MECHANISM OF INSTABILITIES OF EXOTHERMIC
HYPersonic BLUNT-BODY FLOWS

by

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ABSTRACT

An experimental and analytical investigation of the regular, periodic instability observed in exothermic hypersonic blunt-body flows has been conducted. A ballistic range instrumented with submicrosecond spark schlieren photographic equipment and ionization probes has been used to identify the distinguishing flow field characteristics about hypersonic spheres fired into lean acetylene-oxygen mixtures and stoichiometric hydrogen-air mixtures. The variation of the instability wavelength with initial gas state, body size, and body velocity has been recorded. The boundaries of the several observed flow regimes have been established. An instability mechanism is proposed and is shown to be consistent with the experimental results and with the significant gas dynamic wave interaction processes. The instability originates in the induction zone which separates the bow shock and the exothermic reaction front in the nose region of the flow field. The instability arises from the effect on the chemical reaction rate of entropy waves which are generated by the interaction of compression waves with the bow shock wave. The compression
waves are generated by the time-varying reaction rate. This type of instability can be expected to be present in any hypersonic combustion process wherein strong shock waves are present.

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The design of the ballistic range and related components was based on procedures developed by the Physics Group, Re-entry Simulation Range of the Lincoln Laboratory, M.I.T. under the direction of Drs. Richard E. Slattery and Wallace G. Clay. The optical system design was suggested by A. P. Ferdinand. The overall arrangement of the ballistic range components and the design details of the mechanical system were based on suggestions of Peter A. Darvirris whose generous interest in this work was deeply appreciated. The reliable design and operation of the electronic components was due to the efforts of Joseph A. Pirroni who devoted much of his personal time to instructing the author and other students in the Sloan Laboratory on the intricacies and practical aspects of high frequency electronic systems. The dedication of the latter two persons to the value of education will be long remembered.

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RESULTS, CONCLUSIONS AND RECOMMENDATIONS

1. The flow field about hypersonic spheres moving through lean acetylene-oxygen and stoichiometric hydrogen-air gas mixtures at speeds on the order of the Chapman-Jouguet speed and at sufficiently high gas pressures is unstable. At moderate pressures, the instability generates a regular, periodic wave pattern easily observed by means of schlieren photography. At higher pressures, the instability generates a turbulent schlieren pattern.

2. The regular, periodic schlieren wave pattern is produced by:

(a) a corrugation of the boundary separating the hotter reacted gas from the colder unreacted gas. The corrugations, which are frozen in the flow, generate a prominent train of vertical striations, nearly equally spaced, and appearing alternately dark and bright on the photographs.

(b) compression waves propagating in the inviscid wake and the entropy waves generated by the interaction of the compression waves with the bow shock wave. The waves produce a fish-scale like pattern in the inviscid wake between the bow shock and the corrugated reacted gas boundary.

3. The periodic instability is generated by the interactions of the reaction front and the bow shock with compression waves and contact discontinuities propagating within the induction zone in the nose region.
of the flow field. The instability arises from the sensitivity of the induction time to perturbations in the state of the shocked gas.

4. The wavelength of the instability is a strong function of the induction time of the exothermic gas mixture. It is a weak function of the sphere diameter. The instability can exist whenever strong shocks appear in exothermic gas mixtures.

5. The several distinctly different schlieren patterns created by the instability define four flow regimes: (1) Wake combustion regime; (2) Turbulent regime; (3) Stable regime; (4) Periodic regime. The boundaries of the periodic regime, within which the regular, periodic instability is observed, are strong functions of the sphere diameter and velocity and of the initial gas pressure and mixture ratio. The range of First Damköhler numbers based on induction time, sphere radius, and velocity for which the instability is observed lies between 0.1 and 1.0. The Chapman-Jouguet speed and the second explosion limit condition do not define any of the flow field regime boundaries. The theoretical and experimental results suggest that the characteristics of the temperature rise marking the end of the induction period is a factor in determining the boundaries.

6. It is recommended that the conditions required for the existence of the regular periodic instability be investigated by an experimental and analytical program. Experiments should be conducted with exothermic gas mixtures which have an induction zone behavior significantly different
from that of the hydrogen-oxygen and hydrocarbon-oxygen reaction; a carbon monoxide-oxygen system is an example. Analytical calculations of the variation of the temperature rise at the end of the induction period with initial gas state should be carried out to determine if this quantity is responsible for the periodic regime boundaries.

7. It is recommended that the degree to which the presence of the instability increases the extent of the exothermic reaction in the flow field about the sphere be investigated. Since the instability is probably nondestructive, the instability, particularly the turbulent mode, can possibly be used to promote the combustion process in hypersonic air-breathing engine combustors.
INTRODUCTION

The characteristics of the combustion process in gaseous systems wherein the reaction front propagates with a supersonic rate with respect to the unburned gas is of interest to at least two fields of study: (1) detonation wave research, the objective of which is to determine the mechanisms responsible for the development, structure and stability of detonation waves; (2) supersonic combustion research, the object of which is to develop a hypersonic air breathing engine in which supersonic flow is maintained at all points within the device.

Research on detonation waves has been carried out for over forty years yet the processes which govern the characteristics of such waves are not fully understood. A survey of the state of detonation wave research which contains an extensive bibliography is found in Ref. 1. In recent years it has become apparent that self sustained detonation waves are intrinsically unstable and that if sufficiently refined instrumentation is employed, the wave will be found to be three-dimensional and time dependent. Such a state of affairs has long been recognized for waves propagating in mixtures near the detonability limits. Under these conditions the scale of the instability is large and easily observed; "spinning" detonation waves are examples. At conditions removed from the detonability limits, the instability scale becomes extremely small. Some experimental evidence indicates the instability retains a regular,
periodic behavior; this has been termed multi-headed spin. Other evidence suggests the gas flow behind the wave front is randomly turbulent. The experimental difficulties involved in examining the structure of such waves, coupled with their apparent structural complexity, has made extremely difficult the identification of the mechanism responsible for the instabilities.

Research and development of supersonic combustion ramjet engines has only recently grown to a significant scale. Ideally, the goal of this program is to develop an engine in which the flow remains at a high supersonic Mach number throughout. The cycle efficiency of such a design can be shown to be higher than for a conventional design and other advantages accrue, such as reduced structural cooling loads. If it were possible to inject, mix and burn the fuel while the through-flow maintains a high velocity, then the static conditions which govern the chemical reaction rates would not be radically different from those found in present designs. However, either by design or unavoidable circumstances, strong shocks will most likely exist in local regions of the flow. In these regions conditions will be generated similar to those found in detonation waves. Thus the nature of the instabilities associated with such conditions will be of interest to the designers of these devices. Furthermore, it may prove that by controlled use of unsteady chemical kinetic-gas dynamic interactions, the overall reaction rate in supersonic combustors can be promoted.
Recently, two groups conducted experiments in which conditions similar to those found in detonation waves were generated by firing projectiles into quiescent detonable gases. Ruegg and Dorsey (Ref. 2) fired spheres into hydrogen-air mixtures and observed the flow field by means of schlieren photography. Later they reported briefly on results obtained with methane-air and pentane-air mixtures (Ref. 3). Struth, Behrens, and Wecken (Ref. 5) performed similar experiments and included work with gases which underwent endothermic reactions (carbon tetrachloride and benzene) (Ref. 5). Later they performed experiments with conical projectiles (Ref. 6). The experiments indicated that under a wide range of conditions the flow field, as in the case of self sustained detonations, is highly unstable. In the case of the spherical projectiles and exothermic gas mixtures, under certain conditions, the instability generates a highly regular, periodic flow field structure which is easily observed (e.g., Fig. 1). The fact that the flow field structure is easily resolved and is so highly regular as opposed to the very small scale, complex structure of self sustained detonation waves suggests that the ballistic range can be used with advantage to study the mechanisms of the instabilities observed in exothermic hypersonic flows. A program of this type is reported herein.

The previous investigators have reported evidence that the length of the induction zone behind the normal segment of the bow shock is the
characteristic length which determines the wavelength of the observed
regular instability. They suggest that under some conditions an
acoustic instability exists and that under other conditions periodic
ignition occurs. (The conclusions drawn by these authors are discussed
in detail in the section entitled, "Instability Mechanism." ) The
fundamental difference between the two types of theories is that the
role of chemical kinetics does not enter into the evaluation of the
frequency behavior of the acoustic (or hydrodynamic) model whereas a
"periodic ignition" model implies by definition that kinetics controls
the instability period or frequency. Thus, if the instability is purely
hydrodynamic, no information concerning the instability mechanism can
be gained by a study of the frequency behavior; the frequency is de-
termined by the geometry of the system. The results of this study indi-
cate that the observed regular instability is the result of strong gas
dynamic-chemical kinetic interactions and is not of the acoustic type.

The type of behavior observed in these experiments can exist in any
exothermic hypersonic flow involving the gas mixtures of the type used
(hydrogen-air, hydrocarbon-oxygen) wherein strong shock waves are present.
In these experiments, the blunt body serves merely to generate and to
stabilize a strong shock in the gas. A sphere generates an axially
symmetric field; such flow fields are particularly well suited to study
by optical methods. As will be shown, the wavelength of the instability
is only a weak function of the sphere diameter; the characteristic
length which determines the wavelength is, as noted previously, the induction zone length.

The similarities and differences between these instabilities and those associated with the most commonly observed exothermic hypersonic flow, detonation waves, should be pointed out. The simplest and most effective method by which the detonation wave instability can be observed is the soot technique wherein a helical pattern is produced on a soot covered wall as the spinning wave propagates through a tube. Detonations exhibiting multi-headed spin produce a "fish-scale" or cellular pattern. The wavelength of the instability is characterized by the size of the cell. The first success in theoretically predicting the instability frequency was achieved by Fay (Ref. 7) and Manson (Ref. 8) who showed that the spin frequency corresponds to frequency of transverse acoustic wave in the hot product gas behind the wave front. The higher spin frequencies observed at higher pressures and for mixture conditions removed from the detonation limits correspond to the higher acoustic harmonics. The characteristic length entering the analyses is the tube diameter; neither the induction zone length nor any other parameter characteristic of the chemistry appears. The transverse vibrations are a manifestation of the instability of the detonation wave structure; however, since the theory is hydrodynamic, no information on the mechanism of the instability is gained.
Numerous mathematical analyses have been performed to determine under what conditions an exothermic plane wave is stable (see Ref. 1). The approach usually involves the determination of whether or not small disturbances existing within the structure of a detonation wave are amplified or damped; the wave structure is usually assumed to be one-dimensional and is specified by an assumed rate chemistry. Since it has now been determined experimentally that self-sustained detonations are always observed to be unstable, what is desired is an analysis which will describe the asymptotic behavior of the small disturbances as they grow to finite amplitude. Lacking this, on the basis of experimented results, various models of the three-dimensional, finite amplitude, time dependent disturbances have been proposed. Much work has been published in the Russian literature and is reviewed in the book by Shchelken and Troshin (Ref. 9). The structure proposed involves collisions of triple shock configurations which generate local regions of intense temperature and pressure leading to the presence of discrete detonations "heads" which generate the observed instability patterns. The application of this experimentally derived structure to the prediction of instability wavelengths for specified gas conditions is still in an embryonic stage. Some authors claim the model supports the connection between the observed cell size and a chemical reaction time (Ref. 10) and that the discrete structure observed is "a characteristic of the front itself and is independent of the external conditions," i.e., tube geometry (Ref. 11).
relationship between this model and the spin frequencies predicted by
the acoustic theory in which the tube geometry is a determining factor
remains to be explained. In the study reported herein the complex
graphy associated with the perturbed plane wave is avoided and the
prediction of the instability wavelength which should be observed for a
given proposed instability mechanism is quite straightforward.

A number of experiments with detonation waves indicate that under
some conditions the flow is turbulent. White (Ref. 12) obtained such
evidence with the use of an interferometer and Opel (Ref. 13) by means
of schlieren photography. Recent experiments employing other techniques
indicate that a regular periodicity persists on a very small scale in
detonations occurring under conditions far removed from the detonability
limits (Refs. 7 and 14). Thus there is a question as to whether the
detonations under any conditions indeed exhibit a random turbulence or
whether the optical records merely appear turbulent due to the tortuous
path followed by the light rays while traversing the detonation front.

In these ballistic range experiments reported herein both turbulent
and periodic flow patterns are observed and there is no question that
the two cases represent a distinctly different behavior of the reaction
front. The appearance of the flow field drastically changes from periodic
to turbulent as the flow conditions pass through well defined boundaries.

In the following text, a description of the ballistic range and
associated equipment used in the experiments is given. The characteristic
flow field features which appear in the schlieren photographs are then identified. A review of the principles involved in schlieren analysis is given in Appendix I. The experimental results concerning the factors effecting the various observed flow field regimes and data on the behavior of the instability wavelength are then presented. Finally, an instability mechanism is proposed and is discussed in light of the experimental results. Supporting gas dynamic and chemical kinetic calculations are given in Appendices II, III, IV.
SECTION I
DESCRIPTION OF EXPERIMENTAL APPARATUS

The experiments were conducted in a ballistic range instrumented with schlieren photographic equipment and an ionization probe. An overall view of the equipment is given in Fig. 2.

The design of the ballistic range components was based on designs developed for use in the Re-entry Simulation Range, Lincoln Laboratory, Massachusetts Institute of Technology. The ballistic range consists of a specially designed .50 caliber smooth-bore gun barrel and electrically operated breech which fires the projectile into a blast tank where the motion of the powder gases is retarded. The projectile passes through a sabot catcher and diaphragm, continues past a shadowgraph station into the test section where the schlieren equipment and ionization probe are located. The projectile then continues through an uninstrumented section and impacts on a steel plate. The overall length of the range is approximately 25 feet; the diameter of the test section is 6 inches.

The projectiles used are solid aluminum spheres having a Grade 200 finish. The majority of the tests were performed with 1/2 inch spheres which did not require the use of sabots. The sabots used with the smaller diameter spheres are 1/2 inch long nylon cylinders having a 1/2 inch diameter and are recessed at one end to accept the projectile. The sabots are cut lengthwise to promote their break-up so they can be easily separated from the projectile. Standard .50 caliber cartridge
cases are filled with fast burning rifle powder; both Hercules 2400 and Du Pont 4227 are used. The muzzle velocities produced in the experiments ranged from 4500 to 7500 feet per second. The maximum attainable velocity with the 1/2 aluminum spheres is probably on the order of 8000 fps. With the use of light-weight plastic projectiles, higher velocities could be attained. The velocity scatter at any given powder charge is approximately 400 fps or 6% under average conditions. This could be improved with the use of wadding to pack the powder in the cartridge cases and thereby maintain a more even burning rate. The barrel is mounted in a recoil absorbing traversing mechanism. The barrel is bore sighted on the impact plate and requires realignment only after the range is dismantled. The scatter of the projectiles on the impact plate is generally limited to a one inch diameter circle. The breech is remotely operated so that the test cell can be vacated during testing.

The blast tank was fabricated from a heavy-walled 18 inch diameter steel pipe. A single replaceable baffle plate having a 3/4 inch diameter opening is located at the mid-plane. During testing, the blast tank is pumped down to pressures of 100 mm Hg or less. If a low test section pressure is to be used, the effectiveness of the blast tank in retarding the powder gases is critical since the retarding effect of the test gas itself is small. Under these conditions, the blast tank pressure is usually raised to approximately the test section pressure. At test section
pressures, above 150 mm Hg, the blast tank is usually evacuated to less than 1 mm Hg. The effect of the blast tank pressure on muzzle velocity is small; the difference in velocities between a blast tank pressure of 100 mm Hg and 1 mm Hg is less than the muzzle velocity scatter cited above.

A second baffle located outside the blast tank and having a replaceable 5/8 inch diameter opening serves as the sabot catcher. If a sabotted projectile is used, the blast tank pressure is raised to 100 mm Hg in order to promote separation of the sabot and the projectile. Separation was effected in approximately 85% of the runs involving sabots.

The test section gas is separated from the blast tank gas by a one mil mylar diaphragm. At test section pressures on the order of 50 mm Hg or less there is evidence that the debris from the ruptured diaphragm remains in the vicinity of the projectile as it passes the schlieren station which is located approximately two feet downstream of the diaphragm holder. Since most tests were performed at pressures higher than 50 mm Hg, this does not present a significant problem. In some of the photographs of runs made at higher pressures, tiny particles and the Mach waves generated by them can be observed. Whether these are particles of unburned powder or debris from the diaphragm has not been determined.

The projectile velocity is determined by measuring the time interval required for the ball to pass from the shadowgraph station to the
schlieren station. The shadowgraph station is comprised of a point spark light source, a single focusing lens and a camera with a pin hole aperture covered by a blue filter. The lens focuses the spark light on the pin hole; the filter eliminates the need for a capping shutter. The purpose of the shadowgraph is to record the location of the projectile at the instant the timing device is triggered. A trigger signal is provided by the projectile's breaking a light beam which is monitored by a phototube. The light beam is produced by a high intensity D. C. lamp.

The schlieren station is located 13 inches downstream of the shadowgraph station. The distances between a reticle on the shadowgraph station window and reticles on the schlieren mirror window were measured by photographing a calibration bar which was positioned along the centerline of the test section. The calibration bar contains accurately machined reference points such that the errors introduced in the velocity measurements due to errors in the measurement of the distance between measuring points is negligible--less than 0.1%.

The schlieren system is a double pass system utilizing a single 6 inch diameter spherical mirror having a 48 inch radius of curvature. The mirror is located inside the test section, three inches off the center line. Because of the severe environment created by the detonating gases, the mirror surface is protected by a one inch thick optical quality glass cover whose surfaces are ground and polished to the same tolerances as the mirror surface. After each set of runs, the glass cover is
cleaned by overnight immersion in aqua-regia. Pitting of the glass surface due to debris churned up by the exploding gases gradually accumulated over the duration of the program, but the quality of the photographs was not markedly effected.

The remaining components of the schlieren system are grouped outside of the test section at the end of a cross arm, approximately 42 inches from the centerline of the test section. An optical arrangement which made possible the exposing of two photographs along virtually the same optical path is employed. The arrangement of the components is shown in Fig. 3. Two complete schlieren systems, one along a vertical axis, the other along the horizontal axis are shown. The light rays are deflected along the third axis to the schlieren mirror by a four-sided pyramidal mirror, not visible in Fig. 3. Light enters the test section through a one inch diameter, $\frac{1}{8}$ inch thick commercial quality glass window. With this system, the rays incident to the and reflected from the schlieren mirror must travel slightly off-axis in order to strike opposite faces of the pyramidal mirror. In such a double pass system, a slight amount of parallax is introduced. The parallax is minimized by having the incident and reflected bundle of rays strike the pyramid as close to the apex as the dimensions of the bundle cross section allows. For this reason, a lens is used to bring the incident rays to focus on the pyramid face, and the knife-edge, where the reflected rays form a second image of the light source, is placed as close as possible
to the opposite face of the pyramid. The latter requirement could be relaxed by the introduction of another lens such that the reflected image of the light source could be successively focused on the pyramid face and then on the knife edge, but it was felt that the gains would be offset by the deterioration of the picture quality accompanying the introduction of another lens in the reflected ray optical path. Because of the geometry of the light source (a line source rather than a point source), the parallax could be reduced if the four-sided pyramid were replaced by a two-sided mirror and a single rather than dual schlieren system used. For this reason, the dual system was used only when wave propagation rates or other time dependent effects were being studied, and the single system used when the flow field geometry was being studied.

The light source is formed by a spark discharge between two 0.020 inch tungsten electrodes embedded in a 0.020 inch groove machined in a fused quartz block. The electrode gap is of the order of 0.10 inches. The groove is used to stabilize the position of the spark discharge. A line source is used rather than a point source in order to achieve greater illumination; the spark illumination is critical for the f/8 schlieren mirror. A low inductance, 1/4 microfarad pulse capacitor charged to 4000 volts is connected to the tungsten electrodes by 1/4 inch diameter copper leads. The light from the line source is focused on interchangeable slits by a 1 inch focal length microscope eyepiece. The slit widths used range from 0.006 to 0.020 inches; the slit used depends on the
balance between light intensity and schlieren sensitivity desired. The
time duration of the spark as measured by the width of the voltage vs.
time output of a type 1P21 phototube was found to be approximately 0.25
\( \mu \)sec. The trace width was measured at the point where the voltage rise
was \( e^{-1} \) of the maximum. This spark duration is satisfactory for the
projectile velocities encountered in these tests although it does not
provide sufficient resolution to measure the detachment distance with
confidence. The triggering signal for the schlieren spark sources is
generated either by use of a preset oscilloscope delayed trigger or by
the output of a phototube which is focused on the intersection of the
test section centerline and the schlieren system optical axis. Under
the conditions of the experiments, the self luminosity in the nose region
of the projectile is sufficient to produce a strong phototube output.

The knife-edge consists of a section of 0.010 inch shim shock
connected to a micrometer drive. The knife-edge can be oriented both
parallel to and vertical to the flow field axis and can be inserted
from either direction to produce either a bright or a dark bow shock.
For most of the runs a vertical knife-edge position, i.e., oriented
normal to the flow field axis, was used.

The schlieren systems are equipped with a high intensity lamp which
can be inserted in the optical path to replace the spark light source
for alignment and focusing purposes. The primary alignment was achieved
by adjustment of the orientation of the schlieren mirror, which is
mounted internally by means of a three-legged telescope-type installation. Despite the use of an extremely rugged construction, the shock and detonation waves generated at the higher pressures occasionally cause misalignment of the mirror. Alignment is easily checked by illuminating the spark gap and viewing the knife-edge and spark gap by eye through the objective lens.

Achromatic telescope objective lenses were used as objective lenses to focus the test section image. A series of lenses having focal lengths from 20 to 34.5 inches were used to provide different magnifications. The cameras were mounted on a track such that the film plane could be positioned at the focal plane. Commercial 4 x 5 view cameras equipped with Polaroid backs were used together with 3000 speed Polaroid Type 57 film. Electrically operated Packard shutters are fitted to the cameras. The shutters are remotely opened in the darkened room test cell immediately before firing.

Since the event to be photographed is highly luminous, considerable effort was expended in developing a reliable optical and photographic system which could produce unfogged photographs. The self-luminosity is characterized by a delay of approximately 500 μsec between the time the projectile passes the schlieren station and is photographed and the time of the appearance of significant radiation from the hot products of reaction. The duration of the self-luminosity is of the order of 0.05 sec. Thus an electrically operated capping shutter having a closing
time of 500 μsec was required to eliminate all fogging. Unfortunately, no convenient system is commercially available. The Beckman-Whitley capping shutter which uses detonator caps to shatter a glass block and the Jarall-Ash collapsing foil shutters were judged to be too bulky, too expensive, and too time consuming to operate. A program to develop a lead vapor capping shutter (Ref. 15) was unsuccessful since the light radiated from the vaporizing lead was itself sufficient to fog the 3000 speed Polaroid film. The system used is a mechanical system in which a taut 0.020 inch piano wire pulls a 0.010 brass shim across the optical path. The shutter is activated by discharging a high voltage capacitor through a 0.006 inch steel wire which is used to restrain the piano string. Approximately 1200 μsec were required for the shutter to block a 3/4 inch diameter aperture. This is sufficiently fast to eliminate most of the fogging for the test conditions used. The range of test conditions was restricted, however, since richer mixtures and higher pressure mixtures produced more self-luminosity.

In addition to the capping shutter, filters are used to reduce the effects of the self-luminosity. Most of the radiation from the hot gases is of a wave length of 5000 angstroms and higher. The radiation from the spark is predominantly in the blue and ultra-violet regions--below 5000 angstroms. Also, the film is more sensitive to the high energy, short wavelength radiation. Unfortunately, glass lenses and windows block
the ultra-violet radiation, hence the useable portion of the spectrum lies between 3000 and 5000 angstroms. A Kodak Wratten Filter No. 34 which has a peak transmittance at 4200 angstroms and zero transmittance between 5000 and 6400 was used in all of the experiments.

The ionization probe which is used to detect the location and, in some cases, the motion of the reacted gas boundary is mounted beside the schlieren phototube such that it is within the field of view of the schlieren system. The probe used in most of the test runs consists of two parallel 0.030 inch steel electrodes spaced 0.060 inches centerline to centerline. The probe projects approximately 1/4 inch from the tip of a 1/2 inch diameter supporting rod. The plane through the electrode centerlines was oriented parallel to the optical axis; as a result the probe silhouette observed in the photographs appears as that of a single wire. The probe electrodes are connected in series with a 45 volt battery and a 500 ohm resistor. The voltage drop across the resistor was recorded by an oscilloscope. Typical outputs were of the order of 1 volt indicating an effective resistance across the electrode gap of 0.025 meg ohms. The capacitance of the circuit was estimated to be 100 picofarads resulting in a calculated circuit rise time on the order of 3 μsec.

The triggering circuit for the schlieren and shadowgraph spark sources is based on the use of a specially designed spark modulator which converts the 10 to 30 volt gate or delay trigger output of the oscilloscope into
a 4000 volt square wave having a duration on the order of 0.1 μsec. The electronic circuitry involves the use of a hydrogen thyratron to control the high voltage discharge and a pulse forming network to shape the square wave. This pulse is applied to a third or "tickler" electrode which, in the case of the line source, projects into the mid-point of the gap formed by the tungsten electrodes. The discharge from the tickler electrode to the ground electrode induces the breakdown of the primary gap. The delay between the input to the oscilloscope and the rise on the output of a phototube viewing the spark light is typically 0.4 μsec. Most of this delay is accounted for by the delay built into the oscilloscope gate output.

Tektronix Type 555 dual beam oscilloscopes are used as the basic timing and recording devices. The oscilloscopes were calibrated periodically by use of crystal controlled Tektronix Type 181 Time Mark Generator. Non-linear sweep rates and errors in measurements of oscilloscope traces are the dominant source of error in the projectile velocity measurements and limit the accuracy to approximately ± 3%. The velocity is determined by dividing the distance the projectile moves between the shadowgraph and schlieren stations by the time interval. The distance is measured from the position of the ball relative to the reticles as recorded by the photographs and the known distance between reticles. The time interval is found by measuring the oscilloscope trace length which is determined by the time of the gate outputs to the
spark modulator. As noted above, there is a delay on the order of 0.4 sec between the gate output and the observation of spark light--the two delays are nearly equal and, therefore, tend to cancel any errors introduced. The time interval was typically in the vicinity of 180 μsec for the gas mixtures used. When the dual schlieren system is used, the time between schlieren sparks is measured by monitoring the sparks by a phototube.

The gases used in the experiments were obtained from Airco commercial cylinders. The gases are mixed in a rotating mixing tank usually within six hours of their use. Previous work on the determination of the detonation wave speeds of the gas mixtures indicated that a mixing time of approximately two minutes was sufficient to insure a homogeneous mixture. Usually much longer mixing times were used in the experiments. The gas volumes are measured by means of partial pressures; the gases are assumed to be ideal. The test section is evacuated to a pressure of from 100 to 300 microns of Hg before each run; the test gases therefore typically contain 0.10% residual gases.
SECTION 2
IDENTIFICATION OF FLOW FIELD FEATURES

The characteristics of exothermic hypersonic blunt-body flow field which are distinguishable on schlieren photographs are shown schematically in Fig. 4. These features are discussed below. Throughout this discussion reference is made to methods of interpreting schlieren photographs of axisymmetric flow fields and to detailed flow field calculations. These details are discussed in Appendix I and IV. Reference is also made to the features of inert or relatively unreactive flows which are described in standard texts (e.g., Ref. 16) and which are outlined in the following paragraph.

The most prominent feature of the hypersonic flow about a sphere is the nearly parabolic bow shock wave which is normal to the flight axis on the axis and which far downstream approaches the Mach angle corresponding to the body velocity. Other than within the viscous boundary layer, the flow is subsonic with respect to the ball only in a limited portion of the nose region (behind the strong segment of the bow shock). The sonic lines which bound the subsonic region can be very roughly approximated by lines extending upstream from the center of the sphere to the bow shock and inclined at an angle of 45 deg with respect to the axis. The gas velocity with respect to the ball immediately downstream of the bow shock on the centerline corresponds to the value behind a normal shock; the velocity decays to zero at the stagnation point in a
very nearly linear manner. The boundary layer separates from the body shortly after the section of maximum body diameter (the body shoulder). The free shear layers converge to form a neck or section of minimum diameter. A wake shock which turns the flow more or less parallel to the flight axis originates at the neck and trails downstream joining the bow shock far downstream. A weak shock also may emanate from the point where the boundary layer separates from the body. This recompression shock is due to overexpansion of the flow about the body; the shock turns the flow parallel to the converging shear layers. The viscous flow, which includes the boundary layer, shear layer and the core of the wake downstream of the neck, may be either laminar or turbulent depending on the Reynolds number of the flow. For the experiments conducted on the study, transition to turbulence appeared to occur at the wake neck. As discussed below, all of the features visible on schlieren photographs of nonreacting flows are retained in the exothermic flows studied herein.

