Analysis of the Operational Impacts of Alternative Propulsion Configurations on Submarine Maneuverability

by

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ABSTRACT

In an effort to develop submarine designs that deliver reduced size submarines with equivalent capabilities of the current USS VIRGINIA (SSN-774 Class) submarine, a joint Navy/Defense Advanced Research Projects Agency (DARPA) called the Tango Bravo (TB) program was initiated in 2004 to overcome technology barriers that have a large impact on submarine size and cost. A focus area of the TB program is propulsion concepts not constrained by a centerline shaft.

This thesis investigates the operational impacts that a conceptual propulsion configuration involving the use of azimuthing podded propulsors has on a submarine. Azimuthing pods have been used commercially for years, with applications on cruise ships being quite common although their use on large naval platforms has been nonexistent to date. The use of such systems on a submarine would allow for the removal of systems related to the centerline shaft; freeing up volume, weight, and area that must be allocated and potentially allowing the submarine designer to get outside the speed-size-resistance circular path that results in large, expensive platforms. Potential benefits include having the pods in a relatively undisturbed wake field -possibly increasing acoustic performance as well as improving operational maneuvering characteristics.

For this thesis a submarine maneuvering model was created based on analytical techniques and empirical data obtained from the DARPA SUBOFF submarine hullform. This model was analyzed for two configurations:

- A centerline shaft configuration utilizing cruciform control surfaces for yaw and pitch control
- A podded configuration utilizing pods for propulsion as well as yaw and pitch control

The maneuvering characteristics for each configuration were investigated and quantified to include turning, depth changing, acceleration, deceleration, and response to casualties.
ACKNOWLEDGEMENTS

I would like to thank my Thesis Advisor, Professor Triantafyllou, for providing me the opportunity to investigate this interesting topic. In addition, I would like to thank the Navy for allowing me to study here at MIT. Lastly, I would like to thank Christine, Derek, Scout, and Cassie for all the support they have provided.
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CHAPTER 1 INTRODUCTION

1.1 BACKGROUND AND OVERVIEW

In recent years, the U.S. Navy has made great strides towards advancing the capabilities of its warships while at the same time reducing acquisition and life-cycle costs. On surface vessels this has involved the increasing use of all-electric ships that employ electric motors powered by a combined ship service/ship propulsion electric plant. This provides flexibility in how loads are shared between propulsion, combat, and auxiliary systems as well as providing significant electrical power margins that allow for future additions of high energy combat systems such as directed energy weapons and advanced radars. Although all-electric ships have been fielded in commercial applications such as cruise ships in recent years, this is a relatively new direction for the Navy. Significant resources are expended used to build an U.S. engineering base for future electric ships through the Electric Ship Research and Development Consortium (ESRDC) as well as the research and development of a land based prototype electric plant for use on naval combatants. Ship classes such as the LEWIS AND CLARK Dry Cargo/Ammunition ships (T-AKE class) and the ZUMWALT Class Destroyer (DDG-1000 class) are two examples of recent all-electric Navy ship designs.

The U.S. Navy has built electric drive submarines in the past. Two such examples are the TULLIBEE (SSN-597) and the GLENARD P. LIPSCOMB (SSN-685). These early electric submarines, designed and built in the 1960's and 1970's, experienced significant operation problems and utilized a centerline shaft propulsion configuration similar to the majority of U.S. and foreign submarines that have been built to date. Because of these problems – mostly attributed to the relatively immature electrical propulsion plant equipment – the Navy continued to design and build primarily mechanical drive submarines.

The advances made in surface ship all-electric designs and equipment in recent years should be transferable to submarine design and construction. Should a reliable all-
electric drive submarine be built, this would present the opportunity to explore means of propulsion other than a centerline shaft configuration that is currently the norm.

1.2 MOTIVATION

In an effort to develop submarine designs that deliver reduced size submarines with equivalent capabilities of the current USS VIRGINIA (SSN-774 Class) submarine, a joint Navy/Defense Advanced Research Projects Agency (DARPA) program was initiated in 2004 to overcome technology barriers that have large impact on submarine size and cost[7]. This program, called Tango Bravo, is focused on five main areas:

1. Propulsion concepts not constrained by a centerline shaft.
2. Externally stowed and launched weapons.
3. Conformal alternatives to the existing spherical sonar array.
4. Technologies that eliminate or substantially simplify existing submarine hull, mechanical and electrical systems.
5. Automation to reduce crew workload for standard tasks.

The first focus area, alternative propulsion configurations to the centerline shaft is the motivation behind this work.

The typical centerline shaft configuration in submarines today locks the submarine designer into a constrained set of propulsion train equipment with shaft seals, vibration reducers, thrust bearings, couplings, and reduction gears. For designs requiring more speed and more horsepower, these components must be increased in size and weight to accommodate the power. This results in a larger submarine that has more resistance, and therefore requires more power. A vicious circle ensues with a resulting submarine design that is not only large, but also very costly to build. By moving away from the centerline shaft propulsion configuration – which an all-electric plant would allow – this circle is broken and the potential exists to design smaller and less expensive submarines for a given speed.

There are many ways in which a submarine can be propelled without a centerline shaft driving a propulsor. Flapping foils, azimuthing podded propulsors, magnetohydrodynamic or water-jet drives are all potential candidates for an alternative
propulsion configuration. All present advantages and disadvantages, as well as technical
difficulties in employing such systems in an actual submarine.

This thesis investigates the operational impacts that a conceptual propulsion
configuration involving the use of azimuthing podded propulsors has on a submarine. Azimuthing pods have been used commercially for years, with applications on cruise ships being quite common. Their use on large naval platforms has been nonexistent to date, but the concept has been considered and investigated [18]. The use of such systems on a submarine would allow for the removal of systems related to the centerline shaft; freeing up volume, weight, and area that must allocated and potentially allowing the submarine designer to get outside the speed-size-resistance circular path that results in large, expensive platforms. Using pods leverages work already completed and being applied in the world of electric ships. Other potential benefits include having the pods in a relatively undisturbed wake field -possibly increasing acoustic performance as well as increased operational maneuvering characteristics. This project focuses primarily of the operational maneuvering characteristics.

1.3 PROJECT GOALS

The goal of this research is to investigate the operational impacts that a podded propulsion system has on a submarine. To do so requires a benchmark – in this case a conventional submarine configuration using a centerline shaft configuration and a standard cruciform control surface configuration with rudders and sternplanes. The concept submarine would replace the control surfaces and propeller with pods. Figure 1.1 provides a conceptual drawing of such a configuration.
The operational characteristics of a submarine in terms of maneuvering can be broken down into a few key areas:

1. Acceleration Performance
2. Deceleration Performance
3. Turning Characteristics
4. Depth Changing Characteristics
5. Response to and Recovery from Casualties

The first four are straightforward and self-explanatory and are quantified in this thesis. The last area, however, is quite complex. The operations of submarines are normally limited in certain speed and depth combinations so that they can recover from casualties that may occur such as flooding or jamming of control surfaces. These limitations are characterized with a Submerged Operating Envelope (SOE). Figure 1.2 shows an example of an SOE [5].
At shallower depths, the limitations on a submarine are such that they can avoid broaching the surface and potentially colliding with a surface ship should a jam occur in the rise direction to the control surfaces. At deeper depths the limitations are in place such that the submarine can recover from seawater flooding, or not exceed the collapse depth of the submarine hull structure should a jam occur in the dive direction to the control surfaces. SOE's are sometimes further analyzed for recovery actions such as whether an emergency main ballast tank blow is initiated or not.

The casualty events that generate the limitations to the SOE for conventional submarine configurations are well known; they are typically jams to the stern planes. For the conceptual podded configuration, the limiting casualty events are unknown. In addition to analyzing the maneuvering performance characteristics, this thesis attempts to identify the limiting casualties, and quantify the differences between the conventional and podded configurations.
CHAPFER 2 THE CONCEPT SUBMARINE

The selection of a base submarine hull form was required to allow for the calculation and use of coefficients needed to analyze the dynamics of a maneuvering submarine. This could be done in one of two ways.

1. Utilize an existing submarine hull form with known characteristics, geometry, and possibly empirical data
2. Utilize a notional scalable and configurable submarine hull form

The first way is preferable, especially if the submarine hydrodynamic coefficients are known through model testing or real world operating characteristics. U.S. submarine designs are normally classified – as is the data associated with them – so the ability to utilize an existing hullform is quite limited. The DARPA SUBOFF program was a Computational Fluid Dynamics (CFD) program that utilized an unclassified submarine hullform. This hullform has been tested extensively through CFD analysis as well as scale model testing at the Naval Surface Warfare Center Carderock Division (NSWCCD). The use of the SUBOFF as a base submarine hull provides the ability to leverage previous work done in determining hydrodynamic coefficients. A drawback of using the SUBOFF hull is that it does not provide the flexibility in analyzing submarines with different geometric configurations.

Using a configurable submarine hull form is useful for conceptual design in that it allows for unique and variable submarine hulls to be analyzed for maneuvering characteristics. There exist geometric parameters used in conceptual submarine design that allow for any shape and size of submarine to be produced. The use of these parameters provides flexibility in modeling, but has the disadvantage of relying on analytical predictions of hydrodynamic coefficients. This can present problems when calculating viscous forces that can be difficult to predict without empirical data.

This thesis utilized both the SUBOFF hullform and a generalized, scalable hullform that allows for testing submarines of various shapes and sizes. The SUBOFF provides
empirical data that can be used to increase the robustness and accuracy of the model, and
the general hullform provides flexibility that results in a useful evaluation tool.

2.1 SUBOFF HULLFORM

The geometric characteristics of the DARPA SUBOFF hullform are identified through
equations that describe the axisymmetric hull, sail, and control surfaces [12]. Figure 2.1
shows the profile of the SUBOFF hullform with control surfaces removed. NSWCCD built
and tested two geometrically identical models at a linear scale ratio of λ=24. A multitude of
experiments were conducted on the hull form by NSWCCD [17]. These experiments
included captive-model testing with a Planar Motion Mechanism (PMM) to predict the
hydrodynamic, stability, and control coefficients of the SUBOFF hullform [23]. The testing
done with the PMM is particularly useful in that it provides empirical hydrodynamic
coefficients for various model configurations including the bare hull with and without the
sail. This provides a good baseline model upon which the propeller, control surfaces, and
azimuthing pods can be added to analyze the effects they have on maneuvering
performance.

![Profile of Submarine](image)

**Figure 2.1: SUBOFF Hullform**
The characteristics of the full scale SUBOFF hullform with sail are shown in table 2.1:

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>Length Overall</td>
<td>LOA</td>
<td>343</td>
<td>ft</td>
</tr>
<tr>
<td>Diameter</td>
<td>D</td>
<td>40</td>
<td>ft</td>
</tr>
<tr>
<td>Length of Forebody</td>
<td>L_F</td>
<td>80</td>
<td>ft</td>
</tr>
<tr>
<td>Length of Parallel Midbody</td>
<td>L_P</td>
<td>175.5</td>
<td>ft</td>
</tr>
<tr>
<td>Length of Afterbody</td>
<td>L_A</td>
<td>87.5</td>
<td>ft</td>
</tr>
<tr>
<td>Length/Diameter Ratio</td>
<td>L/D</td>
<td>8.57</td>
<td></td>
</tr>
<tr>
<td>Length to Center of Buoyancy</td>
<td>LCB</td>
<td>158.5</td>
<td>ft</td>
</tr>
<tr>
<td>Seawater Submerged Displacment</td>
<td>Δ</td>
<td>9753</td>
<td>LT</td>
</tr>
<tr>
<td>Hull Wetted Surface</td>
<td>W_S</td>
<td>36701</td>
<td>ft</td>
</tr>
</tbody>
</table>

Table 2.1: SUBOFF Hull Characteristics

The SUBOFF hullform is similar in size and L/D ratio to the USS SEAWOLF (SSN-21 Class) submarines (LOA=353ft, D=40ft, L/D=8.825), but has a much smaller L/D ratio than the USS LOS ANGELES (SSN-688 Class), VIRGINIA, and USS OHIO (SSBN-726 Class) submarines that have L/D ratios of 10.9, 11.09, and 13.3 respectively. This means that the hullform coefficients may be comparable to the SEAWOLF class submarines, but may differ from the majority of the submarines that make up the current U.S. Naval submarine force. The L/D ratio does, however, lend itself well to the assumption of length being significantly larger that the diameter which will make the slender body approximations outlined in chapter 4 reasonable.

The appendages of the SUBOFF hullform include the sail and control surfaces. The sail is a faired foil section located top dead center. There is no taper from the root to the tip of the sail section. The control surfaces consist of identical rudder and sternplanes located in the aft section of the hull. The two rudders are located top and bottom dead center of the hull centerline and the stern planes are located left and right dead center of the hull – the typical cruciform configuration. There is a taper from the root of the control surfaces out to the square tips. The standard control surfaces of the SUBOFF hullform are undersized, resulting in the submarine being unstable in both horizontal and vertical planes. Because of this, the size of the control surfaces were increased by using parametric relationships based on submarine displacements. The geometric characteristics of the appendages used in this study are listed in table 2.2. The SUBOFF hullform was also tested with a ring fin appendage to simulate the effects of a pump-jet propulsor. Because the
configurations tested in this thesis were modeled with a propeller, the data from testing with the ring fin appendages were not used.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>SAIL</td>
<td>$S_{\text{sail}}$</td>
<td>492.5</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>Sail Planform Area</td>
<td>$X_{\text{sail}}$</td>
<td>87.3</td>
<td>ft</td>
</tr>
<tr>
<td>Sail Chord</td>
<td>$\text{Chord}_{\text{sail}}$</td>
<td>29</td>
<td>ft</td>
</tr>
<tr>
<td>Sail Span</td>
<td>$\text{Span}_{\text{sail}}$</td>
<td>17.5</td>
<td>ft</td>
</tr>
<tr>
<td>Sail Aspect Ratio</td>
<td>$\text{AR}_{\text{sail}}$</td>
<td>0.603</td>
<td></td>
</tr>
</tbody>
</table>

**CONTROL SURFACES**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Surface Planform Area</td>
<td>$S_{\text{cs}}$</td>
<td>206.4</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>Control Surface Mid-chord Location (aft of bow)</td>
<td>$X_{\text{cs}}$</td>
<td>323.9</td>
<td>ft</td>
</tr>
<tr>
<td>Control Surface Root Chord</td>
<td>$\text{Root}_{\text{cs}}$</td>
<td>16.9</td>
<td>ft</td>
</tr>
<tr>
<td>Control Surface Tip Chord</td>
<td>$\text{Tip}_{\text{cs}}$</td>
<td>12</td>
<td>ft</td>
</tr>
<tr>
<td>Control Surface Span</td>
<td>$\text{Span}_{\text{cs}}$</td>
<td>14.4</td>
<td>ft</td>
</tr>
<tr>
<td>Control Surface Aspect Ratio</td>
<td>$\text{AR}_{\text{cs}}$</td>
<td>0.74</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2 SUBOFF Appendage Characteristics

### 2.2 GENERALIZED HULLFORM

The bodies of submarine hulls are usually made up of ellipsoidal and parabolic shapes for the fore and afterbody shapes. The equations for true ellipsoids and parabolas result in shapes that are too fine for submarines. There are modified equations that provide for fuller shapes that develop more useful geometries for modern submarine designs [16]. The shape of a submarine can therefore be modeled by:

$$y_f = \frac{D}{2} \left[ 1 - \left( \frac{x_f}{L_f} \right)^{\eta_f} \right]^{\frac{1}{\eta_f}}$$

$$y_a = \frac{D}{2} \left[ 1 - \left( \frac{x_a}{L_a} \right)^{\eta_a} \right]$$

where $x_f$ and $x_a$ are the distances from the maximum diameter, $y_f$ and $y_a$ are the hull radius at $x_f$ and $x_a$, and $L_f$ and $L_a$ are the lengths of the fore and after bodies. The exponent’s $\eta_f$ and $\eta_a$ are used to change how full the shapes are. Their values typically range from 2 to 4 depending on how full of an entrance or exit run is needed for the submarine. Parallel mid
body (PMB) is typically added to increase displacement, which is required to fit the combat and machinery systems needed to have a functional submarine. The geometries associated with developing a submarine hull form with this method are shown in figure 2.2

Figure 2.2: Geometry of Generalized Hullform

A sail and control surfaces can be added to the bare hull form. The size of the sail is typically driven by the requirements to fit mission related masts and antennas, ventilation systems, and to ensure a safe height above water for the officer of the deck when conducting surface transits. Control surfaces are located and sized to allow for adequate maneuverability for operational requirements.

The use of the generalized hull form and ability to add a sail and control surfaces as needed provides flexibility in the model created for this project – allowing for new concept designs to be analyzed for maneuvering characteristics. Any results that are yielded through this hullform are based entirely on analytical derivations of the hydrodynamic coefficients and not empirical data - conclusions drawn from those results must therefore be tempered by that fact.
CHAPTER 3 GOVERNING EQUATIONS OF MOTION

There are several sources of information regarding the methodologies for simulating the trajectories and responses of submerged bodies. David Taylor Model Basin has developed standard equations of motion for use in simulations regarding submarines [10][8]. Abkowitz and Fossen have both developed general equations for underwater vehicles [2][9]. Recent research in Autonomous Undersea Vehicles (AUVs) have employed these methodologies in various forms [20][21][22]. All of these techniques are related - based on fundamental principles of kinematics and dynamics. This project follows a similar approach.

3.1 COORDINATE SYSTEM

Six degrees of freedom (DOF) are required to determine the position and orientation of a submarine. A summary of the DOFs used in this thesis and their notations are provided in table 3.1. These are the standardized SNAME notations [25].

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Translation/Rotation</th>
<th>Force/Moment</th>
<th>Linear and angular velocities</th>
<th>Position and angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>Motion in x axis</td>
<td>X</td>
<td>u</td>
<td>x</td>
</tr>
<tr>
<td>Sway</td>
<td>Motion in y axis</td>
<td>Y</td>
<td>v</td>
<td>y</td>
</tr>
<tr>
<td>Heave</td>
<td>Motion in z axis</td>
<td>Z</td>
<td>w</td>
<td>z</td>
</tr>
<tr>
<td>Roll</td>
<td>Rotation about x axis</td>
<td>K</td>
<td>p</td>
<td>θ</td>
</tr>
<tr>
<td>Pitch</td>
<td>Rotation about y axis</td>
<td>M</td>
<td>q</td>
<td></td>
</tr>
<tr>
<td>Yaw</td>
<td>Rotation about z axis</td>
<td>N</td>
<td>r</td>
<td>ψ</td>
</tr>
</tbody>
</table>

Table 3.1: Coordinate System Notation

Two coordinate frames are used. The first, known as the body-fixed frame is fixed to the submarine, in this case with the origin located at the center of buoyancy:

\[
r_B = [x_B, y_B, z_B]^T = [0, 0, 0]^T
\]  

(3.1.1)

The selection of the center of buoyancy as the origin allows for simplifications due to vehicle symmetry about the x-z plane. The other coordinate frame is the inertial frame. All motion of the body-fixed frame is relative to the inertial frame. The inertial frame is used to determine the actual position and orientation of the submarine while the body-fixed
frame is used to determine the linear and angular velocities. Figure 3.1 shows a graphical depiction of the two coordinate frames and their relation to each other.

![Figure 3.1: Body-Fixed and Inertial Coordinate Frames](image)

Using standard SNAME notation the position, orientation, and velocities of the submarine can be described by vectors:

\[
\eta = [\eta_1^T, \eta_2^T]^T \quad \text{where } \eta_1 = [x, y, z]^T \text{ and } \eta_2 = [\phi, \theta, \psi]^T
\]

\[
\nu = [\nu_1^T, \nu_2^T]^T \quad \text{where } \nu_1 = [u, v, w]^T \text{ and } \nu_2 = [p, q, r]^T
\]  

(3.1.2)

The body-fixed translational velocity vector can be expressed in the inertial frame using a transformation matrix \(J_1(\eta_2)\) such that:

\[
\dot{\eta} = J_1(\eta_2)\nu
\]

where \(\eta = [\dot{x}, \dot{y}, \dot{z}]^T\)  

(3.1.3)
The transformation matrix $J_1(\eta_2)$ is obtained by translating the inertial frame until its origin is the same as the body-fixed frame, then performing three rotations of the inertial frame about $\phi$, $\theta$, and $\psi$ until the body-fixed frame is obtained. The order in which these rotations occurs will yield different transformation matrices. Abkowitz [2] and Fossen [9] present two different transformations to link the two coordinate frames - this thesis utilizes those presented by Fossen whereby the order of rotation is first the yaw angle about the z-axis, then the pitch angle about the y-axis, then finally the roll angle about the x-axis. The resulting transformation matrix is shown below, with the notation $s(*)$ representing $\sin(*)$, $c(*)$ representing $\cos(*)$ and $t(*)$ representing $\tan(*)$.

$$
J_1(\eta_2) = \begin{bmatrix}
  c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\theta \\
  s\psi c\theta & c\psi c\phi + s\phi s\theta \psi & -c\psi s\phi + s\theta s\psi c\phi \\
  -s\theta & c\theta s\phi & c\theta c\phi
\end{bmatrix}
$$

(3.1.4)

Similarly, the body-fixed angular velocity vector can be expressed in the inertial frame as a Euler rate vector using a different transformation matrix $J_2(\eta_2)$ such that:

$$
\dot{\eta}_2 = J_2(\eta_2)v_2 \quad \text{where} \quad \eta_2 = [\phi, \theta, \psi]^T
$$

(3.1.5)

The transformation matrix $J_2(\eta_2)$ is obtained once again by rotating the inertial frame with respect to the body-fixed frame, yielding:

$$
J_2(\eta_2) = \begin{bmatrix}
  1 & s\phi \theta & c\phi \theta \\
  0 & c\phi & -s\phi \\
  s\phi / c\theta & c\phi / c\theta
\end{bmatrix}
$$

(3.1.6)

It should be noted that this transformation produces a singularity when pitch angle ($\theta$) reaches a value of +/- 90 degrees as $\tan(90^0)$ is undefined. Other transformations will present singularities in either roll or yaw. A singularity in yaw would be unacceptable, however a submarine is not expected to reach roll or pitch values that exceed 90 degrees so this transformation is adequate for the intended purpose.

