Acoustic Bragg Reflectors for $Q$-Enhancement of Unreleased MEMS Resonators

by

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ABSTRACT

In this thesis, the author introduces the first fully unreleased Micro-Electro-Mechanical (MEM) resonator, and the design of acoustic Bragg reflectors (ABRs) for energy localization and quality factor ($Q$)-enhancement for unreleased resonators.

Two of the greatest challenges in MEMS are those of packaging and integration with CMOS technology. Development of unreleased MEMS resonators at the transistor level of the CMOS stack will enable direct integration into front-end-of-line (FEOL) processing, making these devices an attractive choice for on-chip signal generation and signal processing. The demonstrated first fully unreleased resonator exhibits a resonance at 39 GHz with a $Q$ of 129, corresponding to the 1st harmonic longitudinal resonance of the unreleased resonator, a silicon Resonant Body Transistor (RBT) fully clad in SiO$_2$. A spurious mode occurs at 41 GHz, which is in good correspondence with simulation results. The $Q$ of 129 at 39 GHz is about 4 times lower than that of its released counterpart.

Enhanced with the ABRs, the unreleased resonator is able to maintain high $Q$, and suppress spurious modes. Analysis on the ABR design for unreleased resonators covers design principles, fabrication variations, and comparison to released devices. In the end, it is demonstrated that the ABR is more favorable than the phononic crystal for acoustic energy localization for unreleased resonators, providing a 9 times higher $Q$.

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INDEX

ABSTRACT ........................................................................................................................................... 2

ACKNOWLEDGMENTS .................................................................................................................... 3

CHAPTER 1. Motivation, Introduction, and Background ........................................................................ 6
  1.1 Motivation .................................................................................................................................. 6
  1.2 Introduction to MEMS Resonators ............................................................................................ 8
    1.2.1 General background of MEMS resonators ........................................................................... 8
    1.2.2 Air gap resonators .............................................................................................................. 11
    1.2.3 Internal dielectrically transduced resonators ...................................................................... 13
    1.2.4 Field effect transistor (FET) sensing—the resonant body transistor (RBT) ......................... 14
  1.3 Energy Localization Structures for MEMS resonators ............................................................ 16
    1.3.1 Acoustic Bragg reflectors (ABRs) ...................................................................................... 17
    1.3.2 Phononic crystals (PnCs) ................................................................................................... 18
  1.4 Goals of the Thesis ................................................................................................................... 19

CHAPTER 2. The First Fully Unreleased MEMS Resonator ................................................................. 21
  2.1 Principle of Operation for RBT .................................................................................................. 21
  2.2 The Unreleased RBT ................................................................................................................ 23
  2.3 COMSOL Finite Element Simulation ......................................................................................... 24
  2.4 Testing Results ......................................................................................................................... 26
  2.5 Discussion .................................................................................................................................. 28

CHAPTER 3. Acoustic Bragg Reflectors for Q-Enhancement of Unreleased MEMS Resonators .... 30
  3.1 Motivation .................................................................................................................................. 30
  3.2 Analysis of ABRs ....................................................................................................................... 31
    3.2.1 Reflection at double interfaces ............................................................................................ 31
    3.2.2 Reflection recursion formula ............................................................................................... 33
    3.2.3 Characterization of the ABR bandgap ............................................................................... 34
  3.3 Unreleased Resonator Bounded by ABRs—One Possible Fabrication ........................................ 37
3.4 Unreleas[ed Resonator Bounded by ABRs—Theory.......................... 39
  3.4.1 One-dimensional analysis of unreleased resonator bounded by ABRs........ 40
  3.4.2 Fabrication variation analysis......................................................... 44
3.5 Unreleas[ed Resonator Bounded by ABRs — Simulations.................... 46
  3.5.1 Comparison of unreleased design and released design .................. 46
  3.5.2 In-plane ABR design for unreleased resonator ......................... 50
3.6 Phononic Crystals vs. Acoustic Bragg Reflectors for Unreleas[ed Resonator Design.... 54

CHAPTER 4. Conclusions and Future Work............................................. 57

LIST OF FIGURES.................................................................................. 59

BIBLIOGRAPHY .................................................................................. 63

APPENDIX ......................................................................................... 66
CHAPTER 1. Motivation, Introduction, and Background

1.1 Motivation

Two of the greatest challenges currently faced by MEMS are those of packaging and integration with CMOS technology. There are two main approaches to integrate MEMS with circuits: to include the electronics and MEMS functionalities on separate chips that are packaged together into a multichip system, or to monolithically integrate the electronics and MEMS functionalities onto a single chip. The motivation for monolithic integration includes enhanced signal transduction, reduced footprint, improved immunity from parasitics and electromagnetic interference, robustness to harsh environments, and potentially lower cost compared to multichip systems. However, monolithic integration introduces challenges. Methods for MEMS-CMOS cofabrication have focused on modular MEMS-first or MEMS-last processes in which the MEMS devices are created in one set of process steps and the electronics are created in a second set of process steps, or vice versa. However, these MEMS-first and MEMS-last processes require an increased number of masks, which decreases yield and increases cost due to added complexity. Moreover, the constraints on the process sequence, thermal budget, and materials require a compromise between the MEMS and electronic devices and lead to reduced performance [1]. Non-modular MEMS-CMOS processing in which the MEMS and CMOS devices share process steps has also been demonstrated using the Back-End-of-Line (BEOL) CMOS stack. This approach reduces mask count but limits the MEMS materials to metals and dielectrics. This method also requires a post-processing release step to suspend the structure [2].

The need for direct integration of MEMS devices with CMOS is critical for successful implementation of high frequency active MEMS resonators. Monolithic integration of these devices can provide basic RF and mm-wave building blocks with high quality factor ($Q$), small footprint, and low power for use in wireless communication, microprocessor clocking, navigation and sensing applications. Nevertheless, the majority of MEMS resonators require a release step to freely suspend the moving structures. The released structures necessitate complex encapsulation and packaging, restricting fabrication to MEMS-last or BEOL processing of large-scale devices.
To overcome these obstacles, it is worth investigating the area of unreleased resonators. Unreleased resonators refer to micro electromechanical resonators that are fabricated without the released step. They are surrounded by the substrate and the capping medium. The use of unreleased resonators directly addresses the issues of monolithic integration and packaging. By fabricating resonators side by side with transistors using exactly the same process, MEMS resonators can be directly integrated into the Front-End-of-Line (FEOL) CMOS processing. This provides RF and microwave CMOS circuit designers with on-chip MEMS building blocks with few if any additional mask steps, making these devices an attractive choice for low power clock generation and high-\(Q\) tank circuits (Figure 1.1).

Development of unreleased resonators is not limited to application in CMOS processes, and can be implemented in applications where a minimal or no-packaging solution is needed.

![Diagram](image.png)

Figure 1.1 Conceptual schematics of the unreleased resonator fabricated side by side with transistors. Energy localization structures could be implemented surrounding the unreleased resonator for \(Q\)-enhancement.

The potential issue with unreleased resonators lies in their quality factors. Because there are no gaps to separate the resonators from the substrate, and the dielectric materials are capped on top, acoustic energy can leak by wave paths into the surrounding medium. The performance of an
unreleased resonator can therefore be degraded relative to its released counterpart. Nevertheless, this energy loss can be mitigated by adding acoustic energy localization structures, achieving $Q$-enhancement for unreleased devices and providing performances comparable to or even superior than the released ones.

Two types of structures have been proposed for the localization of acoustic vibrations in place of $\lambda/4$ suspension beams found in most released structures: the acoustic Bragg reflector (ABR) [3] and the phononic crystal (PnC) [4]. Both of them can be borrowed for unreleased resonator design. In this thesis, the discussions focus on the former.

1.2 Introduction to MEMS Resonators

1.2.1 General background of MEMS resonators

MEMS resonators are developed for application in radio frequency circuits in place of their electrical counterparts. Traditionally, these components are created with a combination of integrated circuits and high-$Q$ off-chip mechanical resonators such as quartz crystals [5]. These off-chip devices limit the miniaturization of radio transceivers and limit power scaling in microprocessors.

Due to the benefits of mechanical vibration, MEMS resonators are able to provide high quality factor, small footprint, integrability with circuits, and low power consumption. These properties make MEMS resonators promising in replacing traditional quartz resonators for signal generation, and in replacing circuit elements for narrow-band filters that can be directly applied to radio front-end. These basic pin-to-pin replacement possibilities and extended functionalities provide MEMS resonators wide application realms including wireless communication, microprocessor clocking, navigation, and sensing etc.

Besides these outstanding properties from mechanical vibration, challenges still exist for MEMS resonators, including temperature stability (the drift in resonator frequency with temperature, which is characterized by temperature coefficient of frequency—TCF), manufacturing yield, the ability to integrate the resonators with circuits, and packaging etc, which are being investigated
and addressed by a lot of works in this area [1] [2] [6] [7]. The ability to co-fabricate MEMS resonators with circuits is complicated by the fact that most MEMS resonators require a release step to freely suspend the resonant mechanical structure. This is achieved by an under-etch such as the removal of underlying oxide beneath a silicon layer in a typical SOI process [8]. Therefore MEMS resonator fabrication calls for custom processes, which are generally not integratable with standardized CMOS processes for circuits. There also exist resonators that do not require release steps in fabrication, such as the Surface Acoustic Wave (SAW) resonator [9] and the Solidly Mounted Resonator (SMR) [3]. Unreleased as they are, they still need free surfaces in contact with air or vacuum to resonate, and their processes are not integratable with CMOS.

From the angle of energy transduction, for a MEMS resonator, the energy is first converted from the electrical domain to the mechanical domain (by coupling forces including electrostatic, piezoelectric, thermal expansion, etc.), then localized by mechanical cavities to form resonance, and lastly recovered to electrical domain (by coupling mechanisms such as electrostatic, piezoelectric, modulation of current by strain, etc.). For electrical engineers, this mechanical physics in the middle can be converted and equivalently represented as circuit models for unified representation and more accessible analysis [10].

Like most devices, there are several figures of merit that are commonly discussed for MEMS resonator characterization and comparison. These figures of merit include resonant frequency, quality factor ($Q$), f.$Q$ product, electromechanical coupling coefficient $k_t^2$, motional impedance, spurious mode suppression, temperature coefficient of frequency (TCF), etc. Their meanings are explained as the following:

- The resonant frequency is the peak frequency of the resonator. It corresponds with the eigenmode frequencies of the mechanical resonant cavity.
- The quality factor ($Q$) is defined as the energy stored in resonance over the energy loss per cycle. It also represents the sharpness of the resonant peak. One resonator design goal is reducing energy loss for higher $Q$. Loss mechanisms for resonator design may include squeeze film damping, anchor loss, thermal elastic damping (TED), phonon-phonon scattering etc.
• The $f\cdot Q$ product is important to be listed alone because it characterizes the theoretical limit for $Q$ at various frequencies. This theoretical limit is constant at relatively low frequency ranges ($< 1$ GHz) in the Akheiser regime [11] where the $f\cdot Q$ product is limited by phonon-phonon scattering instead of anchor loss.

• As a characterization for the coupling efficiency between the two physics domains, electromechanical coupling coefficient $k_t^2$ is commonly used. It is defined as the actual energy converted over the total input energy.

