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LEG-BASED HEURISTIC METHODS TO NETWORK  
SEAT INVENTORY CONTROL

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# Leg-Based Heuristic Methods to Network Seat Inventory Control

by

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## ABSTRACT

Driving the development of increasingly sophisticated methods of seat inventory control are the complex fare and route structures evident in airline operations today. Most control methods are currently based on an individual flight leg level. However, with the increased presence of multi-leg traffic flows, it becomes necessary to extend control methods beyond the flight leg level in order to maximize network revenues. While utilizing a full network optimization approach is not a suitable solution to satisfying airline desires for immediate revenue gains, the development of leg-based network seat inventory control algorithms is viewed as an applicable alternative approach.

In this thesis, two distinct components of seat inventory control are addressed. First, control structures that recognize multi-leg traffic flows are introduced and incorporated into leg-based network seat inventory control algorithms. Secondly, the development of local displacement cost logic is made and implementation issues within existing and newly developed control algorithms are explored. In order to quantify the effectiveness of inventory control algorithms to provide incremental revenue gains over existing control methods, an optimization/booking simulation was utilized to test the performances of these algorithms under a variety of demand patterns. Extensive simulation results in addition to discussions on practical implementation issues of different control approaches are made within the thesis.

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# Chapter 1

## Introduction

### 1.1 Motivation for Thesis

In 1938, the Civil Aeronautics Board (CAB) became the regulatory agency of the commercial airline industry in the United States. The formation of the CAB marked a period of dramatic expansion experienced by the airline industry. However, in the early 1970s, with growing airline inefficiency coupled with a period of severe economic recession, an industry wide drop in growth and profits occurred. Since airline fares were then based on an accounting of the airline's cost and investment; in order to provide a fair rate of return on investment, the CAB at that time began approval of higher fares to counter increasing costs.

Mounting criticism of the CAB's pricing policy began as airline fares increased. It was a common belief the CAB protected airlines from competitive forces and therefore fostered inefficiency that leads to higher costs and higher prices. As a results, advocates of "deregulation" preached that a deregulated environment would provide the competition among airlines necessary to bring about the objectives of efficiency, innovation and ultimately lower fares. In 1978, Congress passed the Airline Deregulation Act,

and during the next five years would phase out the CAB and government regulation of most commercial and economic activities within the domestic airline industry.

The passage of the Airline Deregulation Act allowed complete freedom for domestic airlines to set fares at will, dramatically altering the pricing strategies of the entire industry. By imposing restrictions on purchase and travel, carriers soon discovered that incremental gains in revenue can be had by offering several different fare products at varying fare levels to the traveling consumer. This practice is known as differential pricing. The advantages of utilizing a differential pricing strategy is twofold. First, providing lower discount fares would stimulate price-sensitive consumers to travel. Secondly, with the lowering of barriers to entry, the availability of lower fares are necessary to maintain a competitive image against low cost new entrant carriers infiltrating dominant major markets. In conjunction with offering different fares, fare restrictions and booking limits imposed on each group of fare products are used to prevent the diversion of passengers willing to pay higher fares. Although the benefits of differential pricing are not immediately recognizable to higher fare passengers, the incremental revenue generated by discount passengers does help to keep average fare levels down by covering a portion of the operating costs for the flight.

The practical difficulty with utilizing a differential pricing scheme is determining how the allocation of seats to different groups of fare products can be conducted in a revenue maximizing fashion. On the one hand, airlines are looking to reduce the number of empty seats on any one flight with the introduction of discounted fares. On the other hand, they want to minimize the displacement of higher revenue passengers by lower revenue



passengers. Ultimately, with an increasingly complicated and dynamic pricing environment, carriers found that it was necessary to develop tactical tools to help monitor and control the inventory of seats to sell at different price levels.

To derive the necessary seat allocations, it was important to first predict the number of seats that would not be sold to higher fare passengers. Driven by the purchasing behavior of different market segments, these seats would then be made available to lower fare passengers earlier in the booking process while a certain number of seats would be protected for later booking higher fare passengers. Moreover, the allocation of seats must also account for passenger demands for a whole range of different fare products and market segments. It is the ultimate goal of a proper inventory control approach to provide a carrier with a substantial increase in expected revenues. This is the basic philosophy behind the development of revenue/yield management or seat inventory control techniques.

Controlling seat inventory was further complicated by significant changes in airline route structures throughout the industry after deregulation. Initially, carriers had only to contend with dividing inventory among passengers traveling on a single flight leg from origin to destination. In a point-to-point service network, there is little need to differentiate among passenger revenue contributions to the network. However, as U.S. carriers sought to improve operational efficiency, many found benefits in a shift to a hub-and-spoke route structure. While the use of hub-and-spoke increases the operational reach of carriers and improves service frequency to many destinations, it also means that some passengers will be traveling on multi-leg journeys from origin to destination, often with a connection at the hub.

