Missing Links: Referrer Behavior and Job Segregation

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Abstract

How does referral recruitment contribute to job segregation, and what can organizations do about it?

Current theory on network effects in the labor market emphasizes the job-seeker perspective, focusing on the segregated nature of job-seekers’ information and contact networks, and leaves little role for organizational influence. But employee referrals are necessarily initiated from within a firm by referrers. We argue that referrer behavior is the missing link that can help organizations manage the segregating effects of referring. Adopting the referrer’s perspective of the process, we develop a computational model which integrates a set of empirically documented referrer behavior mechanisms gleaned from extant organizational case studies. Using this model, we compare the segregating effects of referring when these behaviors are inactive to the effects when the behaviors are active. We show that referrer behaviors substantially boost the segregating effects of referring. This impact of referrer behavior presents an opportunity for organizations. Contrary to popular wisdom, we show that organizational policies designed to influence referrer behaviors can mitigate most if not all of the segregating effects of referring.
Introduction

Most organizations recruit new workers via informal referrals, along with other more formal methods (Marsden and Gorman 2001). Reliance on referral recruitment is particularly strong among smaller firms without dedicated recruiting budgets or personnel – firms which constitute about half of the labor market (Bartram et al. 1995; Barber, Wesson and Roberson 1999; Mencken and Winfield 1998). Indeed, it is a common and encouraged strategy for firms to pay bonuses to employees who refer candidates who are successfully recruited to the firm (Berthiaume and Parsons 2006; SHRM 2001).

Recruitment via employee referrals has long been theorized as contributing to job segregation1 (Doeringer and Piore 1971; Fernandez and Sosa 2005; Kanter 1977; Marsden 1994; Marsden and Gorman 2001; Moss and Tilly 2001; Mouw 2002; Reskin, McBrier and Kmec 1999). Job segregation has numerous organizational and societal costs. Perhaps the most prominent of these effects is gender wage inequality – most of which is attributable to the sex segregation of jobs (e.g., Baron and Bielby 1986; Bayard et al. 2003; Kmec 2003; Petersen and Morgan 1995; Tomaskovic-Devey 1993). In addition, segregation introduces harmful labor market rigidities – where fluctuations in demand can result in shortages (e.g., recent shortages in engineers and nurses) or gluts for particular jobs rather than being absorbed by job mobility (Anker 1997; Kahn 2000; Padavic and Reskin 2002:58-9). From an organizational and legal perspective, job segregation contributes to a “reasonable cause” determination for litigating complaints of violations in U.S. Equal Employment Opportunity (EEO) laws (Gutman 2000; regarding sex and race discrimination cases, see Hirsh and Kornrich [2008]), and is associated with the imposition of mandated policy changes on organizational defendants in EEO lawsuits (regarding sex discrimination cases, see Hirsh [2008]). In light of these perceived organizational and societal perils, the numerous calls to reduce or even eliminate recruitment via employee

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1 Although referral recruitment is viewed to have associations with both the race and sex segregation of jobs, in this paper we focus on sex segregation. The simplifying choice to focus on sex segregation was a pragmatic one: modeling the dynamics of a dichotomous variable such as sex is more straightforward than a multi-valued (and potentially multi-dimensional) variable as race. We recognize this choice as a limitation. Sex and race are both important and omnipresent social markers of individuals in organizational contexts (Ashforth and Humphrey 1995; Heilman 1995; Hirschfield 1999), and these particular markers can interact in complex ways (Chafetz 1997), especially in the context of the labor market (Browne and Misra 2003; Robinson et al. 2005). This simplification neither makes any assumptions of the relative importance of one type of segregation over another, nor reflects a view that the homogenous treatment of groups by race or sex, and not both simultaneously, is unproblematic. We plan to expand future versions of the model to include race and sex simultaneously.
referrals (Braddock and McPartland 1987; LoPresto 1986; Padavic and Reskin 2002; Roos and Reskin 1984) are understandable.

Contrary to popular wisdom, we argue that organizations can manage referral recruitment practices to virtually eliminate their segregating effects. Extant views of referral processes and their relation to job segregation suffers from two major deficits. The first is this literature’s almost exclusive focus on job-seekers and their networks. The referral process requires a dyad. The referral applicant (also simply referral) is the job-seeker using her networks to identify job opportunities. The other half of the dyad is the referrer – necessarily an organizational member aware of a job opportunity who shares that information with the referral. From the perspective of organizations, there is little the organization can do to influence either the network structure or job-search behaviors of the job-seeker. For this reason, organizations have largely been absolved from any responsibility for the segregating outcomes of referral recruitment (e.g., National Research Council 2004: 43). This absolution is misplaced. Half of the dyad, the referrer, is an organizational member and therefore is subject to organizational influence. This is demonstrated by the fact that firms often encourage employees to refer people by offering cash and other kinds of bonuses for successful referrals. In addition, the literature provides examples of informal organizational policies to manage referrer behaviors (e.g., Waters 2001: 106). Still, precious little research has addressed referrers’ behavior and its subsequent impacts on job segregation.

The second deficit characterizing this literature is the lack of a process-based understanding of the segregating effects of referring. Although some empirical work has established a link between referral processes and job segregation (Braddock and McPartland 1987; Fernandez and Fernandez-Mateo 2006; Fernandez and Sosa 2005; Mouw 2002; Petersen, Saporta and Seidel 2001), there has been no formal mechanism-based theory of how one is associated with the other. Mechanism-based theories can serve as invaluable aids in designing organizational interventions and moving research forward (Davis and Marquis 2005; Hedström 2005; Reskin 2003; Schelling 1998). Without a better understanding of referrers’ behavior,

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2 In this paper, we define referral applicants as those applicants who can identify an in-firm referrer by name. Indeed, referral bonus policies are predicated on precisely this type of dyadic relationship. We recognize that some definitions of referral applicants are more broad and could include contacts from non-employees e.g., those who learned of a job advertisement through a friend not employed by the hiring company.
and the mechanisms by which these behaviors lead to segregation, we can offer organizations little guidance on how to mitigate the allegedly segregating effects of referring.

This paper addresses both of these limitations of past research in this area. We identify a set of previously under-theorized mechanisms – missing links – in the understanding of how network processes affect job segregation. We operationalize a mechanism-based theory of referral processes as a formal computational model. A key feature of our model is that it is designed from the perspective of the referrer. In particular, we focus on the firm’s hiring yield from referring employees, and the impact of referrer-referral ties on referring and turnover behaviors. We use this model to address the following three questions. (1) What degree of job segregation results from referral recruitment processes? (2) How do various referrer behaviors contribute to these segregating effects? (3) Can firms mitigate the segregating effects of referring through policies targeting referrer behavior?

To answer these questions, we organize our paper as follows. In the first section, we present a systems view of job segregation and review the empirical findings from the literature documenting actual referrer behaviors and referrer-referral mechanisms that can influence job segregation. Our second section describes our computational model, how we operationalize the identified mechanisms, and how we plan to use our model to draw inferences. In section three, we present the results of our analysis. Finally, in section four we specify a set of propositions based on our findings, and discuss their implications.

1. Referring and Job Segregation

The composition of a job is the net result of hires (inputs) and exits (outputs). Thus job segregation – that is, a biased job composition – is the net result of biases in personnel inputs and outputs (Sørensen 2004:628). A myriad of mechanisms contribute to the biasing of job inputs and outputs, including many over which the organization itself has little to no control. Skills and job-specific qualifications, education and other human capital, job-type preferences and other important factors are rarely distributed with perfect equity across the labor market (Mincer and Polachek 1974; Polachek 1981; Zellner 1975). These and related “supply-side” factors (Okamoto and England 1999) create the context of sex biased inputs within which organizations operate. Similarly, many factors are beyond the organization’s putative control, such as market-wide
expansions or contractions of occupations or industries (e.g., DiPrete and Nonnemaker 1997), unequally distributed domestic labor and family care responsibilities (Hochshild and Machung 2003), societal norms and social control (Jacobs 1989). Such factors can exert differential pressures on male and female job holders to exit. These factors and processes can yield job segregation, even absent any biases in the input or output processes controlled by the organization itself. We set aside these processes to focus on the segregating impact of referring processes with particular attention to the behavior or organization-based referrers.

In order to isolate referring processes, we also set aside a variety of “demand-side” biasing processes over which organizations have direct control. Organizations screen the set of job applicants to determine which of those applicants are offered the job. This screening process is often seen as the main opportunity and locus for organizations to contribute to job segregation through biased inputs (Petersen and Saporta 2004; Reskin and Bielby 2005). Firms also influence the demographic composition of their job applicants through recruitment practices. Where and how job opportunities are advertised can influence the demographic composition of applicants via formal recruitment methods (Bloch 1994:16-17, Gorman 2005). But even choices such as where a firm physically locates itself can have large effects on applicant pool compositions (Fernandez and Su 2004).

People exit from their jobs for a variety of reasons, both voluntary and involuntary. Few if any of these exits could be claimed to be completely free from a firm’s influence. Further, there is a wide body of evidence for sex biases in job exits. Glass ceilings (Abraham 2003; Hymowitz and Schellhardt 1986; Morrison, White and Van Velsor 1987; U.S. Department of Labor 1991), sticky floors (Booth, Francesconi and Frank 2003), revolving doors (Chan 1999; Jacobs 1989) and glass escalators (Budig 2002; Hultin 2003; Maume 1999; Williams 1992) are some of the common metaphors used to describe phenomena of biases in advancement (necessarily, exiting one job for another) and turnover. Biases in exit alone are sufficient to segregate a job. As with our consideration of the factors affecting personnel inputs, we acknowledge the presence and importance of the myriad biasing mechanisms, but focus our attention on empirically-supported processes related to referring.
1.1. Biased Inputs & Outputs from Referring Processes: Homophily

Recruiting via word-of-mouth referral processes has long been viewed as a contributor to the segregation of jobs (Doeringer and Piore 1971; Kanter 1977; Marsden 1994; Marsden and Gorman 2001; Moss and Tilly 2001; Mouw 2002; Reskin, McBrier and Kmec 1999). These segregating effects are largely seen as a result of homophily (McPherson, Smith-Lovin and Cook 2001) – the tendency of people to associate with others like themselves – in the associational choices of referrers and job-seekers alike. Such homophily in referrer-referral networks can bias the composition of firm’s personnel inputs by shifting the sex composition of the job applicant pool.

In terms of sex segregation, male (female) referrers generate referral applicants that are more male (female) than non-referral applicants. One case study examining referral homophily found that while non-referral applicants were 65% female, female referrers generated referral applicants that were 75% female, and male referrers generated referral applicants that were 56% female (Fernandez and Sosa 2005: 878). Thus, employee referral processes in this setting produced an application pool that departed from the baseline of non-referral applicants in the following way: applicants referred by female referrers were 10 percentage points more female, and applicants referred by male referrers were 9 percentage points more male than the baseline sex distribution of non-referrals. A similar pattern was found in another setting with a similar level of scrutiny on referring homophily (Fernandez and Fernandez-Mateo 2006). In this case, non-referral applicants were 57% female, with the referral applicants from female referrers being 68% female (11 percentage points more female than non-referral applicants), and the referral applicants from male referrers being 44% female (13 percentage points more male than non-referral applicants).

Operationalizing referring homophily as a shift in the referral applicant composition from the baseline of non-referral applicants, we illustrate the biasing effects of homophily with an example. Consider a 1000-person job that is 70 percent female, and that the job’s non-referral applicants are also 70 percent female. A quarter of job holders generate referral applicants (175 women, 75 men), and each referrer generates one referral applicant. In this 70% female job, let us say referring homophily generates a 10

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3 This paper focused on racial homophily, and the figures regarding sex homophily were taken from an earlier draft.
percentage point shift from the baseline composition of non-referral applicants. Referral applicants generated by female referrers will then be 80% female and 20% male, and referral applicants generated by male referrers will be 40% male, and 60% female. Referring men are generating more women than men, but men are generating more men among their referral applicants than would be expected from referring without any homophily. The 175 female referrers generate 140 female referral applicants and 35 male applicants, while the 75 male referrers generate 30 male and 45 female applicants. In this scenario, referral applicants are 74% female – more female than job holders or the baseline composition of non-referral applicants. So ceteris paribus, the probability of filling a job vacancy with a female hire is 0.74 if filled with a referral hire, versus the 0.7 probability of a female hire when hiring a non-referral applicant. The actual probability of filling a vacancy with a female hire falls somewhere between these two values, depending on the composition of referrals and non-referrals in the applicant pool. As this example shows, homophilous referring can serve to bias the inputs to the job and thus contribute to job segregation. In the conventional wisdom concerning the role of labor market networks on job segregation, this is the end of the story. By recognizing the missing link of organization-based referrers and how their behaviors affect these labor market dynamics, we will show there is much more to this story.