• The motional impedance is defined as the input voltage divided by the output current at resonance. If the MEMS resonator is put into an equivalent circuit as a series RLC resonator, the motional impedance equals the equivalent resistance.

• For filter and oscillator applications of RF MEMS resonators, it is required that there are no spurious peaks (modes) near the intended peak. Spurious modes can be very common for mechanical resonance due to the diversity of mechanical modes, and non-idealities in the mechanical structure's geometry definition during fabrication processes. Therefore, spurious mode suppression is an important factor in resonator design, which is commonly achieved by proper anchor design.

• Temperature fluctuation affects the material properties of the resonator including Young’s modulus, density etc, which consequently varies the resonant frequency of the resonator. This effect is characterized by the temperature coefficient of frequency (TCF), which is defined as the coefficient of the second Taylor expansion term (the 1st derivative term) of frequency as a function of temperature.

High $Q$ resonance of the MEMS resonator is originated from the mechanical vibration, which is able to provide equivalent inductance orders of magnitude larger than that from integrated circuits. This is the fundamental reason why MEMS resonators can outperform their electrical counterparts. The extra energy transductions between the electrical domain and mechanical domain, however, require high efficiency transduction mechanisms that can couple energy conversion between two domains. Capacitive (electrostatic) and piezoelectric mechanisms are the most extensively explored transduction mechanisms for MEMS resonators owing to their own benefits. The capacitive transduction provides large quality factor and the best mode purity, while the
piezoelectric transduction offers excellent coupling efficiency and low motional impedance [12]. At high frequency ranges, mechanisms with less parasitics, such as the Field Effect Transistor (FET) mechanism can be implemented for sensing instead of either capacitive of piezoelectric effects.

This introduction focuses on the capacitive transduction mechanism due to its relevance to this work, and the FET sensing is discussed for high frequency applications. Some recent developments concerning capacitive transduced MEMS resonators are discussed in the following.

### 1.2.2 Air gap resonators

The most classic and well-known capacitive transduction based MEMS resonator would be the comb-drive resonator invented by Bill Tang and Roger Howe in 1989 [13]. It relies on lateral attraction forces of an array of capacitors in parallel to drive a bulk silicon structure. The structure is suspended by silicon beams anchored on the substrate acting as springs for vibration.

The analysis and design of this type of resonators is comparatively straightforward. It is equivalently a mass-spring-damper system, with the central bulk being the mass, and the suspended flexural beams providing restoring force. Damping mechanisms can include squeeze-film damping, thermal elastic damping (TED), anchor loss, and other internal damping mechanisms. The fabrication of these devices commonly involves deep reactive ion-etching (DRIE) etching to form air-gaps and under-etching to release the structure.

The resonance based on flexural modes is limited in terms of resonance frequency and quality factor at higher frequencies. Loss mechanisms such as TED are especially evident for flexural modes [14]. To overcome this issue, bulk-mode resonators are developed [15]. These devices employ in-plane modes (most of the time longitudinal modes) instead of flexural modes. Their performances are not limited by TED, and therefore they can be pushed to higher working frequencies with higher $Q$. The reason for higher frequencies for in-plane mode resonators can be attributed to larger velocity of longitudinal elastic waves compared to transversal elastic waves (almost a factor of two). The 1st eigenmode of a resonant cavity typically corresponds to half-wavelength inside the resonator. Accordingly, for the same dimensions of the resonator (therefore
the same wavelength of the resonant mode), the resonant frequency would be larger for larger wave velocities.

To drive air-gap capacitors, first a DC bias voltage is applied on both electrodes, and then an AC voltage is superposed as the driving input. This generates a DC bias force, a 2nd harmonic force, and the driving force at the 1st harmonic, which can be expressed as the following:

\[ F_{\text{cap,\alpha}} = \frac{1}{2} (V_{\text{DC}} + v_{\alpha} e^{i\omega t})^2 \frac{\partial C}{\partial x} \]

\[ = V_{\text{DC}}^2 \varepsilon_0 A_{\text{cap}} + \frac{v_{\alpha}^2 \varepsilon_0 A_{\text{cap}}}{2 g^2} + V_{\text{DC}} v_{\alpha} \frac{\varepsilon_0 A_{\text{cap}}}{g^2} \]

in which \( V_{\text{DC}} \) and \( v_{\alpha} \) are the DC and ac bias voltages respectively, \( \varepsilon_0 \) is the dielectric constant, \( A_{\text{cap}} \) is the capacitor area, and \( g \) is the gap length of the capacitor. In this formula, the 1st term is the static bias force, which does not contribute to resonance; the 2nd term is at 2nd harmonic frequency, and is small considering \( v_{\alpha} \ll V_{\text{DC}} \); the last term represents the driving force for resonance, and its frequency equals the frequency of the ac voltage.

Assuming the resonant cavity to be a one-dimensional bar of length \( L \), then the mode shape can be expressed as:

\[ u(x) = U_0 \sin\left(\frac{n\pi x}{L}\right) \]

in which \( n \) is an positive integer, \( U_0 \) is the amplitude, and \( x \) is the coordinate with the origin being the center of the bar. Additionally, the frequency of the 1st eigenmode can be expressed as:

\[ f_{\text{1st}} = \frac{1}{2L} a(v) \sqrt{\frac{E}{\rho}} \]

in which \( E \) is the Young's modulus, \( \rho \) is the density, and \( a(v) \) is a constant expressed with Possion's ratio. The constants \( a(v) \) for longitudinal waves and transversal waves are different.

On the sensing side, the capacitive sensing can be expressed as AC output current:
\[ i_{out} = \frac{dQ}{dt} = V_{DC} \frac{dC}{dx} \frac{dx}{dt} = V_{DC} \frac{\varepsilon_0 A_{cap}}{g^2} \omega_{res} U_0 \] (1-4)

in which \( Q \) is the total charge, \( \omega_{res} \) is the resonance angular frequency, and \( U_0 \) is the resonance amplitude at the sensing electrode, which equals mode amplitude because sensing capacitor is placed at point of max displacement.

With these basic analyses, the air-gap resonator can be further modeled as equivalent RLC circuit elements. The complete modeling and analysis of air-gap resonators can be found in [16].

To increase driving efficiency and reduce the motional impedance for air-gap resonators, direct methods would be increasing the driving area and reducing the gap. However, the gap cannot be too small due to limit of the aspect ratio of DRIE etching. To address this issue, it is necessary to introduce dielectrically filled resonators.

1.2.3 Internal dielectrically transduced resonators

Dielectric capacitive transduction has several benefits over common air-gap transduction:

- In dielectric transduction, the capacitor gap is created by thin film deposition instead of etching. As a result, the gap can be much smaller, which increase driving efficiency as shown in equation (1-1).
- The dielectrically filled gaps eliminate drawbacks of air-gaps such as stiction and pull-in.
- The driving force is further enhanced due to the increase of dielectric constant, as shown in equation (1-1).

An example of the dielectrically transduced resonator is the Internal Dielectrically Transduced resonator [17], as displayed in Figure 1.2. To optimize the performance of this type of resonator, direct analogy from air-gap resonators is incorrect. Instead of sensing at the point of maximum displacement, the internal dielectrically transduced resonator senses at the point of maximum strain. In addition, for dielectrically transduced resonators, the capacitors are part of the resonant
cavity, which must be considered in design. And this is exactly the reason why it is called the 'internal' dielectric transduction.

\[
\begin{align*}
\text{(a) Air gap resonator} & \quad \text{(b) Dielectrically filled resonator} \\
-L/2 & \quad x = 0 & L/2 \\
\vcenter{\hbox{v_{in}}} & \quad \text{i}_{\text{out}} & \quad \text{i}_{\text{out}} \\
\text{u}_3(x) & \quad \text{g} & \quad \text{V}_{\text{DC}} \\
\end{align*}
\]

Figure 1.2 (a) Schematic of an air-gap resonator. The signal is sensed at the point of max displacement on the edge, and one of the capacitor plates is not part of the resonant cavity. (b) Schematic of an internal dielectrically filled resonator. The signal is sensed at the point of max strain, and the capacitors are part of the resonant cavity. Both of these device form longitudinal resonance, and the 3rd harmonic is exhibited for both devices for comparison.

The internally dielectrically transduced resonators have yielded the highest acoustic resonant frequencies (up to 4.5 GHz) and among the highest frequency-quality factor products (up to $5.1 \times 10^{13}$) published to date in silicon [5], [17]. In addition, these dielectrically transduced resonators demonstrate improved efficiency as frequency is scaled up, providing a means of scaling MEMS resonators to previously unattainable frequencies. However, as the frequency is further scaled up above 10 GHz frequency or even into the mm-wave frequency range (>30 GHz), the parasitics through driving and sensing capacitors become inevitable. This imperfection motivates new sensing mechanisms.

1.2.4 Field effect transistor (FET) sensing--the resonant body transistor (RBT)

FET sensing is by nature applicable for high frequency operation, due to its excellent isolation between input and output which results from the gate-oxide-channel structure. Furthermore, this sensing mechanism can take advantage of industrial transistor development, and therefore make use of high cut-off frequency and small scales of transistors for superior performance in MEMS resonators.
The application of FET sensing for MEMS resonators can be traced back to 1967, when Nathanson et al. demonstrated the Resonant Gate Transistor (RGT) [18], driving resonance in a conductive cantilever with an air-gap capacitive electrode, and sensing by the resonance-modulated current in the FET channel.

In contrast, the Resonant Body Transistor (RBT) is a recently-demonstrated MEMS resonator that forms resonance inside the body of the transistor rather than the gate [19]. It inherits the geometry of the internal dielectrically filled resonator, with the addition of the FET channel inside the fin (Figure 1.3, grey region) for sensing instead of capacitor sensing. The RBT can also be seen as a sensing transistor directly integrated in the resonator body, and its geometry is very similar to that of an Independent-Gate FinFET [20]. The FinFET is a candidate for the next-generation transistor in CMOS. This similarity in geometry indicates the potential for RBT to be integrated in CMOS.

As a result of the benefits of FET sensing, the RBT can amplify the mechanical signal prior to any feed-through parasitics, pushing operating frequencies to previously inaccessible ranges in silicon, demonstrating the highest resonant frequency in silicon to date [20].

Figure 1.3 shows a top-view schematic of the RBT principle of operation. In the vertical dimension, it is the structure of an internal dielectrically filled resonator, except that one of the capacitors (dielectric near G2) is now not used to generate capacitive current, but acts as the gate to drive FET sensing in the channel S-D.

The region in light grey represents the undoped active area of the FET, while the blue region is highly doped. The active area near the drive electrode G1 is biased into accumulation (red), so that a capacitive force acts across the thin dielectric film (yellow) driving longitudinal waves in the fin. A DC gate voltage is applied to G2, generating an inversion channel (blue) which results in a DC drain current. At resonance, elastic waves modulate the drain current by the piezoresistive effect. The gain of the transistor reduces the output impedance of the device, providing high transduction efficiency at RF and mm-wave frequencies.
Both the geometry and the fabrication process of the RBT are similar to that of the Independent-Gate FinFET. This indicates the possibility of this type of device to be fabricated side-by-side with transistors, achieving FEOL CMOS integration. However, the fabrication of RBT still requires a release etching step, which is not CMOS compatible. By removing this release step, the device will be fully unreleased and the fabrication process will be exactly the same as the Independent-Gate FinFET. Consequently, it becomes very necessary to investigate how these devices behave in the unreleased form. In the Chapter 2 of this thesis, the fully unreleased RBT is demonstrated, indicating possibility of FEOL CMOS integration of resonators. This unreleased design is significant and differs from existing unreleased SAW resonators and face-mounted SMR in its ability of CMOS integration and complete embedding in the unreleased medium.