Bow Shock Wave

For the exothermic mixtures employed in these experiments, the bow shock retains the more or less parabolic shape characteristic of non-reactive flows. For relatively nonreactive gases such as air it has been shown that the bow shock shape for supersonic flow fields about spheres can be correlated over a wide range of conditions by simple expressions involving the Mach number and the density ratio across a normal shock.
at that Mach number. Korkan (Ref. 17) gives the following correlation:

\[
\frac{r}{D} = A \left( \frac{\rho}{D} \right)^n
\]

\[
A = \frac{1.52}{(M_c^2 / \rho)}^{0.2} + \frac{1}{M_c^2}
\]

\[
n = \frac{0.53}{(M_c^2 / \rho)^2} + \frac{1}{2M_c^2} + 0.44
\]

This correlation was found to apply also in the case of exothermic mixtures if a density ratio, \(\frac{\rho_{\text{ave}}}{\rho_p}\), indicative of the extent of reaction in the nose region were used in evaluating the parameters \(A\) and \(n\). For example, for Run No. 87 (Fig. 5) the density ratio for which the correlation provides a good representation of the observed shock shape is 0.200 whereas the unreacted value is 0.182 and the fully reacted value is approximately 0.265. A comparison of the shock profile given by the correlation with the experimental profile for conditions of Run No. 87 (Hydrogen-air, \(\Phi = 1.0, M_c = 5.10, \rho = 0.396\) atm) is shown in Fig. 6.

This shock shape correlation is used in the analysis of the instability mechanism to obtain an estimate of the velocity in the inviscid wake near the body shoulder.

In all cases, at points sufficiently far downstream of the shock apex, the angle of the shock wave with respect to the flight axis approaches the Mach angle corresponding to the given ball velocity. An oblique detonation wave stabilized by the body was never observed. Such waves could theoretically exist for cases where the projectile velocity is greater than the Chapman-Jouguet speed. These waves would be charac-
terized by a "bow shock wave" whose minimum inclination would be that for which the normal component of the free stream gas velocity with respect to the bow shock wave is equal to the Chapman-Jouguet speed. These shock angles would be much greater than the Mach angle and would be readily observed.

In cases where the projectile velocity is less than the Chapman-Jouguet speed, a self-sustained detonation could develop in the wake of the projectile and eventually overtake the projectile. This was observed in several cases; Fig. 7 is an example. The detonation wave is observed to be somewhat curved and to exhibit a turbulent appearance in the schlieren photograph. The interaction of the detonation wave and the partially reacted wake gases was not studied; Behrens, et. al., present some data on this phenomena (Ref. 4). In some cases, photographs were obtained after the detonation wave had passed the ball. In these cases, no bow shock wave was observed indicating the Mach number of the reacted gas behind the detonation wave relative to the projectile was less than unity. This is consistent with the Chapman-Jouguet model which specifies sonic flow (with respect to the wave front) behind a plane detonation wave.

In cases where the schlieren photographs do not indicate the presence of turbulence in the inviscid region of the flow field, the bow shock retains the smooth, continuous shape characteristic of nonreactive flows. In cases where a turbulent effect does exist, the bow shock be-
comes distinctly wrinkled and the contour in the nose region becomes fuzzy indicating the presence of strong disturbances (Figs. 7 and 8). The wrinkling is probably due to the impingement of weak shock waves generated by nonuniform, locally intense exothermic reaction.

Shear Layers, Viscous Wake, Wake Shock and Recompression Shocks

As is the case of non-reactive supersonic blunt-body flows, for the flow fields having non-turbulent reaction zones, the boundary layer is observed to separate shortly after the section of maximum diameter. The resulting shear layer converges to form a neck followed by a viscous wake (Figs. 9 and 10). A wake shock originates in the neck region and trails downstream. The shear layer cannot be observed in the case of turbulent reaction because it is masked by the strong schlieren effects generated by the turbulence (Fig. 8).

For those runs where the Mach number is sufficiently low that the fluid in the boundary layer does not undergo significant exothermic reaction until after the neck region is reached, the schlieren effect of the hydrodynamic turbulence becomes significantly enhanced as heat release occurs (Figs. 11 and 12). The effect of the chemical reaction on the hydrodynamic turbulence was not studied in this program.

In many of the photographs (e.g., Fig. 12), recompression shocks resulting from overexpansion of the flow near the shoulder of the body can be seen. In many cases these shocks are located asymmetrically
with respect to the flight axis because of imperfections in the body surface caused by the wearing action of the gun barrel.

**Reacted Gas Boundary**

Chemical reaction occurs in the stagnation region, where the residence time of the gas elements is infinite, and exists in a limited region of the inviscid wake. The extent of the region depends essentially on the temperature and pressure levels in the shocked gas and the reactivity of the gas mixture. The reaction zone, or more precisely the region wherein the gas has undergone exothermic reaction, is observed to be a region bounded by a smooth surface or a corrugated surface, or it is observed to be a turbulent region with an indefinite boundary. These reaction zone configurations, which provide a basis for classifying the various flow regimes, are illustrated in Figs. 8, 12, 13, and 14 which are discussed below.

The experimental basis for identifying this region as a reaction zone are the results of the ionization probe measurements, the boundary velocity measurements made by use of the dual schlieren photographs, and the interpretation of the schlieren photograph shadings.

The ionization probe was used in conjunction with the photographs to determine in what region of the flow field the ionization level reached detectable limits. In these experiments, ionization can be produced either thermally or by the production of ionized species in
the chemical reaction process. Both of these conditions exist behind a reaction front. For those runs where the probe was positioned such that the tip did not intersect the reaction zone boundary, no signal was recorded; for those cases where probe intersected the boundary, a finite deflection of the oscilloscope beam was recorded for times later than the intersection time.

An example of a case where the reaction zone does not extend to the probe and thus no probe signal is recorded is given in Fig. 15. No deflection of the oscilloscope trace is noted at the time the probe passes through the bow shock. The shock heating is insufficient to produce detectable ionization levels. The normal component of the approach flow at the intersection point is estimated to be 3.2 in this case. (The top trace in the oscillograph is used to measure the ball velocity and is not pertinent to this discussion.)

An example of a case where the reaction zone does not initially extend to the probe but where it reaches the probe after a delay is shown in Fig. 16. Here a probe signal is recorded beginning approximately 70 μsec (or equivalently 9 ball diameters) after the ball passes the probe station. The probe output is recorded by the lower traces on the oscillographs. During the delay period the reaction zone has grown laterally either through expansion or through the molecular transport processes associated with subsonic flame propagation. The initial oscillating nature of the signal is caused by the corrugated surface of the reaction
zone immersing varying lengths of the probe electrodes with ionized gas. The normal component of the bow shock Mach number, which determines the temperature and pressure of the gas immediately behind the shock, is estimated to be 3.5.

An example of a case where the gas immediately behind the bow shock wave undergoes exothermic reaction is shown in Fig. 17. Here a signal is recorded approximately 2 μsec after the second picture was taken. The normal component of the bow shock Mach number at the intersection point in this case is estimated to be 3.6.

If the corrugated boundary of Fig. 13 is a reacted gas boundary, it should move in the streamwise direction with the local gas velocity. On the other hand, if this region represents a series of propagating compression waves, the boundary should move in the streamwise direction with a velocity equal to or greater than the sum of the acoustic speed and local gas velocity. From the dual schlieren photographs such as Fig. 17, it is possible to obtain the average velocity with which the reaction zone boundary moves in the direction of flight between the time the two photographs were taken. As discussed in the following paragraphs, it is indeed observed that the boundary moves with the local gas velocity. For Run 157 (Fig. 17) a typical local gas velocity in the inviscid wake as measured by a stationary observer will be shown to be approximately 700 fps in the direction of the motion of the ball. On the other hand, the acoustic speed in a gas having a temperature of
2500 R, molecular weight of 31.6 and a specific heat ratio of 1.4 is on the order of 2300 fps. Thus the difference between the gas velocity and the propagation rate of rate of compression waves is quite large and it is possible to distinguish between the motion of a contact discontinuity and a compression wave with certainty.

It should be noted that in the inviscid wake downstream of the missile, the unreacted gas is relatively cool due to the temperature reduction associated with the expansion of the gas about the missile. Therefore, the propagation rate of the reaction front in the inviscid wake, measured relative to the local gas velocity, is quite small. A reaction zone boundary in the wake should therefore appear to be frozen in the wake. Based on the calculations of Appendix IV, at all locations downstream of the major diameter of the ball, expansion should have cooled the gas sufficiently to freeze the induction reactions associated with shock heating; any propagation of the reaction front would be due to molecular transport and can be expected to be quite small.

By identifying the peculiarities in the reacted gas boundary in Fig. 17, a specific point on the boundary can be located on both photographs. In this particular case, the boundary irregularities are quite distinct, particularly on the original photographs, and the corresponding points can be located with reasonable certainty. The photographs were exposed at a 12 µsec interval. This is the time required for the ball
to move approximately 1.5 diameters. In Fig. 18 the average boundary streamwise velocity during this time interval is plotted as a function of the average position downstream of the shock apex. The end points of the horizontal bars represent the positions of the given point in the two photographs. The velocity is measured relative to the laboratory observer, not relative to the ball. If the reacted gas boundary is frozen in the flow, this is the velocity with which the disturbed gas drifts past the stationary observer. Note that the measured velocities of from 400 to 700 fps are small relative to the ball velocity of 5540 fps. In this coordinate system, the variation of the streamwise velocity due to expansion and to the effect of the wake shock is emphasized. The points corresponding to the lowest three plotted values of $\left(-\frac{U}{C}\right)$ are located upstream of the wake shock in both of the photographs and therefore the indicated velocity variation can be due only to expansion. The expansion of the gas causes an increase in the velocity of the gas measured relative to the ball but a decrease velocity measured relative to the stationary observer. The effect of the wake shock is to abruptly increase the wake velocity measured by the stationary observer. The rounded shape of the curve shown in Fig. 18 is due to the averaging associated with the measurements being taken over a relatively large time interval.
The velocity of the reacted gas boundary measured relative to the ball (not relative to the stationary observer as plotted in Fig. 18), is the quantity which relates the wavelength of the instability (as measured by the separation of the boundary corrugations) to the instability frequency:

\[ f = \frac{U_w}{\lambda} \].

Under a given set of conditions, the instability frequency is a constant but the wake velocity, \( U_w \), is a function of the axial and radial coordinates. Therefore, the measured wavelength can be expected to vary with distance downstream of the ball. The spacing of the striations appears in the photographs, however, to be remarkably constant. This is a consequence of the fact that the variation of the wake velocity relative to the ball in the inviscid wake downstream of the body shoulder is actually quite small. For Run 157 (Fig. 18), the wake velocity varies by 300 fps compared to the ball velocity of 5540 fps, a variation of 5.4 percent. This is undetectable in the photographs. Based on a velocity of defect of 700 fps, the appropriate wake velocity to be used to relate frequency and wavelength is, for the case of this illustration, 87 percent of the ball velocity. The error introduced in any analysis by use of the ball velocity rather than an appropriate wake velocity is small. As detailed in the section entitled "Instability Mechanism," a wake velocity estimated by use of the bow shock correlation given earlier in this section is used as the appropriate wake velocity.

The magnitude of the reacted gas boundary velocity measured by means of the dual schlieren photographs is confirmed by the probe measurements.
As mentioned previously, in cases where the probe tip grazes the reacted gas boundary, the probe signal oscillates due to the periodically varying depth of penetration of the probe into the ionized gas. This results from the motion of the corrugated boundary. The average boundary velocity can be found by dividing the measured wavelength by the measured oscillation period. For the case of Run 157 (Fig. 17) the average of the peak to peak and trough to trough periods is approximately 15.9 μ sec and the measured wavelength to diameter ratio is 0.211 (Table I). This results in an average velocity of 630 fps which compares favorably with the 400 to 700 fps detected by the dual schlieren technique. For the case of Run 139 (Fig. 16), the probe measurements yield a boundary velocity of 570 fps.

The boundary velocity was also measured by monitoring the output of a phototube which viewed the image produced by a continuous schlieren light source. That is, the camera film-holder was replaced by a phototube, the field of view of which was limited to a narrow vertical slit of a height approximately equal to the width of the reacted gas region. The spark light source was replaced by a high intensity lamp. The phototube output oscillates as the striations produced by the schlieren effect drift past the slit. The phototube output for Run 142 is given in Fig. 19. In this case, the average period of oscillation is approximately 30 μ sec and the average wavelength to diameter ratio is 0.57. The resulting wake velocity is 780 fps.
The measured values of reacted gas boundary velocity, which are in the neighborhood of 700 fps, agree with the analytically predicted value of the gas velocity at the corresponding region in the flow field of an unreacting gas. The analytical prediction is based on the flow field calculations detailed in Appendix IV and summarized in Fig. 20. The Mach number in the inviscid wake at a radial distance of 0.8 diameters (the reacted gas boundary radius for Run 157) at an axial position of, for example, 1.3 diameters downstream of the bow shock apex is approximately 3.2. For the acetylene-oxygen mixture used in Run 157 this corresponds to a velocity of 4730 fps. The wake velocity observed by a stationary observer would be 630 fps which agrees fairly well with the experimental observations. The wake velocity corresponding to a Mach number of 3.4, the limit of the calculations shown in Fig. 20, would be 530 fps.

In view of the large difference between these calculated gas velocities and the boundary velocity which would be observed if the boundary were produced by propagating compression waves, it is apparent that the boundary is a contact discontinuity, fixed to the fluid particles.

Further evidence that the boundary under consideration is a boundary between reacted and unreacted gas is provided by the schlieren photographs. The direction of the density change across the corrugated boundary can be deduced from examination of the schlieren shadings; see Figs. 13 and 21. The knife-edge used in Fig. 13 was inserted in the direction
opposite from that in Fig. 21, accounting for the bright bow shock in Fig. 13 compared to the dark bow shock in Fig. 21. (The dark outline on the upper half of the bow shock in Fig. 13 is produced either by an inadvertant horizontal knife-edge effect caused by one of the apertures not being large enough to pass the light rays which are strongly deflected in the vertical plane by the bow shock, or by a shadowgraph effect caused by the film not being positioned precisely at the objective lens focal point.) As pointed out in Appendix I, the very prominent alternating dark and bright bands characterizing the reaction zone are accounted for entirely by its corrugated boundary and the density change across the boundary. Streamwise periodic density changes within the volume of reacted gas need not exist to produce this strong regular schlieren pattern; indeed such density fluctuations do not exist. If a segment of the corrugated boundary is chosen for examination in which the width of the boundary is increasing in the downstream direction, then, if the density on the inside of the surface is less than that on the outside, the shading should be opposite that of the bow shock. (The bow shock is an envelope whose width is increasing in the downstream direction but whose density change is in the opposite sense from that across the reaction front.) This is seen to be the case in the photographs, confirming that the density inside the corrugated boundary is less than that outside the boundary, as would necessarily be the case for an exothermic reaction front boundary.
In certain cases where the instability wavelength is short, striations are observed even in the nose region (e.g., Fig. 22). It was originally thought, early in the program before the cause of the vertical striations had been determined, that these waves observed in the nose region were oscillating shock waves. However, early work with the dual schlieren apparatus (Runs 30 through 47, Table I), where the time between exposures was held to a fraction of the instability period, failed to produce an example of the wave propagating upstream; i.e., from the ball to the bow shock. This, of course, is consistent with the fact that the striation is caused by the corrugated edge of the reaction front and that such striations will always flow downstream.

In addition to the above experimental evidence that the boundary under consideration is the reaction zone boundary, there is analytical confirmation, for the nonturbulent cases, that the boundary profile is determined by the ignition delay characteristics of the fuel-oxidizer combination used. This is discussed in Appendix IV where the experimentally observed and analytically predicted profiles for the case of a Mach 5.0 sphere moving through a stoichiometric hydrogen-air mixture are compared.

**Inviscid Wake**

For those cases where the reaction zone boundary is corrugated or where the reaction zone is turbulent, there exists in the inviscid wake
a pattern of two sets of smoothly curved wave fronts which extend from the reaction zone to the bow shock and which cross one another to yield a net-like pattern. These waves have been identified as pressure waves and entropy waves as noted in Fig. 4. As discussed below, this identification is based on the results of the schlieren analysis, on the magnitudes of the wave propagation rates as determined by the dual schlieren photographs, and on the results flow field analysis (Appendix IV).

As noted in the previous section, the corrugated reaction zone boundary is fixed in the flow and moves with the local gas velocity. Inspection of photographs such as Fig. 13 and 21 indicates that one set of waves (the entropy waves) attaches to the reaction zone boundary at a given point on each corrugation. The other set (the compression waves) appears indistinct near the reaction zone boundary in all the photographs, and it is not possible to establish by the photographs that these waves do or do not attach to that boundary. Both sets of waves extend to the bow shock surface. The actual joining of the waves at the bow shock is, in general, indistinct on the photographs. By examination of the photographs wherein the instability is observed to be irregular (e.g., Fig. 23), it is concluded that each wave in one set corresponds to a particular wave in the second set, and that the corresponding waves have a common point at the bow shock. The waves which are fixed to the reaction zone boundary, which moves with the local gas velocity, must themselves
move with the gas velocity and, therefore, must be contact discontinuities or entropy waves. Such waves could be generated by the interaction of compression waves with the bow shock. This would account for the one-to-one correspondence between waves in each set and the apparent joining of the waves at the bow shock. If the waves of the second set are compression waves, their propagation speed relative to the gas velocity must be equal to or greater than the acoustic speed. This is found to be the case by examination of the dual photographs such as Fig. 17.

The pertinent data from Fig. 17 is reproduced schematically in Fig. 24. The positions of a single point on the reaction front boundary have been located in the dual photographs and are sketched in Fig. 24 together with the entropy waves emanating from that point. Compression waves have been sketched in the figure at the intersection of the entropy waves and the bow shock. The angle of inclination of the entropy waves, bow shock and compression waves are those measured in Fig. 17. The distance \( \Delta S_{\theta} \), moved by the intersection point along the bow shock is then measured; the distance, \( \Delta S_{\perp} \), moved by the compression wave normal to itself can then be calculated from the geometry of the interaction. The average propagation rate of the compression wave relative to the ball can then be calculated using the known time interval of 11.9 \( \mu \) sec. The gas velocity behind the bow shock measured relative to the ball can be approximated by use of the oblique shock relations using the average value of the bow
shock inclination along the intersection path. The compression wave
propagation rate relative to the gas velocity is the difference between
the velocity of the wavefront with respect to the ball and the gas
velocity relative to the ball. These values calculated for
points located 2.5 and 4.9 diameters downstream of the bow shock apex
for Run 157 are 1,400 fps and 1,000 fps, respectively. The acoustic
speed at these locations was calculated to be 1,375 and 1,345. The
agreement between the observed and calculated values is reasonable con-
sidering the error amplification which results when taking the difference
between two large quantities. The measured propagation rates of the two
sets of waves thus support the contention that they are entropy waves and
compression waves.

The direction of the density change across such waves is known--
the density must increase across a compression wave and decrease across
the contact discontinuity generated by the interaction compression wave
and the bow shock. (This interaction is treated in detail in Appendix II.)
The direction of the density change can also be deduced from the shadings
of the schlieren photographs. As discussed in Appendix I, the shading
of the compression wave must be opposite from the shading of the bow
shock since both are compression waves but have opposite slopes measured
with respect to the flow field axis. Examination of the photographs
indicates this to be the case. The shading of the entropy wave should
be the same as that of the bow shock since both have slopes of the same
sign, and in both cases the light ray passes from a medium of low density to a medium of high density when crossing the wave. (In Appendix II it is shown that the gas on the upstream side of the contact discontinuity is hotter than that on the downstream side.) In the photographs the entropy wave appears as a double band, the downstream band having the proper relative brightness. The double band is probably related to the structure of the wave as discussed in Appendix I.

It is apparent from the symmetry of the flow field that the waves originate in the nose region. A further test of whether these waves represent compression and entropy waves would be to construct the contours which such waves would exhibit as time progresses. Such a calculation has been carried out; the details are given in Appendix IV. An infinitesimal pressure pulse was assumed to be generated at the stagnation point and its location at later times corresponding to an arbitrarily chosen time interval was plotted. The location of the infinitesimal contact discontinuity generated by the interaction of the pressure pulse with the bow shock was also plotted. The results of the calculation are shown in Fig. 25.

The developing shapes of the compression and entropy waves are seen to correspond to the shapes observed in the photographs. The entropy waves remain attached to the stagnation point and undergo a severe bending in the downstream region as they traverse the inviscid wake. This bending is particularly noticeable in the schlieren photographs of the low Mach
number runs (e.g., Fig. 26) where the reaction zone does not extend far into the inviscid wake.

The shape of the compression waves is determined primarily by the location of the source of the disturbance. The distortion of these waves due to the variation of acoustic speed and flow velocity throughout the flow field is small. The waves have the appearance of a set of concentric rings centered about the nose of the ball.

Based on the above observations that the compression waves appear to radiate almost as concentric rings from the origin of the disturbance and that the entropy waves owe their shape to the "stretching" caused by their being attached to the rapidly moving compression wave-bow shock intersection point while simultaneously being fixed to the slowly moving fluid elements, it is concluded that any periodic disturbance distributed symmetrically and located in the nose region of the flow field will produce a wave pattern in the inviscid region having the characteristics observed in the photographs.

If the two sets of waves do represent compression waves and entropy waves, then a specific relationship should exist between the angles of intersection of the bow shock, the compression wave and the entropy wave or contact discontinuity. Also, another wave, the reflection of the compression wave from the bow shock, should exist. A schematic representation of the shock interaction is given in Fig. 27. The reflected wave is not shown in this figure. In Fig. 27a the shock interaction
is shown as seen in a coordinate system fixed with respect to the ball. The steady state configuration, i.e., in a coordinate system fixed with respect to the point of intersection, is shown in Figs. 27b and 27c. The details of the interaction are considered in Appendix II where it is shown that the reflected wave is a rarefaction wave and that all the waves are weak. A weak rarefaction wave cannot be expected to be visible in the schlieren photographs. As discussed in Appendix II, a discrepancy of from 10 to 30° exists between the analytically predicted and experimentally observed inclinations of the entropy wave. The experimentally observed configuration can be analytically generated for an interaction of the type assumed in Fig. 27, only if the propagation rate of the compression wave with respect to the local gas velocity is assumed to be less than the acoustic speed. Such an assumption does not correspond to physical reality and therefore some other, as yet undetermined, explanation must exist for the discrepancy. This is further discussed in Appendix II.

Although the interaction analysis fails to explain the precise observed angular relationship among the waves at the intersection point, it is felt that the discrepancy is not severe enough to suggest that the observed waves are not compression and entropy waves. The discrepancy is probably due to some unaccounted for effect such as distortions caused by the following powder gases or the effects of the exothermic reactions in the wake. The conclusion drawn from the measurements reported in this section is that the waves indeed represent propagating compression waves.
and the entropy waves generated by the interaction of the compression waves with the bow shock.
SECTION 3

EXPERIMENTAL DATA

The recorded experimental data is composed primarily of schlieren photographs of the hypersonic flow field about spheres fired into lean acetylene-oxygen mixtures and stoichiometric hydrogen-air mixtures. Argon and helium were used as diluents in several runs. Velocity measurements were recorded for each run and ionization probe data were recorded in a large number of cases.

The primary objective of the tests was to obtain data on the variation of the instability wavelength with the controllable experimental parameters (initial gas state, sphere velocity and diameter). It became apparent early in the program that the regular instability could exist only under a limited range of conditions; therefore, a number of runs were made also with the objective of determining the boundaries of the various flow regimes.

The principal data obtained from the experiments are presented in Table 1. A selected number of photographs from which the data have been taken are reproduced herein and are discussed throughout the following text. Most of the data of Table 1 has been used in the preparation of figures which summarize the data. Some of the data is questionable because of suspected unreliable oscilloscope calibrations or weak or irregular schlieren patterns which makes measurement of the flow field characteristics difficult. Such data is so identified by notes in the
"Remarks" column of Table 1. Those runs in which dual schlieren photography was employed and those in which probe measurements were recorded also are specified. Unless otherwise noted, the data of Table 1 was obtained with the use of a half-inch unsaboted sphere.

**Flow Field Regimes**

Four different flow field regimes can be distinguished for hypersonic blunt bodies moving in a gas mixture capable of undergoing exothermic reaction. The regimes are characterized by the nature of the chemical reaction zone as observed on schlieren photographs. The regimes are defined as follows: (A discussion of the details observed in the photographs referred to is included in the text below.)

1) Wake Combustion Regime. Only the fluid in the boundary layer is heated sufficiently to permit the exothermic reaction to be initiated. The effects of the reaction are observed in the turbulent viscous wake where the chemical heat release results in the enhancement of turbulent schlieren effect. See Fig. 11.

2) Turbulent Regime. Chemical reaction spreads throughout a large portion of the inviscid wake and exhibits a definitely turbulent appearance. See Fig. 8.

3) Stable Regime. Chemical reaction spreads throughout a large portion of the inviscid wake but the flow within the reaction zone is non-turbulent and the reacted gas boundary is smooth. See Fig. 14.
4) Periodic Regime. The region of reacted gas is characterized by strong regularly spaced, dark and bright vertical striations. The reacted gas boundary has a corrugated appearance and compression and entropy waves can be observed propagating in the inviscid wake. See Fig. 1.

The location of these flow regimes in terms of the laboratory controlled parameters mixture pressure and the ball Mach number is given in Fig. 28 for the case of 1/2 inch diameter spheres fired into a mixture of 15 moles of \( \text{O}_2 \) to 1 mole of \( \text{C}_2\text{H}_2 \). This gas mixture was used to generate a regime plot because the controllable parameters could be easily varied over a range wide enough to observe all the significant boundaries. Richer acetylene-oxygen mixtures generated excessive self-luminosity which produced fogging of the film. In the case of the hydrogen-air mixtures, the pressure level required to observe the high pressure boundaries exceeded the levels where the effects of the shock waves disturbed the alignment of the optical equipment. The data obtained for the hydrogen-air mixtures (Fig. 29) indicates the same general regime boundaries exist for that mixture as for the acetylene-oxygen mixture although, of course, the locations of the boundaries on a pressure-Mach number plot differs from those of Fig. 28.
Acetylene-Oxygen; 1/2 Inch Diameter Spheres

Fig. 28 indicates, as expected, that at low Mach numbers, regardless of the pressure levels, the chemical reaction is confined to the viscous wake. As seen in Fig. 11, there appears to be no effect of the reaction on the flow field other than the increased wake temperature made apparent by the enhanced turbulent schlieren effect. For most of the runs made in this program, the Reynolds number of the flow was such that the viscous wake should become turbulent at the neck. In Fig. 11 the schlieren sensitivity is relatively weak such that the turbulence cannot be observed immediately downstream of the neck. However, approximately two diameters downstream of the neck exothermic reaction becomes sufficiently great that turbulent flow is clearly observed. The effect of the heat release on the turbulence intensity and on the turbulence statistics was not investigated. The effect observed in Fig. 11 is primarily an enhancement of the schlieren effect caused by the substantially increased density fluctuations caused by the heat release.

At somewhat higher speeds the reaction zone begins to spread laterally into the inviscid wake. The boundary of the reacted gas downstream of the ball can in some cases be distinguished from the shear layers. In Fig. 12 the faint shear layers lie inside of the smooth reacted gas boundary. The hot reacted gases again enhance the schlieren effect of the turbulent gases. Also observed in this photo are weak recompression
shocks emanating from the body shoulder; the existence of these shocks is, of course, not dependent on the exothermic reaction.