1.2. Biased Inputs & Outputs from Referring Processes: Referrer Behaviors

The purpose of this paper is to point out that the segregating effects of referring are neither from homophily alone, nor beyond the scope of organizational influence. Revealing the opportunities for organizations to mitigate the segregating effects of referring requires a focus on referrer behavior. With few exceptions, however, the scholarship positing that referral recruitment contributes to job segregation mentions no mechanisms other than homophily between the referrer and the referral applicant.

We describe three empirically-identified referrer behavior mechanisms – referring asymmetries by sex, by referral status, and referrer-referral exit chains, and detail how each mechanism could also contribute to job segregation. We propose to investigate their role in job segregation and whether policies managing referrer behavior could be used to reduce the segregating effects of referring. Although organizations are unlikely to be able to have much influence over homophily in referring, firms can and do enact policies to
influence the behavior of referrers. Using a combination of mathematical and computational modeling, we elucidate the role of these referrer behavior mechanisms on job segregation.

We selected these three referrer behavior mechanisms in particular because they were the only ones we identified that satisfied the following criteria for inclusion. First, the mechanisms must directly involve the behavior of referrers either in their generation of referral applicants as personnel inputs, or their turnover as personnel outputs. Second, the mechanisms needed to have been empirically documented in such a way as to allow quantitative estimates for their operation. Third, the mechanisms must be able to be modeled in a relatively simple simulation model. As the purpose of this paper is an analytical approach to theory building, our desire was not to create an exhaustive listing of documented referrer behaviors, but rather to generate a set of such behaviors that could be modeled simply and still reveal the consequential nature of referring behavior on job segregation.

1.2.1. Referring Asymmetries by Sex

In the presence of homophily in referring, mechanisms which act to bias the composition of referrers relative to job holders can further contribute to biasing the personnel inputs to a job. Continuing with our 1000-person, 70% female job example from above, let us consider the effect if women were more likely to engage in referring than men. Rather than a quarter of both men and women becoming referrers, let us say that 35% of women and 15% of men on the job become referrers. Without homophily, referrers generate referral applicants with the same composition as non-referral applicants, and it doesn’t matter whether men or women are doing the referring. With homophily, those 245 referring women generate 196 women and 49 men as referral applicants, and the 45 referring men generate 27 women and 18 men. Whereas this level of homophily alone yielded a referral applicant composition that is 74% female, adding in an asymmetry in referring behavior results in a 77% female group. Given a constant composition of referrals versus non-referrals in the applicant pool, the probability of filling a job vacancy with a female hire increases with the addition of this asymmetry in who refers.

4 Other referrer behaviors documented in the literature would require models featuring multi-job firms (Leicht and Marx 1997; Mencken and Winfield 2000), or productivity and performance dynamics (Castilla 2005; Yakubovich and Lup 2006). Integrating these more complex dynamics would require a commensurately more complex model.
Notably, an asymmetry in the opposite direction would have a mitigating effect. Consider the situation where the under-represented group was over-represented among referrers such that 15% of women and 35% of men on the job become referrers. In this case, the 105 referring women would generate 84 women and 21 men referral applicants, and the 105 referring men would generate 63 women and 42 men referral applicants. The referral applicants would be 70% female – less female than results from homophily alone, and the same composition as non-referral applicants. This conceptual example shows that even in the presence of homophilous referring, changes in the behavior of referrers can mitigate the segregating effects of referring.

Two empirical studies provide estimates for the level of sex asymmetries in referring. One case study found referring-eligible women within a firm were 20% more likely than the referring-eligible men to actually generate referral applicants (25.7% of women generated at least one referral applicant while 21.4% of men did \(p < 0.003, \text{likelihood ratio } \chi^2=8.753, \text{df}=1\); Fernandez and Sosa 2005: 876). Another case study found referring-eligible women in a different firm were 6% more likely than the referring-eligible men to actually generate referral applicants – an asymmetry not significantly different from zero (36.6% of women in the firm generated at least one referral applicant while 34.6% of men did \(p > 0.5, \text{likelihood ratio } \chi^2=0.228, \text{df}=1\); Fernandez and Fernandez-Mateo 2006:53).\(^5\) So although the men and women in a given firm may engage in referring at significantly different rates, we have no evidence that this asymmetry is a general trend.

1.2.2. Referring Asymmetries by Referral Status

A second type of empirically-demonstrated asymmetry in referring is referral status. Simply put, referral applicants hired to the job may be more likely to go on to generate referral applicants than job holders who were non-referral applicants. For this asymmetry to contribute to biasing the personnel inputs to a job, two conditions must be met. First, job holders hired via referral processes would need to be disproportionately male or female relative to the sex composition of the job or that of non-referral applicants. Second homophily must affect referrers’ production of referral applicants. In the presence of these two conditions, an asymmetry whereby referral hires are more likely to refer can further bias personnel inputs.

\(^5\) Although we are not focusing on the racial segregation of jobs in this paper, we note that this case study found significant differences in referring probabilities by the racial category of the job holders.
Continuing the example of our 1000-person job from above, sex asymmetries in referring rates with women being over-represented among referrers yielded 223 (196+27) female referral applicants and 67 (49+18) male referral applicants. In this hypothetical example, we define a hiring process with no sex bias and no referral status bias (either of which could exacerbate biased inputs) and with no differences in applicant quality such that these female and male referral applicants are equally likely as their non-referral applicant counterparts to be hired. So the referral applicants, which are now 77% female, are equally likely to be hired as the non-referral applicants, which have been defined as being 70% female. Let us imagine referral hires are 20% more likely to engage in referring than non-referral hires. A hired female referral applicant will have a 42% chance of referring, and a hired male referral applicant will have an 18% chance of referring, compared to the 35% and 15% referring chances of non-referral female and male hires, respectively, in the presence of referring asymmetries by sex. Given that a hired referral applicant is female with a probability of 0.77 compared to 0.7 for hired non-referral applicants, and given that referring generates disproportionately gender homophilous referrals, this asymmetry in referring makes for even greater numbers of female referral applicants in the next generation of referrals. Asymmetries in referring behavior based on referral status can thus exacerbate the biasing of personnel inputs by amplifying existing asymmetries in the system.

One empirical case study illustrates this asymmetry. In their hazard-rate model estimating the factors affecting the risk of employees engaging in referring, Fernandez and Castilla (2001) found that having been a referral hire increases a job holder’s likelihood to refer by about 300% relative to comparable non-referral hires (Fernandez and Castilla 2001: Table 3 and Figure 1). In two different job categories (customer service representatives [CSRs] and non-CSRs), non-referral hires had a probability of referring of about 0.06 and 0.04, respectively; while referral hires had a probability of referring of about 0.25 and 0.16, respectively. We did not find any other empirical estimates of this association in any other published works.

### 1.2.3. Referrer-Referral Exit Chains

Several theories and empirical studies argue that referrer behaviors can affect job exits. Referrer-referral ties are consequential in forming exit chains (Sgourev 2007). Simply put, a referrer-referral exit chain is the phenomenon that when a referrer or referral exits a firm, their referrals and referrers, respectively,
become more likely to exit the firm as well (Fernandez et al. 2000). Conversely, referrers and referrals are less likely to exit the firm (i.e., more strongly anchored within the firm), while their referrals and referrers, respectively, remain in the same firm with them (Fernandez et al. 2000). This form of differential turnover, interacting with largely homophilous referrer-referral ties, may bias the composition of job exits. To date, the implications of this “intriguing finding” (Marsden and Gorman 2001:489) has not been explored.  

2. Methods

Explanations of the dynamic relationship between referral processes and job segregation have remained elusive because of conceptual and methodological difficulties involved in bridging the micro-macro gap (Coleman 1986: 1320). Job segregation is a macro phenomenon affected by (among other things) the micro-level behaviors of referrers and ties between referrers and referrals. Tools enabling the analysis of organizational-level outcomes emerging from network-mediated individual-level dynamics are recent additions to the organization science toolbox (Anderson 1999; Ibarra, Kilduff and Tsai 2005; McKelvey 1997; Meyer, Gaba & Colwell 2005). Theory-building from idealized models can complement empirical findings to advance understandings of organizational phenomena (Anderson 1999; McKelvey 1997; Van de Ven and Poole 2005). Following McKelvey’s (1997) methodological guidance, we create an idealized model of referral processes, translating previous empirical findings into mechanisms defined as rates and probabilities, and implementing micro-level interaction dynamics in an agent-based simulation. We use this model as a tool to understand the segregating effects of referring, to explain the contributions of referral processes to job segregation, and to compare referring’s segregating effects with those of another major segregating mechanism – hiring biases (Perry, Davis-Blake and Kulik, 1994).

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6 In addition to one case study, the presence of referrer-referral exit chains is also supported by several lines of empirical and theoretical research. If referrer-referral ties can contribute to job embeddedness, then the literature associating job embeddedness with turnover (Felos et al. 2009; Lee et al. 2004; Mitchell et al. 2001; Mitchell and Lee 2001; Moosholder, Settoon and Henagan 2005) would predict this dynamic. Additionally, social niche theory, as presented by Popielarz and McPherson (1995), posits an analogous dynamic based on voluntary organizations. A person in a voluntary organization was more likely to remain in the organization when tied to others in the organization, and more likely to leave when tied to others outside the organization. Finally, the stream of research on the organizational outcomes of recruitment source (Blau 1990; Rynes 1991; Breaugh et al. 2003) has identified empirically that turnover rates among referral hires can differ significantly from those for non-referral hires (Wanous [1992:36] for a summary of 12 such studies; Griffeth et al. 1997; Saks and Ashforth 1997; Williams, Labig and Stone 1993).
Our research questions and our approach to answering them are a canonic example of “forward problem” research in organization science (Lomi and Larsen 2001:4; Burton 2003: 97). That is, given the identified and defined referral recruitment mechanisms, what is the nature of any job segregation resulting from these mechanisms? A useful answer to this question requires a step beyond an analysis of the output from a single simulation model. While it is broadly accepted that referral recruitment can be a segregating mechanism, we would like to try to assess the level or severity of this mechanism’s segregating effects. How can we use a computational simulation model to answer this question with any generalizability beyond our model? Our solution is to draw upon and extend the validation tool of model docking (Burton 2003) to support the generalizability of our findings as well. After docking the basic mechanics of our computational simulation model with an analogous mathematical model, we use the well-researched mechanism of sex bias in job screening to serve as a more general yardstick for our simulation findings. We expand upon this effort towards establishing inter-subjective validity for our findings below.

As mentioned above, we investigate the segregating effects of referrer behaviors in isolation from other biasing mechanisms. We model a job where all other biasing mechanisms are inactive. Some of those biasing mechanisms are not subject to organizational influence – in particular, supply-side aspects of job segregation – and are taken as a given and invariant over the duration of our simulations. However, other biasing mechanisms – e.g., screeners’ preferences for one gender over the other, or biases for or against referred applicants (see Fernandez and Sosa 2005) – are under the firm’s control. Although these are not the central focus of this paper, as discussed in more detail below, we will incorporate the sex and referral biases in screening into the model for the purpose of model validation and as a referent for assessing how consequential referral processes are in segregating jobs.

2.1. Model Definition

We model a firm with only one job. Doing so helps to control for a number of processes and allows for a simpler model. The matching of applicants to different jobs within a firm is itself a sparsely studied segregating process both via organizational behavior (Fernandez and Mors 2008) and referrer behavior (Leicht and Marx, 1997; Mencken and Winfield, 2000). A firm with one job obviates any additional
segregating effects of the matching process. Similarly, the well-documented segregating effects of promotions and internal job ladders (Baldi and McBrier, 1997; Diprete and Soule, 1988; Doeringer and Piore, 1971; James, 2000; Kelley, 1982; Maume, 2004; Miech, Eaton and Liang, 2003; Smith, 2005; Thomas and Gabarro, 1999; Wilson, Sakura-Lemessy and West, 1999) are controlled for by using a one-job firm. Second, to control for human capital and other individual-characteristics explanations for disparate labor market outcomes, all agents in the model are defined as being identically qualified for the job and performing equally well while on the job. Additionally, the simulated job provides uniform compensation to all workers, so there is no bias in on-the-job rewards. Finally, except for the purposes of model docking and validation, we exclude biases in hiring and exits from our model, as they are not referrer-referral processes.