For fully unreleased resonators, energy loss can be an issue, which may significantly decrease the $Q$ of the resonator. Therefore, designing energy localization structures around the unreleased resonator is necessary for $Q$-enhancement. This topic will be analyzed theoretically and by simulations in Chapter 3.

**1.3 Energy Localization Structures for MEMS resonators**

Even for released resonators, there is strong motivation for the topic of energy localization. The energy localization structures have been investigated to reduce anchor loss, which is a major loss
mechanism for MEMS resonators. Adding reflectors around the anchor reduces wave propagation into the substrate and enhances $Q$ [3], [21].

For unreleased face-mounted resonators such as the previously mentioned SAW resonator and the SMR, energy localization structures are also implemented for $Q$-enhancement.

These energy localization structures include acoustic Bragg reflectors and phononic crystals, which are further discussed in the following.

1.3.1 **Acoustic Bragg reflectors (ABRs)**

The acoustic Bragg reflector (ABR) is named after its optical analogue, which is used as thin film optical mirrors to improve reflectivity at a designed wavelength. The ABR is composed of periodic layers of two materials of low and high acoustic impedance. This periodicity induces bandgaps for acoustic waves. At each layer interface of the ABR, a fraction of the acoustic wave energy is reflected. To form coherent superposition of these reflected waves, these layers can be optimized at odd multiples of one quarter wavelength, resulting in an overall reflectivity identical to that of free or rigid boundary conditions.

![Acoustic Bragg Reflectors](image)

**Figure 1.4** (a) Sideview of the ABR created by material deposition underneath the Solidly Mounted Resonator (SMR); (b) Sideview of lithographically defined ABR around the resonator in a released plate.

In 1965, W. E. Newell introduced the idea of applying ABRs to thickness mode piezoelectric resonators for wireless applications in high frequency integrated circuits [21]. This work was further developed in fully integrated surface micromachining technology for solidly mounted resonators (SMRs) [3] (Figure 1.4(a)). These ABRs are composed of multiple depositions of
alternating materials, resulting in isolation in one dimension at a single frequency per wafer. In-plane isolation can be achieved using lithographically defined ABRs, as demonstrated in a suspended plate [22] (Figure 1.4(b)). This configuration enables resonators of multiple frequencies to be fabricated side by side on the same chip.

1.3.2 Phononic crystals (PnCs)
An alternative to the ABR for acoustic energy localization is the phononic crystal (PnC), which is mostly lithographically defined. They have been recently explored by several groups for microscale applications, including acoustic mirrors for resonators in suspended plates [4], acoustic waveguides, and filters [23].

Like the ABR, the phononic crystal is also a periodic structure, with periodicity in 2D or even 3D. It is analogous to photonic crystals, or even to atoms in a crystal lattice. And similar to the ABR, this periodicity induces a bandgap that can reject waves over a range of frequencies.

Figure 1.5 shows the most commonly seen PnC resonator design [4]. It is composed of a silicon plate etched with periodic holes through microprocessing. A wave is excited on the very left and transmitted and sensed to the very right. Resonance occurs inside the cavity defined by the absence of phononic crystal. This PnC resonator works exactly like a Fabry-Perot interferometer in optics.

![Resonant Cavity Diagram]

Figure 1.5 Structure of a resonator isolated by phononic crystals in a suspended plate. The resonant cavity is defined by the absence of phononic crystals.
Typically, air holes are etched as scattering elements in a silicon slab (or slab made of another material) to form a PnC structure. There are also solid scattering approaches to fill these holes with materials with acoustic impedance that is distinct from that of the slab [24]. This type of solid scattering design can be borrowed for unreleased resonator energy localization.

1.4 Goals of the Thesis

Toward the goal of unreleased resonator integrated into the FEOL in CMOS processing, two questions are asked: (1) Does the resonator work when it is unreleased? (2) If it works, how well can it work compared to the released resonator? Can the unreleased resonator work better?

To address these questions, two major topics are discussed in this thesis:

(1) Demonstration of the unreleased version of the RBT as a prototype of the fully unreleased resonators. This is the unreleased resonator without energy localization structures. The fabrication of these devices was completed previously in the same set as the released RBTs at the Cornell Nanoscale Science and Technology Facility (CNF) except for the release step. Testing of these existing unreleased devices provides a quick demonstration of the feasibility of fully unreleased resonators.

(2) Analysis of the performances of the unreleased resonator with the ABR. This part investigates how well unreleased resonators can perform compared to released ones. The analysis was completed through theoretical calculation and finite element simulation in COMSOL. In detail, these topics are covered:

- Analysis of the energy localization structures alone. It includes analysis of the ABR from the wave propagation point of view, and comparison of ABRs formed by different CMOS materials.
- Analysis of the unreleased resonators combined with the ABR. Its performance is compared with that of the ones without the ABR, and with that of the released resonators.
- Investigation of different in-plane ABR design for a longitudinal mode resonator. This is useful for layout design for these unreleased resonators.
• Comparison of the ABR and the phononic crystal for energy localization of unreleased resonators.

Of these goals, (1) is discussed in Chapter 2, and (2) is explained in detail in Chapter 3. Fabrication and testing of these fully unreleased resonators with the integrated ABR belong to future work.
CHAPTER 2. The First Fully Unreleased MEMS Resonator

2.1 Principle of Operation for RBT

The RBT was previously introduced in Chapter 1, along with the concept for a fully unreleased MEMS resonator based on its design. This chapter presents an analytical model for the RBT along with a circuit representation of its operation.

As discussed in section 1.2.4, the RBT inherits the structure of the internal dielectrically transduced resonator. Therefore, the analytical model for the RBT builds on the model for the internal dielectrically transduced resonator as derived elsewhere [17]. The derivation steps for the motional impedance of the RBT are outlined here.

From the internal dielectrically transduced resonator model, the resonant amplitude for the fin structure of the RBT can be easily acquired under a given DC bias $V_{DC}$ and a given ac input signal $v_{ac}$ at the driving gate. As a result, in this analysis, we assume the amplitude of vibration $U_0|_{RBT}$ has already been calculated from $v_{ac}$, and focus the discussion on the derivation of the FET sensing.

Assuming a known amplitude of vibration $U_0|_{RBT}$, the strain can be calculated by taking the derivative of the amplitude. The strain induced in the resonator piezoresistively modulates the drain current running through the inversion layer at the sensing gate. Assuming a piezoresistive coefficient of $\pi_{110}$ for current traveling along $<110>$, the change in mobility can be calculated as

$$\left.\frac{d\mu_n}{\mu_n}\right|_{\text{inversion}} = \pi_{110}E_k U_0|_{RBT} \cos\left(\frac{1}{2} k_n g\right)$$

(2-1)

where $g$ is the dielectric thickness, $E$ is the Young’s modulus and $k_n$ is the wave number of the nth harmonic. The piezoresistive mobility modulation of Equation (2-1) generates an AC current linearly dependent on the drain current

$$i_{out}|_{RBT} \approx I_D \frac{\partial \mu_n}{\mu_n}$$

(2-2)

With the output current, the motional impedance can be calculated from
The gain of the transistor amplifies the output current, and therefore reduces the output impedance of the device, providing high transduction efficiency at RF and mm-wave frequencies. This amplification through transistor transconductance can also be applied to identify mechanical vibration peaks of the RBT. For mechanical resonance peaks, with the increase of inversion gate (G2 in Figure 1.3 and Figure 2.1) voltage, the sensed current increases due to transistor amplification, and therefore the amplitude of the resonant peak increases.

\[ R_{\text{RBT}} = \frac{V_{\text{ac}}}{i_{\text{out}}_{\text{RBT}}} \] (2-3)

More intuitively, the operation of the RBT can be described by modifying the small signal model of a traditional field effect transistor, as shown in Figure 2.1. Analogous to the FET equivalent circuit, the model includes transconductance terms \( g_m \) and \( g_{mb} \) of front and back gate respectively, defining the electronic amplification of the RBT. In addition, a mechanical transconductance \( g_{ma} \) is introduced to express the electromechanical amplification of the mechanical signal which exhibits a Lorenzian frequency response, with center frequency and quality factor defined by acoustic resonance. Under DC biasing condition of the inversion gate G2 (Figure 1.3), the small signal current due to the front gate \( (g_{mvgs}) \) is suppressed. Additionally, the current due to \( g_{mb} \) is engineered to be small, particularly in standard CMOS technology. The dominant AC signal is attributed to the electromechanical \( g_{ma} \). From (2-3), \( g_{ma} = 1/R_{\text{RBT}} \), showing a direct dependence on
the $g_m$ of the FET. The dynamic range of the RBT is therefore limited by the back gate contribution to the drain current modulation and by the inherent electrical gain of the transistor. The FET $g_m$ is also the primary limitation in frequency scaling of the resonator, as the drain current modulation cannot be measured reliably beyond the transistor's cutoff frequency $f_T$. The performance of these active MEMS resonators is therefore defined by the quality of the transistor technology, and can benefit greatly from direct integration with cutting edge CMOS circuits.

### 2.2 The Unreleased RBT

As the first step toward the goal of CMOS integratable unreleased MEMS resonators encompassed in acoustic isolation structures, the first fully unreleased MEMS resonator without energy localization structures is demonstrated, comprising a Resonant Body Transistor (RBT) at 39 GHz, bounded on all sides by a thick SiO$_2$ layer. These structures were fabricated from a released resonator design in previous work at Cornell [19]. The original, released RBT fabrication relies on a SOI process in which the fin is etched out of the SOI wafer's device layer, followed by dielectric deposition and deposition of poly-silicon gate. Chemical mechanical polishing (CMP) is used to planarize the structure, and different dopings are applied to the gates, source, drain and the fin (blue and grey in Figure 2.2(a)). Finally, an under etch is applied to release the structure.

![Figure 2.2](image_url)

**Figure 2.2**: (a) Schematics of the RBT working principle, as described in section 1.2.4. (b) Scanning electron micrograph (SEM) of the unreleased RBT. The 1 μm thick field oxide is visible on top of the unreleased resonator, showing contact holes to the silicon underneath. (Inset) Device with oxide cladding removed to show the underlying RBT.
The resonators tested here are identical to those in the previous work, except that the structure is covered with a field oxide and there is no final release etch. An SEM of the unreleased RBT (before metallization) is found in Figure 2.2 (b), showing the field oxide (FOX) layer on top of the device with silicided holes for electrical contact to the silicon device layer underneath. The inset of Figure 2.2 (b) depicts the device with oxide cladding removed to show the underlying RBT. Figure 2.2 (a) recapitulates the schematics of the RBT working principle.