### 3.2 VEHICLE DYNAMICS

Dynamic problems are governed by Newton’s second law:
\[ F = m \frac{dv}{dt} \]  \hspace{1cm} (3.1.7)

With the body-fixed coordinate system of the submarine moving at a velocity \( v_o \) relative to the inertial frame and rotating at an angular velocity \( \omega \) the velocity of the submarine relative to the inertial frame becomes

\[ v_l = v_o + \omega \times r_G \]  \hspace{1cm} (3.1.8)

where \( r_G = [X_G, Y_G, Z_G]^T \) is the vector of the center of gravity of the submarine relative to the origin which was selected to be at the submarines center of buoyancy. From this follows

\[ F = m \frac{dv_o}{dt} + m \frac{d}{dt} (\omega \times r_G) \]  \hspace{1cm} (3.1.9)

Using the expansion for the total derivative,

\[ \frac{df}{dt} = \frac{df}{d\tau} + \omega \times f \]  \hspace{1cm} (3.1.10)

the inertial force on the submarine becomes:

\[ F = m \left\{ \frac{dv_o}{dt} + \omega \times v_o + \frac{d\omega}{dt} \times r_G + \omega \times (\omega \times r_G) \right\} \]

(3.1.11)

Use of the vector triple product, \( \omega \times (\omega \times r_G) = (\omega \cdot r_G)\omega - (\omega \cdot \omega) r_G \) then yields:

\[ F = m \left\{ \frac{dv_o}{dt} + \omega \times v_o + \frac{d\omega}{dt} \times r_G + (\omega \cdot r_G)\omega - (\omega \cdot \omega) r_G \right\} \]

(3.1.12)

This can be expanded to develop the equations of motion for the three forces \( X, Y, \) and \( Z \):

\[ \sum X_{ext} = m \left[ \dot{u} + q w - rv - x_G(q^2 + r^2) + y_G(pq - r) + z_G(rp + q) \right] \]

\[ \sum Y_{ext} = m \left[ \dot{v} - wp + ur + x_G(qp + r) - y_G(r^2 + p^2) + z_G(qr - p) \right] \]

\[ \sum Z_{ext} = m \left[ \dot{w} - uq + vp + x_G(rp - q) + y_G(rq + p) - z_G(p^2 + q^2) \right] \]

(3.1.12)

Using a similar derivation for angular momentum yields the equations of motion for the three moments \( K, M \) and \( N \) (equations 3.1.13):
The moments of inertia for submarines can be estimated using rules of thumb for the gyradius provided by the Marine Vehicle Weight Engineering handbook[6]. The gyradius is the virtual point located from the origin where the entire mass of the body appears to be located. It is defined as

\[
gyradius = \sqrt{\frac{I_{\text{axis}}}{\Delta}}
\]  

(3.1.14)

Knowing gyradius and the mass displacement (\(\Delta\)) for the submarine allows for the calculation of weight moment of inertias about the principle axes (\(I_{\text{axis}}\)). For submarines the rules of thumb for gyradius are:

<table>
<thead>
<tr>
<th>Gyradius Axis</th>
<th>Rule of Thumb</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>40% of Beam</td>
</tr>
<tr>
<td>Y</td>
<td>25% of LOA</td>
</tr>
<tr>
<td>Z</td>
<td>25% of LOA</td>
</tr>
</tbody>
</table>

Table 3.2: Gyradius Thumbrules

The submarines cross-inertia terms \(I_{xy}, I_{yz},\) and \(I_{zx}\) are all very small compared to the weight moment of inertia terms \(I_{xx}, I_{yy},\) and \(I_{zz}\). The assumption is made that the cross-inertia terms are zero. This is a valid assumption due to the fact that the submarine has symmetry in the x-z plane and, with the exception of the sail, symmetry in the x-y plane. Since the sails of submarines are typically not part of the pressure hull, have relatively little weight implications for the submarine. The assumption of x-z plane symmetry also allows for a simplification where \(y_G\) is zero. This greatly simplifies the equations of motion to:

\[
\sum X_{\text{ext}} = m \left[ \ddot{u} + qw - rv - x_G (q^2 + r^2) + z_G (pr + q) \right]
\]

\[
\sum Y_{\text{ext}} = m \left[ \ddot{v} - wp + ur + x_G (qp + r) + z_G (qr - p) \right]
\]
\[ \sum Z_{\text{ext}} = m \left[ \dot{w} - uq + vp + x_G (rp - q) - z_G (p^2 + q^2) \right] \]

\[ \sum K_{\text{ext}} = I_{xx} \ddot{p} + (I_{zz} - I_{yy}) \dot{r} q - m z_G (v - wp + ur) \]

\[ \sum M_{\text{ext}} = I_{yy} \ddot{q} + (I_{xx} - I_{zz}) \dot{r} p \dot{r} + m [z_G (u - vr + wq)] - x_G (w - uq + vp) \]

\[ \sum N_{\text{ext}} = I_{zz} \ddot{r} + (I_{yy} - I_{xx}) \dot{p} q + m x_G (v - wp + ur) \] (3.1.15)

These inertial force equations equal the summation of the external forces and moments that are developed on the submarine from various sources. These include:

- Hydrostatic forces due to weight, buoyancy and submarine orientation
- Hydrodynamic forces from added mass effects, viscous drag and lift
- Propulsion forces from propulsion
- Control surface forces from control planes and rudders
- Environmental forces from wind and waves

In the context of the equations of motion, these external forces are usually expressed in terms of coefficients. The analytic derivation and empirical estimation of these coefficients are described in Chapter 4.
CHAPTER 4 EXTERNAL FORCES AND MOMENTS

4.1 HYDROSTATIC FORCES

The static forces of weight \( W \) and buoyancy \( B \) act through the submarine's center of gravity \( r_G \) and center of buoyancy \( r_B \). Weight and buoyancy are defined as:

\[
W = mg
\]

\[
B = \rho V g
\]

Using the outline presented by Fosen [9], these static forces which occur in the inertial frame can be transformed onto the body of the submarine based on orientation. The gravitational force, \( f_G(\eta_2) \), from \( W \) and the buoyant force, \( f_B(\eta_2) \), from \( B \) can be expressed using the inverse of the coordinate transformation matrix outlined by equation 3.1.4 in Chapter 3 and the property that \( J_i^{-1}(\eta_2) = J_i^T(\eta_2) \).

\[
f_G(\eta_2) = J_i^{-1}(\eta_2) \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}
\]

\[
f_B(\eta_2) = -J_i^{-1}(\eta_2) \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}
\]

The vector of hydrostatic forces and moments becomes:

\[
g(\eta) = \begin{bmatrix} f_G(\eta) + f_B(\eta) \\ r_G \times f_G(\eta) + r_B \times f_B(\eta) \end{bmatrix}
\]

This vector is expanded to provide the individual hydrostatic forces and moments:

\[
X_{HS} = -(W - B) \sin(\theta)
\]

\[
Y_{HS} = (W - B) \cos(\theta) \sin(\phi)
\]

\[
Z_{HS} = (W - B) \cos(\theta) \cos(\phi)
\]

\[
K_{HS} = -(y_G W - y_B B) \cos(\theta) \cos(\phi) - (z_G W - z_B B) \cos(\theta) \sin(\phi)
\]

\[
M_{HS} = -(z_G W - z_B B) \sin(\theta) - (x_G W - x_B B) \cos(\theta) \cos(\phi)
\]

\[
N_{HS} = -(x_G W - x_B B) \sin(\theta) - (y_G W - y_B B) \sin(\theta)
\]

(4.1.4)
4.1.1 EMERGENCY MAIN BALLAST TANK BLOW

A crucial survivability feature for submarines is the emergency main ballast tank (EMBT) blow system. Such systems allow for the rapid deballasting of water in the main ballast tanks by blowing pressurized air into the tanks forcing water out through the bottom of the tank. This allows for buoyancy to be quickly added to the submarine creating hydrostatic forces that allow for the submarine to surface. This system can be utilized to recover from flooding or control surface casualties, and normal surfacing operations for some submarines. The speed at which deballasting occurs is a function of external seawater pressure and the pressure and capacity of the pressurized air banks in the submarine such that the volumetric rate at which water is discharged from the ballast tanks must equal the volumetric rate at which the pressurized air is expanded. Figure 4.1 shows how this concept works.

![Figure 4.1: EMBT Blow Conceptual Diagram](image-url)
It can be shown that this flow rate is proportional to the square of the pressure differential[5]:

\[ EMBT_{FlowRate} \propto \sqrt{\Delta p} \]  \hspace{1cm} (4.1.5)

Figure 4.2 shows a graph of the EMBT flow rate as a function of air bank pressure based on a design depth of 800 feet, and an air bank design pressure of 4500 psi. It is clear that as air bank pressure approaches the ambient seawater pressure of 352 psi, the flow rate greatly diminishes. For most of the pressure range of the air bank the flow rate is significant – only reaching 50% of design flow at around 1500 psi. For shallower depths where seawater pressure is even lower, the flow rates of the EMBT system are higher.

![Figure 4.2: EMBT Blow Deballasting Rate at 800 feet](image)

For most submarines, EMBT blow systems are designed to recover from a flooding casualty of an assumed severity that occurs at design depth. Although the capacity and size of the system are designed for flooding casualties, the system may also be used to combat control surface casualties, especially jammed stern plane events at deep depths. In this case the forward main ballast tanks may be blown with air in order to obtain positive
buoyancy as well as impart a positive moment in pitch to assist in arresting further downward depth excursions due to forward velocity. Typical specifications for an EMBT air flask system would be a design air pressure of 4500 psi (310 Bar) with a total capacity of approximately 0.3 ft³ (8.5 liters) per ton of normal surface displacement [15]. The volume of ballast tanks are typically sized to provide 12.5% reserve buoyancy when on the surface – called the normal surface condition (NSC); the sum of the NSC and the ballast tanks is equal to the submerged displacement of the submarine. This allows for an estimation of the size of the ballast tanks as a function of submerged displacement:

\[ MBT_{\text{volume}} = \frac{\Delta}{9} \]  

(4.1.6)

The volume of the ballast tanks as well as the volume of the air flasks are typically split with 60% of the total volume in the forward tanks and 40% in the aft tanks.

To estimate the deballasting rate of the EMBT system, a parametric relationship between submerged displacement and initial blow rate was utilized where the initial blow rate (IBR) at test depth in m³ air/sec is:

\[ IBR = 0.0003 \frac{m^3 \text{air}}{\text{sec}. \text{LT}} \times \Delta_{\text{submerged}} \]  

(4.1.7)

Combining equations 4.1.5 and 4.1.7 allows for the blow rate of the system to be found for a given seawater, and air bank pressure. The air bank pressure varies as a function of the amount of air cumulatively blown into the ballast tanks. For simplicity this can be modeled as an ideal gas where:

\[ P_{\text{bank}}(t) = \frac{P_{\text{bank, initial}} - \int_0^t P_{sw}(\text{depth}) \times BR(t) dt}{V_{\text{bank}}} \]  

(4.1.8)

where \( P_{\text{bank}} \) is the air bank pressure as a function of time, \( P_{\text{bank, initial}} \) is the initial pressure of the air banks, \( P_{sw} \) is the ambient seawater pressure in the ballast tanks, \( BR(t) \) is the blow rate of the system as a function of time, and \( V_{\text{bank}} \) is the volume of the air banks.
When actuated the EMBT blow system will add buoyancy to the submarine at a rate equal to the EMBT blow rate since this results in water being displaced from the ballast tanks. This added buoyancy also moves the longitudinal center of buoyancy, $x_B$, depending on how much buoyancy has been added as well as the location of the ballast tanks. This added buoyancy and change in $x_B$ will affect the hydrostatic forces in equations 4.1.4.

### 4.2 ADDED MASS

When an object in a fluid accelerates, the body moves some volume of the surrounding fluid. Added mass is the measure of the additional inertia generated by this moving water as the body accelerates or decelerates. The added mass can be expressed as a matrix, $M_a$:

$$
M_a =
\begin{bmatrix}
X & X & X & X & X & X \\
\_u & \_v & \_w & \_p & \_q & \_r \\
Y & Y & Y & Y & Y & Y \\
\_u & \_v & \_w & \_p & \_q & \_r \\
Z & Z & Z & Z & Z & Z \\
\_u & \_v & \_w & \_p & \_q & \_r \\
K & K & K & K & K & K \\
\_u & \_v & \_w & \_p & \_q & \_r \\
M & M & M & M & M & M \\
\_u & \_v & \_w & \_p & \_q & \_r \\
N & N & N & N & N & N \\
\_u & \_v & \_w & \_p & \_q & \_r
\end{bmatrix}
$$

(4.2.1)

Due to potential flow theory, the added mass matrix is symmetric such that $M_a = M_a^T$. This along with port/starboard and top/bottom symmetry allows for simplification of the added mass matrix:

$$
M_a =
\begin{bmatrix}
X & 0 & 0 & 0 & 0 & 0 \\
0 & Y & 0 & 0 & 0 & N \\
0 & 0 & Z & 0 & M & 0 \\
0 & 0 & 0 & K & 0 & 0 \\
0 & 0 & Z & 0 & M & 0 \\
0 & Y & 0 & 0 & 0 & N \\
\_u & \_v & \_w & \_p & \_q & \_r
\end{bmatrix}
$$

(4.2.2)

The kinetic energy of a moving object is a function of $\frac{1}{2}$ times mass times velocity squared. Using the vector of $q = (u,v,w,p,q,r)^T$ we get the kinetic energy of the fluid, $E_k$: 

Page 32
\[ E_k = -\frac{1}{2} \dot{q}^T M_k \dot{q} \]  
(4.2.3)

From Triantafyllou [29] Kirchoff's relations state that for a velocity vector \( \vec{v} \) and angular velocity vector \( \vec{\omega} \) the inertia terms expressed on a fixed body coordinate system are:

\[
\text{Force} = -\frac{\partial}{\partial t} \left( \frac{\partial E_k}{\partial \vec{v}} \right) - \vec{\omega} \times \left( \frac{\partial E_k}{\partial \vec{v}} \right)
\]

\[
\text{Moment} = -\frac{\partial}{\partial t} \left( \frac{\partial E_k}{\partial \vec{\omega}} \right) - \vec{v} \times \left( \frac{\partial E_k}{\partial \vec{\omega}} \right)
\]

(4.2.4)

When Kirchoff's relations are applied to the symmetrically simplified added mass matrix the following force and moment equations are yielded:

\[
X_A = X. u + Z. wq + Z. q^2 - Y. vr - Y. r^2
\]

\[
Y_A = Y. v + Y. r + X. ur + Z. wp - Z. pq
\]

\[
Z_A = Z. w + z. q - X. uq + Y. vp - Y. rp
\]

\[
K_A = K. w
\]

\[
M_A = M. w + M. q - (Z. - X.) uw - Y. vp + (K. - N.) rp - Z. uq
\]

\[
N_A = N. v + N. r - (X. - Y.) uv - Z. wp - (K. - M.) pq - Y. ur
\]

(4.2.5)

### 4.2.1 AXIAL ADDED MASS

The added mass in the \( x \) direction, \( X_u \), due to an acceleration in the \( x \) direction, \( u \), was determined using empirical formulas presented by Blevins[4] where the submarine is represented as an ellipsoid of revolution of length \( L \), and hull diameter \( D \). \( X_u \) is a function of \( L, D \) and the ratio of \( L/D \):

\[
X_u = -\alpha \frac{4}{3} \rho \pi \left( \frac{L}{2} \right) \left( \frac{B}{2} \right)^2
\]

(4.2.6)
Alpha, $\alpha$, can be found by the following table:

<table>
<thead>
<tr>
<th>$L/B$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>0.1</td>
<td>6.14800</td>
</tr>
<tr>
<td>0.2</td>
<td>3.00800</td>
</tr>
<tr>
<td>0.4</td>
<td>1.42800</td>
</tr>
<tr>
<td>0.6</td>
<td>0.90780</td>
</tr>
<tr>
<td>0.8</td>
<td>0.65140</td>
</tr>
<tr>
<td>1</td>
<td>0.50000</td>
</tr>
<tr>
<td>1.5</td>
<td>0.30380</td>
</tr>
<tr>
<td>2</td>
<td>0.21000</td>
</tr>
<tr>
<td>2.5</td>
<td>0.15630</td>
</tr>
<tr>
<td>3</td>
<td>0.12200</td>
</tr>
<tr>
<td>5</td>
<td>0.05912</td>
</tr>
<tr>
<td>7</td>
<td>0.03585</td>
</tr>
<tr>
<td>10</td>
<td>0.02071</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

**Table 4.1: Added Mass Parameter $\alpha$ for Ellipsoid of Revolution**

Values from this table can be estimated by fitting an equation through regression analysis. The equation $\alpha=0.4466 \left( \frac{L}{D} \right)^{1.235}$ allows for the added mass of any ellipsoid to be estimated. For this thesis, it was assumed that the added mass imparted by the control surfaces and sail would be much less than the added mass from the hull and were subsequently neglected for simplicity.

### 4.2.2 CROSSFLOW ADDED MASS

To estimate the added mass resulting from the flow of fluid across the hull, the strip theory technique presented by Newman was used [19]. Strip theory relies on two basic assumptions. The first is that the flow over a strip of a body is two dimensional and the second is that the interaction between strips that are adjacent to each other is small. For bodies that have a characteristic dimension much larger than another such as a submarine with a length $L$ much greater than its diameter $D$, strip theory works quite well. As a body or submarine becomes less slender (such as when $L$ is no longer much greater than $D$), strip theory will not be accurate due to the flow becoming three dimensional around the ends of the body. For the submarines analyzed in this project, strip theory should provide an accurate estimation of the crossflow added mass terms.
For a cylindrical two-dimensional slice of a submarine, the added mass per unit length of the slice is:

\[ m_a(x) = \pi \rho R(x)^2 \]  

(4.2.7)

where \( R(x) \) is the hull radius as a function of the submarine’s axial position \( x \). For sections that have control surfaces, the added mass is calculated differently. Blevins [1] provides a relationship for the added mass of a circle with multiple equally spaced fins.

\[ m_{af}(x) = 2\pi \rho a_{\text{fin}}(x)^2 \left( \frac{1 + (R(x)/a_{\text{fin}}(x))^n}{2} \right)^{4/n} - \frac{1}{2} \left( \frac{R(x)}{a_{\text{fin}}(x)} \right)^2 \]

(4.2.8)

where \( n \) equals the number of fins and \( a_{\text{fin}}(x) \) is the maximum height of the control surfaces above the centerline of the submarine as a function of axial position \( x \). For a cruciform control surface configuration with 4 control surfaces, the added mass becomes:

\[ m_{af}(x) = \pi \rho \left( a_{\text{fin}}(x)^2 - R(x)^2 + \frac{R(x)^4}{a_{\text{fin}}(x)^2} \right) \]

(4.2.9)

To estimate the cross-flow added mass due to the sail it was modeled as a separate rectangular plate. Blevins provides an empirical estimate for a plate based on the length (chord of the sail \( C_{\text{sail}} \)) and width (height of the sail, \( H_{\text{sail}} \)) of a plate:

\[ m_{as}(x) = \frac{\alpha \pi}{4} \rho \left( C_{\text{sail}} H_{\text{sail}} \right)^{1/2} \]

(4.2.10)

In this equation, \( \alpha \) is a function of \( \frac{C_{\text{sail}}}{H_{\text{sail}}} \) represented by the following table:

<table>
<thead>
<tr>
<th>( \frac{C_{\text{sail}}}{H_{\text{sail}}} )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.579</td>
</tr>
<tr>
<td>1.25</td>
<td>0.6419</td>
</tr>
<tr>
<td>1.59</td>
<td>0.7038</td>
</tr>
<tr>
<td>2</td>
<td>0.7568</td>
</tr>
<tr>
<td>2.5</td>
<td>0.8008</td>
</tr>
</tbody>
</table>
Table 4.2: Added Mass Parameter $\alpha$ for a Rectangular Plate

For intermediate values of $\frac{C_{\text{tail}}}{H_{\text{tail}}}$, $\alpha$ can be estimated by fitting an equation through regression analysis. In this case:

$$\alpha = -0.0003 \left( \frac{C_{\text{tail}}}{H_{\text{tail}}} \right)^4 + 0.0088 \left( \frac{C_{\text{tail}}}{H_{\text{tail}}} \right)^3 - 0.0835 \left( \frac{C_{\text{tail}}}{H_{\text{tail}}} \right)^2 + 0.3584 \left( \frac{C_{\text{tail}}}{H_{\text{tail}}} \right) + 0.3043 \tag{4.2.11}$$

Using the relationships outlined above and integrating over the length of the submarine by strip theory, the following expressions are derived for the cross-flow added mass terms.

$$Y_v = \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_{af}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_{a}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_{a}(x) \, dx$$

$$Z_w = \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_{af}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} m_{a}(x) \, dx$$

$$M_w = \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_{af}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_a(x) \, dx$$

$$N_v = \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_{af}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x m_{af}(x) \, dx$$

$$Y_r = N_v$$

$$Z_q = M_w$$

$$M_q = \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x^2 m_a(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x^2 m_{af}(x) \, dx - \int_{x_{\text{gin}}}^{x_{\text{gfn}}} x^2 m_a(x) \, dx$$
4.2.3 ROLLING ADDED MASS

To account for the added mass due to the submarine rolling about the x axis, it was assumed that the cylindrical sections of the hull do not incur any added mass in roll. This leaves only the sections containing control surfaces and the sail as sources of added mass in roll. For a section with four equally spaced fins in roll, Blevins [4] provides the following formula for added mass in roll.

\[ m_{a f, \text{roll}}(x) = -2 \frac{\rho}{\pi} a_{fn}(x)^4 \]  

(4.2.13)

where \( a_{fn}(x) \) is the fin height above the vehicle centerline.

The added mass for the sail was determined by using the added mass from crossflow multiplied by mean height of the sail above the submarine centerline, \( Z_{\text{sail}} \), to apply the appropriate lever arm:

\[ m_{a s, \text{roll}}(x) = \frac{\pi}{4} \rho \left( C_{\text{saif}} H_{\text{saif}} \right)^2 r_{\text{sail}} \]  

(4.2.14)

When combined and integrated over the appropriate area, the following added mass equation is obtained for roll about the x axis:

\[ K_p = -2 \frac{\rho}{\pi} \int_{s_{\text{fn}}}^{s_{\text{sail}}} a_{fn}(x)^4 dx - \frac{\pi}{4} \rho \left( C_{\text{saif}} H_{\text{saif}} \right)^2 \int_{r_{\text{sail}}}^{r_{\text{saif}}} r_{\text{sail}}(x) dx \]  

(4.2.15)

4.2.4 ADDED MASS CROSS TERMS

The coefficients from the rest of the added mass cross-terms are evaluated from terms that are already derived (equations 4.2.16):

\[
\begin{align*}
X_{wq} &= Z_{wq} \quad & X_{qq} &= Z_{qq} \quad & X_{vr} &= -Y_{v} \quad & X_{rr} &= -Y_{r} \\
Y_{wa} &= X_{wa} \quad & Y_{wp} &= -Z_{wp} \quad & Y_{pq} &= -Z_{pq} \\
Z_{wa} &= -X_{wa} \quad & Z_{vp} &= -Z_{wp} \quad & Z_{rp} &= Y_{rp}
\end{align*}
\]
4.3 HYDRODYNAMIC DAMPING FORCES AND MOMENTS

Analytically calculating the hydrodynamic damping on a submarine is a difficult task. The forces involved are highly nonlinear and coupled. To simplify the prediction of damping forces, some assumptions are usually made. For this project, they are:

- Coupling velocities and accelerations are neglected. This assumption is based on Newton’s second law, from which we expect the submarine’s inertia forces to be linearly dependent on acceleration.
- The submarine is assumed to have port/stbd (x-z plane) and top/bottom (x-y plane) symmetry with the exception of the sail. This allows for the elimination of many hydrodynamic coefficients that are negligible. Forces and moments caused by the asymmetry of the sail will be addressed as external forces.
- Damping terms greater than second-order are considered very small and negligible.

There are certain observations that can be made on the damping forces on a submarine. The damping is comprised of many different components including linear friction due to laminar and turbulent boundary layers, damping from vortex shedding, radiation-induced potential damping from body oscillations, and damping from waves. For the case of a submarine, the damping is dominated by the first two.

The non-dimensional Reynolds number provides a measure of the ratio between inertial and viscous forces for a body and is defined as:

\[
Re = \frac{uL}{v}
\]  

(4.3.1)

where \( u \) is the speed of a body, \( L \) is the characteristic length of a body, and \( v \) is the kinematic viscosity of the fluid. Seawater at 15\(^\circ\) C has a kinematic viscosity of 1.19 X 10\(^{-6}\).
Full scale submarines are of several hundred feet in length and typically operate anywhere from 2.5 to greater than 13 m/s. This means the Reynolds number is on the order of $10^8$ to $10^9$ - well beyond the transition range from laminar to turbulent flow that occurs at Reynolds numbers between $10^5$ and $2 \times 10^6$ [19]. Subsequently, the assumption that the submarine will experience turbulent flow is an appropriate one.

4.3.1 AXIAL DRAG FORCE

The axial drag of a body moving through a fluid can be expressed by the following equation:

$$AxialDrag = \frac{1}{2} \rho AC_d u^2$$

where $A$ is the frontal area of the body, $\rho$ is the fluid density, $C_d$ is the drag coefficient and $u$ is the fluid velocity. Drag coefficients are estimated from experiments such as tow tank testing, or from empirical formulas. Several sources are available for empirical formulas of streamlined body of revolutions [2][13][15][29]. Hoerner's [13] equation for a body of revolution,

$$C_d = C_f \left[1 + 1.5 \left(\frac{D}{L_a}\right)^{1/2} + 7 \left(\frac{D}{L_a}\right)^3\right]$$

is modified by Jackson [15] for submarines based on actual submarine resistance data:

$$C_d = C_f \left[1 + 1.5 \left(\frac{D}{L_a}\right)^{1/2} + 7 \left(\frac{D}{L_a}\right)^3 + 0.002(C_p - 0.6)\right]$$

In these equations $D$ is hull diameter, $L_a$ is the length of the submarine afterbody, $C_p$ is the prismatic coefficient, and $C_f$ is the frictional drag coefficient. This equation was determined based on the area in the axial drag equation being the wetted surface area, $A_{ws}$. Within the bracket of the equation, the second term accounts for dynamic pressure, the third term accounts for flow separation, and the fourth term – Jackson's modification – accounts for the effects of parallel mid body. The frictional drag coefficient, $C_f$ can be estimated from the ITTC 1957 Line [26]:

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The frictional drag coefficient is dependent on Reynolds number – determined by speed and length of the submarine. For simplicity in modeling, the drag coefficient can be linearized about an assumed speed. In reality the frictional drag coefficient may vary from as high as 0.0022 for slower speeds of 1 m/s to around 0.0016 for higher speeds of 13 m/s.

