the counterfactual parameter $c$ is determined by the job but does not change with the composition of the job itself. Our chosen model structure has been a discrete-time Markov model. We test the robustness of our findings in the context of each of these choices in turn.

Sex-invariance of $h$

One notable assumption was that our homophily adjustment parameter, $h$, was sex-invariant. We lack the empirical data necessary to establish whether the observed values of $h$ for men and women in our two cases (Tables 2a and 2b) vary significantly by sex. Our main analysis proceeded under the assumption of sex-invariance to avoid placing causal importance on a relative uninvestigated parameter. Our analysis then identified the tipping point phenomenon that serves as the paper’s main finding. Do we still find a tipping point if we relax the sex-invariance assumption? In short, yes.
We modify our model to allow for sex-varying homophily adjustment just as we did to allow for sex-varying referring rates. We introduce a new parameter, \( h_{\text{asym}} \), to capture the sex-asymmetry in homophily adjustment, and write the homophily adjustment parameter term for women as \( h + h_{\text{asym}} \) and the homophily adjustment term for men as \( h - h_{\text{asym}} \). Comparing this operationalization with the empirical findings in Tables 2a and 2b, we can see that in the former case, \( h = 9.4 \) and \( h_{\text{asym}} = 0.7 \); and in the latter case, \( h = 11.95 \) and \( h_{\text{asym}} = -1.15 \). Although this variation \( h_{\text{asym}} \) in could be a result of random noise, it could also reflect some underlying differences in the two cases. Using this modified model, we see how variation in \( h_{\text{asym}} \) could affect our calculation of a job’s tipping point.

Although the job still has a tipping point, the new formula for the tipping point is no longer independent of \( h \). It is now a function of four parameters: \( r, c, h, \) and \( h_{\text{asym}} \). This new formula is given below in equation (9).

\[
\text{tp}(h_{\text{asym}}) = \frac{r(h_{\text{asym}} + 2c, h - h)}{h_{\text{asym}} - 2ch_{\text{asym}} - h}
\] (9)

It is easy to see that when \( h_{\text{asym}} = 0 \), equation (9) reduces to the original formula for the tipping point, equation (8). Does this dependence on \( h \) and \( h_{\text{asym}} \) undermine our presentation of an easy-to-calculate tipping point? Certainly, it is more difficult for a firm to calculate its own \( h \) and \( h_{\text{asym}} \) along with \( r \) and \( c \) than simply to calculate \( r \) and \( c \) alone. We present Figure 5 to illustrate the affect \( h_{\text{asym}} \) has on changing the calculated tipping point. Figure 5 graphs a new function, \( \Delta(r,c,h, h_{\text{asym}}) \) defined as the difference between equations (8) and (9). For any values of its four parameters, \( \Delta \) gives the difference between the tipping point calculation under the sex-invariant assumption (8) and the tipping point calculation allowing non-random sex-variance (9).

Because \( \Delta \) is a formula with four parameters, its graphic representation presents a minor challenge. Using two parameters as independent axes, and fixed values for the other two parameters, we can make a surface graph of \( \Delta \). By observing how changes in the fixed values change the surface, we can show how all four parameters affect the change in the tipping point calculation. Because our focus is the new parameter \( h_{\text{asym}} \), we set that parameter as one of the axes. For the other axis, we use the percent
female in the counterfactual of non-referral applicants, \( c \), to demonstrate the effects of \( h_{asym} \) across the spectrum of male-dominated to female-dominated jobs. We vary the other two parameters, \( r \) and \( h \), in a two-by-two table, providing the structure for Figure 5.

All four graphed surfaces of \( \Delta \) in Figure 5 share the same surface shape, save for variation in the vertical scale of the surfaces. Two observations follow immediately from Figure 5: that as jobs are more segregated (as reflected by counterfactual parameter \( c \) approaching 0 or 1), the \( h_{asym} \) parameter has a diminishing effect on the calculation of the tipping point. Similarly, the \( h_{asym} \) parameter has the greatest effect for jobs with parity (i.e., 50:50) in their counterfactual applicant composition. The other two variables, \( r \) and \( h \), do not alter these relationships; they only serve to scale the magnitude of their effects. Increasing the referring rate, \( r \), serves to magnify the effects of \( h_{asym} \) linearly – doubling \( r \) doubles the value of \( \Delta \), shown by comparing the two pairs of surface graphs in the same columns of Figure 5.