The present work represents the first time that these or any other resonators have been demonstrated in a fully-unreleased state. By removing the final release step, the fabrication process of the unreleased RBT becomes exactly the same as that of the independent-gate FinFET. Accordingly, this unreleased RBT demonstrates the feasibility of direct integration into FEOL CMOS processing, providing RF and microwave CMOS circuit designers with on-chip MEMS building blocks, making these devices an attractive choice for low power clock generation and high-\(Q\) tank circuits.

2.3 COMSOL Finite Element Simulation

COMSOL simulations of both released and unreleased (fully-clad) RBTs were performed to determine the distortion of the 1st harmonic longitudinal resonance in the presence of buried oxide (340 nm BOX) under the device and field oxide (1 \(\mu\)m FOX) above it (Figure 2.3).

In this side view simulation, elastic wave pattern is presented in this unreleased device with red and blue representing peak and trough of the wave pattern. The device geometries are as following: the single crystal silicon fin has a width of 114 nm (x-direction) and thickness of 220 nm (y-direction). The \(\text{Si}_3\text{N}_4\) gate dielectrics have thicknesses of 15 nm, and the polysilicon gates have widths of 400nm and thicknesses of 400nm. In addition, due to etching of the BOX during an over-etch of the device layer, the fin was offset by 50 nm from the bottom of the poly gate according to fabrication data. Longitudinal waves are excited at the accumulation gate dielectric (dielectric on the left) and sensed piezoresistively at the FET channel (next to the dielectric on the right). The dielectric on the left was loaded by a pair of equal and opposite external forces to represent the capacitive driving force. Meanwhile, the stress at the interface between the fin and
the dielectric on the right was integrated to generate the output signal, indicating the strength of piezoresistive modulation on the drain current. Perfectly Matched Layers (PMLs) are applied to absorb waves coming at the outside boundaries, representing energy sinks of infinity.

![Diagram of COMSOL simulation of side view of unreleased resonant mode in RBT](image)

Figure 2.3 COMSOL simulation of side view of unreleased resonant mode in RBT, showing contour plot of stress along x-axis. A force applied across one dielectric (15 nm silicon nitride) generates a 1st harmonic longitudinal mode detected by strain induced in the FET channel. PMLs indicate regions of no reflected energy.

In order to determine frequency response properties of this unreleased resonator and extract figures of merit such as $Q$, a frequency sweep from 30 to 50 GHz was performed in frequency response analysis in COMSOL structural mechanics module, shown in Figure 2.4. Resonance peaks were observed at 42 and 44 GHz for both released and unreleased RBTs. The similarity in frequency response between the released and unreleased devices indicates the capacity to design and realize bulk-mode unreleased resonators at RF and mm-wave frequencies.
Figure 2.4 Simulated frequency response of unreleased resonator showing resonance for both released and unreleased devices at the targeted frequency.

2.4 Testing Results

The unreleased RBTs were tested in a standard two-port configuration at room temperature in a vacuum probe station, as shown in Figure 2.5. Vacuum measurement was only necessary to prevent oxidation of the Ni metallization (metal conducting lines to the resonator), and does not affect device performance.

Figure 2.5 Schematic of 2-port measurement for frequency characterization of RBT. The drive and sense gates of the RBT are biased into accumulation and inversion, respectively. An AC excitation is superimposed on the drive gate, while the piezoresistive modulation of the drain current is detected at the output.
The measured frequency response of the unreleased RBT is given in Figure 2.6. The DC bias of the inversion gate \( V_{G2} \) was set at 3 V, 4 V, and 4.3 V successively, with a DC accumulation gate voltage \( V_{G1} \) of -1.12 V, drain voltage \( V_D \) of 4.31 V, and an input RF power of -13 dBm. The device exhibits one peak at 39 GHz and a spurious mode at 41 GHz, corresponding well with simulation results. The slight shift in resonance frequency compared to simulation can be attributed to variation of fin length and damping mechanisms. The \( Q \) of 129 at 39 GHz (\( fQ \) product of \( 5 \times 10^{12} \)) is \( \sim 4 \times \) lower than that of its released counterpart [20]. As discussed in the next chapter, for unreleased resonator design, both \( Q \) and spurious mode suppression can be improved by introducing energy localization structures.

![Figure 2.6 Measured frequency response of the unreleased RBT after standard short-open de-embedding.](image)

The signal increases with sensing gate voltage. This effect strongly validates that the signal originates from mechanical resonance. With the increase of \( V_{G2} \) from 3 V to 4.3 V, the resonance frequency shifts by 0.1 GHz and 0.2 GHz for 39 GHz and 41 GHz peaks respectively. The observed frequency shift results from local resistive heating due to increased DC drain current at higher gate voltage, which alters the Young's modulus of silicon. Engineering of the temperature coefficient of frequency (TCF) has been demonstrated in other systems using composite Si-SiO\(_2\) resonators to reduce temperature sensitivity well below 1 ppm/°C [6]. Similarly, the TCF of this
unreleased resonator can be compensated and reduced by adjusting the oxide thickness, making the resonator temperature insensitive. In a technology with limited control over layer thickness, this can be achieved by strategically patterning material around the resonant structure.

### 2.5 Discussion

The resulting $fQ$ product of the unreleased RBT is $5 \times 10^{12}$, which is plotted in Figure 2.7 against the fundamental limit of silicon defined by phonon-phonon scattering. At high frequencies in the Landau-Rumer regime [11], resonator $Q$ is limited primarily by anchor loss where the acoustic wavelength is comparable to and can be smaller than the dimensions of the supporting structure.

![Figure 2.7 Frequency-quality factor products of various silicon-based electrostatic resonators, plotted against the theoretical limit of phonon-phonon scattering.](image)

Anchor loss in released resonant cavities at GHz frequencies has been addressed by use of phononic crystals surrounding the device. This method has been shown to approach the phonon-phonon scattering limit in this frequency range. The quality factor of unreleased resonators can be
significantly improved by localization of acoustic energy using acoustic Bragg reflectors or phononic crystals. These designs can provide near-perfect acoustic mirrors in place of the lossy support beams required for released resonators. At mm-wave frequencies, unreleased resonators with energy localization may therefore achieve better performance than their released counterparts.
CHAPTER 3. Acoustic Bragg Reflectors for $Q$-Enhancement of Unreleased MEMS Resonators

3.1 Motivation

Due to energy leakage by wave paths into the surrounding medium, the performance of an unreleased resonator is degraded relative to its released counterpart, as demonstrated by experimental results from Chapter 2. This energy loss can be mitigated by adding acoustic energy localization structures, such as the acoustic Bragg reflector (ABR) or the phononic crystal (PnC).

This chapter studies the fully unreleased resonator surrounded by lithographically defined ABRs, embedded in a homogeneous cladding layer (Figure 3.1). This one-mask design enables resonator banks of various frequencies on the same chip, providing multiple degrees of freedom in the ABR design. With the goal of direct integration into the FEOL CMOS processing, resonator performance is investigated for materials commonly found in the CMOS stack. The characteristics of these unreleased structures are compared with freely suspended resonators, released resonators isolated with lithographically defined ABRs, and PnC based unreleased resonators.

![Figure 3.1](image_url)

Figure 3.1 Side view of unreleased resonator and ABRs lithographically defined in the same mask process. The resonant mode is longitudinal in the horizontal direction, exciting plane waves which are confined by the ABRs. Oxide cladding the resonator and ABRs illustrates the structures achievable in standard SOI CMOS processing.
3.2 Analysis of ABRs

3.2.1 Reflection at double interfaces

Existing ABR analysis is based on a transmission line analogy, borrowing results from the impedance transforming properties of cascaded transmission lines [21]. However, a more fundamental analysis can be performed conveniently from the wave propagation point of view through superposition.

To characterize more complicated reflectance properties of multilayer structures such as the ABR, it is helpful to begin with the element of a three-layer composite and acquire the lumped reflectance over two interfaces, using reflectance and transmittance of both interfaces in both directions.

Reflectance is defined by the acoustic impedance of two materials. For the case of longitudinal waves, the acoustic impedance $Z$ is defined by material properties as:

$$ Z = \frac{E}{c} = \sqrt{\frac{(1+v)(1-2v)\rho E}{(1-v)}} $$

where $E$, $\rho$, $v$ are Young's modulus, density, and Poisson's ratio respectively.

![Figure 3.2 Reflectance of longitudinal wave at an interface. The amplitude of the incident wave is normalized at 1.](image)
As exhibited in Figure 3.2, at a single interface, the reflectivity and transmittivity from material I into material II are:

\[
R = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{3-2}
\]

\[
T = \frac{2Z_1}{Z_1 + Z_2} \tag{3-3}
\]

Notice we have limit cases of \( R=1 \) for \( Z_1 \gg Z_2 \) (free boundary condition), and \( R=-1 \) for \( Z_2 \gg Z_1 \) (fixed boundary condition).

Similarly, the reflectivity and transmittivity from material II into material I are:

\[
r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{3-4}
\]

\[
t = \frac{2Z_2}{Z_2 + Z_1} \tag{3-5}
\]

Moving one step forward, we add one more interface. It becomes more complicated due to multiple reflections in the middle layer (Figure 3.3).

The model is built as follows. An acoustic wave travelling to the right in material I is incident on an interface with material II and is separated into reflected and transmitted components of \( R \) and \( T \). At the next interface, the transmitted component \( T \) is reflected at an assumed lumped reflectivity of \( R' \) from the following stack of materials. The reflected wave now travelling to the left with amplitude \( TR' \) is then reflected (\( r \)) and transmitted (\( t \)) at the first interface. As a result, infinite reflections occur inside material II with their amplitudes decreasing in geometric progression. By summation of all the reflections, we acquire the lumped reflectivity:

\[
R_{\text{total}} = R + TR'e^{-i2\delta} + TR'^2rte^{-i4\delta} + TR'^3r^2te^{-i6\delta} + ... \\
= R + TR'e^{-i2\delta}(1 + R're^{-i2\delta} + (R're^{-i2\delta})^2 + (R're^{-i2\delta})^3 + ...) \tag{3-6}
\]

\[
= R + \frac{Ttr'e^{-i2\delta}}{1 - rR'e^{-i2\delta}}
\]
in which $\delta$ is the phase change over the propagation length in material II.

![Diagram showing the derivation of recursion formula for ABRs through wave superposition.](image)

Figure 3.3 Derivation of recursion formula of reflectivity for ABRs through wave superposition. A summation of all the reflected components in the dashed line box generates relationship of lumped reflectivity $R(n)$ and $r(n)$ of the $n$th ABR bi-layer.

### 3.2.2 Reflection recursion formula

With the relation of lumped reflectivity between two adjacent interfaces, we can obtain the reflectivity recursion formula for ABR:

\[
R(1) = R
\]

\[
R(n) = R + \frac{Ttr(n)e^{-i2\delta}}{1 - rr(n)e^{-i2\delta}} \tag{3-7}
\]

\[
r(n) = r + \frac{tTR(n-1)e^{-i2\delta}}{1 - RR(n-1)e^{-i2\delta}}
\]
in which $\delta_1$ and $\delta_2$ are the phase change over the propagation length in material I and material II respectively, and $r(n)$ and $R(n)$ are the lumped reflectivities of the nth ABR bi-layer as shown in Figure 3.4. With these two relations, the properties of the ABR can be extracted numerically in MATLAB.

\[
\begin{align*}
\text{Material I} & \quad Z_1 = \frac{E_1}{c_1} \\
\text{Material II} & \quad Z_2 = \frac{E_2}{c_2}
\end{align*}
\]

Figure 3.4 Schematic on acoustic Bragg reflectors, with notation of lumped reflectivity at each interface.