To calculate the axial drag of a submarine, additional terms must be applied. Jackson adds an additional allowance, $C_a$ to the drag calculation to account for roughness of the hull, seawater suction and discharge pipes, and additional inaccuracies. He found that $C_a$ values can range from 0.0002 to 0.0015 for submarines. In addition to the allowance, the resistance added by appendages such as the sail and control surfaces must be accounted for. Both the sail and the control surfaces have their own areas and drag coefficients based on their geometries. When combined, the total axial drag for a submarine can be expressed as:

$$AxialDrag = -\frac{1}{2} \rho |u| \left[ A_{ws}(C_d + C_a) + A_{sail} C_{d\_sail} + A_{control\_surface} C_{d\_control\_surface} \right]$$ (4.3.6)

It is from this equation that the axial drag coefficient is found:

$$X_{u|u|} = \frac{1}{2} \rho \left[ A_{ws}(C_d + C_a) + A_{sail} C_{d\_sail} + A_{control\_surface} C_{d\_control\_surface} \right]$$ (4.3.7)

### 4.3.2 CROSSFLOW DRAG AND MOMENTS

To calculate the drag and moments generated by the flow of fluid across the submarine, a strip theory approach – similar to that used in the calculation of crossflow added mass in section 4.2.2 is utilized. Although the prediction of axial drag has been well studied and there are empirical equations that allow for a fairly accurate prediction of the axial drag analytically, it is very difficult to predict the crossflow drag of a submarine. There are many non-linearities and complexities such as vortex shedding that are difficult to predict without doing experimentation on either a scale model or a full size submarine. Because of this, it is not uncommon to have significant inaccuracies in the prediction of
crossflow drag without model testing. The strip theory approach does, however, allow for the calculation of the terms found in the equations of motion.

To use the strip theory approach, the cylindrical sections of the submarine are modeled as 2-D circular discs. The drag coefficient, \( C_{dc} \) for such discs are dependent upon Reynolds number, but for Reynolds numbers of around \( 10^4 \), Blevins finds the value of \( C_{dc} = 1.1 \) [3]. The crossflow drag from the control surfaces and sail must also be accounted for. Since the control surfaces and sail are low aspect ratio foil sections, the drag coefficient for these appendages, \( C_{dc,cs} \) and \( C_{dc,sail} \), were estimated using empirical equations developed by Whicker and Fehlner [30]. They found the drag coefficient to be dependent upon the shape of the tips as well as the taper ratio, \( \lambda \), which is the ratio of the chord at the tip of the foil to the chord at the base of the foil. For faired tips that are typical for the design of a submarine sail:

\[
C_{dc,sail} = 0.1 + 0.7\lambda
\]  
(4.3.8)

For square tips which are often seen on control surfaces:

\[
C_{dc,cs} = 0.1 + 1.6\lambda
\]  
(4.3.9)

Applying strip theory and adding crossflow drag from appendages results in the following expressions for drag coefficients:

\[
Y_{|x|} = -\frac{1}{2} \rho C_{dc} \int_{x_{bow}}^{x_{tail}} 2R(x)dx - \frac{1}{2} \rho S_{cs} C_{dc,cs} C_{dc,sail} - \frac{1}{2} \rho S_{sail} C_{dc,sail}
\]

\[
Z_{|w|} = -\frac{1}{2} \rho C_{dc} \int_{x_{bow}}^{x_{tail}} 2R(x)dx - \frac{1}{2} \rho S_{cs} C_{dc,cs} C_{dc,sail}
\]

\[
N_{|v|} = -\frac{1}{2} \rho C_{dc} \int_{x_{bow}}^{x_{tail}} 2xR(x)dx - 2x_{cs} \left( \frac{1}{2} \rho S_{cs} C_{dc,cs} C_{dc,sail} \right) - \frac{1}{2} \rho S_{sail} C_{dc,sail}
\]

\[
M_{|w|} = \frac{1}{2} \rho C_{dc} \int_{x_{bow}}^{x_{tail}} 2xR(x)dx + 2x_{cs} \left( \frac{1}{2} \rho S_{cs} C_{dc,cs} C_{dc,sail} \right)
\]

\[
Y_{|v|} = -\frac{1}{2} \rho C_{dc} \int_{x_{bow}}^{x_{tail}} x|R(x)dx - 2x_{cs} x_{cs} \left( \frac{1}{2} \rho S_{cs} C_{dc,cs} C_{dc,sail} \right) - \frac{1}{2} \rho S_{sail} C_{dc,sail}
\]
\[ Z_{q[y]} = \frac{1}{2} \rho C_{dc} \int_{x_{cs}}^{x_{sail}} 2x|\alpha| R(x) \, dx + 2x_{cs}\left| x_{cs}\right| 2 \frac{1}{2} \rho S_{cs} C_{dc \_cs} \]  
\[ N_{r[r]} = -\frac{1}{2} \rho C_{dc} \int_{x_{cs}}^{x_{sail}} 2x^{3} R(x) \, dx - 2x_{cs}\left| x_{cs}\right| 2 \frac{1}{2} \rho S_{cs} C_{dc \_cs} - \frac{1}{2} x_{sail}^{3} \rho S_{sail} C_{dc \_sail} \]  
\[ M_{q[q]} = -\frac{1}{2} \rho C_{dc} \int_{x_{cs}}^{x_{sail}} 2x^{3} R(x) \, dx - 2x_{cs}\left| x_{cs}\right| 2 \frac{1}{2} \rho S_{cs} C_{dc \_cs} - \frac{1}{2} x_{sail}^{3} \rho S_{sail} C_{dc \_sail} \]  

(4.3.10)

where \( \rho \) is seawater density, \( R(x) \) is the radius of the hull as a function of axial length \( x \), \( x_{cs} \) is the mid-chord axial location of the control surfaces, \( x_{sail} \) is the mid-chord axial location of the sail, \( S_{cs} \) is the planform area of the control surfaces and \( S_{sail} \) is the planform area of the sail. The effect that crossflow drag from the sail will have on roll moments can be accounted for in a similar manner:

\[ K_{v[v]} = -\frac{1}{2} z_{sail} \rho S_{sail} C_{dc \_sail} \]  
\[ K_{r[r]} = -\frac{1}{2} z_{sail} x_{sail} |x_{sail}| \rho S_{sail} C_{dc \_sail} \]  

(4.3.11)

where \( z_{sail} \) is the mid-span vertical location of the sail.

### 4.3.3 ROLLING DRAG

The submarine will also encounter rolling resistance due to rotation about the \( x \) axis. This resistance will be made up of frictional drag from the hull as well as the crossflow drag from the sail and control surfaces. It is assumed that the drag in roll due to the control surfaces and sail will be much greater that that of the hull; because of this the rolling drag from the hull is neglected. Assuming four control surfaces, the following rolling drag coefficient is obtained where \( z_{cs} \) is the mid-span vertical location of the control surfaces:

\[ K_{p[p]} = -4 \left( \frac{1}{2} \rho S_{cs} C_{dc \_cs} \right) z_{cs}^{3} - \frac{1}{2} \rho S_{sail} C_{dc \_sail} z_{sail}^{3} \]  

(4.3.12)
4.4 BODY LIFT AND MOMENTS

When a slender body moves through a fluid at an angle of attack, helical body vortices form which create a low pressure suction force. This suction force is usually located aft of the bodies center of gravity which generates not only a lifting force, but also a stabilizing moment due to the offset in the location of the force. For small angle of attacks, these vortices are usually symmetric and stable – allowing for them to be modeled with a certain degree of accuracy. At higher angles of attack however, the vortices become very large and may shed asymmetrically which makes them very hard to model and predict. There are different methods available for estimating the body lift due to angle of attacks, especially at smaller angles. This project leverages work by Hoerner [14].

4.4.1 BODY LIFT FORCES

Hoerner provides experimental data from streamlined bodies and airplane fuselages that can be used to predict the lifting forces and moments that are seen by a submarine moving at an angle of attack. The lift on a body can be characterized by the equation:

\[
Lift = \frac{1}{2} \rho AC_y u^2
\]  

(4.4.1)

where \(A\) is the area referenced by the diameter of the body squared \((A=d^2)\), \(C_y\) is the body lift coefficient, \(\rho\) is fluid density and \(u\) is the forward velocity of the body. The body lift coefficient can be expressed as:

\[
C_y = C_{yd} \beta = \frac{\partial C_{yd}}{\partial \beta} \beta
\]  

(4.4.2)

where \(\beta\) is the angle of attack in degrees. Hoerner found that for streamline bodies with length to diameter ratios from 5 to 10, the body lift coefficient is roughly constant for modest angles up to 8 to 15 degrees dependent upon body shape, and in terms of degrees is roughly:

\[
C_{yd} = 0.003 \left( \frac{l}{d} \right)
\]  

(4.4.3)

or in radians
The angle of attack is a function of the bodies translational velocities. In reference to the x-y plane with angle of attack $\alpha$ and the x-z plane with angle of attack $\beta$ respectively it is defined as:

$$\tan \alpha = \frac{v}{u} \quad \tan \beta = \frac{w}{u}$$  \hspace{1cm} (4.4.5)

The assumption that $v$ and $w$ are small compared to $u$ allows for small angle approximations and the linearization of the angle of attack:

$$\alpha = \frac{v}{u} \quad \beta = \frac{w}{u}$$  \hspace{1cm} (4.4.6)

Using these relationships allows for the calculation of body lift force in the $y$ and $z$ directions due to angle of attack:

$$Lift_{ybody} = -\frac{1}{2} \rho d^2 C_{yd} uv$$

$$Lift_{zbody} = -\frac{1}{2} \rho d^2 C_{yd} uw$$  \hspace{1cm} (4.4.7)

which results in the hydrodynamic body lift coefficients:

$$Y_{wt} = -\frac{1}{2} \rho d^2 C_{yd}$$

$$Z_{wz} = -\frac{1}{2} \rho d^2 C_{yd}$$  \hspace{1cm} (4.4.8)

### 4.4.2 BODY LIFT MOMENTS

Hoerner found that for a round shaped streamlined body, the location of lift force is between 60 to 70% of the length of the body as measured from the leading edge. This is because the flow of fluid goes smoothly around the forward end of the body and only develops a force on the leeward side of the after section of the body. For this project, it is estimated that the center of lift occurs at a location of 65% from the submarine’s bow.
Taking into account the origin of the submarine being located at the center of buoyancy, the moment arm generated by the lift force can be defined as:

\[ x_{lift} = -0.65l - x_{cb} \]  

(4.4.9)

where \( x_{lift} \) is the location of the lift force, \( l \) is the submarine length, and \( x_{cb} \) is the location of the origin (center of buoyancy) measured from bow.

When combined with the lift force, the coefficients for moments caused by body lift are obtained:

\[ N_{w} = \frac{1}{2} \beta d^2 C_{yd} x_{lift} \]

\[ M_{w} = \frac{1}{2} \beta d^2 C_{yd} x_{lift} \]  

(4.4.10)

4.5 CONTROL SURFACE LIFT AND MOMENTS

Submarines typically use movable control surfaces to impart forces and moments that allow for changes in pitch and/or yaw. There are various control surface configurations that may be used, some of which are graphically depicted in Figure 4.3 [5].
The baseline submarine uses a standard cruciform configuration (configuration (a) in figure 4.3). In this setup, pitch is controlled with two horizontal stern planes and yaw is controlled with two vertical rudders. These control surfaces are linked such that both rudders move together, and both stern planes move together.
4.5.1 CONTROL SURFACE LIFT

The lift generated by a control surface can be expressed as:

\[ \text{Lift}_{cs} = \frac{1}{2} \rho C_{Ls} S_{cs} V^2 \delta_e \]  \hspace{1cm} (4.5.1)

where \( C_L \) is the lift coefficient of the control surface, \( S_{cs} \) is the planform area of the control surface, \( V \) is the total inflow velocity of the fluid, and \( \delta_e \) is the effective effective angle of attack for the control surface. Hoerner [14] and Triantafyllou [27] provide empirical equations for estimating the lift coefficient of a control surface:

\[ C_L = \frac{dC_{L}}{d\delta} \]  \hspace{1cm} (4.5.2)

and

\[ C_{L\delta} = \frac{dC_L}{d\delta} = \frac{1}{\frac{2\pi}{\bar{\alpha}} + \frac{1}{\pi(AR_s)} + \frac{1}{2\pi(AR_s)^2}} \]  \hspace{1cm} (4.5.3)

where \( \bar{\alpha} = 0.9 \), and \( AR_s \) is the effective aspect ratio of the control surface taking into account mirroring effects due to proximity to the hull. It can be found by:

\[ AR_s = 2(AR) = 2 \frac{\text{Span}_{cs}}{\text{Chord}_{cs}} = 2 \frac{\text{Span}_{cs}}{\text{Area}_{cs}} \]  \hspace{1cm} (4.5.4)

The effective angle of attack of the control surfaces (\( \delta_{re} \) for rudders and \( \delta_{se} \) for stern planes) is comprised of the summation of two components: 1) the angle of the control surface relative to zero angle (\( \delta_r \) for rudders and \( \delta_s \) for stern planes) and 2) the effective inflow angle of the fluid (\( \beta_{re} \) for rudders and \( \beta_{se} \) for stern planes).

\[ \delta_{re} = \delta_r - \beta_{re} \]

\[ \delta_{se} = \delta_s - \beta_{se} \]  \hspace{1cm} (4.5.5)

The geometry of these components are illustrated in figures 4.4 and 4.5.
The effective inflow velocities to the control surfaces are functions of the velocity of the submarine origin, and the velocity caused by the angular rotation of the submarine coupled with the distance of the control surface from the submarine origin. In the case of the rudder these are:

---

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\[ u_r = u + z_r q - y_r r \]
\[ v_r = v + x_r r - z_r p \]
\[ w_r = w + y_r p - x_r q \]  \hspace{1cm} (4.5.6)

where \( x_r, y_r, \) and \( z_r \) are the distances of the rudder from the submarine origin. For this submarine, the values of \( y_r \) and \( z_r \) are very small relative to the velocities of the submarine and are therefore neglected. Because the rudders are located far aft in the submarine, the \( x_r \) terms can be significant when coupled with rotational velocity. This simplifies these terms to be:

\[ u_r \approx u \]
\[ v_r \approx v + x_r r \]
\[ w_r \approx w - x_r q \]  \hspace{1cm} (4.5.7)

When combined with the geometric property,

\[ \tan(\beta_{re}) = \frac{v_r}{u_r} \]  \hspace{1cm} (4.5.8)

with the assumption of small angles and performing a similar derivation for the stern planes, the effective inflow angles can be approximated by:

\[ \beta_{re} = \frac{v_r}{u_r} \approx \frac{1}{u} (v + x_r r) \]
\[ \beta_{st} = \frac{w_s}{u_s} \approx \frac{1}{u} (w - x_f q) \]  \hspace{1cm} (4.5.9)

With the effective angle of attack for the control surfaces determined, the total lift forces generated by those surfaces can be approximated by substituting equations 4.5.9, 4.5.5, and 4.5.3 into 4.5.1:

\[ Y_{rudder} = \frac{1}{2} \rho C_{L_r} S_r (u^2 \delta_r - uv - x_r u_r) \]
\[ Z_{sternplanes} = -\frac{1}{2} \rho C_{L_s} S_s (u^2 \delta_s + uw - x_s u_s) \]  \hspace{1cm} (4.5.10)
Taking into account two rudders and two stern planes, these can be broken down into the individual hydrodynamic coefficients:

\[
\begin{align*}
Y_{\text{rudder}} &= -Y_{\text{rudder}} = \rho C_{Lb} S_r x_r \\
Y_{\text{rudder}} &= -\rho C_{Lb} S_r x_r \\
Z_{\text{rudder}} &= Z_{\text{rudder}} = -\rho C_{Lb} S_r x_s \\
Z_{\text{rudder}} &= \rho C_{Lb} S_r x_s
\end{align*}
\]

(4.5.11)

### 4.5.2 CONTROL SURFACE MOMENTS

The axial location of the control surfaces generates a moment on the submarine when the coupled with the lift force. This yields the following expressions for moments due to lift:

\[
\begin{align*}
N_{\text{rudder}} &= \frac{1}{2} \rho C_{Lb} S_r x_r (u^2 \delta_r - uv - x_r u) \\
M_{\text{rudder}} &= \frac{1}{2} \rho C_{Lb} S_r x_r (u^2 \delta_r + uw - x_r uq)
\end{align*}
\]

(4.5.12)

As in section 4.5.1, counting for two control surfaces each, the coefficients become:

\[
\begin{align*}
N_{\text{rudder}} &= -N_{\text{rudder}} = \rho C_{Lb} S_r x_r \\
N_{\text{rudder}} &= -\rho C_{Lb} S_r x_r \\
M_{\text{rudder}} &= M_{\text{rudder}} = \rho C_{Lb} S_r x_s \\
M_{\text{rudder}} &= -\rho C_{Lb} S_r x_s
\end{align*}
\]

(4.5.13)

### 4.5.3 CONTROL SURFACE DRAG

For conditions in which the submarine is not turning, it is appropriate for the drag from the control surfaces to be included as part of the axial drag – as is done in section 4.2.1. When the control surfaces are at an angle of attack due to either deflection or submarine motions, the drag coefficient for control surfaces changes as a function of effective angle of attack. Whicker and Fehlner found that the drag coefficient, \(C_D\) can be computed with reasonable accuracy based on the following equation [30]:

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\[ C_D = C_{d_{0}} + \frac{C_L^2}{\pi (AR_e e)} \]  

(4.5.14)

where \( C_{d_{0}} \) is the minimum section drag coefficient (equal to .0065 for NACA 0015 section, \( AR_e \) is the effective aspect ratio of the control surface taking into account mirroring due to hull interaction, and \( e \) is the Oswald efficiency factor (\( e=0.90 \)). As \( C_L \) is a function of effective angle of attack, so is \( C_D \). This allows for the drag of the control surfaces to be calculated:

\[ \text{Drag}_{\text{controlsurfaces}} = \frac{1}{2} \rho S_{c} C_D V^2 \]  

(4.5.15)

The total inflow velocity, \( V \), can be determined from the effective inflow velocities described in section 4.5.1. Using the simplified expressions for effective inflow velocities of equations 4.5.7 we obtain:

\[ V_{\text{rudder}} = \sqrt{u^2 + (v + x_r)^2} \]

\[ V_{\text{sternplanes}} = \sqrt{u^2 + (w - x_g)^2} \]  

(4.5.16)

Although the approximations of equations 4.5.10 and 4.5.12 are relatively accurate for predicting the lift and moments generated by the control surfaces, they do not account for the induced drag that is imparted on the submarine due to lift. Lift is always generated perpendicular to the inflow velocity of the fluid. Because the effective inflow angle of fluid is not always in line with the direction of the hull, the lift generated will have a component in the axial direction which contributes to the axial force on the control surface. Figure 4.6 shows these force components for the rudder.
Figure 4.6: Forces on Rudder

The axial and normal forces, as well as the moment ($Q_{cr}$) from the rudders can be determined from simple geometry such that:

$$X_{rudder} = F_{sr} = \text{Lift}_{rudder} \sin(\beta_{re}) - \text{Drag}_{rudder} \cos(\beta_{re})$$

$$Y_{rudder} = F_{yr} = \text{Lift}_{rudder} \cos(\beta_{re}) + \text{Drag}_{rudder} \sin(\beta_{re})$$

$$N_{rudder} = Q_{cr} = Y_{rudder} X_r$$ \hspace{1cm} (4.5.17)

Similarly for the stern planes:

$$X_{sternplanes} = F_{sx} = \text{Lift}_{sternplanes} \sin(\beta_{se}) - \text{Drag}_{sternplanes} \cos(\beta_{se})$$

$$Z_{sternplanes} = F_{zs} = \text{Lift}_{sternplanes} \cos(\beta_{se}) + \text{Drag}_{sternplanes} \sin(\beta_{se})$$

$$M_{rudder} = Q_{cr} = -Z_{sternplanes} X_s$$ \hspace{1cm} (4.5.18)
4.6 SAIL LIFT AND MOMENTS

The sail of a submarine is foil section, not unlike a control surface. They are however, much larger, asymmetrically located, and not movable. The same basic principles outlined in section 4.5 apply to the sail with the major difference being that the effective angle of attack of the sail is comprised entirely by the effective inflow velocities at the sail. An approach similar to section 4.5.3 is applied to the sail with the following exceptions:

1. The vertical location of the sail, \( z_{sail} \) cannot be neglected.
2. The asymmetry of the sail imparts a moment about the \( x \) axis in roll.
3. The effective angle of attack, \( \delta_{sail} \), is equal to the effective inflow angle, \( \beta_{sail} \).

The effective inflow velocities for the sail,

\[
\begin{align*}
    u_{sail} &= u + z_{sail}q - y_{sail}r \\
    v_{sail} &= v + x_{sail}r - z_{sail}p \\
    w_{sail} &= w + y_{sail}p - x_{sail}q
\end{align*}
\]  

(4.6.1)

can be simplified knowing that \( y_{sail} \) is zero:

\[
\begin{align*}
    u_{sail} &= u + z_{sail}q \\
    v_{sail} &= v + x_{sail}r - z_{sail}p \\
    w_{sail} &= w - x_{sail}q
\end{align*}
\]  

(4.6.2)

This leads to the angle of attack for the sail being defined as,

\[
\beta_{sail} = \tan^{-1}\left(\frac{v_{sail}}{u_{sail}}\right) = \tan^{-1}\left(\frac{v + x_{sail}r - z_{sail}p}{u + z_{sail}q}\right)
\]  

(4.6.3)

and the effective inflow velocity:

\[
V_{sail} = \sqrt{(u + z_{sail}q)^2 + (v + x_{sail}r - z_{sail}p)^2}
\]  

(4.6.4)

With these parameters defined, the same fundamental equations in section 4.5 for Lift and Drag – as well as their coefficients can be used to calculate those values for the sail. Using the same geometry defined in figure 4.4 and applying it to the sail yields:

\[
X_{sail} = F_{xs} = Lift_{sail} \sin(\beta_{sail}) - Drag_{sail} \cos(\beta_{sail})
\]
\[ Y_{\text{si}} = F_y = Lift_{\text{si}} \cos(\beta_{\text{si}}) + Drag_{\text{si}} \sin(\beta_{\text{si}}) \]

\[ N_{\text{si}} = Q_{\text{si}} = Y_{\text{si}} x_{\text{si}} \]

\[ K_{\text{si}} = Q_{\text{xi}} = -Y_{\text{si}} z_{\text{si}} \]  \hspace{1cm} (4.6.5)

### 4.7 PROPELLER FORCES AND MOMENTS

Propellers are typically used to develop the axial thrust force required to propel a submarine at forward speed. To achieve a steady-state speed, the thrust force generated by the propeller must equal the resistance generated by the hull, appendages, and resistance effects due to the interactions between the hull and the propeller. There is standard nomenclature to describe the parameters and metrics that are used to define the operational characteristics of a propeller. Table 4.3 lists the basic parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Advance Coefficient</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>KT</td>
<td>Thrust Coefficient</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>KQ</td>
<td>Torque Coefficient</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>n</td>
<td>Propeller rotational speed</td>
<td>rev/s</td>
</tr>
<tr>
<td>V</td>
<td>Vessel Speed</td>
<td>m/s</td>
</tr>
<tr>
<td>VA</td>
<td>Advance Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>T</td>
<td>Thrust</td>
<td>N</td>
</tr>
<tr>
<td>Q</td>
<td>Torque</td>
<td>N·m</td>
</tr>
<tr>
<td>D</td>
<td>Propeller Diameter</td>
<td>m</td>
</tr>
<tr>
<td>(\eta_0)</td>
<td>Open Water Efficiency</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>CT</td>
<td>Thrust Coefficient</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>CQ</td>
<td>Torque Coefficient</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Advance Angle at 70% Radius</td>
<td>Deg</td>
</tr>
<tr>
<td>w</td>
<td>Wake fraction</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>t</td>
<td>Thrust deduction factor</td>
<td>Non-Dim</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Seawater Density</td>
<td>Kg/m³</td>
</tr>
</tbody>
</table>

Table 4.3: Propeller Nomenclature

The advance ratio is defined as,

\[ J = \frac{V_A}{nD} \]  \hspace{1cm} (4.7.1)

where the advance velocity is:

\[ V_A = V(1 - w) \]  \hspace{1cm} (4.7.2)
From equation 4.7.2, we see that the wake fraction provides a measure of determining the incoming velocity into the propeller as a function of submarine speed. The wake fraction is highly dependent upon the hull shape and is often derived from model testing and working trial results backward based on data from full scale ships and submarines. Parametric studies for single screw submarines have been done resulting in the following equation that provides a means of estimating the wake fraction [15]:

\[ w = 1 - 0.371 - 3.04 \frac{D}{\sqrt{W_s}} \]  

(4.7.3)

where \( W_s \) is the wetted surface of the submarine. The thrust that must be generated by the propeller must equal the resistance of the hull to achieve a required speed. When a propeller is placed behind a submarine it increases the apparent resistance of the hull due to a reduction in pressure seen at the forward end of the propeller blades. This increased resistance is accounted for by the thrust deduction coefficient, \( t \), such that:

\[ T = \frac{R}{(1 - t)} \]  

(4.7.4)

where \( R \) is the hull resistance at a given speed. Similar to the wake fraction, regression equations are available that allow for the prediction of the thrust deduction coefficient [15]:

\[ t = 1 - .632 - 2.44 \frac{D}{\sqrt{W_s}} \]  

(4.7.5)

The thrust and torque coefficients, \( K_T \) and \( K_Q \), are the coefficients normally used when describing propellers. They are defined as:

\[ K_T = \frac{T}{\rho n^2 D^4} \]

\[ K_Q = \frac{Q}{\rho n^2 D^5} \]  

(4.7.6)

These coefficients, along with \( \eta_o \), can be plotted as a function of advance coefficient. The resulting graph, known as an open water diagram to signify the propellers performance
when not in interaction with a hull, is dependent upon the specific propeller geometry and characteristics. A representative open water diagram is provided in figure 4.7.