Increasing the value of \( h \) decreases the effect of \( h_{asym} \) on the tipping point, as shown by comparing the two pairs of surface graphs in the same rows of Figure 5.

In those jobs for which promoting integration is the most important – highly segregated jobs – the \( h_{asym} \) parameter has the smallest effect, and firms can reasonably use the simpler equation (8). For other jobs, the \( h_{asym} \) parameter may render the actual tipping harder or easier to attain, depending on the sign of \( h_{asym} \), but this deviation is likely to be large only in firms with high levels of referring and that are close to being integrated already. Firms wishing a more accurate estimate of their own tipping point may use equation (9), where they will need to construct homophily tables similar to those in Tables 2a and 2b to estimate their \( h \) and \( h_{asym} \) parameters. In most cases, the simpler equation for the tipping point (8) should provide a close estimate sufficient to inform organizational efforts.

**Setting the counterfactual as a job-specific constant**

Our model has assumed that the counterfactual composition of non-referral applicants is constant for each job; specifically, that this counterfactual composition does not vary with the actual composition of the job. It is possible that the current composition of the job affects the composition of applicants
including the counterfactual composition of applicants that would be generated by unbiased referring. Indeed such a mechanism has been proposed as a one that may contribute to the sex segregation of jobs (Reskin, McBrier & Kmec, 1999:351). Empirical research supporting the existence of this mechanism is limited (we identified only Martin, 1990:498 - referenced in Reskin, McBrier & Kmec, 1999). Adding this additional feedback loop to our model, where the counterfactual is a function of the current (or lagged) job composition, would result in the undesirably extreme behavior. That is, jobs that are compositionally majority male - however slight that majority, would have an equilibrium composition of 100% male, and any job with a majority female composition would become 100% female at equilibrium. The only jobs that would avoid these two corner solutions would be those that begin at exactly a 50:50 male:female composition.

Although the dynamics resulting from this modification to the model are trivial, the actual model structure changes significantly because a model parameter becomes a function of the current state of the model, changing the Markov process from a homogeneous model to a non-homogeneous (i.e., the transition probabilities vary with time) one. Since we are not adopting this modification, we present only a numerical/graphical analysis of the implications of this additional feedback. Figure 6a shows the modeled percent female over time in a model where the counterfactual composition depends upon the current job composition for five different initial compositions of the job (75%, 51%, 50%, 49%, and 25% female). Each of these paths leads to an equilibrium of 100% female, 0% female or remaining exactly 50%. In this model, there are no other solutions to this model. Compare this result to that illustrated in Figure 6b - the same five job lines using our model with an invariant counterfactual. Referring still segregates in the direction of the majority, but in a more muted fashion and with a distinct equilibrium for each job line. It is clear that when trying to understand the dynamics by which referring segregates, we prefer the model that is both simpler and that yields non-trivial solution sets.

Most importantly regarding the robustness of our model, even had we adopted this additional feedback to our model, the calculated value for the tipping point would not change. At the tipping point, the equilibrium composition of the job would equal the counterfactual composition of the job. Below the
tipping point, the job would have a final percent female of 0%, and above the tipping point, the job would have a final percent female of 100%.

**Discrete-time Markov model structure**
To ensure that our choice of using a discrete-time Markov model was not the source of our results, we developed an analogous continuous-time model using as system of differential equations. This model produced mathematically identical results. Thus, we are confident that our findings are a product of referring dynamics as we have defined them, and not merely of the model structure we chose to use to present our findings.

**Open empirical questions**
In addition to providing a new understanding of the segregating effects of referring, our analysis has revealed a host of new research questions needing resolution to further our understanding of referring dynamics. For example, is referring actually memoryless process? That is, do the characteristics of one’s referrer influence the type of person one is likely to refer? What is the empirical evidence regarding the structuring of the homophily adjustment parameter, $h$? What are its determinants? Does it vary by job, sex, or other demographic characteristics? How good a proxy for unbiased referring is the population of non-referral applicants? If it is in fact a very good proxy, then firms could pursue a two-pronged approach to get towards managing towards the tipping point. For example, when referring rates are high, the tipping point can be large in magnitude. By moving the percent female among non-referral applicants towards 50%, the tipping point is brought closer to zero, making other efforts at managing the asymmetry in referring rates more likely to be successful.