If the pressure is sufficiently low, the reaction zone continues to spread laterally into the inviscid wake as the Mach number is increased and no instability of the flow is observed. Thus, at low pressure there is no sharp demarcation between the wake combustion regime and the stable combustion regime. It was observed, however, that at the very low pressures required to bypass the regime of periodic oscillations in the acetylene-oxygen mixtures (below 0.066 atm) the stability of the flow was easily upset by influences of the gunpowder gases. At these low pressures it was necessary to maintain a pressure of at least 0.1 atm in the blast tank in order to retard the motion of the powder gases and prevent these disturbances which cause the reacted gas boundary to wrinkle and thereby exhibit a turbulent appearance.

At higher pressures, as the Mach number is increased above a certain minimum, the periodic instability regime, of which Fig. 1 is typical, is encountered. Within this regime, as the Mach number is further increased, the wavelength of the instability decreases. (The variation of the wavelength with ball speed and mixture pressure is discussed in detail in the following sub-section.) Throughout most of the periodic regime the schlieren photographs exhibit a quite regular structure. The deviations from this regular behavior which occur near the regime
boundaries are discussed in the following paragraphs. At the lowest Mach numbers at which the regular instability is observed, the wavelength becomes quite long and the wave form takes on a somewhat different appearance (Figs. 19 and 30) from that observed at the higher speeds. This different appearance is solely the result of the increased wavelength and does not represent a change in the mode of oscillation. This is further discussed in the section on the instability mechanism.

At these low Mach numbers, and at the highest pressures at which tests were run with the acetylene-oxygen mixture, a gross distortion of the wave form is observed (Fig. 23). The wavelength of the instability becomes random as does the transverse dimension of the reacted gas boundary. The strength of the compression and entropy waves in the inviscid wake as inferred from the distinctness of their schlieren traces is also variable. It is observed that the widest regions of the reacted gas region are joined with the strongest entropy waves.

At the boundary between the periodic regime and the wake combustion and stable combustion regimes, the amplitude of the boundary corrugation is observed to be markedly reduced, and the schlieren pattern produced by the corrugations is relatively weak. In some cases the instability is observed to die out and then reappear. This is illustrated in Fig. 5. These low pressure regime boundaries are fairly well defined, although as indicated by the arrows in Figs. 28 and 29, there is a certain amount of scatter.
As indicated in the regime plots, the regular instability is observed to exist at velocities both above and below the Chapman-Jouguet speed. In other words, from the results of these experiments, the Chapman-Jouguet speed has no significance in determining the shape of boundaries of any of the flow regimes. For the two mixtures studied it does provide an indication of the approximate missile speed required to observe the periodic instability. As discussed in the section on the instability mechanism, the instability is due to nonequilibrium effects; the parameter governing the conditions under which the instability can exist is the First Damköhler number based on the induction time of the reaction. On the other hand, the Chapman-Jouguet speed is determined by purely equilibrium calculations. Therefore, it should not be expected that the Chapman-Jouguet speed is a quantity useful in the interpretations of these experiments. The similar location of the regime boundaries with respect to the Chapman-Jouguet speed for the two mixtures cannot be construed as an indication that this will be the case for all exothermic mixtures.

At the higher pressures, as the Mach number is increased an abrupt transition from the periodic to the turbulent regime takes place. To produce such a strong schlieren pattern (as observed in Fig. 8) this turbulent schlieren effect must be produced by a disturbance of the reacted gas boundary across which a large change in the index of refraction occurs.
This disturbance could be created by a wrinkling of the boundary, similar to flame wrinkling, in which case the reacted gas itself need not necessarily be turbulent. On the other hand, the entire volume of reacted gas could be turbulent with the strong schlieren effect being produced by the hot turbulent gas ingesting the cold outer gas at the boundary. A schlieren photograph cannot distinguish between the two cases; i.e., it is not known whether or not the instability actually generates hydrodynamic turbulence.

The pattern of entropy waves and compression waves in the inviscid wake, a characteristic of the regular instability, is again observed in the turbulent regime; however, the spacing of the waves is random. The compression waves must be stronger than those generated in the periodic case since a noticeable distortion of the bow shock profile occurs due to the wave interaction process (Fig. 8). Also characteristic of the turbulent case is the fuzzy appearance of the bow shock wave in the region. The fact that the bow shock wave is so strongly effected in a random manner suggests that the gas passing through the shock does gain a significant random velocity component and that all throughout the volume of the reacted gas region does possess a certain turbulent component. On the other hand, the turbulent appearance of the schlieren photographs is certainly not a random turbulence.

As the boundary of the periodic regime is approached by lowering the pressure at constant Mach number, the turbulent pattern more and
more suggests the evenly spaced vertical striations of the periodic regime. This is illustrated by Fig. 31 where conditions, except for the reduced pressure, are nearly equal to those for Run 128, pictured in Fig. 8. Figure 31 suggests that the turbulent pattern is caused by a distortion of a corrugated boundary. Again, however, such photographs actually provide no information on whether the distortion is caused by the growth of turbulence within the reacted gas or by a wrinkling of the gas interface (an instability of the reacted gas boundary itself).

As the pressure is further reduced, other conditions remaining fixed, the flow becomes periodic. This is illustrated in Fig. 33 where the characteristic periodic oscillation is observed under conditions similar to those in Fig. 31 except for the reduced pressure. Note, however, that the wavelength of the instability is decidedly shorter than the wavelength associated with the turbulence of Fig. 31.

Another example of a run near the boundary of the periodic and turbulent regimes is shown in Fig. 32. Here the flow is observed to be intermittently turbulent and periodic.

Within the turbulent regime, if the velocity is increased with other conditions remaining the same, the characteristics of the flow change to only a small degree. The turbulence spreads through a greater lateral extent as would be expected and the scale of the turbulence decreases (Fig. 34). In Fig. 34 there is also a noticeable difference
between the schlieren effect produced by the turbulence near the body and that produced downstream of the body. This suggests that the wrinkling of the reacted gas boundary does not persist with time but instead is damped out. If the wrinkling were due to the spreading of hydrodynamic turbulence, the wrinkling should persist because it is unlikely that the decrease in temperature and pressure due to expansion would greatly effect the turbulence intensity.

Above a certain Mach number, the regime plot, Fig. 28, indicates the periodic instability is no longer observed; the flow appears to be either turbulent or stable. The data trend indicates that the slope of the turbulent/periodic boundary becomes small at the high Mach numbers and that there is a minimum pressure below which the turbulent regime is not encountered.

Finally, it is interesting to note that at velocities in the vicinity of the Chapman-Jouguet speed, as the pressure is increased, the character of the flow changes from a stable flow to one exhibiting a well ordered periodic behavior and then to a turbulent behavior. Studies of the structure of detonation waves have produced evidence that such waves are unstable and that at low pressures a coarse structured, well organized, periodic behavior exists (spin). As the pressure is increased, the characteristic multi-headed spin instability wavelength decreases until the scale becomes so small that the structure can no longer be
resolved. At the same time, other experimental evidence has been obtained that suggests that the flow becomes turbulent. The question arises as to whether the turbulent detonations and those exhibiting multi-headed spin are one and the same, the classification depending upon the resolving power of the instrumentation. For the experiments conducted in this program, however, it is apparent that for exothermic blunt body flows, the turbulent regime and the high frequency periodic regime are fundamentally different and are sharply divided.

Acetylene-Oxygen; \(\frac{3}{8}\) and \(\frac{1}{4}\) Inch Diameter Spheres

A limited number of runs were performed with spheres of \(\frac{3}{8}\) and \(\frac{1}{4}\) inch diameter. It was observed that the diameter has a strong effect on the location of the regime boundaries. It was found that if a regime plot is constructed using the product of the pressure and diameter as the ordinate, it is possible to fit the few data points for the smaller diameter spheres onto the same plot with the half-inch spheres (Fig. 35). The arrows again indicate that the specified data points, according to the observed character of the reaction zone, should lie on the other side of the boundary. The boundaries shown are those of Fig. 17. The agreement is very good, although there are too few data points to draw any firm conclusions. The agreement suggests, however, that the induction time, which is inversely proportional to pressure, plays the major role in determining the conditions under which the periodic instability
can exist. It further suggests that the other factors which determine the shape of the regime boundaries are relatively insensitive to changes in pressure and diameter. These other factors are discussed in the section on instability wavelength. The fact that the induction time does not by itself determine the regime boundary is illustrated by a comparison of the contour of a line of constant Damkohler number based on induction time, shown as a dashed line in Fig. 35, with the regime boundaries.

**Hydrogen-Air 1/2 Inch Diameter Spheres**

The regime plot for the stoichiometric hydrogen-air mixtures (Fig. 29) indicates that relatively higher pressures are required to observe the periodic instability in this mixture than in the acetylene-oxygen mixture. As shown, no runs were made at pressures sufficiently high to observe the turbulent regime. Photographs exhibiting the turbulent regime were obtained in lean hydrogen-oxygen mixtures \((H_2:O_2, \Phi = 0.25)\) in which no diluent was used. The periodic behavior is observed for the stoichiometric hydrogen-air mixture again at velocities both above and below the Chapman-Jouguet speed.

The published results of Ruegg and Dorsey (Ref. 2) and Struth, Behrens, and Wecken (Ref. 4) illustrate the effect of pressure and diameter on the regime boundaries for this mixture. The data are shown in Fig. 36 where the ordinate is the pressure-diameter product and the solid regime boundaries are taken from Fig. 29. The turbulent regime
was observed by Ruegg and Dorsey with 0.785 inch diameter spheres fired into gas at a pressure of 0.55 atm. At the low Mach numbers, long wavelength surges were observed which created strong deformation of the bow shock. Struth, et. al., did not observe the instability above the Chapman-Jouguet speed with their 9mm spheres. This is in contrast to the observations reported herein (Fig. 29).

**Instability Wavelength**

The instability wavelength, measured in ball diameters, for the lean acetylene-oxygen mixtures is shown in Fig. 37, and for stoichiometric hydrogen-air mixtures in Fig. 38. The wavelength is defined as the distance between striations on the schlieren photographs measured at axial locations in the vicinity of the rear edge of the ball. The gas velocity in this region of the inviscid wake, which is of the order of 80 to 90 percent of the free stream velocity, further increases due to expansion by only a small fraction of the free stream velocity and therefore the measured wavelength is relatively insensitive to the measurement position chosen. The wavelength appears to the eye to be constant downstream of the ball.

**Acetylene-Oxygen; 1/2 Inch Diameter Spheres**

The details of the variation of the wavelength with pressure and velocity will be discussed with reference to the data for half-inch
spheres fired into the 1:15 acetylene-oxygen mixtures. As noted previously, this mixture was chosen for study since the pressure and velocity could be varied over relatively wide ranges within the limits of capability of the equipment. The data obtained for an ambient gas pressure of 0.132 atm (100 mm Hg) is shown by the circles in Fig. 37. As noted on the regime plot, at this pressure the periodic instability was observed up to a Mach number of 6.0. Under this condition, the pressure behind the bow shock, assuming chemically frozen flow in thermal equilibrium, is 5.75 atmospheres and the temperature $1940$ K ($3500$° R). As the velocity is decreased, the wavelength increases steadily until a Mach number of 5.28 is reached. At this point a gap in the data exists, the next data point being at Mach 5.0 where the wavelength is 25 percent higher than at the previous point. There is no apparent change in the waveform across the gap. It is not certain whether the gap actually exists or whether it is a result of scatter in the data. As discussed below, the gap does appear to exist at other pressure levels.

As the velocity is further decreased, the wavelength continues to increase until at a Mach number of 4.92, where the wavelength is approximately three times longer than at Mach 6.0, a sudden decrease in wavelength is observed. A marked change in the schlieren effect is also observed since the anomalously short wavelength or high frequency is accompanied by a reduced amplitude of the boundary corrugation. (Compare
As will be noted in the section on the instability mechanism, this anomalously high frequency may be caused by the effect of reflected waves which generate a pattern having a wavelength twice the expected wavelength. This high frequency behavior persists for a short Mach number interval during which the wavelength rapidly increases. Then another abrupt increase in the frequency apparently occurs. The amplitude of the boundary corrugations is so small and the striations so extremely weak that the wavelength cannot be measured with accuracy; two such points corresponding to Run Nos. 251 and 220 are located near Mach 4.9 on Fig. 37. In these cases the entropy waves in the inviscid wake produce a stronger effect than the boundary corrugation and it is the relatively close spacing of these waves which suggests that a second abrupt increase change in frequency has occurred. This effect was not observed at other pressure levels.

As the Mach number is decreased below Mach 4.75, the instability is no longer observed for runs made at 100 mm Hg. The normal shock conditions corresponding to this Mach number are a pressure downstream of the shock of 3.6 atm and a temperature of $1360^\circ$ K ($2470^\circ$ R).

As indicated on the regime plot, at lower pressures the Mach number interval through which the instability is observed is shorter than at 100 mm Hg. At pressure of 0.098 atm (75 mm Hg) the instability was observed at a Mach number of 5.55 but was not observed in the case of two
runs made between Mach 5.55 and Mach 5.34. In one of these runs the flow was stable; in the other a weak irregular, intermittent pattern was observed. Thus a gap in the data exists similar to that observed at a pressure of 100 mm Hg. Between Mach 5.34 and 5.20 the instability wavelength increases; the wavelength is observed to be longer than that for a pressure of 100 mm Hg. As the Mach number is further decreased, the anomalous high frequency behavior is observed and then the flow becomes stable.

At a pressure of 0.198 atm (150 mm Hg), the highest Mach number at which the regular instability is observed is Mach 5.33. At this high Mach number range there is virtually no separation between the wavelengths observed at this pressure and at a pressure of 100 mm Hg. This is in conflict with the proposed model of the instability mechanism which predicts that the wavelength is inversely proportional to pressure. As the Mach number is decreased, once again there is evidence of an abrupt increase in the wavelength (near Mach 5.15). The increase in the wavelength magnitude is a smaller percentage than observed at the lower pressure, and it is therefore even more difficult to determine if this apparent jump is real or is due to data scatter. In one of the photographs (Fig. 39), the instability exhibits an intermittent behavior in which two different wavelengths appear to exist at different times.

Unfortunately, a horizontal knife edge was used in this run and resolution
of the flow field structure is quite poor. However, a change in the form of the corrugated reacted gas boundary can be observed in Fig. 39 in the vicinity of the point where the trailing wake shock crosses the reacted gas boundary. Other photographs wherein a horizontal knife edge was used do not exhibit this peculiarity. This is the only photograph out of over 250 which suggests that a transition from one frequency to another can occur; previous investigators have reported observing such a transition (Ref. 4).

As the Mach number is further decreased, the wavelength increases but again there is no significant separation between this data and that for the lower pressure. Between a Mach number of 4.80 and 4.65 the anomalous high frequency is observed; three such data points are included on Fig. 37. For the run at Mach 4.71, shown in Fig. 15, the schlieren effect is very weak and the wavelength measurement is questionable. The value plotted in Fig. 37 is less than that of the other two points suggesting that this is a case corresponding to the second abrupt frequency jump cited in the discussion of the runs made at 100 mm Hg pressure. However, the schlieren characteristics of the photograph do not correspond to those runs, and it is believed that the shorter wavelength is due to measurement errors. It is observed in Fig. 15 that the spacing of the vertical striations in the reacted gas appears to change intermittently from the small value corresponding to the anomalously high frequency to twice that value. This is indicated in Fig. 37 by the vertical line
joining two points, one having twice the wavelength of the other. As discussed in the section of the instability mechanism, this effect is believed to be due to the varying effectiveness of the entropy waves in generating the boundary corrugation. In Fig. 40, this double-valued effect is again observed in a section of the wake, this time with alternately spaced uneven wavelengths. Measurements indicate that the sum of the uneven lengths is twice the wavelength in the region of the wake where the spacing is even. Both values joined by a vertical line are again shown in Fig. 37.

At the very low Mach numbers the long wavelength is observed (Figs. 19 and 30). The spacing of the waveforms is somewhat irregular as indicated by the vertical lines in Fig. 37. Below Mach 4.4 the flow was observed to be stable.

At higher pressures, the regular periodic instability is observed only over a short Mach number range due to the transition to the turbulent regime. There again is very little separation of the wavelength data from that taken at the lower pressures, particularly in the Mach 4.86 to 5.0 range. Also it is apparent from this data that the transition to the anomalous high frequency mode occurs at lower Mach numbers as the pressure is increased. At a pressure of 0.230 atm (175 mm Hg), Fig. 41 indicates transition occurs at a Mach number in the vicinity of 4.65. Note that two sets of entropy waves are observed in the invi-
scid wake but the reacted gas boundary, the corrugations of which are formed by the action of the entropy waves, still displays the fundamental wavelength. This phenomenon is further discussed in the section on the instability mechanism.

At the highest pressure for which tests were made in the acetylene-oxygen mixtures, 0.264 atm (200 mm Hg), the regular instability waveform was observed in only one case. In other runs the waveform exhibited severe transverse distortions as shown in Fig. 23.

To summarize the most important points of the foregoing, it was observed in the tests made with half-inch spheres fired into 1:15 $C_2H_2:O_2$ mixtures that the wavelength of the regular instability increases with decreasing Mach numbers, the increase becoming quite rapid at the lowest Mach numbers for which the instability was observed. At a given Mach number, the wavelength was in general smaller for high pressures, but this was not a strong effect. Two significant anomalies were observed in the otherwise regular trend of increasing wavelength with decreasing Mach number:

(1) In the Mach number range from 5.0 to 5.4, a gap exists in the data wherein the observed wavelength appears to undergo an abrupt increase of 15 to 25 percent as the Mach number is decreased. This apparent effect may be generated by data scatter, but the effect was observed at three different pressure levels.
At a certain low Mach number, which decreases with increasing pressure, an apparent transition to a higher frequency occurs. As discussed later, this is believed to be merely an effect of wave reflections on the reacted gas boundary corrugations, the observed wavelength being twice the value that would be observed in the absence of the reflections.

**Acetylene-Oxygen; 1/4 Inch Diameter Spheres**

The instability wavelength data obtained for one-quarter inch diameter spheres fired into \( \text{C}_2\text{H}_2:0 \) mixtures at a pressure of 0.264 atm (200 mm Hg) is shown in Fig. 42. Shown for comparison purposes is the data for one-half inch spheres at 100 mm Hg pressure. In the latter data, the wavelengths measured in the cases of the anomalous high frequency were doubled to produce the continuous trend shown.

According to theory, the data should lie on a common curve since the Damkohler number is the same for both sets of runs. It is seen that the data for the smaller spheres lies slightly above the other data. The slight separation is small but is significant when compared to the separation obtained when varying pressure at constant diameter.

**Hydrogen-Air; 1/2 Inch Diameter Spheres**

The instability wavelength data obtained for stoichiometric hydrogen-air mixtures is shown in Fig. 38. Again it can be seen that the separation of the low pressure data from the high pressure data is less
than would be expected from the inverse proportionality relationship of the induction times. One case of the anomalous high frequency was observed. No runs were made at Mach numbers sufficiently low to observe the long wavelength instability.
SECTION 4

INSTABILITY MECHANISM

This section contains a discussion of the proposed mechanism which generates the regular, periodic instability. The similarities and differences between this flow and that of a one-dimensional detonation are first discussed. The probable physical cause of the instability of these exothermic flows is reviewed. The suggestions of previous investigators regarding the instability mechanism is then reviewed. Equations describing the wavelength of these acoustic theories are derived and the results of the experiments and theory are compared. The gas dynamics of the pertinent wave interaction processes is examined and an instability mechanism consistent with the results of these considerations is proposed. The experimental wavelength behavior is then compared with theoretical wavelength prediction. The proposed mechanism is shown to be consistent with physical processes which could lead to the existence of the several flow regimes discussed in the previous section. The probable process by which the reacted gas boundary corrugations are formed is discussed. Finally, an expression for the induction time of hydrogen-oxygen mixtures is derived by comparison of theory with experiments. This expression is then compared with induction time correlations derived by other experimental methods.

In this discussion it will be shown that the induction zone length along the central streamline is the proper length over which occur the
interactions responsible for the periodic striations. On reviewing the work that has been done on determination of induction times, it is apparent that there is significant scatter in the experimental data and often the conditions (mixture ratio and percent diluent) used in shock tube and other types of experiments differ significantly from the conditions of the ballistic range experiments. Therefore, apparent agreement between the experimentally observed instability wavelength and that predicted theoretically based on induction time correlations cannot alone be regarded as sufficient evidence that a proposed instability mechanism is indeed the correct mechanism. At the very least, the model must also be consistent with the laws of gas dynamics. In particular, the interaction of pressure waves or entropy waves with the boundaries of the induction zone should be examined to determine if the wave reflections at the boundaries are consistent with the model of, for example, a closed-ended or open-ended organ pipe. Also, the proposed mechanism should contain, or at least be consistent with, some feedback mechanism by which the instability is maintained. Therefore, in addition to comparing the theoretically predicted and experimentally observed wavelength behavior, a significant portion of this section is devoted to examining these other considerations.

Shock Induced Combustion

As noted in the section on identification of flow field features, examination of the wave patterns in the schlieren photographs indicates
that the instability originates in the nose region of the flow field. A gas particle in the free stream approaching the nose region, specifically a particle on the axis of symmetry, first encounters a normal shock, then after a time interval equal to the induction time, passes through a reaction front at which point there is an abrupt rise in temperature. The gas continues to the stagnation point, where it reaches the equilibrium temperature due to its infinite residence time. The principle feature of the flow, the shock separated from the reaction front by an induction zone is similar to that which would exist in a one-dimensional model of a normal detonation wave.

The criteria which determine the stability of the exothermic hypersonic blunt-body flow field cannot be expected to be the same as that for the normal detonation because of the radically different boundary conditions characterizing the two problems. In the case of the flow field about the sphere, the existence of the bow shock is determined primarily by the presence of the body; the shock exists whether or not the gas is capable of exothermic reaction. In the case of the detonation wave, the existence of the shock and of the reaction front are interdependent--the two are closely coupled. The strong interdependence leads to a complex instability of the detonation wave apparently involving severe departures from one dimensionality (Ref. 1). This instability, now believed to be always present in self-sustained detonations, is as
yet not understood satisfactorily. The flow field in the nose region of a blunt body moving in a detonable mixture more closely resembles that of an overdriven detonation than that of a self-sustained detonation. In the case of the overdriven wave, some agent other than the chemical reaction, such as a piston or an expanding driver gas, is to a large part responsible for the existence of the primary shock wave. This other agent is relatively insensitive to effects of nonuniformities of the primary shock, and thus there is a weaker feedback mechanism through which instabilities can be sustained. Strongly overdriven detonations can exhibit stable, one-dimensional structures (Ref. 12).

Although the detailed criteria which determine the stability of the flow field about the ball and the flow field of a self-sustained detonation probably differ, the most likely physical cause of the instability is the same. This is the sensitivity of the reaction front location or, equivalently, the sensitivity of the instantaneous rate of chemical energy release, to disturbances effecting the strength of the bow shock. The reaction front location is determined by the induction time. The induction time is quite sensitive to changes in strength of the bow shock and typically exhibits a dependence on the temperature and pressure of the shocked gas of the form:

\[
\zeta = \frac{A}{P} \exp\left(\frac{E}{RT}\right)
\]

\(A\) - constant
\( p \) - pressure

\( T \) - temperature

For small disturbances and for an isentropic relationship between pressure and temperature, the following relationship exists (Ref. 1):

\[
\frac{\Delta T}{c} = - \left( \frac{\gamma}{\gamma - 1} + \frac{E}{\rho T} \right) \frac{\Delta T}{T} \tag{2}
\]

Under conditions of these experiments, which are close to the conditions existing behind the shock in self-sustained detonations, the bracketed term is of the order of 10. Thus temperature fluctuations behind a disturbed bow shock will produce induction time fluctuations of a ten-times larger magnitude. This suggests that the observed instability could be caused by the amplification of disturbances in the induction zone which separates the reaction front and the bow shock in the nose region of the flow field.

**Previous Work**

Before pursuing this reasoning, the work of previous investigators will be reviewed. Ruegg and Dorsey (Ref. 2) suggest that the instability is due to longitudinal oscillations "between the sphere and the (bow shock) wave" and that such an oscillation would exhibit a period:

\[
\frac{T}{f} = \frac{\pi \Delta}{C_3} \tag{3}
\]
They suggest that the value of $C_3$ to be used should lie between the value behind an adiabatic shock wave and that behind a Chapman-Jouguet detonation. They indicate that under the conditions of their experiments this model gives a period of oscillation on the order of $10^{-5}$ sec which is on the same order as the observed periods. This period is somewhat larger than but of the same order as the induction time. They do not attempt to deduce the mechanism of the instability from these observations. (The primary result of their work is quantitative data on induction times derived from the measured separation between the bow shock and the reaction front.)

In a later discussion (Ref. 18), Ruegg and Dorsey identify the regular striations observed in their schlieren photographs as being the effect of a "strong acoustic oscillations" which generates waves in the nose region which "propagate between the flame front and shock wave, and are there reflected and returned to the flame in proper phase to prevent damping of the disturbance." A linear relationship between the wavelength/diameter ratio and the period, which is calculated by dividing the separation distance between the reaction front and the
shock by the speed of sound in the unreacted gas behind the shock, is cited as evidence that the oscillation is an acoustic effect. The plot contains seven data points. The "visible propagation of waves between the combustion and shock fronts" (observed by means of schlieren photographs) is given as evidence that "acoustic waves at the front of the missile are responsible for the oscillations." The "waves" referred to, e.g., Fig. 22 of this report, have been identified herein as the schlieren effect caused by the corrugated edge of the reacted gas boundary and, as pointed out in the part of Section 2 dealing with the reacted gas boundary, were never observed (by the dual schlieren technique) to oscillate but were observed only to propagate from the shock to the reaction front.

Behrens, Struth and Wecken (Ref. 4) compared the period of the oscillation observed in their experiments with the induction times obtained for stoichiometric hydrogen-air mixtures by Ruegg and Dorsey. The oscillation period was obtained by dividing the observed wavelength by an appropriate flow velocity (the missile velocity less the wake velocity defect (Ref. 5)). They note that under certain conditions the comparison is favorable, but that under other conditions (other hydrogen-oxygen mixtures) the comparison is not favorable. For those cases where the comparison is favorable, they refer to the vertical striations in the reacted gas as a shock pattern which results from a buildup of an acoustic oscillation; the buildup being due to
"resonance. . . (between) the induction frequency and the vibration frequency." To explain the wide range of frequencies observed, they "have to assume that there exists quite a continuous range, or dense spectrum, of acoustic frequencies in the gas column before the missile."

They do not elaborate by describing a specific model but rather prefer to present their observations in these general terms. For those cases where the comparison between the period of oscillation and the induction time is in poor agreement, they suggest that acoustic oscillations exist. The frequency of these oscillations is determined by the expression for the frequency of acoustic oscillations in a pipe closed at both ends:

\[ \nu = \frac{nC}{2(\Delta - \Delta \nu)} \]  

\( \nu \) - frequency

\( n \) - integer

\( \Delta - \Delta \nu \) - distance between the combustion wave and the sphere

\( C \) - the mean velocity of sound between these space limits

They do not attempt to make a quantitative comparison between experiment and theory because they were not able to measure \( \Delta - \Delta \nu \) in their photographs. Note that the length over which the oscillations occur is stated to be
the distance between the sphere and the reaction front as opposed to the
distance between the shock and the reaction front.

In summary, previous workers suggested that the wavelength of the
vertical striations is related to the period of oscillations of one-
dimensional acoustic waves propagating over a length equal to either
the induction zone length, the detachment distance, or the distance
between the sphere and the reaction front.

**Acoustic Oscillations**

The period of oscillation, $T'$, of an acoustic wave on the central
streamline in the nose region is the sum of the time required to
propagate in the forward direction or toward the bow shock, $t_f$, plus
the time required to propagate in the backward direction, $t_b$. The
period of an acoustic wave oscillating in the unreacted gas between
the reaction front and the bow shock will be designated $T_a$ and the
period of oscillation in the reacted gas $T_R'$.