We conceptualize the central element of our simulated job as two stocks: a stock of male job holders and a stock of female job holders. We constrain the sum of these two stocks to be constant, to represent a stable (rather than growing or shrinking) job. The levels of these stocks change as a result of the probability that job holders exit the job, and with the probability that the next agent hired to fill the resulting vacancy is female. The mechanisms described above and operationalized below modify these probabilities as functions of the mechanisms’ governing parameters and the state of the system. We record the percent female among job holders over the course of the simulation to investigate the segregating effects of these mechanisms. Even with our restricted focus, we are able to demonstrate that referrer behaviors substantially boost the segregating effects of referring beyond that which would be produced by homophily alone. Moreover, we show that organizations can mitigate these segregating effects through policies targeting referrer behaviors.

2.2. Model Setup: Personnel Flow with Homophilous Referring

The first four parameters of our model define the basic dynamics for a one-job firm that hires new workers via both referring and non-referring pathways. These four parameters are, $c$: the baseline sex composition of the job; $k$: the baseline exit rate of job holders; $p$: the proportion of referral applicants in the job applicant pool; and $h$: the level of homophily evident in referrer-referral ties. We define each in turn.
2.2.1. Baseline sex composition of non-referral applicants: \( c \)

For a wide variety of reasons, the non-referral applicant pool for a particular job may not be perfectly representative of the labor market as a whole. We assume this net result of many processes is constant over the time of our simulation. Thus, a particular job has a set sex composition of non-referral applicants. This composition baseline parameter is \( c \). Absent any biasing mechanisms, and regardless of the initial sex composition of the job, the final stable sex composition of the job will eventually be \( c \). In our simulations, we set the starting point for the sex composition of the job to \( c \), thus \( c \) serves as the baseline composition for purposes of measuring deviations. For our simulations we set \( c \) to 0.65, based on one of the empirical case studies reviewed above (Fernandez and Sosa 2005), but allow \( c \) to vary during our model validation tests.

2.2.2. Baseline exit rate: \( l \)

Agents exit the job with a baseline probability \( l \) that is identical for both male and female agents. In our simulations, we set \( l=0.01 \) for a one-week time step. So in each simulated week, each job holder has a one percent chance of exiting. This exit rate is equivalent to an average expected tenure of 100 weeks or just under two years. Each step of the simulation begins with allowing current job holders to exit. The vacancies created by these exits are filled with new agents. Making new hires contingent upon vacancies keeps the size of the job stable over the course of the simulation. Our simulations use a job size of 333 agents. Absent any referring homophily or other biases, the expected sex composition of the newly hired agents is also \( c \), so the sex composition of the job would not change over time. Our simulation of biased inputs focuses on mechanisms that may serve to change the probability that a new hire is female to something other than \( c \).

2.2.3. Composition of referral applicants in the applicant pool: \( p \)

A vacancy is filled with a non-referral applicant with probability \( 1-p \), and thus with a referral applicant with probability \( p \). In our simulations, \( p \) takes on values in the set \{0.25, 0.5, 0.75, 1.0\}.

2.2.4. Homophily in referring: \( h \)

The sex composition of referral hires may differ from that of non-referral hires in the presence of homophily. If referrers referred randomly, the composition of referrals should closely resemble the composition of non-referrals. That is, the people contacted by referrers should be representative of the labor
market, and the same set of forces that yield a non-representative non-referral applicant pool should affect the composition of the randomly-referred applicant pool. Non-random referring, specifically homophilous referring, would result in a shift away from this randomly-referred baseline. That organizations cannot be expected to influence homophily in referring networks is precisely the claim that has been used to exempt organizations from responsibility for the segregating effects of referring. Stipulating that managing homophily is not feasible, the purpose of this paper is to reveal how other mechanisms that are subject to organizational influence can and do moderate these effects. Because we do not explore the effects of varying homophily (except in model validation), we implement homophily very simply. Given a homophily parameter $h$, a female referrer yields a female referral applicant with probability $c+h$, and yields a male referral applicant with a probability $1-c-h$. Similarly, a male referrer yields a male referral applicant with a probability $1-c+h$ and a female referral applicant with a probability $c-h$. Based on the previous empirical results discussed above, we set $h$ to 0.1 in our model for all purposes except model validation, where we allow $h$ to vary (including the value 0.2). We note that when $c$ (the sex composition of non-referral applicants, and the starting composition of the job) equals 0.5, referring with homophily alone does not generate any segregation, as the over-production of female referral applicants by female referrers is exactly balanced by the male over-production of male referral applicants by male referrers.

2.3. Model Focus: Referrer Behavior Mechanisms

When a job vacancy is to be filled in the presence of referring, the simulation first randomly selects whether the new hire will be a referral or non-referral hire based on the parameter $p$. If the new hire is to be a referral, the simulation picks a random referring-eligible job holder. Once selected, the sex of the referring job holder along with the parameters $c$ and $h$ governing homophily, determine the probability that the new hire will be female. Once determined, the proper stock of male or female job holders is incremented appropriately to reflect the hire, and the referring eligibility of the new hire is calculated and added as a value associated with the particular agent hired.

As suggested by this summary, referrer behaviors are determined in no small part by referring eligibility. If all employees are referring eligible, the sex composition of actual referrers would mirror that of
all job holders. To implement the asymmetries in referring described above, we restrict referring eligibility so a random draw from referring eligible job holders will yield the desired non-representative distribution of job holders. Not only is the restriction of referring eligibility a convenient method for implementing the two referring asymmetry mechanisms, but it is also consistent with the empirical evidence. Two published case studies detailing many aspects of referrer behaviors (Fernandez and Fernandez-Mateo 2006; Fernandez and Sosa 2005) present data consistent with the proposition that referrers are unlikely to be a random and unbiased selection of job holders. Appendix A presents the details of the analysis leading to this conclusion.

Based on these two studies, we proceed with our simulation investigating two levels of referring eligibility. For one set of our simulations, we set the baseline likelihood that any given job holder may be a referrer (parameter: $r$) to a higher value of 0.78. We repeat these simulations with otherwise identical parameters but using a lower value of 0.38 for the baseline likelihood that any given job holder may be a referrer. That is, absent other mechanisms affecting referring likelihood, in one exploration of the model parameter space, 78% of job holders are potential referrers and 22% are not, and in a parallel exploration, 38% of job holders are potential referrers and 62% are not.

We now operationalize the three empirically-identified referrer behavior mechanisms that are the focus of this study. We identify each mechanism with the variable we use as its governing parameter, and provide a formal definition for the mechanism’s operation. As each mechanism is based on an empirical finding from at least one case study, we have at least one empirical estimate for a plausible value for the parameter. In addition to exploring the implications of these empirically-based starting points for these parameters, we also investigate the effects of twice the parameters’ values, half their values, and when their null value (i.e., the mechanism is “off”). At the end of this section, we detail how our defined parameter space translates into different probabilities for referring eligibility and exit.

2.3.1. Asymmetries in referring by sex: Mechanism A

Although the limited empirical evidence on gender and referrer behavior finds no differences in the numbers of referral applicants generated by male and female referrers (Fernandez and Sosa 2005: 877), gender differences in generating at least one referral applicant are apparent in that case. In the call center case,
female job holders were about 20% more likely to be a referrer than male job holders (Fernandez and Sosa 2005: 876). To implement this mechanism, we make the probability that a new agent is eligible to refer sex-dependent. Letting $r$ be the baseline or central tendency percentage of potential referrers (in one set of cases, 78%, in the other, 38%), and $A$ be the sex asymmetry in referring (e.g., the female rate is 120% of the male rate), we calculate the probability that a new agent is a potential referrer as follows:

$$
\Pr(\text{an agent is eligible to refer}) = \begin{cases} 
    p_1 = 1 - \frac{1-r}{A} = \frac{A+r-1}{A}, & \text{if the agent is female;} \\
    p_2 = \frac{p_1}{A} = \frac{A+r-1}{A^2}, & \text{if the agent is male.}
\end{cases}
$$

(1)

The logic of this formulation is based on several criteria. First, and as before, we would like to keep the probability of becoming a potential referrer bounded between 0 and 1. A simple $rA$ implementation would not satisfy this first criteria. However, $r/A$, when scaling down (as in the case of an asymmetry in the opposite direction) is asymptotic to zero for all $A > 1$. Rather than scaling up the potential referrer rate, $r$, by $A$, we scale its complement (1-$r$, or the non-potential referrer rate) down by $A$, and then take the complement. This calculation gives the potential referrer rate for female agents. That rate divided by $A$ becomes the potential referrer rate for male agents. Given $r = \{0.78, 0.38\}$ and $A \geq 1$, this method yields probabilities falling between 0 and 1, as desired. As described above, empirical estimates of $A$ have included 1.2 (Fernandez and Sosa 2005) and 1 (Fernandez and Fernandez-Mateo 2006). In our simulations, we explore the following values for $A$: $A \in \{0.8, 0.9, 1.0, 1.1, 1.2, 1.4\}$. Note that $A=0.8$ is a reversal of the asymmetry such that male job holders are more likely to refer than female job holders.

2.3.2. Asymmetries in referring by referral status: Mechanism $M$

We implement the asymmetries by referral status in the probability that a new agent is eligible to refer such that they may co-occur with asymmetries by sex. In this implementation, we use an analogous construction logic as we did with the previous asymmetry. With both referring eligibility mechanisms active, and letting $M$ be the governing parameter for the referring asymmetry by referral status mechanism (i.e., the mechanism where referrals refer more), the probability that a new agent is eligible to refer is given below:
Pr(an agent is eligible to refer) = \begin{align*}
p_1 &= \frac{A+r-1}{A}, \text{ if the agent is a female nonreferral;} 
p_2 &= \frac{p_1}{A} = \frac{A+r-1}{A^2}, \text{ if the agent is a male nonreferral;} 
p_3 &= 1 - \frac{1-p_1}{M} = \frac{AM+r-1}{AM}, \text{ if the agent is a female referral;} 
p_4 &= 1 - \frac{1-p_2}{M} = \frac{A^2M-A^2+A+r-1}{A^2M}, \text{ if the agent is a male referral.}
\end{align*}

As discussed above, one empirical case study found referrals to be 300% more likely to refer than non-referrals (Fernandez & Castilla 2001). As a result, we model $M$ to reflect referring likelihood increases of 0%, 150%, 300% and 600%, or $M \in \{1.0, 2.5, 4.0, 7.0\}$. Table 1 shows how the intersection of all of simulated parameter values of $A$ and $M$ as defined above affect the referring eligibility of newly hired agents. As indicated above, referring eligibility determines who refers, and the sex of the referrer in combination with homophily stochastically determines the sex of the referral hire.

Several patterns are apparent from Table 1. First, when $A=1.0$ and $M=1.0$, the referring eligibilities are the given baseline levels (either 0.38 or 0.78) for men and women, referrals and non-referrals. When $A=1.0$, indicating no referring asymmetry by sex, referring eligibilities for men and women are identical regardless of the value of $M$ or referring status, as can be seen by comparing the $A=1.0$ rows of the upper and lower panels of Table 1. When $M=1.0$, indicating no referring asymmetry by referral status, the referring eligibility of referrals and non-referrals are identical, regardless of the value of $A$, as can be seen by comparing the first two columns of both male and female referring eligibility values in each of the panels of Table 1.

