Towards Intelligent Structures:
Active Control of Buckling

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Abstract

The buckling of compressively-loaded members is one of the most important factors limiting the overall strength and stability of a structure. I have developed novel techniques for using active control to wiggle a structural element in such a way that buckling is prevented. I present the results of analysis, simulation, and experimentation to show that buckling can be prevented through computer-controlled adjustment of dynamical behavior.

I have constructed a small-scale railroad-style truss bridge that contains compressive members that actively resist buckling through the use of piezo-electric actuators. I have also constructed a prototype actively controlled column in which the control forces are applied by tendons, as well as a composite steel column that incorporates piezo-ceramic actuators that are used to counteract buckling. Active control of buckling allows this composite column to support 5.6 times more load than would otherwise be possible.

These techniques promise to lead to intelligent physical structures that are both stronger and lighter than would otherwise be possible.

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Professor of Electrical Engineering

Dedicated to Dorit
with Love

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Chapter 1:

Introduction

I show that it is possible to increase the load-bearing strength of a structure by incorporating structural elements that actively resist buckling through careful adjustment of their dynamical behavior. Just as people can support significant weight by balancing, compressively-loaded structural elements can be made stronger by wiggling them so that they cannot decide which way to fall down. I have developed a variety of implementation and control strategies, and have conducted analysis, simulation, and experimentation to show that active control of buckling can be achieved. I have constructed a railroad-style truss bridge that is composed of compressive members that actively resist buckling through the use of piezo-electric actuators. I have also constructed a prototype actively controlled column in which the control forces are applied by tendons, as well as a prototype composite column that is stabilized against buckling through the use of piezo-electric actuators. Active control of buckling allows this composite column to support a factor of 5.6 times more load than would otherwise be possible.

Motivation

Structures built by man bear little resemblance to structures that occur in nature. Man-made structures attain their strength through the use of materials that provide rigidity, and through the use of clever architectural topologies that act to increase structural stability. Nature, on the other hand, builds structures (such as humans) that contain many flexible joints, bend fairly easily, and are not even secured by foundations. Yet these naturally occurring structures manage to stand up, support loads, and move around with grace and precision. Even considering the huge advances that man has made in transportation technology, we still fall far short of building anything with the performance characteristics of a cheetah (or even a jellyfish!).

The key advantage that nature has over us is its use of inherently unstable structures, that stabilize themselves by actively modifying their dynamic behavior. These "intelligent structures" incorporate an embedded computing system that senses their current dynamical state, predicts likely future states, and applies controlling forces that alter the structure's behavior. The advent of "smart" materials and embedded computation technology holds the promise of allowing mankind to create our own intelligent structures.

This dissertation is intended to move us a step closer to a world in which computation is embedded as an integral part of the objects around us. Research already underway ([13],[14],[16]) promises to create "smart paint", containing sensors and actuators, that can be applied to an object to alter its behavior or characteristics. In the long run, it may even be possible to "stir in" a smart paint-like substance when mixing up a batch of steel, or to embed computational elements within a structural member [39], [26]. In the future, the behavior of a material will be determined not only by its composition, but also by what dynamic behaviors and other structural properties are programmed into it.

Buckling is an attractive domain in which to study the impact of embedded computation both because buckling is an important phenomenon that is a limiting factor in many of the structures designed today, and because the very nature of the buckling phenomenon provides opportunities for exchanging computational control for material strength, mass, and distribution. I show that even using the primitive embedded computation technology available today, active control can significantly increase the load-bearing strength of a structure. This work both brings into focus the long-term potential of embedded computation to make it possible to build structures that are stronger and lighter than would otherwise be possible, and suggests potential applications that would benefit from the use of the embedded computation technology available today.

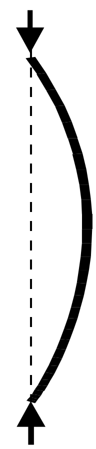

Figure 1.1: Shape of the first mode of a column with pinned ends. The shape of a column can be expressed as a combination of mode shapes. For small deflections, the modes are independent of one another, each moving at its own characteristic frequency. From a structural engineering point of view, the first mode is the most important, since buckling in the first mode occurs at a smaller load than does buckling in higher modes. The particular shape of the modes depends on how the column is supported and on the material and geometrical properties of the column

Origins of Buckling Behavior

Buckling[1] results from dynamical instability: below a certain critical load, an axially-loaded column has enough stiffness to restore itself to its original shape when subjected to small perturbations. Above that critical load, the column can no longer resist perturbations and becomes unstable, leading to an exponential increase in deflection over time that causes the column to fail. Although there are many shapes, known as modes, in which columns can buckle, buckling in the first mode is the factor that limits how much load can be applied to a column (see Figure 1.1), since buckling in the first mode occurs at a lower load than does buckling in any other mode. The load at which a column buckles in the

[1]In this dissertation, the word **buckling** refers to in-plane buckling of a structural element. Such 3-dimensional effects as torsional buckling and simultaneous buckling in two different planes are beyond the scope of this document.

first mode, known as the *critical buckling load*, is related to the geometric and material properties of the column by the following formula:[2,3]

$$P_{\text{critical}} = \alpha \frac{\pi^2 E I}{L^2}$$

The critical buckling load ($P_{critical}$) is a force that depends on the length of the column (L) , the Young's modulus of the material[4] (E), the way the column is supported[5] (α), and the geometry of the column as measured by the moment of inertia[6] (I). In modern engineering practice, structures are designed such that the load applied to each compressive member is always much less than the critical buckling load. To satisfy this constraint, the equation for the critical buckling load leaves a designer with the following set of choices:

1. Limit the maximum load that will ever be applied to the member.

2. Increase the moment of inertia by using more material to make a wider beam, or by using a specially-shaped element such as an I-beam.

3. Limit the length of the member.

4. Use a material that has a higher Young's modulus; i.e., a stiffer material.

Embedded computation coupled with active control makes it possible to load a column beyond the load at which buckling begins to occur. The active control system prevents structural failure by detecting the onset of buckling and applying control forces that

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[2]The formula shown is for an axially loaded column having pinned ends (i.e. simply supported), and neglects any loading associated with gravity acting on the column material.

[3]See Chapter 3 for a description of the mathematics that lead to this formula for the critical buckling load, as well as a description of the higher buckling modes.

[4]Young's modulus is a numerical constant that describes the “springiness” of a material. For instance, spring steel having Young's modulus E = 206 GN/m^2 is about 3 times springier than aluminum, which has Young's modulus E = 70.9 GN/m^2.

[5]α is a factor determined by the way the column is supported. For a column supported by pins at its ends, $\alpha = 1$. For information about other types of end supports, see [40], page 670-681 or [10] page 590.

[6]For a column of rectangular cross-section, the moment of inertia **I** = ($^1/_{12}$) (width) (thickness^3).

counteract the buckling motion. Active control allows designers to circumvent the constraints normally imposed by buckling, allowing computation to replace material strength and to overcome architectural limitations.

In theory, a perfectly straight, unperturbed column will never buckle, and can support loads well above the critical buckling load. In practice, columns are never perfectly straight, and there is always some sort of disturbance going on. The key idea exploited in this dissertation is that if the onset of buckling is detected very early, while the column is still nearly straight, the column will still be supporting almost all of the applied load. Thus the control forces do not need to support the applied load directly, but merely need to oppose disturbances and compensate for material imperfections.

Dissertation Overview

Novel results introduced in this dissertation include:

1. Real world evidence that active control of buckling can be achieved, provided by a composite column that actively resists buckling through the use of embedded piezo-ceramic actuators. This column is capable of supporting **5.6** times more load than would otherwise be possible, through active stabilization of the first and the second buckling modes.

2. Demonstration of the first compound structure composed of compressive members that actively resist buckling. This structure, a railroad-style truss bridge, consists of two active compressive members which are controlled independently of one another. Experimental results show that the interaction between the two active members in the truss structure is minimal.

3. Demonstration of a column stabilized against buckling through the use of variable-tension tendons connected via a yard.

4. Novel actuation strategies for cancellation of buckling motion, including: An active pin joint that simulates a fixed end support; a driven inertial mass; various tendon-based approaches; distributed bending moments; and the use of external combustion (explosives).

5. Dynamic analysis and finite-element simulations illustrating the tradeoffs involved in choosing material and actuator technologies for use in actively stabilized compressive members.

6. Development of and experimentation with a variety of control strategies for active control of buckling.

Chapter 2: Related Work

Part of the inspiration for my work is a wonderful book written by William Zuk in 1970, entitled Kinetic Architecture [43]. In his book, Zuk argues that the same forces that drove evolution towards creating active "kinetic" structures, will eventually lead man down the same path. In particular, Zuk proposes that it is theoretically possible to actively hold a column in its third buckling mode, thereby increasing its strength by a factor of 9. Zuk goes on to speculate that one day it may be possible to build an entire city on top of an existing city, supported by actively stabilized columns.[7]

Perhaps the earliest work related to active control of buckling is the 1968 Ph.D. thesis of Raymond Jefferis (1968) [20]. Jefferis was interested in controlling the deformation of steel plates as they cool during the manufacturing process. He experimented with the use of externally mounted electromagnets to stabilize the first buckling mode of a column. Since that time, extensive work has been performed on active control of vibration, but until recently, little attention has been given to the possibility of controlling buckling for the purpose of increasing the load-bearing strength of a structure.

Vibration Control

Most work on active structural control has focused on control of vibration ([23],[33]). Notable applications of active vibration control include alignment of optical systems, damping of wind-induced vibration of tall buildings, and damping of thermally-induced vibration of space structures. Control of vibration is closely related to control of buckling

[7] [43], page 41.

in that a vibration controller also seeks to reduce the amplitude of motion of the fundamental modes of a beam or column. The key difference is that a system undergoing vibrations is inherently stable, with active control being used primarily to remove energy from the system over time, so as to reduce the amplitude of oscillations. In contrast, a system undergoing buckling is inherently unstable, and will collapse rather than oscillate. As a consequence of this instability, the response of the control system to the onset of buckling must be to add energy to the system, not only to counteract motions induced by external perturbations, but also to forcibly restore the beam to the equilibrium position. Thus buckling control systems must deal with potentially unstable violent behaviors, such as *snap-through* (see Chapter 5), that do not arise when controlling vibration.

Some of the actuation techniques suitable for use in vibration control may also be applied to the control of buckling. One common technique used for vibration control in large buildings is active mass damping, in which an inertial mass is accelerated to apply forces that counteract vibrations. In Chapter 6, I suggest that an analogous technique may be used to counteract the buckling of a column by attaching a small linear motor and reaction mass to the midpoint of a column.

Distributed Electromagnet Array

In 1968, Raymond Jefferis [20] suggested the possibility of using an array of electromagnets to apply control forces that stabilize a column against buckling. Jefferis suggested that an array of electromagnets, mounted adjacent to a column, could apply stabilizing forces that counteract buckling motion. Jefferis published a theoretical analysis that shows that under static loading conditions, the first mode of column buckling can be stabilized by applying a distributed restoring force to *every* location along the length of a column, proportional to the deflection (from the vertical axis) of the column at each point. This approach requires that an independently supported array of electromagnets be mounted adjacent to the column, which may prove practical in controlled environments such as a steel mill, but is not very feasible for use in structural applications where an independent support for the magnet array is not available.

Electromagnetic Bracing

Jefferis also conducted an experiment, in which a single electro-magnet was used to apply a restoring force to the midpoint of the column. The magnitude of the restoring force was proportional to the deflection of the column at the midpoint. This approach is similar to bracing the center of the column, in that the electromagnet exerts restoring forces on the center of the column, and the location of the electromagnet is rigidly fixed by an independent support. Indeed, since an independent support is available, it is possible to simply connect the center of the column to the external support, thereby bracing the midpoint of the column. The advantage provided by the use of an electromagnet rather than a physical brace is that the electromagnet can apply forces to materials where a physical connection is not possible, such as in the case of steel that is cooling during the manufacturing process.

The experimental results reported by Jefferis are somewhat difficult to interpret, in that his results indicate that by controlling the first buckling mode, an increase in strength was achieved well beyond that at which failure due to the second buckling mode should have occurred. This discrepancy is apparently related to the unusual combination of pinned and fixed end support conditions used in the experiment, as illustrated in Figure 2.1. In the presence of these unusual end supports, it is difficult to know to what extent first-mode buckling was prevented by the electro-magnets applying a control force, as opposed to the onset of buckling causing a deflection that changed the system from a column with pinned ends to a column with fixed ends, a configuration which has a substantially higher critical buckling load. It appears that these unusual end-conditions substantially altered the mode shapes of the column, thereby preventing observation of the effects of higher buckling modes and partially restraining the column against first-mode buckling.

I have found that in practice, when stabilizing column buckling by applying a restoring force at the column's midpoint, a PD (proportional+derivative) controller is needed to reliably stabilize column buckling, particularly in the presence of time-varying external loads. Perhaps Jefferis was able to make do with a simple “P” controller because friction in the experimental system provided the velocity-damping “D” term. Another possibility is that Jefferis's experiment lost kinetic energy when the buckling column collided with the fixed restraint, as illustrated in Figure 2.1.

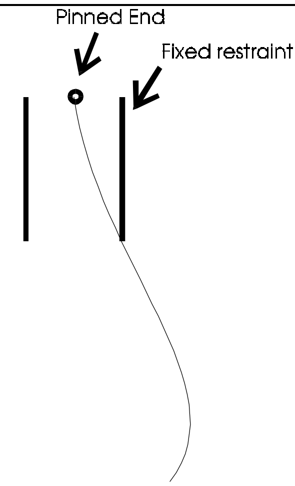

Figure 2.1: Support conditions used by Jefferis. The experiment conducted by Jefferis utilized an unusual end-condition that exhibits both pinned and fixed-end behavior depending on the deflection of the column.

Load Redistribution

Amr Baz [3] has developed a way to actively redistribute the load in a structure so as to keep supporting columns from being loaded above their buckling load. Baz's work involves sensing when buckling is about to occur, at which point Nitinol shape memory actuators reduce the effective length of the column, thereby shifting the load to other supporting members in the structure. This technique does not actually increase the load-bearing capability of any particular member, but rather improves the ability of the overall structure to withstand large, concentrated loads.

Baz [4] has also used wires attached to external “ceiling” and “floor” supports to adaptively brace a column against buckling. In this approach, wires constructed of the Nitinol shape memory alloy are embedded within the column material itself. The tension in the Nitinol wires can be adjusted by heating them with an electric current, thereby adjusting the amount of external support provided to the column. This approach is similar to the load redistribution approach described above, in that this method provides a mechanism whereby neighboring structural members that have additional load-bearing capability may, via the Nitinol wire braces, act as external supports so as to increase the load-bearing capability of an overloaded column.

Theoretical Work

During the final stages of my work, two independent papers were published concerning the possibility of actively stabilizing the dynamics of buckling. One of these papers, by Meressi and Paden [27], presents a theoretical analysis suggesting that resistive strain gages and piezo-electric PVDF film may be used to actively stabilize the first buckling mode of a column through the use of proportional feedback. Their analysis assumes the use of a continuous piece of PVDF film covering both sides of the beam, which has the effect of applying bending moments at each end of the beam. Their analysis goes on to show that using an a Linear Quadratic Regulator (LQR) control strategy, the system will be stable for any load below 4.1 times the first critical buckling load. An alternative approach that would be worth investigating would be to use a piece of PVDF film whose thickness varied with location on the beam, so as to form a modal actuator that would actuate the first buckling mode directly. This would avoid the spillover of actuation energy into higher-order buckling modes that is associated with the application of bending moments at the ends of the column.

A second paper by Chandrashekhara [8] presents the results of finite-element simulations that suggest that 2-D plates can be actively stiffened against uniaxial (first mode) buckling through the use of a single piezo-ceramic actuator patch located in the center of the plate, controlled using a proportional control algorithm. This work models a graphic-epoxy composite plate, using a more intricate finite-element model than I used that models the plate using 9-noded isoparametric quadrilateral elements. The applied load is modeled as slowly increasing over a span of approximately 30 seconds. Their simulations indicate that the buckling load of the column can be increased by roughly 10%.

A third paper by Su and Tadjbaksh [36] discusses optimal control laws for the control of “buckling” beams, but in fact is dealing with bending in the buckling direction for beams loaded below their critical buckling load, rather than with the strength-limiting buckling instability problem addressed in this dissertation.

Collapse Control

An interesting related development is the discovery by Kitagawa, Hagiwara, and Tsuda [21] that the ability of automobile side members to absorb crash energy can be increased through the use of beads that alter buckling behavior. Placing an array of beads along the length of a member effectively subdivides the member into smaller elements, each of which collapses due to local buckling in an accordion-style collapse mode, rather than in the bending collapse mode that would result from buckling of the overall member. Although this technique does not significantly alter the load-bearing capability of the member during normal operation, it does alter the buckling behavior of the member, thereby allowing it to absorb more energy during a crash.

Chapter 3:

Dynamics of Column Buckling

Modifying the dynamics of a beam so as to counteract buckling requires that control forces be applied that oppose the motion caused by buckling, but that do not induce other shape changes that could cause undesirable or even violent behaviors. Sensing the onset of buckling requires the ability not only to measure the shape of a beam, but also to distinguish shape changes caused by relatively harmless vibrations from shape changes caused by buckling. Thus the selection of what types of sensors and actuators to use, and where to place them, is critically dependent on the inherent dynamical behavior of a beam and on the forces and shape changes associated with buckling. In this chapter, I describe the dynamics of beam motion that relate to buckling, as well as the techniques that I used to reason about feasibility and design trade-offs that arise in the various sensing, actuation, and control issues associated with active control of buckling.

When an undriven, unloaded beam is subjected to a small perturbation, it vibrates in a set of well-defined mode shapes, each of which has a fundamental frequency associated with it. This vibration is described by Euler's equations of motion, which are presented later in this chapter. Subjecting a beam to an applied axial load reduces the ability of the beam to resist shape changes, thereby lowering its vibrational frequencies. Eventually, as the applied axial load is increased beyond the buckling load of a mode, the frequency of vibration in that mode becomes imaginary, causing the beam to “buckle”, i.e. to exhibit an exponential increase in deflection over time, rather than a sinusoidal vibration. Euler's equations of motion can be extended to represent the frequency changes associated with applied axial loading, as shown below.

Equations of Motion of a Beam

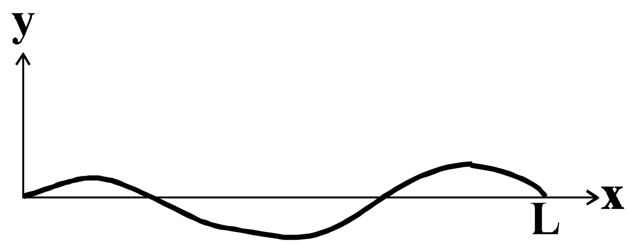

Figure 3.1: Coordinate System. In the coordinate system used to represent Euler's equation of motion, the **y** axis measures the deflection of the beam at each point, while the **x** axis indicates the location along the length of the beam. The beam shown above has length **L**.

Using the rectangular coordinate system shown in Figure 3.1, the dynamic behavior of an undamped beam undergoing small deflections can be described by Euler's beam equation:[8]

$$(\frac{EI}{\gamma})\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Equation 3.1)}$$

In this equation, $\boldsymbol{E}$ represents the Young's modulus of the material used to form the beam; $\boldsymbol{I}$ represents the moment of inertia of the beam; and γ represents the mass per unit length of the beam. The overall solution to Euler's beam equation describes how the deflection of each point on the beam varies with time:

$$y(x,t) = \sum_{n=1}^{\infty} \phi_n(x) q_n(t)$$

This overall solution, $\boldsymbol{y(x,t)}$, is expressed as a sum of independent "modal" solutions. Each modal solution consists of a function $\boldsymbol{\phi_n(x)}$ describing the mode shape of the nth mode, and a function $\boldsymbol{q_n(t)}$ describing how the amplitude of this mode shape varies over time. Solutions to Euler's equation of motion for a variety of support conditions are readily available in the literature.[9] For the case of a beam of length $\boldsymbol{L}$ supported by pins at

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[8]The discussion of Euler's dynamic equation is primarily based on Chapter 8 of Vibration of Mechanical and Structural Systems [19].

[9]For instance, see the table of solutions to Euler's equation in [19], page 608.

its endpoints, solution of Euler's equation via separation of variables leads to the following solution for the behavior of each mode:

$$\phi_n(\mathrm{x}) = \sin\left(\frac{n\pi \mathrm{x}}{\mathrm{L}}\right) \qquad \text{(Equation 3.2)}$$

$$\ddot{q}_n + w_n^{\,2} q_n = 0 \qquad \text{(Equation 3.3)}$$

$$w_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\gamma}} \qquad \text{(Equation 3.4)}$$

where $\boldsymbol{w_n}$ is the natural frequency of mode $\boldsymbol{n}$. Equation 3.3 is an ordinary differential equation having solutions of the form:

$$q_n(t) = A\sin(w_n\, t) + B\cos(w_n\, t) \qquad \text{(Equation 3.5)}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are integration constants determined by initial conditions. Thus the amplitude of each mode varies sinusoidally over time. For a pinned column, the shape of each mode is determined by Equation 3.2, as illustrated in Figure 3.2.

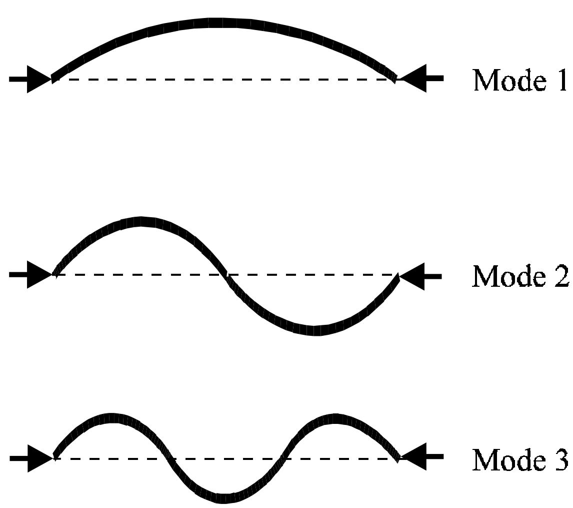

Figure 3.2: The first three mode shapes of a beam with pinned ends, as determined by Equation 3.2. The shape of the beam at any instant in time is expressed as a sum of mode shapes. The amplitude of each mode shape varies with time as determined by Equation 3.5.