The separation of the reaction front from the bow shock is fixed
by the induction time $\tau$. A linear velocity decay behind the bow shock
(a good approximation in an unreacting gas) with a velocity gradient
in the unreacted region equal to that found in the case of no reaction
is assumed such that (Fig. 43a):

$$v = \frac{\Delta' \cdot \Delta}{\Delta} u_s$$  \hspace{1cm} (5)
then

\[
\frac{\Delta'}{\nu_0} \int_0^{\Delta u} \frac{dx}{\Delta' - x} = \int_0^\tau dt
\]

(6)

and

\[
1 - \frac{\Delta u}{\Delta'} = e^{-\frac{\nu_0}{\Delta'}}
\]

(7)

The characteristic flow time appearing in this expression can be written as

\[
\frac{\Delta'}{\nu_0} = \frac{\Delta' P_x R}{\nu_0 \cdot \frac{\rho_0}{\rho_o}^{0.53}}
\]

(8)

Employing an empirical correlation for detachment distance (Ref. 19)

\[
\frac{\Delta'}{R} = 0.88 \left( \frac{\rho_0}{\rho_o} \right)^{0.053}
\]

(9)

this becomes

\[
\frac{\Delta'}{\nu_0} = \frac{R}{\nu_0} \cdot 0.88 \left( \frac{\rho_0}{\rho_o} \right)^{0.053}
\]

(10)

Over the Mach number range of interest the density ratio across a normal shock is nearly constant meaning that the induction zone length depends almost exclusively on the first Damköhler number, \( D = \frac{\tau}{\rho R \nu_0} \):

\[
\frac{\Delta u}{\Delta'} = 1 - e^{-\frac{\nu_0}{\Delta'} B} = 1 - e^{-\frac{\nu_0}{\Delta'} D}
\]

(11)

where \( B \equiv 0.88 \left( \frac{\rho_0}{\rho_o} \right)^{0.053} \approx \text{const. for} \ 4 < M_o < 6 \)
Since the acoustic speed in the unreacted gas, $c_u$, varies by a negligible amount with $\lambda$, the velocity with which the wave propagates in the forward direction is

$$v_{\text{prop}} = - [c_u - \nu_s (1 - \frac{c_u}{\lambda})]$$  \hspace{1cm} (12)

and the time required for the wave to propagate from the reaction front to the shock is

$$t_f = - \int_{\Delta_u}^{\Lambda} \frac{d\lambda}{c_u \left[ 1 - \frac{\nu_s}{c_u} (1 - \frac{c_u}{\lambda}) \right]} = \frac{\Delta_1}{v_s} \ln \frac{1 - M_5 \left(1 - \frac{\Delta_u}{\lambda} \right)}{1 - M_5}$$  \hspace{1cm} (13)

where $M_5 = \text{Mach number downstream of the bow shock}$

Similarly,

$$t_b = - \frac{\Delta_1}{v_s} \ln \frac{c_u + (1 - \frac{\Delta_u}{\lambda})}{\frac{c_u}{v_s} + 1} = - \frac{\Delta_1}{v_s} \ln \frac{1 + M_5 \left(1 - \frac{\Delta_u}{\lambda} \right)}{1 + M_5}$$  \hspace{1cm} (14)

and thus

$$T_u = \frac{\Delta_1}{v_s} \ln \frac{1 + M_5}{1 - M_5} \frac{1 - M_5 \left(1 - \frac{\Delta_u}{\lambda} \right)}{1 + M_5 \left(1 - \frac{\Delta_u}{\lambda} \right)}$$  \hspace{1cm} (15)

To determine the period of acoustic waves oscillating in the reacted gas, it is necessary to approximate the distance of the reaction front from the surface of the sphere, $\Delta_R$ in Fig. 43b, or equivalently, the detachment distance $\Delta$, taking account of the effects of heat release.
Assuming the effective density between the shock and the sphere to be:

\[ \rho_{\text{eff}} = \rho u \frac{\Delta u}{\Delta} + \rho R \frac{\Delta R}{\Delta} \]

which assumes that pressure and gas constant undergo no change across the reaction front such that \( \frac{\Delta u}{\rho u} = \frac{\Delta R}{R} \), then the ratio of detachment distances can be approximated as

\[ \frac{\Delta}{\Delta'} = \frac{\rho u}{\rho_{\text{eff}}} = \frac{\Delta u}{\Delta} \left( 1 - \frac{\Delta u}{\rho u} \right) + \frac{\Delta R}{R} \]

or by rearranging terms,

\[ \frac{\Delta'}{\Delta} = \frac{\Delta R}{\rho u} - \frac{\Delta u}{\rho u} \left( \frac{\Delta R}{\rho u} - 1 \right) = \frac{\Delta R}{\rho u} \frac{1 - \frac{\Delta u}{\rho u}}{1 - \frac{\Delta u}{\rho u}} \]  

(18)

The gas velocity in the reacted gas is assumed to decay linearly to zero from the value \( \nu_R \) found immediately behind the reaction front:

\[ \nu = \nu_R \left( 1 - \frac{\Delta'}{\Delta} \right) \]  

(19)

Neglecting compressibility effects and gas constant changes as before, and evaluating Eq. (1) at \( \nu = \Delta' \)

\[ \nu_R = \frac{\Delta R}{\rho u} \nu_R = \frac{\Delta R}{\rho u} \left( 1 - \frac{\Delta u}{\rho u} \right) \nu_S \]

\[ = \frac{\Delta'}{\Delta} \left( 1 - \frac{\Delta}{\Delta'} \right) \nu_S \]  

(20)

where Eq. (18) has been used.

Assuming the acoustic speed in the reacted gas, \( c_R \), does not vary with \( \chi \), the forward propagation speed of an acoustic wave is
\[ v_{prop} = - \left[ c_R - \frac{\Delta'}{\Delta} v_s \left( 1 - \frac{\Delta}{\Delta'} \right) \right] \]  \hspace{1cm} (21)

and the forward propagation time is then
\[ t_f = - \frac{1}{v_s} \int_{\Delta}^{\Delta'} \frac{dx}{c_e \frac{v_s}{v_e} - \frac{\Delta}{\Delta'} \left( 1 - \frac{\Delta}{\Delta'} \right)} \]  \hspace{1cm} (22)

\[ = \frac{\Delta'}{v_s} \ln \frac{c_e \frac{v_s}{v_e} - \frac{\Delta - \Delta u}{\Delta'}}{c_e \frac{v_s}{v_e}} \]

Similarly, the backward propagation time is
\[ t_b = \frac{\Delta'}{v_s} \ln \frac{c_e \frac{v_s}{v_e} + \frac{\Delta - \Delta u}{\Delta'}}{c_e \frac{v_s}{v_e}} \]  \hspace{1cm} (23)

and the period of an acoustic oscillation in the reacted gas is
\[ T_R = \frac{\Delta'}{v_s} \ln \frac{c_e \frac{v_s}{v_e} + \frac{\Delta - \Delta u}{\Delta'}}{c_e \frac{v_s}{v_e} - \frac{\Delta - \Delta u}{\Delta'}} \]  \hspace{1cm} (24)

Assuming \( c_R = \sqrt{\frac{\gamma}{M} c_a} \) this can be written
\[ T_R = \frac{\Delta'}{v_s} \ln \frac{\frac{c_e}{v_e} + M_5 \frac{\Delta - \Delta u}{\Delta'}}{\frac{c_e}{v_e} - M_5 \frac{\Delta - \Delta u}{\Delta'}} \]  \hspace{1cm} (25)

As will be discussed later in this section, the instability produces a periodic fluctuation in the chemical reaction rate, the end result of which is to create the corrugated reacted gas boundary. This boundary, as noted in Section 2, is fixed to the fluid elements and therefore moves downstream with the local velocity of the inviscid wake, \( v_{sw} \).
The wavelength of the corrugations is then related to the period of oscillation by the simple relation:

$$\lambda = T \nu_w$$  \hspace{1cm} (26)

The applicability of this expression can easily be visualized by considering the process as viewed in a coordinate system fixed with respect to the ball.

As noted in Section 2, the inviscid wake velocity downstream of the ball at radial distances typical of the reacted gas boundary radius varies little with downstream distance and is on the order of 85 percent of the free-stream velocity. The error introduced by deriving the instability wavelength from the period by use of the free-stream velocity, \(\nu_c\), instead of \(\nu_w\) would be small. The error in the wavelength trend (variation of \(\lambda\) with free-stream Mach number) would be negligible because of the similarity of the flow field over the Mach number range of interest. Nevertheless, an estimate of \(\nu_c\) based on the correlation for bow shock profiles discussed in Section 2 was used in calculating the instability wavelength. The wake velocity was assumed to be equal to the velocity behind the bow shock at the axial coordinate of the body shoulder; at \(\frac{X}{D} = \frac{1}{2} \left( 1 + \frac{X}{R} \right)\). For a typical case, the value of \(\frac{\nu_w}{\nu_c}\) so calculated varied from 0.82 at \(M_o = 4.4\) to 0.84 at \(M_o = 5.6\).
By combining Eqs. (15) and (25) with Eq. (26) the wavelength to diameter ratios can then be written for acoustic oscillations in the unreacted and reacted gases as:

\[
\left( \frac{A}{D} \right)_u = \frac{v_w}{v_0} \frac{B}{2} \ln \frac{1 + \Delta s}{1 - \Delta s} \frac{U - \frac{\Delta u}{\Delta s}}{U - \frac{\Delta u}{\Delta s}}
\]

\[
\left( \frac{A}{D} \right)_R = \frac{v_w}{v_0} \frac{B}{2} \ln \frac{\sqrt{\frac{\Delta u}{\Delta t} + \Delta s \frac{\Delta u}{\Delta t}}}{\sqrt{\frac{\Delta u}{\Delta t} - \Delta s \frac{\Delta u}{\Delta t}}}
\]

The wavelength/diameter ratios calculated by these equations for the case of a one-half inch diameter sphere fired into a 1:15 mol ratio acetylene-oxygen mixture at a pressure of 0.132 atmosphere are given in Fig. 44. The induction time was estimated from the correlation

\[
\log_\text{e} \left( [O_2] T \right) = -9.88 + \frac{17,100}{2302 R T}
\]

\([O_2]\) - oxygen concentration, mol/liter

\(\dot{e}\) - induction time, sec

\(T\) - temperature, °K

which was obtained by extrapolation of the shock tube data of Kistiakowsky and Richards (Ref. 20). They suggest the expression

\[
\log_\text{e} \left( [O_2] T \right) = -10.58 + \frac{17,100}{2302 R T}
\]

for stoichiometric \(\text{C}_2\text{H}_2:O_2\) mixtures. Their experimental data indicates
a trend toward increasing $\varepsilon$ as the oxygen-acetylene ratio is increased. The maximum ratio reported in their work was 7.7:1; Eq. (29) was obtained by extrapolating to a ratio of 15:1.

The ratio of the temperature of the reacted gas to the temperature of the unreacted gas, $\frac{T_R}{T_u}$, along the central streamline was found by assuming the temperature rise due to reaction to be 50 percent of the ratio of the equilibrium temperature rise. The magnitude of the assumed ratio has a small effect on $(\frac{A}{D})_u$, entering only in the calculation of the wake velocity. The effect on $(\frac{A}{D})_R$ is relatively strong but does not alter the trend exhibited—$(\frac{A}{D})_R$ increasing with increasing Mach number.

The lack of agreement between predicted wavelengths (Fig. 44) and observed wavelength (Fig. 37) for the case of acoustic oscillation in the reacted gas is severe. The calculations show that the thickness of the reacted gas layer in the nose region increases rapidly with Mach number above Mach 4.0 due to the rapid decrease in induction time. The decrease of induction time with increasing temperature is much faster than the increase in acoustic speed; hence the length/acoustic speed ratio increases with Mach number resulting in the wavelength trend shown. It is concluded that the regular instability observed in the photographs is not associated with an acoustic oscillation in the reacted gas.
Since the thickness of the layer of unreacted gas in the nose region decreases rapidly with increasing Mach number, the theoretical wavelength characteristic of an acoustic oscillation in the unreacted gas resembles the experimental trend, particularly at the higher Mach numbers (Fig. 44). The discrepancy between the calculated and observed wavelengths at any given Mach number could easily be accounted for by an inaccuracy in the induction time correlation. It is apparent, however, that \( \frac{A}{D} \) does not exhibit the sharp rise in magnitude observed in the experiments at low Mach numbers. This is a result of the fact that below a certain Mach number, approximately Mach 4.0 in this case, virtually the entire nose region is composed of unreacted gas, and therefore there is practically no increase in the length over which acoustic oscillations occur as the Mach number is further decreased. The very small increase in calculated wavelength is due to the decrease in the acoustic speed resulting from the lower adiabatic shock temperature and to the slight increase in detachment distance which results from the decreasing Mach number.

The apparent agreement between experiment and theory at high Mach number and poor agreement at low Mach numbers led previous investigators (Ref. 4) to the unsatisfactory suggestion that different mechanisms prevail in the high and low Mach numbers ranges. As shown below, however, the purely acoustic model is incompatible with the physics of
the interactions between a compression wave and a shock wave. An equally simple model which gives good agreement between experiment and theory and which is consistent with the physics of the interaction processes is easily developed.

**Gas Dynamics of Wave Interactions**

The interaction between a compression wave and the bow shock wave for conditions typical of the experiments is shown schematically in Fig. 45. In Fig. 45a a weak shock wave of Mach number 1.05 is depicted as approaching a stationary Mach 5.0 bow shock. (A Mach 1.05 shock was selected for these sample calculations since the changes across such a shock were large enough to provide reasonable numerical accuracy and yet the wave could be considered a weak shock. The magnitude of 1.05 is arbitrary and has no special significance.) The conditions downstream of the bow shock, Region 2, and downstream of the propagating weak shock, Region 1, given in the table of Fig. 45 were obtained from normal shock tables for a constant $\gamma = 1.4$ process. The situation after the interaction is depicted in Fig. 45b. The interaction of two normal shock waves of the same family, (the over-taking of shock waves) in gases having an adiabatic exponent $\gamma \leq 5/3$ always results in a transmitted shock wave and a reflected (in general weak) rarefaction wave with a contact discontinuity between them (Ref. 21, p. 178).
The interaction is shown on an $x-t$ diagram in the lower right section of Fig. 46. In this diagram, time increases in the downward direction. The right-hand boundary of the diagram represents the position of the bow shock. The incident weak shock, labelled $\rho$, interacts with the bow shock reflecting a weak rarefaction, $R$, and a contact discontinuity, $CD$. The other waves shown in Fig. 46 will be discussed later.

The conditions in the region between the reinforced bow shock and the reflected wave, Region 3, given in the table, were calculated by the method outlined in Ref. 21, p. 182. The weak rarefaction wave propagating back into Region 1 causes a slight reduction in temperature and pressure. Across the contact discontinuity which separates the gas which has passed through the undisturbed Mach 5.0 bow shock (Region 3B) from that which has passed through the reinforced (Mach 5.064) shock (Region 3A) there is a sharp rise in temperature. The effect of these disturbances on the induction time of the shocked gas is also shown in the table. There is a substantial decrease in induction time due to the effect of the incident weak shock ($\frac{C_1}{C_2} = 0.80$) and a further substantial reduction after the interaction occurs ($\frac{C_{30}}{C_2} = 0.68$). The effect of the reflected rarefaction wave is very small ($\frac{C_{38}}{C_2} = 0.82$ compared to $\frac{C_1}{C_2} = 0.80$).

This example illustrates that because the reflected acoustic wave is extremely weak, it can play only a minor role in the maintenance of
the instability. On the other hand, the contact discontinuity generated by the interaction is a relatively strong disturbance which, taken together with the sensitivity of the induction time to this disturbance, provides a mechanism through which the instability can be sustained. Indeed, as discussed below, the effect of the shortened induction time of the hot gas behind the contact discontinuity is to generate a second compression wave which begins the next cycle of the instability.

Studies of one-dimensional interactions of pressure and entropy waves with flame fronts have shown that the effects of the interactions, i.e., the nature of the reflected and transmitted waves, can be deduced by determining the effect of the incident waves on the heat release at the reaction front (Ref. 22). If the heat release at the front is increased, pressure waves will propagate in both the upstream and downstream directions. The pressure waves are created by the increased volume occupied by the gases passing through the front after the increase in the energy release rate has occurred. In the case where the Mach number of the gases approaching the reaction front is small (in a coordinate system fixed to the reaction front) and where the change in the specific heat ratio, \( \gamma \), across the reaction front is small, the strength of the shocks so generated is related to the change in the reaction rate by (Ref. 22):
\[ \Delta \omega = \frac{r-1}{\gamma} \Delta P (u_2 - u_1) + \frac{r}{r-1} \left( \frac{1 + \frac{\Delta P}{P_1}}{2} \right) \frac{c_{2,1} c_{\omega}}{P_1} \]  

where 

\( \omega \) = heat release per unit reaction front area

\( \Delta P \) = pressure rise across shock

\( 1, 2 \) = conditions upstream and downstream of the reaction front

Under the restrictions given, the upstream and downstream propagating shocks are equal strength.

In the problem under study, the change in the energy release rate, which creates the compression wave, is caused by the effect of the reduced induction time of the gases which have passed through the reinforced bow shock; the situation is shown schematically in Fig. 47. The conditions a short time after a compression wave has interacted with the bow shock are shown in Fig. 47a. (The reflected rarefaction wave has been neglected.) The reaction front is located downstream of the bow shock at the distance which the gas behind the undisturbed bow shock moves in a time, \( \zeta \), equal to its induction time. The contact discontinuity produced by the shock interaction is shown propagating downstream. After the contact discontinuity has moved a distance determined by the induction of the gas behind the contact discontinuity, \( \tilde{\zeta} \), a reaction front will exist at that location as
well as at the location determined by $t = \frac{\rho}{c}$. The reaction rate is thereby effectively doubled. The situation a short time later is depicted in Fig. 47b. The shock waves generated by the doubling of the reaction rate are shown propagating both upstream and downstream. Later, after the contact discontinuity passes the location determined by $t = \frac{\rho}{c}$, no unreacted gas exists at that position and a single reaction front exists at the location determined by $t = \frac{\rho}{c_0}$. The reaction rate is halved when the front located at $t = \frac{\rho}{c}$ is extinguished and rarefaction waves are generated (Fig. 47c).

An $x-t$ representation of the interaction is also shown in Fig. 47. The times corresponding to the three schematic representations discussed above are indicated by the hash marks on the left-hand edge of the plot; this edge represents the position of the undisturbed reaction front. The effect of the contact discontinuity, $CD$, with the hotter gas behind it is to generate a second reaction front, $RF$. At the instant the second reaction front appears, weak shocks, $P$, are produced which propagate both upstream and downstream. The upstream propagating shock further raises the temperature and pressure of the unreacted gas causing the second reaction front to move upstream. This effectively increases the reaction rate and therefore is accompanied by the generation of a train of weak compression waves. The effect of these waves is small and is neglected. The downstream propagating shock also raises the temperature and pressure of the gas behind itself, and thus causes
the originally undisturbed reaction front also to begin to move upstream after the shock reaches the front. The compression waves so generated are shown in the figure but their effect is small and is neglected. The contact discontinuity now separates unreacted and reacted gas. After the contact discontinuity reaches the original reaction front, no additional unreacted gas reaches this front, and the reaction at this location is extinguished. The sudden decrease in energy release rate causes a rarefaction wave, $R$, to be generated.

According to this simple model, the strength of the shock wave generated by the effect of the contact discontinuity on the reaction front would be independent of the strength of the control discontinuity. The shock strength depends on the change in the rate of energy release at the reaction front; according to the model this change results from the doubling of the rate which would exist in the absence of the instability. The strength of the shock reaching the bow shock would, however, depend on the strength of the contact discontinuity. For example, if the contact discontinuity were weak, then $\frac{L_{3\alpha}}{L_1} \approx \frac{L_2}{L_1}$, and the shock would be immediately followed by the rarefaction which is generated by the halving of the reaction rate. The two waves would interact, and the shock would decay before the shock reaches the bow shock. On the other hand, if $\frac{L_{3\alpha}}{L_1} \gg \frac{L_2}{L_1}$, the compression wave would reach the bow shock before the rarefaction had caught up--its strength would be undiminished.
Regardless of the exact details of the interactions, it is clear from the standpoint of the physics of the interactions that it is much more likely that the strong effects of the contact discontinuity on the reaction front are responsible for the feedback which closes the cycle of events than are the weak effects of the rarefaction wave which is reflected from the bow shock.

**Proposed Instability Mechanism**

The complete cycle of events for the instability model composed of the effects of the interaction of a compression wave with the bow shock and the subsequent effects on the reaction front of the contact discontinuity so generated is shown in Fig. 48. An x-t diagram, not drawn to scale, is shown in Fig. 46. The beginning of the cycle is shown at a time when the contact discontinuity approaches the undisturbed reaction front. A zone of decreasing entropy follows the contact discontinuity as indicated later in Steps 5 and 6. In Step 2 the hot gas behind the contact discontinuity begins to react at a second front upstream of the original reaction front position, thereby increasing the overall rate of reaction and generating shock waves which propagate both upstream and downstream. At a somewhat later time, the contact discontinuity, now with reacted gas behind it, reaches the position of the downstream or original reaction front, causing the reaction at this front to be extinguished. Rarefaction waves are generated as a result
of the decreased overall reaction rate. A short time later, Step 3, the entropy zone, begins to penetrate the remaining reaction front causing the reaction rate to diminish and causing the front to begin to recede. The weak rarefaction wave which results from the gradually decreasing reaction rate is neglected. At a later time, the compression wave interacts with the bow shock reflecting a weak rarefaction and generating a contact discontinuity (Step 4). The reflected rarefaction has a negligible effect on the gas through which its propagates, as illustrated in the example of Fig. 45. In Step 5 the incident rarefaction generated by the extinguishing of the original reaction front penetrates the bow shock, restoring the bow shock to its initial strength and generating a zone of decreasing entropy. A chain of weak pressure waves are reflected during the penetration; their effects are negligible. The cycle of events is completed as the contact discontinuity followed by the zone of decreasing entropy approaches the undisturbed reaction front (Step 6).

For purposes of clarity, the contact discontinuity, CD, and zone of decreasing entropy, Z, (a broadened entropy wave) have been shown as being separate and distinct; however, as noted at the end of the preceding subsection, it is just as likely that the two will be merged into a single wave having steep front and a shallow tail. In this case the incident compression wave, P, and rarefaction wave, R, would also
be merged; the rarefaction would catch up with the weak shock causing it to decay. This situation was first suggested during analysis of the shadings of the schlieren photographs when it was observed that the entropy waves in the inviscid wake were composed of a double band—-a dark band followed immediately by a light band (or vice versa).

The period of the cycle of events for this model (hereafter referred to as the P-C D model) is longer than that of the acoustic model, since the time required for the wave front to propagate from the bow shock to the reaction front is determined by the gas velocity alone. Using Eq. (5) for the gas velocity:

\[ t_b = \int_0^{\Delta u} \frac{d\tau}{v_s (1 - \frac{\Delta u}{\Delta})} \]

\[ = - \frac{\Delta u}{v_s} \ln \left(1 - \frac{\Delta u}{\Delta}\right) \]  

(32)

The forward propagation time is given by Eq. (13). Thus the period is

\[ T_{P-CO} = \frac{\Delta u}{v_s} \ln \frac{1 - \alpha s (1 - \frac{\Delta u}{\Delta})}{(1 - \alpha s)(1 - \frac{\Delta u}{\Delta})} \]  

(33)

and the wavelength observed in the wake is given by

\[ \left(\frac{1}{D}\right)_{P-CO} = \frac{v_w}{v_0} B \ln \frac{1 - \alpha s (1 - \frac{\Delta u}{\Delta})}{(1 - \alpha s)(1 - \frac{\Delta u}{\Delta})} \]  

(34)

In Fig. 44 the wavelength trend predicted by this model for the lean acetylene-oxygen mixtures is compared with those predicted by the
acoustic models. The wavelength is observed to increase much more rapidly with decreasing Mach number for the P-C D model than for the acoustic model. This is due to the rapid increase in the time required for the contact discontinuity to reach the reaction front as the reaction front recedes into the low velocity region near the stagnation point. The period of oscillation predicted by the P-C D model thus remains a strong function of the induction time as the induction time becomes long. As noted previously, this is not so for the acoustic model in which case the period is more or less proportional to the induction zone length which asymptotically approaches the detachment distance as the induction time becomes long.

The predicted wavelength trend for the P-C D model is compared with experimental measurements in lean acetylene-oxygen mixtures in Fig. 49. The experiments indicate an even more rapid increase of the wavelength with decreasing Mach number than does the theory. This could easily be due to the induction time exhibiting a faster increase with temperature at the low temperatures than that given by Eq. (29). The linear dependence of \( \log_{10}(\left[ \frac{C_z}{T} \right]) \) on reciprocal temperature was observed by Kistiakowsky and Richards (Ref. 20) to apply over a temperature range from 2200 K to 1300 K. The temperature at the Mach number of 4.7 where the experimental curves steepen is 1350 K. It is quite possible that at lower temperatures a three body chain breaking reaction becomes important and causes faster increase in induction time than
would otherwise be the case. This is observed in the hydrogen-oxygen reaction. Kistiakowsky and co-workers identify the rate limiting reaction for the acetylene-oxygen induction zone reactions as the same as that for the hydrogen-oxygen reaction in the temperature range from 1300 to 2200°K (Refs. 20 and 23). The gratifying quantitative agreement between the magnitudes of the theoretical and experimental wavelengths at the higher Mach numbers must be regarded as fortuitous in view of the crudity of the expression for induction times used in the theoretical model.

Although there is good agreement between the experiments and theory with respect to the behavior of the wavelength with changing Mach number, the experiments fail to exhibit clearly the strong pressure dependence prescribed by any theory which specifies the induction time as the characteristic time. According to the induction time correlations, there is an inverse relationship between induction time and pressure. The lack of separation of the data in the Mach number range from 4.8 to 5.0 as the pressure is varied from 100 to 175 mm Hg is particularly noticeable in Fig. 37. For the runs made at pressure of 100 and 150 mm Hg where there is sufficient data to represent the experimental trend in Fig. 49, the separation indicated by the theory is significantly greater than that experimentally observed in the Mach number range from 4.7 to 5.3. It is difficult to perform conclusive tests on the
dependence of the wavelength on the induction time by conducting experiments over a wide pressure range with Mach number fixed, however, because of the transitions to the turbulent and stable flow regimes which occur (Fig. 28).

It is possible to demonstrate the dependence of instability wavelength on induction time by plotting the wavelength as a function of the reaction zone width, Fig. 50. The reaction zone width is a direct measure of the induction time as discussed in Appendix IV, "Flow Field Calculations." The shorter the induction time, the wider the reaction zone appears on the schlieren photographs. For purposes of Fig. 50, the reaction zone width is defined to be the distance between the envelopes drawn through the peaks of the corrugated reacted gas boundary, measured perpendicular to the axis of symmetry two diameters downstream of the nose of the ball.

The data shown in Fig. 50 includes shots made in hydrogen-oxygen mixtures and acetylene-oxygen mixtures for both one-half inch and one-quarter inch diameter projectiles. Most of the data lies in a band which bends sharply upward as the width to diameter ratio decreases to unity. A significant number of data points lie below the band in this region; these represent runs exhibiting the anomalous high frequency behavior noted earlier in the section on experimental results. The fact that there is a relatively small scatter in the data irrespective
of changes in pressure, Mach number, ball diameter and gas mixture indicates that the reaction zone width and the instability wavelength are strong functions of the same variable; i.e., the induction time.

It should be pointed out with regard to the instability mechanism that although the sphere diameter, \( D \), is a convenient length to use in nondimensionalizing the wavelength measurements, the sphere diameter has only a small influence on the wavelength. The significant length in the problem is, as pointed out previously, the induction zone length, which is determined by the induction time and by the velocity of the gas behind the shock on the centerline. The induction time depends on the temperature and pressure of the gas behind the shock; these properties change by only a very small degree as the gas proceeds from the shock to the stagnation point. Therefore the influence of the ball diameter enters only through its effect on the gas velocity on the centerline. The velocity was assumed to exhibit a linear decrease from the shock to the stagnation point (Eq. 1). In cases where the induction zone length is small, the change in velocity in the induction zone would be small and thus the wavelength should be found to be nearly independent of the ball diameter. This is illustrated by evaluating the analytical expression for the wavelength, Eq. (34), at the limit where \( \Delta u \) becomes small.
By algebraic rearrangement, Eq. (34) can be written

\[
\frac{\lambda}{D} = \frac{\nu u - B}{\nu e} \ln \left[ 1 + \frac{\Delta u}{(1 - M_s)(1 - \frac{\Delta u}{A'})} \right]
\]  

(35)

In the limit as \( \frac{\Delta u}{A'} \to 0 \),

\[
\frac{\lambda}{D} \to \frac{\nu u - B}{\nu e} \cdot \frac{\Delta u}{A'}
\]

(36)

Substituting the shock detachment distance correlation, Eq. (2), this becomes

\[
\lambda \to \frac{\nu u}{\nu e} \cdot \frac{\Delta u}{\left(\frac{\rho}{\rho_0}\right)(1 - M_s)}
\]

(37)

The factor \( \frac{\nu u}{\nu e} \) introduces only a slight dependence on diameter; the other factors in the expression are independent of diameter.