Two of the empirically observed values for $A$ and $M$ discussed above are 1.2 and 4, respectively. When $A=1.2$ and $M=1.0$ in the low referring eligibility case ($r=0.38$, shown in the upper panel of Table 1), meaning a referring asymmetry by sex but not referral status, women’s referring eligibility is 0.483 and men’s referring eligibility is 0.403. The female to male referring eligibility ratio is 1.2. In the high referring eligibility case ($r=0.78$), women’s referring eligibility is 0.817 and men’s referring eligibility is 0.681. Again, the female to male referring eligibility ratio of the two is 1.2. Our implementation of this asymmetry behaves as expected for this modest level of asymmetry, but it also serves as a robust and conservative implementation allowing large asymmetry parameters. The $M=4$ parameter value represents a 300% increase in the referring probability of referral hires relative to non-referral hires. Such an increase for the two referring baselines
(r=0.38 and r=0.78) would both yield probabilities greater than one. Indeed the maximum direct increases for the two probabilities are 163% and 28%, respectively. Equation (2) yields changes in probabilities such that $M=2$ (a 100% increase) corresponds to an increase of half the maximum (81.5% and 14%, respectively) possible increase. When $M=4$ and $A=1.0$, the increase is three-quarters of the maximum possible increase (122% and 21%, respectively), to a referring eligibility of 0.845 and 0.945, respectively, as shown in Table 1. These probabilities illustrate our conservative approach in modeling this mechanism. The effects of the mechanism when increasing the parameter $M$ are not only modest, but also show decreasing marginal returns, and referring eligibility approaches 1 only as $M$ approaches infinity.

When $A=1.2$ and $M=4$ and the baseline referring eligibility is $r=0.38$, equation (2) yields a referring eligibility for men of 0.851 and a referring eligibility for women of 0.871, giving a female to male referring eligibility ratio of 1.02. For the same $A$ and $M$ values with the higher baseline referring eligibility of $r=0.78$, equation (2) yields a referring eligibility for men of 0.920 and a referring eligibility for women of 0.954. The female to male referring eligibility ratio in this scenario is 1.04. Again, this example shows our choices implement the mechanisms conservatively. When both asymmetries interact, the apparent impact of either asymmetry in isolation is reduced.

2.3.3. Referrer-Referral Exit Chains: Mechanism $X$

Referrer-referral exit chains have dichotomous effects on turnover. Referrers and referrals with links to individuals who have exited the job or firm in turn have a higher probability of exiting, while those with links to individuals still within the job or firm have a lower probability of exiting. Letting $t_{in}$ represent the number of referral ties to alters within the organization, $t_{out}$ represent the number of referral ties to alters outside the organization, $l$ represent the exit probability for job holders without ties, and $X$ represent our exit chain mechanism governing parameter, we define an agent’s exit probability as:

$$\text{Pr(}\text{an agent will exit in a given time step}) = lX^{\frac{(t_{out}-t_{in})}{\max(t_{out},t_{in})}}$$

(3)

The goal of this formulation was to implement the mechanism conservatively. Obviously, when $X=1$, there is no biasing effect on exit likelihoods. When $X > 1$ and an agent’s $t_{in} > t_{out}$, that agent will be less likely to exit than the baseline likelihood of $l$ as the exponent term will be negative. When $X > 1$ and an
agent’s $t_{in} < t_{out}$, that agent will be more likely to exit. The denominator in the exponent term serves two purposes. The first is there to ensure that the increase (decrease) in exit likelihoods as $t_{out} - t_{in}$ becomes more positive (negative) at a decreasing rate. That is, an agent’s first alter to exit the firm increases that agent’s exit likelihood more than that agent’s fourth alter to exit the firm. The second purpose is to ensure that the effect of an alter leaving (or staying) is scaled by a job holder’s total number of ties. That is, the effect of an alter’s exit on an agent will be larger for an agent with only two total ties than for an agent with four total ties. (E.g., an agent with 3 alters out and 2 alters in will have a smaller exit likelihood: 0.0136, than an agent with 1 alter out and 0 alters in: 0.0198.) Because the functional form of this mechanism has not been established in empirical research, we tried to model the mechanism as conservatively as possible while maintaining the effects suggested by the theoretical and empirical findings.\footnote{We don’t know the functional form of this exit chain dynamic. We have been asked whether there should be some time-dependent decay for this mechanism (e.g., if an employee’s referrer leaves the firm and the employee remains on the job for the year after that, the absent referrer should no longer have an effect on her exit likelihood). The empirical evidence actually suggests there is no decay (Fernandez, Castilla and Moore 2000:1341, table 10).} The one empirical case study with some estimate of $X$ found job holders whose referrals had left had an increase in their exit likelihood of 98% (Fernandez et al. 2000). Based on this finding, our simulations explore increases of 0%, 49%, 98% and 200%, or $X \in \{1.0, 1.49, 1.98, 3.0\}$. Unlike the asymmetry mechanisms, this mechanism is not asymptotic to one, but could conceivably yield exit likelihoods greater than one. At our largest modeled value of $X$, 3.0, it would require 18 alters to have exited the firm and no alters within the firm to exceed an exit probability of one. Given our implementation of our model, the probability that an agent has 18 alters is indistinguishable from zero.

Table 2 shows the exit likelihoods for agents with $t_{in}$ and $t_{out}$ values ranging from zero to five for the three simulated values of $X$ that are greater than 1: 1.49, 1.98, and 3.0. When $X=1$, it is clear from equation (3) that the exit probability is always $l$. Similarly, when $t_{in} = t_{out}$; that is, when has an equal number of referrer or referral ties to others who are in the firm as those who have left the firm, the exponent in equation (3) is zero, and thus the exit probability is also always $l$. The diagonals of all three panels of Table 2 show this equivalence. The upper triangles of these panels show exit probabilities when an individual has more referrer or referral ties to others who have left the firm than remain in the firm. These individuals have higher exit probabilities. Looking across the rows of these upper triangles, it is clear that the biggest increase in exit
probability comes from the exit of the one alter who tips the balance towards having more alters outside the firm than inside. Additional exits by an individual’s alters increases that individual’s exit probability, but at a decreasing rate. Further, this effect is scaled by an individual’s total number of alters. There are many ways to have a net difference of one alter outside the firm. If such a difference arises from having only one alter total (i.e., 1 out and 0 in), the effect on exit probability is greater than if the difference arises in someone with three alters (2 out and 1 in), which in turn is greater than for someone with five alters (3 out and 2 in), and so on. This trend can be seen in the diagonal of values above the main diagonal of the three panels of Table 2. The corresponding pattern of effects regarding reductions in exit probabilities may be seen in the lower triangles of the three panels of Table 2, when an individual has more alters in the firm than out. Table 2 shows the exit probabilities realized in our simulation.8

3. Analysis

Our analysis proceeds by investigating the segregating effects produced by the simulation mechanisms defined above over the set of parameter values indicated above. Table 3 summarizes the full set of parameters and the sets of their values we use in our study, defining the parameter space of our simulation.

3.1. Measuring Segregating Effects

Absent any biasing mechanisms in personnel inputs or outputs, the sex composition of the job remains at $c$ – the initial sex composition of the job and the sex composition of non-referral applicants. The introduction of referral process mechanisms or other biases may shift the job away from $c$. Similarly, the removal of these mechanisms will result in the job sex composition returning to $c$. For this reason, we call $c$ the baseline sex composition.

To compare the deviating (that is, segregating or integrating) effects of the mechanisms under study, we need a quantifiable definition of “deviating effect.” We operationalize the deviating effect of a set of mechanisms as the area between the baseline job composition and the curve describing the composition of the job over a specified length of time in the presence of the mechanisms. If the mechanisms have a stable

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8 The likelihood of an agent having the number of alters with the appropriate distribution of exits to fall outside the values given in Table 2 is small. Empirical settings also show the number of referral applicants a referrer generates tends to be quite small with few exceptions (Fernandez and Fernandez-Mateo 2006: 54; Fernandez and Sosa 2005: 876-877).
equilibrium - that is, if the job composition curve has an asymptote - then as time tends towards infinity, the area between the curve and baseline is perfectly correlated with the difference between the equilibrium composition and the baseline composition. Despite this eventual similarity, we use neither the equilibrium composition of the job nor the composition at the end of the simulation as our deviating effect indicator.

The problem is that the speed with which a curve approaches its asymptote can be consequential. For example, consider a mechanism with an equilibrium deviation of 5 percentage points (say from 50:50 to 55:45) that takes about 100 years to get close to that level, and a second mechanism with just a 2 percentage point equilibrium deviation (from 50:50 to 52:48) that takes only 2 years to reach that level. With an infinite time horizon, the first mechanism obviously has a greater deviating effect than the second. Organizations don't have infinite time horizons, and in this example, the mechanism with the 2 percentage point equilibrium deviation will have a stronger deviating effect starting from time zero for all observation windows shorter than 19 years. Figure 1 illustrates over a window of 20 simulated years these two scenarios. The dividing line between time windows where the first mechanism has a larger deviating effect and those where the second mechanism has a larger deviating effect. This example demonstrates the importance of specifying a window of time for comparing the deviating effects of different mechanisms or sets of mechanisms. In this paper, we use a 10-year time horizon for our comparisons. We wanted a time horizon long enough to include mechanisms that deviate more slowly, but not so long as to need to consider generational effects.

We have defined eight parameters governing our model, and a time horizon for observing the model’s behavior. The outcome of concern for a given set of parameter values is the deviation between the percent female of job holders and the baseline sex composition. The deviation is measured as the area between these two curves (the latter curve, the baseline sex composition, being the horizontal line at \(c\)). To reduce the effect of stochastic noise in our simulation output, the curve describing the percent female of the simulated job setting over the time horizon is the mean output from 1000 runs of the simulation model with the same set of eight parameters, differing only in the random number seed.\(^9\) Defining \(SIM(*)\) as the function giving the mean output from 1000 iterations of our simulation model for a given set of parameters, we can

\(^9\) The code for the model, implemented using the RePAST agent based modeling libraries for java (North, Collier & Vos 2006), is available from the authors upon request.
represent our outcome of concern – the segregating effect of referring as a deviation from the baseline sex composition of the job – as equation (4):

\[
\text{Segregating Effect} = \int_0^{520} [SIM(c, l, p, h, r, A, M, X, t) - c] dt
\]

(4)

3.1.1. Docking & Model Validity

If the simulation model could be implemented as a mathematical model, our goal could be achieved analytically. Although a mathematical implementation of our three referrer behavior mechanisms operating simultaneously is not practical, a more constrained version of our model is not only practical but also very useful. This mathematical model includes the referring dynamics governed by parameters \( p, h, \) and \( r \) (percent of referrals among applicants, homophily, and baseline referring eligibility, respectively), but not the referrer behavior mechanisms governed by \( A, M, \) and \( X \) (asymmetries by sex, referral status, and exit chains, respectively). We use this mathematical version of our model to validate our simulation model by “docking” (Axtell et al. 1996; Burton 2003). That is, we implement the two versions of our model, one computational and one mathematical, and we compare their outputs under comparable parameter settings. If similar behavior is generated by the same set of mechanisms implemented in two very different styles of models, we can be more confident that the dynamics are a result of the mechanisms themselves, and not some artifact of the particular method of modeling.

The full details of the mathematical version of our model, implemented using differential equations and the Maple (Maplesoft 2003) software package, are provided in Appendix B. Figures B1 and B2 in Appendix B show the output of both our computational simulation and the mathematical model produce very similar dynamics under an wide array of parameter settings. This docking not only helps to validate our simulation model’s implementation of referring dynamics, but also presents the opportunity to report the output of our simulations in terms of a metric with inter-subjective validity – sex bias units.

3.1.2. Sex Bias Units

When docking our computational simulation model with the mathematical model, we did so over a broader set of the common parameter space than we explore below (with the important exception of the
In addition, we explored the behavior of the two models in the presence of explicit hiring biases. The two biases explored in Appendix B are sex biases in hiring and hiring biases based on referral status. Because both models generate a measure of segregating effects, it is possible to not just pose but answer the question: Given a set of referring processes implemented in the simulation model, what level of sex bias in screening in the mathematical model produces the equivalent segregating effect for job with an otherwise identical set of governing parameters? In answering this question, we can identify the segregating effect of a particular level of sex bias in screening (parameter \( \delta \)). This measure of sex-bias units becomes our primary dependent variable, and a basis for comparison of model outputs. To represent this idea with an equation, we define \( MATHs(*) \) as the function giving the output of the mathematical version of our model of personnel flow through a one-job firm that includes no referring\(^{10} \) but has a level of sex bias in hiring determined by the sex bias parameter, \( s \). To identify the segregating effects of referring for a given set of parameter values in terms of sex bias units, we look for the value of the sex bias parameter, \( s \), that solves the following equation:

\[
\text{Segregating Effect} = \int_0^{520} [SIM(c, l, p, h, r, A, M, X, t) - c] dt = \int_0^{520} [MATHs(c, l, s, t) - c] dt. \tag{5}
\]

We use the sex bias parameter, \( s \), solving equation (5) to summarize the segregating effects of any given set of referring mechanism parameters. More than a convenient summary measure, the sex bias parameter has a clear meaning, and gives a kind of effect size measure for our simulation output. If the segregating effect of a set of referring mechanisms is equivalent to a level of sex bias that would be of concern to organizations, then the segregating effect of those referring mechanisms should be of similar concern.