### 3.2.3 Characterization of the ABR bandgap

Figure 3.5 shows the MATLAB results of ABR reflectivity as a function of the frequency of the incident wave, with the total number of ABR bi-layers as a reference variable. Si and SiO$_2$ are used as constitutive materials, and the length of each ABR layer is designed as the quarter wavelength of a 1 GHz longitudinal wave.
Figure 3.5 Frequency response of ABR amplitude (top) and phase (bottom). The bandgap approaches ideal shape with increasing number of ABRs. When the ABR pair number is small, the bandgap is attenuated and bandwidth increases.

As approximated by the result of 100 ABR pairs, an ideal ABR of infinite pairs is able to provide total reflection over several frequency ranges, called the bandgaps. The center frequency of each bandgap is determined by the length of each ABR layer relative to the wavelength. Total reflectance occurs around the frequencies where odd multiples of quarter wavelengths fit in each ABR layer, which also explains the periodicity at odd harmonics.

For practical application and consideration of footprint, only a finite small number of ABR pairs can be used. Although this bandgap shape is attenuated for a finite number of ABR pairs, the numerical result demonstrates that 7 pairs are sufficient to provide a satisfactory approximation, resulting in a reflectivity of 0.99 at the center of bandgap. With smaller number of pairs, the reflectivity decreases and the equivalent bandwidth gets wider.

The width of the bandgap is dependent on the material set, specifically, the acoustic impedance contrast of the two constitutive materials. The material pair with higher acoustic impedance
contrast ratio \((Z_1/Z_2)\) forms a wider bandgap (Figure 3.6). This contrast ratio also determines how fast the reflectivity converges to 1 with respect to the number of ABR pairs, and therefore the minimum footprint consumed during the design. The larger the acoustic impedance contrast ratio, the faster the reflectivity converges.

![Bandgap of Si/SiO₂ ABR—Effect of Acoustic Impedance Mismatch](image)

**Figure 3.6** Effect of acoustic impedance contrast ratio on the ABR bandgap width. The bandgap decreases with the acoustic impedance contrast ratio.

Figure 3.7 shows the convergence curve at the center of the bandgap of various commonly seen CMOS material pairs, with reflectivity plotted against the number of ABR pairs. The sign of reflectivity is determined by the order of materials in the ABR bi-layer. If the first material in the bi-layer has a higher acoustic impedance, the overall ABR will approach a free boundary condition (positive reflectivity). On the other hand, if the first material has a lower acoustic impedance, it will approach fixed boundary condition (negative reflectivity).
Materials of large acoustic impedance contrast such as W/SiO$_2$ [25], AlN/SiO$_2$ [3], Mo/Al [26] have previously been used to form ABR pairs in SMRs. This large contrast minimizes the number of pairs required to achieve good acoustic isolation, reducing the number of deposition steps during fabrication. In contrast, this will not complicate fabrication of the unreleased resonator-ABR structure, since the entire ABR structure can be defined lithographically in one step. Si/SiO$_2$ ABRs are selected for the design and analysis in this chapter, due to their FEOL CMOS compatibility, low internal loss, and small achievable footprint.

### 3.3 Unreleased Resonator Bounded by ABRs—One Possible Fabrication

The conceptual structure of the unreleased resonator bounded by lithographically defined ABRs in Figure 3.1 can be fabricated through a standard SOI process (Figure 3.8). Starting from an SOI wafer, the lithographically defined resonator and ABRs are etched over the same step by DRIE,
and then field oxide is filled on top, followed by a Chemical Mechanical Polishing (CMP) step to create the complete ABR structures and the unreleased capping medium. Lastly, contact holes to the devices and the substrate are created followed by dielectric deposition and metallization to create the capacitive driving and sensing structures.

Figure 3.8 One possible fabrication of the unreleased resonator bounded by ABRs. (a) Start from a SOI wafer; (b) DIRE etch to create the mesa structures of the resonator and the ABR Si layers; (c) Deposit field oxide on top, followed by CMP to create complete ABR structure and the unreleased medium; (d) Open contact holes, deposit dielectrics and then define metal conduct structure on top.
From this fabrication process, the unreleased resonator is bounded by the ABRs on both sides, embedded in oxide on top and bottom. The geometry can be almost one dimensional, i.e., it has a relatively large width in the out-of-plane direction. Longitudinal waves are launched in the horizontal direction and confined by the ABRs to form resonance inside the resonant cavity in the middle. In this conceptual design, the device is driven on top, which couples into the horizontal direction through Poisson's effect. The driving efficiency can be improved by creating capacitor structures vertically (like the case of the internal dielectrically transduced resonator), but the fabrication process would be much more complicated.

From the discussion of the ABR properties from the previous section, we understand that the ABR has to be designed at quarter wavelength for maximum reflectivity. For the resonator cavity, half wavelength exists inside it at the 1st harmonic mode. This means in the first etching step, the length of the resonator cavity should be double of the length of each Si ABR layer. The spacing between ABR layers is determined by the quarter wavelength of longitudinal wave inside SiO₂.

3.4 Unreleased Resonator Bounded by ABRs—Theory

With this conceptual structure of the unreleased resonator bounded by ABRs, we can continue to apply a basic one-dimensional analysis to determine its performance.

The acoustic energy loss analyzed here in the unreleased resonators is analogous to anchor loss in freely suspended devices. In most released and unreleased resonators, this energy localization can be the limiting mechanism for Q. Accordingly, this analysis is focused on the mechanical domain and neglects the physics of electro-mechanical coupling. Mechanical forces are directly applied to drive the structure instead of starting the analysis by applying a voltage; maximum or average strain is used to represent the signal output.

This mechanical-only analysis in turn gives a result that can be generalized to any driving or sensing mechanisms, as long as this mechanism relies on a strain modulated electrical signal as the output. These mechanisms may include dielectric capacitive, piezoelectric, piezoresistive, and Field Effect Transistor sensing, etc.
Silicon and silicon oxide are used as ABR materials in this analysis; the resonator itself is made of silicon.

3.4.1 One-dimensional analysis of unreleased resonator bounded by ABRs

The calculation results on ABR reflectivity can be applied as a known boundary condition to the analysis of an ABR-bounded unreleased resonator. Unlike simple free or rigid boundary conditions whose reflectivity is constantly 1 or -1 respectively over the entire frequency spectrum, the reflectivity of ABR is strongly frequency dependent, making it challenging for analytical expressions. As a result, all the analysis was carried out numerically in MATLAB through the superposition method.

The theoretical base for this analysis is as follows. The analysis is one-dimensional, with superposition steps shown in Figure 3.9:

![Superposition method](image)

Figure 3.9 Superposition method to calculate strain at center of the unreleased resonator bounded by ABRs. A pair of equal and opposite harmonic forces are applied on both boundaries to generate longitudinal waves. Infinite reflections take place with amplitude decreasing in geometric progression inside the resonator.
The resonator is modeled as a one-dimensional bar of length \( L \), with a pair of equal and opposite harmonic forces applied on both boundaries, generating longitudinal waves inside the resonator. The waves propagate and get reflected back and forth inside the resonant cavity bounded by the ABRs. The amplitude \( A \) of the excited longitudinal wave can be derived from the force balance relation at the boundary as:

\[
A = \frac{f_0}{ikE}
\] (3-8)

in which \( f_0 \) is the amplitude of the sinusoidal excitation force per area, \( k \) is the wave number, and \( E \) is the Young's modulus.

Waves excited by both forces propagate in opposite directions and form a standing wave, with complex amplitude of \(-2iA\), as shown in Figure 3.9. As the waves propagate to the ABR boundary, they are reflected with a decrease in amplitude and change in phase, represented by a complex reflectivity of \( R(\omega) \). In addition, taking phase change during propagation into account, the amplitude of next set of standing waves is found to be multiplied by a factor of \(-e^{i(\omega L)}R(\omega)\). Continuing with this process an infinite number of times, we are able to calculate the total amplitude of the standing wave from the summation of the geometric progression:

\[
A_{total} = -2iAe^{i\omega t} \sin(\omega t) \left( 1 + \left( e^{ikL}R(\omega) \right) + \left( e^{ikL}R(\omega) \right)^2 + ... \right)
= \frac{-2iAe^{i\omega t} \sin(\omega t)}{1 + e^{ikL}R(\omega)}
\] (3-9)

To obtain the strain at the center as the output signal, we can take derivative on \( x \):

\[
\varepsilon = \frac{dA_{total}}{dx}
= \frac{-2iAe^{i\omega t} \cos(\omega x)}{1 + e^{ikL}R(\omega)}
= \frac{-2iAe^{i\omega t}}{1 + e^{ikL}R(\omega)}
= \frac{E}{1 + e^{i\omega L/c}R(\omega)} \cos(\omega x) e^{i\omega t}
\] (3-10)
In these expressions, $R(\omega)$ is the reflectivity of the ABR, and it can be determined numerically from the previous section.

Note that the output strain is independent of the wave number, because the $k$ from the derivative and the $k$ from the amplitude cancel each other. It is also notable that the signal amplitude is dependent on the excitation frequency at two places: the reflectivity of the ABR which is frequency dependent; and the phase delay due to the propagation inside the resonator itself.

For a resonant cavity designed at half wavelength of a 1 GHz longitudinal wave and each ABR layer designed at quarter wavelength at the same frequency, the numerical analysis result is shown in Figure 3.10, with the x axis being the excitation frequency, and the y axis being the amplitude and phase of the max strain output. Resonant peaks appear periodically at the odd harmonics, which agrees well with the bandgap shape of the ABR. The $Q$ of the peaks increases rapidly with the increase in the number of ABR pairs as the reflectivity approaches 1. These results are consistent with the results of the ABR analysis (Figure 3.5).

![Normalized Max Strain of Unreleased Resonator with ABR](image)

Figure 3.10 Numerical calculation results of maximum strain output of unreleased resonator embedded in ABR. The resonator is of half wavelength of a 1 GHz longitudinal wave, and each ABR layer is of a quarter wavelength of the same wave. The strain is normalized to the maximum strain at 7 ABR pairs.
This result describes the performance of an optimally-designed device, which agrees with the commonly used design in literature, in which the center frequency of the ABR bandgap is the same as the resonator eigenfrequency. However, it is worth exploring other design options, such as when the two elements (the unreleased resonator and the ABR) are targeting different frequencies. For consistent comparison, the length of the resonator is kept constant at one half wavelength of 1 GHz, which is the 1st harmonic, while the length of two ABR layers varies simultaneously to target different bandgap frequencies. This varying ABR length is described by the ratio of the ABR bi-layer length to the wavelength at 1 GHz; this ratio is referred to as the normalized length. For example, for the optimal design, a normalized length of 0.5 is achieved by selecting 0.25 in each layer (quarter wavelength). For ABR designs away from the optimal number, the peak frequency drifts away from 1 GHz at the 1st harmonic, and the signal level reduces.