**Development of Reacted Gas Boundary Corrugations**

The most likely process by which the corrugations of the reacted gas boundary are produced is discussed below. Before proceeding, the considerations which determine the location of the reacted gas boundary in the absence of the instability will be outlined. The calculation of this reaction front profile is considered in detail in Appendix IV.

The positions of the reaction front, the bow shock and a single streamline for typical flow conditions are shown in Fig. 51. The dashed
lines in this figure, which give the experimental profiles, will not be considered in this discussion. The streamline shown is the limiting streamline, outside of which no streamlines intersect the reaction front and inside of which all streamlines intersect the reaction front. Neglecting diffusion, the limiting streamline, therefore, is coincident with the reacted gas boundary downstream of the point labeled F where the induction zone reactions freeze. For streamline inside the limiting streamlines, the end-of-induction condition is reached before the reactions freeze due to expansion. By following the progress of the induction zone reactions along several of these streamlines, the points where the end-of-induction condition are reached can be plotted and the reaction front profile sketched by joining the points. See Appendix IV for further details.

In general terms, the corrugations are formed, in the region upstream of the body shoulder, by the reaction front proceeding outward from the flow field axis to a greater extent in the hot gas behind the contact discontinuity than it does in the cold gas ahead of the contact discontinuity. Two hypothetical reacted gas boundaries are shown in Fig. 52: (1) the (inner) boundary representing that boundary which would exist if the flow were stable; (2) the (outer) boundary which represents the maximum transverse extent to which the reacted gas boundary could proceed for a given decrease in induction time. The decrease in
induction time of concern is that due to the increased temperature behind the contact discontinuities associated with the instability.

The points at which the reacted gas boundaries becomes coincident with a streamline due to freezing of the reaction is indicated by the points labeled F.

The contours of compression waves, $P_1$ to $P_4$, generated by the instability are shown as solid lines in the figure. The contact discontinuities generated by the interaction of the compression waves with the bow shock wave are shown as dashed lines. The contact discontinuities extend from the bow shock to the central streamline near the stagnation point; only the segment from the bow shock to the stable reacted gas boundary is shown in the figure. The compression wave contour $P_1$ was generated by assuming the wavefront to be coincident with the bow shock along a short segment near the axis. Another segment was then located by determining the progress of the assumed segment over a time interval equal to the period of the instability. In accordance with the P-C D model, motion of the contact discontinuities, from the bow shock to the outer reacted gas boundary was determined by the gas motion; motion of the compression wave from the reaction front to the bow shock was determined by application of Huygens principle. The instability period was determined from the assumed separation on the stagnation streamline of the bow shock and outer reacted gas boundary.
As shown, the compression wave contour $P_1$ is found to be practically coincident with the bow shock over a significant transverse distance, then it begins to bend away from the shock at an increasing rate. The point where $P_1$ intersects the outer reacted gas boundary locates a point on the contact discontinuity generated by $P_2$. (It is the interaction of this contact discontinuity with the reaction front that generates the compression wavelets which form the wavefront $P_1$.) The intersection of the contact discontinuity with the bow shown determines a point on $P_2$, etc. The wave contours for values of $\frac{\alpha}{D} < 1.0$ were developed by following the reaction of selected segments as described above; those shown for $\frac{\alpha}{D} > 1.0$ were sketched. It can be seen that the compression wave rapidly develops from the normal wave on the axis of symmetry into the wave shape observed in the downstream region of the photographs.

The reacted gas is shown in the figure by the cross-hatched areas. The protrusions of reacted gas beyond the inner reacted gas boundary are bounded on the downstream edge by a contact discontinuity which separates hot gas on its upstream side from cold gas on its downstream side. The location of the upstream edge of the protrusion would be determined by the rate of decrease of the temperature of the hot gas behind the contact discontinuity. The upstream edge shape shown in the figure was sketched arbitrarily, but the spear-shaped protrusions of reacted gas must result.
The protrusions would generate a saw-tooth silhouette on a schlieren photograph as shown by the insert. That is, the undercutting of the protrusion by unreacted gas would not appear on the photographs (although the undercutting would determine the lightness or darkness of the schlieren effect). This general shape of the reacted gas boundary is observed in Fig. 1.

It is apparent in photographs such as Figs. 1 and 13, that the shape of the boundary corrugations changes as the flow expands downstream of the body shoulder. The slope of the downstream edge of the corrugation (the edge having a slope opposite that of the bow shock) becomes smaller and the slope of the upstream edge becomes larger. This effect is due to the difference in the velocity of the gas at the peak of the corrugation and at the trough—the velocity at the trough being the greater. This difference in velocity is due to the hot reacted gas attaining a higher velocity when expanded to a given pressure than the colder unreacted gas. Thus the base of the protrusion of hot gas moves downstream faster than the peak, causing the protrusion to change its appearance from that of leaning downstream to that of leaning upstream. The beginning of this development is illustrated in Fig. 52 by the two downstream corrugations, the configurations of which were sketched on the basis of the above reasoning—they were not calculated.

The effect of the hot gas closer to the flow field axis moving faster than the cold gas exterior to the reacted gas boundary is particularly
noticeable in the photographs of the long wavelength instability such as Figs. 19 and 30. In these cases the amplitude of the corrugation is large and a large difference exists between the velocity of the peak and of the trough of the "corrugation." For the first several wavelengths downstream of the ball there is a distinct separation between the protrusions, but soon thereafter the base of one protrusion appears as though it is drawn into the next.

It is of interest to note that the corrugations serve as a means of joining the expanding cold outer gas with the expanding hot inner gas without the formation of a vortex sheet. If the boundary were a streamline, a shear layer would necessarily exist at the boundary. This is the case for the stable flow; e.g., Fig. 14. However there is no means of detecting the shear layer in the schlieren photographs.

Examination of the photographs indicates that the amplitude of the corrugation decreases as the Mach number increases. Because the wavelength decreases with increasing Mach number, the amplitude trend also manifests itself as a decrease in amplitude with decreasing wavelength (compare Figs. 10 and 13). As the Mach number increases, the two reacted gas boundaries shown in Fig. 52 would move farther out in the inviscid wake. A reduction in the corrugation amplitude implies that the separation distance between the two boundaries decreases as they move outward. The separation distance is determined by the strength of the
contact discontinuities as they sweep over the region between the two boundaries; the stronger the contact discontinuity, the larger the separation. The results of the shock interaction calculations, Appendix II, do indeed indicate that the strength of the contact discontinuity is smaller in the outer region of the inviscid wake. The strength decreases due to the change in the geometry of the interaction between the compression wave and the bow shock. For a given bow shock strength, the contact discontinuity generated by the interaction is strongest when the compression wave is parallel to the bow shock; the contact discontinuity strength decreases as the angle between the two waves increases. As can be seen in Fig. 52, this angle does increase as the interaction point moves outward along the bow shock.

The foregoing theory of how the corrugations are formed can be extended to suggest a plausible cause of the anomalous high frequency behavior reported previously and the observation of two characteristic instability wavelengths in the same photograph. These effects may be a manifestation of wave reflections which have been neglected in the cycle of events listed in Fig. 48. In Fig. 41, the corrugated reacted gas boundary exhibits a well defined wavelength of a magnitude which is in agreement with that predicted by the P-C D model. However, a double set of entropy waves appears in the inviscid wake. In the usual case,
each entropy wave in the inviscid wake corresponds to one striation in the reacted gas region since the corrugation of the reacted gas boundary is caused by the greater transverse advance of the reaction front into the hot gas behind the contact discontinuity. In Fig. 41 one set of entropy waves apparently has a greater effect on corrugating the reacted gas boundary than does the other. The effect of the second set in producing a corrugation can be seen immediately behind the ball, but the effect is washed out as the flow proceeds downstream.

The uneven spacing of the entropy waves and the dominance of one set in producing the prominent striations in the reacted gas leads one to suspect the effect is due to a weak and previously neglected wave reflection. If circumstances were such that the spacing were even and that the extent of the corrugation produced by each set were the same, then it would appear as if an oscillation with twice the expected frequency were occurring. Figure 40 illustrates such a case. (Notice that to the right of the reticle the striations are, indeed, evenly spaced. The wavelength is half the expected value. To the left of the reticle, the striations are unevenly spaced. The wavelength, defined as the distance between similarly appearing points on the boundary, is equal to the expected value. At the far left of the photo, the striations again appear to be evenly spaced.) Cases where the wavelength is observed to be regular throughout have been referred to as those
exhibiting an anomalously high frequency; e.g. Fig. 9. If the data for these runs were replotted in Figs. 37 and 50, using a wavelength twice that recorded in Table I, then a more regular data distribution would result. As noted previously these data can be identified not only by the anomalously high frequency but also by the low amplitude of the boundary corrugation observed in the photographs.

The wave most likely responsible for producing the second set of entropy waves is the compression wave which is generated in Step 2 of Fig. 48 and which propagates from the reaction front toward the ball surface. This compression wave is reflected from the ball surface and would be amplified upon passing through the reaction front (Ref. 22). It would then interact with the bow shock as does the primary compression wave in the P-C D model. From the limited number of runs in which the high frequency behavior was observed (less than 10 runs out of 200), it is apparent that only under certain circumstances are conditions suitable for the reflected wave to produce an effect on the reacted gas boundary and to generate the second set of entropy waves observed. Thus it may be that under certain circumstances a resonant condition exists which results in an abnormally strong reflected wave reaching the bow shock. This speculation is supported by the observation that the anomalously high frequency exists in only a very small Mach number range for any given pressure, and by the observation that, except for
the single case cited (Fig. 41), the timing is always such that the reflected compression wave and the primary compression wave reach the bow shock at equally spaced time intervals such that the entropy waves are observed to be equally spaced.

**Existence of the Periodic Instability**

As indicated by the regime plot, Fig. 28, it is not possible to conduct experiments over a wide pressure range since the flow becomes stable at low pressures and turbulent at high pressures. This sensitivity of the existence of the regular instability mode to the pressure level is a problem different from that of the prediction of the frequency of the instability under given conditions. However, if the proposed model is to be consistent with the actual gas dynamic processes, then some plausible explanation of this effect should be contained within the framework of the model.

Energy is fed into the instability at that point in the cycle when the reaction rate abruptly increases due to the onset of reaction in the hot gas behind the contact discontinuity. The magnitude of the energy release is reflected by the strength of the shock waves generated, Eq. (31). To explain the presence of the lower boundary of the regime plot below which the flow is stable, the value of $\Delta \omega$ in Eq. (31), the units of which are $(\text{energy}) (\text{length})^{-2} (\text{time})^{-1}$, must be pressure sensitive. The value of $\Delta \omega$ depends on the magnitude of the temperature.
rise which occurs at the end of the induction period. This temperature rise must indeed be pressure sensitive, since the exothermic termolecular recombination reactions increase more rapidly with rising pressure than do the bimolecular endothermic dissociation reactions. This effect is, of course, responsible for the increase in the equilibrium temperature rise of exothermic reactions with increasing pressure. Whether this effect is strong or weak in the highly nonequilibrium condition which exists at the end of the abrupt temperature rise is not known. Little work has been published on solutions of the equation governing the exothermic region of the hydrogen-air or acetylene-oxygen reactions.

According to the regime plot, the instability dies out as pressure is decreased, but as pressure is increased a turbulent appearance is generated. Since it is not possible to differentiate between true turbulent flow and a wrinkled reacted gas boundary by use of schlieren photography, it is difficult to speculate on the mechanism by which such turbulence is generated. However, as mentioned in the section describing the flow field regimes, several of the photographs suggest that the turbulent appearance is produced by a wrinkling of the boundary of the reacted gas, just as the vertical striations characteristic of the regime of regular oscillations are a schlieren effect produced by a regular, periodic distortion (a corrugation) of the surface of the reacted gas boundary. The wrinkling of flame fronts has been studied
extensively (Ref. 24, 25), however, there is not complete agreement on the cause. The two causes most widely subscribed to are, (1) that the instability of the flame front is due to the hydrodynamic instability of a fluid containing an interface across which there is a density change; and (2) that the instability is caused by differences in the diffusion coefficients of the fuel and oxidizer which tend to produce inhomogeneities in the mixture. Both phenomena would be intensified by an increase in the magnitude of the property changes across the reaction front boundary. If the turbulent appearance of the reacted gas in the schlieren photographs is due to a wrinkling of the boundary, then the same reasoning used to explain the presence of the low pressure boundary in the regime plot would also apply at the high pressure boundary. The increased pressure results in a greater magnitude of the property changes across the reaction front which results in a greater instability of the reaction front.

**Induction Time of Hydrogen-Oxygen Mixtures**

By use of Eq. (34) for the theoretical instability wavelength for the P-C D model, it is possible to obtain an expression for induction times based on the experimental ballistic range data. In Fig. 53, the experimental data for the hydrogen-air runs are shown together with the theoretical curves predicted by use of Eq. (34) with an induction time correlation given by:
\[
\log_{10}(\text{[O}_2] \text{ cm}^2) = -9.90 + \frac{15100}{R \cdot 303} 
\]  

(38)

This correlation is that which yields the best fit to the data for runs made at an ambient pressure of 300 mm Hg. The conditions behind the shock generated by these runs lie in the following ranges:

\[
1400 < T \text{(°K)} < 1800 \\
9.9 < p \text{(atm)} < 13.6 \\
0.0126 < \text{[O}_2] \text{ (mols/liter)} < 0.0135 \\
[\text{H}_2]/[\text{O}_2] = 2
\]

A comparison between this expression and other experimentally determined correlations is given in Fig. 54. There is good agreement between these experiments and the data obtained by Ruegg and Dorsey, Strehlow and Cohen, and an extrapolation of the data of Miyama and Takeyama. The data of Ruegg and Dorsey (Ref. 2) was obtained by observing by means of schlieren photography the separation distance between the shock wave and reaction front in ballistic range experiments carried out in stoichiometric hydrogen-air mixtures with pressures behind the shock of from 3 to 12 atmospheres. The data of Strehlow and Cohen (Ref. 26) is based on use of schlieren photography to determine the separation between the shock front and the reaction zone in shock tubes containing hydrogen-oxygen mixtures highly diluted with inert gases. The pressure range covered by their experiments was not specified.
The data of Miyama and Takeyama (Ref. 27) is based on the detection of OH absorption and the detection of the pressure change associated with chemical reaction in shock tube. Hydrogen-oxygen mixtures highly diluted with inert gases were studied. The pressure behind the shock varied from 4.5 to 5.6 atm. Among their results, they obtained evidence that under the conditions of their experiments at temperatures above 1100 K the induction times determined by OH absorption detection and by pressure change were the same.

The induction times obtained from these experiments lie significantly above the data of Schott and Kinsey and of White and Moore. Schott and Kinsey (Ref. 28) based their measurements on the detection of OH absorption in highly dilute hydrogen-oxygen mixtures in a shock tube with pressures behind the shock on the order of 0.1 atmosphere. White and Moore (Ref. 29) based their measurements on the use of an interferometer to measure the separation between the shock and reaction fronts in undiluted hydrogen-oxygen mixtures. A shock tube having a converging-diverging section was used to produce a stable wave structure. The pressure behind the shock front was on the order of 0.12 atm. The data of Mullaney et al. (Ref. 30) is based on experiments similar to those of White and Moore; pressures behind the shock are less than 0.1 atmosphere.
The induction time correlation required to bring about agreement between the theoretically predicted and experimentally observed instability wavelengths is thus seen to be in good agreement with induction time correlations obtained by other experimental techniques, particularly with those obtained by experiments at relatively high pressures.

In summary, in this section a model of the high frequency regular periodic instabilities associated with exothermic hypersonic blunt-body flows has been developed. The model is composed of the interaction of a compression wave with the bow shock wave in the nose region of the flow field and the subsequent effect of the contact discontinuity generated by this interaction on the rate of chemical energy release at the reaction front. The model has been shown to predict a wavelength trend that is consistent with the experimental measurements carried out in stoichiometric hydrogen-air mixtures and lean acetylene-oxygen mixtures with the exception that the experiments do not clearly produce the pressure effect which the model predicts. The model has been shown to be consistent with the probable explanation for the existence of the corrugated reacted gas boundary and with effects which could possibly be responsible for the existence of the flow field regime boundaries.
REFERENCES


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<tr>
<td>$A$</td>
<td>Constant Bow shock correlation parameter, Eq. (1)</td>
</tr>
<tr>
<td>$a$</td>
<td>Acoustic speed Shock reflection coefficient parameter, Eq. II-(3)</td>
</tr>
<tr>
<td>$B$</td>
<td>Density function, Eq. (11) Relative brightness (APPENDIX I)</td>
</tr>
<tr>
<td>$B_S$</td>
<td>Bow shock</td>
</tr>
<tr>
<td>$b$</td>
<td>Shock reflection coefficient parameter, Eq. II-(4)</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Contact discontinuity</td>
</tr>
<tr>
<td>$c$</td>
<td>Acoustic speed Concentration</td>
</tr>
<tr>
<td>$c^*$</td>
<td>Acoustic speed at $M = 1.0$</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter First Damkohler similarity group</td>
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<td>$E$</td>
<td>Activation energy</td>
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<td>$F$</td>
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<td>$K_q(T)$</td>
<td>Equilibrium constant</td>
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<td>$k'$</td>
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<td>$M$</td>
<td>Mach number</td>
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<tr>
<td>$M^*$</td>
<td>Dimensionless speed, $v/c^*$</td>
</tr>
<tr>
<td>$n$</td>
<td>Integer Bow shock correlation parameter, Eq. (1) Index of refraction (APPENDIX I)</td>
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</tbody>
</table>
Pressure

Body radius
Gas constant
Rarefaction wave

Reaction front

Radial coordinate

Compression wave, shockwave

Path length
Entropy

Temperature

Period

Time

Velocity

Reaction zone width

Axial coordinate measured from shock apex
Mol fraction

Ratio of O₂ to H₂

Transverse coordinate

Entropy zone

Transverse coordinate

Angle of incidence (APPENDIX I)
Ratio of Reaction rates, Eq. III-(17)

Ratio of specific heats

Detachment distance
Detachment distance, unreacted gas, Fig. 43

Streamline inclination with respect to flowfield axis

Transformation constant, Eq. III-(2)

Angular deflection

Wave inclination with respect to flowfield axis

Wavelength

Time constant

Mach angle

Frequency

Density

Induction time

Equivalence ratio

Rate of chemical energy release

Subscripts

b Backward rate

f Forward rate

i Condition at end of induction period

3 Third body

ns1 Normal to compression wave 1

R Condition behind reaction front

s Condition behind shock

u Condition in unreacted gas

w Wake

0 Condition in undisturbed gas
The schlieren photographs reproduced in this report were used to determine the location of the various wave fronts in the flow field and the direction of the gas density change across these fronts. Location of the shock waves or entropy waves which produce a given schlieren pattern in an axisymmetric flow field is straightforward since the light rays which are bent to the greatest extent are those tangent to the fronts. The view produced is thus a cross-sectional view of the flow field in a plane through the axis of symmetry, normal to the undisturbed light rays. Care must be exercised, however, in determining the direction of the density change; i.e., whether the density increases or decreases across a given discontinuity. The necessary considerations are detailed below.

The schlieren system employed in the experimental apparatus is a double pass system utilizing a single spherical mirror. As illustrated in Fig. 55a, the light source and the knife edge are located slightly off the optical axis near the center of curvature of the mirror. Both the light rays incident to and reflected from the mirror are shown in this figure; in the remaining sketches only the reflected rays are shown. When a region of changed index of refraction is located in the light path, the rays are bent in accordance with the laws of optics.
A brightened or darkened image is produced on the viewing screen depending on the direction of bending and on orientation of the knife edge. For example, the effect of placing a glass wedge in the test area is shown in Fig. 51b. The light rays are deflected onto the knife edge and are cut off, thus producing a darkened imagine on the screen. If the knife edge were inserted from the right instead of from the left, a brightened image would be produced. Also, if the wedge were pointed in the opposite direction, the image would be brightened rather than darkened.

The bending of the light rays can be either due to (1) refraction at a boundary separating two media of different refractive index or (2) to the curvature developed by a light ray on passing through a medium having a refractive index gradient in a direction normal to the ray (Ref. 31).

The magnitude of the former effect is governed by Snell's law of refraction

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \quad \text{I-(1)} \]

where:

- \( n \) - refractive index
- \( \alpha \) - angle between normal to boundary and light ray

and the subscripts refer to the two media. It is this effect which produces the deflection of a light ray on passing through a glass wedge. The direction of the deflection which produces the brightening or darkening on the schlieren screen is thus dependent on the boundary
geometry and on the direction of the change in the refractive index on
crossing the boundary. It is this effect which is primarily responsible
for the patterns observed on schlieren photographs of axisymmetric super-
sonic flows. In the gas dynamic case, the variation of the index of
refraction with gas density is given by an expression of the form

\[ n = 1 + k \rho \]  

I-(2)

where

\[ \rho = \text{gas density} \]

\[ k = \text{constant (typical value = 0.000292 for air at STP.)} \]

The magnitude of the angular deflection, \( \varepsilon \), produced by a refractive
index gradient in a direction, \( \mathbf{z} \), normal to the direction of the light
ray, \( \mathbf{Z} \), is governed by the expression (Ref. 31, p. 9):

\[ \varepsilon = \int_{Z_1}^{Z_2} \frac{\partial}{\partial z} \ln h \, dZ \]  

I-(3)

where the integration is performed throughout the test area or, effecti-
vously, between the flow boundaries. In the case of the glass wedge,
no deflection due to this cause would be produced if the glass were
homogeneous. In the aerodynamic case, however, streamwise density
gradients are usually present which can produce a ray deflection.

These two effects which produce schlieren patterns both derive
from the basic law of physical optics known as Fermat's law. The law
states that light propagation takes place along a path such that the
first variation of the optical path is zero (Ref. 32): \[ \int_{s_i}^{s_f} n ds = 0. \]

Eq. I-(3) is the form taken by the Euler's equation for the variational equation. Snell's law replaces the Euler equation at a surface of discontinuity (Ref. 31). Since both effects derive from the same basic law, it can be expected that under certain conditions both effects can be evaluated by a single calculation. That this is so for the aerodynamic case is demonstrated below.

In the gas dynamic case, which is the Euler equation of the variational problem, Eq. I-(3) can be written as

\[
\mathcal{E} = \int_{z_i}^{z_f} \frac{1}{\mathcal{A}} (\mathcal{H} - \mathcal{H}_0) \, dz
\]

where: \( \mathcal{H}_0 \) is the refractive index of undisturbed gas

since the value of \( \mathcal{H} \) given by Eq. I-(2) is very close to unity. The form of Eq. I-(4) and the fact that Snell's law deals with the boundaries of the flow field suggests that the expression

\[
\mathcal{E}_{\text{total}} = \frac{1}{\mathcal{A}} \int_{z_i}^{z_f} (\mathcal{H} - \mathcal{H}_0) \, dz
\]

where the derivative has been brought outside the integral sign, can be used to evaluate the total angular deflection of the light ray. Note that Leibnitz's rule cannot be rigorously applied since the integrand is discontinuous at the boundaries. This is readily demonstrated in the following paragraph for the case of the wedge (Fig. 55c).
Considering the deflection of the ray through one surface of the wedge (Fig. 55c), Eq. I-(1) can be written as

\[ n_1 \sin \alpha_1 = n_2 \sin (\alpha_1 - \varepsilon) \]  

where: \( \varepsilon = \alpha_2 - \alpha_1 \)

Expanding and rearranging terms:

\[ \varepsilon = \sin \varepsilon = \frac{(n_2 - n_1 \cos \varepsilon) \sin \alpha_1}{\cos \alpha_1} = (n_2 - n_1) \tan \alpha_1 \]  

for \( \varepsilon \ll 1 \).

Evaluating Eq. I-(5) for the upper surface of the wedge

\[ \varepsilon_{\text{tot}} = \frac{\partial}{\partial x} \int_{z_1}^{z} (n_2 - n_1) \, dz \]

\[ = - (n_2 - n_1) \frac{\partial z}{\partial x} \]

\[ = (n_2 - n_1) \tan \alpha_1 \]  

which is the same expression as obtained by application of Snell's law, Eq. I-(7).

From the foregoing it is concluded that the total angular deflection of a light ray can be evaluated by determining the gradient of the total optical path of the ray. By substituting Eq. I-(2) into Eq. I-(5), and noting that the brightness of the image on the viewing screen is proportional to the angular deflection of the ray, the following expression
is obtained:

\[ B = \text{const} \frac{2}{2\pi} \int_{z_1}^{z_2} \Delta \phi \, dz \]  

where \( \Delta \phi \) = Relative brightness

Use of Eq. I-(9) to predict the characteristics of a schlieren photograph of a hypersonic sphere is illustrated in Fig. 56. A vertical knife edge orientation is assumed. Only the effect of the bow shock wave on the image plane brightness is examined. Fig. 56a shows the section produced by the plane \( \xi = 0 \) which is effectively the plane seen on the viewing screen. The location of a typical plane \( \gamma = \gamma_0 \) containing the light rays to be examined in detail is specified.

The plane \( \gamma = \gamma_0 \) is shown in Fig. 56b. The intersection of the bow shock with the plane \( \gamma = 0 \) is shown as a dashed line; the intersection with the plane \( \gamma = \gamma_0 \) is shown as a solid line. The paths of selected light rays are shown with the deflection angles much exaggerated. The path of the undeflected ray is shown as a dashed line. The basis for sketching the ray paths as shown is provided in Figs. 56c and 56d. In Fig. 56c is shown the integrated light path in the plane \( \gamma = \gamma_0 \) for all values of \( \chi \). This curve is constructed by considering how the term \( \int_{z_1}^{z_2} \Delta \phi \, dz \) must behave based on the known behavior of the flow field. Between \( \chi = 0 \) and \( \chi = \chi_1 \), \( \Delta \phi \) is zero since the flow is undisturbed. At \( \chi = \chi_1 \), the bow shock is encountered and \( \int_{z_1}^{z_2} \Delta \phi \, dz \) begins to take on con-
tinuously greater positive values due to the progressively greater values of $z_2 - z_1$, as defined by the solid contour of Fig. 56b. The $\int_{z_1}^{z_2} \beta \, dz$ grows at a decreasing rate with increasing $\chi$ because of the parabolic shape of the contour. At the same time, the average value of $\beta$ must decrease with increasing $\chi$ due to expansion of the flow. At large values of $\chi$ the mean value of $\beta$ must decrease to zero or become slightly negative as the pressure decays to ambient (the gas temperature will be greater than ambient and the density less due to the heightened entropy caused by the shock). Hence the value of $\int_{z_1}^{z_2} \beta \, dz$ must be zero initially, must be positive in the intermediate region, must have smaller gradients with increasing $\chi$, and must finally decrease to zero.

Having established the shape of the $\int_{z_1}^{z_2} \beta \, dz$ curve, the streamwise gradient can be sketched as shown in Fig. 56d. The principal features are the abrupt change in brightness at $\chi = \chi_1$, and the brightness inversion in the downstream region. If the knife edge were positioned such that the bow shock is dark, then somewhat downstream the inviscid flow field should appear brightened; far downstream it should appear to have the brightness of the background. The brightening can be seen in Fig. 21. If the knife edge were reversed such that the bow shock is bright, the streamwise expansion causes a darkening as illustrated by Fig. 13. The inversion is most noticeable in the vicinity of the sphere where the streamwise density gradients are large due to rapid expansion about the ball.
The utility of using Eq. I-(9) to predict the relative brightness characteristics of the photographs becomes apparent if one would attempt to predict the separate effects of boundary refraction (Snell's law) and of ray curvature due to continuous streamwise density gradients for the example of Fig. 56. To estimate which effect predominates along a path such as \( \zeta = \zeta' \), would require numerical calculations.

**Corrugated Reacted Gas Boundary**

The alternate dark and bright bands which characterize the reacted gas zone in the case of the regular periodic instability (e.g., Fig. 21) are accounted for in the analysis given in Fig. 57. The distinct, highly contrasting bands are a result of the sawtooth edge of the reaction zone refracting the light in the manner accounted for by Snell's law. The boundary geometry and the direction of density change across the boundary cause the refraction. Streamwise density gradients within the volume and the refraction at the bow shock boundary affect the magnitude of the \( \frac{\partial \zeta}{\partial x} \) but have little effect on its streamwise gradient.