From the perspective of managing seat inventory, the increase in multi-leg origin-destination passengers on any one flight leg means greater complexity in evaluating the revenue contributions coming from any passenger. On any one flight leg into a hub, seat inventory allocations must not only taking into consideration the total fare paid by the passenger but also the travel itinerary of the passenger. If a multi-leg booking request was to be accepted, the travel itinerary of the passenger will allow the control approach to evaluate the potential of displacing higher revenue passengers on other flight legs. To maximize network revenues, inventory control algorithms must look beyond a single flight leg perspective and consider the network as a whole, making necessary the development of network/OD seat inventory control algorithms.

## **1.2 Thesis Objectives**

Driving the development of increasingly sophisticated methods of seat inventory control are the complex fare and route structures evident in airline operations today. A part of current development is the need for seat inventory control methods that extend beyond the individual flight leg environment. At the same time, the ease of implementing newly developed control approaches into current inventory control practices must also be taken into account. As airlines are looking for immediate revenue gains from a control methodology, a primary objective of this research is to introduce several practical alternatives to full network seat inventory control through the development of several leg-based network seat inventory control

algorithms that incorporate some key design changes to current inventory control approaches.

In order to accomplish this task, control structures that recognize multi-leg traffic flows must first be introduced and incorporated into any leg-based methods to network seat inventory control. Additionally, the development of local displacement cost approximations and implementation issues in utilizing displacement costs must also be considered. Armed with the components necessary to develop such control algorithms, several approaches are designed and implemented into a simulated hub-and-spoke environment to test their effectiveness and to illustrate their ease of implementation. The control algorithms developed represent a range of control possibilities differing in their effectiveness and required investment.

It is important to stress that the purpose of this research is not to provide an exhaustive set of empirical results to be used in comparing one inventory control approach with another with the intent of determining the "best" approach. On the contrary, the empirical results are used as a means to quantify the effectiveness of a particular approach to generate incremental revenue gains over current practices and to serve as incentives to implement such approaches. The empirical results also serve to illustrate the ease of implementing such an approach in practice and to encourage the user to draw more relevant, case-specific conclusions as to which approach will work best under their own demand environments.

### 1.3 Structure of Thesis

The remainder of this thesis is divided into five chapters. Chapter 2 introduces the basics of seat inventory control. We begin with a generic problem definition whereby the complexities of controlling seat inventories are discussed on both a leg and network level. Next, current solutions to the inventory control problem are discussed through the use of leg-based fare class control methodologies. The methodology is outlined in addition to problems associated with implementing such approaches in a network environment. We then justify the need to develop network control approaches through the use of a simple example. Typical mathematical approaches to network inventory control solutions are also discussed along with their limitations in the "real-world" environment. Finally, leg-based heuristic methods to origin-destination control are introduced and a basic design framework of a leg-based OD control algorithm is established.

Through the discussion of the current body of literature available, Chapter 3 develops a timeline in the development of seat inventory control approaches, ending with the most current thinking on network inventory control. Breaking down inventory control approaches into three distinct categories, we introduce different methods associated with each category in addition to the advantages and disadvantages of use in a network environment.

In Chapter 4, approaches to leg-based network seat inventory control utilizing OD control structures are discussed at length. We begin by defining the OD control structure and the role it plays in controlling inventory on a

network level. Next, a review of current fare class nesting control structure is made to contrast two different control structures specifically designed to control seat inventory on a network level. "Virtual Nesting" and "Stratified Bucketing" are then introduced and their advantages relative to basic fare class nesting are discussed in detail. The remainder of Chapter 4 illustrates and analyzes empirical results obtained from simulation runs utilizing several seat inventory control algorithms developed in this research that employ the previously mentioned control structures. Simulation runs are performed at a various levels of demand based on real demand data obtained from an airline.

Chapter 5 extends the discussion of developing OD or network seat inventory control algorithms with the introduction of local displacement cost logic. In this chapter, a detailed discussion on the definition of local displacement cost and several approximations to local displacement cost is presented. An approximation to local displacement cost is established and local displacement cost logic is implemented into the control algorithms developed in Chapter 4. Static and dynamic applications of displacement cost logic are also discussed and incorporated into the development of control algorithms. Simulation runs of several network seat inventory control algorithms with displacement cost are then performed under a variety of demand scenarios, and revenue impacts are illustrated and analyzed. In addition to extending displacement cost logic to leg-based OD control algorithms, an attempt to implement such logic into a leg-based fare class control algorithm is also made. Empirical results obtained from simulations are also presented at the end of the chapter to this effect.

Finally, in Chapter 6 we summarize the findings of this research and suggest future directions for further research. The need for some form of network or OD inventory control is growing in the airline industry. Within this research, the development of several applicable leg-based algorithms to network seat inventory control can provide a first step towards achieving the current goals of the industry.

# Chapter 2

## The Basics of Seat Inventory Control

### 2.0 Introduction

In allowing new freedoms to set fare levels and routes, deregulation in the U.S. airline industry has triggered an explosion of different fare products available to the traveling public. While the public benefits from increased fare competition, airlines are increasingly forced to determine new tactical methods of remaining competitive without compromising profitability. The development of an automated seat inventory control system is one of many operational measures adopted by airlines to help achieve greater levels of revenue.