Figure 3.11(a) plots the calculated stress at the center of the resonator as a function of wave frequency and ABR normalized length in a two parameter sweep. Figure 3.11(b) re-plots Figure 3.11(a) as a top view (intensity plot) to better distinguish the key characteristics at different ABR designs. Different horizontal cross-sections represent different designs. For example, the cut of Figure 3.11(c) represents the optimal design at ratio 0.5. As for Figure 3.11(d), at ratio of 0.4, the signal reduces, and higher harmonics are suppressed relative to the first harmonic. Cross-section at (e) is also a good design, where the signal level at 3rd harmonic is the same as those in (c), and it has an extra advantage in the mode suppression of the 1st and the 5th harmonics.
Figure 3.11 Numerical calculation results on frequency responses of different ABR designs at fixed unreleased resonator length. The length of the resonator is fixed at half wavelength of a 1 GHz longitudinal wave, and the length of the ABR bi-layer is normalized to the wavelength at this same frequency (the normalization has taken effect of different wavelengths is different in different materials into account). (a) the two parameter sweep result on wave frequency and normalized ABR length; (b) topview of (a), with different cross-sections denoting different ABR designs; (c) cross-section at normalized ABR length of 0.5; (d) cross-section at normalized ABR length of 0.4.

3.4.2 Fabrication variation analysis
As mentioned in the previous section, the signal output is affected by both the length of the resonator and the length of the ABR bi-layer. Because lengths of both the ABR layers and resonant cavity are lithographically defined, the dimensions are subject to fabrication variations, such as over-exposure and under-exposure in photolithography, or over-etching and under-etching. Figure 3.12(a) shows how over-etching erodes the Si ABR and the resonator, thereby decreasing
the length ratio of the Si layer relative to the total bi-layer length. This subsequently shifts performance of the entire system including frequency, $Q$, and signal output.

As a result of the frequency bandgap, the frequency selectivity of the ABR is not singular. Therefore it is tolerant to fabrication variations, i.e. after variation, the targeted frequency will still stay inside the new bandgap. Similar to the previous section, to study this variation effect, the two parameter sweep method is used. The new element in this study is that the two layers of the ABR are now varying in opposite directions instead of the same direction because the periodicity of the overall structure is unchanged by over or underetch of the lithographically-defined lines as shown schematically in Figure 3.12(a).

The performance of the device was calculated for the case of fabrication variations using a two-parameter sweep in wave frequency and the fractional variation $\Delta l/l$ of the final linewidth of the silicon layers in the ABR (Figure 3.12(b)). This calculation was repeated for a series of different numbers of ABR pairs.

Figure 3.12(c) shows different excerpts from the resulting variation data set of the 1st harmonic. The topmost plot describes normalized strain at the center of the resonator as a function of the fractional variation of the silicon layers. Fabrication variation within 10-15% results in deviation of strain output up to 10 % from the as-designed ABR geometry. The plot in the middle of Figure 3.12(c) shows the frequency drift as a result of the ABR variation. When the device operates at a deviated frequency, it is no longer half wavelength inside the resonant cavity. This is caused by the phase change as the wave gets reflected at the ABR boundaries. The result agrees with the bandgap plot in Figure 3.5, in which the phase of reflectivity is not zero beyond the center of the bandgap. The plot at the bottom of Figure 3.12(c) gives the change of $Q$ as a result of the ABR variation. Again, fabrication variation within 10-15% results in degradation in performance up to 10%.

The variation affects larger signal more compared to smaller signals, i.e., for larger numbers of ABR pair, where the peak is larger, it is more susceptible to variations. With this being said, the
performance can still be guaranteed due to the relative flat top of these curves. For example, for 7 pairs of ABR layers, which is chosen for design, it is still tolerant to ~10% of variation.

![Diagram](image)

Figure 3.12 Fabrication variation study of unreleased resonator with ABR. (a) Over-exposure or over-etching may cause resonator length and Si ABR lengths to vary by $\Delta l$. (b) This variation can be studied by a two-parameter sweep on frequency and $L_0/L_{ABR}$. (c) The variation results in decreased performance including signal output, $Q$; and shift in resonance frequency.

3.5 Unreleased Resonator Bounded by ABRs — Simulations

3.5.1 Comparison of unreleased design and released design

The models above describe the predicted performance of the ABR-enhanced unreleased resonators, but they alone cannot identify how much of their performance is attributable to the unreleased capping medium and how much is attributable to their ABR isolation. To address this question and demonstrate the contribution of ABRs to unreleased resonator performance, two dimensional finite element simulations were performed in COMSOL Multiphysics 4.1.

The analysis is first carried out on a cross-section of the resonator as viewed from the side, assuming the width of the resonators to be sufficiently long that this 3rd dimension does not distort the mode shape (Figure 3.13). A plane strain model in the COMSOL mechanics module is selected to represent this assumption.
Figure 3.13 Schematics showing the construction of simulation. To better analyze the unreleased resonator bounded by ABRs, the simulations are decomposed into two steps: first the cross-section view, and then the top view.

The frequency response of ABR-enhanced unreleased structures is compared with freely suspended resonators, simple unreleased resonators (no ABRs), and released resonators isolated with lithographically defined ABRs (Figure 3.14). All of the resonators considered here are driven by a pair of equal and opposite forces on both edges over a range of frequencies. The resonators are designed for the 1st harmonic longitudinal vibrations at 1 GHz. The output signal is taken as the average strain across the entire resonant cavity. Perfectly matched layers (PMLs) are imposed at boundaries corresponding to energy sinks for waves propagating to infinity.

When defining PML parameters in COMSOL, it is recommended to match the PML to the desired acoustic wavelength. Although energy absorption is only optimized around the targeted wavelength, comparison of simulations with varying PML settings has shown robustness of PML performance over a broad range of frequencies for a single matched wavelength. For example, simulations of the resonator shown in Figure 3.14(b, c) were performed with PMLs targeted at 1 GHz and 3 GHz wavelength, demonstrating maximum variation of less than 2% between the two simulations across a 3 GHz frequency sweep.
Figure 3.14 Frequency sweep in COMSOL for cross-section views of free bar (a), released resonator-ABR plate (b), unreleased resonator-ABR structure embedded in oxide (c), and simple unreleased resonator embedded in oxide (d). The curves plot average strain of the resonators against the excitation frequency, and the resonators used are of thickness-length ratio of 6.5 and ABR number of 7; the characteristic contour plots describe the x component of the strain tensor at resonators of aspect ratio of 3.5 and ABR number of 3.

From these simulations, results are acquired as follows. The released bar (Figure 3.14(a)) provides the sharpest peak, which diverges for infinitesimal frequency steps due to the absence of internal damping in simulation. It is evident that due to the excitation of various plate modes, there are numerous spurious modes around the targeted frequency. On the other extreme, the unreleased resonator in oxide provides almost no peak at all (Figure 3.14(d)), due to homogeneous energy dissipation into the surrounding medium. Note that resonant peaks exist for the unreleased RBT due to the existence of dielectrics and poly gates, acting as a simple reflector. Comparison is made focusing on the released resonator-ABR plate (Figure 3.14(b)) and the unreleased resonator-ABR structure (Figure 3.14(c)). The output signal of the unreleased structure approaches that of the
released one with increasing thickness-length aspect ratio. At an aspect ratio of 6.5, the unreleased resonator-ABR structure provides an output signal only 20% less than its released counterpart. The unreleased device also exhibits clear spurious mode suppression, with a single peak of $Q=143$ at the desired frequency of 1 GHz.

Of all configurations, the unreleased resonator-ABR structure provides the purest mode due to damping of undesired plate modes in the non-resonant direction. Depending on the thickness-length aspect ratio of the device (Figure 3.14(a)), this out-of-plane damping may contribute to a reduced $Q$ of the targeted mode, with high aspect ratio favorable for low loss. Nevertheless, it only requires an aspect ratio of approximately 7 for the output of unreleased resonator-ABR structure to match that of the released resonator-ABR plate. This effect results from the higher thickness uniformity of the mode present in the unreleased resonator (Figure 3.14(c)), providing a larger effective area for sensing.

To get a better overview of the unreleased resonator-ABR structure, we can broaden the frequency sweep range into higher harmonics, as shown in Figure 3.15. At higher harmonics, the signal level of the unreleased structure matches that of the released one. Even for the 1st harmonic, the degradation compared the released design can be mitigated by increasing the thickness-length aspect ratio. This result is intuitive because the higher the aspect ratio, the larger the reflecting area and the less the leakage for acoustic energy. This also explains why the 3rd harmonic has a better performance. For higher harmonics, same thickness fits in more wavelengths, therefore the equivalent reflection area is even larger.

With these approaches, the unreleased design is able to match the performance of released resonator-ABR structure with extra advantage in spurious mode suppression.
3.5.2 In-plane ABR design for unreleased resonator

To this point, analysis and optimization has been addressed for the side view structure of both released and unreleased resonators, assuming the structure to be infinitely wide (Figure 3.13). However, in reality the width of the resonators would be finite, which introduces non-idealities in their performances. To study and minimize effects of this non-ideality, we have to perform the in-plane ABR design, which is from the top view. In this analysis, the out-of-plane dimension is again assumed to be sufficiently long (i.e., the thickness of the devices is sufficiently large).

Despite the non-ideality caused by this finite width, it also provides a third degree of freedom in design, i.e. the in-plane dimension perpendicular to the dimension where the longitudinal resonance is formed. For convenience, we name the ABRs in the resonant direction (ones discussed in the previous section) as the 'primary ABRs', and this extra set of ABRs as the 'side ABRs'. All these ABRs are lithographically defined at the same step. These side ABRs can be
applied to enhance $Q$ and suppress spurious modes. In addition, for a complete 2D in-plane ABR design, ABRs can even be added at the four corners, which we name as the 'corner ABRs' (Figure 3.16(b)).

In previous sections, to focus the investigation on the physics of unreleased resonator bounded by the ABRs, we did not mention how the devices is driven and sensed, assuming driving and sensing to be in the same bulk, i.e., driving mechanical vibration on both edges of the resonator, and sensing the strength of resonance by taking strain at the max point or integration over the entire area. In practice, driving and sensing take place at different parts of the same resonant cavity, i.e. part of the resonator is used for driving, while the other part for sensing.

Figure 3.16(a) illustrates a top view simulation of an unreleased resonator divided into driving and sensing. Force is applied on half the area on the left, and strain is integrated over half the area on the right to represent the sensed signal. To fit with this driving and sensing configuration, 2nd harmonic is used instead of the 1st harmonic. Therefore, a full wavelength lies inside the resonator instead of half wavelength, and the length of the Si ABR layer would be a quarter of the length of the unreleased resonator. The longitudinal mode is still targeted, but as a result of finite width, the signal amplitude is decreased compared to the ideal device of infinite width (which has a uniform 1D mode). This can be explained by the wave leakage on both sides of the resonator, which adds to damping loss in the unreleased medium.