**Summary and Conclusion**
This paper offers a more detailed investigation into and explanation of the segregating effects of referral recruitment than has previously been available. In doing so, we have subverted the prevailing understanding of the role of referral processes in contributing to job segregation. Notably, the prevailing

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21 Details on this model are available from the authors upon request.
view has focused on the networks of job seekers, and has assumed that short of ending informal recruitment via referrals, firms can do little to influence its segregating effects (e.g., National Research Council 2004:43). This false assumption has served to exempt firms from responsibility concerning the segregating effects of referring processes. We have not only shattered the assumption, but revealed a tool through which firms can mitigate the segregating effects of referring. Not only can firms use the management of referrer behavior to retain informal recruitment while eliminating its segregating effects, but referring can actually be managed such that it is a source of integration, not segregation.

Sex asymmetries in referring rates are vitally important in job segregation outcomes, and require some level of referral homophily with which to interact. The previous understanding has been that the demographic composition of referral applicants is a result of the demographic composition of the job combined with homophilous contact networks. Our analysis suggests a new spin: it is the demographic composition of referrers interacting with job-specific adjustments to homophily that determines the demographic composition of referrals. These characteristics, not the composition of the job, determine whether the effects of referring are segregating or integrating.

Importantly, there is a tipping point for this referring rate asymmetry where referring processes can continue without any segregating or integrating dynamics. On one side of this point, referral processes tip towards segregation, and on the other, they tip towards integration. Contrary to current thinking, the segregating effects of referring are subject to organizational intervention via the lever of sex differences in referring rates. For example, while recruitment policies to attract certain groups are routine, our findings direct firms to explore similar appeals to potential referrers as a means of promoting integration via this lever (e.g., “women and minorities are especially encouraged to refer.”). Because the tipping point can be identified using within-firm data (the sex composition of non-referral applicants and the rate of referring), not only does this policy lever have enormous potential, but it is also quite easy to operationally define.

The segregation arising in our model occurs absent the action of what is considered the crucial mechanisms for segregation – biases in personnel decisions (Bielby 2000; Kaufman 2002; Petersen and
Saporta 2004; Reskin and Bielby 2005). In addition, some scholars have posited that a screening bias favoring referral applicants is related to the segregating effects of referring (e.g., Fernandez and Sosa 2005). Our findings reveal that such biases are not required for referring processes to segregate.

Our model has focused on job segregation by sex. Having identified asymmetries in referring rates as the key contributor to job segregation, we posit that similar dynamics influence job segregation by race. Indeed, previous empirical research has documented racial asymmetries in referring rates (Fernandez and Fernandez-Mateo 2006) and positive adjustments to referral homophily (Fernandez and Fernandez-Mateo 2006). Future work will extend the model to explicitly investigate racial segregation outcomes from referring dynamics as well as those for race-sex interactions. Further, we claim that any groups showing asymmetries in referring rates and referral homophily are likely to be segregated by referral processes.

Finally, we have demonstrated that the mechanisms that we have identified here can exacerbate or reduce job sex segregation, even in the absence of any biases in hiring or exit from the firm. Thus, the findings of this paper serve as a complement to the current policy instrument of choice for reducing of job sex segregation, i.e., the elimination of biases in personnel decisions. This paper establishes the existence and direction of these segregating processes. A companion paper, using an agent-based modeling approach (Rubineau and Fernandez 2006), explores the relative magnitudes of the segregating effects of referring dynamics and bias in personnel decisions. We find that for significant portions of the labor market, the segregating effects of referring are non-trivial in magnitude, and actually can exceed those due to hiring biases. Those findings, too, serve to further demonstrate the power of the theoretical distinctions we have drawn here.
References