The sex bias parameter, \( s \), ranges from zero to infinity, and gives the \textit{ceteris paribus} level of preference for hiring female applicants\(^{11} \). When \( s = 1.0 \), \textit{ceteris paribus}, otherwise equivalent male and female applicants are equally likely to be hired. When \( s = 2.0 \), \textit{ceteris paribus}, a female applicant is twice as likely to be hired as an otherwise equivalent male applicant, and when \( s = 0.5 \), \textit{ceteris paribus}, a female applicant is half as likely to be hired as an otherwise equivalent male applicant. The simulation output presented below is given in terms of

\(^{10}\) In Appendix B, we also provide a mathematical model including referring, \( MATHr(*) \), used in docking our simulation model, but this second model is not used in determining the segregating effects in terms of sex bias units.

\(^{11}\) A full description of the implementation of sex bias parameter \( s \) is given in Appendix B.
these sex bias units, using the value of $s$ solving equation (5) for the appropriate set of model parameters.

Figure 2 illustrates the application of sex bias units to summarize the behavior of our computational model for a sample set of parameter values. The two thick lines in Figure 2 show the mean behavior over the 520-week time horizon of 1000 runs each of our computational simulation model with the following two parameter settings: \{$c=0.65, l=0.01, p=0.5, b=0.1, r=0.78, A=1.0, M=1.0, X=1.0\}$ as the thick darker gray line, and \{$c=0.65, l=0.01, p=0.5, b=0.1, r=0.38, A=1.0, M=1.0, X=1.0\}$ as the thick lighter gray line. These two simulations depict the segregating effects of homophilous referring alone in a 65% female job setting with no active referrer behaviors, and with an applicant pool half composed of referral applicants. The two simulations differ only in that the former assumes a high level of referring eligibility within the firm ($r=0.78$), while the latter assumes a low level of referring eligibility within the firm ($r=0.38$).

Our computational simulation calculates the area-under-the-curve measure for the thick darker line in Figure 2. This area is given by $SIM(c=0.65, l=0.01, p=0.5, b=0.01, r=0.78, A=1.0, M=1.0, X=1.0)=7.0$. Using the $MATHs()$ model, we find that segregating effect of $MATHs(c=0.65, l=0.01, r=1.077)=7.0$. That is, a sex bias parameter of 1.077 (equivalent to a 7.7% preference for a female applicant over the equivalent male applicant) generates the equivalent segregating effect as that particular simulation model with referring.

Repeating this process for the lower referring eligibility scenario ($r=0.38$), we find that the segregating effect of $SIM(c=0.65, l=0.01, p=0.5, b=0.01, r=0.38, A=1.0, M=1.0, X=1.0)=6.9$, and the segregating effect of $MATHs(c=0.65, l=0.01, r=1.076)=6.9$. In other words, the segregating effect of the simulation with those parameter settings is 1.076 in sex bias units.

Figure 2 illustrates the successful docking of our simulation and analytical models as well as the usefulness of these sex bias units. The thick dark and light lines are the simulation results showing percent female on the job over time when $r=0.78$ and $r=0.38$, respectively. The dashed line is the prediction of the mathematical model of referring, $MATHr$ (defined in Appendix B). The thin dark and light lines are the predictions of the mathematical model including sex bias only when $s=1.077$ and $s=1.076$, respectively. Note that we intentionally depict a job scenario with modest segregating effects, as revealed by the range of the y-axis in Figure 2. No referring-related mechanisms other than homophily are active, and even these effects are
diluted by the half of all applicants coming from sources other than referring. Even in this very detailed view of the percent female on the job over time, our simulation output, the output predicted by our mathematical model of referring, $MATH_r$, and the curves depicting our use of a single sex-bias term to summarize this behavior, are remarkably similar. Our claim in presenting our simulation results as bias units is essentially that the thin lines in Figure 2 are a reasonable way to summarize the simulation results as shown in the thick lines in Figure 2. The interpretation of the bias units is that the two referring scenarios defined above create a deviation in the percent female on the job equivalent to the deviation created by a comparable job pipeline with levels of sex bias in screening such that women are 7.7 percent and 7.6 percent more likely to be hired than men, respectively. Using sex-bias units we can summarize the segregating effects of referring from a wide set of model parameters with a single number. The remainder of this paper uses these sex bias units when discussing model results.

### 3.2. Segregating Effects of Referring & Referrer Behaviors

#### 3.2.1. Referring Homophily Alone

We explore the segregating effects of referring using the parameter space described above and summarized in Table 3. Each specific set of parameter settings can be thought of as a different job setting. We begin by considering a set of jobs with referring taking place, but absent any of the three referrer behavior mechanisms (A, M or X) described above. Such jobs represents the traditional conception of how referring contributes to job segregation – that is, exclusively via homophily in referring. The segregating effect (in sex bias units) of referring with homophily depends in part upon the percent of hires coming from referring, as is illustrated in column (1) of Table 4, panels A and B. Panel A reveals the segregating effects in sex bias units for jobs with high levels of referring eligibility ($r=0.78$), and Panel B the same for otherwise identical jobs with low levels of referring eligibility ($r=0.38$). Column (1) in Table 4 shows that homophilous referring operating alone in our simulated job pipelines yields biasing effects ranging from the equivalent of a 1.04 to a 1.18 sex bias in hiring (i.e., a 4% to 18% preference for female applicants), depending upon the percent of

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12 Because of the noise evident in our simulation results, we present our sex bias unit measures as whole-number percentage points. The example simulations shown in Figure 2 would thus both be 8 sex bias units.
referral hires. Thus, consistent with the conventional wisdom, referring homophily alone serves to bias the sex composition of jobs, and more so as the firm increasingly relies on referral recruitment (cf. the rows of column 1 in Panels A and B, as the percent of referrals in the applicant pool varies).

Another notable result from Table 4 is that this large difference in the two referring eligibility baseline parameter values modeled, \( r=0.38 \) and \( r=0.78 \), did not appreciably change the segregating effects of referring (cf. the values in Panel A with those in Panel B). Our mathematical model, being independent of \( r \), anticipated this result. An important implication of this finding is our first proposition:

**Proposition 1:** Organizational policies modifying the uptake of referring – such as offering a referring bonus or not – are unlikely to have much of an effect on the segregating effects of referring.

As discussed above, even in a setting where referral bonuses are not offered (Fernandez and Fernandez-Mateo 2006), clearly some employees refer. To the degree that referring bonuses provide incentives to expand the number of employees who are willing to refer, changes in this incentive do not appear to affect the segregating effects of referring. Rather, it is the extent to which a firm’s pool of applicants is stocked with referrals that greatly affects the segregating effects of referring for that firm.

**Proposition 2:** Changes in the degree to which an organization relies on referral recruitment is likely to affect the segregating effects of referring.

### 3.2.2. Integrating Referrer Behaviors (A, M and X)

Table 4 also shows the biasing effects of the referrer behavior mechanisms, at their empirically-derived parameter values, operating singly (columns 2 through 4), in pairs (columns 5 through 7), and all together (column 8). The biasing effects shown in column 8 of Table 4 represent the effects of documented referring processes operating at the levels identified in empirical research and implemented conservatively. To the extent that sex biases in hiring ranging from 5 to 20 percent (values of 1.05 to 1.20) would be considered non-trivial, these results show the biasing effects of referring are also non-trivial.

The biasing effects shown in column 8 of Table 4 are consistently greater than the biasing effects that would have been produced by homophily alone. Column 3 of Table 5 shows that the increase in the biasing effect of having all the mechanisms turned on over referring homophily alone in percentage terms.
Irrespective of whether the referring eligibility is high or low, these increases in the biasing effect of referring range from 12% to 47%. Isolating those segregating effects NOT attributable to homophily alone (column 4), we find that between 11% and 32% of the segregating effects of referring are directly attributable to referrer behavior processes. That is, if the segregating effects of referring in an actual firm were measured, this measure would include the effects of homophily AND various referrer behavior mechanisms. Neglecting the segregating effects of referrer behaviors neglects (in the case of the mechanisms implemented in this simulation) the mechanisms contributing between one-tenth and one-third of the total segregating effects of referring. Whereas previous scholarship has focused exclusively on homophily as the culprit in the segregating effects of referring, we find that referrer-behavior processes are also substantial contributors to this outcome.

Careful examination of the results presented in Table 4 also reveals that the three referrer behavior mechanisms interact in complex ways. For example, the greatest increase in biasing effects over the “homophily only” scenario is not in the presence of all three referrer mechanisms, but in the pairing of two: sex asymmetries in referring and referrer-referral exit chains, shown in column (6). Interestingly, the exit chain mechanism does not have much of an effect in isolation (as shown in column [4] of Table 4), but appears to interact with the sex asymmetry mechanism synergistically to yield more bias. The comparison between columns (8) and (6) of Table 4 shows that introducing the asymmetry in referring by referral status mechanism \(M\) reduces the biasing effects of referring in every row of Table 4. This latter finding reveals a potential policy lever: referrer behavior mechanisms can be used intentionally to reduce the biasing effects of referring.

3.3. Potential Interventions: Managing Referrer Behavior

The differences in the columns in Table 4 come from the referring mechanisms being either present or absent. Organizational policies are unlikely to be able to completely turn off any of these mechanisms as we can in a simulation. We now explore the biasing effects of various policy interventions targeting the three referrer behaviors \(A, M\) and \(X\) by varying their governing parameters.

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13 We conducted additional explorations of these synergistic dynamics not shown here. The exit chain mechanism builds off the sex asymmetry in referring, along with homophily, the exit rate, and the baseline sex composition. These latter three parameters determine the sex asymmetry that renders the exit chain mechanism “neutral.” That is, changing the \(X\) parameter has no effect on job segregation. When the asymmetry is greater than this threshold, increases in \(X\) have a strong and positive segregating effect, and when the asymmetry is below the threshold, increases in \(X\) have a strong and negative segregating effect.
3.3.1. Changes in $A$: Asymmetries in Referring by Sex

We found two case-study estimates of $A$, one with women referring more than men by 20% and another with no sex differences. The fact that these two conditions both appear in low-wage female-dominated jobs suggests that this asymmetry is unlikely to be a strong general tendency. If so, then approaches similar to those commonly suggested for EEO hiring (e.g., especially encouraging a particular group to apply for a job), might successfully be applied within an organization to especially encourage a particular under-represented group to take advantage of the company’s referral bonus policy, for example. It is thus plausible that such interventions could actually result in the under-represented group referring more – thereby reversing the direction of the asymmetry.

Towards this end, we explore the range of values in $A$ shown in Table 3, beyond the simple “on” or “off” values previously presented, looking for values that reduce the segregating effects of referring. Figure 3 shows the effects in sex-bias units of varying $A$ for eight scenarios (four values of $p$: the percentage of referrals in the applicant pool, and two values of $r$: the level of referring eligibility), holding the other mechanisms constant at their empirically estimated values. Across all scenarios, there is a direct association between $A$ and the segregating effects of referring. Increases in the referring behavior of the underrepresented group (i.e., moving from right to left on Figure 3, from reducing to reversing the asymmetry in referring), reduce those effects. Indeed, the greatest reduction in segregating effects in Figure 3 occurs when the $A=0.8$, when the sex asymmetry in referring is reversed. This finding buttresses the intuition we tried to cultivate with our earlier conceptual example: that asymmetries in referring behavior can modify the segregating effects of referring. Although our simulation has focused on sex segregation, these interaction of asymmetries and homophily can be expected to be robust to other ascriptive group categories.