The Effects of Applied Axial Loads

Applying an axial load to a column has the effect of applying a bending moment to every location along the column. This applied bending moment opposes the natural tendency of the column to restore itself to an undeflected position, thereby reducing the effective stiffness of the column. Specifically, for the coordinate system illustrated in Figure 3.1, in which the variable y denotes the deflection of the column at each point, the bending moment[10] resulting from the application of a force P at each end of the column is: Moment = - P y. Incorporating this bending moment into Euler's equation of motion (Equation 3.1) leads to the equation of motion for a column supporting an axial load:[11]

$$\left(\frac{EI}{\gamma}\right)\frac{\partial^4 y}{\partial x^4} + \frac{\mathrm{P}}{\gamma}\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Equation 3.6)}$$

The application of an axial load reduces the effective stiffness of the column, altering the column's natural circular frequencies (w_n) as follows:

$$w_{n_{loaded}} = w_{n_{unloaded}} \sqrt{1 - \frac{P}{P_{critical_n}}} \qquad \text{(Equation 3.7)}$$

$$P_{critical_n} = \frac{n^2 \pi^2 EI}{L^2} \qquad \text{(Equation 3.8)}$$

where from Equation 3.4, $$w_{n_{unloaded}} = \frac{n^2 \pi^2}{L^2}\sqrt{\frac{EI}{\gamma}}$$

Equation 3.7 shows that when a column is loaded above the critical buckling load, the frequency of vibration becomes imaginary. In other words, the column no longer has sufficient strength (stiffness) to overcome the bending moments being applied by the axial load. Unless an external control force intervenes, the amplitude of deflection in that mode will grow exponentially over time, leading to failure of the column. Equation 3.8 predicts

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[10]In this document, the sign convention used for bending moments applied to each end of a beam is that the bending moments are positive when they tend to make the beam bend downwards so as to hold water: $\cup$, or in the case of columns, when they tend to make the column bend leftwards: $\subset$.

[11]Note that for incorporation into Equation 3.6, the applied bending moment term **- P y** is differentiated twice with respect to x to produce the net acceleration in the x direction resulting from the applied load P. For a detailed description of the differential geometry of a beam element, see [19], page 601.

that in theory, preventing buckling in the first mode will yield a factor of 4 increase in load-bearing strength, since the critical buckling load for the second mode (n=2) is 4 times larger than the critical buckling load for the first mode (n=1). Similarly, preventing buckling in each of the first two modes will (theoretically) result in a factor of 9 increase in strength.

In practice, beams are never perfectly straight, but rather have some degree of built-in eccentricity, the direction and magnitude of which may vary along the length of the beam. Consider a built-in curvature that increases the effective displacement of the column from the equilibrium position by an amount χ. This imperfection results in dynamics that differ from those of an ideal column, due to an increase the magnitude of the destabilizing bending moment induced by the axial load: Moment = - P (y + χ). Built-in curvature reduces the force available to restore the column to the ideal "straight" equilibrium position. Since the column "prefers" to be somewhat curved, additional bending in the curved direction results in less stress than in the ideal case. Thus in the real world, a column will begin to buckle at loads somewhat smaller than the theoretical buckling load predicted by Equation 3.8. In modern engineering practice structures are typically designed in such a way that the maximum load applied to a column will never come within 5% of the idealized critical buckling load.

It is important to note that the dynamics of each mode are entirely independent of one another (see Equation 3.3). In other words, motion in one mode does not affect motion in any other mode. Ideally, actuation forces applied to counteract buckling would also have this orthogonality property: The forces applied to counteract buckling in one mode would not affect any of the other modes. Since the bending moments induced by an axial load vary continuously along the length of a column, this requires the use of an actuator capable of applying a different amount of force to every point along the length of the column. Although there are unusual situations where it may be practical to construct such a distributed force actuator, in most applications it is only feasible to apply independently controlled actuation forces to a few points or regions of a member, making it difficult to affect a single buckling mode without exciting other buckling or vibrational modes.

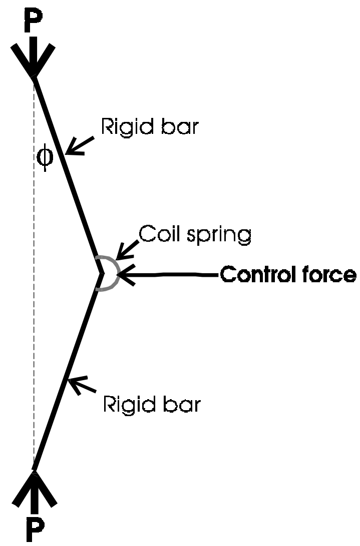

Figure 3.3: A simple model for the first buckling mode of a column, consisting of two rigid bars connected together by pin joint and a coil spring. The coil spring models the bending stiffness of the column , while a lumped mass located at the center hinge joint models the inertial mass of the column.

Active Control of Buckling: Feasibility Analysis

A crude estimate of the forces required to cancel the first buckling mode can be obtained from the very simple model shown in Figure 3.3. This model uses two hinged rods coupled by a coil spring to represent buckling in the first mode. The spring constant is chosen to represent the 1st-mode bending stiffness of the column. Although this model ignores the continuous nature of the column material and the effects of higher-order modes, it provides quick insight into the relative magnitudes of the forces required to counteract buckling. For a column of length **L**, the potential energy (**V**) of the pinned model system is:

$$V = PL\cos(\phi) + 2K_{coil}\phi^2 + 2M_{applied}|\phi|$$

At static equilibrium, $\frac{\partial V}{\partial \phi} = 0$. Hence with the column slightly deflected, the restoring moment $\mathbf{M_{applied}}$ required to counteract the destabilizing moment exerted by the axial load **P** is:

$$M_{applied} = P\sin(\phi)\frac{L}{2} - 2K_{coil}\phi$$

which for small angles reduces to:

$$M_{applied} = \left(P\frac{L}{2} - 2K_{coil}\right)\phi$$
$$\text{with } K_{coil} = \frac{P_{critical}L}{4} \qquad \text{(Equation 3.9)}$$

Equation 3.9 shows that the size of the restoring moment required to stabilize a column against buckling is linearly related to the angular deflection ϕ. Thus the amount of force required to stabilize a column against buckling directly depends on how quickly the onset of buckling can be detected by sensors.

An inexpensive sensor, such a resistive strain gage, can detect strains[12] (ε) on the order of 5×10^{-6}. For a column of thickness t_s, strain is related to curvature (κ) as follows:

$$\varepsilon = \kappa\frac{t_s}{2}$$

Measurement of a curvature κ by a sensor corresponds in the hinge model to an angle ϕ of approximately:

$$\phi = \frac{2}{L\kappa}\left(1 - \cos(\frac{L\kappa}{2})\right)$$

which for small angles reduces to:

$$\phi \approx \frac{L\kappa}{4}$$

Combining equations, the restoring moment required to stabilize a column against buckling is:

$$M_{applied} = \left(P\frac{L}{2} - 2K_{coil}\right)\frac{L\varepsilon}{2t_s}$$

[12]Strain is a measure of the change in length of an object, expressed as a fractional quantity: Strain = (Change in length) / (Original length)

Torque actuators such as rotary motors mounted at the endpoints of the column, or piezo-ceramics bonded onto the column, can apply the restoring moments directly. However, in some situations it is desirable to apply a restoring force **F** to the midpoint of the column, as illustrated in Figure 3.3, producing restoring moments that can be approximated as:

$$M_{applied} = F\frac{L}{2}$$

Combining equations:

$$F = \left(P - P_{critical}\right)\frac{L\varepsilon}{2t_s}$$

For a column that is 10 meters high and 0.1 meters thick, a strain gage can detect a deflection of approximately 2.5 mm. At this deflection, based on the simple hinge model for a steel column of square cross section, the first buckling mode of a column can be stabilized through the application of a restoring force to the midpoint of the column of only 2.5 Newtons for every 10,000 Newtons of axial loading in excess of the column's critical buckling load. Complete cancellation of the first buckling mode would increase the effective critical buckling load of the column by a factor of 4 from 169360 Newtons to 677440 Newtons, through the use of a restoring force of only 127 Newtons.

Although the two-element hinge model is useful for producing rough estimates, the analysis presented above somewhat underestimates the force required to counteract buckling. Part of this underestimate is caused by the fact that buckling is a dynamic phenomenon requiring that control forces be applied to counteract the buckling motion of the column (thereby absorbing the kinetic energy) as well as to oppose the static forces associated with the compressive load. In the early stages of designing active column experiments, I used a rule of thumb to account for dynamical effects. Specifically, I selected materials, geometries, and actuation techniques so as optimize the ratio between the maximum deflection at which sufficient control authority is available to *statically* counteract buckling, and the deflection at which buckling can first be detected. In other words, I measured how much room the column had to fall down before it passed beyond the point at which even if the column had no velocity, buckling could no longer be prevented.

Another factor contributing to underestimation in the two-element hinge model is that in reality the column material is continuous, and applying a force at the midpoint will affect other modes in addition to the first. Basically, the two-element hinge model is useful as a quick way of performing rough feasibility estimates. For more detailed work, such as the selection of where on a column to place actuators, and the simulation of control strategies, a more complex discrete-element method had to be used, as described below.

Discrete-element Modeling of a Column

Interactions with active control components can alter the dynamics of a beam, affecting both its fundamental frequencies and its mode shapes, even when no control forces are applied. For instance, bonding piezo-ceramic actuators onto a segment of a beam modifies the effective stiffness, moment-of-inertia, and mass of that segment, even when the actuators are not energized. Expressed in continuous form (Equation 3.1), Euler's equations of motion are valid only for a beam having uniform material properties over its entire length. In order to study the behavior of composite beams of non-uniform thickness, I used a discrete-element approximation to Euler's equations, in which the material properties of each element were chosen to account for the effects of the active control components on the material properties of each beam segment.

Unlike the simple 2-element pinned model, which made use of rigid elements, the discrete-element model uses beam elements that are permitted to bend, representing the shape of each element by a cubic polynomial. By accounting for the continuous nature of the column material, the discrete-element model allows study of the interactions between control forces and higher-order vibrational and buckling modes, as well as the modeling of actuator dynamics and of composite beams whose material properties may be discontinuous. Simulations based on this model, presented in Chapter 4, proved essential for analyzing the trade-offs involved the design of a composite beam stabilized by piezo-ceramic actuators. The basis for the discrete-element model I used is the discrete-element model described in [19], with extensions to account for the applied axial load as described in [30].[13]

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[13]Note that this model was also used by Baz [3] to study the effectiveness of Nitinol actuators in reducing the axial load applied to a buckling column.

Nonlinear Postbuckling Behavior of a Column

The dynamics of column buckling, based on a model developed by Stoker [35], are frequently used as an example in nonlinear systems texts. It is important to note that the equations of motion presented earlier in this chapter are linear differential equations, accurately describing the motion of a column for small deflections. The discrepancy between the linearized equations and the actual nonlinear behavior of a column arises from the fact that in the derivation of the linearized equations of motion, the term $\frac{\partial^2 y}{\partial x^2}$ is used to approximate the curvature of the column. For large deflections, this approximation becomes invalid, resulting in a more complex, nonlinear behavior pattern.

Specifically, when loaded beyond the critical buckling load, an initially straight column will begin to buckle. Initially, buckling leads to an exponential increase in deflection with time. However, as the deflection grows large, nonlinear effects come into play and the column finds a new stable, deflected position. In other words, at the critical load the stable equilibrium point associated with zero deflection becomes *unstable*, and leads to the creation of two new *stable* equilibrium points, one on either side of the column. This phenomenon, where one equilibrium point loses stability while giving rise to two stable equilibrium points, is known as a pitchfork bifurcation.

Zhao [42] has suggested the possibility of using a very clever control strategy, based on the Stoker model, that takes advantage of the nonlinear dynamics that arise in the vicinity of the stable post-buckled equilibrium points. Figure 3.4 shows the location of the post-buckled equilibrium points as a function of applied load. Note how even a tiny increase above the critical buckling load leads to a large deflection. Such a large deflection of a supporting member would lead to failure of many structures, and could even cause material failure (due to excessive stress) of the column itself.[14] For this reason, when seeking to increase the load-bearing strength of a column, it is not practical to use a control strategy that permits the column to approach the post-buckled equilibrium

[14]For a detailed explanation of the origin of the non-linear effects of post-buckling behavior, see [12], page 80]. For an equation governing the location of the post-buckling equilibrium points, as well as a discussion of why such large deflection lead to material failure of a column, see [6], page 6.

points.[15] Instead, both to avoid material failure and to keep the magnitude of the actuation forces small, it is essential to control the column in such a way that the magnitude of the deflection remains small at all times.

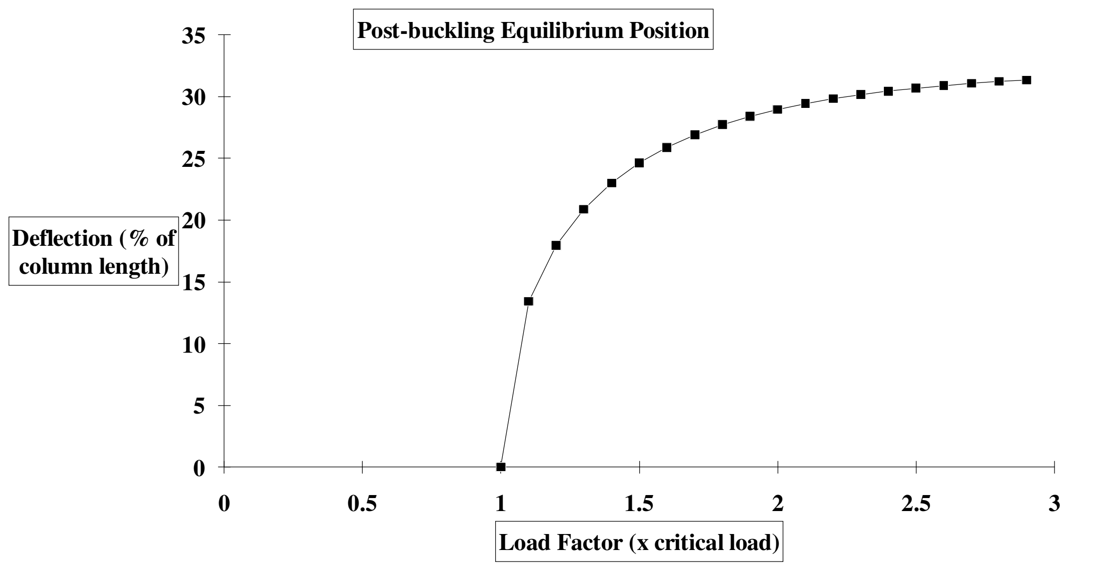

Figure 3.4: Location of the stable post-buckling equilibrium points. Even a tiny increase of 1% above the critical buckling load leads a huge deflection on the order of 13% of the length of the entire column.

[15]One area for future investigation would be to try applying nonlinear control techniques in the spirit of those suggested by Zhao to the new system dynamics that arise in the actively controlled column. As discussed later in this document, measurement noise in the actively controlled system leads to the existence of two stable equilibrium points located very close to the unstable equilibrium point at zero deflection.

Stability Analysis

In theory, provided that the deflection of the column is kept small, linear control techniques may be used to control buckling. In the buckling control experiments (described in Chapters 4 and 5), the control strategy that experimentally worked best was a nonlinear variant of a P.I.D. (Proportional + Integral + Derivative) linear control strategy. Intuitively, P.I.D. control can be thought of as a combination of three goals:

1. If the column is deflected, apply a restoring force proportional to the deflection.

2. If the column is moving, apply a force that opposes the motion.

3. If over a period of time, the column continually buckles in the same direction, apply a balancing force to counteract the apparent asymmetry.

The stability of P.I.D. control applied to the first buckling mode of a column may be analyzed using linear control theory. The modal equation of motion (see Equation 3.3) is:

$$M\ddot{q} + Kq = F$$

where $\boldsymbol{q}$ is the modal amplitude, $\boldsymbol{M}$ is the modal mass coefficient for the first mode, $\boldsymbol{K}$ is the modal stiffness coefficient for the first mode, and $\boldsymbol{F}$ represents the restoring control force that is applied to the first mode. Converting to the frequency domain and substituting in a P.I.D. control law that seeks to maintain a deflection $\boldsymbol{q_0}$ results in the following equation of motion, for an equation having proportional gain constant **P**, derivative gain constant **D**, and integral gain constant **I**:

$$Ms^2 q + Kq = P(q - q_0) + Ds(q - q_0) + I\frac{1}{s}(q - q_0)$$

which in turn leads to the system function:

$$\frac{q}{q_0} = \frac{-D\,s^2 - P\,s - I}{Ms^3 - Ds^2 + (K - P)s - I} \qquad \text{(Equation 3.10)}$$

According to linear control theory, the controlled system will be stable provided that all roots of the denominator of Equation 3.10 have negative real parts. According to the Routh test for cubic polynomials, this requires:

1. $D < 0$

2. $P < K$

   (Note that K < 0 when the column is loaded beyond its critical buckling load.)

3. $I < 0$

4. $\frac{(K - P)}{M} > \frac{I}{D}$

In other words, the permissible values for the ***P***, ***I***, and ***D*** control coefficients are interrelated: Increasing the magnitude of ***P*** may require simultaneously decreasing the magnitude of ***I*** or increasing the magnitude of the damping coefficient ***D***. In practice I have found that sensor noise limits the maximum amplitude of the damping coefficient ***D***, which in turn places limits on the permissible values for ***P*** and ***I***. Note that as the applied load on the column increases beyond the critical buckling load, the modal stiffness coefficient ***K*** becomes increasingly negative, requiring that the magnitude of ***P*** be increased to compensate.

Chapter 4:

Control of Buckling using Piezo-Ceramic Actuators

I have constructed a composite column that actively resists buckling through the use of piezo-ceramic actuators. Active control of buckling allows this column to support 5.6 times more load than would otherwise be possible. This corresponds to complete stabilization of the first mode of column buckling, as well as partial stabilization of the second buckling mode. I first describe the structure of the experimental composite column, and then discuss the design issues that led to this choice of structure. Finally, I describe the control strategy used to counteract buckling and present the results of experiments performed using the active column.

Structure of the Composite Column

The experimental active column is a composite composed of steel and piezo-ceramic materials. The column is approximately 1 foot in length, and has a theoretical buckling load of 9 Newtons. In practice, buckling occurs at a load of 5.27 Newtons, which is somewhat less than the theoretical buckling load due to eccentricity and other small imperfections in the material and support structure of the column.

The onset of buckling is detected by an array of 10 strain gages mounted along the length of the column in pairs, with five strain gages on each side of the column. Forces that counteract buckling are supplied by an array of piezo-ceramic actuators also located along the length of the column. Strain measurements are transmitted to a digital control computer, which determines the control actions required to counteract buckling, and sends control signals to high-voltage amplifiers that drive the piezo-ceramic actuators.

The composite column is constructed from a base material of 0.010 inch thick spring steel, 12.75 inches long and 2 inches wide.[16] A total of 10 piezo-ceramic actuators are mounted on the column in pairs, with 5 actuators on each side. Early in the design process, finite-element simulations indicated that local bending would occur in the gaps between the actuators, since that region is more flexible than the rest of the column. To prevent localized bending, small 0.010 inch thick steel plates that act as stiffeners are mounted in the gaps between adjacent actuators, as illustrated in Figure 4.1.

[16] At each end of the column, 3/8 inch of material is encased in a hinge assembly, thereby reducing the effective length of the column to 12 inches.

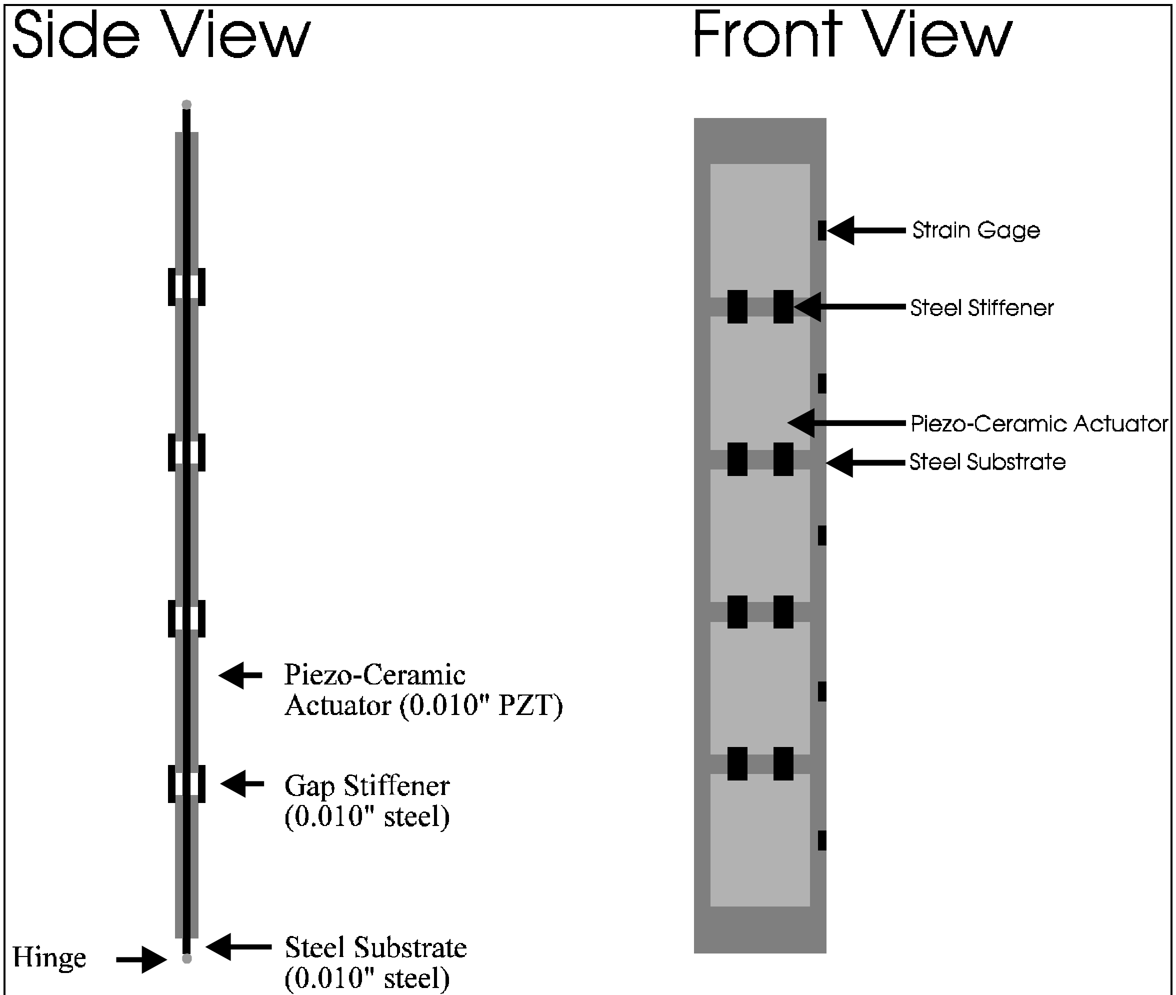

Figure 4.1: Front and side views of the composite steel/piezo-ceramic column. The column is 12.75 inches in length. Steel stiffeners are used to increase the stiffness of the area between adjacent actuators that is not covered by piezo-ceramic material.