The direction of the density change across the sawtooth or corrugated boundary can be determined by noting whether the image is brightened or darkened at a given location. Choosing a zone in which the sawtooth edge diverges in the downstream direction, it is seen that the shading is in the opposite sense from that of the bow shock. The bow shock diverges in the downstream direction and the density increases when proceeding from the free stream into the inviscid wake. To produce the
observed shading, the density must decrease when passing across the corrugated boundary from the inviscid wake. The suggestion that the volume within the corrugated boundary contains reacted gas is thus supported.

**Inviscid Wake - Pressure Waves and Entropy Waves**

The two distinct families of waves observed in the inviscid wake in the case of the regular periodic instability are shown schematically in Fig. 58a. A knife edge position producing a darkened bow shock is assumed. The waves identified as compression waves in the schematic appear as bright bands on the photographs. They are bounded on either side by a background brightness equivalent to that produced by an undisturbed inviscid wake. On the other hand, the entropy waves appear as dark bands bounded on the downstream edge by the background brightness and on the upstream side by a bright band (e.g., Fig. 21). The pressure waves extend from the bow shock to the vicinity of the corrugated boundary of the reaction zone where they become indistinct. The entropy waves extend from the bow shock to the reaction zone boundary where they assume a definite position with respect to the boundary corrugations.

The contours of the waves in the plane $\phi = \phi$ are shown in Fig. 58b and the relative density profile along the centerline of this plane is shown in Fig. 58c. In the latter figure, the decay of the density change caused by the entropy wave is shown to be rapid, whereas the
decay of the density change due to the pressure wave is shown to be gradual. A rapidly decaying wave will produce both positive and negative streamwise density gradients having somewhat the same magnitudes. The schlieren record will then be a double band, one bright and one dark. A gradually decaying wave will produce a single band since the density gradient in the decay region will not differ greatly from that of the background, and the schlieren record caused by the decay will be too weak to be detectable. Although the assumed decay rates will generate the observed schlieren patterns, it is questionable that the two waves can have significantly different decay characteristics, particularly near the bow shock. The entropy wave is generated by the interaction of the compression wave with the bow shock and therefore its spatial structure would be determined by the compression wave structure; the structures of the two waves would be expected to be similar.

The integrated optical path and the relative brightness derived from Figs. 58b and 58c are given in Figs. 58d and 58e.
SHOCK INTERACTION ANALYSIS

The prominent inviscid wake wave pattern which characterizes the regular periodic instability is produced by the interaction with the bow shock wave of compression waves (or weak shock waves) generated by the unstable nose region flow. The compression waves and the entropy waves (slip lines, contact discontinuities) produced by the interaction are visible on the schlieren photographs and form the observed net-like wave pattern. The reflections of the compression waves are not visible in the photographs. The calculations carried out to determine the geometry of the interaction process, the nature of the reflected wave, and the strengths of the several waves, are detailed hereunder. The calculations indicate that the compression wave must be a very weak wave, virtually an acoustic wave, and that the reflected wave is a rarefaction wave and cannot be expected to be visible in the schlieren photographs. The predicted and observed wave interaction geometries are not in agreement; there is a small but significant discrepancy, the reason for which is as yet not known. Calculations of the relative magnitudes of the density changes across the compression waves and across the contact discontinuities do not explain the greater schlieren effect produced by the contact discontinuities. The observed decrease in the amplitude of the reacted gas boundary corrugations with in-
creasing projectile velocity is shown to be consistent with the predicted shock interaction behavior.

Geometry of Shock Interaction

The interaction is shown schematically in Fig. 27. The interaction as observed in a coordinate system fixed to the ball, or as depicted in one of the instantaneous schlieren photographs, is sketched in Fig. 27a. The compression wave, labeled an acoustic wave, propagates downstream, from left to right, sweeping along the bow shock and generating an infinitesimal contact discontinuity. The contact discontinuity is fixed to the fluid elements and therefore trails behind the faster propagating acoustic wave. The process as observed in this coordinate system is unsteady; to facilitate the analysis, the interaction is transformed to a stationary system.

Before proceeding, the parameters which characterize the interaction and which are measured in the unsteady system will be indicated. They are:

- $M_0$ - Free stream Mach number
- $\theta_1$ - Bow shock angle
- $\theta_3$ - Compression wave inclination
- $\theta_0$ - Contact discontinuity inclination

These four quantities completely determine the interaction; i.e., all flow field quantities can be calculated with these values given. How-
ever, in order to proceed with the calculation in the most straightforward manner, it is convenient to assume the magnitude of the propagation rate (or the strength) of the compression wave and to reserve one of the measured quantities as a check on the assumption. The compression wave speed was measured by the dual schlieren pictures and found to be on the order of the acoustic speed, and therefore this value is assumed for the first step in the iteration. The angle $\theta_0$ is used to check the assumption.

The steady state coordinate system is that system in which the interaction point, $O$, is stationary. Therefore, the transformation involves subtracting the velocity of the interaction point from all other velocities. The direction of the velocity vector is given by the slope of the bow shock at the intersection point. The magnitude of the velocity of the intersection point depends on the gas velocity and acoustic speed behind the bow shock and on the angle between the bow shock and compression wave, $\theta_1 \theta_3$. It can be seen by reference to Fig. 27a that if the acoustic wave is nearly parallel to the bow shock, as would be the case in the nose region of the ball, then the velocity of the intersection point becomes nearly infinite. In this case, of course, one has a case of the interaction of normal waves which is treated by other means. (See Fig. 45.)
The velocity of the intersection point is given by

\[ \frac{\vec{v}}{a_0} = \frac{\vec{V}_{\text{ns}}}{\sin (\Theta, \Theta_s)} \]  \quad \text{(II-1)}

\[ \vec{V}_{\text{ns}} = \text{Velocity of compression wave normal to itself.} \]

The interaction in the transformed coordinate system is shown in Fig. 27b and again in Fig. 27c where the system has been rotated such that the transformed free stream velocity is parallel to the x-axis. The variables in this system are denoted by primes. The interaction then takes on the more familiar geometry of the interaction of two waves of the same family. For the interactions typical of these experiments, however, the primary shock inclination, \( \Theta \), is quite steep and the flow deflection across the shock approaches the maximum possible deflection for the given free stream Mach number. The weak compression wave appears to be inclined upstream with respect to the free stream, but must, of course, be inclined downstream with respect to the flow immediately ahead of it. (This is discussed further below.) The contact discontinuity must be parallel to the streamlines downstream of the bow shock. Shown in Fig. 27c is the calculated inclination of the contact discontinuity and the position of the contact discontinuity as observed in the schlieren photograph. There is a discrepancy of 9.3°. In the case assumed here, where the compression wave is an acoustic wave, the flow in all regions downstream of the bow shock is uniform.
Since all waves are stationary in the transformed coordinate system of Fig. 27c, the normal component of Mach number of the gas approaching the acoustic wave (Wave 3) must be unity ($M_{ns} = 1.0$) since this is the assumed basis of the transformation. Furthermore, the inclination of the compression wave to the approaching flow must be between the normal to the flow and the Mach angle. In the case where the compression wave is assumed to be an acoustic wave, the Mach angle is $90^\circ$, hence the acoustic wave must be normal to the flow. This is seen not to be the case in the construction of Fig. 27c where the angle between a streamline and the wave front is calculated to be $99^\circ$. This is indicative either of errors in the measurement of the wave angles or that the compression wave is not an acoustic wave; both possibilities are considered below.

The most likely measurement error would involve the inclination of the compression wave, since the schlieren effect produced by this wave is relatively weak near the bow shock (see Figs. 13 or 26). The bow shock angle is easily measured and the free stream velocity measurements are considered reliable. The measured inclination angle of the contact discontinuity is subject to question, but this value has not as yet entered the analysis. With the compression wave assumed to be acoustic, then a value of the inclination, $\Theta_3$, can be calculated for which the wave is normal to the flow. This angle is found to $31.7^\circ$ in
contrast to the measured value of 41.3° (Fig. 27a). The calculated parameters in the primed coordinate system corresponding to this compression wave angle of 31.7° compared to those given in Fig. 27c are:

<table>
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<th>Quantity</th>
<th>Compatible Figure</th>
<th>System</th>
<th>27c</th>
</tr>
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<tbody>
<tr>
<td>Shock inclination, $\theta'_1$</td>
<td>62.8°</td>
<td>62.3°</td>
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<tr>
<td>Acoustic wave inclination, $\theta'_3$</td>
<td>59.0°</td>
<td>49.9°</td>
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<tr>
<td>Mach No. after shock, $M'_1$</td>
<td>1.0</td>
<td>1.03</td>
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<tr>
<td>Contact discontinuity inclination, $\theta'_0$</td>
<td>31.0°</td>
<td>31.1°</td>
<td></td>
</tr>
<tr>
<td>Observed C.D. inclination</td>
<td>22.3°</td>
<td>21.8°</td>
<td></td>
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</table>

It is seen that the change of 9.6° in the inclination of the acoustic wave produces changes in the calculated angles of less than 1°. The discrepancy between the observed and calculated inclinations of the contact discontinuity remains essentially unchanged. That a change in the inclination of the acoustic wave should indeed produce a very small change in the calculated inclination of the contact discontinuity can be inferred by reference to Fig. 27a. The contact discontinuity inclination is primarily determined by the rate at which the intersection point moves along the bow shock. This in turn depends strongly on the propagation rate of the compression wave but only weakly on its inclination except when the angle between the bow shock and the compression wave is quite small. It is therefore concluded that the discrepancy between the observed and measured inclinations of the contact dis-
continuity cannot be attributed to errors in measurements of the compression wave inclination.

If the compression wave were assumed to be of finite amplitude rather than of infinitesimal amplitude as heretofore assumed, there exists a wide range of values of compression wave inclination angle which satisfy the requirement that the compression wave lie between the normal to the flow and the Mach line. This is a result of the rapid decrease of the Mach angle as the flow Mach number increases from unity. For example, if the compression wave is assumed to be a Mach 1.1 shock, the geometry conditions are compatible for all compression wave angles, \( \theta_3 \), below 35°. In these cases, however, another consideration becomes important. In the steady state reference system for compression waves of finite strength, the flow downstream of the compression wave remains supersonic for only a very limited range of wave strengths. If the flow does not remain supersonic throughout, no solution to the interaction problem exists in which the various regions can be joined by a wave configuration of the type considered. The configuration is shown in detail in Fig. 59; the construction of which is similar to that of Fig. 27c but wherein the reflected wave has been included. No satisfactory configuration has been found which can provide for the possibility of subsonic flow behind the compression wave and which is at the same compatible with the observation that the waves are only slightly curved.
and therefore of nearly uniform strength throughout their transverse extent. Furthermore, no deflection of the bow shock is observed at the intersection point; this would necessarily be the case if the wave were a stronger shock. Such a break is observed in the cases where the reaction zone appears turbulent; apparently much stronger disturbances are generated in this case. (See Figs. 7 and 8.) From these considerations it is concluded that the compression wave strength is sufficiently small such that the flow behind the compression wave remains supersonic.

Using the measured values of $\Theta$ and $\mu$ for any given run and values of $\Theta_3$ in the neighborhood of the measured value, the range of compression wave strengths for which the flow remains supersonic throughout was found to be extremely small. Typically, values of $\mu$ smaller than 1.03 were required to satisfy the supersonic flow requirement. It is, therefore, concluded that the compression wave is extremely weak; it is virtually an acoustic wave. The discrepancy between the observed and calculated contact discontinuity slopes cannot therefore be the result of any difference between the actual compression wave strength and the assumed infinitesimal strength.

Regardless of the above considerations, the discrepancy could not be resolved by assuming a value for the compression wave propagation speed higher than the acoustic speed. This can be visualized in Fig. 27a by observing that if the propagation rate of the compression wave were higher, then the interaction point 0 would move along the bow shock at
a faster rate. The velocity of the gas which has passed through the
disturbed portion of the bow shock would increase by only a small amount
due to the increased compression wave strength. The contact discontinuity
would therefore be effectively stretched and would lie closer to the bow
shock. This change in slope is in the opposite sense to that required
to explain the discrepancy. A decrease in the propagation rate below
the acoustic speed would be required. This, of course, is not possible
from a physical standpoint.

Calculations were carried out to determine if the discrepancy were
due to the perfect gas assumption or to the assumption of two-dimensional
flow. As expected, however, calculations using thermodynamic data
accounting for specific heat variation produced results which differed
only insignificantly from the constant $\gamma$ calculations. The two-dimen-
sional relations apply strictly only in the immediate vicinity of the
intersection point. The effect of the bending of the contact discontinuity
due to three-dimensional effects was estimated by determining its
contour for a conical flow field having the same shock radius as the
bow shock. The effect was small and tended to incline the contact dis-
continuity even more, slightly increasing the discrepancy.

The variation of the measured and calculated wave interaction para-
meters with distance downstream of the shock apex for a typical case is
shown in Fig. 60. It is seen that the measured contact discontinuity
slope increases continuously, whereas the calculated value remains nearly constant. Thus the discrepancy increases with increasing downstream distance. As noted previously, to explain the discrepancy in terms of the preceding analysis, an effect must occur which produces the same result as would a decrease in the acoustic speed. The effect has not been determined.

Reflected Wave

Since the strength of the compression wave is small, the character of the reflected wave can be determined by use of expressions for the reflection coefficients of disturbances which interact with shock waves. Analytic expressions for the reflection coefficient for a perfect gas with constant specific heats for a two-dimensional interaction with uniform flow upstream of the bow shock are given by Chernyi (Ref. 33) as:

\[ \lambda = \frac{a - b}{a + b} \]  
\[ \alpha = \frac{2 \tan(\theta_i - \delta_i)}{\tan \mu_i} \left[ 1 - \frac{\gamma - 1}{2} \frac{M_i^2 \sin^2(\theta_i - \delta_i) \left( \frac{\rho_i}{\rho_0} - 1 \right)}{\gamma M_i^2 \sin^2(\theta_i - \delta_i) \left( \frac{\rho_i}{\rho_0} - 1 \right)} \right] \]  
\[ b = \frac{1 + \frac{\gamma \cos^2 (\theta_i - \delta_i)}{\rho_0}}{\tan^2 \mu_i} - \frac{\gamma M_i^2 \sin^2(\theta_i - \delta_i) \left( \frac{\rho_i}{\rho_0} - 1 \right)}{\gamma M_i^2 \sin^2(\theta_i - \delta_i) \left( \frac{\rho_i}{\rho_0} - 1 \right)} \]  

where \( \lambda = \frac{P_i - P_0}{P_i - P_0} \) = reflection coefficient
\[ \mu = \text{Mach angle} \]

and the other symbols are defined in Fig. 59.
The parameters in these expressions can be related to the free stream Mach number, $M_0'$, and bow shock angle, $\Theta'$, through the following:

\[
\frac{A}{F_0} = \frac{(\gamma+1)M_0'^2 \sin^2 \Theta'}{(\gamma-1)N_0'^2 \sin^2 \Theta' + 2}
\]

II-(5)

\[
M_0'^2 \sin^2 (\Theta' - \delta') (F_0 - 1) = \frac{(\gamma+1)N_0'^2 \sin^2 \Theta'}{2 \gamma N_0'^2 \sin^2 \Theta' - (\gamma-1)}
\]

II-(6)

\[
\tan^2 M' = \frac{1}{M_0'^2 - 1}
\]

II-(7)

\[
\tan (\Theta' - \delta') = \frac{\tan \Theta'}{F_0}
\]

II-(8)

A plot of the variation of the reflection coefficient as a function of the upstream Mach number, $M_0'$, and the flow deflection angle, $\Theta' - \delta'$, is given in Fig. 61 (Fig. 4.7 of Ref. 33). It is seen that the reflection coefficient can be either positive or negative and that the value decreases rapidly near the limiting value of $\Theta' - \delta'$. This limit is approached as the Mach number behind the bow shock approaches unity. It is this branch of the curves that is of interest in the problem at hand. In terms of Eqs. II-(2) to II-(4), the rapid decrease in $\lambda$ is the result of the rapid increase to infinity of $\tan \psi'$ as $M'$ approaches unity. From the expressions for the parameters $a$ and $b$, it is apparent that $a$ approaches zero and $b$ remains finite and non-zero as $\tan \psi'$ becomes large. Thus $\lambda$ approaches -1 in the limit. In this case the compression wave is reflected as an expansion wave of equal strength.
The value of the reflection coefficient on the nearly vertical leg of the curves of Fig. 61 can be determined approximately by evaluating the flow parameters required by Eqs. II-(2) and II-(3), except for \( \tan \mu' \) in the limiting case where \( \hat{M} = 1 \). The exact value of \( \tan \mu' \) is then used with the resulting expression to evaluate \( \lambda \). The limiting value of the shock angle for sonic flow used in Eqs. II-(5) to II-(7) is given by the relation

\[
\sin^2 \vartheta = \frac{1}{\hat{M}_0^2} \left\{ \frac{(\hat{M}_0^2 - 3 - \theta) + \left[ (\hat{M}_0 + 1) \hat{M} \right]^2}{-\alpha (3 - \theta) \hat{M}_0^2 + \left[ (\hat{M}_0 + 1)^2 \right]} \right\}^{\frac{1}{2}} \quad \text{II-(9)}
\]

The results of calculations for a typical case are given in Fig. 16. The values of \( \hat{M}_0 \) and \( \vartheta \) used are those given in Fig. 27c. The compression wave inclination, \( \vartheta \), was assumed to be 25\(^\circ\) and the strength of the compression wave corresponds to a Mach 1.005 shock. Using conditions at the \( M = 1 \) limit, the parameters \( a \) and \( b \) can be written as:

\[
a = \frac{1.085}{\tan \mu'} \quad \text{II-(10)}
\]

\[
b = 0.158 + \frac{1.200}{\tan^2 \mu'} \quad \text{II-(11)}
\]

The condition for which no reflection occurs, \( a = b \), corresponds to \( \hat{M} = 1.016 \). Under the assumed conditions of a \( M = 1.005 \) shock, \( \hat{M} \) is found to be 1.0122. The reflection coefficient is calculated to be \( \lambda = -0.0682 \); that is, the reflected wave for the assumed conditions in a very weak rarefaction wave.
The Mach number after the Mach 1.012 shock is only slightly greater than unity, $M'_2 = 1.005$, such that the Mach angle of the flow, which determines the position of the head of the reflected expansion wave, is quite large, $\theta'_2 = 86.0^\circ$. Figure 59, therefore, illustrates that the incident compression wave and the reflected expansion wave are nearly coincident.

As mentioned previously, no reflected wave is observed in the schlieren photographs. This could be due to the fact that the incident and reflected waves are nearly coincident; and, therefore, the two waves cannot be resolved. Even if the separation were sufficient for resolution, however, the schlieren effect produced by a rarefaction wave is extremely weak and such waves could not be expected to be visible. Fig. 16.29 of Ref. 34 illustrates this point.

**Wave Strengths**

The calculated density changes across the wave fronts, which influence the strengths of the schlieren effect produced by the waves, are noted in Fig. 59. The density ratio across an oblique shock wave is given by the expression:

$$\frac{\rho_2'}{\rho_1} = \frac{(\gamma+1) \frac{P_2'}{P_1'} + (\gamma-1)}{(\gamma-1) \frac{P_2'}{P_1'} + (\gamma+1)}$$  \hspace{1cm} \text{II-(12)}
which in the weak shock limit can be differentiated to give the density change across the compression wave in terms of the pressure change:

$$\frac{\Delta \rho}{\rho_0} = \frac{1}{\gamma} (\frac{\Delta P}{P})_3$$  \hspace{1cm} \text{(II)-(13)}

This same expression is also obtained by differentiation of the isentropic relation which applies to the density change across the expansion wave:

$$\frac{\rho_2}{\rho_1} = (\frac{P_2}{P_1})^{\frac{1}{\gamma}}$$  \hspace{1cm} \text{(II)-(14)}

To evaluate the density change across the contact discontinuity, the change in the density behind the bow shock due to the disturbance of the incident compression wave is calculated by use of Eq. II-(12). For weak disturbances, differentiation yields:

$$\frac{\rho_2 - \rho_1}{\rho_1} = \left[ \frac{\gamma+1}{(\gamma+1)
\frac{\rho_0}{P_0} + (\gamma-1)} - \frac{\gamma-1}{(\gamma-1)
\frac{\rho_0}{P_0} + (\gamma+1)} \right] \frac{\rho_2 - \rho_1}{P_1}$$  \hspace{1cm} \text{(II)-(15)}

Since the pressure difference across the contact discontinuity is zero, from Fig. 59

$$\frac{\rho_2 - \rho_1}{\rho_1} = \frac{P_2 - P_1}{P_1} + \frac{\rho_2 - \rho_1}{P_1}$$

$$= (1 + \lambda) \left( \frac{\Delta P}{P_1} \right)$$  \hspace{1cm} \text{(II)-(16)}

The net change in density across the incident compression wave and reflected expansion wave is given by:

$$\frac{\rho_2 - \rho_1}{\rho_1} = \frac{1}{\gamma} \frac{\rho_2 - \rho_1}{\rho_1}$$

$$= \frac{1}{\gamma} (1 + \lambda) \left( \frac{\Delta P}{P_1} \right)$$  \hspace{1cm} \text{(II)-(17)}
The density change across the contact discontinuity is then given by:

\[ \frac{\rho_2 - \rho_1}{\rho_1} = (1 + \lambda) \left( \frac{dP}{\rho} \right)_3 \left\{ \frac{1}{\gamma - 1} - \left[ \frac{\gamma^2}{\gamma + 1} \right] \right\} \]

This expression indicates that the value of the density change increases with increasing bow shock strength. This trend is caused by the greater irreversibility associated with stronger shocks. The expression also indicates that for negative values of the reflection coefficient, the density change across compression wave is always greater than the change across the contact discontinuity. This is not indicated by the schlieren photographs where it is observed that the schlieren effect produced by the contact discontinuities is stronger than that produced by the compression waves. Since the schlieren effect is related to the density gradients through the wave rather than to the total density change, a difference in wave structure may be responsible for the stronger schlieren effect of the contact discontinuity.

The temperature change across the contact discontinuity can be found directly from Eq. II-(18), since the pressure change across the wave is zero. Thus the temperature in Region 3B is higher than that in 3A. As discussed in the section on the instability mechanism, the magnitude of this temperature change is important in determining the amplitude of the corrugations of the reacted gas boundary. The amplitude of the corrugations is observed to diminish with increasing reaction zone
width, implying that the magnitude of the temperature change across the contact discontinuity decreases with increasing distance from the flow field axis. This could be due either to a decay in the strength of the incident compression wave with distance from the axis or to a decrease in the strength of the contact discontinuity generated by a compression wave of a given strength as the wave moves outward from the axis.

Equation II-(18) indicates that the latter effect does indeed occur; i.e., the magnitude of the temperature change for a given \( \frac{\Delta T}{T} \) decreases with decreasing \( \frac{P}{P_0} \) and increasing \( \lambda \). The pressure rise across the bow shock, \( \frac{P}{P_0} \), is maximum on the axis and decreases to unity far from the axis. The reflection coefficient value will lie on the vertical leg of the curves of Fig. 61, but whether it increases or decreases with increasing distance from the axis must be determined from Eq. II-(3) and II-(4) which requires knowledge of the change in form taken by the interaction configuration (Fig. 59). The qualitative arguments given in the following paragraph indicate that \( \lambda \) does increase.

Reference to Fig. 27a and to Fig. 52, which shows the change in the compression wave contour with distance from the axis indicates that angle between the compression wave and the bow shock becomes smaller as the axis is approached; on the axis this angle is zero and the interaction is one-dimensional. As noted previously, this results in the velocity of the interaction point becoming very large such that in the
transformed coordinate system (Fig. 27c), $M_1'$ becomes large. But reference to compressible flow charts, e.g., Ref. 35, indicates that as the upstream Mach number becomes large, the value of the shock angle at the limit of sonic flow downstream, $\theta' = \alpha^*$, increases by only a few degrees from the value given in Fig. 27c. The density ratio across the bow shock, $\rho_0/\rho_1$, increases from a value of approximately 3 at the downstream location represented in Fig. 27 to a typical value of 5 on the flow field axis. The deflection angle $\delta'$ is related to $\theta'$ and $\phi'$ by Eq. II-(8). The changes in $\theta'$, $\phi'$ and $\delta'$ thus are small relative to the change in $\tan \psi'$. Therefore, $\tan \psi'$ dominates the behavior of the parameters $a$ and $b$. As the angle between $S_1$ and $S_3$ becomes small in Fig. 27c, the component of $M_1'$ normal to $S_3$ becomes small. This component determines the strength of the compression wave. For a given compression wave strength then, the deviation of $M_1'$ from unity must increase as the interaction point approaches the axis. Thus $\mu'$, determined by Eq. (6), decreases and the reflection coefficient, determined by Eqs. II-(2) to II-(4) increases. The coefficient does not increase sufficiently to become positive, since as indicated in Fig. 45, the one-dimensional interaction is shown to produce a reflected rarefaction wave.

The predicted change in the strength of the contact discontinuity with distance from the axis is therefore consistent with the observed decrease in amplitude of the boundary corrugation with increasing distance from the axis.
APPENDIX III

CHEMICAL KINETICS OF HYDROGEN-OXYGEN INDUCTION REACTIONS

Hydrogen-oxygen gas systems are particularly suited to experimental investigations of exothermic flows since the kinetic mechanism of this system has been extensively studied in the high temperature range of interest, particularly the induction zone reactions (Ref. 36 to 39). Numerical values of the specific reaction rate constants are thought to be fairly well established. The induction zone kinetic mechanisms can be represented by a set of linear first order differential equations; therefore, detailed calculations of the progress of the reactions in the induction region are easily performed. Presented in this appendix are the details of the kinetic calculations. The results are used to generate an induction time correlation which is consistent with the experimental instability wavelength observations. The results are also used in Appendix IV to compare the experimental reaction front profile with that predicted analytically.

The reactions which govern the hydrogen oxidation process can be classified as initiation reactions, induction reactions and exothermic reactions. The initiation reactions are those which reduce a small fraction of the oxygen and hydrogen molecules to atoms and radicals which are in turn required to initiate the chain branching mechanism of the induction zone. The time required to complete these reactions
is usually not significant compared to the induction time. The induction
time, however, does depend strongly on these initial atom and radical
concentrations; therefore, the initiation reactions must be included in
the analysis. The initiation reactions together with the assumed rate
constants used in the numerical calculations are given in Table II.

During the induction period, the atom and radical concentrations
grow rapidly due to a branched chain mechanism. The chain carrying and
branching reactions specified in Table II, Reactions I, II, and III, are
important at all temperature levels of interest in the hydrogen-oxygen
system (Ref. 24, p. 22). The chain breaking reaction, Reaction VI,
is important at temperatures below 1200° K and, being a termolecular
reaction, is more effective at higher pressures. Brokaw (Ref. 40) also
includes the reaction,

\[ \text{XI} \quad \text{H}_2\text{O}_2 + \text{H}_2 \rightarrow \text{H}_2\text{O}_2 + \text{H} \]

which is important only at low temperatures where induction times are
of the order of milliseconds or greater. For purposes of these hyper-
sonic flow experiments, only induction times on the order of microseconds
are of consequence; therefore, Reaction XI was not included. Nicholls
(Ref. 36) includes the reactions

\[ \text{H}_2\text{O} + \text{M} \rightarrow \text{OH} + \text{H} + \text{M} \]
\[ \text{OH} + \text{M} \rightarrow \text{O} + \text{H} + \text{M} \]
\[ 2\text{OH} \rightarrow \text{H}_2\text{O} + \text{O} \]

but shows that they have a negligible effect in the temperature range
of interest.
The exothermic reactions become important, by definition, for times greater than the induction period. These reactions, mostly three-body recombination reactions, are much slower than the chain branching reactions, and therefore the time required for the overall reaction to be completed is generally an order of magnitude greater than the induction period (Ref. 29, 37, 39). Since the primary purpose of these calculations is to determine the induction time, these reactions are not included herein. In reality, the behavior of these reactions determines the end of the induction period; however, as will be discussed later, the knowledge that the chain branching reactions are much faster than the exothermic reactions is sufficient to establish a criterion which signifies the end of the induction period.