**Proposition 3:** Ascriptive group asymmetries in referring behavior within a firm, in the presence of ascriptive group homophily in generating referral applicants, is directly related to the segregating effects of referring for that firm. As a result, getting the under-represented group in an organization to refer more will likely reduce the segregating effects of referring.
3.3.2. Changes in $M$: Asymmetries in Referring by Referral Status

As is clear from column 3 of Table 4, the impact of $M$ alone is not appreciably different from homophily alone. Figure 4 shows further explorations of the parameter $M$, and reveals that lower values of $M$ tend to be associated with greater segregating effects. To this end, increasing this asymmetry may be a useful organizational policy to reduce the segregating effects of referring.

Asymmetries in referring by referral status could be managed directly via incentives. For example, to increase the asymmetry, a firm could modify traditional referral bonus policies in two steps. First, the policy could be changed so the referral bonus money is shared between both the referrer and the referral hire. Second, the referral bonus could be made such that each successive referral hire yields a larger bonus for the referrer and referral according to an increasing schedule of bonuses. (To be practical, the increase would need to be asymptotic, not linear growth.) As a result, referral hires would have a higher reward for their first referral hire than non-referral hires. Thus, referral hires would have larger incentives to refer. In contrast, if the goal were to reduce the asymmetry, a firm could again share the bonus between the referrer and the referral hire, with the amendment that any job holder can only receive referral bonus money only once. These paired modifications would effectively create direct incentives only for non-referrals to become referrers.

3.3.3. Changes in $X$: Referrer-Referral Exit Chains

Finally, referrer-referral exit chains could be managed as well. To reduce the impact of the exit-chain mechanism (focusing on retention), retention bonuses could be given to workers with many links or to those whose alters have left. Because the effectiveness of direct incentives for retention has been questioned (Capelli 2001), it is unclear whether such an intervention would work as desired. To increase the exit-chain effect, a firm would have monetary and social options. The previous suggestion of sharing a referral bonus between the referrer and her referral hire could strengthen the relationship of the dyad. In addition, the firm could promote interactions between referrers and their referrals during their tenure at the firm in a myriad of ways. Strengthening these relationships should increase the role of referrer-referral ties in forming exit chains and job anchors, if the exit chain mechanism operates according to the theories presented above (an open empirical question). Figure 5 demonstrates that although sometimes increases in $X$ are directly associated with
increases in segregating effects, the mechanism also often appears neutral to any segregating effects. As we saw above, this mechanism in particular appears to interact synergistically with sex asymmetries in referring. We explore this interaction more below.

3.3.4. Changes in $A$, $M$, and $X$

As we saw in Table 4, these mechanisms can interact with each other synergistically. For this reason, the single-mechanism analysis presented above is likely to show an incomplete picture of the potential gains from manipulating these referrer behaviors. To address this concern, we explored the parameter space of the three referrer mechanisms $A$, $M$, and $X$ as defined in Table 3. We summarize the part of the explored parameter space most successful in reducing the segregating effects of referring in Column (5) of Table 5. This represents the local minimum in segregating effects of referring. To achieve this minimum, the sex asymmetry in referring ($A$) is reversed such that $A$ is 0.8; referral asymmetry in referring ($M$) is eliminated such that $M=1.0$; and the effect of the exit-chain mechanism ($X$) is increased such that $X=3$. Column 6 of Table 5 shows that the substantial reductions in the segregating effects under these conditions range from 62% to 75% of the segregating effects identified in column 8 of Table 4. These reductions actually counteract not just the added segregating effects of referrer behaviors, but also much of the segregating effects of homophily that are beyond an organization’s control (cf. columns[1] and [5] of Table 5). While our simulation cannot be used to make predictions about the expected reductions any actual firm may expect, the results of our simulation raise the possibility that managing referrer behavior is a viable tool for organizations to mitigate most of the segregating effects of referring, even if firms cannot change referrers’ homophily tendencies or the degree to which the firm relies upon referring for recruitment.

Achieving these large reductions in the segregating effects of referring took advantage of referring asymmetries by referral status ($M$) and referrer-referral exit chains ($X$): two mechanisms that operate independent of the sex of the agents in the pipeline, but interact with homophily and sex asymmetries in referring. As a result, when the sex asymmetry in referring is reversed, increases in the exit-chain mechanism and decreases in referral asymmetry in referring come to have integrating effects. These effects were not apparent in the exploration of single-mechanism dynamics shown in Figures 4 and 5, but were identified by
exploring their simultaneous interactions. The advantage of these two mechanisms is that because they operate independent of sex, opportunities for their management may be less constrained than efforts to manage asymmetries in referring by sex, for example.

**Proposition 4:** Even those referrer behavior mechanisms that operate independent of ascriptive groups can have segregating effects and can present opportunities for organizations to reduce the segregating effects of referring.

4. Summary and Conclusion

Current theory on network effects in the labor market emphasizes the job-seeker perspective, focusing on the segregated nature of job-seekers’ information and contact networks, and leaves little role for organizational influence. But employee referrals are necessarily initiated from within a firm by referrers. This simulation model study is aimed at building theory about the under-theorized dynamics of referring in the labor market from the firm’s and referrer’s perspectives. We argue that referrer behavior is the missing link that can help organizations manage the segregating effects of referring. Adopting the referrer’s perspective of the process, we develop a computational model which integrates a set of empirically documented referrer behavior mechanisms gleaned from extant organizational case studies. Using insights from the model, we develop a set of novel propositions about how the various referring processes are likely to affect job segregation in empirical settings.

Referring can have quantifiable and considerable segregating effects on a job even absent any biases in screening. In docking our simulation model to a mathematical model, we developed a novel intersubjective measure of segregating effects: sex-bias units. To the extent that sex biases in hiring ranging from 5 to 20 percent would be considered non-trivial, our results show the biasing effects of referring are of a similar order. The degree to which a firm’s applicant pool is stocked with referral applicants is an important factor in the segregating impact of referring. While homophily in referrer-referral ties is a driver of these segregating effects, contrary to the current understanding, homophily alone provides only a partial explanation for the segregating effects of referring. Empirically documented referrer behaviors can substantially exacerbate the segregating effects of referring. Also contrary to the conventional understanding of referral processes, these
referrer behaviors present an opportunity for organizations, because they can be managed by organizational policies.

Our study questions traditional understandings and identifies new levers for organizations to manage the segregating effects of referring. For example, one of the main policies available to firms – offering a referral bonus or not – is unlikely to have much of an effect on the segregating effects of referring. Rather, asymmetries in which groups within the firm tend to be the most active in generating referral applicants is a key determinant – and potential lever – affecting these effects. Use of this lever first requires that an organization attend to those employees who generate referral applicants, and consider how they resemble or differ from all job holders. In addition, attending to other aspects of the referral process observable by an organization, such as an individual employee’s referral status and the referrer-referral network formed by referring ties, would assist an organization in managing the segregating effects of those referrer behavior processes that can interact with other segregating mechanisms to either exacerbate or mitigate job segregation.

Although the mechanisms we have identified here do not exhaust the list of possible policy levers, this study has shown the management of referrer behaviors has the potential to mitigate most if not all of the segregating effects of referring without needing to eliminate the practice. Referrer behavior merits more systematic study so the nature and extent of these and other referrer behavior mechanisms can be better understood. Using this understanding to inform organizational policies to manage referrer behaviors, the segregating effects of referring can be mitigated, or conceivably, even reversed.
References


Table 1: Full set of simulated referring eligibility probabilities as modified by the asymmetries in referring by sex (A) and referral status (M) mechanisms.

**Upper Panel: Using a baseline likelihood that 38% of job holders are eligible to refer**

<table>
<thead>
<tr>
<th>A: Referring Asymmetry by sex</th>
<th>Female Non-Referrals</th>
<th>Female Referrals</th>
<th>M: Referring asymmetry by referral status</th>
<th>Male Non-Referrals</th>
<th>Male Referrals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2.5</td>
<td>4</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.225</td>
<td>0.225</td>
<td>0.690</td>
<td>0.806</td>
<td>0.889</td>
</tr>
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<td>0.9</td>
<td>0.311</td>
<td>0.311</td>
<td>0.724</td>
<td>0.828</td>
<td>0.902</td>
</tr>
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<td>1</td>
<td>0.380</td>
<td>0.380</td>
<td>0.752</td>
<td>0.845</td>
<td>0.911</td>
</tr>
<tr>
<td>1.1</td>
<td>0.436</td>
<td>0.436</td>
<td>0.775</td>
<td>0.859</td>
<td>0.919</td>
</tr>
<tr>
<td>1.2</td>
<td>0.483</td>
<td>0.483</td>
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<td>0.871</td>
<td>0.926</td>
</tr>
<tr>
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<td>0.557</td>
<td>0.823</td>
<td>0.889</td>
<td>0.937</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.9</td>
<td>1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.5</td>
<td>4</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.281</td>
<td>0.281</td>
<td>0.713</td>
<td>0.820</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.346</td>
<td>0.346</td>
<td>0.738</td>
<td>0.836</td>
<td></td>
</tr>
<tr>
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<td>0.380</td>
<td>0.752</td>
<td>0.845</td>
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</tr>
<tr>
<td></td>
<td>0.397</td>
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<td>0.849</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.403</td>
<td>0.403</td>
<td>0.761</td>
<td>0.851</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.398</td>
<td>0.398</td>
<td>0.759</td>
<td>0.849</td>
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</tr>
<tr>
<td></td>
<td>0.977</td>
<td>0.987</td>
<td>0.987</td>
<td>0.977</td>
<td></td>
</tr>
</tbody>
</table>

**Lower Panel: Using a baseline likelihood that 78% of job holders are eligible to refer**

<table>
<thead>
<tr>
<th>A: Referring Asymmetry by sex</th>
<th>Female Non-Referrals</th>
<th>Female Referrals</th>
<th>M: Referring asymmetry by referral status</th>
<th>Male Non-Referrals</th>
<th>Male Referrals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2.5</td>
<td>4</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.725</td>
<td>0.725</td>
<td>0.890</td>
<td>0.931</td>
<td>0.961</td>
</tr>
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<td>0.9</td>
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<td>0.756</td>
<td>0.902</td>
<td>0.939</td>
<td>0.965</td>
</tr>
<tr>
<td>1</td>
<td>0.780</td>
<td>0.780</td>
<td>0.912</td>
<td>0.945</td>
<td>0.969</td>
</tr>
<tr>
<td>1.1</td>
<td>0.800</td>
<td>0.800</td>
<td>0.920</td>
<td>0.950</td>
<td>0.971</td>
</tr>
<tr>
<td>1.2</td>
<td>0.817</td>
<td>0.817</td>
<td>0.927</td>
<td>0.954</td>
<td>0.974</td>
</tr>
<tr>
<td>1.4</td>
<td>0.843</td>
<td>0.843</td>
<td>0.937</td>
<td>0.961</td>
<td>0.978</td>
</tr>
<tr>
<td></td>
<td>0.906</td>
<td>0.906</td>
<td>0.963</td>
<td>0.977</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>0.840</td>
<td>0.840</td>
<td>0.936</td>
<td>0.960</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>0.780</td>
<td>0.780</td>
<td>0.912</td>
<td>0.945</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>0.727</td>
<td>0.727</td>
<td>0.891</td>
<td>0.932</td>
<td>0.961</td>
</tr>
<tr>
<td></td>
<td>0.681</td>
<td>0.681</td>
<td>0.872</td>
<td>0.920</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>0.602</td>
<td>0.602</td>
<td>0.841</td>
<td>0.901</td>
<td>0.943</td>
</tr>
</tbody>
</table>
Table 2: Agent exit probabilities in a given timestep (simulated week) as modified by the referrer-referral exit chain mechanism ($X$) and the number of alters in and out of the firm when the baseline exit probability, $l=0.010$.