Principles of Piezo-Ceramic Actuation

Applying an electric field to a piezo-ceramic material causes it to undergo stress. If the ceramic is able to move freely, it will seek to alleviate this stress by either growing or shrinking, depending on the polarity of the applied electric field. However, if the motion of the ceramic is blocked, then it acts as a stressed material, exerting forces that are directed so as to induce motion that will alleviate the electrically-induced stress. Thus when a piezo-ceramic actuator is bonded onto another material, the application of an electric field causes the piezo-ceramic to apply forces to that other material in an attempt to affect the length change necessary to relieve the stresses induced by the electric field.

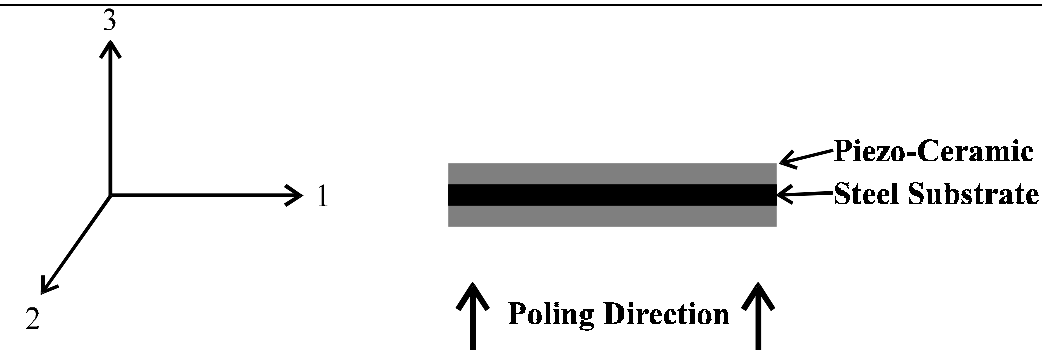

Figure 4.2: Poling axis of a piezo-ceramic. During manufacture, piezo-ceramics are poled so as to create a distinguished "3" direction. In the composite piezo/steel composite beam, the piezo-ceramics are mounted such that the "3" direction is perpendicular to the steel substrate.

During manufacture, a strong electric field is used to polarize the molecules within the ceramic material along a particular distinguished axis, referred to as the "3" direction. In the composite column, the piezo-ceramics are poled such that the "3" direction is oriented perpendicular to the steel substrate, as illustrated in Figure 4.2. When an electric field is applied along the axis of poling, stresses are induced that cause forces to be exerted that cause the ceramic material to expand in the "1" and "2" directions, and shrink in the "3" direction. For the purpose of applying bending moments that oppose buckling, the forces of interest are those directed along the "1" axis.

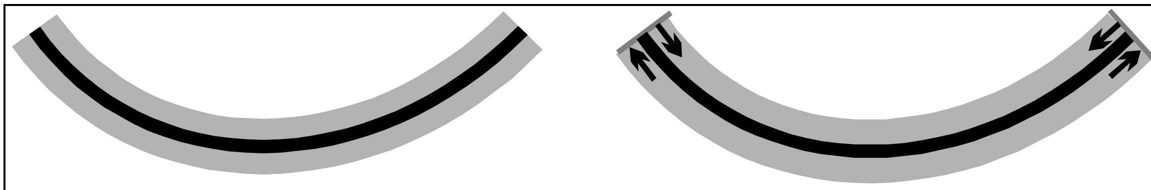

Figure 4.3: Piezo-Ceramic Actuation. *Left*: The piezo-ceramic mounted on one side of the beam substrate seeks to grow, while the piezo-ceramic mounted on the other side seeks to shrink, thereby causing the composite assembly to bend. The piezo-ceramics are shown in gray, with the beam substrate material shown in black. *Right*: Actuation of a piezo-ceramic can be modeled as the application of an extensional (or compressive) force at each end of the actuator. For the purpose of computing bending moments, the location of the force is approximated to be midway through the actuator, at a distance $t_a/2$ from the surface of the beam substrate.[17] Thus, the bending moments produced at each end of the actuator can be modeled as a force couple, whose forces are a distance t_b+t_a apart. (t_b denotes the thickness of the beam substrate, while t_a denotes the thickness of each piezo-ceramic actuator.)

As illustrated in Figure 4.3, piezo-ceramic actuators applying forces along their "1" axis can be used in pairs to apply bending moments to a beam, by having the actuator on one side of the beam try to grow while the actuator on the other side of the beam tries to shrink. Specifically, for a beam having thickness t_b, the application of an extensional force **+F** on one side of the beam by an actuator of thickness t_a, with a compressive force **-F** being simultaneously applied by an identical actuator mounted on the other side of the beam, results in a bending moment being applied to the beam having magnitude:

$$Moment = F\ (t_b + t_a) \qquad \text{(Equation 4.1)}$$

Thus complementary excitation of a pair of piezo-ceramic actuators results in the application of a force couple at each end of the piezo-ceramic actuator pair. Each force in the couple has magnitude **F**, is located a distance $\mathbf{t_a/2}$ away from the surface of the beam

[17]Note that this model of piezo-ceramic behavior differs from that commonly encountered in the literature. This model, based on work by Anderson [1] accounts for the piezo-ceramic material itself forming an integral part of the beam and undergoing Bernoulli-Euler bending. In contrast, the model typically encountered in the literature is intended for use with a configuration in which the thickness of the piezo-ceramics is negligible with respect to the thickness of the beam being controlled, and hence the bending of the piezo-ceramic can be neglected.

substrate, and is directed along the "1" axis so as to alter the length of each piezo actuator. The magnitude of the force **F** exerted by a piezo-ceramic at its endpoints is related to the magnitude of the applied electric field by the proportionality constant $\mathbf{d_{31}}$:

$$F = d_{31}\, E_{field}\, E_p\, (t_p\, w_p)$$

where $\boldsymbol{E_{field}}$ represents the strength of the applied electric field; $\boldsymbol{E_p}$ is the Young's modulus of elasticity of the piezo-ceramic material; $\mathbf{t_p}$ represents the thickness of the piezo-ceramic (along the "3" axis); and $\boldsymbol{w_p}$ represents the width of the piezo-ceramic (along the "2" axis). Expressed in terms of the voltage applied across the "3" axis of the piezo-ceramic, the magnitude of the force **F** is related to the applied voltage $\boldsymbol{V}$ as follows:[18]

$$F = d_{31}\, E_p\, w_p\, V \qquad \text{(Equation 4.2)}$$

Summarizing, mounting a piezo-ceramic actuator on a beam has two effects. First, the piezo-ceramic itself (which is about as elastic as aluminum) constitutes additional material that modifies the dynamics of the beam by increasing its mass and its effective bending stiffness. Second, as described above, the application of an electric field to an actuator causes it to exert forces both on itself and on any material to which it is attached. The use of a pair of actuators mounted on opposite sides of a beam and driven with opposite electric field polarities results in bending moments being applied to the beam at each end of the piezo-ceramic actuators.

Hysteresis and other Nonlinearities of Piezo-Ceramics

Piezo-ceramics exhibit behavior that is far from ideal. For one thing, they exhibit hysteresis. In other words, even for small deformations, piezo-ceramics do not behave entirely elastically, but rather take somewhat of a plastic "set" when deformed even for very short periods of time. Thus although a piezo-ceramic will change its length when an electric field is applied, when the field is removed it will not completely return to its original length. In the buckling column experiment, hysteresis in the piezo-ceramic actuators was particularly troubling whenever the column was allowed to buckle, either

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[18]Note that ($E_{field} = V/t_p$)

due to a bad control strategy or due to intentional turning off of the control system for demonstration purposes. Although a safety mechanism prevented buckling from progressing more than a small amount (about an inch of deflection at the center of the column), this relatively large excursion caused a significant set in the piezo-ceramic actuators. In order to restart the system after buckling occurred, it was necessary to buckle the beam in the other direction for a few moments, and then to make minor manual adjustments to the control law to account for the shift in the dynamical equilibrium position caused by residual forces in the piezo-ceramics.

Another important nonlinearity occurring in piezo-ceramics is that the coefficient $\mathbf{d_{31}}$, a key component of the relationship between force and voltage (Equation 4.2), varies depending on how much total strain the piezo-ceramic is undergoing (i.e. how deformed the material is). The value for d_{31} normally reported in material specifications is only valid for the strains resulting from electric field strengths of less than about 10 V/mm.[19] When responding to an external perturbation, the buckling control system undergoes high strains, requiring the use of voltages as high as 200 Volts, which corresponds to a field strength of 787 V/mm. At these high strains, the effective value of d_{31} is approximately double the value of d_{31} for low strains. When designing the composite column used for the buckling experiments, I used the larger, high stress value for d_{31}, on the grounds that at the time that the maximum force the piezo actuators can supply is needed, the column will be buckling and hence the actuators will be undergoing high stress.

Design Criteria

A variety of design alternatives were considered prior to constructing the steel-piezo-ceramic composite column. The primary design goals were:

1. Even with actuators and sensors incorporated into the composite, the dynamical behavior would still approximate that of a column.

2. The amount of load required to make the column buckle was to be relatively small, so as to avoid the expense and hazards associated with large-force buckling experiments.

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[19]See [1], page 26.

The first phase of the design process was to gain intuition about the trade-offs associated with actuator placement. I used finite-element simulations to determine whether it is necessary to completely cover the beam substrate with actuators, or whether having actuators in only certain select locations along the length of the column would be sufficient to counteract buckling. Considering only the first buckling mode, one would tend to place the actuators towards the midpoint of the column, since that is where the greatest deflection occurs, and is thus the portion of the column that undergoes the greatest de-stabilizing moment as a result of axial loading. On the other hand, if one hopes to stabilize both the first and the second buckling modes, it may be more desirable to use two independent sets of actuators, located at the maximum deflection points of the second buckling mode. Driving both sets of actuators in the same direction would primarily affect the first buckling mode, while driving the actuators in opposite directions would primarily affect the second buckling mode.

To measure the effectiveness of each actuator placement strategy, simulations were performed to determine the static deflection (in each mode) that an *unloaded* column would undergo as a result of the full-strength energizing of the actuators. The first and third mode displacement was measured with the actuators on the top and bottom of the beam driven in the same direction, while the second mode displacement was measured with the actuators on the top and bottom of the beam driven in opposite directions. Figure 4.4 shows some typical results obtained during the first phase of the design process.

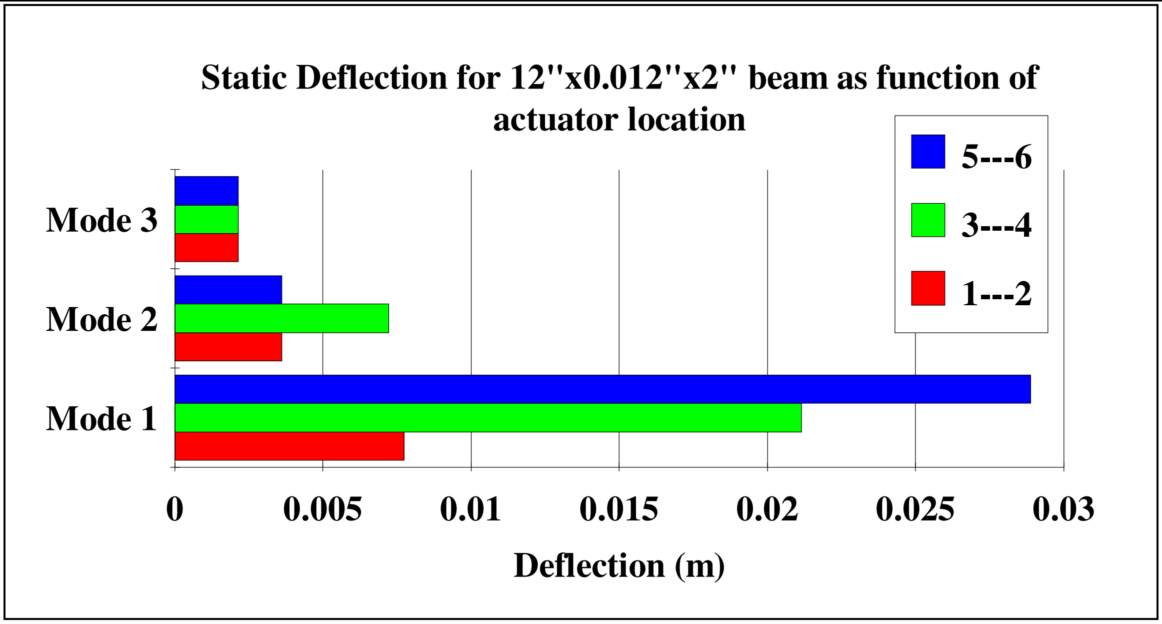

Figure 4.4: Actuator Location Tradeoffs. This chart shows the maximum static deflection that can be obtained in each mode from piezo-ceramic actuation of an unloaded column. The inherent stiffness of the piezo-ceramic material itself was not included in the model, in order to distinguish changes to the system dynamics caused by the location at which actuation forces are applied changes in dynamics associated with the stiffness (and consequent mode shape changes) associated with the placement of the piezo-ceramic actuator material itself. Simulation "1--2" refers to piezo-ceramics covering inches 1,2,11, and 12 of the 12 inch-long column; simulation "3-4" refers to piezo-ceramics covering inches 3,4,9, and 10; and simulation "5-6" refers to piezo-ceramics covering inches 5,6,7, and 8. The results clearly show that to affect the first mode, placing actuators at the center (5,6,7,8) of the column is best. On the other hand, placing actuators at (3,4,9,10) is somewhat (around 30%) less effective on the first mode, but is a factor of 2 times more effective at actuating the second mode. The steel column being simulated was 12 inches long, 2 inches wide, and 0.012 inches thick, undergoing applied moments of 0.119 Newton-meters.

In the second phase of the design process, I took into account the effects of piezo-ceramic actuator stiffness on the dynamical behavior of the column. Mounting piezo-ceramics in certain locations along a substrate results in a composite column that has non-uniform thickness, as well as bending stiffness and buckling mode shapes that will differ from that of the isolated substrate. Figure 4.5 shows how the modal displacement data (originally presented in Figure 4.4) changes when the stiffness of the piezo-ceramic material itself is taken into account. Note that although the benefits of the various actuation sites did not change substantially relative to one another (i.e. mounting actuators in the center is still the best way to affect the first mode), the magnitude of the deflection that can be induced through actuation was greatly reduced.

A portion of the reduction in deflection that results from taking actuator thickness into account is simply due to the composite structure being stiffer overall than the steel substrate material alone. However, there is another effect being exhibited in Figure 4.5 that at first seemed minor, but later on turned out to be critically important. The effect is that the nonuniform thickness of the column allows some regions of the column to bend more easily than others. As a result, a portion of the actuation energy that would otherwise have gone into the first mode instead results in significant local bending of the column in the third mode. This effect can be seen by comparing Figure 4.4 to Figure 4.5. Note how in the case of a column of uniform thickness (Figure 4.4), the displacement in the third mode is independent of which portion of the column is covered by actuators.[20] However, in the case of a column of non-uniform thickness (Figure 4.5), placing the actuators at the center of the column has far more effect on the third mode than does placing the actuators towards the ends of the column. This local bending in the third mode comes at the expense of actuation of the first mode, as illustrated by the case of actuators mounted at the center of the column, where in the non-uniform case the 3rd mode displacement is much larger (relative to the 1st mode displacement) than in the uniform case. In other words, non-uniformities in column thickness can result in spillover of actuation forces from lower modes into higher modes.

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[20]This is not true in general. For all three actuator locations being considered, the boundaries between actuators fell on nodes of the 3rd mode shape. Thus two half-waves of the third mode shape were covered in each case, resulting in modal displacements that are independent of actuator location. When some portions of the column are stiffer than others, the displacement achievable depends on which half-wave is covered by actuators.

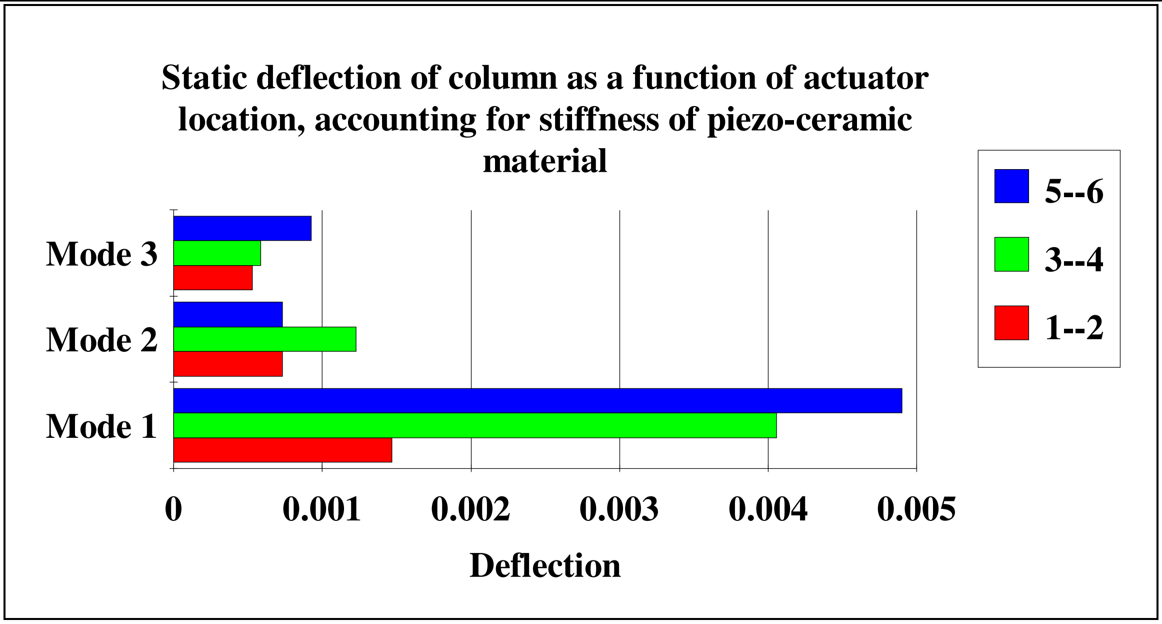

Figure 4.5: Effect of Piezo-Ceramic Stiffness. This chart shows the maximum static deflection that can be obtained in each mode from piezo-ceramic actuation, accounting for the change in the dynamical properties of the beam caused by the material properties (stiffness) of the piezo-ceramic material itself. Comparison to Figure 4.4 shows that accounting for the change in bending stiffness caused by the surface bonding of the piezo-ceramics dramatically reduces the absolute deflection attainable. Note the variations in the amplitude of the third mode, which were not present in Figure 4.4. As in Figure 4.4, the steel column being simulated is 12" long, 2 inches wide, and 0.012" thick. In each case, the piezo-ceramics cover 4 inches of the column length (inches {1,2,11,12}; {3,4,9,10}; or {5,6,7,8}), and are 1.5 inches wide and 0.010 inches thick.

Both to avoid the problems associated with non-uniform column thickness, and to provide the ability to experiment with a variety of actuation locations, I decided upon the composite design presented earlier (Figure 4.1), in which the surface of the column is covered with 5 sets of piezo-ceramic actuators. This was as close as manufacturing constraints would permit to fully covering the surface of the column with piezo-ceramics. The use of five independent pairs of piezo-ceramic actuators has the following advantages:

1. All 5 piezo-ceramics may be driven with equal amplitude, as if they were electrically connected to one another. The forces applied by neighboring piezo-ceramics cancel each other out, so that the only bending moments applied are located at the

endpoints of the column. In essence, this actuation strategy simulates an active pin joint, in which an actuator applies moments at the endpoints to actively alter the support conditions from pinned ends to rigidly fixed ends, thereby increasing the ability of the column to resist buckling.

2. The ideal actuator for canceling the first buckling mode would apply bending moments whose magnitude varied continuously over the length of the column, thereby affecting the first mode directly without affecting any other modes. Covering the beam with five pairs of independently excitable actuators allows a piecewise approximation to an ideal actuator. By driving the actuators near the midpoint of the column with a larger voltage than the actuators located near the endpoints of the column, the bending moments associated with buckling in the first mode can be counteracted without significant overflow of actuation energy into higher buckling modes. This approach differs from the active pin joint approach described above in that it affects the first buckling mode directly, thereby avoiding the significant mode shape changes that are associated with a change from pinned ends to fixed ends. Experimentation revealed that both actuation approaches work, although the pin-joint approach proved to be more sensitive to perturbations than the piecewise actuation approach.