11/8/07


Table 1

Data: Berger, 1995 note: post-hire data

<table>
<thead>
<tr>
<th>Job Title</th>
<th>Men referring men</th>
<th>Men referring women</th>
<th>Women referring women</th>
<th>Women referring men</th>
<th>P Pr(SSR) for men</th>
<th>Q Pr(SSR) for women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janitor, cleaner</td>
<td>61</td>
<td>12</td>
<td>26</td>
<td>27</td>
<td>0.84</td>
<td>0.49</td>
</tr>
<tr>
<td>Misc. machine oper.</td>
<td>78</td>
<td>&lt; 10</td>
<td>29</td>
<td>23</td>
<td>&gt; 0.89</td>
<td>0.56</td>
</tr>
<tr>
<td>Sales worker</td>
<td>32</td>
<td>20</td>
<td>42</td>
<td>6</td>
<td>0.62</td>
<td>0.88</td>
</tr>
<tr>
<td>Cashier</td>
<td>&lt; 32</td>
<td>32</td>
<td>56</td>
<td>6</td>
<td>&lt; 0.5</td>
<td>0.90</td>
</tr>
<tr>
<td>Secretary</td>
<td>&lt; 32</td>
<td>47</td>
<td>77</td>
<td>&lt; 6</td>
<td>&lt; 0.41</td>
<td>&gt; 0.92</td>
</tr>
</tbody>
</table>
I. Table 2
Table 2a
Job: 65.7% Female  Data: Fernandez & Sosa, 2005

<table>
<thead>
<tr>
<th>Male Applicants</th>
<th>Female Applicants</th>
<th>Total Applicants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Referrer Male</td>
<td>43.7 %</td>
<td>56.3 %</td>
</tr>
<tr>
<td>Referrer Female</td>
<td>24.9 %</td>
<td>75.1 %</td>
</tr>
<tr>
<td>Non-referral applicants</td>
<td>35.0 %</td>
<td>65.0 %</td>
</tr>
<tr>
<td>Homophily adjustment</td>
<td>+ 8.7</td>
<td>+10.1</td>
</tr>
</tbody>
</table>

Associated parameter values (rounding to 2 decimal places):
Same-Sex Referring Probability for men, \( P = 1 - c + h \)
\[ 0.44 = 1 - 0.65 + 0.09 \]
Same-Sex Referring Probability for women, \( Q = c + h \)
\[ 0.75 = 0.65 + 0.10 \]

Table 2b
Job: 63.3% Female  Data: Fernandez & Fernandez-Mateo, 2006

<table>
<thead>
<tr>
<th>Male Applicants</th>
<th>Female Applicants</th>
<th>Total Applicants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Referrer Male</td>
<td>55.7 %</td>
<td>44.3 %</td>
</tr>
<tr>
<td>Referrer Female</td>
<td>31.8 %</td>
<td>68.2 %</td>
</tr>
<tr>
<td>Non-referral applicants</td>
<td>42.6 %</td>
<td>57.4 %</td>
</tr>
<tr>
<td>Homophily adjustment</td>
<td>+13.1</td>
<td>+10.8</td>
</tr>
</tbody>
</table>

Associated parameter values (rounding to 2 decimal places):
Same-Sex Referring Probability for men, \( P = 1 - c + h \)
\[ 0.56 = 1 - 0.57 + 0.13 \]
Same-Sex Referring Probability for women, \( Q = c + h \)
\[ 0.68 = 0.57 + 0.11 \]
II. Figure 1: Markov diagrams

1a

- Men
- Women
- P
- Q
- 1 - P
- 1 - Q

1b

- Men
- Women
- rs(1 - P)
- rs(1 - Q)
- rsP - x
- rsQ - x

1c

- Men
- Women
- (r - r_{asym})s(1 - P)
- (r + r_{asym})s(1 - Q)
- (r - r_{asym})sP - x
- (r + r_{asym})sQ - x
Equilibrium job composition
e.g., 35% Female, 65% Male

These male-dominated jobs segregate
These female-dominated jobs integrate

Initial %Female for all labor market jobs (dots)
And their direction of change (arrows) due to referring
IV. Figure 3

Increasing homophily yields an *increasing* slope

Increasing heterophily yields a *decreasing* slope
V. Figure 4
VI. Figure 5

$h = 0.1$

Vertical axis: Change in Tipping Point

$r = 0.25$

$h = 0.15$

Vertical axis: Change in Tipping Point

$r = 0.5$
VII. Figure 6
A – With feedback where counterfactual depends on job composition. X-axis is time.

b. The main model from the paper.
Appendix: Proofs of claims made in the paper

Claim 1 from page 17:
The referring transition matrix constructed only from the same-sex referring probability parameters, \( P \) \& \( Q \), will always have an eigenvalue equal to 1.