<table>
<thead>
<tr>
<th>Referrer-referral exit chain: $X = 1.49$</th>
<th>Alters Out ($t_{out}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Alters</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.010</td>
</tr>
<tr>
<td>In</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.007</td>
</tr>
<tr>
<td>($t_{in}$)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
</tr>
<tr>
<td>5</td>
<td>0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Referrer-referral exit chain: $X = 1.98$</th>
<th>Alters Out ($t_{out}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Alters</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.010</td>
</tr>
<tr>
<td>In</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.005</td>
</tr>
<tr>
<td>($t_{in}$)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.004</td>
</tr>
<tr>
<td>3</td>
<td>0.003</td>
</tr>
<tr>
<td>4</td>
<td>0.003</td>
</tr>
<tr>
<td>5</td>
<td>0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Referrer-referral exit chain: $X = 3$</th>
<th>Alters Out ($t_{out}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Alters</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.010</td>
</tr>
<tr>
<td>In</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.003</td>
</tr>
<tr>
<td>($t_{in}$)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Table 3: Model parameters and their ranges of simulated values.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Personnel flow dynamics</strong></td>
<td></td>
</tr>
<tr>
<td>$c$ Sex composition (% female) of non-referral applicants</td>
<td>${0.5, 0.65^a}$</td>
</tr>
<tr>
<td>$l$ Baseline exit rate</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Referring Processes</strong></td>
<td></td>
</tr>
<tr>
<td>$p$ Proportion of hires coming via referrals</td>
<td>${0, 0.25, 0.5, 0.75, 1}$</td>
</tr>
<tr>
<td>$h$ Homophily</td>
<td>${0.1^a, 0.2}$</td>
</tr>
<tr>
<td>$r$ Baseline probability a job holder is eligible to refer</td>
<td>${0.38, 0.78}$</td>
</tr>
<tr>
<td>$A$ Asymmetry in who refers by sex</td>
<td>${0.8, 0.9, 1, 1.1, 1.2, 1.4}$</td>
</tr>
<tr>
<td>$M$ Asymmetry in who refers by referral status</td>
<td>${1, 2.5, 4, 7}$</td>
</tr>
<tr>
<td>$X$ Referrer-referral exit chains</td>
<td>${1, 1.49, 1.98, 3}$</td>
</tr>
<tr>
<td><strong>Screening Biases (used for validation)</strong></td>
<td></td>
</tr>
<tr>
<td>$s$ Sex bias (favoring female applicants) in hiring</td>
<td>${1^a, 1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2}$</td>
</tr>
<tr>
<td>$b$ Referral bias (favoring referral applicants) in hiring</td>
<td>${1^a, 1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2}$</td>
</tr>
</tbody>
</table>

$^a$ We varied these parameters during the model validation process described in Appendix B, but for the purpose of simplicity did not vary them during the exploration of referrer behavior processes. The indicated value was used for our analysis simulations.
Table 4: Segregating effects of referrer-behavior mechanisms in sex bias units. Whole numbers represent the percentage-point bias favoring female applicants for hire yielding the equivalent segregating effects.

<table>
<thead>
<tr>
<th>r: Initial referring eligibility</th>
<th>p: % referrals among applicants</th>
<th>(1)</th>
<th>Solo Mechanisms</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>All 3 (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Referring Homophily Alone</td>
<td>Referring Asymmetry by Sex A</td>
<td>Referring Asymmetry by Referral M</td>
<td>Referral Exit chains X</td>
<td>AM</td>
<td>AX</td>
<td>MX</td>
<td>AMX</td>
</tr>
<tr>
<td>38%</td>
<td>100%</td>
<td>18</td>
<td>23</td>
<td>18</td>
<td>17</td>
<td>20</td>
<td>24</td>
<td>18</td>
<td>20</td>
<td>15</td>
</tr>
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<td>Panel A</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>75%</td>
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<td>8</td>
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<td>9</td>
<td>12</td>
<td>9</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>50%</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>4</td>
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<td>18</td>
<td>17</td>
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<td>24</td>
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<td>78%</td>
<td>100%</td>
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<td>24</td>
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<td>Panel B</td>
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</tr>
<tr>
<td>75%</td>
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<td>50%</td>
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<td>11</td>
<td>11</td>
</tr>
<tr>
<td>25%</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Constant parameters: \( c=0.65, \ell=0.01, b=0.1 \);
Varied parameters: \( p,r,A,M,X \)
Table 5: Illustrating the impact of referrer behavior mechanisms on the segregating effects of referring. This impact may be a substantial increase in segregating effects, or, through the careful management of referrer behaviors, a major opportunity for mitigation. Whole numbers are segregating effects in sex-bias units, representing the percentage-point bias favoring female applicants for hire yielding the equivalent segregating effects.

<table>
<thead>
<tr>
<th>r: Initial referring eligibility</th>
<th>p: % referrals among applicants</th>
<th>(1) Homophily Alone</th>
<th>(2) AMX Observed</th>
<th>(3) Percent Change</th>
<th>(4) Segregation from Referrer Behaviors</th>
<th>(5) $SIM()$ Local Minimum</th>
<th>(6) Potential Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Table 4 Col (1)</td>
<td>Table 4 Col (8)</td>
<td>(2)/(1)</td>
<td>(2)-(1)/(2)</td>
<td>$A = 0.8$</td>
<td>(2)-(5)/(2)</td>
</tr>
<tr>
<td>38%</td>
<td>100%</td>
<td>18</td>
<td>20</td>
<td>114%</td>
<td>12%</td>
<td>8</td>
<td>62%</td>
</tr>
<tr>
<td><strong>Panel A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>12</td>
<td>15</td>
<td>123%</td>
<td>18%</td>
<td>6</td>
<td>63%</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>8</td>
<td>11</td>
<td>146%</td>
<td>32%</td>
<td>3</td>
<td>71%</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>4</td>
<td>5</td>
<td>125%</td>
<td>20%</td>
<td>2</td>
<td>71%</td>
<td></td>
</tr>
<tr>
<td>78%</td>
<td>100%</td>
<td>18</td>
<td>20</td>
<td>112%</td>
<td>11%</td>
<td>7</td>
<td>63%</td>
</tr>
<tr>
<td><strong>Panel B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>12</td>
<td>16</td>
<td>129%</td>
<td>23%</td>
<td>5</td>
<td>66%</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>8</td>
<td>11</td>
<td>140%</td>
<td>28%</td>
<td>3</td>
<td>73%</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>4</td>
<td>6</td>
<td>147%</td>
<td>32%</td>
<td>1</td>
<td>75%</td>
<td></td>
</tr>
</tbody>
</table>
In the shaded region (i.e., about 19 years and on), the slower-to-the-asymptote curve has the greater deviation from the baseline. In the non-shaded region, the lower curve that gets to its asymptote more quickly has the greater deviation from the baseline.
Figure 2: Illustration of summarizing the behavior of both the simulation and analytical model behaviors in terms of a single “sex-bias units” metric. The graph shows the sex composition of the job during the defined time horizon (520 modeled weeks) for two simulations (thick lines), their associated \textit{MATHr()} model (dashed line), and the two \textit{MATHs()} models (thin lines) producing equivalent segregating effects as the simulations.

<table>
<thead>
<tr>
<th>Model</th>
<th>Segregating Effect</th>
<th>Sex-Bias Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIM($c=0.65$, $l=.01$, $p=.5$, $b=.01$, $r=0.78$, $A=1$, $M=1$, $X=1$)</td>
<td>7.0</td>
<td>\textit{MATHs}($c=.65$, $l=.01$, $s=1.077$)</td>
</tr>
<tr>
<td>\textit{MATHs}($c=0.65$, $l=.01$, $p=.5$, $b=.01$, $r=0.38$, $A=1$, $M=1$, $X=1$)</td>
<td>6.9</td>
<td>\textit{MATHs}($c=.65$, $l=.01$, $s=1.076$)</td>
</tr>
<tr>
<td>\textit{MATHr}($c=.65$, $l=.01$, $p=.5$, $b=.01$, $b=1$)</td>
<td>6.8</td>
<td>\textit{MATHs}($c=.65$, $l=.01$, $s=1.075$)</td>
</tr>
</tbody>
</table>
Figure 3: Segregating Effect, in sex-bias units, of varying the sex-asymmetry in referring parameter $A$ over the range \{0.8, 0.9, 1, 1.1, 1.2, 1.4\}, while holding the other parameters of the simulation constant ($M = 4, X = 1.98$). Parameter values less than 1 represent a reversal of the asymmetry.

Low Referring Eligibility Condition, $r = 38\%$

* Note: For sex-asymmetries in referring, $A=1.2$ is referred to as the “observed” value of the parameter, and is based on the findings presented by Fernandez and Sosa (2005). $A=1.0$, the “null” value for the parameter was also empirically observed by Fernandez and Fernandez-Mateo (2006).

High Referring Eligibility Condition, $r = 78\%$
Figure 4: Segregating Effect, in sex-bias units, of varying the asymmetry in referring by referral status parameter $M$ over the range $\{1, 2.5, 4, 7\}$, while holding the other parameters of the simulation constant ($A = 1.2, X = 1.98$).

Low Referring Eligibility Condition, $r = 38\%$

High Referring Eligibility Condition, $r = 78\%$
Figure 5: Segregating Effect, in sex-bias units, of varying the exit chain parameter $X$ over the range $\{1, 1.49, 1.98, 3\}$, while holding the other parameters of the simulation constant ($A = 1.2, M=4$).

**Low Referring Eligibility Condition, $r = 38\%$**

**High Referring Eligibility Condition, $r = 78\%$**
Appendix A: Empirical Estimates for Referring Eligibility

The empirical findings concerning referrer behavior summarized above sometimes used as an analytical sample the set of referring employees who produced at least one referral applicant (e.g., Fernandez and Sosa 2005), and sometimes the sample of all employees assumed to be referring-eligible (e.g., Fernandez and Castilla 2001; Fernandez and Fernandez-Mateo 2006). This distinction is consequential. The latter analytical approach assumes that all employees are equally at risk of engaging in referring, while the former approach allows for the possibility that referrers are somehow a distinctive subset of job holders, and not all job holders are truly at risk for referring. Using the data presented in the case studies, we can investigate the implications of these two perspectives for our simulation.

If all job holders are truly at risk for referring, then we can imagine a fixed probability that each job holder attempts to refer (with possible modifiers as described in the discussion of asymmetries above), and another set of probabilities that their referring targets successfully becoming referral applicants. In this scenario, there is a mean combined probability that any job holder generates a referral applicant. It is this perspective upon which our above hypothetical example was based, and also served as the basis for the analysis of referring as a repeated event using the full set of job holders (Fernandez and Castilla 2001: Table 3). Turning this perspective around, when a job applicant identifies herself as a referral applicant by naming the current job holder who referred her, every job holder is at risk of being named. If every job holder is at risk of being named with a positive mean probability, then given the number of referral applicants and total job holders eligible to refer, we can calculate the number of actual referrers that should be identified. This calculation is one based on simple probability.

If there is an $d$-sided fair die, such that each face on the die from 1 to $d$, inclusive, has an equal probability of showing on a given roll, then we can calculate the number of unique faces $f$ expected to turn up after $n$ rolls. The problems are identical. If all job holders can be represented as having the same mean probability of generating a referral applicant, then given the number of referral applicants ($a$, the number of die rolls), and the number of job holders eligible to refer (the $d$ sides on the die), we can calculate the number
of unique referrers \((f\text{ faces})\) expected to be identified by the referral applicants. This number is given by the following recurrence relation:\(^{14}\)

\[
f(n+1,d) = f(n,d) + \left(1 - \frac{f(n,d)}{d}\right); \quad f(0,d)=0.
\] (A1)

The data presented by Fernandez and Sosa (2005:864-5) reveal 1539 referral applicants named 1223 referrers out of 4114 job holders. Given 1539 referral applicants, if each of the 4114 job holders had the same mean probability of being named as a referrer, we would expect to see 1284 unique referrers named. Instead those 1539 referral applicants named 1223 referrers, or the number of referrers expected if only 3213 job holders had the same mean probability of being named as a referrer. The empirical setting documented by Fernandez and Sosa (2005) – in a firm that offered referral bonuses to the referrers of referral hires – operated as if fully 22% of job holders were not truly at risk of referring, and all the referring activity was limited to 78% of job holders.

A second test is provided by the case study documented by Fernandez and Fernandez-Mateo (2006:53), where 580 referral applicants name 200 referrers out of 557 job holders. Given the 580 referral applicants, if all 557 job holders had the same mean probability of being named as a referrer, we would expect to see 361 unique referrers named. The 200 unique referrers actually named is the number we would expect to see named by 580 referral applicants if only 214 job holders had the same mean probability of being named as a referrer. The empirical setting documented by Fernandez and Fernandez-Mateo (2006) – in a firm that did not offer any referral bonuses to the referrers of referral hires – operated as if fully 62% of job holders had zero risk of engaging in referring, and all of the referring activity were concentrated within just 38% of job holders.