3. Covering the beam with segmented actuators allows the actuators to be used out of phase, so as to affect the second buckling mode directly. Ideally, one would like to have completely independent actuation of the top and bottom halves of the column so as to have all available actuators capable of being used to affect the second mode. However in the prototype composite column, manufacturing constraints required the use of piezo-ceramics that were two inches in length, such that the actuator pair located at the midpoint of the column is placed such that half of the actuator is located in the top half of the column and half is located in the bottom half. Thus in the prototype, only four of the five pairs of actuators are available to be driven out of phase so as to affect the second buckling mode, while all five pairs of actuators are available to affect the first buckling mode.

4. Energizing only selected actuators allows experimentation with the effects of applying actuation forces at only selected locations along the length of the column.

The final phase in the design process was to select an appropriate material to serve as the substrate on which the piezo-ceramic actuators were to be mounted. Material selection turned out to be very non-intuitive, since several factors came into play simultaneously. For instance, holding the thickness of the piezo-ceramic actuators fixed, and adjusting the thickness of the column to keep the buckling load fixed as well, consider the effects of material selection: Choosing a more flexible substrate material requires that a thicker beam be used to achieve the buckling load. Use of a thicker beam in turn gives the piezo-ceramic actuators more leverage by moving them farther from the center of the column, thereby increasing the moment that can be achieved through piezo-ceramic actuation. However, moving the piezo-ceramics farther from the center of the column also increases the relative effect of the actuators themselves on the stiffness of the system. In addition, use of a thicker substrate allows more accurate measurement of curvature, since the strain gages are mounted on the substrate, and a thicker substrate will undergo more strain for a given curvature than will a thinner substrate.

In order to evaluate these tradeoffs, I developed three metrics that I used to evaluate each proposed configuration:

1. The *stabilization position*, which measures the maximum static deflection of the midpoint of the column at which sufficient control authority is available to counteract buckling in the first mode, for a column supporting 2x its critical buckling load. It is important that this maximum stabilizable deflection be large, because eccentricities and other imperfections tend to make the column act as though it is slightly deflected even when it is in its equilibrium position, and small external perturbations act to increase the deflection of the column.

2. The *control ratio*, which measures the amount of "falling down" room that the column has. The control ratio is the ratio of the maximum statically stabilizable deflection (mentioned above) to the minimum deflection at which the onset of buckling can be detected. For comparison purposes, it is assumed that the column is axially loaded to 2x its critical buckling load, and that the onset of buckling is detected by a single strain gage located at the center of the column that is capable of detecting 20 microstrain. At the deflection at which that buckling motion is detected, the control system will seek to counteract that motion. The control ratio measures how much safety margin the control system has during which it can slow the column down, before the column will have buckled beyond the point at which it can no longer be stabilized.

3. The overall buckling load of the composite beam.

In designing the composite column, I considered a wide variety of materials, column lengths and substrate thicknesses. Figure 4.6 presents a summary of some of the material selection design tradeoffs associated with the design of a 12 inch long composite column. For instance, a column using a fiberglass substrate that is 0.010 inches thick has a larger stabilization position than does a column that uses a thicker 0.030" thick fiberglass substrate, and is thus less sensitive to eccentricities than the thicker column is. However, the 0.030" thick fiberglass provides a larger control ratio than does the 0.010" fiberglass, since the increased thickness of the substrate increases the ability of the sensors to detect the onset of buckling. The final choice of substrate material for the prototype column was 0.010 inch thick steel. Steel was chosen because it provides both a high control ratio and a relatively large stabilization position, while still providing sufficient stiffness to support substantial loads. In addition, spring steel has the advantage of being resistant to taking a "set" (e.g. undergoing plastic deformation), a factor which permits the prototype column to be reused once it has been permitted to buckle.

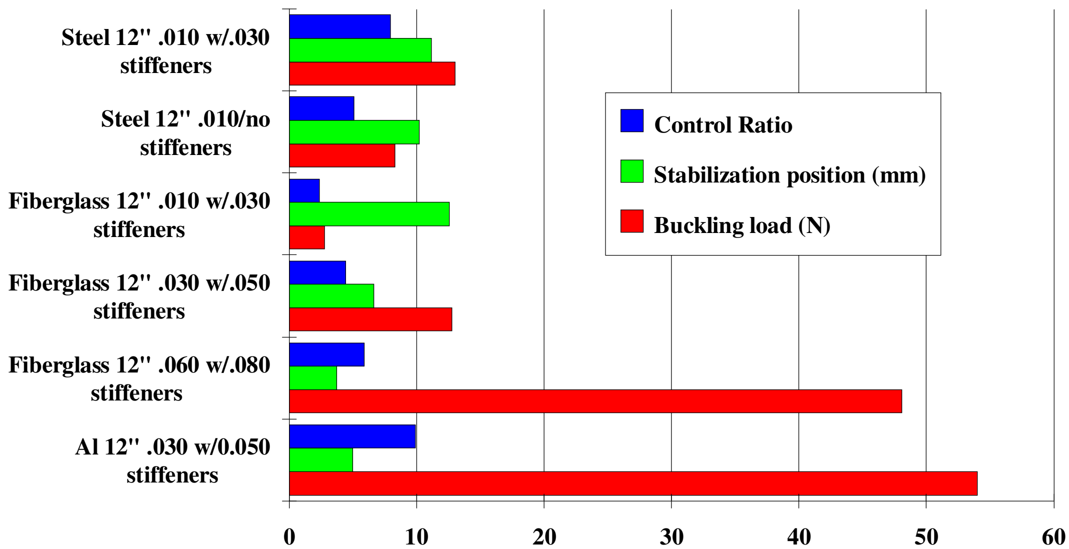

Figure 4.6: Substrate Material Selection Tradeoffs. This chart illustrates the tradeoffs that arise in selecting a suitable substrate material and thickness for use in a piezo-ceramic composite column. All of the configurations shown above are for a 12 inch long column that has five pairs of piezo-ceramic actuators mounted along its length. Note how the steel column that includes stiffeners mounted between adjacent actuators has a larger stabilization position and control ratio than the column that does not have stiffeners. The control ratios shown refer to a column loaded to 2x its critical buckling load, with a minimum detectable deflection that corresponds to 20 microstrain.

Experimental Apparatus

As pictured in Figures 4.8, each end of the column is held in place by a pinned end-support. The end-support consists of a hinge constructed using low-friction stainless steel ball-bearings encased in an anodized aluminum housing, which provides a nearly ideal pinned end-condition.[21] The column itself is mounted in a test jig that applies compressive load using a rod held in place by a linear ball-bearing assembly. The linear ball-bearing ensures that the load force is directed parallel to the column. An aluminum clamp

[21]Each hinge covers 3/8 inch of column material, reducing the effective length of the steel column from 12.75 inches to 12 inches. Additionally, the distance between the top of the column and the pin joint is approximately 0.32 inches at each hinge.

mounted above the linear ball bearing acts as a limit stop, preventing total collapse of the column when buckling occurs.

Each actuator consists of a rectangular 2 inch long, 1.5 inch wide, 0.010 inch thick piece of nickel-plated lead-zirconate-titnate (PZT), a piezo-ceramic material. A local company, Active Control Experts, was hired to package each actuator in a polyimide flex-circuit and to bond the resulting package onto the steel substrate material of the column using epoxy. The polyimide package provides contacts for making electrical connection to the nickel plating on the PZT actuator, insulates the actuator from the steel base material of the column, and acts to improve the quality of the bond between the actuator and the steel base material, since polyimide bonds to steel more easily than does piezo-ceramic. During packaging, the piezo-ceramics were baked into the polyimide package to ensure a tight fit that reliably transmits forces between the ceramic actuator and the steel substrate. The actuator package is attached to the steel substrate with epoxy; the stiffeners are attached to the packaged actuators by epoxy; and the strain gages are attached to the steel substrate using cyanoacrylate.[22]

[22]In modelling the dynamics of the composite column for use in finite-element analysis, I neglected the thickness of the polyimide package and of the epoxy bonding layer.

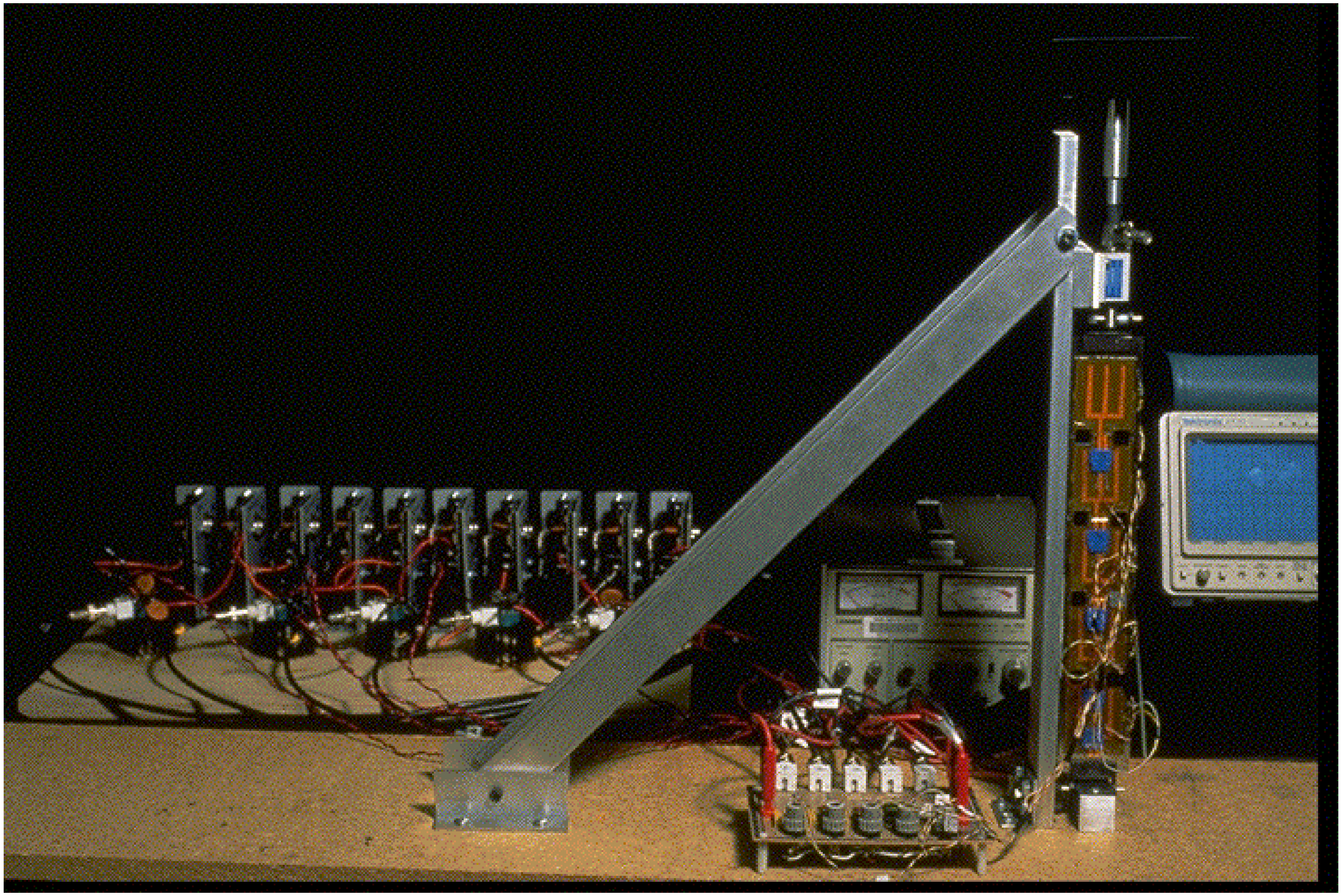

Figure 4.7: Experimental Apparatus. The electronic apparatus in the background on the left are the high voltage power supplies and amplifiers that are used to drive the piezo-ceramic actuators. The strain gage amplifiers are located on the small circuit board in the foreground. The control computer is not shown in the photograph.

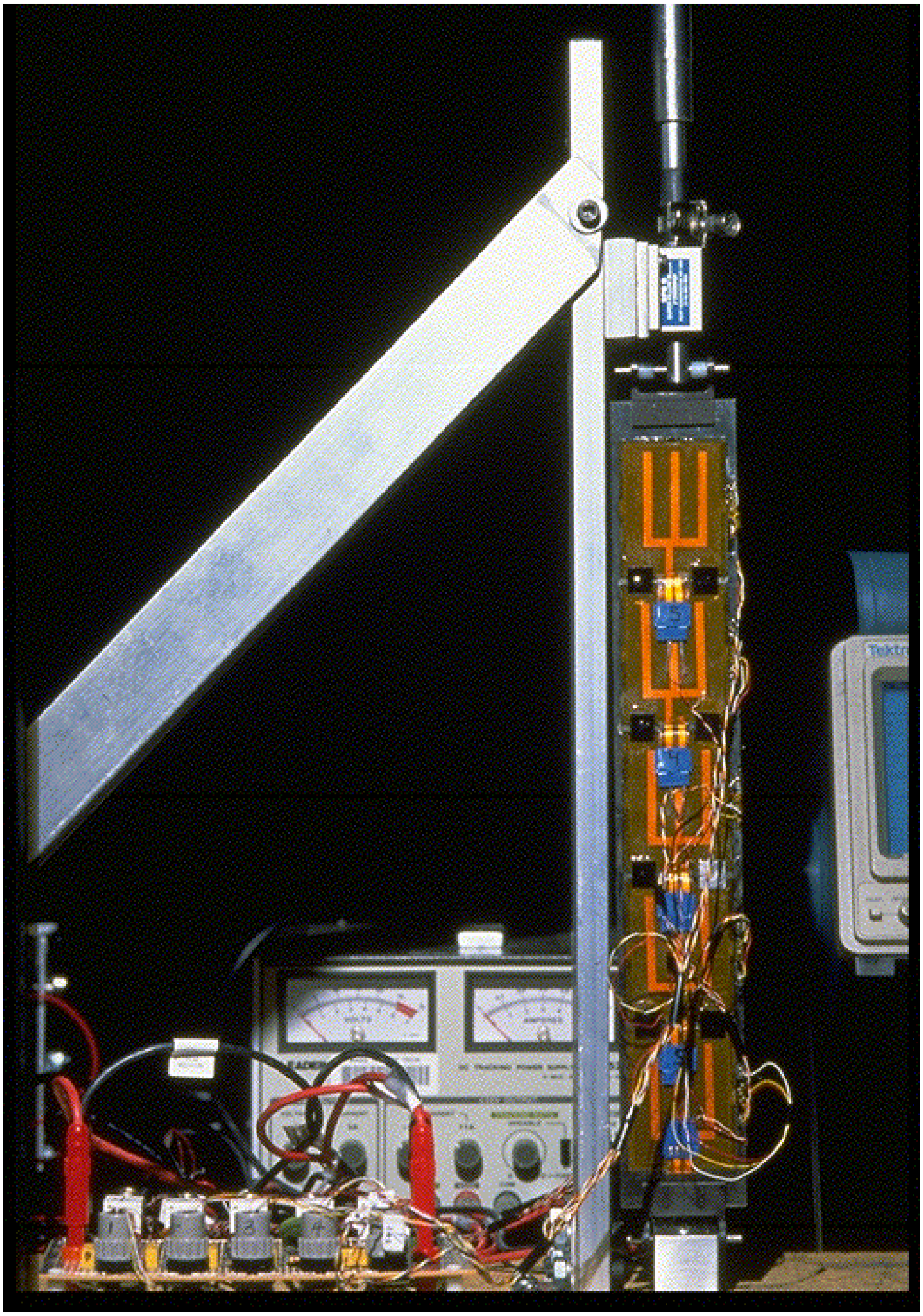

Figure 4.8: Front view of composite steel/piezo-ceramic column. Each end of the column is held in place by a pinned end support. The compressive load is applied to the column via a rod running through a linear ball bearing. A clamp mounted above the linear ball bearing acts as a limit-stop that prevents collapse of the column when buckling occurs.

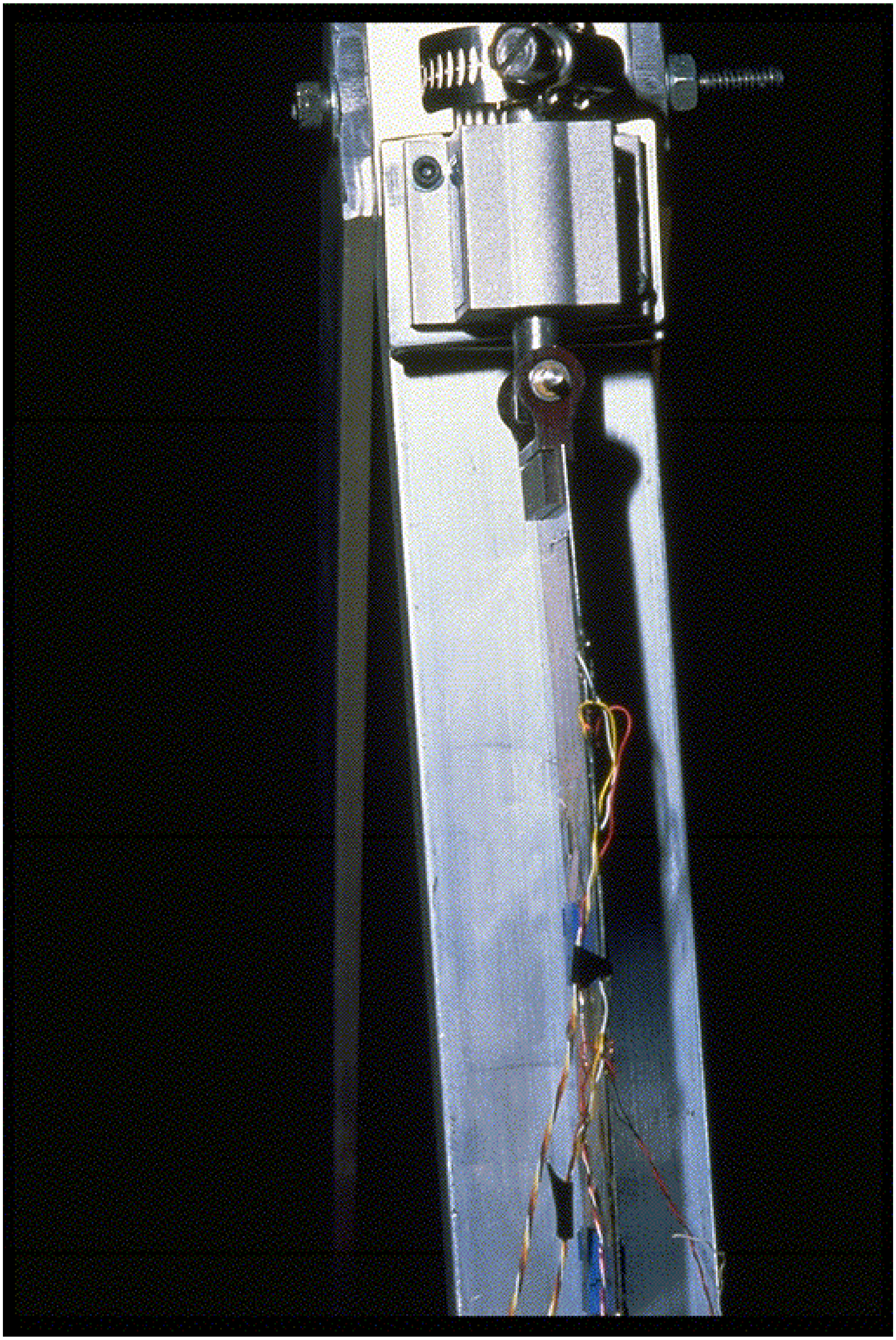

Figure 4.9: Side view of composite steel/piezo-ceramic column.

Due to manufacturing constraints, there is a gap of 0.3 inches between actuators. Early in the design process, finite-element simulation showed that these gaps are much more flexible than the rest of the column, and that as a result the moments applied by the piezo-ceramics would lead to local bending in these gaps rather than to overall bending of the column. To prevent bending in the gaps, 0.010 inch thick spring steel stiffeners are mounted on both sides of the column between adjacent actuators. Were it not for the stiffeners, the bending action induced by the piezo actuators would be localized, merely inducing a local (5th and 11th mode) deformation in the shape of the column. The stiffeners impose the constraint that the edges of adjacent piezo's be tangent to one each other, enabling the forces exerted by the actuators to induce a global rather than a local shape change in the member.

A total of ten strain gages are mounted on the column in pairs, with five strain gages on each side of the column. Each pair of strain gages is excited by a bridge circuit[23] and the resulting signals are sent to a set of instrumentation amplifiers (Figure [4.7]) that amplify the signals for transmittal to the control computer. The instrumentation amplifiers (based on the Analog Devices AD05) include a single-pole low-pass filter on their output in order to reduce the influence of high-frequency (above 1 Khz) measurement noise. The instrumentation amplifiers are low noise and extremely sensitive, producing a noise level on the first mode deflection amplitude of only ±40 microns.[24] The control computer consists of an Intel 486-based PC with an Omega DAS-16G1 analog input card and an Omega DDA-06 analog output card. High-Voltage amplifiers based on the Apex PA-88 amplify the signals produced by the control computer, generating up to ±200 volts to drive the piezo actuators.[25]

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[23]The strain gage excitation voltage was 2 volts, which was the maximum permitted voltage for the ultra-slim strain gages used in the active column. The strain gages had 120 ohm resistance and a gage factor of 2.

[24]The gain of the strain gage amplifiers is 2000.

[25]The gain of the high-voltage amplifiers is 20.