To reduce this part of the energy loss, a straightforward design would be to add another identical set of ABRs in this dimension, i.e., the side reflectors, or better, adding corner ABRs. The localized wave patterns are displayed in Figure 3.16(b) in the ABR geometries named 'cross', 'square', and 'circle'. Surprisingly, instead of increasing the signal amplitude, these designs tend to decrease signal amplitude and $Q$ (Figure 3.16(c)), and introduce spurious modes. Why? The exact reason why unreleased resonator-ABR design is able to completely suppress spurious modes is because of this side damping, which eliminates the standing wave that can be possibly form in this direction, therefore eliminating the plate modes. By adding the side ABRs, these reflectance boundary conditions are recovered. As a result, plate modes occur, causing spurious modes around the desired peak (inset of Figure 3.16(c)).
Figure 3.16 Frequency sweep around the 2nd harmonic in COMSOL on layout designs of cross, square, circular and 1D ABR structures (topview). Adding ABRs for longitudinal waves on the sides of the resonator does not enhance $Q$. Instead, it tends to introduce spurious modes (circled in red). The inset presents the spurious mode for the square layout.

Does this mean that both sides have to be left open to avoid introducing spurious modes? Actually, it is not a trade-off between $Q$ and spurious modes in this design. Adding reflectors targeting at shear waves instead of longitudinal waves as side ABRs can enhance $Q$ without introducing spurious modes (insets of Figure 3.17). Imagining the targeted mode to be purely one-dimensional, there would be shearing force on both sides of the resonator where it meets the unreleased medium. These forces generate shear waves that act a source of energy loss. By adding shear reflectors, this part of energy can be localized, therefore enhancing $Q$. The reason why no spurious mode is
introduced can be attributed to the fact that longitudinal wave velocity is double of that of the shear wave. As a result, ABRs designed for total reflectance for shear waves would be transparent to longitudinal waves (demonstrated by the ABR bandgap plot in Figure 3.5). In this way, it can still act as a boundary condition that eliminates plate modes.

The simulation results (Figure 3.17) compare the designs of no side ABRs, longitudinal side ABRs, and shear side ABRs. It indicates that the longitudinal side ABR design decreases the performance, and on the contrary, the shear ABR design increases the performance compared to the simple design of primary ABRs only.

![Figure 3.17 Frequency sweep in COMSOL including the 2nd and the 6th harmonic on unreleased resonator with side reflectors (top view). The width-length aspect ratio of the unreleased resonator is 1.5. Designing side reflectors at quarter shear wavelengths provides more uniform mode and larger signal output.]

This enhancement is more evident for higher harmonics. The reduced wavelength of the 6th harmonic (x3 of the fundamental 2nd harmonic) in Figure 3.17 results in a more uniform mode.
along the width of the resonator. In this case, the vibrations near the sides of the resonator are increased such that the contribution from the side ABRs is larger. As can be seen in Figure 3.17, the 6th harmonic at 3 GHz exhibits significantly larger amplitude and higher $Q$ than the 2nd harmonic at 1 GHz due to the increased width to wavelength ratio. In this case, the shear ABRs also serve to enhance the $Q$ by a factor of 2 relative to the simple 1D ABR design and by a factor of 4 relative to the longitudinal side ABR design.

3.6 Phononic Crystals vs. Acoustic Bragg Reflectors for Unreleased Resonator Design

The ABR can be understood as one dimensional case of the phononic crystal (PnC). The PnC is an acoustic wave analogue of photonic crystals, where a periodic array of scattering elements in a homogeneous matrix material causes complete rejection of acoustic waves at a band of frequencies.

As mentioned in Chapter 1, solid scattering designs can be utilized to localize vibrations in unreleased resonators. To compare the performance of a 1D ABR structure to that of a PnC, a silicon square lattice PnC with oxide scattering elements is considered. The fabrication process would be identical to that of the ABR, except for a change in layout, with holes in place of trenches defined by lithography. Similarly to the ABR, the central frequency of the bandgap for the PnC is dominantly dependent on periodicity, or lattice constant 'a', which approximately equals a half wavelength, while the width of the bandgap is related to the filling ratio $r/a$ (Figure 3.18, inset). A COMSOL parameter sweep of the bandgap with respect to filling ratio shows the widest bandgap at a filling ratio of $\sim 0.3$ for Si/SiO$_2$. 
Figure 3.18 Acoustic bandgap comparison between phononic crystal and acoustic Bragg reflectors. The ABR provides a wider bandgap with no eigenmodes beyond the bandgap, and it requires a smaller footprint. The inset shows a 2x2 unit cell for the PnC used in this analysis. The contour plots show the topview of the transmission line structure, with amplitude of displacement decreasing with penetration depth.

To compare the bandgaps formed by the ABR and PnC, a transmission line configuration is investigated in COMSOL, with acoustic excitations generated on the left-hand side and sensing on the right, enclosed in PMLs on both ends (Figure 3.18). Periodic boundary conditions are added on top and bottom so that the bandgap property is not distorted by the width dimension. The comparison result shows that the ABR has several advantages over the PnC:

- ABRs provide a much wider bandgap than that of PnCs for given impedance contrast of materials and for the same footprint.
To achieve a near-perfect bandgap, it requires many fewer layers for the ABR compared to the PnC, requiring a smaller footprint.

At frequencies beyond the bandgap, there exist spurious eigenmodes for the PnC, which result in undesired strong resonance. On the other hand, the ABR provides a perfect bandgap without introducing spurious modes.

Overall, for the same footprint and material set, if we compare the PnC or ABR applied to an unreleased resonator driven on both ends, the ABR offers clear benefits, with a 9x higher $Q$, 20x larger signal output, and suppression of spurious modes at both low and high frequency (Figure 3.19).

Figure 3.19 Comparison of performance of unreleased resonator embedded in PnC and ABR. The contour plot is a topview simulation result in COMSOL with periodic boundary condition added to top and bottom. A pair of equal and opposite forces is applied on both edges of the resonator.
CHAPTER 4. Conclusions and Future Work

In this thesis, toward the objective of monolithic integration and no-packaging design of MEMS resonators, the feasibility of unreleased resonators is demonstrated through the first fully unreleased resonator in the form of an unreleased RBT, and energy localization, specifically using acoustic Bragg reflectors, is investigated through calculation and simulation.

Although the unreleased resonator alone has a disadvantage in $Q$ compared with released resonators, the design of unreleased resonators with ABRs can be optimized to suppress spurious modes and enhance quality factor. With sufficient thickness-to-length aspect ratios, the unreleased design is able to provide signal level and $Q$ that are comparable to their released counterparts. At high frequencies (>1 GHz) where resonator damping is dominated by anchor loss, the unreleased resonator with ABRs can outperform released devices.

Since the ABRs are lithographically defined in the same step as the resonator itself, their fabrication does not require any additional depositions or masks. In addition, it enables flexibility in ABR layout configuration, which can be implemented to optimize the ABR geometry for a single mode, resulting in high $Q$ design with a finite footprint. Compared with the PnC, the ABR is more efficient in acoustic energy localization both in performance and footprint, and does not introduce any spurious modes.

The design of unreleased resonators enhanced with acoustic Bragg reflectors can provide basic building blocks for RF circuit designers with high-$Q$ on-chip signal generation and processing that can be directly integrated into the FEOL in CMOS processing. Unreleased design also provides high yield, low cost, robustness in harsh environments, and minimal or no packaging for MEMS resonators.

Looking forward, several things can be addressed as a continuation of this work:

(1) As an experimental demonstration of these analyses, these unreleased resonators enhanced with ABRs can be fabricated and tested, and their performance compared to released resonators and to and unreleased resonator without ABRs.
(2) These devices can be integrated into the FEOL in CMOS. In particular, one can try to tape out unreleased resonators in industrial foundries to demonstrate their capability of FEOL integration. In this demonstration, RBT type of devices should be used due to their fabrication similarities to transistors.

(3) The coupling of unreleased resonators can be studied. The ABRs can work not only as reflectors, but also as transparent transmission lines at even harmonics. This indicates that the ABRs can be potentially engineered to transmit or reflect waves at designated directions. This property makes it enticing for application in resonator coupling.
LIST OF FIGURES

Figure 1.1 Conceptual schematics of the unreleased resonator fabricated side by side with transistors. Energy localization structures could be implemented surrounding the unreleased resonator for $Q$-enhancement. .................................................................................................................. 7

Figure 1.2 (a) Schematic of an air-gap resonator. The signal is sensed at the point of max displacement on the edge, and one of the capacitor plates is not part of the resonant cavity. (b) Schematic of an internal dielectrically filled resonator. The signal is sensed at the point of max strain, and the capacitors are part of the resonant cavity. Both of these device form longitudinal resonance, and the 3rd harmonic is exhibited for both devices for comparison. ................................. 14

Figure 1.3 Top-view schematic showing principle of operation of a bulk-mode dielectrically transduced Resonant Body Transistor. The accumulation gate (G1) capacitively drives longitudinal waves in the fin while a FET channel generated by the inversion gate (G2) piezoresistively senses resonance. ....................................................................................................... 16

Figure 1.4 (a) Sideview of the ABR created by material deposition underneath the Solidly Mounted Resonator (SMR); (b) Sideview of lithographically defined ABR around the resonator in a released plate. ................................................................................................................................ 17

Figure 1.5 Structure of a resonator isolated by phononic crystals in a suspended plate. The resonant cavity is defined by the absence of phononic crystals. ........................................................ 18

Figure 2.1 Small signal model of the RBT. G1, G2, D and S correspond to schematics in Figure 1.3. $g_m$ is the transistor transconductance through the front gate G2, and $g_{mb}$ is that of back gate G1. Current due to the front gate is suppressed by DC biasing. The back gate effect is small, and defines the signal floor. The Lorenzian frequency response of the piezoresistive transconductance $g_{ma}$ describes the high-$Q$ electromechanical amplification at resonance ............................ 22

Figure 2.2 (a) Schematics of the RBT working principle, as described in section 1.2.4. (b) Scanning electron micrograph (SEM) of the unreleased RBT. The 1 $\mu$m thick field oxide is visible on top of the unreleased resonator, showing contact holes to the silicon underneath. (Inset) Device with oxide cladding removed to show the underlying RBT. .............................................................................. 23

Figure 2.3 COMSOL simulation of side view of unreleased resonant mode in RBT, showing contour plot of stress along x-axis. A force applied across one dielectric (15 nm silicon nitride) generates a 1st harmonic longitudinal mode detected by strain induced in the FET channel. PMLs indicate regions of no reflected energy. .......................................................... 25

Figure 2.4 Simulated frequency response of unreleased resonator showing resonance for both released and unreleased devices at the targeted frequency. .......................................................... 26
Figure 2.5 Schematic of 2-port measurement for frequency characterization of RBT. The drive and sense gates of the RBT are biased into accumulation and inversion, respectively. An AC excitation is superimposed on the drive gate, while the piezoresistive modulation of the drain current is detected at the output. ................................................................. 26

Figure 2.6 Measured frequency response of the unreleased RBT after standard short-open de-embedding. ..................................................................................................................... 27

Figure 2.7 Frequency-quality factor products of various silicon-based electrostatic resonators, plotted against the theoretical limit of phonon-phonon scattering.............................................. 28

Figure 3.1 Side view of unreleased resonator and ABRs lithographically defined in the same mask process. The resonant mode is longitudinal in the horizontal direction, exciting plane waves which are confined by the ABRs. Oxide cladding the resonator and ABRs illustrates the structures achievable in standard SOI CMOS processing. ........................................................................... 30

Figure 3.2 Reflectance of longitudinal wave at an interface. The amplitude of the incident wave is normalized at 1.................................................................................................................. 31