![Open Water Propeller Diagram](image)

**Figure 4.7: Open Water Propeller Diagram**

For most propellers, these graphs are developed only for the condition where there is a positive ship speed and a positive propeller speed - a condition known as the first quadrant. Other quadrants exist according to ship and propeller speeds, and are summarized in table 4.4.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Description</th>
<th>Ship Speed (V)</th>
<th>Propeller Speed (n)</th>
<th>Beta (β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ahead</td>
<td>+</td>
<td>+</td>
<td>0-90</td>
</tr>
<tr>
<td>2</td>
<td>Crashahead</td>
<td>-</td>
<td>+</td>
<td>270-360</td>
</tr>
<tr>
<td>3</td>
<td>Crashback</td>
<td>+</td>
<td>-</td>
<td>90-180</td>
</tr>
<tr>
<td>4</td>
<td>Backing</td>
<td>-</td>
<td>-</td>
<td>180-270</td>
</tr>
</tbody>
</table>

**Table 4.4: Propeller Operating Quadrants**

Here the advance angle at 70% radius, $\beta$ is defined as:

$$\beta = \tan^{-1} \frac{V_A}{0.7\pi n D} \quad (4.7.7)$$

To determine the performance characteristics at quadrants other than the 1st requires the open water diagrams to be expanded to cover all four quadrants such as those shown in figure 4.8. This is not typically done for propellers and doing so presents problems since
when propeller speed approaches zero, the advance coefficient, $K_T$, and $K_Q$ all approach infinity and have a discontinuity.

![Propeller Operating Quadrants](image)

**Figure 4.8 Propeller Operating Quadrants**

To resolve the characterization of four quadrant thrust and torque performance curves are usually presented using the $C_T$ and $C_Q$ coefficients a function of $\beta$. $C_T$ and $C_Q$ are defined as:

$$
C_T = \frac{T}{\frac{1}{2} \left( V_A^2 + (0.7 \pi n D)^2 \right) \frac{\pi}{4} D^2} = \frac{8K_T}{\pi (J^2 + (0.7 \pi)^2)}
$$

$$
C_Q = \frac{T}{\frac{1}{2} \left( V_A^2 + (0.7 \pi n D)^2 \right) \frac{\pi}{4} D^3} = \frac{8K_Q}{\pi (J^2 + (0.7 \pi)^2)}
$$

(4.7.8)

The Maritime Research Institute Netherland (MARIN) has conducted 4 quadrant propeller data on several of the Wageningen B-Screw Series of propeller blades. The data from this testing has been faired by regression to allow for the $C_T$ and $C_Q$ coefficients to be determined for 14 of these propellers covering a range of blade numbers, pitch/diameter ratios, and expanded area ratios [24]. The coefficients can be determined through a summation of regression coefficients:
\[ C_T = \frac{1}{100} \sum_{k=0}^{30} \{ A_T(k) \cos(k\beta) + B_T(k) \sin(k\beta) \} \]

\[ C_Q = \frac{-1}{1000} \sum_{k=0}^{30} \{ A_Q(k) \cos(k\beta) + B_Q(k) \sin(k\beta) \} \]

(4.7.9)

where the regression coefficients are unique to the particular blade series. The regression coefficients for the 5-bladed B5-75 series with a pitch/diameter ratio of 1.0 as an example are:

<table>
<thead>
<tr>
<th>K</th>
<th>( A_T(k) )</th>
<th>( B_T(k) )</th>
<th>( A_Q(k) )</th>
<th>( B_Q(k) )</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>2.38E+00</td>
<td>0.00E+00</td>
<td>-3.62E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>1</td>
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<td>1.16E+02</td>
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<td>3</td>
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<td>-5.45E+00</td>
<td>-1.82E+01</td>
</tr>
<tr>
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<td>3.13E+00</td>
</tr>
<tr>
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<td>4.48E+00</td>
<td>6.63E+00</td>
<td>-8.56E+00</td>
</tr>
<tr>
<td>6</td>
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<td>-1.17E+00</td>
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</tr>
<tr>
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<td>-6.17E+00</td>
<td>1.62E+00</td>
</tr>
<tr>
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</tr>
<tr>
<td>9</td>
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<td>-2.55E+00</td>
</tr>
<tr>
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<td>-9.20E-01</td>
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<td>1.03E+00</td>
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<td>-1.13E+00</td>
</tr>
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</tr>
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<tr>
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<td>-1.36E-03</td>
</tr>
<tr>
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<td>-6.83E-02</td>
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</tr>
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<td>-4.54E-01</td>
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</tr>
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<td>3.86E-01</td>
<td>6.71E-01</td>
</tr>
<tr>
<td>24</td>
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<td>7.01E-02</td>
<td>4.36E-01</td>
<td>-2.53E-01</td>
</tr>
<tr>
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<td>-1.06E-01</td>
<td>4.31E-02</td>
</tr>
<tr>
<td>26</td>
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<td>-2.08E-01</td>
<td>2.05E-01</td>
</tr>
<tr>
<td>27</td>
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<td>5.49E-01</td>
<td>3.82E-01</td>
</tr>
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<td>3.41E-01</td>
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<td>3.56E-02</td>
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<td>-5.43E-02</td>
<td>-8.52E-02</td>
<td>2.54E-01</td>
</tr>
</tbody>
</table>

Table 4.5: Four Quadrant Coefficients for Wageningen B5-75, P/D=1.0 Propeller
This allows for a regression fit of the four quadrant graph for the B5-75 P/D=1.0 series as shown in figure 4.9:

![Four Quadrant Diagram](image)

**Figure 4.9 Four Quadrant Diagram**

This information allows for the thrust and torque to be found for any submarine speed and shaft rpm, including crashback maneuvers that would occur as part of the response to a jammed control surface casualty. Any of the 14 Wageningen B-Screw Series that have four quadrant regression coefficients can be modeled. The thrust and torque can be expressed in the vehicle force notation as:

\[
X_{\text{prop}} = T
\]

\[
K_{\text{prop}} = Q
\]

(4.7.10)

### 4.8 AZIMUTHING PODDED PROPULSOR FORCES AND MOMENTS

Azimuthing propulsion pods have been used with increasing frequency on surface ships, especially on cruise ships. They generally provide improved maneuverability due to their ability to provide forces at any angle [18]. On submarines their use has been limited to units that assist in slow speed maneuvers. Their application as the primary propulsion
has not been done on submarines. To do so would allow the forces and moments created by the azimuthing pods to replace the forces that are typically generated by the aft control surfaces and the propeller.

Studies on the force and moment predictions from azimuthing pods have been done. Stettler characterized the quasi-steady vectored maneuvering forces associated with an podded propulsor to include the unsteady transient maneuvering forces caused by rapid changes in azimuth angle and propeller speeds [27]. In his work he developed non-dimensionalized coefficients to predict the axial (F_x) and normal (F_y) forces for a range of advance ratios from 0 to 0.58, and azimuth angles over the range of +/- 180 degrees. He also developed pod-only force coefficients over an azimuth angle of +/- 180 degrees to account for the forces associated only with the pods’ orientation. Figure 4.10 shows the basic geometry involved with the azimuthing pods in the x-y plane. In this case δ is the propulsor azimuth angle and β is the effective inflow angle of the fluid.

![Figure 4.10: Azimuthing Pod Geometry](image)

The effective angle of attack of the pod, α, is found in a similar fashion as the effective angle of attack of the control surfaces derived in section 4.5.

\[
\alpha_{pod} = \delta_{pod} - \beta_{pod}
\]  

The non-dimensional force coefficients for surge and sway are:

\[
K_{F_x} = \frac{F_x}{\rho n^2 D^4}
\]
Knowing the axial distance of the pod from the submarine’s origin, $x_{pod}$, also allows for the moment generated by the pod about the $z$ axis to be calculated. These coefficients are applicable for the $x$-$z$ plane as well with the exception that sway force is in the $z$ direction and the moment is about the $y$ axis. In this case the $K_{Fx}$ coefficient is the same but the $K_{Fy}$ coefficient would be $K_{Fz}$ and the angle of attack for the pod would be based on the $x$-$z$ velocities. If the pods operate independently of one another, a moment about the $x$ axis is also generated dependent upon the pods radial position from the vehicle centerline – expressed as either $y_{pod}$ or $z_{pod}$. These moments, along with the forces in the vehicle force notation are:

\[
X_{pod} = F_x \\
Y_{pod} = F_y \\
Z_{pod} = F_z \\
N_{pod} = Q_z = Y_{pod}x_{pod} \\
M_{pod} = Q_y = -Z_{pod}x_{pod} \\
K_{pod} = Q_x = -Y_{pod}z_{pod} = Z_{pod}y_{pod} \tag{4.8.3}
\]

Figure 4.11 shows graphs of the force coefficients against azimuth angle. Figure 4.12 shows a graph of the pod-only axial and normal force coefficients against azimuth angle, which allow for the modeling of the pods when the propeller is not turning. It should be noted that for this data the azimuth angle is equal to the angle of attack of the pod.
Quasi-steady surge force ($F_s$) vs. azimuth angle vs. advance coefficient

Quasi-steady sway force ($F_s$) vs. azimuth angle vs. advance coefficient

Figure 4.11: Azimuthing Pod Force Coefficients
The bare hull hydrodynamic coefficients are assumed to be the same for the conventionally propelled submarine and one with azimuthing podded propulsion. The significant differences in modeling are the lack of aft control surfaces and the propeller being replaced by the pods. Since the radial position of the pods are away from the centerline of the submarines, the wake fraction used in calculating the advance coefficient will be a function of the distance from the hull. The wake fraction determined in section 4.7 is the average wake fraction behind the hull. Because of this it is assumed that this will also be representative of the wake fraction seen by the pods.

A limitation in the data is that it does not allow for crashback conditions of the pods. In practice, this might be an immediate response by the crew for certain casualties. To address this, in situations where reverse thrust is required, the pods will be rotated to the 180 degree position. This would likely provide improved thrust characteristics in slowing forward movement of the submarine since the propellers will be rotating in their designed forward rotations.

Figure 4.12 Pod-Only Force Coefficients
4.9 SUMMATION OF FORCES AND MOMENTS

4.9.1 CROSS TERMS

The different cross-terms from equations 4.2.16, 4.4.8, and 4.5.11 must be combined to get the total hydrodynamic coefficients on the submarine.

\[
Y_{uw} = Y_{wul} + Y_{wuf} \\
Y_{wr} = Y_{ural} + Y_{wrf} \\
Z_{uw} = Z_{wul} + Z_{wuf} \\
Z_{ur} = Z_{ural} + Z_{urq} \\
M_{uw} = M_{uwa} + M_{wuf} + M_{wul} \\
M_{ur} = M_{ural} + M_{urq} \\
N_{uw} = N_{uwa} + N_{wuf} + N_{wul} \\
N_{ur} = N_{ural} + N_{urq}
\] (4.9.1)

Since the control surfaces and sail are accounted for as external forces, they are dropped from the equations above, leaving only the added mass and body lift force and moment contributions to the hydrodynamic coefficients.

4.9.2 TOTAL FORCES AND MOMENTS

All the forces and moment coefficients associated with the vehicle are combined with the external forces from the various components. Equations 4.9.2 below show the summation of the external forces and moments for the conventional submarine configuration with propeller and aft control surfaces:

\[
\sum X_{ext} = X_{HSu} + X_{u} + X_{wq} + X_{qq} + X_{vr} + X_{wr} + X_{wul} + X_{wuf} + X_{wul} + \{X_{rudder} + X_{sternplanes} + X_{prop}\}
\]

\[
\sum Y_{ext} = Y_{HSv} + Y_{v} + Y_{r} + Y_{ur} + Y_{wp} + Y_{pq} + Y_{uv} + Y_{vq} + Y_{ur} + Y_{sail} + \{Y_{rudder}\}
\]

\[
\sum Z_{ext} = Z_{HSw} + Z_{w} + Z_{q} + Z_{wq} + Z_{wp} + Z_{rp} + Z_{uw} + Z_{wuf} + Z_{wul} + Z_{q} + \{Z_{sternplanes}\}
\]
These equations can be modified to account for the azimuthing pods by replacing the control surface and prop forces/moments in \{\} with the forces/moments associated with the pods in section 4.8.

4.10 EMPIRICAL DATA

The selection of the DARPA SUBOFF Hullform allowed for the utilization of many non-dimensional hydrostatic coefficients obtained from captive model measurements obtained from planar motion mechanism (PMM) testing by NSWCCD [23]. Empirical coefficients were utilized to the maximum extent possible. For those coefficients that were not available, the coefficients were obtained using the analytical approach described in this chapter.
CHAPTER 5 MODEL ARCHITECTURE

To simulate the maneuvering characteristics and response of the submarine, a MATLAB program was created that utilized the SIMULINK modeling tool. This MATLAB program consists of two scripts that serve different functions and one SIMULINK model:

1. A MATLAB script is utilized to generate and define the geometric characteristics of the submarine and the hydrodynamic coefficients, and store them in a matrix.
2. A separate “Driver” MATLAB script is used to load the data from the first script as well as the four quadrant propeller data. This script runs the SIMULINK model simulation for a set period of time and is used to retrieve and display data obtained during the simulation.
3. A SIMULINK model is used as the main tool during the simulation. This model links together different force inputs and solves the equations of motion in discrete time steps to predict the motion of the submarine as a function of time. A schematic representation of the SIMULINK model is shown in figure 5.1.

Figure 5.1: Schematic of SIMULINK Model
5.1 SIMULINK MODEL

Several distinct blocks are used to create the complete SIMULINK model. Each of these blocks consist of separate input variables that change as a function of time and an embedded MATLAB script that utilizes the input variables to calculate output variables that are used in other blocks. The variables are calculated in separate time steps as the SIMULINK model is sequentially analyzed from start to finish.

5.1.1 CONSTANT BLOCK

The constant block is utilized to define input variables for use in other blocks. This block is defined in a previously run hydrostatic coefficient MATLAB script and loaded by the ‘Driver’ script for use in SIMULINK. It is also used to define four quadrant propeller data.

5.1.2 COORDINATE TRANSFORMATION BLOCK

The coordinate transfer block utilizes the coordinate transformation matrices derived in chapter 3.1. The body fixed submarine translational and rotational velocities are used as input variables and transformed into inertial frame translational and rotational velocities. These inertial frame translational and rotational velocities are then integrated to obtain an inertial frame location and orientation, which is then used in other blocks.

5.1.3 INTEGRATOR BLOCK

This block takes the submarine vehicle accelerations that are solved for in the maneuvering block and integrates them to determine the submarine vehicle velocities.

5.1.4 PROPELLER DYNAMICS BLOCK

This block is based on the theory described in chapter 4.7. It uses submarine velocity, data from the constant block, and a user configurable propeller RPM scheme as input variables. The embedded MATLAB script then calculates the resultant thrust and torque that serve as inputs to the maneuvering block.

5.1.5 CONTROL SURFACE EFFECTS BLOCKS

The control surface effects blocks are used to calculate the external forces and moments generated by the rudder, sternplanes, bowplanes, and sail described in chapter
4.5. Separate blocks are used for each control surface. These blocks use vehicle translational and rotational velocities as well as a user configurable scheme that defines the control surface deflections as inputs. The embedded MATLAB script in these blocks then calculate effective inflow velocities, angle of attacks, lift and drag. The forces and moments generated on the submarine by the control surfaces are provided as outputs which are then used in the maneuvering block.

5.1.6 AZIPOD DYNAMICS BLOCKS

These blocks are used to calculate the forces and moments generated by the podded propulsion units. These characteristics are outlined in chapter 4.8. For inputs the blocks utilize the submarine velocities as well as two user defined and configurable variables: 1) The RPM scheme of the pod and 2) The azimuth angle of the pod. These inputs are initially used to calculate the effective azimuth angle and the advance ratio. These values are then used to determine the quasi-steady surge and sway force coefficients that are then used to calculate the forces and moments generated on the submarine by the pods, which are input to the maneuvering block.

5.1.7 EMBT BLOCK

The EMBT Block is used to calculate the various dynamics associated with an EMBT blow. This block utilizes several inputs, including data from the constant block, submarine depth, as well two inputs that are determined by the output of the EMBT block:

1. The amount of air cumulatively blown by the EMBT system
2. The current air bank pressure

An additional input is the user decision of the time during the simulation at which to initiate the EMBT blow. The dynamics of this block are described in chapter 4.1.1. The outputs of buoyancy and longitudinal center of buoyancy are used as inputs to the maneuvering block. The outputs of blow rate and air bank pressure rate are integrated to obtain the inputs of blown air and air bank pressure, respectively.

5.1.8 MANEUVERING BLOCK

The maneuvering block receives multiple types of input used to solve the nonlinear equations of motion to determine the submarine's translational and rotational
accelerations for a given time period during a SIMULINK simulation. The various inputs into this block include:

1. Submarine translational and rotational velocities \((u, v, w, p, q, r)\): These are obtained from the output of the integrator block.

2. Submarine orientation \((\phi, \theta, \psi)\): The inertial frame orientation of the vehicle is obtained from the output of the coordinate transfer block.

3. Submarine constants: These inputs, loaded from the constant block, include the geometric properties, mass properties, moments of inertia, and hydrodynamic coefficients of the submarine.

4. External forces and moments \((F_x, F_y, F_z, Q_x, Q_y, Q_z)\): These inputs are the summation of the forces and moments due to the combined outputs of the propeller dynamics block, the control surface effect’s blocks, and azipod dynamics blocks.

5. Buoyancy characteristics \((B, x_B)\): These inputs, obtained from the EMBT block, take into account the change in buoyancy and the longitudinal center of buoyancy as a function of an EMBT blow.

The maneuvering block solves for the accelerations of the submarine using the combined nonlinear equations of motion. These accelerations are the output of this block which serve as the input to the integration block.

### 5.2 NONLINEAR EQUATIONS OF MOTION

The governing equations of motion for translation and rotation (3.1.15) can be combined with the external force and moment equations (4.9.2) to develop the combined nonlinear equations of motion in all six degrees of motion.

For translation in the \(x\) direction:

\[
\begin{align*}
& m \left[ u + qw - rv - x_G (q^2 + r^2) + z_G (pr + q) \right] = \\
& X_{HS} + X \cdot u + X_{wu} wq + X_{qq} qq + X_{vr} vr + X_{rr} rr + \\
& X_{dual} [u] + X_{sal} + \{ X_{rudder} + X_{sternplane3} + X_{prop} \}
\end{align*}
\]  

\((5.2.1)\)
For translation in the y direction:

\[
m \left[ \dot{v} - wp + ur + x_G(qp + r) + z_G(qr - p) \right] = \\
Y_{HS} + Y_v \dot{v} + Y_r \dot{r} + Y_{ur} ur + Y_{wp} wp + Y_{pq} pq + Y_{uv} uv + \\
Y_{v|r} |r| + Y_{sail} + \{Y_{rudder}\} \tag{5.2.2}
\]

For translation in the z direction:

\[
m \left[ \dot{w} - uq + vp + x_G(rp - \dot{q}) - z_G(p^2 + q^2) \right] = \\
Z_{HS} + Z_w \dot{w} + Z_q \dot{q} + Z_{uw} uq + Z_{wp} wp + Z_{rp} rp + Z_{uv} uv + \\
Z_{w|w} |w| + Z_{q|q} |q| + \{Z_{sternplanes}\} \tag{5.2.3}
\]

For rotation about the x-axis:

\[
I_{xx} \dot{p} + (I_{zz} - I_{yy})rq - mz_G(\dot{v} - wp + ur) = \\
K_{HS} + K_v \dot{v} + K_r \dot{r} |r| + K_{uv} |v| + K_{p|r} |p| + K_{sail} + \{K_{prop}\} \tag{5.2.4}
\]

For rotation about the y-axis:

\[
I_{yy} \dot{q} + (I_{xx} - I_{zz})rp + m \left[ z_G(\dot{w} - vr + wq) - x_G(\dot{w} - uq + vp) \right] = \\
M_{HS} + M_w \dot{w} + M_q \dot{q} + M_{uw} uw + M_{uv} uv + M_{wp} wp + M_{rp} rp + \\
M_{w|w} |w| + M_{q|q} |q| + \{M_{sternplanes}\} \tag{5.2.5}
\]

For rotation about the z-axis:

\[
I_{zz} \dot{r} + (I_{yy} - I_{xx})pq + mx_G(\dot{v} - wp + ur) = \\
N_{HS} + N_v \dot{v} + N_r \dot{r} + N_{ur} ur + N_{uv} uv + N_{wp} wp + N_{pq} pq + \\
N_{v|r} |r| + N_{sail} + \{N_{rudder}\} \tag{5.2.6}
\]

As in chapter 4.9, these equations are modified for the azimuthing pods by replacing the control surface and propeller force and moment terms in \{\} with the forces and moments.
from the pods. To allow for the acceleration terms of the submarine to be calculated, they are separated from the rest of the terms such that the combined equations of motion are rewritten as:

For translation in x direction:

\[
\begin{align*}
(m - X_u)\ddot{u} + mz_G \dot{q} &= X_{HS} + (X_{wq} - m)\dot{w}q + (X_{qq} + mx_G)qq \\
+(X_{vr} + m)\dot{v}r + (X_{rr} + mx_G)\dot{r}r + X_{u[u]}\dot{u}u - mZG pr + X_{sail} + \\
\{X_{rudder} + X_{sternplanes} + X_{prop}\}
\end{align*}
\]  \ (5.2.7)

For translation in y direction:

\[
\begin{align*}
(m - Y_v)\ddot{v} - mz_G \dot{p} &= Y_{HS} + (Y_{ur} - m)\dot{u}r + \\
(Y_{wp} + m)\dot{w}p + (Y_{pq} - mx_G)pq + Y_{uw}uv + Y_{vr}v|v| + Y_{r|r}|r| + \\
mz_G qr + Y_{sail} + \{Y_{rudder}\}
\end{align*}
\]  \ (5.2.8)

For translation in z direction:

\[
\begin{align*}
(m - Z_w)\ddot{w} - (mx_G + Z_q)\dot{q} &= Z_{HS} + (Z_{wq} + m)uq + \\
(Z_{vp} - m)\dot{v}p + (Z_{rp} - mx_G)rp + Z_{uw}uw + Z_{w[w]}w|w| + \\
Z_{q[q]}q|q| + mz_G (p^2 + q^2) + \{Z_{sternplanes}\}
\end{align*}
\]  \ (5.2.9)

For rotation about x-axis:

\[
\begin{align*}
-mz_G \ddot{v} + (I_{xx} - K_p) \dot{p} &= K_{HS} + K_{a[p]}r|p| + K_{a[v]}v|v| + K_{p[p]}p|p| - \\
(I_{zz} - I_{yy})qr + m(uq - vp) - mz_G (wp - ur) + K_{sail} + \{K_{prop}\}
\end{align*}
\]  \ (5.2.10)

For rotation about y-axis:

\[
\begin{align*}
mz_G \ddot{u} - (mx_G + M_q)\dot{w} + (I_{yy} - M_w)\dot{q} &= M_{HS} + M_{uw}uw + \\
(M_{wq} - mx_G)uq + (M_{vp} + mx_G)vp + (M_{rp} - (I_{xx} - I_{zz}))rp + \\
M_{w[w]}w|w| + M_{q[q]}q|q| + mz_G (vr - wq) + \{M_{sternplanes}\}
\end{align*}
\]  \ (5.2.11)

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For rotation about z-axis:

\[
(mx_G - N_v)\dot{v} + (I_{zz} - N_r)\dot{r} = \dot{N}_{HS} + (N_{ur} - mx_G)ur + N_{uv}uv + \\
(N_{wp} - mx_G)wp + (N_{pq} - (I_{yy} - I_{zz}))pq + N_{vq}vq + N_{rr}rr + N_{sail} + \{N_{rudder}\}
\]