Control Strategy

In the experiments I conducted using the piezo-ceramic composite column, it was only necessary to control the first two buckling modes. The higher modes were inherently stable since the applied loads were well below the critical buckling load of the third mode. Control of the first buckling mode is achieved through the use of all five pairs of piezo-ceramic actuators acting in parallel. The actuators located near the midpoint of the column were excited using a higher voltage than the actuators near the ends of the column, so as to match the relative magnitudes of the bending moments associated with the first mode. To actuate the second mode, the topmost two pairs of actuators were driven in one direction, while the bottommost two pairs of actuators were driven in the opposite direction. The center actuator pair was not used for actuation of the second mode, since it lies partially in the top half and partially in the bottom half of the column.

The control system samples the five pairs of strain gages sequentially at 3000 Hz, acquiring the full set of 5 strain gage readings and performing control operations at a rate of 600 Hz. A modal filter, based on the mode shape estimates produced by finite-element analysis, converts these curvature measurements into modal amplitude estimates.[26] Control is performed based on the modal amplitude estimates for the first and second buckling modes. This approach enables the control system to distinguish curvature changes associated with buckling in the first two modes from curvature changes associated with higher-order mode vibrations. The first and second buckling modes are treated as being independent of one another, and are controlled by independent controllers.

Each mode is controlled by a variant of a P.I.D. control strategy that I refer to as **PD+IV** control. I developed this variant based on experimental observation in order to overcome problems associated with sensor noise, built-in eccentricity of the column itself, and hysteresis in the piezo-ceramic actuators. The final form of the control law includes terms that operate at two distinct time scales to allow the response to high-frequency

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[26]Modal amplitude is expressed in units of 1/5 micron, as measured by the deflection of the maximally deflected point on the modal shape. Thus for the first mode, the modal amplitude would be expressed as the deflection at the midpoint of the column that is due to bending in the first mode. Similarly, for the second buckling mode, the modal amplitude would be expressed as the deflection at the midpoint of the upper half of the column that is caused by bending in the second mode..

perturbations to be specified independently of the response to low-frequency, long lasting effects. For *each* mode, a control voltage is computed as follows:

$$\text{Control Voltage} = \mathbf{20}\ \Big(\ (\mathbf{P}\ \mathit{hf\_position}) + (\mathbf{D}\ \mathit{hf\_velocity}) + (\mathbf{I}\ \mathit{lf\_position}) + (\mathbf{V}\ \mathit{lf\_velocity})\ \Big) \qquad \text{(Equation 4.3)}$$

hf_position refers to the actual modal amplitude data produced by the modal filter based on an average of the two most recent strain gage readings.

hf_velocity is the high-frequency modal velocity estimate produced by a weighted average of multiple sample-to-sample velocity estimates. Specifically:

$$hf\_velocity = \frac{raw\_velocity + (gamma3)(hf\_velocity)}{(1 + gamma3)}$$

where raw_velocity is computed on a sample-to-sample basis:

$$raw\_velocity = \frac{current\_hf\_position - previous\_hf\_position}{amount\_of\_time\_between\_samples}$$

The equation for *hf_velocity* shown above implements an exponential decay to give older velocity estimates less importance than newer velocity estimates. This weighted averaging technique represents a balance between the need to average velocity estimates together so as to reduce noise and the need to respond to velocity changes as quickly as possible.

The low frequency terms are also computed using the exponential decay technique. However, in computing the low-frequency terms, the decay rate is made to be much slower, so that only long-term (about 1/4 second) variations in system behavior are observed:

$$lf\_position = \frac{hf\_position + (gamma2)(lf\_position)}{(1 + gamma2)}$$

$$lf\_velocity = \frac{raw\_velocity + (gamma4)(lf\_velocity)}{(1 + gamma4)}$$

The multiplicative factor of 20 that arises in the equation for control voltage (Equation 4.3) reflects the gain of the high-voltage amplifiers that drive the piezo-ceramic actuators.

The control law parameters are manually adjusted each time the system is operated. Typical values of the control parameters are shown in the table below:

Parameter	**Value**	**Description**
Gamma2	150	L.F. Position decay rate
Gamma3	3	H.F. Velocity decay rate
Gamma4	20	L.F. Velocity decay rate
Mode 1:		
P_{mode1}	-.000395334	H.F. Position Feedback Factor
I_{mode1}	-.000655298	L.F. Position Feedback Factor
D_{mode1}	-.000405908	H.F. Velocity Feedback Factor
V_{mode1}	-.0004	L.F. Velocity Feedback Factor
Mode 2:		
P_{mode2}	-.0001	H.F. Position Feedback Factor
I_{mode2}	-.00001	L.F. Position Feedback Factor
D_{mode2}	-.0004	H.F. Velocity Feedback Factor
V_{mode2}	0.0	L.F. velocity feedback is not used for control of the second buckling mode.

Intuitively, the controller works by using the proportional term (**P**) to counteract small high-frequency disturbances in the vicinity of the equilibrium point. The short time-scale derivative term (**D**) provides damping that helps to prevent overshoot and to ensure that oscillations associated with proportional feedback will die out. The integral term (**I**) is used to provide a long-term (nearly DC) offset that corrects for eccentricity in the column and for any offset of the controller's target position from the true undeflected position of the column. A long time-scale derivative term (**V**) acts to damp out any oscillations induced by the integral feedback.

The control equations shown above produce a single control voltage for each mode. The modal control voltage for the first mode is designated $\mathbf{V_1}$, while the modal control voltage for the second mode is designated $\mathbf{V_2}$. The actual voltage applied to each actuator is determined as follows, where the actuators are numbered sequentially along the length of the column:

Actuator1 = $\mathbf{V_1 C_1 + V_2}$
Actuator2 = $\mathbf{V_1 C_2 + V_2}$
Actuator3 = $\mathbf{V1\ C_3}$
Actuator4 = $\mathbf{V_1 C_4 - V_2}$
Actuator5 = $\mathbf{V_1 C_5 - V_2}$

In other words, each actuator has a constant multiplier associated with it. This constant multiplier determines how much voltage will be applied to a particular actuator in response to a given first mode control voltage. When all five of the actuator-specific multipliers are set to the same value, the five actuators act in tandem, emulating a single continuous piezo-ceramic that applies bending moments to each end of the column. However, by varying the actuator-specific multiplier constants, it is possible to adjust which buckling modes are affected the most by the actuation forces.[27] The values of the constant multipliers that worked best are shown in the table below. Note that these values lie in between those that would come closest to actuating primarily the first mode, and those that would use the full power of the actuators to emulate a column with fixed ends. These coefficients were determined experimentally, by starting with coefficients that affected primarily the first mode[28] and then increasing the magnitude of the coefficients until interactions with higher-order modes became a problem. When C_1 is turned all the way up to 1.0, a loud ringing noise is heard and the system can visibly be seen to be resonating (the lead wires shake!).

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[27]Note that the second mode does not have scaleable actuation forces. This is because with only two actuators available with which to affect the mode, it was not desirable to reduce the size of the actuation force, even at the cost of some spillover of actuation energy into higher buckling modes.

[28] C_1=C_5=.44; C_2=C_4=.814

Actuator Strength Multiplier Constants	
C_1	.87
C_2	1.0
C_3	1.0
C_4	1.0
C_5	.87

To prevent the controller from reacting to measurement noise when the column is in the vicinity of the equilibrium position, position and velocity deadbands were implemented. The deadband forces the modal amplitude (or velocity) to zero any time that it becomes smaller than a threshold value. The threshold values for the deadbands are shown in the table below:

Controller Deadband Thresholds	
1st Mode Amplitude	120 microns
1st Mode Velocity	240 microns/sec
2nd Mode Amplitude	40 microns
2nd Mode Velocity	240 microns/sec

Control Law Tuning

The control coefficients were tuned manually in real-time until suitable values were found. I created a computer keyboard mapping that allowed each of the control coefficients to be adjusted up or down with a single keystroke. Tuning the system was similar to playing a musical instrument: When the controller reached the verge of becoming unstable, a high pitched buzzing was heard due to the excitation of higher order modes. Tuning the system was achieved by considering a real-time phase portrait, a real-time plot of control force and position vs. time, and listening to the sounds made by the system itself.

Due to hysteresis and other non-elastic properties of the system, the equilibrium position varied slightly from one use of the column to the next. At system power-on time, I manually adjusted the target position of the controller to be in the vicinity of the equilibrium position. It turned out that the most reliably control target position is slightly to one side of the equilibrium position, as will be discussed in more detail later in this chapter.

Experimental Results

By controlling both the first and second buckling modes, the active column was able to support a load of 29.88 Newtons, a factor of 5.6 times greater than the experimentally observed uncontrolled buckling load of 5.27 Newtons. The system was able to withstand external perturbations, as illustrated in Figure 4.10. When the column was loaded above 29.88 Newtons, failure occurred in the second buckling mode. Even after failure occurred and the rod applying the load had fallen down to the limit-stop, the first mode remained controlled, with the column coming to rest in an "S" shape characteristic of the second buckling mode. Only after the controller was turned off did the column return to its first-mode shape.

The primary factors preventing the column from supporting even more load were measurement noise and limited control authority. The limited control authority is partially due to the placement of the actuators, which is such that the centermost actuator cannot be used to actuate the second buckling mode. Figure 4.11 shows a phase-space plot of the controlled system. The plot clearly shows that the controlled system contains two stable fixed points, one on either side of the equilibrium position. In other words, measurement noise prevents true stabilization of the system,[29] leaving the zero deflection point unstable and creating two stable fixed points located at very small deflections to either side of the zero deflection point. If measurement noise was reduced, the two stable fixed points would move closer to the equilibrium position.

A factor of 5.6 increase in load-bearing capability is quite significant from a structural engineering point of view. As sensor, control, and actuator technology improves, it is

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[29]True stabilization of the system cannot be achieved because measurement noise prevents the controller from knowing when the system is exactly at the equilibrium position.

clear that even greater increases in strength will become achievable. In the long run, I expect that the factor that limits how much strength increase can be attained through active control of buckling will be some sort of material failure of the column, probably due either to yielding or to plastic deformation.

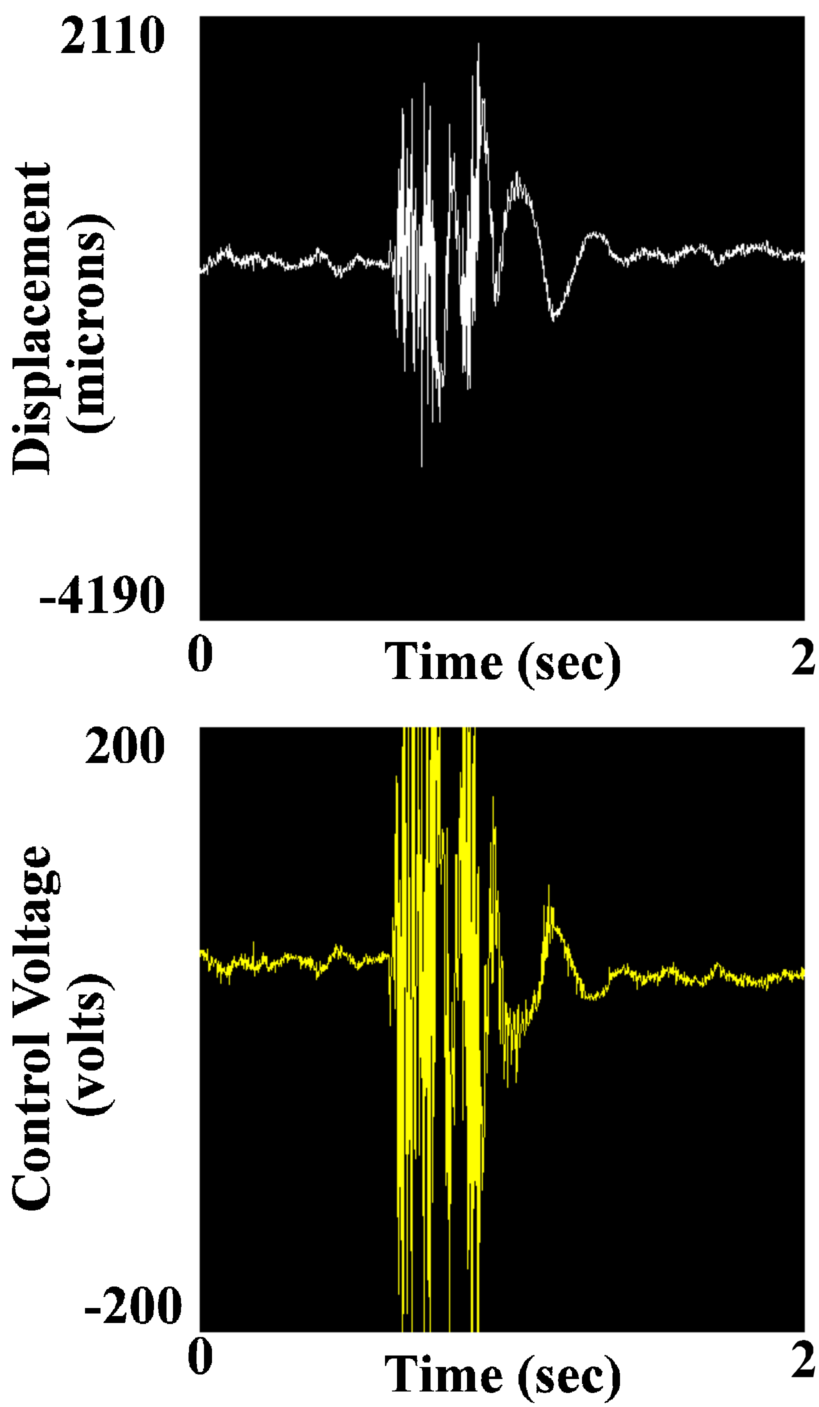

Figure 4.10: Response to perturbation. The upper plot shows the 1st-mode deflection of the column vs. time, while the lower plot shows the control voltage vs. time. The perturbation was induced by banging my fist on the table as hard as I could!

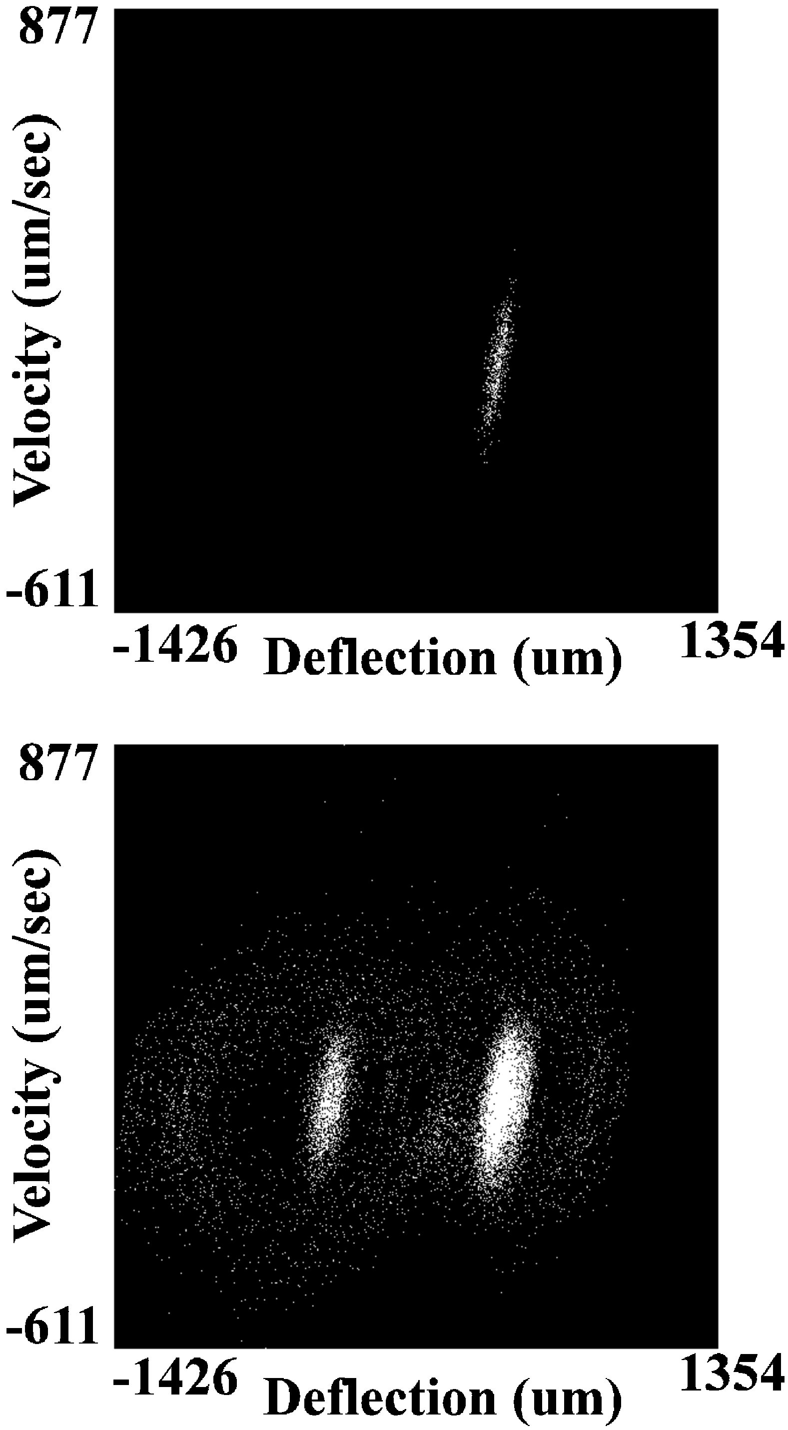

Figure 4.11: Phase plots of the system in operation. The top plot shows the velocity vs. position of the system during normal operation. The bottom plot shows the velocity vs. position of the system undergoing oscillations. This view was created by artificially adjusting the control law so as to induce oscillations that would depict the structure of phase space.

Chapter 5:

Control of Buckling Using Tendons

Prior to conducting the piezo/steel composite experiments, I constructed a much simpler prototype intended to test the feasibility of actively controlling buckling. This early prototype, pictured in Figure 5.1, has a configuration that resembles a boat mast. Actuation forces are supplied by a permanent-magnet electrical motor applying a torque that varies the relative tension between the two tendons, thereby applying a restoring force at the midpoint of the column. A single pair of strain gages measure the curvature of the column at the midpoint, thereby detecting the onset of buckling. Both P.I.D. and bang-bang control strategies were shown to be capable of increasing the load-bearing strength of the column.

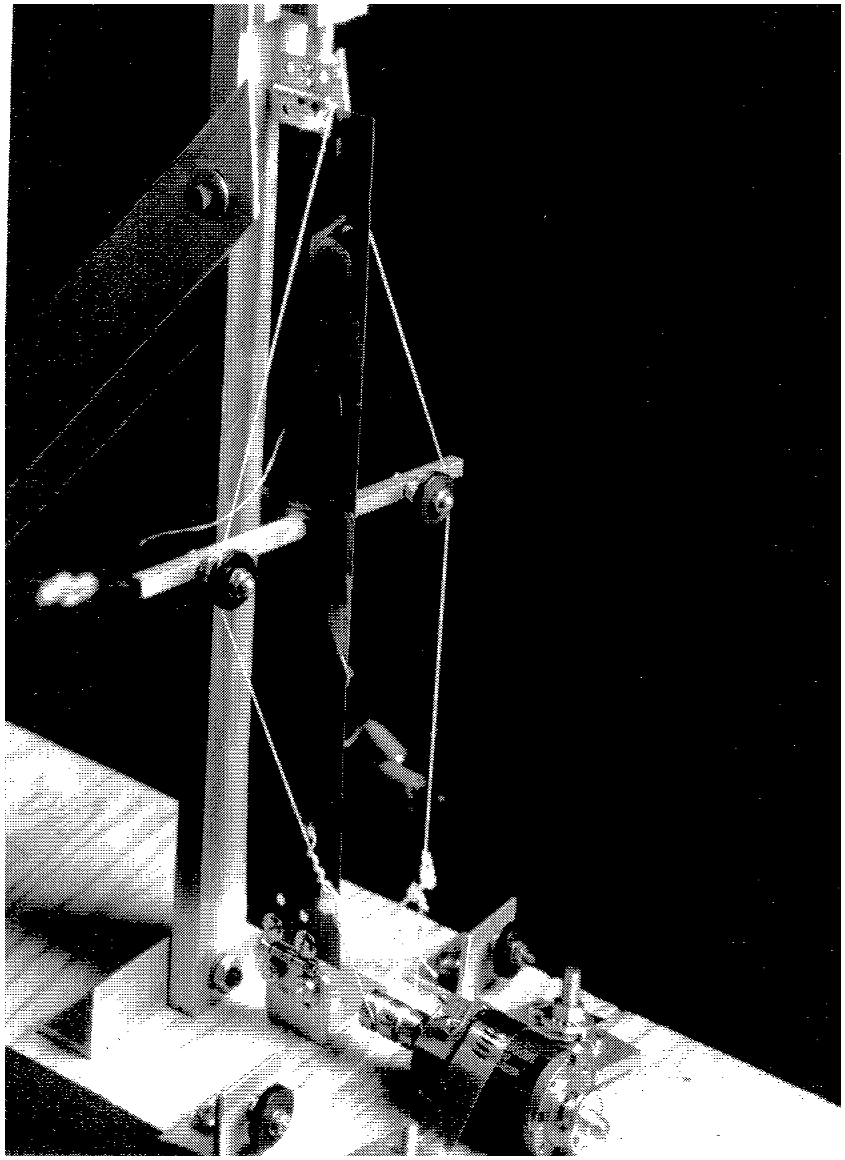

Figure 5.1: Prototype active column. A permanent-magnet motor is used to vary the tension in the tendons so as to apply a net force to the column that counteracts buckling. Strain gages mounted at the center of the column measure the deflection, allowing the controller to stabilize the column against buckling. A yard which passes through the center of the beam is held in place by epoxy. The compressive load is applied by a rod which is supported by a linear ball bearing assembly. A pin mounted on the rod acts as a limit stop that prevents collapse of the column when buckling occurs.