Proof:
Let \( M \) be a 2 x 2 matrix constructed from only two parameters, \( P \) and \( Q \) as defined in equation (1) above in the paper. Both \( P \) \& \( Q \) can take on any values in the range \([0,1]\), inclusive. \( M \) will have two eigenvalues: \( \lambda_1 \) \& \( \lambda_2 \), each satisfying the definition of an eigenvalue:

\[
\text{det}(M - \lambda I) = 0.
\]

Given that \( M \) is a 2 x 2 matrix, the eigenvalues may be found by solving the quadratic formula below:

\[
0 = (P - \lambda) \cdot (Q - \lambda) - (1 - P) \cdot (1 - Q) = PQ - \lambda Q - \lambda P + \lambda^2 - (1 - P - Q + PQ) = \lambda^2 - \lambda(P + Q) + (P + Q - 1)
\]

The roots of this quadratic formula show that \( \lambda_1 = 1 \), and \( \lambda_2 = (P + Q - 1) \). For all possible values of \( P \) \& \( Q \) except \( P = Q = 1 \), \( \lambda_1 > \lambda_2 \). (As we showed in the paper, when \( P \) \& \( Q \) both equal 1, there is no equilibrium composition – all jobs retain their initial compositions.)

Thus 1 is always the largest eigenvalue of \( M \) as defined in equation (1) in the paper. Q. E. D.

Claim 2 from page 18:
If \( v \) is an eigenvector of \( M \), where \( M \) is a function exclusively of \( P \) \& \( Q \), defined as above, then \( v \) is also an eigenvector of \( M' \), the transition matrix defined in equation (4) in the paper, with the additional \( r \), \( s \), and \( x \) parameters to allow for varying probabilities in referring, selection for hire, and exiting the job. As with \( P \) \& \( Q \), the parameters \( r \), \( s \), \& \( x \) are constrained to the range \([0,1]\), inclusive.

Proof:
\[
Mv = \lambda v \quad \text{by definition of eigenvector};
M' = (rsM - xl) \quad \text{from equation (4)};
M'v = (rsM - xl)v \quad \text{multiplying both sides by} \ v, \text{the eigenvector of} \ M;
= rsMv - xv \quad \text{right distributivity property of matrix multiplication};
= rs(\lambda v) - xv \quad \text{substitution from above};
= (rs\lambda - x)v \quad \text{distributive property of equality}.
\]

Thus, we see that if \( v \) is an eigenvector of \( M \) associated with eigenvalue \( \lambda \), then \( v \) is also an eigenvector of \( M' \) associated with the eigenvalue \((rs\lambda - x)\). As we showed above, the eigenvalue of interest of \( M \) is always equal to 1, therefore the corresponding eigenvalue for \( M' \) is \((rs - x)\).

Appendix: expanded function \( g \):

\[
g(r, r_i^{\text{asymp}}, c, h) = \begin{cases} -(r + r_i^{\text{asymp}} + \sqrt{[4r_i^{2h^2} + 8rc r_i^{\text{asymp}} h - 4r_i^{h^2} - 4r_i^{h^2} - 4r_i^{r_i^{\text{asymp}} c^2 + 4rc r_i^{\text{asymp}} - 4r_i^{\text{asymp}} c^2 + r^2 - 2r_i^{\text{asymp}} + 8r_i^{\text{asymp}} h] + 2rc + 2r_i^{\text{asymp}} c}) / (-r + 2rh - 3r_i^{\text{asymp}} + 2r_i^{\text{asymp}} c - \sqrt{[4r_i^{2h^2} + 8rc r_i^{\text{asymp}} h - 4r_i^{h^2} - 4r_i^{h^2} - 4r_i^{r_i^{\text{asymp}} c^2 + 4rc r_i^{\text{asymp}} - 4r_i^{\text{asymp}} c^2 + r^2 - 2r_i^{\text{asymp}} + 8r_i^{\text{asymp}} h]}) & r_i^{\text{asymp}} > 0 \\ -(r + r_i^{\text{asymp}} - \sqrt{[4r_i^{2h^2} + 8rc r_i^{\text{asymp}} h - 4r_i^{h^2} - 4r_i^{h^2} - 4r_i^{r_i^{\text{asymp}} c^2 + 4rc r_i^{\text{asymp}} - 4r_i^{\text{asymp}} c^2 + r^2 - 2r_i^{\text{asymp}} + 8r_i^{\text{asymp}} h]}) & r_i^{\text{asymp}} < 0 \\ (r + r_i^{\text{asymp}}) & r_i^{\text{asymp}} = 0 \end{cases}
\]