Equations 5.2.7 through 5.2.12 can be expressed in matrix form:

\[
\begin{bmatrix}
(m - X_v) & 0 & 0 & 0 & m z_G & 0 \\
0 & (m - Y_v) & 0 & -m z_G & 0 & (m z_G - Y_r) \\
0 & 0 & (m - Z_w) & 0 & -(m z_G + Z_q) & 0 \\
0 & -m z_G & 0 & (I_{xx} - K_p) & 0 & 0 \\
m z_G & 0 & -(m z_G + M_w) & 0 & (I_{yy} - M_q) & 0 \\
0 & (m z_G - N_v) & 0 & 0 & 0 & (I_{zz} - N_r)
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w} \\
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
\ddot{X} \\
\ddot{Y} \\
\ddot{Z} \\
\ddot{K} \\
\ddot{M} \\
\ddot{N}
\end{bmatrix}
\]

This allows for the acceleration terms to be solved for by the following equation:

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w} \\
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
(m - X_v) & 0 & 0 & 0 & m z_G & 0 \\
0 & (m - Y_v) & 0 & -m z_G & 0 & (m z_G - Y_r) \\
0 & 0 & (m - Z_w) & 0 & -(m z_G + Z_q) & 0 \\
0 & -m z_G & 0 & (I_{xx} - K_p) & 0 & 0 \\
m z_G & 0 & -(m z_G + M_w) & 0 & (I_{yy} - M_q) & 0 \\
0 & (m z_G - N_v) & 0 & 0 & 0 & (I_{zz} - N_r)
\end{bmatrix}
\begin{bmatrix}
\ddot{X} \\
\ddot{Y} \\
\ddot{Z} \\
\ddot{K} \\
\ddot{M} \\
\ddot{N}
\end{bmatrix}
\]
5.3 NUMERICAL SOLUTION TO THE NONLINEAR EQUATIONS OF MOTION

In the case of the nonlinear equations of motion in equation 5.2.14, the accelerations can be expressed as a vector:

\[ \dot{x}_n = f(x_n, I_n) \]  

(5.3.1)

where \( x_n \) is the state of the submarines kinematics at a time step \( n \):

\[ x_n = [u', v', w, p, q, r, x, y, z, \phi, \theta, \psi] \]  

(5.3.2)

and \( I_n \) is the input forces, moments, and other dynamics at a time step \( n \):

\[ I_n = [F_x, F_y, F_z, Q_x, Q_y, Q_z, B, x_g] \]  

(5.3.3)

The accelerations in the nonlinear equations of motion are solved by using numerical ordinary differential equation (ODE) solvers inherent in MATLAB. There are several solvers in MATLABs library that are available and are grouped in two main types: fixed-step and variable-step solvers. In their most basic form, these solvers take the state of the submarine at a given time, \( x_n \), and calculate the rate of change of those states, \( \dot{x}_n \), at that time. This rate of change \( \dot{x}_n \) is then multiplied by a time step \( \Delta t \) and added to original state to determine the state of the submarine at the next time step. This can be expressed mathematically by a first order Euler's Method solver:

\[ \dot{x}_{n+1} = \dot{x}_n + \dot{x}_n \Delta t \]  

(5.3.4)

A fixed-step solver uses a defined time step that does not vary while a variable-step solver adjusts the time-steps such that larger time steps are used during times when the dynamics of the model are not rapidly changing and smaller time steps are used when the dynamics of the model are rapidly changing. A variable-step solver is typically used to reduce computational simulation time, however for this thesis, a fourth order fixed-step solver using the Runge-Kutta integration technique was used.
5.3.1 RUNGE-KUTTA SOLVER

The Runge-Kutta method improves on the accuracy of the Euler’s Method by using a weighted average of a slope over a time step. For this method, the slope at four points are calculated:

\[
k_1 = x_n + \left( x_{n+1} - x_n \right) I_n
\]
\[
k_2 = f \left( x_n + \frac{\Delta t}{2} k_1, I_{n+\frac{1}{2}} \right)
\]
\[
k_3 = f \left( x_n + \frac{\Delta t}{2} k_2, I_{n+\frac{1}{2}} \right)
\]
\[
k_4 = f \left( x_n + \Delta t k_3, I_n \right)
\]

where \( k_1 \) is the slope at the beginning of the time step, \( k_2 \) is the slope at the midpoint of the time step using \( k_1 \), \( k_3 \) is the slope at the midpoint of the time step using \( k_2 \) and \( k_4 \) is the slope at the end of the time step using \( k_3 \), and the input vector:

\[
I_{n+\frac{1}{2}} = \frac{1}{2} \left( I_n + I_{n+1} \right)
\]

These slopes are then weighted to find an average slope and the state at the next time step:

\[
x_{n+1} = x_n + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4)
\]
CHAPTER 6 MANEUVERING ANALYSIS

The maneuvering performance of the two submarine configurations is detailed in this chapter. Key performance aspects looked at include:

1. Powering
2. Acceleration
3. Deceleration
4. Turning
5. Depth changing maneuvers
6. Response and recovery from casualties

For the analyses contained in this chapter, the hydrodynamic coefficients were based on the DARPA SUBOFF hullform model outlined in chapter 2.1.

6.1 POWERING

The powering requirements of the DARPA SUBOFF hull form were estimated using the MIT 2N Submarine Mathcad model, based on the theory and equations developed by Jackson [15]. The SUBOFF hull form was geometrically generated in the Mathcad program, and the effective horsepower (EHP) requirements at given speeds were determined to serve as the baseline against which the MATLAB program generated in this thesis would be validated.

6.1.1 STANDARD CONFIGURATION

Calculating the required power versus speed in the MATLAB program required a specific propeller to be selected. The 7 meter diameter, Wageningen B4-70 series propeller with a pitch to diameter (P/D) ratio of 1.4 was selected for use in this analysis. This propeller was selected because it was the propeller with the highest P/D ratio for propellers that had four quadrant data available as outlined in 4.7. With this data, the steady state speed of the submarine is controlled by the propeller RPM. This was done in an iterative manner until the desired speed was obtained. At this point the thrust (and power) from the propeller was determined from the output of the propeller dynamic block
described in chapter 5.1.4. The results of this, and the comparison against the Mathcad predictions are shown in table 6.1 below:

<table>
<thead>
<tr>
<th>Submarine Speed (Knots)</th>
<th>Propeller RPM</th>
<th>MATLAB EHP (hp)</th>
<th>MATHCAD EHP (hp)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>42.5</td>
<td>189</td>
<td>211</td>
<td>11.0%</td>
</tr>
<tr>
<td>10</td>
<td>85</td>
<td>1491</td>
<td>1577</td>
<td>5.6%</td>
</tr>
<tr>
<td>15</td>
<td>128</td>
<td>5071</td>
<td>5121</td>
<td>1.0%</td>
</tr>
<tr>
<td>20</td>
<td>171</td>
<td>12069</td>
<td>11825</td>
<td>2.0%</td>
</tr>
<tr>
<td>25</td>
<td>214</td>
<td>23626</td>
<td>22645</td>
<td>4.2%</td>
</tr>
</tbody>
</table>

Table 6.1: Standard Propulsion Configuration Powering Requirements

The powering requirement predicted by the MATHLAB SIMULINK model is consistent with values predicted by the Mathcad program, indicating that the modeling of the resistance is valid.

6.1.2 PODDED CONFIGURATION

The procedure for determining the power requirement for the podded configuration was similar to the standard configuration with a few exceptions. The added drag from the control surfaces is not present since they are not part of the pods. The powering estimation in the MIT 2N Submarine Mathcad program was adjusted to account for this. Although the pods themselves do have drag associated with them, the method used to calculate the thrust and forces associated with the pods takes this resistance into account.

The data source and equations outlined in chapter 4.8 for the pods leave only the selection of the size of the pods (determined by the diameter of the pod), the selected RPM of the pod, and the azimuth angle as user configurable parameters. For the case of steady state speed, the azimuth angle is fixed at zero degrees. This leaves only pod RPM and diameter as variables that affect the thrust developed by the pods. Although an actual specific pod in industry was not part of this thesis, a pod design representative of those currently available was desirable such that a feasible choice of pod in terms of size, power, and RPM was selected.
Having the extreme ends of the pods within the hull envelope would be advantageous in that the pods would have some means of protection against debris in the water or during docking operations. Using azipod product information from ABB Marine Solutions [1] information revealed that this was impossible if a realistic pod was to be used. The thrust developed in the pod is proportional to diameter, and also to RPM$^2$. For the power requirements for the submarine, the RPM limits for ABB pods are roughly 400-450 RPM. Using this RPM as the design flank speed RPM left only the pod diameter to be changed to ensure the appropriate thrust was developed at speed. A design pod diameter of 4.5 meters was ultimately selected. This size, along with a longitudinal location of the pods at 50 meters aft of the LCB meant that the pods would protrude beyond the extent of the submarine envelope. A suitable offset from the ends of the propeller to the hull section was also required to minimize the potential for adverse interactions between the hull and propeller occurring. In this case a value of 20% of the pod diameter was selected for the offset. The selection of the longitudinal location was chosen to locate the pods as far aft as possible, while still having enough space in the hull to house the required bearings, steering units and structure to support the pods.

Using a procedure similar to the standard configuration propeller, the required RPM and power at given speeds were determined and compared to the prediction from the MIT 2N Submarine Mathcad model. These results are consistent with each other and are provided in table 6.2.

<table>
<thead>
<tr>
<th>Submarine Speed (Knots)</th>
<th>Pod RPM</th>
<th>MATLAB EHP (hp)</th>
<th>MATHCAD EHP (hp)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>83</td>
<td>181</td>
<td>203</td>
<td>11.4%</td>
</tr>
<tr>
<td>10</td>
<td>165</td>
<td>1431</td>
<td>1513</td>
<td>5.5%</td>
</tr>
<tr>
<td>15</td>
<td>248</td>
<td>4851</td>
<td>4908</td>
<td>1.2%</td>
</tr>
<tr>
<td>20</td>
<td>330</td>
<td>11452</td>
<td>11319</td>
<td>1.2%</td>
</tr>
<tr>
<td>25</td>
<td>412</td>
<td>22313</td>
<td>21682</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

Table 6.2: Podded Configuration Powering Requirements

Both configurations have very similar propulsion requirements in terms of effective horsepower. Although from a maneuvering perspective this is fine, it does not tell the whole story in regards to the amount of power that will have to be produced. Shaft
horsepower (SHP) is a function of propulsive coefficient, the thermal and mechanical efficiency of the propulsion plant, and the electrical inefficiencies in the electric motors of the pods. Although not directly calculated as part of this thesis, it is reasonable to assume that a submarine using pods would be able to have a smaller hull form, and therefore smaller power requirements due to the removal of reduction gears and the external location of the motors outside of the pressure hull.

6.2 ACCELERATION

Straight line acceleration simulations were conducted on both configurations. This required a limit on the rate at which propulsion can change. A nominal value of 10% of total power per second was applied to the simulation model to account for the rate at which different bells can be answered. For both configurations, a series of jump bells from a steady operating condition were analyzed. These transients included:

1. Ahead 1/3 to Ahead Standard (5 knots to 15 knots)
2. Ahead 1/3 to Ahead Full (5 knots to 20 knots)
3. Ahead 1/3 to Ahead Flank (5 knots to 25 knots)
4. Ahead 2/3 to Ahead Flank (10 knots to 25 knots)

The change of speed and distance traveled were both tracked versus time from initiation of the bell change. Although propeller cavitation is typically a concern during rapid maneuvering transients, this dynamic was not modeled. Due to the typically deep operating depth of submarines, the ambient seawater pressure is large enough such that cavitation is not a major concern even during extreme changes in bells.

6.2.1 STANDARD CONFIGURATION

The standard submarine configuration shows a similar trend for all bell changes. There is an initial slow acceleration as the RPM increases to the new bell. This is followed by a more rapid acceleration once the new RPM is reached. This acceleration rate then decreases as the new steady state speed is approached. In general roughly 90% of the desired speed is reached within 2-3 minutes of the initiation of the bell change. The majority of the remaining speed is reached in the following 2-3 minutes. Another expected trend is that the acceleration rate of the submarine is very dependent upon the jump bell,
such that the submarine's max acceleration rate is much higher for an Ahead 1/3 to Ahead Flank bell change than it is for an Ahead 1/3 to Ahead Full bell change. The performance of the standard configuration submarine is shown in figures 6.1 and 6.2 below:

Figure 6.1: Standard Configuration Acceleration Run (Speed vs. Time)
6.2.2 PODDED CONFIGURATION

The podded configuration displayed a similar trend in acceleration as the standard configuration, but with a significantly faster response. For the podded configuration, 90% of new speed was reached in typically 30-50 seconds after the initiation of the bell change, and the majority of the remaining speed obtained after approximately 90 seconds. These acceleration runs are shown in figure 6.3.
This acceleration rate is much quicker than expected and initially was a cause for concern with the modeling of the azimuth dynamics, specifically the axial force coefficient as a function of advance coefficient obtained from the data in figure 4.11. Figure 6.4 shows a comparison of the two configurations for an Ahead 1/3 to Ahead Full bell change. In the case of the podded configuration the advance coefficient drops from 0.41 to 0.15 corresponding in an increase in the axial force coefficient from 0.017 to 0.091; an increase by a factor of 5. In comparison, for the standard configuration the thrust coefficient jumps from 0.248 to 0.295; an increase of ~1.2. These differences, along with RPM, affect the delivered thrust for each configuration. A plot of the axial thrust force as a function of time between the two configurations for the same jump bell is shown in figure 6.5.
The standard configuration simulation provides response times that are consistent with the accelerations for a real submarine. The podded configuration, however, is much
quicker. The type of propeller used to determine the pod dynamics was considered as a possible reason for the significant difference in performance. The data used in Stettler’s work for the podded configuration was based on a trolling motor propeller [27]. These propellers are designed for low speed, high thrust situations similar to a bollard condition, whereas the B-series propellers are designed for steady state conditions at specific operating speeds[28]. This would explain the results seen in figure 6.4 – when a rapid jump bell is initiated and the advance coefficient drops significantly, the propellers on the pods are operating in conditions they were designed for. The B-series propeller in the standard configuration is not designed for such conditions; subsequently the acceleration performance is not as good. The fundamental difference between the B-series propeller and the pod propeller make it impossible to make any definitive conclusion regarding the acceleration differences between the two configurations, however it is reasonable to expect that a pod utilizing a propeller geometry similar to the B-series would have a much slower acceleration performance that what is modeled here.

6.3 DECELERATION

The deceleration performance of the two submarine configurations was conducted for several initial speeds to measure the ability of the submarine to respond to a back emergency bell. Four initial conditions from which back emergency was ordered were examined for each submarine configuration:

1. Ahead Flank (25 knots)
2. Ahead Full (20 knots)
3. Ahead Standard (15 knots)
4. Ahead 2/3 (10 knots)

6.3.1 STANDARD CONFIGURATION

To decelerate the standard configuration, a crashback on the propeller was used. This is a condition, outlined in table 4.4, is where the submarine speed is positive but the propeller RPM is negative. Typically submarines are not capable of generating full horsepower (or full RPM) in the reverse direction. For the case of a nuclear steam propulsion plant this is due to the fact that the astern turbines are typically only one or two
stages and not optimized for that direction. To model this condition, the RPM was limited in the reverse direction to 171 RPM – the same RPM corresponding to Ahead Full. Similar to chapter 6.2, the propulsion rate of change was also limited to 10% of total power per second. The performance of the submarine response to a back emergency bell are shown in figures 6.6 and 6.7.

Figure 6.6: Standard Configuration Deceleration Run (Speed vs. Time)

Figure 6.7: Standard Configuration Deceleration Run (Distance vs. Time)
The response of the submarine is such that the deceleration rate is initially small since it takes time for the propeller to first slow to zero, then reverse in speed. The deceleration rate of the submarine increases until the submarine is answering the Back Emergency condition of negative 171 RPM after 12-19 seconds depending on the initial bell. It is at this point that the deceleration rate is at a max. This deceleration rate remains roughly constant until all headway is taken off the submarine, which occurs roughly one to two minutes after the bell order. As expected, figure 6.7 shows that the distance to stop is highly dependent on initial speed. In the worst-case scenario (Ahead Flank), it takes approximately 7 ship’s lengths to stop. It should be noted that the stopping characteristics of the submarine can be improved by cycling the control surfaces to increase drag and/or put an angle of attack on the hull which will further increase drag.

6.3.2 PODDED CONFIGURATION

There are two ways in which the podded configuration can answer a back emergency. The first way, similar to the standard configuration, involves reversing the propeller direction to obtain a crashback condition. The second way, is to change the azimuth angle of the pods 180 degrees and drive the propeller in its ahead direction. The deceleration runs were conducted by using the later method for two reasons. First, four quadrant propeller data is not available for the pods – they were only analyzed in the positive rotation. Secondly, propeller performance is typically better when rotating in the positive direction which means that the stopping performance should be better by rotating the pods 180 degrees and answering ahead RPMs on all pods.

The manner in which the azipods were reversed was consistent for all initial speeds and followed the timeline below:

- Time = 0 to T1: The RPM of the pods is decreased to zero
- Time = T1 to T2: The pods are rotated to 180 degrees
- Time = T2 to T3: The RPM of the pods is increased to Ahead Flank
- Time = T3 to T4: Headway is taken off the submarine until it stops

The times above vary depending upon the initial bell of the submarine as well as the azimuthing rate of the pods. For submarines, control surface rate limits are typically +/- 4
degrees per second for rudders and +/- 7 degrees for sternplanes. These rates are adequate for a movement of +/- 25 or 35 degrees typically seen with control surfaces, but would be unsatisfactory for a pod that is required to rotate +/- 180 degrees particularly during Back Emergency conditions. Commercially available pods have a maximum azimuthing rate of 12 degrees per second. To be conservative, a rate of +/- 10 degrees per second for the pods was utilized.

Although the deceleration performance of the submarine in the podded configuration can be improved by starting the pod rotation to 180 degrees before the propeller stops and increasing the pod rotation to Ahead Flank before the pod reaches 180 degrees, this adds significant complexity to the maneuver and can result in extremely large side forces as the pod passes through azimuth angles ~ 90 degrees. For simplicity of the analysis, the timeline above was followed. Figures 6.8 and 6.9 depict the performance of the submarine while decelerating.

![Podded Configuration Diagram](image)

*Figure 6.8: Podded Configuration Deceleration Runs (Speed vs. Time)*
The deceleration of the submarine with the pods follows an expected pattern.

Initially, as the advance coefficient of the propellers increase with the pods aligned forward negative thrust will be generated when the submarine starts to 'outrun' the pod propellers. The pods are then rotated through 180 degrees to the reverse orientation. During this time submarine is decelerated due to the resistance of the hull and the added resistance of the non-spinning pods as they rotate from 0 to 180 degrees. During this time the deceleration rate is relatively constant. Once at 180 degrees and the pods' RPM start to increase, the deceleration rate rapidly increases as thrust from the pods are generated in the reverse direction. The submarine subsequently stops fairly quickly from this point.

It is clear that the deceleration times and distances are much shorter for the podded configuration than for the standard configuration. These are shown quite clearly in figures 6.10 and 6.11. It should be noted that the same fundamental propeller design issue that caused significant differences between acceleration runs for the two configurations are still at work in this case – particularly when the pod propellers are driving in the positive rotation at the 180 degree orientation. Because of this the deceleration rates of the podded configuration are most likely higher than they would otherwise be. Regardless, the podded configuration should result in a shorter stopping distance and stopping time.
Figure 6.10: Deceleration Distance Comparison

Figure 6.11: Deceleration Speed Comparison
6.4 TURNING CHARACTERISTICS

The turning characteristics of the submarine configurations were characterized based on a circle turning maneuver in which the submarine – answering a bell at a steady speed – initiates a turn by way of a constant rudder angle (or in the case of the podded configuration, a steady azimuth angle of the rudder pods). The standard parameters measured were the advance, transfer and steady turning diameter. These characteristics are shown in figure 6.12.

Figure 6.12: Distance Related Turning Characteristics (From [26])
6.4.1 STANDARD CONFIGURATION

A series of runs was conducted to characterize the turning performance of the submarine. For turns the rudder deflection rate was limited to +/- 4 degrees per second and the deflection limit was +/-35 degrees; typical rates for submarines. All runs used a constant RPM input, which result in a speed loss as the turn progresses. This speed loss is expected and due to the combined effects of the added drag on the hull due to cross flow as well as the added drag due to the rudder deflection. The speed loss is highly dependent upon the rudder angle and in the case of a hard-over rudder a speed loss approaching 50% is seen. This speed loss increases the time it takes to complete a turn. In practice this can be mitigated by increasing shaft RPMs in an attempt to maintain a constant speed during a turn.

The submerged turning characteristics of the submarine are displayed in figure 6.13 for a 5 knot initial speed. The distance related parameters for submerged submarines are mostly dependent upon the rudder angle, with the characteristics of different speeds being quite similar. Because of this the 10 knot initial speed distance related parameters look very similar to the 5 knot initial speed. Larger rudder angles obviously result in shorter turning distances. The figure also shows that at large rudder angles there is a quickly diminishing effect of turning distances.

![5 Knot Turning Parameters](image)

**Figure 6.13: 5 Knot Distance Related Turning Parameters**
Although initial speed has little effect on distance related performance, it does have a dramatic effect on the time to turn. Figure 6.14 shows the time related turning characteristics for a 10 degree rudder angle for various speeds.

![Figure 6.14: 10 Degree Rudder Time Related Turning Characteristics](image)

It should be noted that during turning maneuvers, the submarine does experience a roll about the x-axis. The amount of this roll is very dependent upon the amount of rudder angle as well as the speed of the submarine. For very small speeds, the roll is quite minimal. For higher speeds involving larger rudder angles this roll can become extreme resulting in what is known as a snap roll. A snap roll is caused due to the forward location of the sail. During a high-speed turn, the asymmetric lift generated by this sail as the submarine executes a turn at an angle of attack causes the submarine to roll into the turn while pitching the submarine down. This roll is also compounded by the fact that the rudder angles begin to act like stern planes causing an even greater downward pitch. The snap roll can be very difficult to control – often requiring a skilled ship’s control party that can apply a rise on the stern planes and/or sail planes at appropriate times to allow for large rudders to be used. Other ways to mitigate the effects are through the use of different control surface configurations such as those in figure 4.3, by projecting more vertical control surface area below the centerline of the ship than above, or by limiting the amount of rudder deflection allowed at high speeds.
Figure 6.15 shows the first 200 seconds of the path of a submarine experiencing a snap roll. The submarine with an initial speed of 20 knots executes a 25 degree rudder change with no additional operator response (such as stern plane or bow plane operation). After 100 seconds from the initiation of the turn, the roll angle is 40 degrees into the turn with the pitch at 42 degrees, causing a rapid increase in depth.
6.4.2 PODDED CONFIGURATION

This configuration requires an angle of attack on the pods to generate the forces and moments required to turn the submarine. To characterize the turning response, both upper and lower rudder pods were simultaneously operated in a fashion similar to the rudder control surfaces in the standard configuration. As described in chapter 6.3.2, the pod deflection rate is +/- 10 degrees per second and the pods are capable of full 360 degree rotations. As with the standard configuration, the effect of speed has little effect on the distance related turning characteristics for a given pod angle. Figure 6.16 shows the distance related turning performance characteristics for a 5 knot initial speed. Figure 6.17 shows the time related turning characteristics for a 10 degree pod deflection for various speeds.

![Figure 6.16: 5 Knot Distance Related Turning Parameters (for Pods)](image)

![Figure 6.17: 10 Degree Rudder Time Related Turning Characteristics (for pods)](image)
The trends are the same for the pod configuration as for the standard configuration. The major difference is the pods are capable of being effectively rotated at angles much greater than the rudders, which experience stall at large deflection angles. This is shown explicitly in figure 6.18, which shows the comparison between steady turning radiiuses for the two configurations. For a given deflection, the rudder is more effective than the pods. This is highly dependent on the size of the control surfaces. Because the pods can deflect at much greater angles than the rudder, the podded configuration is capable of much tighter turns. The standard configuration is capable of turns within approximately 3 ship's lengths using a 35 degree rudder while the podded configuration is capable of turning in just over one ship’s length with a 90 degree pod angle.

![Figure 6.18: Comparison of 5 Knot Steady Turning Radius](image)

It should be noted that at such high pod angles there might be many dynamics that are not modeled. These may cause many adverse vibration and acoustic effects that would operationally limit pod deflections to angles much less than 90 degrees. The podded configuration is also subject to the same snap roll phenomena described in chapter 6.4.1. This will also contribute to pod angle limitations being operationally imposed.
6.5 DEPTH CHANGING MANEUVERS

At depths greater than periscope depth, submerged submarines are typically operated at a neutral trim and buoyancy. This means that the submarines must rely on a combination of a change in ship’s angle, \( \theta \), and vertical forces created by control surfaces to change depths. The change in ship’s angle allows the submarine to drive itself up or down in the water column, while the vertical forces from control surfaces such as bow planes or sail planes assist the depth change. A major factor in determining the speed at which a depth change can occur is the time it takes to reach different ship’s angles. For normal depth changes, submarine angles are typically limited to +/- 20 degrees so as to not limit crew mobility or cause equipment and stores to shift or move. The different submarine configurations were analyzed for the time and distance changes to execute different submarine angles.