This early experiment was intended to provide a quick proof of concept demonstration that active control of buckling is possible. As such it made use of readily available, non-precision components such as kitchen cabinet hinges and a previously used permanent magnet motor whose characteristics were far from ideal. The column itself consists of a 12 inch long, 1.78 inch wide, 0.010 inch thick piece of spring steel,[30] with a 4-inch aluminum yard mounted perpendicular to the column at the midpoint. The onset of buckling is detected by a single pair of strain gages mounted at the midpoint of the column. The use of only a single pair of strain gages provides a shape measurement that combines the effects of bending in multiple modes, making it difficult to distinguish the shape changes associated with buckling from the shape changes associated with vibration and actuation-induced bending. Nevertheless, active control of buckling was able to increase the load-bearing strength of the column by approximately a factor of 2x.[31]

Control Strategies

I used the prototype column to experiment with a variety of "off-the-shelf" control algorithms. Both bang-bang, and P.I.D. (proportional+integral+derivative) control worked well enough to allow the column to be loaded above the experimentally measured buckling load of 1.5 pounds. Even when using an ad-hoc bang-bang controller that only considers the curvature at the center of the column (ignoring velocity), the column was able to reliably support 2.25 pounds, reflecting a factor of 1.5 increase in the load-bearing capability of the column. A relatively straightforward P.I.D. control strategy, that was much less sophisticated than the P.I.D. variant developed for the piezo/steel composite experiments, enabled the column to support just under 3 pounds, corresponding to a factor of 2x increase in load-bearing strength. These early results showed that using active control to increase the strength of a member is both viable and realizable using existing sensor and actuator technology, and laid the groundwork for the more precise (and more expensive) piezo/steel composite experiment described earlier in Chapter 4.

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[30]The column is 1200 times taller than it is thick!

[31]One potential drawback to this approach is that applying tension to the tendons increases the compressive load on the column itself. For small deflections, this effect becomes insignificant for yards lengths that are a few percent of the column length.

The controller is currently implemented using a programmable computer equipped with an analog interface card that allows it to sense and respond to shape changes in real time. The controller counteracts buckling motion by applying a voltage to the permanent-magnet motor. The force generated as a result of this applied voltage varies depending on the actuation frequency and position of the motor shaft, due to inductance and various non-linearities of the motor assembly itself.

The first experiments were conducted using a bang-bang control law that considers only the curvature κ at the midpoint of the column. Whenever the measured curvature exceeds a certain threshold τ, the actuation voltage $V_{actuation}$ is set to a constant value ν is applied so as to counteract the shape change. When the curvature no longer exceeds the threshold, the applied voltage is removed. In other words:

$$V_{applied} = \begin{cases} +v & (\kappa > \tau) \\ -v & (\kappa < -\tau) \\ 0 & otherwise \end{cases}$$

The controller is tuned by adjusting τ and ν until values are found that add just enough energy to the system each time it passes the threshold to drive the column towards the undeflected position for a variety of loading conditions. Figure 5.2 illustrates the behavior of the system in operation. For a given actuation voltage constant ν, choosing τ to be small permits buckling to be reacted to quickly, thereby allowing large axial loads to be supported. However, since the same actuation voltage ν is applied even if the column is lightly loaded, choosing τ too small can cause the system to overreact to a perturbation, pushing the column too far in the opposite direction, leading to an unstable oscillation. Thus choosing the appropriate actuation voltage and trip-point involves a trade-off between the range of loads over which the control law will be stable, and the maximum load to which the system will be able to react.

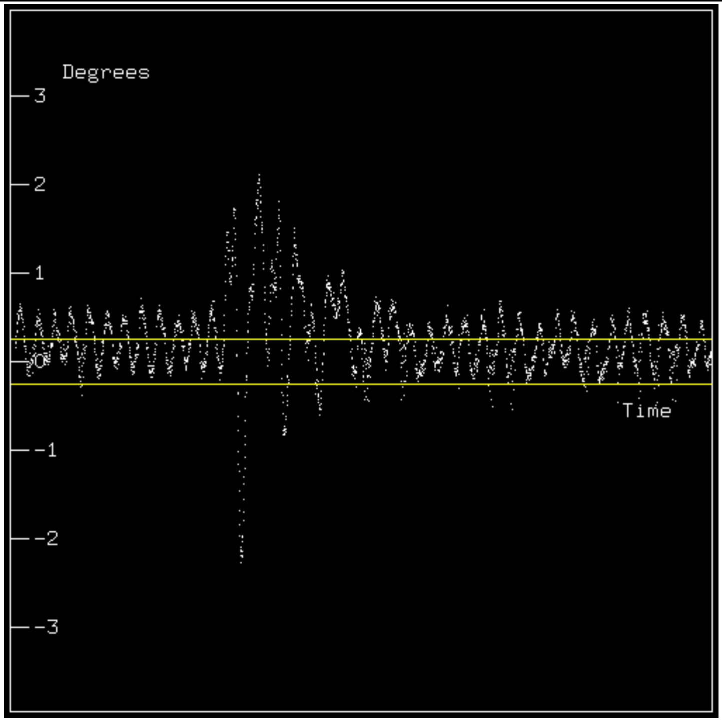

Figure 5.2: Behavior of the column stabilized by bang-bang control. The figure shows the curvature[32] at the midpoint of the prototype column[33] as a function of time, for a time interval of 0.5 seconds. The horizontal lines show the trip points for the bang-bang controller. Most of the time the column oscillates near the equilibrium point. The large excursion shows the response to a perturbation induced by banging my fist on the table. A better controller, such as P.I.D., would result in a more damped response to the perturbation.

Measurement noise is a major factor limiting the performance of both the bang-bang and the P.I.D. control strategies. In both cases, the raw sensor data was sampled at a high rate and multiple samples were averaged together to form the sensor reading that was actually used by the control laws. Nevertheless, sensor noise both due to thermal, electromagnetic, and higher-order vibration effects significantly reduced the amount of curvature that could be reliably measured. In the case of the bang-bang controller,

[32]The curvature is expressed in terms of the angular deflection ϕ of the hinge model for column buckling.

[33]The applied axial load was 2.25 pounds, a factor of 1.5x greater than the uncontrolled buckling load of the column.

measurement noise forces the curvature threshold τ to be set sufficiently large to ensure that the controller does not react to the measurement noise, as this would tend to destabilize the column by turning on the actuator even when the column is not deflected.

The P.I.D. control strategy performed better than the bang-bang strategy primarily because it takes velocity and long-term errors into account. In P.I.D. control, the control voltage $V_{actuation}$ is in part based on the integration of the curvature sensor readings over a period of time, and in part based on a velocity estimate obtained by taking the difference between adjacent sensor readings:

$$V_{actuation} = P\kappa + I\int \kappa \, \mathrm{dt} + D\dot{\kappa}$$

The P.I.D. controller sampled the position of the column at 3000Hz, and averaged together 3 samples to produce a position estimate that was passed on to the P.I.D. controller. Thus the signal going to the actuator was at 1000Hz. Five position estimates were summed together to form the integrated position associated with the **I** term of the control law.

In tuning the P.I.D. controller, the goal was to make P as large as possible without creating so much restoring force that the system becomes unstable by overshooting. Similarly, a goal is to make D as large as possible, since that increases the damping in the system, thereby allowing the P term to be made very large without the risk of drastically overshooting the undeflected position of the column. In practice, measurement noise resulted in very noisy estimates of velocity, thereby limiting the amount of damping (D) that could be introduced into the system. Observation of the system indicated that the proportional (P) term primarily responded to small, high-frequency perturbations, while larger perturbations, such as banging a fist on the table, were counteracted primarily by the integral (I) term. The presence of the damping term enabled the system to recover from external perturbations with less oscillation than was encountered by the bang-bang controller.[34]

The integration time period was adjusted manually until an acceptable response was achieved over the range of perturbations that the system was subjected to. While this

[34]The velocity was estimated based on the difference between position measurements.

approach did work, integrator wind-up proved to be somewhat of a problem. Integrator wind-up occurs when a perturbation causes the column to spend significant time buckled towards one side, allowing the integrator term to build up to a large value. Unless there is a great deal of damping, the integrator term will still have some residual value ("wind-up") even after the actuator has pushed the column back to the undeflected position, which in turn forces the column to spend some time buckled in the opposite direction so as to eliminate the wind-up effect. Experience with integrator wind-up in these early experiments led to the development of the more complicated P.I.D. control strategy used in the piezo/steel column experiments, in which two independent integral terms having different time periods are used, and greater weight is given to more recent sensor readings than to older readings.

Factors Limiting Performance

Significant static friction[35] in the hinges and in the motor itself complicate the system dynamics, causing the system behavior to approximate that of a column with pinned end supports when the column is in motion, but to approximate a column with rigidly fixed ends when standing still. Thus the load at which buckling occurs once the column is slightly in motion is much less than the load which the column can support when it is stationary, since static friction acts to increase the buckling resistance of the stationary column. In general, active control of buckling keeps the column in motion, thereby providing a strength increase relative to the buckling load of a *moving* column. Thus in a system that suffers from a large amount of static friction, it is conceivable that the actively stabilized system will not be able to support as much load as in the uncontrolled static situation, in which the column is simply "stuck" in the unbuckled position.[36] Experimental

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[35]Friction in the system was primarily the result of friction in the hinges (kitchen cabinet hinges are not an instance of precision machining) and friction in the actuating motor itself, which did not rotate very smoothly. Experience with friction in this system is the reason that so much care was taken to avoid friction in the steel/composite experimental apparatus, in which high-precision ball bearings were used to form the hinge assembly.

[36]It is interesting to note that this is the opposite situation than was encountered in Raymond Jefferis' experiment (see Chapter 2). In Jefferis' experiment which used a combination of a peculiar combination of a pinned and a fixed end support, once buckling begins the column switches over from pinned to fixed ends, thereby *increasing* its buckling load and making it difficult to distinguish between the effects of active control and the effects of the change in end-support conditions. On the other hand, in the case of purely pinned ends that suffer from friction, once buckling motion begins the buckling load of the column

measurements showed that the strength increase provided by active control of buckling was sufficient to increase the load-bearing capability of the column even beyond the buckling load of the *stationary* column by approximately a factor of 2x.

One way to measure how significant static frictional effects are is to compare the static buckling load of the column to the theoretical buckling load. In the absence of friction, the experimentally measured buckling load will be less than the theoretical buckling load due to small imperfections and built-in eccentricity in the column itself. The presence of friction increases the experimentally measured buckling load. In the case of the prototype column, the uncontrolled buckling load of the prototype column was measured to be approximately 1.5 pounds. This is *greater* than the theoretical buckling load of approximately 0.3 pounds for a pinned column having the same dimensions, indicating that static friction dramatically increases the ability of the column to resist the onset of buckling motion.

The excitation of higher-order modes limited the strength increase attainable in the prototype column setup. Applying a restoring force to the midpoint of the column does indeed act to stabilize buckling in the first mode, but as illustrated in Figure 5.3b, it also causes the column to bulge in the middle, corresponding to excitation of higher-order[37] buckling modes. This creates two problems. First, since only one sensor was used, the bulge at the middle of the column causes the sensors to incorrectly report that the column is buckling in the other direction, thereby confusing the control system.[38] Second, the energy stored in higher-order modes can produce violent behaviors if the midpoint of the column is restored to the undeflected position too quickly.

Specifically, the frequencies associated with the controller restoring the midpoint of the column to the undeflected position must be kept well below the frequencies at which vibration occurs in the higher-order modes. Otherwise, a violent shape change known as "snap-through" can occur, as illustrated in Figure 5.3, in which the column suddenly snaps

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decreases, thereby making it even more difficult for active control to stabilize buckling, and ensuring that any stabilization that is provided is entirely due to active control.

[37]Primarily excitation of the 3rd and 5th modes was observed.

[38]In the design of the piezo/steel composite column, this problem was avoided by measuring the curvature of the column at five different sensing locations, thereby permitting the amplitude of deflection in each of the first five modes to be determined independently of one another.

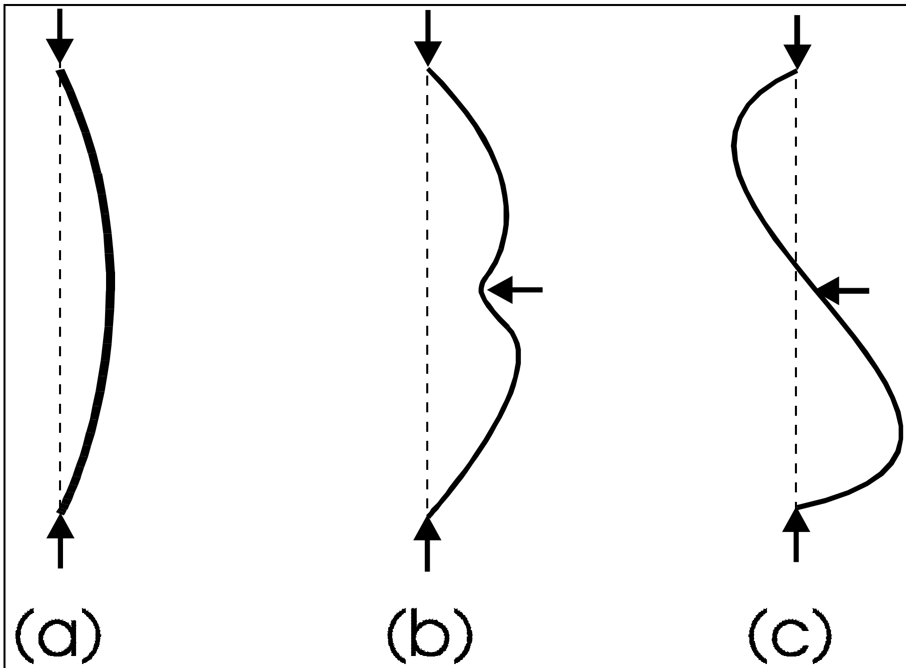

Figure 5.3: Excitation of higher-order modes. **(a)** The column begins to buckle in the first mode. **(b)** A restoring force is applied at the midpoint of the column, causing the column to begin to bulge, exciting the 3rd and the 5th buckling modes. **(c)** The midpoint of the column is quickly moved towards the undeflected position, inducing a high curvature at the midpoint of the column that causes the shape of the column to "snap-through" towards that of the second buckling mode.

from being dominated by the first, third, and fifth modes to being dominated primarily by the second mode. In several experiments involving the bang-bang control law, snap-through was the factor that limited how large a control force could be applied.

Alternative Control Strategies

Both the bang-bang and P.I.D. control strategies are "off-the-shelf" control laws that do not incorporate a model of the dynamics of column buckling. As a result, the "D" term will seek to slow the column down even when it is moving in the appropriate direction, and the "P" term will seek to accelerate the column any time it is deflected, even if it already has enough velocity to return to the undeflected position on its own. These problems can be overcome by using a model-based control strategy in which the forces applied are based on a model of the inherent dynamical behavior of the system. However, such an approach would require knowledge of the system parameters (such as the size of the axial load) which may vary during operation of the system. Such parameters may be measured (using sensors such as load-cells), or may be inferred by observing the dynamical behavior of the system using parameter estimation techniques

Conclusions

Experimental observation of the tendon-based system showed the importance of such effects as static friction, as well as the violent snap-through behavior that can result from the excitation of higher-order modes. These experiments provided valuable insights that were applied to the design of the piezo/steel composite column. Specifically, in the piezo/steel composite, a large number (5) of independently controllable actuators were employed so as to reduce the excitation of higher-order modes; high-precision ball-bearings were employed to provide nearly ideal pinned support conditions; multiple sensing locations were used so as to permit the shape changes associated with the first five modes to be distinguished from one another; and a more complex P.I.D. control strategy was developed to avoid the integrator wind-up problems encountered in the tendon-based experiment.

The prototype column based on tendon actuation clearly demonstrated that active control of buckling can be achieved. Ideally, applying a restoring force at the midpoint of the column would act as a virtual brace, effectively dividing the column into two halves and thereby completely canceling the first buckling mode. One possible direction for future work would be to iterate this approach, using additional actuators to further reduce the effective length of the column. For instance, using two actuators located so as to divide the column into thirds, would allow actuation of the first mode when the actuators work in parallel, while actuation of the second mode would occur when the actuators apply forces in opposite directions. In the next chapter, I describe a number of alternative actuation strategies for which prototypes have not yet been constructed.

Chapter 6: A Compound Structure: Active Control of a Truss Bridge

In order to show that it is possible to incorporate multiple actively stabilized members into a structure, I constructed a small-scale railroad-style truss bridge whose two compressive members actively resist buckling through the use of piezo-ceramic actuators. This bridge, pictured in Figure 6.1, is composed of steel/piezo-ceramic composite beams identical in structural design to the piezo/steel composite column described in Chapter 4. Each active member has its own dedicated control computer and is controlled entirely independently of the other active beam in the structure. In addition to demonstrating that multiple active members can work together to support a structure, this prototype bridge demonstrates that buckling can be prevented even in beams that are oriented non-vertically and are therefore subjected to the laterally destabilizing influence of gravity.

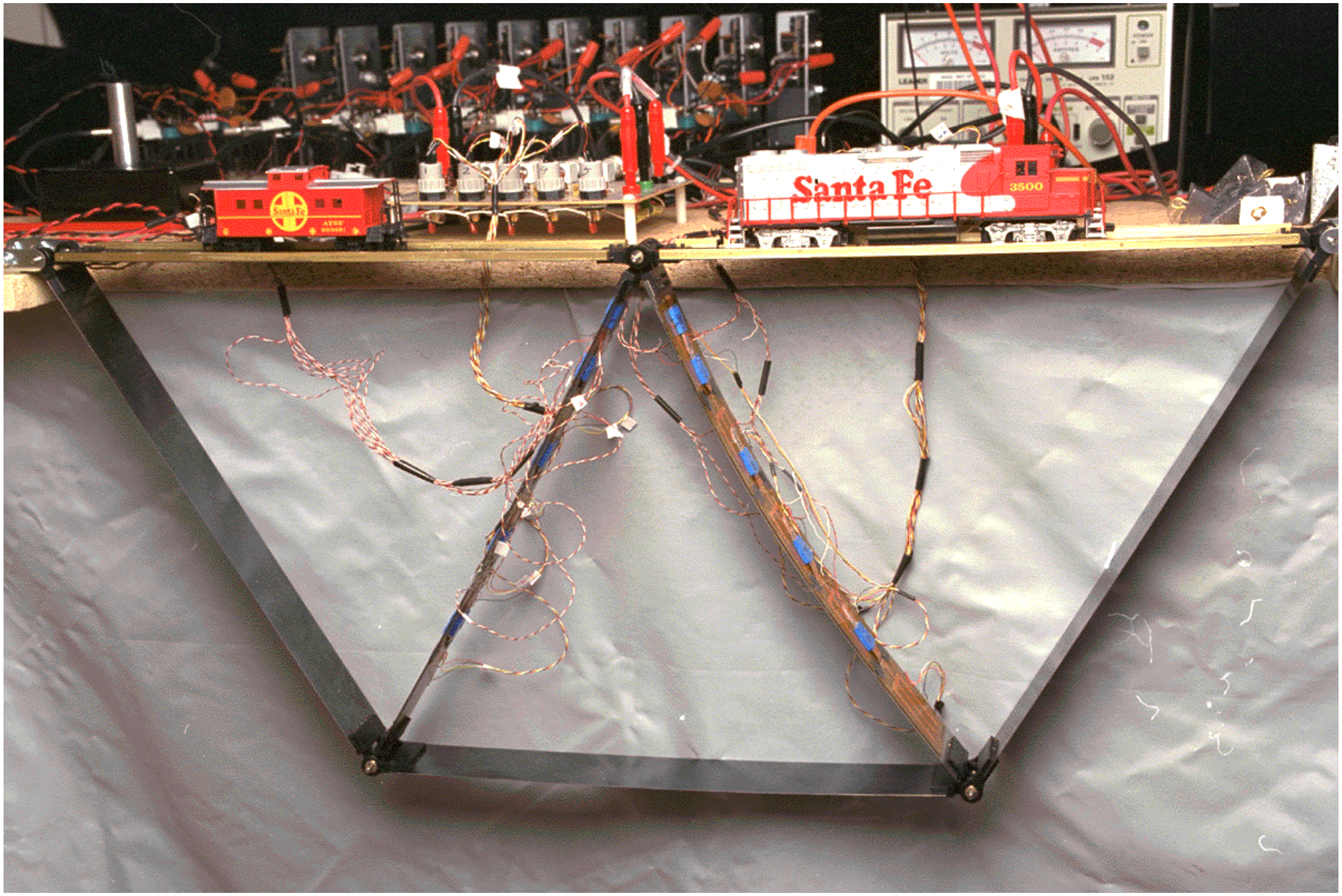

Figure 6.1: Actively Controlled Truss Bridge. This bridge includes two compressive members that actively resist buckling through the use of piezo-ceramic actuators.

Motivation

Compound structures have interactions that are complex and troublesome to model, making it difficult to predict how control actions taken to prevent buckling of one member will influence other members in a structure. Will it be possible to use independent, local strategies to control each active member, or will it prove to be essential that the various active members in a structure communicate with one other to coordinate control actions? It is conceivable that the various active members in a structure will act as coupled oscillators, creating system-level dynamical instabilities similar to those that can arise due to the wind-induced pumping of energy into the vibrational modes of a structure.[39]

Compound structures such as bridges have many structural vibrational modes that involve interactions between members. In most designs, dynamic instability is avoided by arranging for the natural frequencies of the structure to be significantly higher than the maximum frequency of any anticipated external perturbation. However, when active members are incorporated into a structure, the fundamental vibration frequencies are dependent upon the control actions taken by the active members, and external perturbations have the potential to interact with the active members as well as with the vibrational modes of the overall structure. The situation is further clouded by the fact that effects such as the spillover of control actions into higher-order modes may cause energy to pumped between the local vibrational modes of individual elements and the overall vibrational modes of the structure.