Figure 3.3 Derivation of recursion formula of reflectivity for ABRs through wave superposition. A summation of all the reflected components in the dashed line box generates relationship of lumped reflectivity R(n) and r(n) of the nth ABR bi-layer. .................................................................................. 33

Figure 3.4 Schematic on acoustic Bragg reflectors, with notation of lumped reflectivity at each interface.......................................................................................................................... 34

Figure 3.5 Frequency response of ABR amplitude (top) and phase (bottom). The bandgap approaches ideal shape with increasing number of ABRs. When the ABR pair number is small, the bandgap is attenuated and bandwidth increases........................................................................ 35

Figure 3.6 Effect of acoustic impedance contrast ratio on the ABR bandgap width. The bandgap decreases with the acoustic impedance contrast ratio. ............................................................... 36

Figure 3.7 For finite number of ABR pairs, the reflectivity increases with the pair number and converges to 1 or -1 rapidly for materials commonly found in CMOS........................................ 37

Figure 3.8 One possible fabrication of the unreleased resonator bounded by ABRs. (a) Start from a SOI wafer; (b) DIRE etch to create the mesa structures of the resonator and the ABR Si layers; (c) Deposit field oxide on top, followed by CMP to create complete ABR structure and the unreleased medium; (d) Open contact holes, deposit dielectrics and then define metal conduct structure on top.......................................................................................................................... 38

Figure 3.9 Superposition method to calculate strain at center of the unreleased resonator bounded by ABRs. A pair of equal and opposite harmonic forces are applied on both boundaries to generate longitudinal waves. Infinite reflections take place with amplitude decreasing in geometric progression inside the resonator.................................................................................. 40
Figure 3.10 Numerical calculation results of maximum strain output of unreleased resonator embedded in ABR. The resonator is of half wavelength of a 1 GHz longitudinal wave, and each ABR layer is of a quarter wavelength of the same wave. The strain is normalized to the maximum strain at 7 ABR pairs.

Figure 3.11 Numerical calculation results on frequency responses of different ABR designs at fixed unreleased resonator length. The length of the resonator is fixed at half wavelength of a 1 GHz longitudinal wave, and the length of the ABR bi-layer is normalized to the wavelength at this same frequency (the normalization has taken effect of different wavelengths is different in different materials into account). (a) the two parameter sweep result on wave frequency and normalized ABR length; (b) topview of (a), with different cross-sections denoting different ABR designs; (c) cross-section at normalized ABR length of 0.5; (d) cross-section at normalized ABR length of 0.4.

Figure 3.12 Fabrication variation study of unreleased resonator with ABR. (a) Over-exposure or over-etching may cause resonator length and Si ABR lengths to vary by Δℓ. (b) This variation can be studied by a two-parameter sweep on frequency and Lsi/LABR. (c) The variation results in decreased performance including signal output, Q; and shift in resonance frequency.

Figure 3.13 Schematics showing the construction of simulation. To better analyze the unreleased resonator bounded by ABRs, the simulations are decomposed into two steps: first the cross-section view, and then the top view.

Figure 3.14 Frequency sweep in COMSOL for cross-section views of free bar (a), released resonator-ABR plate (b), unreleased resonator-ABR structure embedded in oxide (c), and simple unreleased resonator embedded in oxide (d). The curves plot average strain of the resonators against the excitation frequency, and the resonators used are of thickness-length ratio of 6.5 and ABR number of 7; the characteristic contour plots describe the x component of the strain tensor at resonators of aspect ratio of 3.5 and ABR number of 3.

Figure 3.15 Comparison of released resonator-ABR plate (left) and unreleased resonator-ABR structure (right) over a broader frequency range. The different curves in the same plot represent different designs with various thickness-length aspect ratio.

Figure 3.16 Frequency sweep around the 2nd harmonic in COMSOL on layout designs of cross, square, circular and 1D ABR structures (topview). Adding ABRs for longitudinal waves on the sides of the resonator does not enhance Q. Instead, it tends to introduce spurious modes (circled in red). The inset presents the spurious mode for the square layout.

Figure 3.17 Frequency sweep in COMSOL including the 2nd and the 6th harmonic on unreleased resonator with side reflectors (top view). The width-length aspect ratio of the unreleased resonator is 1.5. Designing side reflectors at quarter shear wavelengths provides more uniform mode and larger signal output.
Figure 3.18 Acoustic bandgap comparison between phononic crystal and acoustic Bragg reflectors. The ABR provides a wider bandgap with no eigenmodes beyond the bandgap, and it requires a smaller footprint. The inset shows a 2x2 unit cell for the PnC used in this analysis. The contour plots show the topview of the transmission line structure, with amplitude of displacement decreasing with penetration depth.

Figure 3.19 Comparison of performance of unreleased resonator embedded in PnC and ABR. The contour plot is a topview simulation result in COMSOL with periodic boundary condition added to top and bottom. A pair of equal and opposite forces is applied on both edges of the resonator.
BIBLIOGRAPHY


APPENDIX

MATLAB code for calculation of the ABR bandgap:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%--------MATLAB code for the ABR bandgap calculation--------% 
%--------------------------Mar, 2010-------------------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Initialize parameters
close all
clear all
zl=1.75984e7;  % acoustic impedance of Si
z2=1.19698e7;  % acoustic impedance of SiO2
cl=9659.94;    % longitudinal wave speed in Si
freq0=1e9;     % reference frequency
lambdal=cl/freq0;  % reference wavelength

% define reflectivity and transimitivity at interfaces
rl=(zl-z2)/(zl+z2);
tl=2*zl/(zl+z2);
r2=-rl;
t2=2*z2/(zl+z2);
RO=rl;

L_res=lambdal/2;  % length of the resonator

% first loop for frequency sweep
for j=1:1:600
    freq(j)=j*1e7;  % current frequency
    lambda(j)=cl/freq(j);  % current wavelength
    L_b=lambdal/4;  % the ABR layer length
    sita=2*pi*L_b*2/lambda(j);  %phase change in Si ABR layer
    sita2=sita;  %phase change in SiO2 ABR layer
    num=100+1;  % max ABR number
    if count==1
        R(count)=RO;  %initial values
        R_temp(count)=r2;
    else
        R_temp(count)=r2+tl*t2*R(count-1)*exp(-i*sita)/(1-r1*R(count-1))*exp(-i*sita));  %recursion formula
```

66
% Initialize parameters
close all
clear all

% MATLAB code for fabrication variation of the unreleased resonator with ABR:

R(count)=r1+t1*t2*R_temp(count)*exp(-i*sita2)/(1-r2*R_temp(count)*exp(-i*sita2)); % recursion formula 2end
der1l(2); % output with 1 ABR bi-layer
der3(2); % output with 3 ABR bi-layers
der5(2); % output with 5 ABR bi-layers
der7(2); % output with 7 ABR bi-layers
derln(2); % output with 100 ABR bi-layersend

% plot generation
unifreq=freq/freq0;
set(gcf,'Color',[1,1,1])
subplot(2,1,1)
plot(unifreq,abs(result1),'--',unifreq,abs(result3),'--'
',unifreq,abs(result5),'--',unifreq,abs(result7),'--'
',unifreq,abs(resultn),'k','Linewidth',2)
set(get(gcf,'CurrentAxes'),'FontName','Arial','FontSize',20)
axis([0 6 0 1.1])
title('ABR Bandgap Displayed by Reflectivity')
xlabel('Resonant Frequency Normalized to 1st Harmonics')
ylabel('Reflectivity Amplitude')
legend('1 ABR','3 ABR','5 ABR','7 ABR','100 ABR')
subplot(2,1,2)
plot(unifreq,phase(result1),'--',unifreq,phase(result3),'--'
',unifreq,phase(result5),'--',unifreq,phase(result7),'--'
',unifreq,phase(resultn),'k','Linewidth',2)
set(get(gcf,'CurrentAxes'),'FontName','Arial','FontSize',20)
axis([0 6 -3 3])
xlabel('Resonant Frequency Normalized to 1st Harmonics')
ylabel('Reflectivity Phase')
% ABR material and geometry
cl=9659.94; % longitudinal wave speed in Si
c2=5848.1; % longitudinal wave speed in SiO2
freq0=1e9; % reference frequency
lambda1=cl/freq0; % wavelength in Si
lambda2=c2/freq0; % wavelength in SiO2
L0=(lambda1+lambda2)/4; % designed length of abr1+abr2

% Reflectivity
z1=1.75984e7; % acoustic impedance of Si
z2=1.19698e7; % acoustic impedance of SiO2
r1=(z1-z2)/(z1+z2);
t1=2*z1/(z1+z2);
r2=-r1;
t2=2*z2/(z1+z2);
R0=r1;

% Resonator geometry
L_res=lambda1/2; % length of the resonator

% Some initial values
p_num=100;
vari_r=l/p_num:l/p_num:l;

% Three loops to sweep the parameters
for j=1:1:500 % frequency sweep
    freq(j)=j*4e6; % current frequency
    lambdas1(j)=cl/freq(j); % current wavelength in Si
    lambdas2(j)=c2/freq(j); % current wavelength in SiO2
    for p=1:1:p_num % Sweep on ABR geometry
        sita_res=2*pi*L_res/lambdas1(j); % phase change in the resonator
        L_b1(p)=L0*vari_r(p);
        sita1=2*pi*L_b1(p)*2/lambdas1(j); % phase change in Si ABR layer
        L_b2(p)=L0*(1-vari_r(p));
        sita2=2*pi*L_b2(p)*2/lambdas2(j); % phase change in SiO2 ABR layer
        num=7+1; % maximum ABR number
        for count=1:1:num % ABR reflectivity found by superposition method
            if count==1
                R(count)=R0;
                R_temp(count)=r2;
            else
                r=(r1+r2)/2;
                t=(t1+t2)/2;
                r1=r;
                t1=t;
                r2=r;
                t2=t;
                R(count)=r;
                R_temp(count)=t;
            end
        end
    end
end
\begin{verbatim}
R_temp(count) = r2 + t1 * t2 * R(count-1) * exp(-i * sita) / (1 - r1 * R(count-1) * exp(-i * sita));
R(count) = r1 + t1 * t2 * R_temp(count) * exp(-i * sita2) / (1 - r2 * R_temp(count) * exp(-i * sita2));
end
end

% Calculation of maximum strain in the center of the resonator
result1(p,j) = 1 / (1 + exp(-i * sita_res) * R(2)); %-%-i!!
result3(p,j) = 1 / (1 + exp(-i * sita_res) * R(4));
result5(p,j) = 1 / (1 + exp(-i * sita_res) * R(6));
result7(p,j) = 1 / (1 + exp(-i * sita_res) * R(8));
end
end

% plot(freq, abs(result3))

unifreq = freq / freq0; % normalized frequency

[XX, YY] = meshgrid(unifreq, varargin);

figure
set(gcf, 'Color', [1, 1, 1])

mesh(XX, YY, abs(result3))
set(get(gcf, 'CurrentAxes'), 'FontName', 'Arial', 'FontSize', 12)
view([0 0 1])
title('3 ABR')
xlabel('Resonant Frequency Normalized to 1st Harmonics')
ylabel('Length of SiABR / Total Length')
\end{verbatim}