To allow for a direct comparison for the ability to change depth, a nominal scenario was also analyzed. This scenario involved a series of events outlined below:

1. The submarine is patrolling at 5 knots at a depth of 200 meters.
2. The submarine starts a depth change to achieve an ordered depth of 50 meters.
3. The submarine changes depth and levels out at 50 meters with a level trim and a speed of 10 knots.

This scenario is used to represent the combined effects that the different configurations have on the ability to quickly change depths from a deep patrol depth to a shallower depth from which the submarine will conduct other operations. For this scenario, both submarines utilize identical bow planes that are otherwise retracted during other analyses.

6.5.1 STANDARD CONFIGURATION

The standard configuration utilizes stern planes to control the pitch of the submarine during depth changes. For this analysis, the deflection rate of the stern planes is +/- 7 degrees per second and the deflection limits are +/- 25 degrees; both are typical values for submarines. Figures 6.19 and 6.20 provide quantitative results displaying the time to execute pitch angles for different speed and stern plane angle combinations. As
expected, higher speeds result in quicker pitch angle changes, as does larger stern plane deflections. Because the pitch response at higher speeds is very quick, it is common for stern planes to be limited to prevent rapid uncontrolled depth excursions that could either cause the submarine to broach the surface (possibly causing a collision with surface ships) or exceed the collapse depth of the submarine.

Figure 6.19: Time to Execute Pitch Angles for Slow Speeds

Figure 6.20: Time to Execute Pitch Angles for High Speeds
When utilizing only stern planes for depth changing, submarines initially experience a small depth change in the opposite direction of the intended path due to the stern plane deflection creating a force in the opposite direction. The moment this force creates takes time to affect the pitch of the submarine. This effect is shown in figure 6.21. At very low speeds this phenomenon causes what is called a stern planes reversal, where the moment generated is not large enough overcome the restoring hydrostatic righting forces to achieve a pitch that will change the depth of the submarine in the desired direction. In this case the force from the stern planes will cause the submarine to continue to change depth in the wrong direction.

![Figure 6.21: Depth Change vs. Pitch Angle for 10 Knot Speed](image)

The effect of speed on the time it takes to change depth is quite pronounced. For a given stern plane angle, a doubling of speed causes a decrease in the time to change depth by approximately half. Doubling the stern plane angle for a given speed results in a modest decrease in the time to change depth. These effects are shown in figures 6.22 and 6.23.
Figure 6.22: Depth Change for 12.5 Degree Stern Plane Angle

Figure 6.23: Depth Change for 12.5 Knot Initial Speed (Stern Planes)
6.5.2 PODDED CONFIGURATION

The pods for controlling pitch had a deflection rate limit of \( \pm 10 \) degrees per second and the ability to rotate 360 degrees. The time to execute a pitch angle for different pod angles is shown for the 10 knot initial speed in figure 6.24. Increasing pod angles significantly decreases the time to execute pitch angles for smaller pod deflections (less than \( \sim 40 \) degrees). This effect gets less pronounced until there is little difference between an 80 degree pod angle and a 90 degree pod angle.

![Figure 6.24: Time to Execute Pitch Angle for 10 Knot Initial Speed](image)

As with the standard configuration, the effect of speed on the ability to change pitch and depth as a function of time is more prominent than the effect of pod angle. This is shown in figures 6.25 and 6.26 which show the depth change for a given pod angle vs. varying speed, and a given speed vs. varying pod angle respectively. The relative difference between the standard configuration and the pod configuration are shown in figure 6.27. It is clear that the pod configuration is capable of a faster change in depth than the standard configuration, but only at pod angles that are greater than the angle limits for the stern planes. As stated in 6.4.2, very large pod angles may introduce adverse vibration and acoustic effects to the submarine – as well as excite dynamics that were not modeled as part of this thesis.
Figure 6.25: Depth Change for 12.5 Degree Pod Angle

Figure 6.26: Depth Change for a 12.5 Knot Initial Speed (Pods)
6.5.3 DEPTH CHANGE SCENARIO

Both configurations were analyzed for the nominal depth changing scenario. During these simulations, the submarine pitch angle was limited to approximately +/- 20 degrees. Identical bow planes were also utilized to assist in the depth change maneuver. Additionally, the ordered bell to accelerate was limited to a full bell instead of a flank bell. These constraints were used so that the runs would realistically represent the manner in which operators would change depth. The bow planes used were parametrically sized based on submarine displacement and have the same operational limitations as the stern planes.

To achieve the depth change, the scenario was run iteratively changing input parameters as necessary to minimize the time to change depth as well as the overshoot at the new depth. The resulting timelines for the configurations are shown in table 6.3 below.
<table>
<thead>
<tr>
<th>Time</th>
<th>Action</th>
<th>Time</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>* Full Rise on Stern Planes</td>
<td>0</td>
<td>* 60 degree Rise on Stern Pods</td>
</tr>
<tr>
<td></td>
<td>* Full Rise on Bow Planes</td>
<td></td>
<td>* Full Rise on Bow Planes</td>
</tr>
<tr>
<td></td>
<td>* Ahead Full</td>
<td></td>
<td>* Ahead Standard</td>
</tr>
<tr>
<td>35</td>
<td>* Zero Stern Planes</td>
<td>9</td>
<td>* Zero Rise on Stern Pods</td>
</tr>
<tr>
<td>50</td>
<td>* Ahead 2/3</td>
<td>20</td>
<td>* Zero Bow Planes</td>
</tr>
<tr>
<td>60</td>
<td>* Zero Bow Planes</td>
<td>50</td>
<td>* 30 Degree Dive Stern Pods</td>
</tr>
<tr>
<td>70</td>
<td>* Full Dive Stern Planes</td>
<td>70</td>
<td>* Ahead 2/3</td>
</tr>
</tbody>
</table>

Table 6.3: Timeline for Depth Change Scenario

Figure 6.28 shows the result of the analyses. The pod configuration reaches the ordered depth in approximately 100 seconds, while the standard configuration reaches the ordered depth in approximately 120 seconds. The driving difference between the configurations was the speed at which the submarine could get to the +20 degree pitch angle. The pods are capable of achieving that pitch angle faster – but again at the possible expense of acoustic and vibration problems.

6.6 CASUALTY RESPONSE

It is impossible to anticipate every possible casualty scenario that may occur. Murphy's law dictates that what can go wrong, will go wrong. There is a significant amount of operational history in naval operations to confirm this. To ensure that submarines can be operated safely in light of the fact that unknown events can potentially occur requires a
rigorous assessment in regards to casualty analysis. This is where the development of the SOE outlined in chapter 1.3 becomes important. Giddings and Louis studied submarine control surface jams and flooding casualties in detail, providing general methods of recovery procedures for such casualties [11]. To analyze the submarine’s ability to respond to, and recover from, the limiting casualties of the SOE certain assumptions must be made. In the case of the control surface casualties analyzed here they are:

1. The submarine is fully operational, at a steady speed, neutrally buoyant, and at level trim at the initiation of the casualty.
2. The submarine is manned by an alert crew that is trained to respond to the casualties that may occur by taking the appropriate immediate and supplementary actions needed to recover the submarine.
3. The extreme motions of the submarine will not affect the propulsion plant or the ability of the ship’s control party to respond to the casualty.
4. There is only one casualty: the control surface casualty. Compound casualties will not occur that adversely affect the systems required to recover the submarine.
5. The affected control surface will jam, and remain in the jammed position throughout the casualty.

In the case of the podded configuration, the control surface jam scenario is replaced by the azimuthing of a pod to a deflection angle without the ability to train it back. It is assumed that the ship’s control party can remove power to the pod motor itself.

An assumed time sequence of the events following a casualty leading to the recovery actions of the ship’s control party is presented in table 6.4 below. The actual immediate actions for casualties are extremely dependent upon the type of casualty. In the case of control surface casualties, they generally involve taking all way off the submarine to remove the affects the control surfaces have. This includes a combination of a propulsion crashback and rudder manipulation to increase drag. In the case of a jam dive casualty where the stern planes are jammed in the full dive position, an EMBT blow can also be initiated if needed to add positive buoyancy forward in the submarine. For this project,
only jam dive casualties were examined. These generally have more dire circumstances that jam rise casualties and due to the general symmetry of the submarine have similar responses and limitations. Flooding casualties that make up another portion of casualties that affect the SOE were not analyzed.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Sequence</th>
</tr>
</thead>
</table>
| T=0           | • Casualty has finished occurring  
• Control Surfaces are at the Jammed at the hard over position |
| T=3.5         | • Ship’s Control Party has recognized the Casualty |
| T=5.5         | • Assumed 2 second reaction time to casualty recognition  
• Code Word to take emergency actions is given, such as ‘Jam Dive’ |
| T=7.5         | • Assumed 2 second delay to begin immediate recovery actions  
• Appropriate unaffected control surfaces begin to respond  
• Propulsion train begins to respond |
| T=9.5         | • Assumed 2 second delay to decide on whether to initiate EMBT Blow  
• EMBT Blow initiated (if required) |

Table 6.4: Assumed Time Sequence of Casualty Identification and Response

6.6.1 STANDARD CONFIGURATION

For the standard configuration submarine, the casualty of jammed stern planes was analyzed. For these casualties, the ship is on a level trim and flight. The stern planes then rotate to the dive position (25 degree dive) where they remain throughout the casualty response. Once the planes are at their jammed position, the timeline in table 6.4 begins. The immediate actions used in the model to analyze the submarine response were to conduct back emergency and a hard rudder at a time 7.5 seconds from the time the stern planes reach full dive. The back emergency bell takes forward speed off the vessel until motion is arrested. The hard rudder adds drag and incurs a roll to the submarine, which serves to put the stern planes at an angle to reduce their affect on generating a downward pitch. At the point that all speed has been taken off the submarine, the downward depth rate of the submarine has ceased and supplementary actions such as adjusting internal ballast to control trim and correcting the cause of the jam dive can be conducted. During
Jammed control surface casualties, ship's motions can be quite extreme, with roll and pitch angles exceeding 45 degrees. In accordance with table 6.4, if an EMBT blow is initiated, it is done so at a time 9.5 seconds from the time the stern planes reach full dive.

At higher speeds, rudder angle deflections are normally limited to prevent snap rolls – explained in chapter 6.4. Additionally, the stern plane angle deflections are also normally limited at high speeds. This is done to limit the severity of the jam dive casualties at higher speeds. Figure 6.29 shows the SOE developed for the standard configuration submarine. There are a total of three operationally limiting lines. The first, used at lower speeds has no restrictions on rudder or stern plane deflections. The other two – used for speeds greater that 20 knots - utilize a 7.5 degree stern plane, and 10 degree rudder deflection limitations. The final differentiating feature between the last two is that one utilizes an EMBT blow while the other does not. For this analysis, the submarine casualties were initiated at an operating depth of 200 meters.

![Figure 6.29: SOE for Standard Configuration](image)

The lines in figure 6.29 represent the depths above test depth that the submarine should be limited to at the given speed and control surface limitations. Doing so will ensure that should the jam dive casualty occur the submarine will not exceed the test depth of the submarine if the immediate actions specified in the scenario are taken properly and in time. It is clear that placing restrictions of stern plane deflections is necessary at higher speeds – otherwise the SOE would become overly limiting. The degree to which the control
surfaces are limited has a large affect on the SOE line. A more restrictive limitation on stern plane angle would shift the SOE line down, however this would come at an expense in the ability of the submarine to maneuver quickly. A balance must therefore be made between an SOE that allows the crew of the submarine the flexibility to operate in as much of the water column as possible, while at the same time ensures the submarine has the capability to maneuver adequately. An additional takeaway from this analysis is that the EMBT blow only provides an approximate 25 extra meters to the SOE – the driving factor in recovering from jammed control surfaces is how quickly the submarine can take speed off, not add buoyancy. An EMBT blow should not be greatly relied upon by the crew to recover the submarine. Not only is it not very effective if recovering the submarine, but it also adds significant complexity in the supplementary actions phase of the casualty to prevent the submarine from rising to the surface in an uncontrolled manner.

6.6.2 PODDED CONFIGURATION

The podded configuration was analyzed in a similar fashion to the standard configuration. The driving casualties analyzed were for both stern pods to azimuth to a dive rotation then remain stuck there with the pod motor driving the propellers at the initial RPM. The immediate actions used in these scenarios were to immediately secure power to the affected pods to remove propulsion from them at a time 7.5 seconds from the pods becoming ‘jammed’. At this same time, the rudder pods’ RPM are reduced at the max rate and they begin to rotate to the 180 degree position. As soon as the rudder pods reach 90 degrees, the RPMs are increased on both to maximum to reduce speed. If needed an EMBT blow is initiated at a time 9.5 seconds from the pods becoming ‘jammed’. For the podded configuration, the ability of the pods to change pitch more quickly than the stern planes would lead to the assumption to a jammed pod being more severe than the jammed stern planes. The ability of the podded configuration to quickly decelerate, however, was able to mitigate the severity of the pod casualty.

Two pod jam scenarios were investigated. The first involving a 90 degree pod jam was chosen since it is at that deflection that a pod has the greatest effect on the change of trim the initial stages of the casualty. Once power is secured to the pod, however, there is very little vertical force or moment generated by the pod at a 90 degree angle of rotation –
most of the pods interaction with the water results in axial drag which helps to slow down the submarine. At angles of approximately 60 degrees, there is a maximum vertical force and moment generated by the ‘pod-only’ forces due to the interaction of the pod with the water. This is seen in figure 4.12. For these reason a scenario of a 60 degree pod jam was also analyzed. The results are shown in figure 6.30 below.

![Figure 6.30: SOE for Podded Configuration](image)

It is clear from figure 6.30 that the 90 degree jam is more severe than the 60 degree jam. Even so it is not as severe as a jam dive for the standard configuration, primarily because of the ability of the podded configuration to quickly decelerate. For example, for the 25 knot scenarios, it the podded configuration casualty reaches its maximum depth excursion at 36 seconds from the initiation of the casualty. In the case of the standard configuration it takes 105 seconds until the maximum depth excursion occurs and forward speed is removed from the submarine. This difference in time has a great impact on the effectiveness of an EMBT blow as well. For the podded configuration, initiation of an EMBT blow only gains an additional 0.5 to 1 meter for the SOE. This is because there is not enough time for the EMBT blow to add enough positive buoyancy to make an appreciable difference.

Drawing conclusions between the configurations must be done with some trepidation. This is because the acceleration and deceleration capabilities of the podded configuration, as addressed in chapters 6.2 and 6.3 are not very conservative. This
overestimation of the ability of the submarine to decelerate has a direct impact on the severity of these casualties and the development of the SOE.
CHAPTER 7 CONCLUSIONS

7.1 SUMMARY

This thesis quantifies the operational implications of utilizing podded propulsors as a primary means powering and control from a maneuvering standpoint. This was accomplished by generating a six degree of freedom maneuvering simulation model that allows for testing any number of scenarios. The basic model is also flexible enough to accommodate additional propulsion concepts and unique configurations – thereby providing a useful capability for the future.

Chapter 2 provided a description of the concept submarines analyzed during this thesis. The actual Darpa SUBOFF geometry is a somewhat unique hullform that might not be ideal for an actual submarine. It has an extremely full stern that might introduce flow separation problems for a propeller, as well as undersized control surfaces that contribute to directional stability issues. This last problem was corrected for this thesis by utilizing larger, parametrically sized control surfaces for the standard non-podded configuration. A key benefit of using the SUBOFF hullform was in that it allowed for the use of empirically determined hydrodynamic coefficients. This is very important for calculation of the damping forces, which not only dominate the dynamics of the submarine maneuvering, but are also difficult to analytically predict. The generalized geometric hullform in chapter 2.2 provides future flexibility of the model to analyze various submarine geometries in addition to different propulsion configurations.

Chapter 3 outlined the governing equations of motion for the six degree of freedom submarine model. The presentation of this material was not unique, but tried to greatly utilize previous work as well as standard ocean vehicle maneuvering texts.

Chapter 4 provided a thorough presentation of the theory and calculation behind the external forces and moments that a maneuvering submarine experiences. This was done methodically with the intention of skipping as few derivational steps as possible in hopes that future users of this thesis will have no doubts or questions regarding the manner in which these forces were calculated. The calculation of forces and moments from
the EMBT blow system, the propeller configuration, and pod configuration in particular represent unique ways in which to model the dynamic effects those systems have on a submarine.

Chapter 5 provided a brief overview of the architecture and operation of the SIMULINK/MATLAB model. It assumes that the reader is familiar with computational programming at a minimum. The actual code of the model is provided in the appendix for future use.

Chapter 6 addressed the question that this thesis set out to answer: What are the operational impacts that placing podded propulsors has on submarine maneuverability? This question was partially answered. Because the Stettler [27] data utilized a propeller that was not ideally suited for the submarine propulsion application, some of the analysis presented in chapter 6 must be looked upon with some uncertainty. The acceleration and deceleration performance of the podded configuration, in particular, is most likely overestimated. This may have secondary effects on the validity of the SOE development. The characterization of the turning and depth changing performance should be representative of actual results, especially for smaller pod deflection angles and speeds.

As expected, the pods provide excellent turning characteristics compared to the standard configuration. In actual operation, however, the ability to quickly turn (or turn in a small diameter) is not typically limiting on a submarine's effectiveness at completing its missions. The ability of the podded configuration to provide superior deceleration performance was also anticipated based on the ineffectiveness of propellers in crashback operating conditions. The ability to turn pods 180 degrees and drive them in that direction provided the superior results that were seen in this thesis.

There are some surprising results that were realized during this thesis, however. The acceleration performance of the pods, although questionable, was unexpected. The relatively small difference in depth changing maneuvers (approximately 20 seconds) between configurations was somewhat surprising as well, but was driven primarily by the fact that submarine depth changes are dominated by the time it takes to get to a desired ship's angle. The difference in time to get to the normal ship's angle limitation or +/- 20
degrees is simply not that great between configurations. Another surprising result was the SOE. Although it was expected that the pods in a jam condition could lead to more severe depth excursions, the ability of the pods to decelerate the submarine proved to be so dominant that the SOE for the podded configuration was not overly restrictive. A final surprising result was the relatively small impact that an EMBT blow had on a control surface casualty recovery. Although a tremendous amount of buoyancy is added during an EMBT blow, the driving factor behind recovery still remains how quickly a submarine can reduce speed.

7.2 FUTURE WORK

The goal of this thesis was to investigate the operational impacts that placing azipods on a submarine has on maneuvering. The impacts of such a configuration, however, goes far beyond the aspects of maneuvering. Most of these impacts were not looked at directly during this thesis, but must be qualitatively and quantitatively answered to determine if utilizing pods is a practical solution to a non-centerline shaft configuration submarine. These impacts, each of which must be analyzed in detail, are outlined below.

7.2.1 SIGNATURES

A driving factor behind a submarine’s effectiveness is the ability to operate stealthily. The acoustic performance of external pods is relatively unknown, but could have a dramatic impact on whether pods are feasible. The fact is that if the acoustic performance of pods is not equal to or greater than the performance of a standard shaft configuration, the use of pods on a submarine is unlikely. The acoustics are vital not only for steady state operations, but also during transient conditions. The vibrations that might be present at high pod deflections could be quite severe, however it is the ability to utilize high pod deflections that provides them their superior maneuvering characteristics.

The electrical and magnetic interference (EMI) that may be emitted from external pods could be detrimental to a submarine if the pods put out exploitable EMI signatures that can be detected by magnetic anomaly detectors (MAD) on anti-submarine aircraft or magnetically triggered mines. This may require significant power conditioning equipment
to remove harmonic frequencies sent to the motors, or special shielding or grounding on
the motors to prevent this.

7.2.2 STRUCTURES

A conventional submarine transfers thrust developed by the propeller to the
submarine hull through a thrust bearing that is mounted to the pressure hull. The
structure surrounding the thrust bearing is very robust to allow for extremely high loads to
be transmitted without deforming the submarine hull. The pressure hull is also very
robust in order to withstand high seawater pressures. The locations of the pods – aft of the
submarine aft main ballast tanks – may present structural problems for the submarine.
The ballast tank structure is not designed to the same strength requirements as the
pressure hull, but with the pods located where they are, this structure must now transmit
forces and moments far beyond what it is designed for. The extreme forces generated by
the pods at 90 degree deflections far surpass the forces generated by control surfaces at
their max deflections.

The aft structure of the submarine will have to be designed appropriately to handle
these increased loads. This will likely add weight to the aft structure in addition to the
added weight from the pods themselves. This will change the overall balance of a
submarine, possibly leading to changes in the submarine such as lead distribution, the
relative location of the pressure hull relative to the ballast tanks, the sizing of main ballast
tanks, and the overall geometry of the stern.

7.2.3 POD MATURITY

Pods used in commercial maritime applications today do not experience the deep
external seawater pressures that they would on a submarine. This may present
engineering problems for sealing bearings, and motor housings. These pods are also not
optimized for reducing signatures. Without a commercial demand to do so, the research
and development (R&D) costs associated with producing pods appropriate for submarine
applications will have to be born by the end users. The magnitude of the R&D costs
associated with producing submarine ready pods will need to be quantified.
7.2.4 MANEUVERING

This thesis identified issues related to the available data for azimuthing pods. Expanding upon Stettler's work on pods to include propellers series more applicable to submarine or ship applications would be worthwhile. The fidelity of the results presented in this thesis would also be greatly improved by performing remote controlled model (RCM) testing on scaled submarine models.
APPENDIX A: MODEL CODE

A.1 MATLAB Scripts

Scripts for defining hydrodynamic coefficients and constants and the driver file for running the SIMULINK model.