These calculations are illustrative. The datasets providing the numbers for the calculations represent complex flows of personnel through two firms over time. Referral applicants can be hired and become referrers themselves within the studies’ observation windows. Similarly, job holders contribute to the total

\(^{14}\) We provide a brief proof by induction. In the base case, it is clear that independent of \(d\), after 0 rolls, 0 unique faces will have been revealed. Assuming that after \(n\) rolls, the number of unique faces revealed on an \(d\)-sided die is \(f(n,d)\), then the next roll \((n+1)\) will show a face that has already occurred with a probability of \(f(n,d)/d\). Thus a new, unique face will be revealed with a probability of \(1- (f(n,d)/d)\). So upon \(n+1\) rolls, the total expected number of unique faces is the sum of the number of unique faces revealed in \(n\) rolls and the probability that the next roll will reveal a newly unique face, or using the defined notation: \(f(n,d) + (1-(f(n,d)/d))\). QED.
count of job holders even if they were in the firm for just one week of the data collection window. In addition, there are likely many mechanisms in addition to the ones we explicitly model affecting referring behavior. Using the same logic underlying our choice to use a constant baseline sex composition for non-referral applicants as a way to capture the net effect of many supply-side and other mechanisms affecting the sex composition of the applicant pool for a particular job, we use these two estimates for the referring eligibility composition of job holders to capture the net effect of other referring-related mechanisms. We set the baseline likelihood that any given job holder may be a referrer (parameter: $r$) to 0.78 for half of our simulations, and 0.38 for the other half. That is, absent other mechanisms affecting referring likelihood, in one exploration of the model parameter space, 78% of job holders are potential referrers and 22% will not refer, and in a parallel exploration, 38% of job holders are potential referrers and 62% are not.
Appendix B: Model Docking & Validation

In this Appendix, we develop a mathematical model implementing the same job personnel flows and referring dynamics (but not the referrer behavior mechanisms) as implemented in our simulation model. By comparing the output from both models under similar parameter inputs, we help to validate our simulation model. Although our primary concern is the implementation of referring processes, we begin first with the simpler mathematical model of a job with some sex bias present at screening. This first mathematical model is simpler, and serves as the basis for our key outcome variable, the segregating effect represented in sex bias units. After presenting this first mathematical version of our model, we expand upon it to integrate referring processes, and present both the docking output and the inter-subjective validity suggested by our sex bias units metric.

B.1. Sex bias model: $MATHs(c, l, s)$

Sex bias in job screening is the difference in likelihood that two individuals identical in every way but sex will be offered a particular job at a particular company. It is a widely researched labor market phenomenon with many job and organization specific bias estimates presented in the literature, albeit often with the caveat that males and females are “observationally equivalent.” Sex bias in job screening has been suggested to be the primary opportunity and mechanism through which a firm may contribute to job sex segregation (Kauffman 2002; Petersen, Saporta and Seidel 2001; Reskin and Bielby 2005).

In the presence of a sex bias in hiring, incoming hires would deviate from the baseline sex composition of nonreferral applicants, $c$, as determined by a sex bias parameter, $s$. Although this bias parameter could be implemented simply as the product, $cs$, this could lead to the situation where for large values of $s$, the probability that the next hire is female exceeds one. To avoid this, we implement the bias as follows. In the presence of a sex bias in hiring, the probability a vacancy is filled with a female agent is $sc/(1-c+sc)$, and the probability a vacancy is filled with a male agent is $(1-c)/(1-c+sc)$. This implementation allows arbitrarily large values of $s$, keeps the resulting probability in the appropriate range, and the probabilities of hire for male and female agents sum to one. A necessary result of this implementation is diminishing effects

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15 Except for some special circumstances, empirical research usually cannot rule out that males and females might differ in unmeasured ways when calculating screening bias (see National Research Council, 2004).
to increases in bias. That is, as bias increases, the probability a female agent is hired asymptotically approaches one. In our model validation simulations, we explore values of \( s \) from the set \{1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2.0\}.

Starting with our basic model without referring (governed by parameters \( c \) [input sex composition] and \( l \) [exit rate] only), we can quantify the deviating impact of a particular level of sex bias in screening (parameter \( \delta \)). This deviating impact measure becomes our basis for comparison. For any deviating impact of a set of referrer-based processes, we can answer the question: Given our modeling of the mechanisms, what level of sex bias in screening produces an identical deviation in an otherwise identical job pipeline? Given the simplicity of this model, specifically the absence of referring dynamics, the deviating impact of sex bias in screening for a set of initial conditions can be found analytically without requiring computational simulation. Because of this fact, we can use the mathematical model to help validate our simulation model.

Given exit rate \( l \), the sex composition of non-referral applicants \( c \), and sex bias parameter \( s \); and given the constraint of a stable job; and letting \( w(t) \) represent the number of women in the job at time \( t \), and \( m(t) \) represent the number of men in the job at time \( t \), the mathematical description of the composition of the job over time is:

\[
\frac{dm}{dt} = \frac{1-c}{1-c+sc} l(w(t) + m(t)) - lm(t), \quad \frac{dw}{dt} = \frac{sc}{1-c+sc} l(w(t) + m(t)) - lw(t).
\]

(B1)

The percent female on the job over time in this model, \( MATHs(c, l, s, t) \), in terms of \( m(t) \) and \( w(t) \) from above is:

\[
MATHs(c, l, s, t) = \frac{w(t)}{(m(t) + w(t))}. \quad \text{(B2)}
\]

The segregating effect of the sex bias is:

\[
\text{Segregating Effect} = \int_0^{520} [MATHs(c, l, s, t) - c] dt \quad \text{(B3)}
\]

The timestep for the model is one simulated week. We run each simulation for 520 steps, or 10 years. The area between the curve plotting the percent female on the job over time for a particular simulated job and the line at \( c \) provides our measure of deviation for that job. Note that if a job’s percent female curve goes below \( c \), the difference in area does become negative.

Using the values of \( s, c, \) and \( l \) discussed above, we can evaluate the expected deviating effects of sex biases in both our computational simulation and our mathematical model. The computational simulation was
implemented in RePAST (North, Collier & Vos 2006), and the mathematical model was solved numerically using Maple (Maplesoft 2003). A comparison of the simulation outputs with the exact values of the deviating impact of different levels of sex bias in screening is given in Figure B1. As Figure B1 shows, our simulation model generates results very close to its mathematical counterpart. This replication of model outputs from two very different realizations of the same underlying model buttresses our confidence in the ability of our simulation model to reveal the behavior of the modeled personnel flow dynamics rather than revealing particular simulation model artifacts.

B.2. Referral bias model: $MATH_{r(c,l,p,b)}$

Given that the focus of our investigation is referring dynamics, the mathematical model above helps to provide a conversion tool for our findings, but is less-suited to validation. Like sex bias in hiring, referral bias is easily modeled mathematically. We implement a hiring bias favoring referral applicants in an analogous manner to our implementation of a sex bias in hiring. That is, for a given bias $b$ ($b > 1$ for a bias favoring referrals), a vacancy is filled with a referral hire with a probability $bp/(1-p+bp)$, and with a non-referral hire with a probability $(1-p)/(1-p+bp)$. In our simulations, $p$ takes on values in the set $\{0.25, 0.5, 0.75, 1.0\}$. We include the parameter $b$ in our model validation tests, using the values $\{1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2.0\}$.

We use variation in referral bias ($b$) and homophily ($h$) to help validate our simulation model of the basic referring dynamics. Our simulation model including basic referring dynamics (but absent the three referrer-behavior mechanisms) augments the previous simulation model with governing parameters $p$ (likelihood a new hire is a referral), $r$ (baseline referring eligibility), $h$ (homophily) and $b$ (referral bias). The comparable mathematical model (which is notably independent of $r$) replaces the system of equations in (B1) with those in (B4), below:

$$\frac{dw}{dt} = \frac{1-p}{1-p+pb}\left(cl(w(t) + m(t)) \right) \quad \text{[incoming non-referral female hires]}$$

$$+ \frac{pb}{1-p+pb}\left((c+h)lw(t) \right) \quad \text{[women referred by women]}$$

$$+ \frac{pb}{1-p+pb}\left((c-h)lm(t) \right) \quad \text{[women referred by men]}$$

$$- lw(t) \quad \text{[exiting women]}$$

(B4)
\[
\frac{dm}{dt} = \frac{1-p}{1-p+pb} (1-c)l(w(t) + m(t)) \quad \text{[incoming non-referral male hires]}
\]
\[
+ \frac{pb}{1-p+pb} (1-c-h)lw(t) \quad \text{[men referred by women]}
\]
\[
+ \frac{pb}{1-p+pb} (1-c+h)lm(t) \quad \text{[men referred by men]}
\]
\[-lm(t) \quad \text{[exiting men]}.
\]

With these changes to the mathematical model, the percent female on the job for the \( MATHr(c,l,p,h,b) \) model remains as represented on the right hand side of equation (B2). Because the set of parameters governing this system differ, the solution to the percent female on the job in \( MATHr(c,l,p,h,b) \) will include this distinct set of parameters. Similar to equation (B3), the segregating effect of the \( MATHr() \) model is represented in equation (B5) in terms of the \( MATHr() \) model representing the solution of the system of equations in (B4):

\[
\text{Segregating Effect} = \int_0^{520} [MATHr(c, l, p, h, b, t) - c] dt \quad (B5)
\]

Figure B2 plots the analytical predictions of \( MATHr() \) along with the simulation results for the deviating effects of varying referral bias and homophily using the following set of parameter values: \( c=0.65, \ l=0.01, \ p=0.5, \ h=\{0.1, 0.2\}, \ r=\{0.78, 0.38\}, \) and \( b=\{1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2.0\} \). These results illustrate the agreement between the mathematical model described of basic referring processes, and our computational simulation of those same processes. Again, this similarity in the results of the two models buoys our confidence that our simulation model successfully implements the job pipeline and referring processes as described.

The three referrer-behavior mechanisms we explore in this paper are not easily modeled mathematically for stable job sizes, as they introduce nonlinearities into the system or are dependent upon a social network structure. Consequently, we continue with an analysis of the three referrer behavior mechanisms using our computational simulation model only.
Figure B1: Deviating effects of the mathematical model of sex bias in hiring, \( MATHs() \) for two initial sex compositions \( c = \{0.5, 0.65\} \), and docking results from the simulation for those same values of \( c \) and selected values of \( s \).
Figure B2: Deviating effects of the mathematical model of referral bias in hiring, $MATHr()$ for two values of homophily, $b = \{0.10, 0.20\}$, and docking results from the simulation for those same values of $b$, high and low referring eligibilities $r = \{0.38, 0.78\}$, and selected values of $b$. 

**Mathematical Model, $MATHr()$**
- $b = 0.20$ high homophily
- $b = 0.10$ observed homophily

**Simulation Model, $SIM()$**
- $b = 0.20$ high homophily, $r=0.78$ high referring
- $b = 0.20$ high homophily, $r=0.38$ low referring
- $b = 0.10$ observed homophily, $r=0.78$ high referring
- $b = 0.10$ observed homophily, $r=0.38$ low referring
APPENDIX C

Program Pseudocode
Do Until 520 weeks of simulated time have elapsed:
   Go through the job holder list and update exit likelihoods based on the exit-chains mechanism.
   Allow job holders to exit based on individual exit likelihoods.
   Iterate through the list of job holders: For each agent(i) (i=1..N)
      If random() < agent(i)’s exit likelihood
         Then remove agent(i)
   Fill Vacancies
      Until vacancies = 0
         If random() < percent of hires recruited formally
            Then add a new non-referral hire
               If random() < percent female among non-referrals
                  Then the new non-referral hire is female
               Else the new non-referral is male
               If random () < probability the new non-referral is eligible to refer
                  Then the new non-referral is eligible to refer
               Else the new non-referral is not eligible to refer
            Else add a new referral hire
               Iterate through the list of current job holders to find a referrer (use the first referring-eligible agent selected at random)
               Use homophily settings to calculate the probability the selected referrerer generates a female referral hire
               Based on sex and referral status of new hire, and the corresponding probabilities from Table 1, determine whether the new agent will be eligible to refer.
         Record job pipeline characteristics (e.g., current percent female, deviation from c)
   End.