Equations and Initial Conditions

Using the reactions listed in Table II, the following expressions can be written for the rate of change of the atom and radical concentrations,

\[
\frac{dc_{o^*}}{dt} = -k_1 c_{o^*} c_{H_2} + k_2 c_{o^*} c_{O_2} + k_3 c_{o^*} c_{H_2} + 2k_4 c_{o^*} c_{O_2} \\
\frac{dc_{o^*}}{dt} = k_1 c_{o^*} c_{H_2} - k_2 c_{o^*} c_{O_2} + k_3 c_{o^*} c_{H_2} - k_6 c_{o^*} c_{o^{'}} + 2k_4 c_{o^*} c_{H_4} \\
\frac{dc_{o^*}}{dt} = k_2 c_{o^*} c_{O_2} - k_3 c_{o^*} c_{H_2} + 2k_6 c_{o^*} c_{O_2} c_{H_4} \tag{III-1}
\]

\[
(c_{o^*})_0 = (c_{o^*})_0 = (c_{o^*})_0 = 0 \\
(c_{H_2})_0, (c_{O_2})_0, (c_{H_4})_0 = \text{given}
\]
Throughout the induction period it may be assumed that the molecule concentrations and the total concentration remain unchanged. Thus Eqs. III-(1) represent a system of first order, linear differential equations in the atom and radical concentrations. The system may be written as a homogeneous set by the transformations,

\[ C'_i \equiv C_i + S_i \]

\[ i = \text{OH, H, O} \]  

Equations III-(2) are substituted into Eqs. III-(1) and the constant terms are set equal to zero. The resulting expressions for the constant terms are

\[-k_1 S_1 + k_2 \bar{x} S_2 + S_3 = -2k_{OH}\]

\[-k_1 S_1 + (-k_2 + k_6) \bar{x} S_2 + S_3 = -2k_{H} \frac{1}{\bar{x}\bar{x}_2}\]

\[k_2 \bar{x} S_2 - S_3 = -2k_{O} \frac{\bar{x}}{\bar{x}_2}\]  

where the equations have been nondimensionalized by dividing by \(k_{3} C_{H^2}\) and the following definitions have been introduced:

\[ S_i = \frac{\delta_i}{C_{H^2}} \quad i = \text{OH, H, O} \]

\[ \bar{x} = \frac{C_{H^2}}{C_{H^2}} \]

\[ k_i = \frac{k_i}{k_{3}} \quad i = 1, 2, 3, \text{OH, H, O} \]

\[ \bar{\chi} = \frac{C_i'}{C_{M}} \quad (\text{mol fraction}) \quad i = \text{OH, H, O, H}_2, \text{O}_2 \]

Equations III-(3) may be solved in terms of the initial concentrations:

\[ \delta_1 = \frac{2\bar{x}}{k_1} \left[ k_{OH} + \frac{k_6}{\bar{x}_2} - 2k_2 \bar{x} \right] \]

\[ \delta_2 = -2\bar{x} \]

\[ \delta_3 = 2\bar{x} \left[ \frac{k_{OH}}{\bar{x}_2} - k_2 \bar{x} \right] \]  

\[ \text{III-(4)} \]
where
\[ X = \frac{k_{CH} + \frac{k_{CH2}}{X_0} + \frac{k_{C2}}{X_0^2}}{2X_0 - 1} \]

The rate equations assume the form:
\[
\begin{align*}
\frac{1}{k_{c'}c_{i'}X} \frac{dX_{CH}}{dt} &= -k_1X_{CH} + k_{CH}X_N + k_2X_2 \\
\frac{1}{k_{c'}c_{i'}X} \frac{dX_{CH2}}{dt} &= k_1X_{CH} - (k_2 + k_6)X_N + k_3X_0 \\
\frac{1}{k_{c'}c_{i'}X} \frac{dX_{C2}}{dt} &= k_2X_N - k_3X_0
\end{align*}
\]

The equations are solved in the usual manner by the substitutions,
\[ X_i = \alpha_i e^{\lambda_i t} \quad i = 0, 1, 2, 0 \]

which result in the expressions,
\[
\begin{align*}
(-k_1 - \lambda_i)\alpha_{CH} + k_2 \alpha_{C2} + \alpha_{H} &= 0 \\
k_1 \alpha_{CH} - \left[(k_2 + k_6)X + \lambda_i\right] \alpha_{CH} + \alpha_{H} &= 0 \\
k_2 \alpha_{C2} - (1 + \lambda_i) \alpha_{H} &= 0
\end{align*}
\]

where the nondimensional time constant
\[ \lambda_i \equiv \frac{\lambda_i'}{k_{c'}c_{i'}X_0} \]

has been introduced. Equations III-(7) possess a solution for the eigenvalues of \( \lambda \) given by the cubic equation:
\[
\lambda_i = \frac{k_1 \left(k_1^2 - k_2 \times X \right)}{k_1 + (k_1 + 1)k_0 + \left[(k_1 + 1) + (k_2 + k_6)X\right] \lambda_i + \lambda_i^2}
\]

As discussed by Brokaw (Ref. 40) this equation, which was first derived...
by Kondratiev (Ref. 41), has three real roots, two of which are negative and one of which is positive. The positive root is responsible for the exponential growth of the radical and atom concentration in the induction period. Brokaw gives expressions for approximate solutions for the positive root; however, the full equation is easily solved numerically since the iteration scheme suggested by the form of Eq. III-(8) converges in two or three steps.

After solving for the $\lambda_j^3$, the values of $R_{n,j}$ and $R_{o,j}$ may be found in terms of $R_{o,n,j}$ by the expressions:

$$R_{n,j} = \frac{(k_i + \lambda_j)(1 + \lambda_j)}{2 + \lambda_j} R_{o,n,j}$$

$$R_{o,j} = \frac{(k_i + \lambda_j)}{2 + \lambda_j} R_{o,n,j}$$

The values of $R_{o,n,j}$ are found by use of the initial condition $(c_{o,n})_0 = 0$ which, with the use of the nondimensional form of Eqs. III-(2), and Eqs. III-(6) can be written:

$$\sum_{i=1}^{3} R_{ij} + \delta_i = 0 \quad i = OH, O, H$$

$$j = 1, 2, 3$$

Having determined the eigenvalues, $\lambda_j$, and the constants $R_{ij}$, the atom and radical mol fractions are given at any time by Eqs. III-(6).

The solution of the equations for conditions corresponding to temperatures behind a Mach 5.0 and Mach 4.0 shock moving in a
stoichiometric hydrogen-air mixture (temperatures of 1611 and 1157°K, respectively) and a pressure of 10 atmospheres is shown in Fig. 62. The dashed lines represent the contribution of the term containing the positive exponent in the expression

$$\chi_{OH} = \sum_{j=1}^{3} A_{OH,j} e^{\lambda^j_t} + \delta_{OH}$$  \hspace{1cm} \text{III-(11)}$$

The induction time is of the order of 1 microsecond for the Mach 5.0 shock and 10 microseconds for the Mach 4.0 shock, and it can be seen that the expression

$$\chi_{OH} = (\chi_{OH})_0 e^{\lambda'_t}$$  \hspace{1cm} \text{III-(12)}$$

where \(\lambda'\) is the positive exponent, adequately represents the radical growth throughout all but the initial stages of the induction period.

A plot of the value of \((\chi_{OH})_0\), which is equal to the value of \(A_{OH,j}\) for the positive root, is given in Fig. 63. The termolecular chain breaking reaction causes a pressure effect to become significant at temperatures below 1400°K. Also shown is the approximate value obtained by use of the expression given by Brokaw (Ref. 40),

$$\frac{\lambda}{\chi_{OH}} = 0.296 \frac{\rho}{\chi_{OH}}$$  \hspace{1cm} \text{III-(13)}$$

which is derived by assuming that the only initiation reaction of significance is the one in which OH is produced and that this reaction
is in equilibrium with Reaction I during initiation. The agreement is excellent for the conditions assumed in the calculations.

The time constant (the positive value of $\lambda$) which determines the radical growth rate in the induction zone is given in Fig. 64. At low temperatures, the constant rapidly approaches zero due to the effect of the chain breaking reaction. Since this is a termolecular reaction, it is more effective at higher pressure, as indicated. The time constant actually remains nonzero at the low temperature; the curves do not intersect the x-axis but instead bend quite sharply and are virtually coincident with the axis for the scale of Fig. 64. The value of the positive root of Eq. III-(8) at temperatures below the critical is then of a smaller order of magnitude than that above the critical. The critical temperature corresponds to that for which

$$2 \lambda_2' = \lambda_6 \cdot C_{11}$$  \hspace{1cm} \text{III-(14)}$$

This is the so-called second explosion limit criterion, since it also describes the boundary between conditions for which explosions either will or will not occur in low pressure, low temperature explosion experiments (Ref. 24, p. 24). In the present experiments it is the condition which would, under certain conditions, determine the freezing point for the induction zone reactions (see Appendix IV).

The radical growth during the induction period is shown on a semi-log plot in Fig. 65 for conditions behind normal shocks in a
stoichiometric hydrogen-air mixtures having an initial pressure of 0.396 atmospheres. The actual conditions during initiation are not shown; the curves have been extrapolated to \( t = 0 \) in accordance with the approximation:

\[
x_{\text{OH}} = (x_{\text{OH}})_0 e^{\lambda t}
\]

Since one of the purposes of this work is to predict the location of the exothermic reaction front, a realistic criterion which signifies the end of the induction period, must be utilized. Much of the experimental work on determining the radical growth rate during the induction period has been based on the measurement of the OH concentration by radiation absorption techniques. Schott and Kinsey, in their experiments (Ref. 28), defined the induction period as the time required for the OH concentration to exceed the limits of detection of their apparatus, which happened to be a concentration of \( 10^{-6} \) mol/liter. There has been a tendency for other authors to apply this criterion to the determination of the time required for the exothermic effects to appear. Obviously, this criterion is not necessarily applicable unless it otherwise can be shown that it does correspond to some basis other than the sensitivity of Schott and Kinsey's apparatus. Miyama and Takeyama, in their experiments (Ref. 27), found that above a certain temperature level a sudden change in the level of OH absorption did occur at the moment when the effects of the exothermic reaction were
detected, but this did not necessarily correspond to a fixed OH concentration level.

Nicholls, Adamson, and Morrison, in their analytic treatment of the equations describing the atom and radical growth in the induction period (Ref. 36), point out that the end-of-induction criterion should be based on the rate controlling reaction in the induction period, Reaction II:

\[ \text{Reaction II:} \quad H + O_2 \xrightarrow{k^i} \text{OH} + O \]

They then define the end of the induction period as that time at which the hydrogen atom mol fraction reaches its maximum. Based on their analytic expressions, they show that this occurs when the hydrogen atom mol fraction is equal to the initial oxygen molecule mol fraction:

\[ t = t^* \text{ when } \chi_H = (\chi_{H0})_0 \]

In his earlier numerical work, Duff (Ref. 37) illustrated the dependence of the induction time on the behavior of the rate limiting reaction, ReactionII, and also illustrated the now well-known overshoot of the atom and radical concentrations which leads to the maximum in the H atom concentration employed in the criterion of Nicholls et al. Duff also showed that the end of induction corresponds to the time when the net rate of the rate limiting reaction becomes slow due to the increase in the backward rate. This is caused by the build-up in the product concentrations. At this point, the exothermic reaction
rate is no longer slow compared to the chain branching rate and the
temperature rise becomes perceptible. Thus the fundamental cause for
the end of the induction period is the approach to equilibrium of
Reaction II. This criterion is easy to apply in the case of the
present numerical calculations.

The rate of production of OH atoms from Reaction II is,

\[
\left( \frac{d[OH]}{dt} \right)_{\text{PRODUCTION}} = \kappa_2' \left[ H \right] \left[ O \right]
\]

whereas the rate of consumption due to the reverse reaction is

\[
\left( \frac{d[OH]}{dt} \right)_{\text{CONSUMP.}} = \kappa_2'' \left[ OH \right] \left[ O \right]
\]

Denoting the ratio of the reverse rate to the forward rate at the end
of the induction period by \( \alpha \),

\[
\alpha \equiv \frac{\left( \frac{d[OH]}{dt} \right)_{\text{PRODUCTION}}}{\left( \frac{d[OH]}{dt} \right)_{\text{CONSUMP.}}}
\]

then

\[
\alpha \frac{\kappa_2''}{\kappa_2'} = \frac{[OH] \left[ O \right]}{\left[ H \right] \left[ O_2 \right]}
\]

where the subscript, \( i \), indicates the value is that at the end of
induction. Introducing the equilibrium constant based on concentrations
and writing in terms of the mol fractions:

\[
\frac{(X_{\text{en}})i (X_{e})i}{(X_{n})i (X_{o})i} = \alpha K_{eq} \tau \quad \text{III-(19)}
\]

By use of Eq. III-(9) and the assumption that the reactant mol fractions are unchanged through the reaction period, the following expression can be written:

\[
(X_{\text{en}})i = \alpha \frac{1 + \lambda}{X} X_{o} K_{eq} \tau \quad \text{III-(20)}
\]

The equilibrium constant is evaluated by standard methods (Ref. 42). The resulting value of \((X_{\text{en}})i\) for stoichiometric hydrogen-air mixtures and for \(\lambda = \infty\) is plotted in Fig. 63. Under the conditions of the experiments, the pressure effect is seen to be negligibly small.

The values of \((X_{\text{en}})i\) can then be entered in Fig. 65, thus determining the induction time. The times so determined are compared in Fig. 66 with those given by the correlation of White and Moore (Ref. 29) for the conditions behind normal shock waves in stoichiometric hydrogen-air mixtures of initial pressure of 0.396 atm. The analytical results are represented by the expression,

\[
\log \left( \frac{[O_2]}{[O]} \right) = -9.90 + \frac{15.100}{2302RT} \quad \text{III-(21)}
\]

This expression is compared with other correlations in Fig. 54.

It was noted above that the atom and radical growth rate in the induction period is controlled by the rate of the chain branching
reaction, Reaction II. Indeed, one of the experimental methods of
determining the specific rate constant for Reaction II, $k_2'$ is based
upon the observation of induction times behind shock waves. Values
of $k_2'$ appearing in the literature are given in the Table III. The
value given by Schott and Kinsey is based upon their measurement of
induction time behind shock waves. The value given by Duff is also
based on shock tube work of Schott and Kinsey. Kaufman and Del Greco's
value is based on discharge flow experiments. Baldwin's value is a best
fit to the data from all the above experiments plus the results of
flame and explosion limit studies.

The value obtained from these experiments and the value used in
the kinetic calculations discussed in this appendix is:

$$k_2' = 2.6 \times 10^{12} \exp\left(-\frac{155000}{R_T}\right)$$

This value, when used in the foregoing analysis together with the other
rate constants given in Table II, yields the induction time behavior
shown in Figs. 54 and 66. This induction time behavior is that which
produces the agreement between experiment and theory illustrated in
Fig. 53. Because the correctness of this value of $k_2'$ depends on the
correctness of the instability model, of the assumed kinetic mechanism,
of the criterion for the end-of-induction and, to a lesser extent, on
the correctness of the other rate constants, this analysis cannot be
viewed as a useful method for measuring this rate constant until the
errors involved in the individual steps have been evaluated. Indeed, the value of activation energy given by the expression for $\Delta'_{2}$, 15.5 kcal/mol, is probably somewhat low since the heat of reaction for Reaction II is approximately 16 to 17 kcal/mol in the temperature range of interest (Ref. 46) and it is unlikely that the reverse bimolecular reaction has a negative activation energy. The somewhat low value is proportional to the slope of the induction time correlation curve (Fig. 54) which is slightly smaller than the values measured by other techniques. For the purpose of this study, which was to determine the mechanism of the observed instability, the agreement between the induction times or the value of $\Delta'_{2}$ derived from these experiments and those reported previously is quite satisfactory.
APPENDIX IV

FLOW FIELD CALCULATIONS

Estimates were made of the properties of the flow field about a hypersonic sphere under conditions typical of the experiments for use in the analytical predictions of the contours of the various wave fronts generated by the exothermic reaction and by the regular periodic instability. These estimates are based on the properties of an unreacting flow field and therefore are valid only for the flow outside of the reacted gas boundary. Previously published experimental data (Ref. 43) on the properties of the subsonic and transonic regions about a Mach 5.0 sphere moving through nitrogen provides the basis for the calculations.

As is well known, the calculation of the properties of a hypersonic blunt body flow field from first principles is extremely tedious because of the mixed elliptic-hyperbolic nature of the equations describing the flow. If the flow is a nonequilibrium flow, as in the case at hand, the complexity of the system of equations is further increased. For purposes of this study it was felt that sufficiently accurate estimates of the flow properties in the regions of interest could be made by utilizing published data on the properties of an unreacting flow field. The schlieren photographs suggest that the effect of the chemical reaction on the flow outside of the reacted gas boundary is small. The bow shock shape is similar to that for unreacting flows; the effect of the exo-
thermic serves primarily to displace the bow shock outward in accordance with the reduction in the average gas density near the surface of the ball.

The unreacting flow field properties upon which the flow field estimates are based are those given by Sedney and Kahl (Ref. 43). By use of a ballistic range instrumented with an interferometer, they obtained the density field about a Mach 5.0 sphere fired into a pure nitrogen atmosphere. The streamline and constant Mach number contours were then found by application of a finite difference scheme to solve the momentum and continuity equations for the case of constant $\gamma$ flow behind the shock. The figures presented in their paper for the streamline and constant Mach number contours in the subsonic and transonic regions were used to generate the data given in Figs. 20 and 66 for the flow field of a Mach 5.0 sphere fired into a stoichiometric hydrogen-air mixture.

The data was extended from the transonic region to a position in the vicinity of the wake neck. This mapped a sufficiently large area of the inviscid wake such that the pattern of the compression and entropy waves generated by the instability could be developed. The data was extended by application of the method of characteristics for the axisymmetric flow of a $\gamma = 1.4$ gas in a region of varying entropy. The boundaries of the region were taken to be the Mach 1.8 contour.
given in Ref. 43, the body surface, the shear layer and the bow shock wave. The specification of the shock wave as well as the body surface and shear layer is theoretically redundant; however, this greatly reduces the calculation time. Incompatibilities introduced by the redundancies were quite small and were smoothed during the calculation. The location and shape of the shear layers was based on those observed in the schlieren photographs; the bow shock was extended downstream according to the correlation, Eq. (1), which was found to be compatible with the experimentally observed contours.

The equations and techniques describing the application of the method of characteristics to rotational axisymmetric flows are described in standard texts (e.g., Ref. 44). The technique used herein was a semi-graphical one in which the streamlines and characteristic lines were extrapolated downstream from previously determined points. The properties at the intersection of two characteristic lines were found from the equation:

\[
\frac{d M^*}{M^* \tan \nu} = \frac{d \delta}{\delta \nu} + \frac{\sin \nu \sin \delta}{\sin (\delta + \nu)} \frac{d r}{r} \quad \text{IV-(1)}
\]

where I denotes right running waves

II denotes left running waves

\[ M^* = \frac{v}{c^*} \]

r = radial coordinate

\[ s/R = \text{dimensionless entropy} \]
and the other symbols have their usual meaning. The entropy distribution is determined by the entropy change across the bow shock and the knowledge that flow along a streamline is isentropic. The mesh size chosen was sufficiently small such that the errors introduced by the extrapolations were quite small. Errors were smoothed by keeping running plots of the significant parameters along each streamline as a function of the axial coordinate. Five mesh points were chosen on the initial Mach contour, a total of 60 points were involved in the calculation. The calculated characteristic net is shown in Fig. 66; the streamline and Mach contours are shown in Fig. 20. Since the flow is homoenergetic, the temperature is a unique function of the Mach number. All other properties of interest can be found from the known values of temperature (constant on a Mach contour) and entropy (constant on a streamline).

With the properties of the flow field known, the development of the wave front produced by infinitesimal disturbance can be traced by application of Huygen's Principle (Ref. 45). The result of such a construction for the case of an infinitesimal point generated at the bow shock apex is shown in Fig. 25. Also shown in the figure are the contours of the contact discontinuities produced by the interaction of the acoustic wave front with the bow shock. The time interval between successive positions of the wave fronts was arbitrarily chosen in this example. A further discussion of Fig. 25 is given in Section 2. A
similar construction for the case of a normal wave front on the axis of
symmetry is shown in Fig. 52; this figure is discussed in Section 4.

The theoretical location of the reaction front can be determined
if the properties in the unreacted gas region are known. In applying
the results of the flow field calculations described above, it is assumed
that the exothermic reaction has no effect on the properties in the
unreacted region.

As detailed in Appendix III, the reaction front location is deter-
mined by the attainment of a specified OH radical mol-fraction \((X_{OH})_1\). The initial value is determined by the strength of the bow shock wave
and the initial pressure. The value \((X_{OH})_1\) is a function of the local
gas properties and is independent of the history of the gas particles.
The radical growth is governed by the expression:

\[
\ln X_{OH} = \ln (X_{OH})_0 + \lambda'(p,T) t
\]

IV-(2)

For any time period that is sufficiently short such that the changes
in gas properties are small, the change in the radical mol fraction
is given by

\[
\Delta \ln (X_{OH}) = 2.302 \lambda'(p,T) \Delta t
\]

IV-(3)

Thus the location of the reaction front on a given streamline may be
found by summing the incremental changes in the radical mol fraction
along the streamline until

\[ \sum_{n=1}^{N} \lambda_n(P, T) \Delta t_n = \frac{\log_{10}(X_{OH})_i - \log_{10}(X_{OH})_o}{2.302} \]  

The value of \( \lambda' \) is found from Fig. 64 and the value of initial and final OH mol fractions are given in Fig. 63.

A comparison of the calculated and experimental reaction front boundaries for a Mach 5.0, 1/2-inch diameter sphere moving in a stoichiometric hydrogen-air mixture with an initial pressure of 0.395 atm. is given in Fig. 51.

The bow shock position assumed for the analytic calculations and the calculated reaction front position are indicated by solid lines; the experimental results are shown by dashed lines. The effect of the exothermic reaction is to displace the shock and reaction front away from the body surface. At points in the vicinity of the body shoulder, the induction zone reactions cease to be significant; the chain branching reaction is essentially frozen. At this point, the reaction front, or more precisely the reacted gas boundary, is assumed to become coincident with a streamline. As discussed below, the hot gas behind the boundary causes the reaction to continue to proceed but at an extremely slow rate.

For a given mixture and projectile velocity, the cessation of the branching reaction can either be a sudden freezing or a relatively gradual approach to zero depending on the level of the initial pressure.
Sudden freezing occurs when the second limit condition, Eq. III-(14), is satisfied. This corresponds to the time constant, $\lambda'$, effectively reaching zero as the temperature decreases due to expansion. If the pressure is high, the slope of the $\lambda'$ vs. $T$ curve is large (Fig. 64) such that the dimensional time constant, $\lambda'$, drops precipitously to near zero as the critical temperature is passed. On the other hand, if the initial pressure is relatively low, the $\lambda'$ vs. $T$ curve has a smaller slope and the dimensional time constant, which is proportional to pressure, gradually approaches zero.

For example, at the point where the particular streamline sketched in Fig. 51 intersects the bow shock, the following conditions exist: $\text{T} = 1368^\circ\text{K}$; $\text{p} = 9.95$ atm. From Fig. 62, the initial OH mol fraction generated by the initiation processes corresponds to $\log_{10}(X_{OH})_0 = -10.4$.

(The initiation time can be estimated from Fig. 63 to be of the order of 0.25 $\mu\text{sec}$.) The time interval chosen for the stepwise calculation of OH radical growth corresponded to a distance along the streamline of 0.025 diameters or approximately 0.3 $\mu\text{sec}$. During the first time interval, immediately behind the shock, the growth in $\log_1(X_{OH})$ is found to be 2.00. At the point where the reaction front and the streamline approach one another, point F in Fig. 51, the conditions are: $\text{T} = 1019^\circ\text{K}$; $\text{p} = 3.16$ atm. The OH radical mol fraction has grown to a value corresponding to $\log_{10}(X_{OH}) = -3.1$, while the value at the end of induction (Fig. 63) corresponds to $\log_{10}(X_{OH})_i = -2.9$. During the last time interval considered,
the growth in $\log_{10}(\chi_{OH})$ is 0.15. At this slow rate of growth the reaction front and the streamline approach one another very slowly and the reaction, for practical purposes, can be considered frozen. Thus, in this case, the cessation of the branching reaction corresponds to a gradual decrease to zero. For this example then, the second limit condition $2k_2' = k_0^C C_n$ is not encountered. Only at higher initial pressures than those used in the experiments will this limit be encountered. For these experiments, the second limit condition thus plays no part in determining the location of the flow regime boundaries.