The importance of these effects is heavily dependent on the particular structure being considered, as well as on the range of external perturbations that the structure may undergo. In the prototype truss bridge experiments, interactions between the active members proved to be virtually undetectable, clearly demonstrating that dynamical interactions do not preclude the use of multiple actively stabilized members in a structure. A promising direction for future research would be to more precisely characterize the potential interactions between members and to derive control laws that act to preserve dynamical stability.

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[39]The failure of the Tacoma Narrows bridge due to wind-induced oscillations is among the most famous caused by dynamic instability.

Bridge Design

As illustrated in Figure 6.2, the prototype truss bridge consists of seven structural elements. The three tension members, as well as the two members that form the roadway, consist of 12.75 inch long, 2 inch wide, 0.010 inch thick steel plates. The members that form the roadway are each reinforced against bending by brass rods that are bonded to the upper surface of the roadway, as pictured in Figure 6.3. The two compressive members are steel/piezo-ceramic composites of the design described in Chapter 4 (See Figure 4.1).

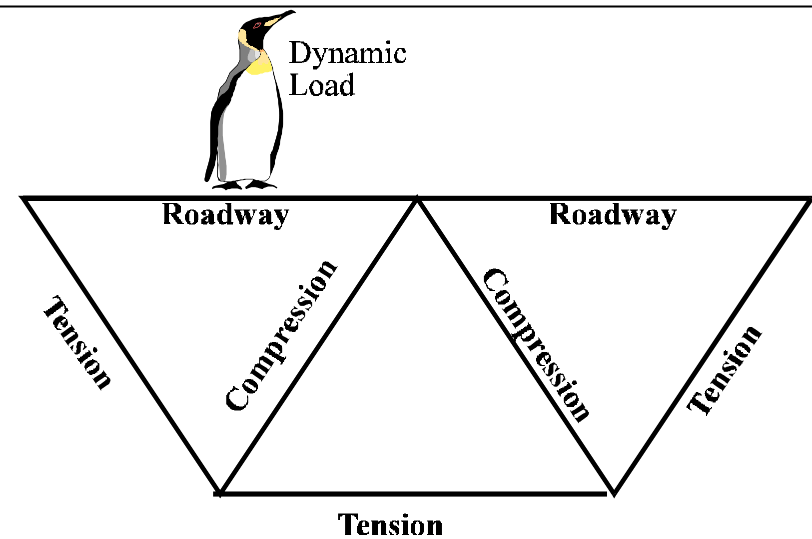

Figure 6.2: Schematic drawing of the prototype truss bridge. All seven elements are the same length, forming three equilateral triangles. The two compressive members are actively stiffened against buckling. Including the hinge mechanisms, each member is approximately 13 inches in length.

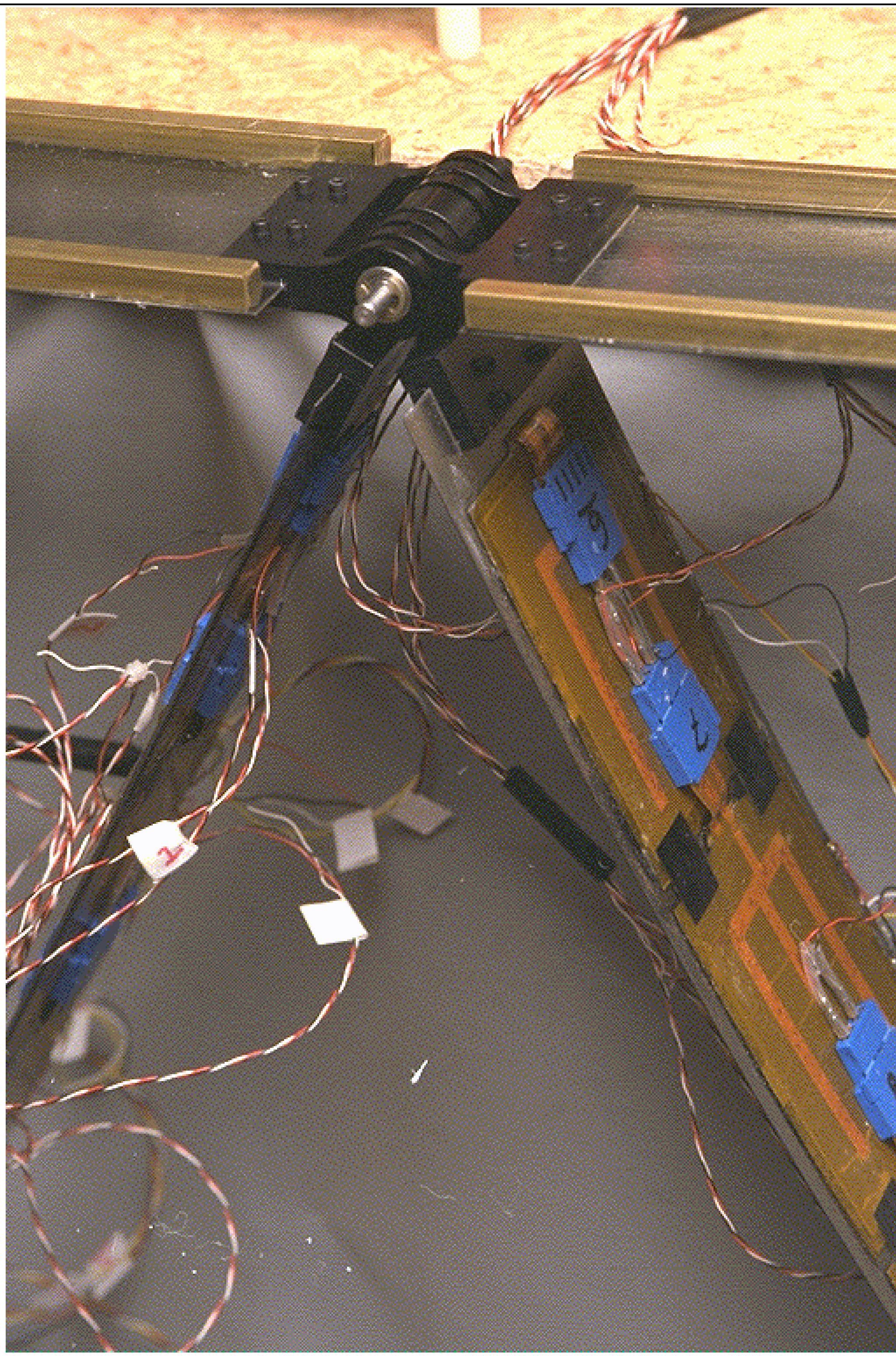

Figure 6.3: Top view of bridge roadway surface. Brass rods bonded to the roadway surface act as stiffeners that prevent bending of the roadway itself.

Although the bridge design called for the compressive members to be identical, one of them was stiffer than the other. (Due to the limited availability of the 0.010" thick piezo-ceramic material, the second column had to be fabricated using slightly thicker 0.012" thick piezo-ceramic.) As a consequence, the compressive member shown on the left in Figure 6.1 is somewhat stiffer than the compressive member on the right. Specifically, the theoretical critical buckling load[40] of the member on the left is 11.7 Newtons, while the theoretical buckling load of the member on the right is 9 Newtons.

Each joint in the bridge is "pinned" using low friction ball-bearings. Each end of the bridge is attached to a wooden board by an aluminum fitting. The aluminum fittings hold the bridge loosely in the horizontal direction, allowing the ends of the bridge to slide slightly towards each other when buckling of the compressive members occurs. The structure of the central hinge causes two pieces of the hinge joint to collide when buckling occurs, thereby acting as a limit stop that prevents total collapse of the bridge when buckling occurs.

Control Strategies

Due to gravity, the dynamics of the active members in the bridge configuration differ considerably from the dynamics in the column configuration. With the controller turned off and no load applied to the bridge, the compressive members sag slightly under their own weight. As compressive load is applied to the column, the eccentricity caused by this sagging increases the bending moments applied to the column, reducing the effective buckling load dramatically, and ensuring that the column will buckle downwards rather than upwards.

Active control of buckling is most effective in the vicinity of an equilibrium position. In the case of a buckling column, the unstable equilibrium position occurs when the column is straight, such that buckling can occur in either direction. However, in the bridge configuration, where each beam is rotated 30 degrees from the vertical axis, the unstable equilibrium point occurs when the beam is bent slightly upwards, such that the bending moment applied by the axial load acts to counteract the lateral destabilizing force of

[40]The critical buckling load specified here is for the vertical column configuration, and hence does not account for the laterally destabilizing role of gravity in the bridge configuration.

gravity. With the column in its slightly bent upwards equilibrium position, a slight perturbation can result in buckling in either direction.

Rather than attempting to counteract buckling with the bridge members in their initial sagging position, I manually elevated the members towards their equilibrium position prior to starting the anti-buckling controller. This manual restoration of the system to the vicinity of its equilibrium point made it possible to use relatively small forces to counteract buckling, rather than the relatively large forces that would be required to counteract the additional bending moments that occur in the case of a downwards-sagging beam.

The two active members were controlled entirely independently of one another, using separate control computers. Each active member was controlled using the same PD+IV control law that was developed for control of the steel/piezo-ceramic composite column described in Chapter 4.[41] As in the case of the actively controlled column, the control law coefficients, including the target deflection, were tuned manually when operation of the system was initiated. The control coefficients were different for each member, primarily as a result of the difference in the thickness of the piezo-ceramic actuators. Values of the control coefficients used are provided below. Note that only the first mode is being controlled: the values of the second mode control coefficient were reduced because there was not sufficient control authority available to control both the first and the second modes in the presence of the destabilizing force of gravity.

[41]The controller for the bridge operated at 400 Hz (sampling the 5 strain gages sequentially at 2000hz) rather than the 600 Hz used for the active column control laws, because the control computer for the second member of the bridge was slower (a 486/33 Mhz) than the control computer that was used for the active column experiments (486/50 Mhz).

Truss Bridge: Control parameters for LEFT member (0.012" piezo-ceramics)		
Parameter	**Value**	**Description**
Gamma2	59	L.F. Position decay rate
Gamma3	2	H.F. Velocity decay rate
Gamma4	20	L.F. Velocity decay rate
Mode 1:		
P_{mode1}	-5.26×10^{-6}	H.F. Position Feedback Factor
I_{mode1}	-.000607841	L.F. Position Feedback Factor
D_{mode1}	$-.00016 \times 10^{-6}$	H.F. Velocity Feedback Factor
V_{mode1}	-1.279×10^{-6}	L.F. Velocity Feedback Factor
Mode 2:		
P_{mode2}	-9×10^{-9}	H.F. Position Feedback Factor
I_{mode2}	-4.5×10^{-9}	L.F. Position Feedback Factor
D_{mode2}	-2.73×10^{-8}	H.F. Velocity Feedback Factor
V_{mode2}	0.0	L.F. velocity feedback is not used for control of the second buckling mode.
C_1 and C_5	0.44	Actuator strength for endmost actuators
C2 and C5	0.814	Actuator strength coefficients
C3	1.0	Actuator strength for center actuator

Truss Bridge: Control parameters for RIGHT member (0.010" piezo-ceramics)		
Parameter	**Value**	**Description**
Gamma2	69	L.F. Position decay rate
Gamma3	2	H.F. Velocity decay rate
Gamma4	20	L.F. Velocity decay rate
Mode 1:		
P_{mode1}	-4.89×10^{-5}	H.F. Position Feedback Factor
I_{mode1}	-0.00176718	L.F. Position Feedback Factor
D_{mode1}	-0.00112515	H.F. Velocity Feedback Factor
V_{mode1}	-2.03×10^{-6}	L.F. Velocity Feedback Factor
Mode 2:		
P_{mode2}	-4.1578×10^{-6}	H.F. Position Feedback Factor
I_{mode2}	-1.5778×10^{-6}	L.F. Position Feedback Factor
D_{mode2}	-2.3682×10^{-6}	H.F. Velocity Feedback Factor
V_{mode2}	0.0	L.F. velocity feedback is not used for control of the second buckling mode.
C_1 and C_5	0.44	Actuator strength for endmost actuators
C2 and C5	0.814	Actuator strength coefficients
C3	1.0	Actuator strength for center actuator

Experimental Results

The truss bridge worked well, with no significant interaction between members observed. A series of external perturbations were applied by hand, by bouncing a finger near the center pin joint on the roadway. For loads[42] below 20.9 Newtons, both active members responded well to the perturbations. The factor that limited how much load could be placed on the bridge was the ability of the thinner active member (the right one) to resist external perturbations.

Figures 6.4 through 6.6 present data plots of the system in normal operation. Note the interesting difference between shape of the phase plots (Figure 6.6) of the behavior of the left member, which tends to circle about the target position, and that of the right member, which tends to cluster at the target position. These same shapes occur for a variety of loading conditions and appear to be characteristic of the behavior of each of the columns for a variety of PD+IV control law parameters. This difference in behaviors may be related to the fact that the left member is thicker, and hence stiffer, than the right column. An interesting direction for future work would be to investigate the underlying causes for these dynamical patterns, as they may provide a clue to potential interactions between the two active members. It would be interesting to see what patterns would arise in a truss bridge composed of two more identical active members.[43]

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[42]Loads applied to the truss bridge are distributed along the roadway surface. The number given in this document to represent the load applied to the bridge is based on the force that the distributed load applies to the pin joint located at the center of the roadway.

[43]Note that the right (thinner) beam exhibited somewhat more sensitivity to measurement noise and was more difficult to control than the left (thicker) beam. It turns out that part of the reason for this effect is that the controller for the right beam in the bridge accidentally differed slightly from the controller for the left beam. In the left beam controller (and in the active column controller described in Chapter 4), the modal amplitudes were computed based on an average of the two most recent strain gage readings. However, for the right beam, the modal amplitudes were computed based on the *single* most recent strain gage reading. As a result, the right beam exhibited somewhat increased measurement noise in comparison to the left beam. This effect was first noticed while the experiment was underway, but the underlying cause was not identified until long after the experiments had been completed.

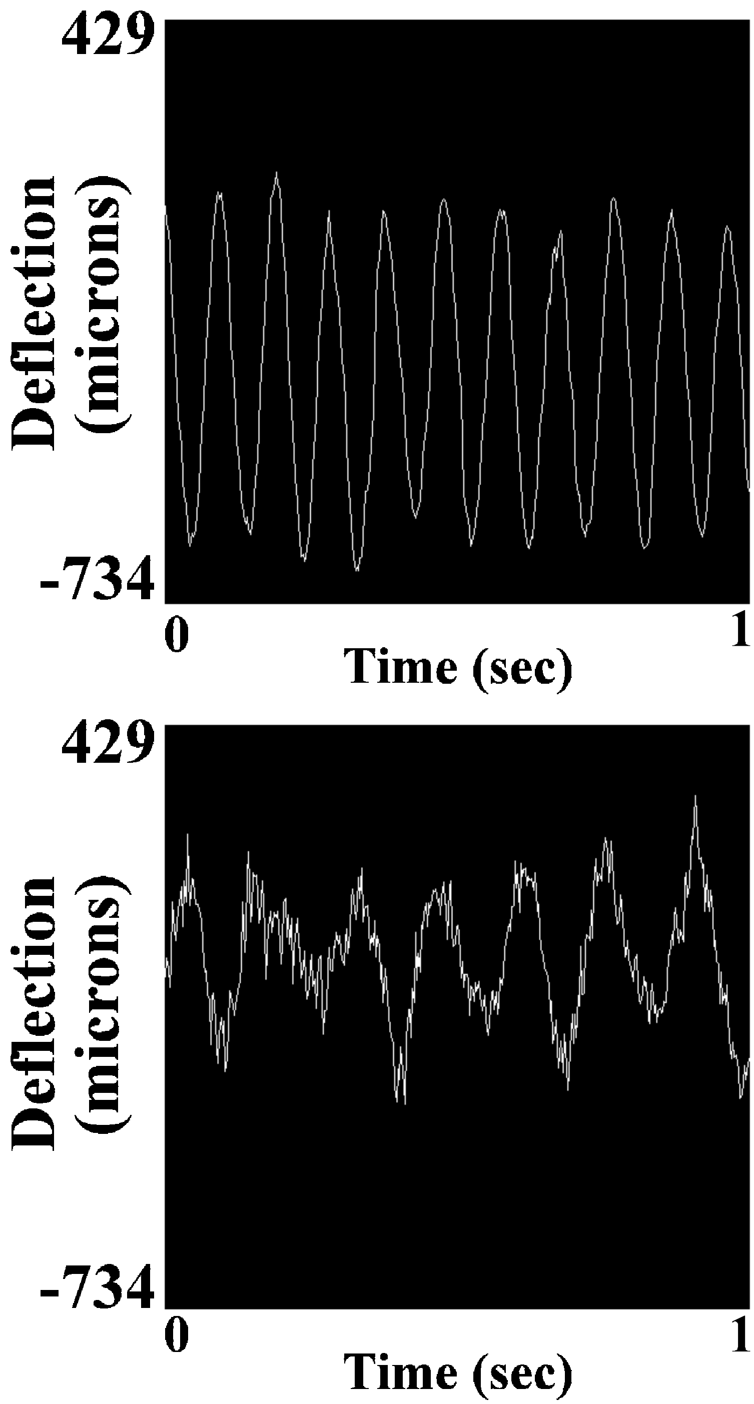

Figure 6.4: Deflection of bridge members vs. time. The plots show the first mode deflection vs. time. The upper plot is for the left (stiffer) member of the bridge, while the lower plot is for the right member of the bridge.

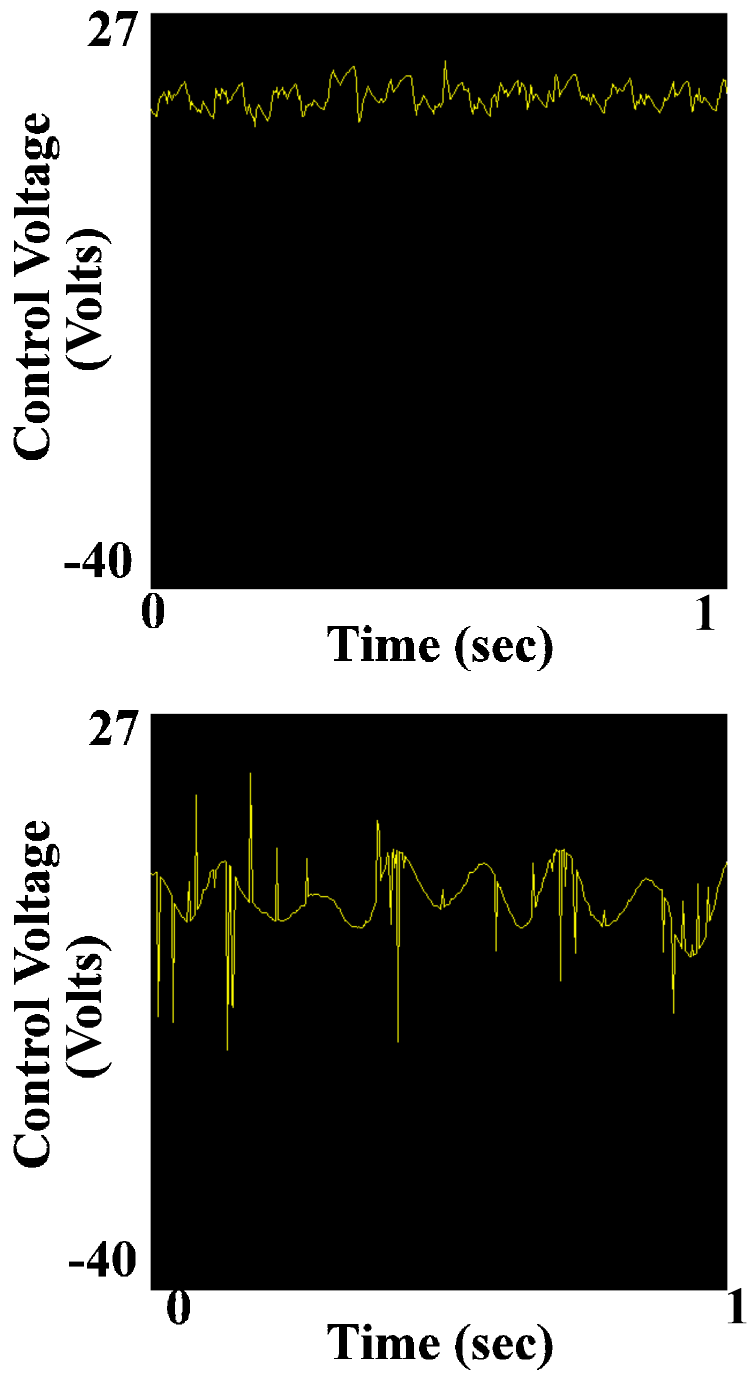

Figure 6.5: Control voltage vs. time. The plots show the control voltage vs. time. The upper plot is for the left (stiffer) member of the bridge, while the lower plot is for the right member of the bridge.