A.1.1 Model_constants.m

%%% Calculation of Submarine Hydrodynamic Coefficients
%%% Utilizing Generic Submarine Hullform ref. Captain Harry Jackson's Notes
%%% on Submarine Design

clear all;
close all; clc;

%% Constants
rho = 1025;  % [kg/m^3] Density of seawater
g  = 9.81;   % [m/s^2] Gravitational constant

%%% Generic Submarine Particulars
% [m] Vessel Length 84.7
L  = 110;
% [m] Vessel Diameter 7.8
D  = 10;
% (non-dim) Entrance Run
Eta_f  = 2.25;
% (non-dim) Exit Run
Eta_a  = 2.75;
% [m] Forebody Length
L_f  = 2*D;
% [m] Afterbody Length
L_a  = 3*D;
% [m] Parallel Midbody Length
L_pmb = L-L_f-L_a;
% [m/s] Vessel Nominal Speed
U  = 10.28;

%%% Sail Particulars
% [m] Sail midchord position (from FP)18.39
X_s  = 28.39;
% [m] Vessel sail mean height above centerline 4.7
R_s  = 5.7*D/2;
% [m] Vessel sail chord 9.7
C_s  = 5.7;
% [m^2] Vessel sail profile area
A_s  = (R_s-D/2)*C_s;
% [m^2] Sail taper ration
T_s  = 1;

%%% Control Surface Particulars
% [m] Vessel control surface midchord position
X_c  = 100;
% [m] Vessel control surface mean height above
(R from FP) 77
R_c  = 3;
% [m] Vessel control surface chord at Root
c baseline at midchord
C_c  = 3;
% [non-dim] Control surface taper ratio
T_c  = .7;
% [m^2] Vessel control surface profile area
A_c  = .5*R_c*(C_c-T_c*C_c)+T_c*R_c*C_c;

%%% Generic Submarine Shape Functions
% Ref. Harry Jackson 'Fundamentals of Submarine Concept Design'

% Submarine Body Definition
% x = 0:.01:L;
yl = zeros(1,length(x));
% Bow Equation for 0 <= x <= L_f
% \( y(\text{bow}) = \frac{D}{2} \left( 1 - \left( \frac{\text{L}_f - x(\text{bow})}{\text{L}_f} \right)^{\text{Eta}_f} \right) \left( \frac{1}{\text{Eta}_f} \right) \); %
% Parallel Mid Body Equation for \( \text{L}_f <= x <= \text{L}_\text{pmb} + \text{L}_f \)
% \( y(\text{pmb}) = \frac{D}{2}; \)
% Afterbody Equation for \( \text{L}_\text{pmb} + \text{L}_f <= x <= \text{L} \)
% \( y(\text{aft}) = \frac{D}{2} \left( 1 - \left( \frac{x(\text{aft}) - (\text{L}_f + \text{L}_\text{pmb})}{\text{L}_a} \right)^{\text{Eta}_a} \right); \)
% \( y_2 = -y_1; \)

%% Sail Profile Definition
% \( x_2 = X_s - C_s/2:.01:X_s + C_s/2; \)
% \( \text{sail} = (X_s - C_s/2 < x_2) \& (x_2 < X_s + C_s/2); \)
% \( y_3(\text{sail}) = R_s; \)
% \( y_3(X_s - C_s/2 == x_2) = y(\text{X}_s - C_s/2 == x); \)
% \( y_3(X_s + C_s/2 == x_2) = y(\text{X}_s + C_s/2 == x); \)

%% Control Surface Definition
% \( x_3 = X_c - C_c/2:.01:X_c + C_c/2; \)
% \( \text{fin} = (X_c - C_c/2 < x_3) \& (x_3 < X_c + C_c/2); \)
% \( y_4(\text{fin}) = R_c + y(\text{X}_c == x); \)
% \( y_4(X_c - C_c/2 == x_3) = y(\text{X}_c - C_c/2 == x); \)
% \( y_4(X_c + C_c/2 == x_3) = y(\text{X}_c + C_c/2 == x); \)
% \( y_5 = -y_4; \)

%% Vehicle Profile Shape
% plot(x,y_1,x_2,y_2,x_3,y_4,x_3,y_5)
% xlabel('Axial Location (m)')
% ylabel('Radial Location (m)')
% title('Profile of Submarine')
% axis equal

%% Submerged Characteristics for Generic Hull Shape
% \( \text{vol} = \pi \text{trapz}(x,y_1.^{\prime}2); \) % [m^3] Vessel Volumetric Displacement
% \( \text{Lcb} = \pi \text{trapz}(x.*y_1.^{\prime}2)/(\pi \text{trapz}(x,y_1.^{\prime}2)); \) % [m] LCB from FP
% \( \text{Aw} = \text{trapz}(x,2*\pi.*y_1); \)
% \( \text{M} = \text{vol}.*\rho; \)
% \( \text{B} = \text{vol}.*\rho.*g; \) % [N] Vessel Bouyancy
% \( \text{W} = \text{B}; \) % [N] Vessel Weight - Neutrally Bouyant Initially

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% DARPA SUBOFF Physical Data (DTRC Model 5470)
% Based on DARPA SUBOFF Model (DTRC Model 5470) captive-model experiments
% and derived hydrodynamic coefficients
%
% \( \text{lm} = 14.2917; \) % [ft] Model length
% \( \text{la} = 3.645833; \) % [ft] Model Afterbody Length
% \( \text{dm} = 1.6667; \) % [ft] Max. model diameter
% \( \text{m} = 1556.2363; \) % [lbm] Model mass in water
% \( \text{um} = 6.5; \) % [kts] Model nominal speed
% \( \text{Awm} = 63.717; \) % [ft^2] Model wetted surface area
% \( \text{Af} = 2.1817; \) % [ft^2] Model frontal area
% \( \text{Volm} = 24.692; \) % [ft^3] Model displaced volume
% \( \text{Asm} = .855; \) % [ft^2] Model sail planform area
% \( T_s = 1; \) % Sail Taper Ratio (Whicker and Felner)
% \( X_{\text{sm}} = 3.637; \) % [ft] Model sail mid-chord location (from FP)
% \( R_{\text{sm}} = 1.19785; \) % [ft] Model sail mean height above centerline
% \( S_{\text{sm}} = .729; \) % [ft] Model sail span with cap
% \( \text{AR}_{\text{sm}} = .603; \) % Sail aspect ratio
% \( \text{Acm} = .267; \) % [ft^2] Model control surface planform area
% \( T_c = .5/.704; \) % Control Surface Taper Ratio (each)
% X_cm = 13.498; % [ft] Model control surface mid-chord location (from FP) 
% R_cm = .3879; % [ft] Model control surface mean height above centerline 
% S_cm = .438; % [ft] Model control surface span 
% Lcbm = 6.6042; % [ft] Model longitudinal center of buoyancy (from FP) 

%% Full-scale Submarine Physical Data (based on Full Size Submarine, D=40 feet) 
%% Converted to Metric 

Lambda = 24; % [non-dim] 
L = 104.5464; % [m = 343 ft] Vessel length 
L_a = 26.67; % [m = 87.5 ft] Vessel afterbody length 
D = 12.192; % [m = 40 ft] Max. vessel diameter 
%M = 9758318; % [kg = 9604.2 LT] Vessel mass in water (displacement) 
%U = 16.38; % [m/s = 31.84 kts] Vessel nominal speed 
%A = 3409.6; % [m^2 = 3701 ft^2] Vessel wetted surface area 
%Af = 116.74; % [m^2 = 1256.6 ft^2] Vessel frontal area 
%vol = 9520.3; % [m^3 = 336147 ft^3] Vessel displaced volume (SW) 
A_s = 45.78; % [m^2 = 36701 ft^2] Vessel sail planform area 
X_s = 26.61; % [m = 87.5 ft] Vessel sail mean height above centerline 
%S_s = 5.334; % [m = 17.46 ft] Vessel sail span with cap 
C_s = 8.832; % [m = 29 ft] Vessel sail chord 
A_c = 14.296; % [m^2 = 153.792 ft^2] Vessel control surface planform area (each) 
X_c = 98.77; % [m = 323.95 ft] Vessel control surface mid-chord location (from FP) 
R_c = 2.839; % [m = 9.311 ft] Vessel control surface mean height above centerline 
%S_c = 3.205; % [m = 158.51 ft] Vessel control surface span 
C_c = 5.15; % [m = 16.89 ft] Vessel control surface chord at Root 
Lcb = 48.32; % [m = 158.51 ft] Vessel longitudinal center of buoyancy (from FP) 
U = 10.28; % [m/s] Vessel Nominal Speed 

%% DARPA SUBOFF Shape Functions 
%% From DARPA SUBOFF Model # 5470 
%% Ref. Geometric Characteristics of DARPA SUBOFF MODELS - DTRC/SHD-1298-01 March 1989 

% Submarine Body Definition 

xm = 0:0.01:1m; 

% Bow Equation for 0 <= x <= 3.333333 
1 = ((1.126395101.*x.*(0.3.*x-1).^4)+0.442874707.*x.^2.*(0.3.*x-1).^3+1-(0.3.*x-1).^4.*(1.2.*x+1)).^(1/2.1); 
% Parallel Mid Body Equation for 3.333333 <= x <= 10.645833 
a2 = '1'; 
% Afterbody Equation for 10.645833 <= x <= 13.976167 
% Afterbody Cap for 13.976167 <= x <= 14.2917 
a4 = '.1175*(1-(3.2.*x-44.733333).^2).^.5'; 

ylm = piecewise_ev (0:0.01:1m,[3.333333 10.645833 13.979167],[a1,a2,a3,a4]); 
y2m = -ylm; 

x = xm*Lambda/3.28; % Full Scale in meters 
y1 = ylm*dm/2*Lambda/3.28; % Full Scale in meters 

% Sail Profile Definition 

x2m = 3.02:.001:4.24; 
y3m = piecewise_ev (3.02:.001:4.24,[3.03 3.031 4.239 4.24],{1,1.808,1.808,1.808,1}); 

x2 = x2m*Lambda/3.28; % Full Scale in meters 
y3 = y3m*dm/2*Lambda/3.28; % Full Scale in meters 

% Vehicle Profile Shape 

plot(x,y1,x2,y3)
xlabel('Axial Location (m)')
ylabel('Radial Location (m)')
title('Profile of Submarine')
axis equal

% Sail Planview Definition
x3m = 3.033:.01:4.24;

% Sail Forebody Equation for 3.032986 <= x <= 3.358507
b1 = .1093750*(((2.094759*2.0372*(x3m-3.032986)).*(3.072*(x3m-3.032986))-1).^4+2.071781.*3.072*(x3m-3.032986)).^2.*((3.072*(x3m-3.032986))-1).^3+1-(3.072*(x3m-3.032986)))/1.4.*((3.072*(x3m-3.032986)+1)).^5);

% Sail Mid Body Equation for 3.358507 <= x <= 3.559028
b2 = .1093750;

% Sail Afterbody Equation for 3.559028 <= x <= 4.241319
b3 = .1093750*(((2.238361*2.41319-x3m)/.6822917).^2.*((2.41319-x3m)/.6822917)-1).^4+3.106529*((2.41319-x3m)/.6822917).^2.*(((2.41319-x3m)/.6822917)-1).^3+(1-(((2.41319-x3m)/.6822917)-1).^4.*(4*(((2.41319-x3m)/.6822917)+1)))

y4m = piecewise_eval(3.033:.01:4.24,[3.358507,3.559028],{b1,b2,b3});
y5m = -y4m;
x3 = x3m*Lambda/3.28;
y4 = y4m*Lambda/3.28; % Full Scale in meters

% Sail Planview Shape
plot(x3*Lambda,y4*Lambda,x3*Lambda,y5*Lambda)
xlabel('Axial Location (ft)')
ylabel('Half-Breadth Location (ft)')
title('Planview of Sail')
axis equal

%% Submerged Characteristics for SUBOFF Hull Form
vol = pi*trapz(x,yl.^2); % [m^3] Vessel Volumetric Displacement
Lcb = pi*trapz(x,x.*yl.^2)/(pi*trapz(x,yl.^2)) % [m] LCB from FP
Aw = trapz(x,2*pi.*yl); % [m] Frontal Area of
M = vol*rho; % [N] Vessel Bouyancy
B = vol*rho*g; % [N] Vessel Weight - Neutrally Bouyant Initially
W = B;

%% Calculate Inertial Tensor
% Taken from SAWE 'Marine Vehicle Weight Engineering' table 8.16 ROT for
% submarines gyradius.
Ixx = (0.40*D)^2*B;
Iyy = (0.25*L)^2*B;
Izz = (0.25*L)^2*B;

% Inertial cross-terms are very small compared primary axis
% (Ixy=Ixz=Iyz=0)
xcgm = 0;
xcg = 0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% NOTE: All following hydrodynamic coefficients are non-dimensionalized %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Axial Drag Coefficient
% Using ITTC 1957 model-ship correlation for Cf
% Non Dimensionalized, but calculated for Full Scale Submarine
nu_sw = 1.83*10^-6; % [m^2/s] Kinematic Viscosity of
Seawater
Rn = L*10/nu_sw; % [non-dim] Reynolds Number for 10
m/s nominal speed full scale submarine
Cf = 0.075/(log10(Rn)-2).^2; % [non-dim] Modeled at 10 m/s
(Varies from ~0.0023 at 1m/s to 0.0015 at 16 m/s)
Af = pi*(D/2).^2; % [m^2] Frontal Area of

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Submarine

\[ C_p = \text{vol}/(L^2\Delta s) \]
\[ C_{rCf} = 1 - 1.5(D/L_a)^{1.5} + 7(D/L_a)^3 + 0.002*(C_p - 0.6) \]
modified by Capt. Jackson to account for parallel mid-body

\[ C_{daxial} = C_{rCf} C_f \]
\[ C_{daxial} = 0.009 \]
from Capt. Jackson's Notes

\[ C_{axial control} = 0.006 \]
from Capt. Jackson's Notes

\[ \text{Coefficient from Capt. Jackson's Notes} = 0.00085 \]

\[ X = \frac{-0.5*rho*(C_{daxial}+Ca)}{(0.5*rho*L)^2} \]

%% Crossflow Drag
% Slender-body theory is used to calculate the crossflow drag coefficients,
% though this is not always the most accurate means to do so.
% Assumes Crossflow relatively small compared to forward velocity -> Low Re
% numbers
% Assumes Cruciform Control Surfaces

\[ C_{dc} = 1.1 \]
\[ C_{ds} = 0.1 + 0.7\times T_s \]
\[ C_{dc} = 0.1 + 1.6\times T_c \]

\[ Y_vv = \text{Yvhull} + \text{Yvvsail} + \text{Yvvcrossflow} \]
\[ N_vv = \text{Nvhull} + \text{Nvvsail} + \text{Nvvcrossflow} \]

%% Rolling Drag
% Drag from sail and control fins

%Generic Hull Form Version
\[ K_{pp} = -0.5*rho*(4\times A_c\times C_{dc} \times (y4(X_c) - R_c/2)^3 + A_s\times C_{ds} \times ((R_s - D/2)/2 + D/2)^3) / (0.5*rho*L^5) \]

$SUBOFF$ Hull Form Version
\[ K_{pp} = -0.5*rho*(4\times A_c\times C_{dc} \times (y4(X_c) - R_c/2)^3 + A_s\times C_{ds} \times (R_s)^3) / (0.5*rho*L^5) \]

%% Added Mass - Axial
% Based off L/B ratio.
% From Blevins "Formulas for Natural Frequency and Mode Shape" Table 14-2
% Use Ellipsoid of Revolution to estimate Axial Added Mass
\[ alpha = 0.4466*(L/D)^{-1.235} \]
\[ X_{udot} = -alpha*(4/3)*rho*P1*(L/2)*(D/2)^2 / (0.5*rho*L^3) \]

%% Added Mass - Crossflow
% Can be calculated using strip theory, Use thin rectangular plate for sail
\[ B_{2Sf} = (X_s - C_s/2) \times (X_s) \]
\[ B_{2Cf} = (X_c - C_c/2) \times (X_c) \]
\[ S_{a2Cf} = (X_s - C_s/2) \times (X_s - C_c/2) \]
\[ C_{f2Ca} = (X_c - C_c/2) \times (X_c - C_c/2) \]
\[ C_{a2S} = (X_c - C_c/2) \times (X_c) \]
SAIL = (X_s - C_s/2 < x) & (x < X_s + C_s/2);
% From Blevin's Rectangular Plate
alpha_sail = -.0003*(C_s/(R_s - D/2))^4 + .0088*(C_s/(R_s - D/2))^3 - .0835*(C_s/(R_s - D/2))^2 + .3584*(C_s/(R_s - D/2)) + .3043;

% Trapezoidal Multiplier for SUBOFF Geometry
mult = .01*lambda/3.28;
% Trapezoidal Multiplier for Generic Hull Form Geometry
mult = .01;

Force_B2Cf = -pi*rho*trapz(mult*y(B2Cf).^2);
Force_Ca2S = -pi*rho*trapz(mult*y(Ca2S).^2);
Force_Cf2Ca = -pi*rho*trapz(mult*((y(Cf2Ca)+R_c)^2 - y(Cf2Ca)).^2 + (y(Cf2Ca)+R_c).^4)/(y(Cf2Ca)+R_c).^2);
Force_SAIL = -rho*alphasail*(pi/4)*(R_s - D/2)^2*C_s;
Moment_B2Cf = -pi*rho*trapz(mult*(Lcb-x(B2Cf)).*y(B2Cf).^2);
Moment_Ca2S = -pi*rho*trapz(mult*(Lcb-x(Ca2S)).*y(Ca2S).^2);
Moment_Cf2Ca = -pi*rho*trapz(mult*(Lcb-x(Cf2Ca)).*((y(Cf2Ca)+R_c)^2 - y(Cf2Ca)).^2 + (y(Cf2Ca)+R_c).^4)/(y(Cf2Ca)+R_c).^2);
Moment_SAIL = -rho*alphasail*(pi/4)*(R_s - D/2)^2*C_s*(Lcb - X_s);

Moment2_B2Cf = -pi*rho*trapz(mult*(Lcb-x(B2Cf)).^2.*y(B2Cf).^2);
Moment2_Ca2S = -pi*rho*trapz(mult*(Lcb-x(Ca2S)).^2.*y(Ca2S).^2);
Moment2_Cf2Ca = -pi*rho*trapz(mult*(Lcb-x(Cf2Ca)).^2.*((y(Cf2Ca)+R_c)^2 - y(Cf2Ca)).^2 + (y(Cf2Ca)+R_c).^4)/(y(Cf2Ca)+R_c).^2);
Moment2_SAIL = -rho*alphasail*(pi/4)*(R_s - D/2)^2*C_s*(Lcb - X_s);

Yvdot = (Force_B2Cf + Force_Ca2S + Force_Cf2Ca + Force_SAIL) / (0.5*rho*L^3);
Zwdot = (Force_B2Cf + Force_Ca2S + Force_Cf2Ca) / (0.5*rho*L^3);

Mwdot = -(Moment_B2Cf + Moment_Ca2S + Moment_Cf2Ca) / (0.5*rho*L^4);
Nvdot = (Moment_B2Cf + Moment_Ca2S + Moment_Cf2Ca + Moment_SAIL) / (0.5*rho*L^4);
Yrdot = Nvdot;
Zqdot = Mwdot;

Mqdot = (Moment2_B2Cf + Moment2_Ca2S + Moment2_Cf2Ca) / (0.5*rho*L^5);
Nrdot = (Moment2_B2Cf + Moment2_Ca2S + Moment2_Cf2Ca + Moment_SAIL) / (0.5*rho*L^5);

% Added Mass - Rolling
% Assume added mass in roll comes primarily from Sail and Control Surfaces
Kpdot = -rho*(alphasail*(pi/4)*(R_s - D/2)^2*C_s*(R_s - D/2/2 + D/2) + C_c*(2/pi)*(R_c^4)) / (0.5*rho*L^5);
% Accounts for Sail & 4 Control Surfaces

% Added Mass Cross-Terms
% Derived using Kirchoff's kinetic energy relations
% Ref: Triantafyllou, M.S., 2.15 Maneuvering and Control of Underwater Vehicles, MIT Class Notes, 2009
Xwq = Zwdot;
Xqq = Zqdot;
Xvr = -Yvdot;
Xrr = -Yrdot;
Yur_a = Xudot;
Ywp = -Zwdot;
Ypq = -Zqdot;
Zuq_a = -Xudot;
Zvp = Yvdot;
Zrp = Yrdot;
Mvp = -Yrdot;
Muq_a = -Zqdot;
Mrp = (Kpdot - Nrdot);
Muw_a = -(Zwdot - Xudot);
Nwp = Zqdot;
Nur_a = Yrdot;
Npq = -(Kpdot - Mqdot);
Nuv_a = -(Xudot - Yvdot);
%%% Body Lift Force
% From Hoerner (13-3)
\[ Cl_{\alpha} = 0.003 \times \frac{L}{D} \times 180/\pi \]
\[ Y_{uv, l} = -0.5 \times \rho \times \frac{D^2}{2} \times Cl_{\alpha} / (0.5 \times \rho \times L^2) \]
\[ Z_{uw, l} = -0.5 \times \rho \times \frac{D^2}{2} \times Cl_{\alpha} / (0.5 \times \rho \times L^2) \]

%%% Body Lift Moment
% From Hoerner (13-4) Lift force location between .6 and .7 of the length
\[ N_{uv, l} = 0.5 \times \rho \times 0.15 \times L \times D^2 \times Cl_{\alpha} / (0.5 \times \rho \times L^3) \]
\[ M_{uw, l} = -0.5 \times \rho \times 0.15 \times L \times D^2 \times Cl_{\alpha} / (0.5 \times \rho \times L^3) \]

%%% Sail Lift Force and Moments
% Sail will induce a side force and moment at angles of attack in the X,Y plane
\[ AR_s = A_s / C_s^2 \]
\[ Alpha$_{bar}$ = 0.9 \]
\[ Cl_{alpha_s} = 1/(1/(2 \pi \times Alpha$_{bar}$)+1/(pi \times 2 \times AR_s)+1/(2 \pi \times (2 \times AR_s)^2)) \]
\[ Y_{uv} = -0.5 \times \rho \times Cl_{\alpha_s} \times A_s / (0.5 \times \rho \times L^2) \]
\[ Y_{ur} = -0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (Lcb-X_s) / (0.5 \times \rho \times L^3) \]
\[ Y_{up} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (-R_s) / (0.5 \times \rho \times L^3) \]
\[ N_{uv} = -0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (Lcb-X_s)^2 / (0.5 \times \rho \times L^4) \]
\[ N_{ur} = -0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (Lcb-X_s) \times (-R_s) / (0.5 \times \rho \times L^4) \]
\[ N_{up} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (-R_s)^2 / (0.5 \times \rho \times L^4) \]
\[ N_{ur} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (-R_s)^2 / (0.5 \times \rho \times L^4) \]
\[ K_{ur} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (Lcb-X_s) \times (-RR_s) / (0.5 \times \rho \times L^4) \]
\[ K_{uv} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (-R_s) / (0.5 \times \rho \times L^4) \]
\[ K_{up} = 0.5 \times \rho \times Cl_{\alpha_s} \times A_s \times (-R_s) / (0.5 \times \rho \times L^4) \]

%%% Control Surface Lift Force and Moments
% Control Surfaces will induce a side force and moment at angles of attack in the X,Y planes
\[ AR_c = A_c / C_c^2 \]
\[ Alpha$_{bar}$ = 0.9 \]
\[ Cl_{alpha_c} = 1/(1/(2 \pi \times Alpha$_{bar}$)+1/(pi \times 2 \times AR_c)+1/(2 \pi \times (2 \times AR_c)^2)) \]
\[ Y_{uc} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c / (0.5 \times \rho \times L^2) \]
\[ Y_{ur} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c \times (Lcb-X_c) / (0.5 \times \rho \times L^3) \]
\[ Z_{uc} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c / (0.5 \times \rho \times L^2) \]
\[ Z_{ur} = 0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c \times (Lcb-X_c) / (0.5 \times \rho \times L^3) \]
\[ M_{uc} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c \times (Lcb-X_c)^2 / (0.5 \times \rho \times L^4) \]
\[ M_{ur} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c \times (Lcb-X_c) \times (-R_s) / (0.5 \times \rho \times L^4) \]
\[ M_{up} = -0.5 \times 2 \times \rho \times Cl_{alpha_c} \times A_c \times (-R_s)^2 / (0.5 \times \rho \times L^4) \]

%%% SUBOFF Captive-Model Experiments
% Non-Dimensional values below are from Captive-Model Experiments referenced to the axes which have their origin 6.6042 feet aft of FP (LCB), along hull centerline.
% Ref. DTRC/SHD-1298-08
% Bare Hull Added Mass
\[ Y_{vdot} = -0.013270 \]
\[ Z_{wdot} = -0.013270 \]
\[ M_{wdot} = -0.000202 \]
\[ N_{vdot} = 0.000202 \]
\[ Y_{rdot} = 0.000060 \]
\[ Z_{qdot} = -0.000060 \]
\[ M_{qdot} = -0.000676 \]
Nrdot = -0.000676;

%Bare Hull Damping
Yuv = -0.005948;
Yur = 0.001811;
Zuw = -0.005948;
Zuq = -0.001811;
Nuv = -0.012795;
Nur = -0.001597;
Muw = 0.012795;
Muq = -0.001597;

%% Save Coefficients to File

savefile = 'coefficient.mat';
coefficient = [L D M U Ixx Iyy Izz xcg;
    Aw vol m Lcb W B 0 0;
    Xudot Yvdot Zwdot Kpdot Nvdot Nrdot 0 0;
    Xuu Yrddot Zqdot Kpp Mqdot Nrdot 0 0;
    Xvr Yuv Zuw Kuv Muw Nuv 0 0;
    Xwq Yur Zuq Kur Muq Nur 0 0;
    Xqq Yvv Zvp Kuv Mvp Nvv 0 0;
    Xrr Ywp Zww 0 Mww Nwp 0 0;
    0 Ypq Zrp 0 Mrp Npq 0 0;
    0 Yrr Zqq 0 Mqq Nrr 0 0];
save(savefile,'coefficient')

A.1.2 driver_submodel.m

close all; clear all; clc;

%% Load Hydrodynamic Coefficients Calculated from Analytical Data

load coefficient.mat
hydrovar = coefficient;
propdata = xlsread('b4_70_14data.xls');
tend = 130;

%% Run Simulation

sim('Sub Model',[0 tend])

figure(1);
plot3(x.signals.values,y.signals.values,-z.signals.values)
xlabel('X (m)')
ylabel('Y (m)')
zlabel('Z (m)')
axis equal

figure(2);
subplot(2,2,1)
plot(t.signals.values,u.signals.values,'r',t.signals.values,v.signals.values,'b',t.signals.values,w.signals.values,'g')
xlabel('Time (s)')
ylabel('Velocities (m/s)')
subplot(2,2,2)
plot(t.signals.values,phi_deg.signals.values)xlabel('Time (s)')
ylabel('Roll (deg)')
subplot(2,2,3)
plot(t.signals.values,theta_deg.signals.values)xlabel('Time (s)')
ylabel('Pitch (deg/s)')
subplot(2,2,4)
plot(t.signals.values,psi_deg.signals.values)
xlabel('Time (s)')
ylabel('Yaw (deg)')
A.2 Propeller Data

This is the 4 quadrant propeller data used in the submarine model for the standard configuration.