Although the chain branching reactions become so slow in the outer streamtubes that the OH concentration marking the end of induction is not attained, the reacted gas boundary (reaction front) must continue to propagate outward into the unreacted gas because of transport effects. The photographs indicate the propagation rate is very small compared to the local gas velocity since the reacted gas boundary appears to have the same contour as a streamline. It should be noted that propagation rates due to diffusion effects can be quite rapid if the property gradients through the front are sufficiently large. Apparently, in the cases where the exothermic flow field about the sphere is stable or is experiencing the regular periodic instability, the magnitude of the property changes across the reaction front is sufficiently small such that the diffusive effects are small. Previous investigators measured the rate
of propagation of the reacted gas boundary in the far wake (Ref. 4) and found under their experimental conditions \((\text{H}_2\text{-Air } \phi = 1.0; p=0.55 \text{ atm})\) a propagation rate of 15 m/sec compared to the ball velocity of approximately 2000 m/sec. A propagation rate of this order is not significant in the time scale of interest.
TABLE I

EXPERIMENTAL DATA

NOTE: Sphere diameter = 0.500 in. unless noted otherwise

Abbreviations: \( P_T \) = Mixture total pressure in mm Hg
\( M \) = Mach number

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<tr>
<th>Regime</th>
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<td>P - Periodic</td>
<td>H - Horizontal Knife Edge</td>
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<td>T - Turbulent</td>
<td>Pr - Probe Data Recorded</td>
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<td>S - Stable</td>
<td>Q - Questionable Data</td>
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<td>W - Wake</td>
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<td>I - Irregular Pattern</td>
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<td>Wk - Weak Schlieren Effect</td>
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- D: Damping number
- W: Wavelength
- Q: quartz
- Ir: Iron
- P: Phase
- T: Temperature
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<td>5580 fps</td>
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<td>257</td>
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<td>&quot;</td>
<td>100</td>
<td>5050 fps</td>
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<tr>
<td>258</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5230 fps</td>
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<td>259</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5120 fps</td>
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182
<table>
<thead>
<tr>
<th>Run</th>
<th>Mixture</th>
<th>Ratio</th>
<th>$P_T$</th>
<th>Velocity</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>$\text{C}_2\text{H}_4\cdot\text{O}_2$</td>
<td>1:15</td>
<td>175</td>
<td>4790</td>
<td>4.46</td>
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<tr>
<td>61</td>
<td>&quot;</td>
<td>&quot;</td>
<td>125</td>
<td>6150</td>
<td>5.73</td>
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<tr>
<td>62</td>
<td>&quot;</td>
<td>&quot;</td>
<td>175</td>
<td>4990</td>
<td>4.65</td>
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<tr>
<td>63</td>
<td>&quot;</td>
<td>&quot;</td>
<td>150</td>
<td>5000</td>
<td>4.67</td>
</tr>
<tr>
<td>64</td>
<td>&quot;</td>
<td>&quot;</td>
<td>75</td>
<td>5615</td>
<td>5.24</td>
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<tr>
<td>65</td>
<td>&quot;</td>
<td>&quot;</td>
<td>175</td>
<td>5140</td>
<td>4.80</td>
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<tr>
<td>66</td>
<td>&quot;</td>
<td>&quot;</td>
<td>150</td>
<td>4820</td>
<td>4.49</td>
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<tr>
<td>67</td>
<td>&quot;</td>
<td>&quot;</td>
<td>175</td>
<td>5300</td>
<td>4.94</td>
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<tr>
<td>68</td>
<td>&quot;</td>
<td>&quot;</td>
<td>100</td>
<td>5260</td>
<td>4.90</td>
</tr>
<tr>
<td>69</td>
<td>&quot;</td>
<td>&quot;</td>
<td>75</td>
<td>5690</td>
<td>5.30</td>
</tr>
</tbody>
</table>
TABLE II

ASSUMED HYDROGEN-OXYGEN KINETIC MECHANISM

<table>
<thead>
<tr>
<th>Reaction Type</th>
<th>Reaction Equation</th>
<th>Specific Rate Constant</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiation Reactions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OH</td>
<td>$\text{OH} + \text{O}_2 \rightarrow \text{2OH}$</td>
<td>$k_{\text{OH}} = 1.0 \times 10^{-14}$ $T^{-1} \exp(-70,000/RT)$</td>
<td>37</td>
</tr>
<tr>
<td>H</td>
<td>$\text{H} + \text{M} \rightarrow \text{2H} + \text{M}$</td>
<td>$k'_{\text{H}} = 1.85 \times 10^{17}$ $T^{-1} \exp(-54,000/RT)$</td>
<td>38</td>
</tr>
<tr>
<td>O</td>
<td>$\text{O}_2 + \text{M} \rightarrow \text{2O} + \text{M}$</td>
<td>$k'_{\text{O}} = 5.8 \times 10^{16}$ $T^{-1} \exp(-60,600/RT)$</td>
<td>38</td>
</tr>
<tr>
<td><strong>Chain Carrying Reactions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>$\text{OH} + \text{H}_2 \rightarrow \text{H}_2 + \text{H}$</td>
<td>$k'_{\text{1}} = 6.3 \times 10^{13}$ $\exp(-5,900/RT)$</td>
<td>46</td>
</tr>
<tr>
<td><strong>Chain Branching Reactions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$\text{H} + \text{O}_2 \rightarrow \text{OH} + \text{O}$</td>
<td>$k'_{\text{2}} = 2.6 \times 10^{14}$ $\exp(-15,500/RT)$</td>
<td>Table III</td>
</tr>
<tr>
<td>III</td>
<td>$\text{O} + \text{H}_2 \rightarrow \text{OH} + \text{O}$</td>
<td>$k'_{\text{3}} = 2.5 \times 10^{12}$ $\exp(-7,700/RT)$</td>
<td>46</td>
</tr>
<tr>
<td><strong>Chain Breaking Reactions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>$\text{H} + \text{O}_2 + \text{M} \rightarrow \text{H}_2 \text{O}_2 + \text{M}$</td>
<td>$k'<em>{\text{6}} = 3.27 \times 10^{21} (x</em>{\text{H}<em>2} + 0.35x</em>{\text{O}<em>2} + 0.43x</em>{\text{N}<em>2} + 0.2x</em>{\text{Ar}}) T^{-1.92}$</td>
<td>40</td>
</tr>
</tbody>
</table>

Units:

- Concentration - mol cm$^{-3}$ sec$^{-1}$
- Temperature - °K
### TABLE III

**KINETIC DATA FOR REACTION II**

II: $\text{H} + \text{O}_2 \rightarrow \text{OH} + \text{O}$

Units: $\text{cm}^3 \text{ mol}^{-1} \text{ sec}^{-1}$

<table>
<thead>
<tr>
<th>Source</th>
<th>Specific Reaction Rate, $k'_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schott and Kinsey, Ref. 28</td>
<td>$3 \times 10^{14} \exp\left[-(17.5 \pm 0.3)\times10^3/RT\right]$</td>
</tr>
<tr>
<td>Duff, Ref. 37</td>
<td>$4 \times 10^{14} \exp(-18,000/RT)$</td>
</tr>
<tr>
<td>Kaufman and Del Greco, Ref. 46</td>
<td>$1.7 \times 10^{16} \exp(-18,100/RT)$</td>
</tr>
<tr>
<td>Baldwin, Ref. 47</td>
<td>$(2.14 \pm 0.3)\times10^4 \exp\left[-(16.6 \pm 0.8)\times10^3/RT\right]$</td>
</tr>
<tr>
<td>Momtchiloff, et al., Ref. 39</td>
<td>$5.64 \times 10^{13} \exp(-15,100/RT)$</td>
</tr>
<tr>
<td>These experiments</td>
<td>$2.6 \times 10^{14} \exp(-15,500/RT)$</td>
</tr>
</tbody>
</table>
FIGURE 1

REGULAR PERIODIC INSTABILITY

Run No. 132: \( \text{C}_2\text{H}_2: \text{O}_2: \text{Ar} = 1:2.5:12.5 \)

Mach No.: 4.80

Pressure: 0.132 atm
FIGURE 2

OVERALL VIEW OF BALLISTIC RANGE
FIGURE 3

CLOSEUP VIEW OF SCHLIEREN SYSTEM COMPONENTS
FIGURE 4

CHARACTERISTICS OF EXOTHERMIC HYPERSONIC BLUNT-BODY FLOW FIELDS

Compression Waves

Entropy Waves

Bow Shock

Corrugated Reaction Zone Boundary

Wake Shock

Turbulent Viscous Wake

Shear Layers

Smooth Reaction Zone Boundary

Recompression Shock
FIGURE 5

INTERMITTENT STABLE/PERIODIC FLOW

Run No. 87: H₂:O₂:N₂  2:1:3.76

Mach No.: 5.10

Pressure: 0.396 atm
FIGURE 6

BOW SHOCK PROFILE CORRELATION

Experiment - Run No. 87 (Fig. 5)

Correlation: \[ \frac{r}{D} = 1.14 \left( \frac{x}{D} \right)^{0.49} \]
FIGURE 7
OVERTAKING DETONATION WAVE

Run No. 12: $\text{C}_2\text{H}_2: \text{O}_2 : 1:8$

Mach No.: 5.94

Pressure: 0.132 atm
FIGURE 8

TURBULENT REGIME

Run No. 128: $\text{C}_2\text{H}_2: \text{O}_2: \text{Ar} = 1:2.5:12.5$

Mach No.: 5.49

Pressure: 0.264 atm
FIGURE 9

ANOMALOUS HIGH FREQUENCY OSCILLATION

Run No. 85: \[ \text{H}_2: \text{O}_2: \text{N}_2: 2:1:3.76 \]
Mach No.: 4.40
Pressure: 0.396 atm
FIGURE 10

VERY HIGH FREQUENCY PERIODIC INSTABILITY

Run No. 108: \( \text{H}_2: \text{O}_2: \text{Ar} : 2:1:3.76 \)

Mach No.: 5.02

Pressure: 0.396 atm
FIGURE 11

WAKE COMBUSTION REGIME

Run No. 144: $C_2H_2:O_2 : 1:15$

Mach No.: 4.05

Pressure: 0.165 atm
FIGURE 12

WAKE COMBUSTION REGIME

Run No. 84: \[\text{H}_2: \text{O}_2: \text{N}_2: 2:1:3.76\]

Mach No.: 4.19

Pressure: 0.264 atm
FIGURE 13
CORRUGATED REACTED GAS BOUNDARY CONFIGURATION

Run No. 88: \( \text{H}_2: \text{O}_2: \text{N}_2: 2:1:3.76 \)

Mach No.: 4.67

Pressure: 0.396 atm
FIGURE 14

STABLE COMBUSTION REGIME

Run No. 106: \( \text{H}_2 : \text{O}_2 : \text{Ar} : 2 : 1 : 3.76 \)

Mach No.: 5.26

Pressure: 0.396 atm
FIGURE 15

IONIZATION PROBE - ZERO SIGNAL

Run No. 151:  \( \text{C}_2\text{H}_2: \text{O}_2 : 1:15 \)

Mach No.:  4.1

Pressure:  0.198 atm

Normal component of bow shock Mach No. at probe intersection point = 3.2

Schlieren Phototube
(40.0 \( \mu \text{sec/cm} \))

Probe (10.4 \( \mu \text{sec/cm} \))

Distance from intersection point to probe - lower trace (diameters)
FIGURE 16

IONIZATION PROBE - DELAYED SIGNAL

Run No. 139: $\ce{C_2H_2:O_2} : 1:15$

Mach No.: 5.01

Pressure: 0.198 atm

Normal component of bow shock Mach No. at probe intersection point = 3.5

Time between photographs: 1.6 $\mu$sec

Schlieren phototube

(40.0 $\mu$sec/cm)

Probe (10.4 $\mu$sec/cm - sweep begins when first Photograph is exposed)

Distance from intersection point to probe - lower trace (diameters)

Spark phototube

(0.52 $\mu$sec/cm)

Probe (53 $\mu$sec/cm - sweep begins when second photograph exposed)
FIGURE 17

IONIZATION PROBE - PROMPT SIGNAL

Run No. 157: $O_2:H_2:O_2$: 1:15

Mach No.: 5.16

Pressure: 0.198 atm

Normal component of bow shock Mach No. at probe intersection point = 3.6

Time between photographs: 11.9 $\mu$sec

Schlieren phototube (40.3 $\mu$sec/cm)

Spark phototube (5.3 $\mu$sec/cm)

Probe (41 $\mu$sec/cm)
Sweep begins 198 $\mu$sec before first photo exposed.

Probe (10.6 $\mu$sec/cm)
Sweep begins when second photo exposed.
FIGURE 18

DUAL SCHLIEREN MEASUREMENT OF REACTED GAS BOUNDARY STREAMWISE VELOCITY

Note: Velocity measured relative to laboratory observer

Distance From Shock Apex - $x/D$

Velocity - $2\tilde{v}$ - fps
FIGURE 19

SCHLIEREN PHOTOTUBE MEASUREMENT OF REACTED GAS BOUNDARY STREAMWISE VELOCITY

Run No. 142: $\text{C}_2\text{H}_2\text{O}_2 : 1:15$

Mach No.: 4.58

Pressure: 0.198 atm

Sweep rate: 51 $\mu$sec/cm

Sweep rate: 21 $\mu$sec/cm

Phototube Output
FIGURE 20

STREAMLINES AND CONSTANT MACH NUMBER CONTOURS

Note: $M_0 = 5.0$
$\gamma = 1.4$
FIGURE 21

REGULAR PERIODIC INSTABILITY

Run No. 86: $H_2:O_2:N_2: 2:1:3.76$

Mach No.: 4.40

Pressure: 0.396 atm
FIGURE 22

EXISTENCE OF STRIATIONS IN NOSE REGION

Run No. 111: $\text{H}_2:\text{O}_2:\text{Ar}: 2:1:3.76$

Mach No.: 4.50

Pressure: 0.396 atm
FIGURE 23
WAVEFORM DISTORTION AT HIGH PRESSURE

Run No. 248: $\text{C}_2\text{H}_2:\text{O}_2 : 1:15$

Mach No.: 4.70

Pressure: 0.264 atm
### FIGURE 24

**PROPAGATION RATE OF COMPRESSION WAVES IN INVISCID WAKE**

*Note: Data from Run No. 157*

<table>
<thead>
<tr>
<th>Measured Values</th>
<th>Calculated Values</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0 = 5540 \text{ fps}$</td>
<td>$v_f = 5180 \text{ fps}$</td>
<td>$a_{\text{meas}} = 1400 \text{ fps}$</td>
</tr>
<tr>
<td>$\Delta \xi = 0.77 \text{ inches}$</td>
<td>$\delta_1 = 13.3^\circ$</td>
<td>$a_{\text{calc}} = 1375 \text{ fps}$</td>
</tr>
<tr>
<td>$\Delta t = 11.9 \mu \text{sec}$</td>
<td>$a_1 = 1375 \text{ fps}$</td>
<td></td>
</tr>
<tr>
<td>$\theta_\infty = 22.3^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_\rho = 45^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{co}} = 72^\circ$</td>
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<td></td>
</tr>
</tbody>
</table>
FIGURE 25

COMPRESSION WAVE AND ENTROPY WAVE CONTOURS

Note: $M_0 = 5.0$
$\gamma = 1.4$

Entropy Wave

Compression Wave
FIGURE 26

REGULAR PERIODIC INSTABILITY

Run No. 62: \( \text{C}_2\text{H}_2: \text{O}_2 : 1:15 \)

Mach No.: 4.90

Pressure: 0.198 atm
FIGURE 27
COMPRESSION WAVE - BOW SHOCK INTERACTION

Note: Numerical data corresponds to Run No. 154, $\gamma = 1.5$

(a) Coordinates Fixed to Ball
(b) Coordinates Fixed to Intersection Point

(c) Coordinates Fixed to Intersection Point, $M_0'$ Parallel to $x$-Axis
FIGURE 28

FLOW FIELD REGIMES  ACETYLENE-OXYGEN  $\Phi = 0.167$

1/2" Diameter Spheres

Pressure - $p$ - atm

Mach Number - $M$

Chapman-Jouguet Mach Number

Viscous Wake

Turbulent

Periodic

(Short $\lambda$)

(1ng $\lambda$)

Stable
FIGURE 29

FLOW FIELD REGIMES
HYDROGEN-AIR $\phi = 1.0$

Note: 1/2 diameter spheres

Viscous Wake
Periodic
Chapman-Jouguet Mach No.
Stable
FIGURE 30
LONG WAVELENGTH REGULAR PERIODIC INSTABILITY

Run No. 260: \( \text{C}_2\text{H}_2: \text{O}_2 : 1:15 \)

Mach No.: 4.46

Pressure: 0.230 atm
FIGURE 31

TURBULENT COMBUSTION NEAR REGIME BOUNDARY

Run No. 129: $C_2H_2:O_2:Ar: 1:2.5:12.5$

Mach No.: 5.59

Pressure: 0.198 atm
FIGURE 32

INTERMITTENT TURBULENT/PERIODIC FLOW

Run No. 254: \( \text{C}_2\text{H}_2 : \text{O}_2 = 1:15 \)

Mach No.: 5.42

Pressure: 0.264 atm
FIGURE 33

PERIODIC INSTABILITY NEAR REGIME BOUNDARY

Run No. 130: $C_2H_2:O_2:Ar: 1:2.5:12.5$
Mach No.: 5.56
Pressure: 0.132 atm
FIGURE 34

TURBULENT REGIME AT HIGH VELOCITY

Run No. 216: \( \text{C}_2\text{H}_2: \text{O}_2 : 1:15 \)

Mach No.: 6.65

Pressure: 0.198 atm
FIGURE 35

EFFECT OF DIAMETER ON REGIME BOUNDARIES
ACETYLENE-OXYGEN $\Phi = 0.167$

Note: 
- $\bigcirc$ - 3/8" Dia Sphere
- $\boxdot$ - 1/4" Dia Sphere

$D = \frac{2\pi d}{R} = 0.167$

Pressure x Diameter (in.)

0.32
0.24
0.16
0.08
0
0.500
0.250
0.100
0.050
0.000

Mach Number - $M$

3.2 3.6 4.0 4.4 4.8 5.2 5.6 6.0 6.4

Viscous Wake
Periodic
Turbulent
Stable
FIGURE 36

EFFECT OF DIAMETER ON REGIME BOUNDARIES
HYDROGEN-AIR \( \Phi = 1.0 \)

Note: Ruegg and Dorsey (Ref. 2)
20 mm dia Nylon Spheres

Behrens, Struth and Wecken (Ref. 4)
9 mm dia Plastic Spheres

Viscous Wake

Turbulent

Periodic

Stable
FIGURE 37

EXPERIMENTAL INSTABILITY WAVELENGTH
ACETYLENE-OXYGEN $\phi = 0.167$

Pressure:

- $0.098\text{ atm (75 mm Hg)}$
- $0.132$ (100)
- $0.165$ (125)
- $0.198$ (150)
- $0.230$ (175)
- $0.264$ (200)
FIGURE 38

EXPERIMENTAL INSTABILITY WAVELENGTH
HYDROGEN-AIR $\Phi = 1.0$

Note: Pressure: 0.264 atm (200 mm Hg), 0.329 (250), 0.395 (300)

Mach Number - M
FIGURE 39

SCHLIEREN EFFECT PRODUCED BY HORIZONTAL KNIFE EDGE

Run No. 161: \( \text{C}_2\text{H}_2:0_2 : 1:15 \)

Mach No.: 5.16

Pressure: 0.198 atm
FIGURE 40
DOUBLE-VALUED BOUNDARY CORRUGATION WAVELENGTH

Run No. 263: $\text{C}_2\text{H}_2:0_2 : 1:15$

Mach No.: 4.67

Pressure: 0.198 atm
FIGURE 41

PERIODIC INSTABILITY EXHIBITING DOUBLE SET OF ENTROPY WAVES

Run No. 262: $\text{C}_2\text{H}_2:\text{O}_2 : 1:15$

Mach No.: 4.65

Pressure: 0.230 atm
FIGURE 42

EFFECT OF BALL DIAMETER ON INSTABILITY WAVELENGTH

Note: Acetylene - Oxygen $\Phi = 0.167$
- $1/2''$ Diameter $p = 0.132$ atm
- $1/4''$ Diameter $p = 0.264$ atm
FIGURE 43
ASSUMED STAGNATION STREAMLINE VELOCITY DISTRIBUTION

Sphere Surface  Reaction Front  Bow Shock

\( v = \frac{\Delta - X}{\Delta'} v_0 \)

(a) Zero Heat Release

\( v = \frac{\Delta}{\Delta'} (1 - \frac{X}{\Delta'}) v_0 \)

(b) Finite Heat Release
THEORETICAL INSTABILITY WAVELENGTHS

Note: Acetylene-Oxygen $\Phi = 0.167$

$P = 0.132 \text{ atm}$

$\log_{10}(\text{OD})^2 = -9.88 + 3.240 \frac{T}{\text{sec} \cdot \text{atm}}$

$\text{OD} = \text{mass/iter}$

$\text{sec} = \text{sec}$

$T = \circ \text{K}$
FIGURE 45

COMPRESSION WAVE - SHOCK WAVE INTERACTION

Coordinates Fixed With Respect To Undisturbed Bow Shock

Note: \( \tau \) - Induction time for stoichiometric \( \text{H}_2\)-air mixture according to correlation: 

\[
\tau = 9.70 - \frac{3.59 \rho}{\gamma}
\]

Mach Number of Incident Compression Wave = 1.05

\( \gamma = 1.4 \)

P - Compression Wave
BS - Bow Shock
R - Rarefaction Wave
CD - Contact Discontinuity

(a) Before Interaction

<table>
<thead>
<tr>
<th>Region</th>
<th>( M )</th>
<th>( \frac{v}{\gamma \rho_0} )</th>
<th>( \frac{P}{\rho_0} )</th>
<th>( \frac{T}{\rho_0} )</th>
<th>( \frac{\sqrt{\gamma}}{\rho_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.0</td>
<td>-5.0</td>
<td>1.00</td>
<td>1.00</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>0.327</td>
<td>-0.80</td>
<td>32.48</td>
<td>5.99</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>0.415</td>
<td>-1.0</td>
<td>29.00</td>
<td>5.80</td>
<td>1.00</td>
</tr>
<tr>
<td>3A</td>
<td>0.312</td>
<td>-0.79</td>
<td>32.10</td>
<td>6.33</td>
<td>0.68</td>
</tr>
<tr>
<td>3B</td>
<td>0.321</td>
<td>-0.79</td>
<td>32.10</td>
<td>5.97</td>
<td>0.82</td>
</tr>
</tbody>
</table>

(b) After Interaction
FIGURE 46

x-t DIAGRAM FOR THE P-CD INSTABILITY MODEL

Note: BS = Bow Shock
RF = Reaction Front
CD = Contact Discontinuity
P = Compression Wave
R = Rarefaction Wave
Z = Entropy Zone
FIGURE 47

EFFECT OF REDUCED INDUCTION TIME ON ENERGY RELEASE RATE

Note: BS = Bow Shock
RF = Reaction Front
CD = Contact Discontinuity
P = Compression Wave
R = Rarefaction Wave
A contact discontinuity followed by a zone of decreasing entropy approaches the reaction front.

The hot gas behind the CD reacts, creating a reaction front upstream of the original front, thereby doubling the reaction rate and generating compression waves which propagate upstream and downstream.

The original reaction front is extinguished, halving the reaction rate and generating rarefaction waves which propagate upstream and downstream. The zone of decreasing entropy begins to penetrate the upstream reaction front.

The compression wave generated in Step (2) interacts with the bow shock producing a weak reflected rarefaction wave and generating a contact discontinuity. The bow shock is strengthened.

The rarefaction wave generated in Step (3) penetrates the bow shock, weakening it and generating a zone of increasing entropy and a chain of weak reflected compression waves.

The weak reflected rarefaction and compression waves have a negligible effect on the reaction front. The next cycle begins.
FIGURE 49

COMPARISON OF EXPERIMENTAL AND THEORETICAL INSTABILITY WAVELENGTHS.
ACETYLENE-OXYGEN $\phi = 0.167$

Note: Theory, P-CD Model
        Experiment (See Fig. 37)

Mach Number - M

Wavelength/diameter - $\frac{\lambda}{D}$

- $P = 0.263$ atm (200 mm Hg)
- 0.197 (150)
- 0.132 (100)
- 0.099 (75)
- 150
- 100
FIGURE 50

CORRELATION OF INSTABILITY WAVELENGTH WITH REACTION ZONE WIDTH

Note:
Mixture:
- C₂H₂:O₂  \( \Phi = 0.167 \)
- Hydrogen:Air  \( \Phi = 1.0 \)
Pressure:
- 0.099 atm (75 mm Hg)
- 0.132 atm (100)
- 0.165 atm (125)
- 0.197 atm (150)
- 0.230 atm (175)
- 0.264 atm (200)
- 0.239 atm (250)
- 0.395 atm (300)
Diameter:
- 1/2 inch
- 1/4 inch

Reaction Zone Width/Ball Diameter - W/D

Wavelength/Diameter - \( \lambda/D \)
FIGURE 51

COMPARISON OF ANALYTICAL AND EXPERIMENTAL REACTION FRONT PROFILES

Note: Hydrogen-Air $\Phi = 1.0$
$p = 0.396$ atm
$M_a = 5.0$

Experimental (Run No. 87, Fig. 21)
Analytical (No Heat Release)
FIGURE 52

DEVELOPMENT OF CORRUGATED REACTED GAS BOUNDARY

- Contact Discontinuity
- Compression Wave
- Bow Shock
- Stable Reacted Gas Boundary
- Reduced Boundary
- Schlieren silhouette of reacted gas boundary
FIGURE 53

COMPARISON OF EXPERIMENTAL AND THEORETICAL INSTABILITY WAVELENGTHS

HYDROGEN-AIR $\Phi = 1.0$

Note: $\log_e \left( \frac{[O_2] I}{[O_2]_{max}} \right) = -9.90 + \frac{2300}{T} - \frac{15}{T} - 0.004$

$p = 0.264$ atm (200 mm Hg)

0.329 (250)

0.395 (300)

$M = \frac{V}{a}$

$p = 0.264$ atm

0.329

0.395

Mach Number - $M$

Instability Wavelength - $\lambda_D$
FIGURE 54

HYDROGEN-OXYGEN INDUCTION TIME CORRELATIONS

\[ \log_{10} \left( \frac{t}{1 \text{ atm}} \right) \]

\[ \frac{1}{10^3} \left( \frac{1}{T} \right) \]

- Miyama and Takeyama
- Ruegg and Dorsey
  - \( p = 10 \text{ atm} \)
  - \( p = 1 \text{ atm} \)
- These Experiments
- Strehlow and Cohen
- Schott and Kinsey
- Mullaney, et al.
- White and Moore
FIGURE 55

SCHLIEREN SYSTEM PRINCIPLES

(a) Arrangement of a Double Pass Schlieren System

(b) Ray Deflection Producing Darkened Image

(c) Refraction Due to a Wedge
FIGURE 56

SCHLIEREN CHARACTERISTICS OF BOW SHOCK WAVE

Note: Vertical Knife Edge

(a) Viewing Plane Image

(b) Plane \( y = y_0 \)

(c) Integrated Optical Path

(d) Relative Brightness
FIGURE 57

SCHLIEREN CHARACTERISTICS OF CORRUGATED
REACTED GAS BOUNDARY

(a) Viewing Plane Image

(b) Plane $y = y_0$

(c) Integrated Light Path

(d) Relative Brightness
FIGURE 58

SCHLIEREN CHARACTERISTICS OF COMPRESSION AND ENTROPY WAVES IN THE INVISCID WAKE

(a) Viewing Plane Image

(b) Plane

(c) Density Profile on

(d) Integrated Optical Path

(e) Relative Brightness
REFLECTED WAVE CALCULATION

\( M_0' = 2.65 \quad \theta_0' = 0 \)

\( M_1' = 1.0122 \quad \theta_1' = 31.2^\circ \)

\( M_2' = 1.005 \)

\( \Theta_1 = 62.8^\circ \)
\( \frac{P_1}{P_0} = 6.32 \)
\( \frac{\lambda_1}{\lambda_0} = 3.16 \)
\( \frac{\rho_1}{\rho_0} = 1.414 \)

\( \Theta_2 = 65.7^\circ \)
\( \frac{(\Delta P)}{P_3} = 0.01167 \)
\( \frac{(\Delta P)}{P_4} = 0.00831 \)

\( \Theta_3 = 62.7^\circ \)
\( \frac{(\Delta P)}{P_3} = -0.00080 \)
\( \frac{(\Delta P)}{P_4} = -0.00057 \)
WAVE INTERACTION GEOMETRY

--- Experimental - Run No. 154
--- Analytical

\[ M_0 = 5.30 \]
\[ C_{2H_2:O_2} \Phi = 0.167 \]

\( \Theta_{ca} \) calculated assuming \( \theta_i, \theta_g \) given

\( \frac{p_2}{p_1} = 1.0 \) (acoustic wave)

Distance Downstream of Shock Apex \( \frac{X}{D} \)
FIGURE 61

SHOCK REFLECTION COEFFICIENT

Note: Data from Ref. 33
\( \gamma = 1.4 \)
FIGURE 62

OH RADICAL GROWTH DURING INITIATION PERIOD

Note: Hydrogen-Air $\bar{\Phi} = 1.0$

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Full solution

Approximation

$M = 5.0$
$T = 1611^\circ K$
$P = 10$ atm

$M = 4.0$
$T = 1157^\circ K$
$P = 10$ atm
FIGURE 63

INITIAL AND FINAL OH MOL FRACTIONS

Note: Hydrogen-Air $\bar{\Phi} = 1.0$
FIGURE 64

INDUCTION PERIOD TIME CONSTANT
HYDROGEN-AIR $\Phi = 1.0$

$$X_{ON} = (X_{ON})_0 e^{\lambda' t}$$

$$\lambda' = \frac{\lambda'}{k_3' C_{H_2}}$$

$$k_3' C_{H_2} = \frac{10^6.96 - 4.69}{P} \frac{P}{T}$$

$$T = 6K \times 10^{-3}$$

$P = 1 \text{ atm}$

1. $P = 1 \text{ atm}$
2. $P = 3$
3. $P = 7$
4. $P = 10$
FIGURE 65

OH RADICAL GROWTH DURING INDUCTION PERIOD

Note: Hydrogen-Air $\Phi = 1.0$
Conditions correspond to those behind a normal shock with $p_{so} = 0.396$ atm
End of induction period based on equilibrium of Reaction II
II: $H + O_2 \rightarrow OH + O$

Log$_{10}$ (OH Mol Fraction - $x_{OH}$)

Time $- t$ ($\mu$sec)

Shock Mach No. - $M = 4.6$
FIGURE 66

INDUCTION TIMES BEHIND NORMAL SHOCK WAVES

![Graph showing induction times behind normal shock waves. The graph plots Mach number (M) on the x-axis and induction time (T) in microseconds (μsec) on the y-axis. There are three curves representing different gases: C$_2$H$_2$:O$_2$, H$_2$, and Air. The curves indicate the induction times for each gas at specific Mach numbers and pressures.](image-url)
FIGURE 67

CHARACTERISTIC NET

Note: $M_0 = 5.0$
$\gamma = 1.4$
BIOGRAPHICAL SKETCH

John B. McVey was born in Allentown, Pennsylvania on March 29, 1933 and received his secondary school education in the Catasauqua, Pennsylvania public school system. He entered Lehigh University, Bethlehem, Pennsylvania in September 1951 and received a B.S. degree in Mechanical Engineering in June 1955 and an M.S. degree in Mechanical Engineering in June 1956. He was then employed by the United Aircraft Corporation Research Laboratories where he performed analytical studies of the performance of hypersonic air-breathing engines and of aircraft environmental control systems.

He entered M.I.T. in September, 1962 and as a Research Assistant in the Department of Mechanical Engineering performed a study of the effects of electric and magnetic fields on the characteristics of laminar flames. The doctoral thesis program reported herein was begun in October 1963.