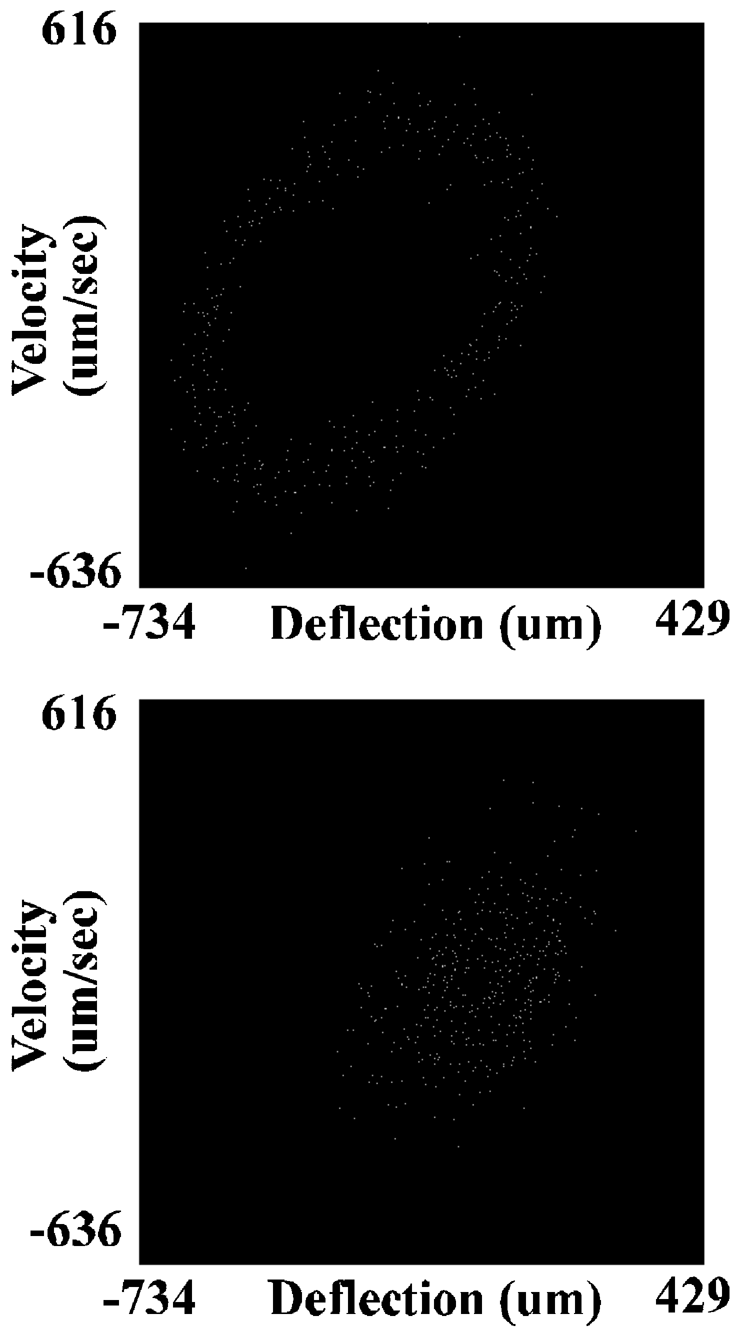

Figure 6.6: Phase plots of bridge in normal operation. These phase plots show the first mode velocity vs. the first mode deflection. The upper plot is for the left (stiffer) member of the bridge, while the lower plot is for the right member of the bridge. Note how the left member's trajectories resemble a circle, while the right member's trajectories appear to clump in the vicinity of an attractor.

Chapter 7:

Actuation Strategies

The experiments performed using the active column and truss bridge laboratory prototypes clearly demonstrate that active control of buckling can be achieved. But how can actuation forces best be applied in a real structure? It would certainly be difficult to imagine running variable-tension tendons from every load-bearing column in a large building to ground-mounted actuators. On the other hand, it is much easier to imagine connecting variable-tension strengthening tendons to the landing struts of an aircraft. In this chapter, I present a variety of potential strategies for applying the actuation forces required to counteract buckling, and suggest some applications for which particular actuation strategies seem well-suited.

Any actuation strategy for active control of buckling must provide a mechanism for applying forces to a structural member. This may involve any combination of applying bending moments to the member; inducing strain in the member; or applying restoring forces at various points along the member. The discussion below suggests a variety of potential strategies for taking advantage of the physical phenomenon that are capable of applying such forces.

Methods of Applying Bending Moments

Active End Supports

I propose a new type of end support for compressive members, an *active pin*, which combines the best features of *fixed* end supports and *pinned* end supports. A pinned end support allows the ends of the column to rotate freely, while a fixed end support does not allow the ends of the column to rotate at all. The advantage to fixed end supports is that they resist buckling: the critical buckling load for a column with fixed end supports is four times larger than the critical buckling load for a column with pinned end supports. Unfortunately, fixed end supports have a serious drawback: when the structure being supported by the column flexes, expands, or is not perfectly aligned, the fixed end support can actually force the ends of the column to rotate, thereby inducing buckling. I propose that an active pin can prevent the ends of a column from rotating, thereby resisting buckling, while isolating the column from rotary motion of the structure being supported.

The active pin consists of a traditional pinned joint coupled with a rotary actuator and sensors. When the active pin senses that the end of the column is beginning to rotate due to the onset of buckling, the rotary actuator applies a restoring torque to counteract the buckling motion. In this way, the ends of the column are *actively* restrained against rotation, but are still pinned. This arrangement allows the active pin to isolate the column from flexure of the supported structure. To achieve this isolation, it is necessary for the active pin to be able to distinguish rotations caused by the onset of column buckling (which should be counteracted) from rotations caused by movement of the structure being supported (which should not be acted upon). This distinction may be detected by measuring the shape of the column directly using a device such as a strain gage, or by measuring the rotation of the ends of the column and subtracting out the rotation of the supported structure. The rotation of the supported structure may be measured against a fixed reference vector such as gravity using a device such as a pendulum. A potential disadvantage to this approach is that the torque applied to the column by the rotary actuator is also applied to the structure being supported. This coupling is not significant in most cases, since the structure being supported tends to be very stiff relative to the loaded column. Nevertheless, coupling would probably prevent this technique from being used to support structures that are locally flexible.

Bending Moments Applied by Tendons Pulling on Anchors

The active end support described above counteracts buckling by applying bending moments to the ends of a column. There are other techniques that may also be used to apply bending moments to a column. For instance, in an actuation arrangement proposed by Zuk [43], linear actuators could be used to induce bending moments in a column by pulling on tendons connected to anchor points on the column, as illustrated in Figure 7.1. Linear actuators are relatively lightweight and cost effective, yet have the potential to apply large forces. Inclusion of anchor points and a linear actuator introduce only very minor changes to the overall column geometry, and have the potential to scale to very large column sizes and load capacities. Applying bending moments via linear actuators and tendons is attractive in that it does not rely on any external supports, and (when used with pinned ends) does not transmit bending moments to the structure being supported. Another feature of this approach is that multiple actuation sites can be included on a single column, so as to achieve independent control of multiple buckling modes.

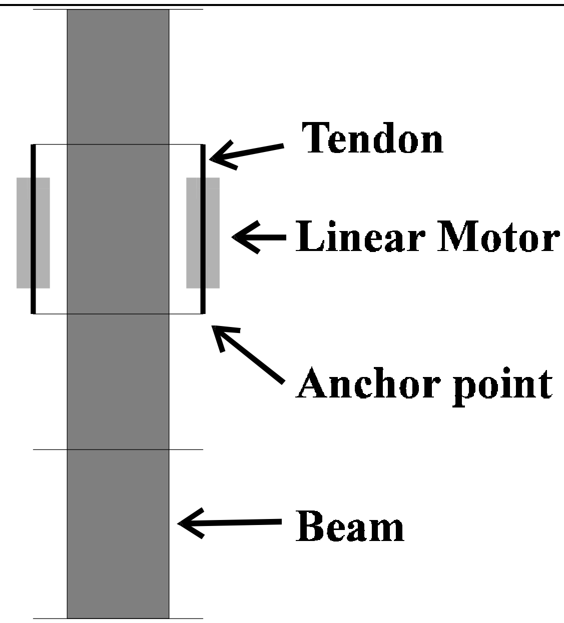

Figure 7.1: Bending moments may be applied by using a linear actuator, such as an electromagnet, to pull on tendons attached to anchors. An alternative would be to use a motorized turnbuckle style rotary actuator rather than a linear actuator to apply forces to the tendons..

Induced Strain Actuation

Piezo-ceramics

Through the use of "smart materials" such as piezo-ceramics, it is possible to apply bending moments to a column by inducing strain on the column's surface. As described in Chapter 4, a smart material may be used to apply bending moments to a column by arranging for the material bonded to one side of the column to grow while material bonded to the other side of the column shrinks. The induced-strain approach is attractive since it maintains the inherent geometry of the member, in that it does not require external tendons, actuators, or supports.

PVDF

PVDF (polyvinylidene flouride) is a film that is similar in consistency to mylar. It is also a piezo-electric material, although its piezo-electric properties are a factor of 20 weaker than those of piezo-ceramic material. PVDF, which is relatively inexpensive, is available in thin sheets that may be applied to a member. As discussed in Chapter 2, Meressi and Paden [27] have recently published a theoretical analysis showing that under ideal conditions (i.e. no sensor noise), PVDF can stabilize the first buckling mode of a highly elastic column. It seems unlikely that PVDF will have a role to play as an actuator in large-scale structures, but may prove effective for use in microdevices and in small-scale laboratory experiments.

Nitinol Film

Another potential way to induce strain in a column is through the use of a film form of the Nitinol shape memory alloy. As Nitinol is based on a thermal effect, the bandwidth available through Nitinol actuation is dependent on how quickly the temperature can be changed. Although Nitinol can be heated up relatively quickly by running an electrical current through it, cooling the material off quickly requires that it be operated in a region well above the ambient temperature, leading to highly inefficient operation. Nitinol is typically found in wire form, where it has a response time on the order of several seconds. The thin film form of Nitinol would have a very small heat capacity, and thus promises to achieve much faster thermal cycling times. However, since in a large-scale buckling control application, the Nitinol would be attached to a (typically metal) substrate, the small heat capacity of the thin film would likely be overshadowed by the large heat capacity of the substrate, resulting in slow response times.

Bracing with External Supports

Hydraulic Ram

A relatively straightforward way to apply stabilizing forces to the midpoint (or other location) of a member is through the use of a linear hydraulic actuator. This option is particularly attractive for use in environments such as aircraft and ships, where reliable hydraulic power is already available.

Electromagnetic Approach

The electromagnetic approach, originally suggested by Raymond Jefferis (see Chapter 2), provides a means of transferring force from an external support to a structural member without the need to make physical contact. This may prove practical in manufacturing applications, where it is feasible to have an external support located near a load-bearing member.[44]

Tendon Actuation (External Actuator)

A tendon based approach was used in the experimental prototype column described earlier in Chapter 5. An externally mounted motor applies restoring forces to the midpoint of the column by varying the tension between the tendons on either side of the column, as discussed in Chapter 5. This approach is attractive in situations where it is desirable to locate the actuators some distance from the member being controlled. For instance, in the case of an actively stabilized bridge that spans a chasm, the actuators could be located on either side of the chasm, transmitting control forces via tendons to compressive members that span the chasm. The advantage that this approach has over the active yard approach is that the column being stabilized need not support the weight of the actuator, thereby reducing the constraints on the size and type of actuator that may be employed. For instance, the actuators that strengthen a portable bridge could be mounted in a truck that is parked at the end of the span.

Bracing Without External Supports

Sand-Filled Support

Nippon Steel has developed a sand-filled cylinder that is used to surround a column so as to provide support that stabilizes the first buckling mode. As the sand weighs a large amount, this approach is most practical for use on the bottommost supporting columns of

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[44]Another possible way to apply force without making contact would be through electrostatics, or perhaps even by using pressurization to blow air or fluid at a member.

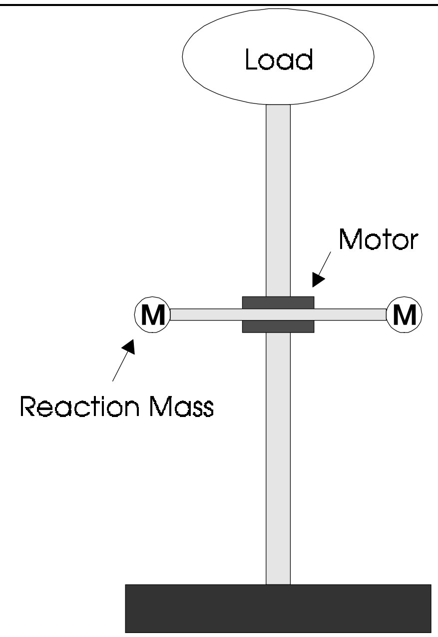

Figure 7.2: Inertial Approach. A linear motor mounted at the midpoint of a column may be used to apply a force that counters the buckling motion by accelerating a reaction mass.

large structures. Although this is a passive technique for stabilizing a column against buckling, I mention it here as it illustrates the importance that control of buckling has in the structural engineering community. An active system promises to provide greater strength improvements at a fraction of the weight of the sand-based passive system.

Inertial-Mass Actuation

Perhaps the simplest way to stabilize a column against buckling is to use the same technique that humans do to balance: moving mass around to generate reaction forces. A linear motor and sliding mass fastened to the midpoint of a column can be used to apply a normal force to the midpoint of the column. When a sensor such as a strain gage indicates that buckling is starting, the motor applies a force that opposes the buckling motion. This force is generated by accelerating the reaction mass, as illustrated in Figure 7.2. Like the electromagnet approach, the reaction mass approach applies a restoring force to the midpoint of the column. However, unlike the electromagnet approach, use of a reaction mass does not require any other members or ground-based anchors for support.

The reaction-mass approach relies on symmetry to limit the range of motion the reaction mass will undergo. By overreacting to perturbations, as in the canonical control theory problem of balancing an inverted pendulum [31], the control system can compensate for asymmetries by arranging for the column to buckle first in one direction and then in the opposite direction, allowing the reaction mass time to return to the center of its motion range. Nevertheless, an unfortunate sequence of unidirectional perturbations can cause the reaction mass to reach the physical limit of its motion. Thus it is be desirable to supplement the reaction-mass force with a method of applying a relatively constant force to the column to correct for asymmetries. The reaction-mass would be used to respond to high-frequency perturbations, allowing the correction force (which corrects for long-term asymmetries) to be applied by a slower-reacting actuation strategy such as the Nitinol deflectors developed by Baz [2].

Tendon Actuation (Active Yard)

A set of tendons arranged in a configuration that resembles a boat mast can be used to apply a restoring force to the midpoint of a column, as illustrated in Figure 7.3. In this configuration, a small beam (the "yard") is mounted perpendicular to the long column. Tendons anchored to the top and bottom of the column are attached to the yard. When buckling is detected, an actuator moves the yard relative to the center of the column. Since the tendons apply forces that resist the motion of the yard, it is the column itself that moves, thereby countering the buckling motion. This approach has a significant advantage over the inertial mass approach in that it is capable of applying a constant force to the beam in order to counter asymmetries. However, it has the disadvantage that additional material is required to form the yard, and that the forces exerted by the tendons vary the compressive load applied to the beam.

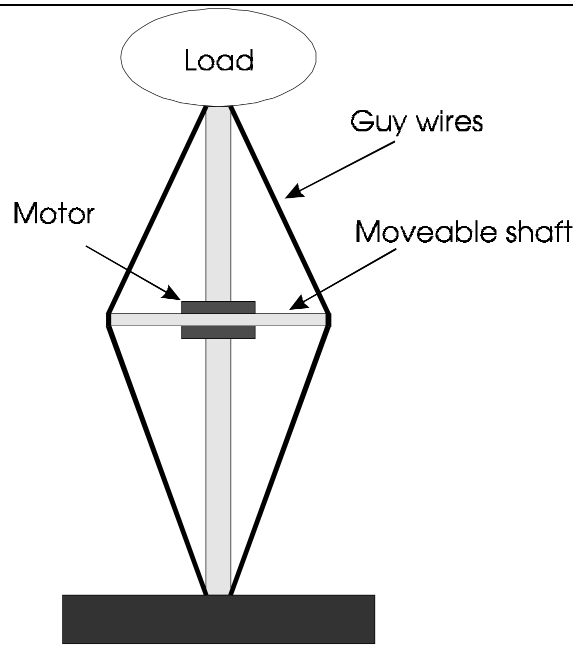

Figure 7.3: Active yard approach. An active yard mounted perpendicular to the column moves the midpoint of the column relative to the yard so as to counteract buckling. Tendons anchored to the top and bottom of the column hold the position of the yard fixed.

External Combustion

Tom Knight [22] has suggested the possibility of filling the center of a column with explosives to be used as a means of counteracting buckling motion. The idea is that when the onset of buckling is detected, a small amount of explosive would be jettisoned and then ignited remotely by a laser. The resulting explosion would apply a restoring force that counteracts the buckling motion. I have not experimented with this approach, although it seems plausible for use as a last resort type of safety measure when collapse of a structure is imminent.

Chapter 8: Conclusions and Suggestions for Future Work

The results presented in this dissertation show that it is possible to increase the load-bearing strength of a structure by incorporating compressive members that actively resist buckling through computer-controlled adjustment of their dynamical behavior. The prototype steel/piezo-ceramic composite column demonstrates that buckling can be prevented without the need for external braces or supports and provides the first experimental evidence that multiple buckling modes can be stabilized simultaneously. The truss bridge experiments demonstrate that it is possible to incorporate multiple independently controlled active members into a structure without inducing undesirable interactions between members.

The prototype columns described in this dissertation performed well. Using resistive strain gages as sensors, piezo-ceramics as actuators, and non-adaptive control strategies, the load-bearing strength of the steel/piezo-ceramic composite column was increased by a factor of 5.6. However, in principle one could achieve even greater strength increases through the use of improved sensing, actuation, and control strategies.

One of the primary factors limiting the performance of the prototype columns was limited sensor accuracy, particularly with regard to the measurement of velocity. While the resistive strain gages used as sensors provided direct measurements of column shape, the velocity estimates derived from these measurements were relatively noisy. In contrast, materials such as PVDF film provide highly accurate measurements of shape *changes*, but do not provide an absolute measurement of shape. One possible direction for future work would be to use both velocity sensing mechanisms, such as PVDF, and shape-measuring devices, such as strain gages, to provide low-noise measurement of both position and velocity. Another possibility would be to use semiconductor-based strain gages, which are more accurate than resistive strain gages.

The prototype active column used five discrete strain gages mounted at different locations along the column to measure column shape. Based on mode shape information obtained through finite-element analysis of the column, these five sensor readings were computationally combined in real-time to obtain an estimate of the amplitude of deflection in each buckling mode. An alternative approach would be to use a continuous modal sensor (see [25,13]) that is specially shaped to measure modal amplitude directly.

Another possible direction for future work, particularly in the domain of constructing larger-scale structures, would be to experiment with alternative actuation strategies. Although the use of piezo-ceramics to apply bending moments to a member is attractive for small-scale structures, such as laboratory prototypes, for larger-scale structures it may prove more practical to use electromagnetic effects to supply the forces required to counteract buckling. One possibility worth investigating would be to use electromagnetic motors to drive tendons that pull on anchors to apply bending moments to a member, or to drive an active pin joint, as described in Chapter 7.

The control strategies used to control the prototype columns were basically “off-the-shelf” control strategies that were not based on knowledge about the dynamics of the system being controlled. For instance, the P.I.D. control strategy seeks to slow down the column even if it is moving in a desirable direction. A model-based control strategy along the lines of that suggested by Zhao [42], which takes the inherent dynamical behavior of the column into account, holds the promise of being better able to control dynamical behavior. Another possibility would be to experiment with adaptive control strategies, which observe the behavior of the system and automatically adjust the control law parameters to eliminate undesirable behavior patterns. For instance, in the case of a bang-bang controller, the bang-bang trip points and actuation voltage could be automatically adjusted to account for variations in the load being applied to the column. A more sophisticated approach could use techniques such as Kalman filtering to estimate the parameters of the dynamical system by observing its behavior.

Although the actively controlled truss bridge shows that it is possible to incorporate multiple actively controlled members into a structure, it does not tell us under what circumstances this will be possible. A design approach needs to be developed to address the structure-level issues associated with the interactions between actively controlled members and the structures in which they are embedded. One possible direction would be to develop control strategies that take into account, or even take advantage of, the interactions among the active members. Such a control strategy could be centralized, requiring that a data network interconnect the various structural elements. Another possibility would be to use purely local control strategies, in which each member is controlled individually using local information. Through clever design, these locally controlled elements could produce globally desirable behaviors, either by using the dynamical behavior of the structure itself to communicate with one another to coordinate actions, or by using techniques for generating complex large-scale behaviors from locally acting agents [41].

As mentioned in Chapter 2, Amr Baz [3],[4] has developed a technique for actively redistributing the load of a structure among multiple redundant support columns, so as to ensure that no particular column becomes loaded beyond its critical buckling load. It would be interesting to create a system that combined Baz's load limiting approach with the dynamical strengthening approach described in this dissertation. In addition to ensuring that the load applied to a member would never exceed the ability of the dynamic control system to counteract buckling, this combination of approaches could be used to simplify the dynamical control problem by limiting the rate at which the load applied to a member is permitted to change.

Potential Applications

One of the most important applications for active control of buckling will be to supplement traditional designs, providing an added factor of safety in the form of "emergency strength" for exceptional situations. For instance, when an airplane makes an unusually hard landing, the landing struts could be given added strength by actively controlling buckling. Another potential application lies in the prevention of metal fatigue. For instance, in a phenomenon known as wave-induced whipping, compressive members supporting the hulls of large ships buckle in heavy sea conditions due to wave action pounding on the hull [17]. Fortunately, the duration of the forces applied by a wave is (usually) short enough that buckling does not progress to the point of causing the immediate failure of the member. However, after repeated loading cycles, the buckling motion causes metal fatigue. Active control would counteract the motion induced by the wave action, thereby preventing buckling-induced metal fatigue.

Active control of buckling promises to make it possible to create structures that are both stronger and lighter than would otherwise be possible. One possible application would be to create very lightweight structures such as a portable bridge. Another would be to create architectural topologies that would not otherwise be possible, such as those that make use of very tall, thin columns, or Zuk's suggestion of building a city on top of an existing city.

Active control of buckling may also have an important role to play in earthquake engineering. One possible application would be to increase the strength of compressive members so as to make them better able to resist earthquake-induced dynamic loads. However, a potentially more important aspect of active control of buckling is that it provides a structural designer with a new option: a compressive member that can actively be made strong during normal operation to provide resistance to wind-induced vibration, but which can be allowed to flex during an earthquake to allow the structure to sway in response to the earthquake.

Much work remains before we reach the point where it is practical to embed active dynamical control components in objects on as routine a basis as we now make use of ordinary paint. The work presented in this dissertation represents a step towards the design possibilities that will become available when active dynamical control components are routinely embedded in the objects and structures around us.

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