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A.3 SIMULINK Model

This provides the embedded scripts for the blocks in the SIMULINK Model

A.3.1 Coordinate Transformation Block
function [xdot, ydot, zdot, phidot, thetadot, psidot] = Global(u, v, w, p, q, r, phi, theta, psi)

% Fossen Transformation
J1 = [cos(psi) * cos(theta) - sin(psi) * cos(phi) + cos(psi) * sin(theta) * sin(phi);
     sin(psi) * sin(phi) + cos(psi) * sin(theta) * cos(phi);
     cos(psi) * sin(theta) + sin(psi) * cos(theta) * sin(phi) -
     cos(psi) * sin(phi) + sin(psi) * sin(theta) * cos(phi);
     -sin(theta) * cos(phi) + sin(2*phi) * cos(theta);
     sin(theta) * sin(phi) + cos(2*phi) * cos(theta);
     cos(theta) * sin(2*phi) + sin(theta) * cos(phi)];

% Alternate Transformation from Cooney thesis
J2 = [cos(theta) * sin(phi) - sin(theta) * cos(phi);
     sin(theta) * cos(phi) + cos(theta) * sin(phi);
     cos(theta) * sin(phi) - cos(theta) * cos(phi);
     sin(theta) * sin(phi) + cos(theta) * cos(phi);
     -sin(theta) * sin(phi) + cos(theta) * cos(phi);
     cos(theta) * sin(2*phi) + sin(theta) * cos(phi)];

zero = [0, 0, 0; 0, 0, 0; 0, 0, 0];

EFCSdot = [J1 zero; zero J2] * [qldot; q2_dot];

xdot = EFCSdot(1, 1);
ydot = EFCSdot(2, 1);
zdot = EFCSdot(3, 1);
phidot = EFCSdot(4, 1);
thetadot = EFCSdot(5, 1);
psidot = EFCSdot(6, 1);

A.3.2 Propeller Dynamics Block
function [Fx,Qx,CT] = Prop(hydrovar,propdata,u,N)

% Propeller Data

D = hydrovar(1,2);
Aw = hydrovar(2,1);
n = N/60;
d = .7;
w = 1-.371-3.04*d/(Aw)^.5;
t = 1-.632-2.44*d/(Aw)^.5;
Va = u*(1-w);

J = Va/(n*d);

% 0.7 Radius Advance Angle

if (Va>0) && (n>0)
beta = atan(Va/.7*pi*n*d);
elseif (Va>0) && (n<0)
beta = pi+atan(Va/.7*pi*n*d);
elseif (Va<0) && (n<0)
beta = pi+atan(Va/.7*pi*n*d);
else
beta = 3*pi/2+atan(Va/.7*pi*n*d);
end

data = propdata;
Ct=0;
Cq=0;
CT=0;
CQ=0;
for i=1:31
Ct=Ct+data(i,2)*cos(data(i,1)*beta)+data(i,3)*sin(data(i,1)*beta);
Cq=data(i,4)*cos(data(i,1)*beta)+data(i,5)*sin(data(i,1)*beta);
end
CT=0;CQ=1000;F
Fx = (1-t)*Ct*.5*(Va^2+.7*pi*n*d)^2*pi*(d^2)/4;
Qx = Cq*.5*(Va^2+.7*pi*n*d)^2*pi*(d^3)/4;

Qx=0;
A.3.3 Control Surface Effects Blocks

```matlab
% Submarine Rudder Data
rho = 1025; % density [kg/m^3]
c = 5.1; % chord [m] (sized by parametrics)
h = 4.4; % height [m]
T_r = .71; % Taper Ratio
Ar = .5*h*(c-T_r*c)+h*c*T_r; % projected area [m^2]
AR = Ar/c^2; % aspect ratio
AR_eff = 2*AR; % effective aspect ratio (due to reflection)
xr = -50; % axial distance from origin/CG (negative aft) [m]

% Calculate effective inflow velocity
UU = sqrt(u^2+(v+xr*r)^2); % effective inflow velocity
beta = atan((v+xr*r)/(u)); % effective inflow angle
```

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% Calculation of Angle of Attack
psi = delta-beta;

% Calculation of rudder lift coefficient slope (Hoerner)
a = 0.9;
CLs = 0.5*3.14159*AReff;81/(1/(2*a*pi)+1/(pi*AReff)+1/(2*pi*AReff^2)); % Hoerner lift theory (low aspect ratio foil)
CL = CLs*psi;
Cd = 0.013 + CL^2/(pi*AReff*.9); % Whicker and Fehlner drag

% Calculate lift and drag forces
L = 2*.5*rho*Ar*UU^2*CL;
D = 2*.5*rho*Ar*UU^2*Cd;

Fx = -L*sin(beta)-D*cos(beta);
Fy = L*cos(beta)+D*sin(beta);
Qz = Fy*xs;

function [Fx,Fy,Qy] = sternplanes(u,w,q,deltafin)

% Submarine Sternplane Data
rho = 1025; % density [kg/m^3]
c = 5.1; % chord [m] (sized by parametrics)
h = 4.4; % height [m]
T_s = .71; % Taper Ratio
Ar = .5*h*(c-T_s*c)+h*c*T_s; % projected area [m^2]
AR = Ar/c^2; % aspect ratio
AReff = 2*AR; % effective aspect ratio (due to reflection)
xs = -50; % axial distance from origin/CG (negative aft) [m]

% Calculate effective inflow velocity
UU = sqrt((u)^2+(w-xs*q)^2)); % effective inflow velocity
beta = atan( (w-xs*q)/(u) ); % effective inflow angle

% Calculation of Angle of Attack
psi = deltafin-beta;

% Calculation of rudder lift coefficient slope (Hoerner)
a = 0.9;
CLs = 0.5*3.14159*AReff;81/(1/(2*a*pi)+1/(pi*AReff)+1/(2*pi*AReff^2)); % Hoerner lift theory
CL = CLs*psi;
Cd = 0.013 + CL^2/(pi*AReff*.9); % Whicker and Fehlner drag

% Calculate lift and drag forces
L = 2*.5*rho*Ar*UU^2*CL;
D = 2*.5*rho*Ar*UU^2*Cd;

Fx = -L*sin(beta)-D*cos(beta);
Fz = (L*cos(beta)+D*sin(beta));
Qy = -Fz*xs;

function [Fx,Fz,Qy] = bowplanes(retract,u,w,q,deltafin)

% Submarine Sternplane Data
rho = 1025; % density [kg/m^3]
c = 3.47; % chord [m] (sized by parametrics)
h = 3.47; % height [m]
T_s = 1; % Taper Ratio
Ar = .5*h*(c-T_s*c)+h*c*T_s; % projected area [m^2]
AR = Ar/c^2; % aspect ratio
AReff = 2*AR; % effective aspect ratio (due to reflection)
xb = 31; % axial distance from origin/CG (negative aft) [m]
% Calculate effective inflow velocity
UU = sqrt((u)^2+(w-xb*q)^2);  % effective inflow velocity
beta = atan( (w-xb*q)/(u ));  % effective inflow angle

% Calculation of rudder lift coefficient slope (Hoerner)
a = 0.9;
CLs = 0.5*3.14159*AReff^2/(1/(2*a*pi)+1/(pi*AReff)+1/(2*pi*AReff^2));  % Hoerner lift theory
CL = CLs*psi;
Cd = 0.013 + CL^2/(pi*AReff^2);  % Whicker and Fehlner drag

% Calculate lift and drag forces
L = retract*2*.5*rho*Ar*UU^2*CL;
D = retract*2*.5*rho*Ar*UU^2*Cd;

Fx = -L*sin(beta)-D*cos(beta);
Fz = (L*cos(beta)+D*sin(beta));
Qy = -Fz*xb;

function [Fx,Fy,Qz,Qx] = Sail(u,v,r,p,q)

% Submarine Sail Data
rho = 1025;  % density [kg/m^3]
c = 8.832;  % chord [m]
h = 5.33;  % height [m]
T_s = 1;  % Taper Ratio
zs = 8.765;  % Mean height above CL [m]
Ar = .5*(c-T_s*c)*h*c*T_s;  % projected area [m^2]
AR = Ar/c^2;  % aspect ratio
AReff = 2*AR;  % effective aspect ratio (due to reflection)
xs = 26.61;  % axial distance from origin/CG (negative aft) [m]

% Calculate effective inflow velocity
UU = sqrt((u+zs*q)^2+(v+xs*r-zs*p)^2);  % effective inflow velocity
beta = atan( (v+xs*r-zs*p)/(u+zs*q ));  % effective inflow angle

% Calculation of rudder lift coefficient slope (Hoerner)
a = 0.9;
CLs = 1/(1/(2*a*pi)+1/(pi*AReff)+1/(2*pi*AReff^2));  % Hoerner lift theory
CL = CLs*psi;
Cd = 0.009 + CL^2/(pi*AReff^2);  % Whicker and Fehlner drag

% Calculate lift and drag forces
L = .5*rho*Ar*UU^2*CL;
D = .5*rho*Ar*UU^2*Cd;

Fx = -L*sin(beta)-D*cos(beta);
Fy = L*cos(beta)+D*sin(beta);
Qz = Fy*xs;
Qx = Fy*zs;

A.3.4 Azipod Dynamics Block

These Blocks replace the rudder and stern plane blocks in the podded configuration (only the stern pod blocks are shown)
Switch1
Rate Limiter

Switch2 Rate Limiter2

Lookup2D
Performs 2-D linear interpolation of input values using the specified table. Extrapolation is performed outside the table boundaries. The first dimension corresponds to the top (or left) input port.

Main Signal Attributes
Row index input values:
-15.0.15.30.45.60.75,90,105,120.135.150.165.180

Column index input values:
[0.0.06.0.12.0.18.0.24.0.3.0.36.0.42.0.48.0.58]

Table data:
[0.15.0.075.0.1.0.065.0.15.0.2.0.245.0.24.0.225.25.10]

Lookup method: Interpolation-Extrapolation

Sample time (-1 for inherited): -1

OK Cancel Help Apply
function [alpha,n,D,xp,J] = SPPod(RPM,u,w,q,delta)

D = 4.5; % [m] Nominal Azipod Prop Diameter
n = RPM/60; % [Rev/S] Propeller Speed
J = u/(n*D); % Advance Ratio
xp = -50; % axial distance from origin/CG (negative aft) [m]

% Calculate effective inflow velocity
UU = sqrt((u)^2+(w-xp*q)^2); % effective inflow velocity
beta = atan((w-xp*q)/(u)); % effective inflow angle

% Calculation of Angle of Attack
alpha = 180/pi*(delta-beta);

function [Fx,Fz,Qy] = SPForce(KFx,n,D,xp,KFz)

rho = 1025; % [kg/m^3] Seawater Density

Fx = 2*rho*n^2*D^4*KFx; % [N] Axial Thrust Force
Fz = 2*rho*n^2*D^4*KFz; % [N] Lateral Force
Qy = -Fz*xp; % [N-m] Moment from Azipod
A.3.5 EMBT Block

function [B,Xcb,BR,PBankdot] = EMBT(z,Switch,hydrovar,Blown,PBank)

% Capacities
Psw = 1025*9.81*z+1.01e5; % [Pa] Seawater Pressure in Main Ballast Tanks
MBT_vol = 1/9*hydrovar(2,2); % [m^3] Based on 12.5% reserve buoyancy of NSC
FBMT_vol = 0.6*MBT_vol; % [m^3] Based on 60/40% split Fwd/Aft for MBT Tanks
AirBank_vol = .005*hydrovar(2,2); % [m^3] Based on Parametric Sizing
Fwd_ABVol = .6*AirBank_vol; % [m^3] Based on 60/40% split Fwd/Aft
Pbank_initial = 31e6; % [Pa] Initial Air Bank Pressure
Xcb_add = -12.5; % [m] Location of Forward MBTs
Lcb = hydrovar(2,4); % [m] LCB aft of FP

% Initial Forward EMBT Blow Rate
% From Parametrics
IBR = .0003*hydrovar(2,2); % [m^3/3] Initial Blow Rate at test depth
BR = Switch*(IBR*(PBank-Psw)^.5)/(29.4465e6)^.5; % [m^3/sec] Proportional to square of pressure differential

if Switch > 0

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% Air Bank Pressure Drop
PBankdot = -Psw*BR/Fwd_ABVol; % [Pa/sec]

if Blown < FMBT_vol
    B = hydrovar(2,5)+Blown*1025*9.81; % [N]
    Xcb = (hydrovar(2,5)*hydrovar(1,8)-Blown*1025*9.81*(Xcb_add-Lcb))/B; % [m] (negative aft of FP)
else
    B = hydrovar(2,5)+FMBT_vol*1025*9.81; % [N] Limit of MBT
    Xcb = (hydrovar(2,5)*hydrovar(1,8)-FMBT_vol*1025*9.81*(Xcb_add-Lcb))/B; % [m] (negative aft of FP)
end
else
    PBankdot = 0;
    B = hydrovar(2,5);
    Xcb = hydrovar(1,8);
end

A.3.6 Maneuvering Block

function [u_dot,v_dot,w_dot,p_dot,q_dot,r_dot] =
Maneuver(hydrovarFx,Fy,Fz,Qx,Qy,Qz,SF,phi,theta,psi,B,xcb)
% Vessel particulars
L = hydrovar(1,1); % [m]
D = hydrovar(1,2); % [m]
Mass = hydrovar(1,3); % [kg]
U = hydrovar(1,4); % [m/s]
Ixx = hydrovar(1,5); % [kg*m^2]
Iyy = hydrovar(1,6); % [kg*m^2]
Izz = hydrovar(1,7); % [kg*m^2]
Xcg = hydrovar(1,8); % [m] (negative aft)
Xcb = xcb; % [m] (negative aft) From EMBT Block
Ycg = 0;
Zcg = -0.4; % [m] (Burcher and Rydel 3-4% diameter for BG)
WV = hydrovar(2,5); % [N]
BV = B; % [N]
rho = 1025; % [kg/m^3]
g = 9.81; % [m/s^2]

% Non-dimensionalize vessel particulars
MS = Mass / (0.5*rho*L^3); % dimensionless vessel mass
IX = Ixx / (0.5*rho*L^5); % dimensionless mass moment of inertia
IY = Iyy / (0.5*rho*L^5); % dimensionless mass moment of inertia
IZ = Izz / (0.5*rho*L^5); % dimensionless mass moment of inertia
xcg = Xcg/L; % = Xcg/L dimensionless c.g. (approx. zero)
xcb = Xcb/L; % = Xcb/L
ycg = Ycg/L; % = Ycg/L
zcg = Zcg/L; % = Zcg/L
wv = WV / (0.5*rho*L^3); % dimensionless vessel weight
bv = BV / (0.5*rho*L^3); % dimensionless vessel bouyancy

% Model particulars
%Lm = hydrovar(2,1); % [m]
%Dm = hydrovar(2,2); % [m]
%Mm = hydrovar(2,3); % [kg]
%Mum = hydrovar(2,4); % [kg/s]
%lm = hydrovar(2,5); % [kg*m^2]
%Im = hydrovar(2,6); % [kg*m^2]
%sxcm = hydrovar(2,7); % [kg*m^2]
%ycgm = 0; % [m] y-center of mass
%zcgm = 0; % [m] z-center of mass

% motion variables
x = [u/u; v/U; w/U; p**L/U; q*L/U; r**L/U];
y = [x(1) x(2) x(3) x(4) x(5) x(6) x(1)*abs(x(1)) x(1)*x(2) x(1)*x(3) x(1)*x(4) x(1)*x(5) x(1)*x(6) ... x(2)*abs(x(2)) x(2)*x(4) x(2)*x(5) x(2)*abs(x(3)) x(3)*x(4) x(3)*x(5) x(3)*abs(x(3)) x(4)*abs(x(3)) x(6)*abs(x(6)) x(5)*x(2) x(6)*x(2)];

% Non-dimensional Coefficients (From Model_constants.m; Linear Terms Only)
X_udot = hydrovar(3,1); % [m/s]
Y_vdot = hydrovar(3,2); % [m/s]
Z_gdot = hydrovar(3,3);
X_u = hydrovar(4,1); % [m]
Y_u = hydrovar(4,2); % [m]
Z_u = hydrovar(4,3);
X_uu = hydrovar(4,1); % [m]
Y_uu = hydrovar(4,2); % [m]
Z_uu = hydrovar(4,3);
X_uv = 0; % [m]
Y_uv = hydrovar(5,2); % [m]
Z_uv = 0;
X_uv = 0; % [m]
Y_uv = 0; % [m]
Z_uv = 0;
X_up = 0; % [m]
Y_up = 0; % [m]
Z_up = 0;
X_uq = 0; % [m]
Y_uq = 0; % [m]
Z_uq = 0;
X_ur = 0; % [m]
Y_ur = hydrovar(6,2); % [m]
Z_ur = 0;
X_vv = 0; % [m]
Y_vv = hydrovar(7,2); % [m]
Z_vv = 0;
X_vp = 0; % [m]
Y_vp = 0; % [m]
Z_vp = 0;
X_yr = hydrovar(5,1); % [m]
Y_yr = 0; % [m]
Z_yr = 0;
X_ww = 0; % [m]
Y_ww = 0; % [m]
Z_ww = hydrovar(8,3);
X_wq = 0; % [m]
Y_wq = hydrovar(8,2); % [m]
Z_wq = 0;
X_wq = 0; % [m]
Y_wq = 0; % [m]
Z_wq = 0;
X_wp = 0; % [m]
Y_wp = 0; % [m]
Z_wp = 0;
X_pq = 0; % [m]
Y_pq = hydrovar(9,2); % [m]
Z_pq = 0;
X_pr = 0; % [m]
Y_pr = 0; % [m]
Z_pr = hydrovar(9,3);
X_yq = hydrovar(7,1); % [m]
Y_yq = 0; % [m]
Z_yq = 0;
X_yr = hydrovar(8,1); % [m]
Y_yr = hydrovar(10,2); % [m];
X_u = 0; % [m]
Y_u = 0; % [m]
Z_u = 0;
\[
\begin{align*}
X_v & = 0; & Y_v & = 0; & Z_v & = 0; \\
X_w & = 0; & Y_w & = 0; & Z_w & = 0; \\
X_p & = 0; & Y_p & = 0; & Z_p & = 0; \\
X_q & = 0; & Y_q & = 0; & Z_q & = 0; \\
X_r & = 0; & Y_r & = 0; & Z_r & = 0; \\
\end{align*}
\]

\[
K_{pdot} = \text{hydrovar}(3,4); \quad M_{wdot} = \text{hydrovar}(3,5); \quad N_{vdot} = \text{hydrovar}(3,6); \\
K_{uu} = 0; \quad M_{uu} = 0; \quad N_{uu} = 0; \\
K_{uv} = \text{hydrovar}(5,4); \quad M_{uv} = 0; \quad N_{uv} = \text{hydrovar}(5,6); \\
K_{uw} = \text{hydrovar}(7,4); \quad M_{uw} = 0; \quad N_{uw} = 0; \\
K_{uq} = \text{hydrovar}(6,4); \quad M_{uq} = 0; \quad N_{uq} = 0; \\
K_{ur} = \text{hydrovar}(6,4); \quad M_{ur} = 0; \quad N_{ur} = \text{hydrovar}(6,6); \\
K_{vr} = \text{hydrovar}(7,4); \quad M_{vr} = 0; \quad N_{vr} = \text{hydrovar}(7,6); \\
K_{wp} = \text{hydrovar}(8,4); \quad M_{wp} = 0; \quad N_{wp} = \text{hydrovar}(8,6); \\
K_{wq} = \text{hydrovar}(4,6); \quad M_{wq} = 0; \quad N_{wq} = 0; \\
K_{pp} = \text{hydrovar}(4,4); \quad M_{pp} = 0; \quad N_{pp} = 0; \\
K_{pq} = \text{hydrovar}(5,4); \quad M_{pq} = 0; \quad N_{pq} = \text{hydrovar}(9,6); \\
K_{qr} = \text{hydrovar}(10,5); \quad M_{qr} = 0; \quad N_{qr} = \text{hydrovar}(10,6); \\
K_{r} = \text{hydrovar}(10,6); \quad \text{Factor to account for speed loss from hull.} \\
X_{rr} = -0.005; \quad \text{Factor to account for speed loss from hull.} \\
X_{qq} = -0.005; \\
\end{align*}
\]

\[
X = \begin{bmatrix} X_u & X_v & X_w & X_p & X_q & X_r & X_{uu} & X_{uv} & X_{uw} & X_{up} & X_{ur} & X_{vv} & X_{vp} & X_{vr} & X_{ww} & X_{wp} & X_{wq} & X_{pp} & X_{pq} & X_{pr} & X_{rr} \end{bmatrix} \\
Y = \begin{bmatrix} Y_u & Y_v & Y_w & Y_p & Y_q & Y_r & Y_{uu} & Y_{uv} & Y_{uw} & Y_{up} & Y_{ur} & Y_{vv} & Y_{vp} & Y_{vr} & Y_{ww} & Y_{wp} & Y_{wq} & Y_{pp} & Y_{pq} & Y_{pr} & Y_{rr} \end{bmatrix} \\
Z = \begin{bmatrix} Z_u & Z_v & Z_w & Z_p & Z_q & Z_r & Z_{uu} & Z_{uv} & Z_{uw} & Z_{up} & Z_{ur} & Z_{vv} & Z_{vp} & Z_{vr} & Z_{ww} & Z_{wp} & Z_{wq} & Z_{pp} & Z_{pq} & Z_{pr} & Z_{rr} \end{bmatrix} \\
K = \begin{bmatrix} K_u & K_v & K_w & K_p & K_q & K_r & K_{uu} & K_{uv} & K_{uw} & K_{up} & K_{ur} & K_{vv} & K_{vp} & K_{vr} & K_{ww} & K_{wp} & K_{wq} & K_{pp} & K_{pq} & K_{pr} & K_{rr} \end{bmatrix} \\
N = \begin{bmatrix} N_u & N_v & N_w & N_p & N_q & N_r & N_{uu} & N_{uv} & N_{uw} & N_{up} & N_{ur} & N_{vv} & N_{vp} & N_{vr} & N_{ww} & N_{wp} & N_{wq} & N_{pp} & N_{pq} & N_{pr} & N_{qr} & N_{rr} \end{bmatrix} \\
\]

% hydrodynamic mass matrix coefficients
\[
A = \begin{bmatrix} MS_{X_{udot}} & 0 & 0 & 0 & MS_{Z_{udot}} \\
0 & MS_{Y_{vdot}} & 0 & 0 & -MS_{Zcg} \\
0 & 0 & MS_{Z_{wdot}} & 0 & MS_{Xcg-Z_{qdot}} \\
0 & 0 & -MS_{Zcg} & 0 & IX_{K_{pdot}} \\
0 & 0 & MS_{Xcg} & 0 & 0 \\
0 & MS_{Xcg-N_{vdot}} & 0 & 0 & MS_{Xcg-Z_{rdot}} \end{bmatrix}; \\
\]

% hydrodynamic mass matrix coefficients
\[
\%A = \begin{bmatrix} MS_{X_{udot}} & 0 & 0 & 0 & MS_{Zcg} \\
0 & MS_{Xcg-Y_{rdot}} & 0 & 0 & MS_{Xcg-Z_{qdot}} \\
0 & 0 & MS_{Xcg} & 0 & MS_{Xcg-Z_{rdot}} \\
0 & -MS_{Zcg} & MS_{Ycg} & 0 & IX_{K_{pdot}} \\
MS_{Zcg} & 0 & -MS_{Xcg-M_{wdot}} & 0 & 0 \end{bmatrix}; \\
\]

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% hydrodynamic coefficients matrix
B = [X; Y; Z; K; M; N];

% hydrostatic Force matrix
C = [-(wv-bv)*sin(theta);(wv-bv)*cos(theta)*sin(phi);(wv-bv)*cos(theta)*cos(phi);-(zcg*wv)*sin(theta);(zcg*wv)*cos(theta)-(xcg*wv-xcb*bv)*cos(theta)*cos(phi);-(xcg*wv-xcb*bv)*sin(theta)];

% forcing matrix
% non-dimensionalize external forces
F = [Xo+Fx/(0.5*rho*U^2*L^2) Fy/(0.5*rho*U^2*L^2) Qx/(0.5*rho*U^2*L^2) Qy/(0.5*rho*U^2*L^2) Qz/(0.5*rho*U^2*L^2)]; % External forces

% x = [u,v,w,p,q,r] ---> dx = [u_dot,v_dot,w_dot,p_dot,q_dot,r_dot]
dx = inv(A)*(F' + B*y'+C);

% dimensionalize outputs
u_dot = dx(1)*U^2/L;
v_dot = dx(2)*U^2/L;
w_dot = dx(3)*U^2/L;
p_dot = dx(4)*U^2/L^2;
q_dot = dx(5)*U^2/L^2;
r_dot = dx(6)*U^2/L^2;
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