The basic seat inventory control problem involves determining the number of seats that should be protected for sale to passengers willing to pay a higher fare away from lower fare passengers, with the intent of maximizing revenue [1]. Inventory control could also provide an airline the opportunity to participate in competitive discount fare initiatives in order to maintain market presence. At a finer level, through an integrated process of historical data collecting, forecasting, optimization and control, seat inventory control

could not only allocate seats between the fare products but between different travel itineraries over the network as well.

## 2.1 Generic Problem Definition

Since deregulation, airlines have adopted the use of differential pricing schemes in conjunction with seat inventory control systems for the purposes of maximizing revenues. By offering seats at a discounted fare, an airline can capture additional demand that otherwise would not be traveling, in turn providing additional revenue. However, differential pricing has also led to an increase in the number of fare products available. As airlines added fare products in order to capture different market demand segments by their willingness to pay, the complexity of controlling the different fares within the reservation system increased. To identify market demand segments, airlines imposed fences or restrictions on purchase to prevent diversion to lower fares by higher paying customers. In effect, by associating these restrictions with different fare levels, airlines are creating a range of level-of-service based products that cater to a variety of market demand segments. Consequently, the ultimate objective of any seat inventory control system is to properly allocate limited resources, in this case aircraft seats, to the different fare products/types in an effort to maximize the expected revenue of future scheduled flights.

Intuitively, the solution to correctly allocating seats may seem to be a simple one. One might suggest allocating as many seats as there is demand for the highest fare product, then continuing the allocation process to the next highest fare products until all demand is satisfied or all seats are filled. This



suggestion would be correct only if all demands for future flights are known with certainty and supply matches demand exactly. In reality, the complexity of the seat inventory control problem extends far beyond the intricacies of matching supply to demand. The sophistication (or lack thereof) of current computer reservations systems (CRS), with their individual system capabilities, must also be taken into consideration. The performances of even the best seat allocation algorithms are bounded by the abilities of the computer reservations system.

The adoption of the hub and spoke route structure by major airlines in the United States presented new levels of complexity to the seat inventory control problem. On any day, a large carrier can serve thousands of origin and destinations (ODs) through its hub and spoke system, while providing a multiplicity of different fare products to each OD market. As traffic flow on any one flight leg to and from a hub will include a mix of passenger OD itineraries, the seat allocation problem can no longer be viewed exclusively from a single flight leg perspective. The interactions between individual flight legs and the implications of their relationship on the entire network must be taken into consideration. These are just a few of the issues that illustrate the complexity of finding a revenue maximizing solution to the seat inventory control problem. The following sections will further detail some of the seat inventory control issues and how it is being addressed by different control methodologies.

## 2.2 Leg-Based Fare/Booking Class Control

Most airlines currently manage their seat inventories at the flight leg level. The process by which seat inventories are managed can be broken down into four basic components: historical data collection, forecasting and optimization, in addition to a control mechanism. Initially, the different fare products are assigned to a booking class. On any flight leg, a booking class can be regarded as an aggregation of demand for a group of fare products in question, irrespective of the flight itinerary or OD. Thus each fare class, representative of a particular fare product defined by the airline, is associated with a booking class for control purposes. For example, a discount ticket classified as "B class" for a Boston to San Francisco through Detroit itinerary would be grouped together in the same booking class as B class demand for a Boston to Detroit itinerary. Although both demands share the same booking class, the revenue contributions from those two itineraries could be significantly different.

Historical data on passenger demand are also collected and analyzed from the perspective of these individual booking classes on each flight leg. Forecasting models are used to obtain demand predictions for the fare products represented by their associated booking classes for future flights. Recommendations for optimal seat allocations to the different booking classes are then determined through marginal seat revenue analysis algorithms. Algorithms such as those presented by Belobaba [2] (Expected Marginal Seat Revenue) or Curry [3] (Optimal Booking Limits) are representative of the seat allocation algorithms utilized by most airline revenue management systems and are discussed in Chapter 3. The recommended seat allocations are

usually presented to the control mechanism as protection levels for seats in higher classes that cannot be sold to lower classes. These protection levels are converted by the control mechanism into booking limits for each individual fare class on each leg independently. In a hierarchical fashion, the booking limits are also nested within each other.

The need for booking limits and a nested control structure is rooted within the seat inventory control problem. A major difficulty in the seat inventory control problem is that requests for discounted fares by leisure travelers have a tendency to be made before higher/full fare passenger demand materializes. If full fare passengers on average booked before lower fare passengers the seat inventory control problem would be trivial. However, this is usually not the case due in part to the nature of leisure or discount customer bookings and the restrictions placed by the airlines on the purchase of these fares. As a result, a major function of seat inventory control is in essence the prediction of the number of seats that must be withheld from lower class passengers for the later booking, higher revenue passengers. This effort requires the determination of protection levels for higher booking classes or booking limits for the lower booking classes. In addition to optimal booking limits, these limits must also be nested hierarchically within each other to prioritize the availability of seats to higher fare passengers.

Effectively, in a leg based booking class inventory control approach, each booking request is evaluated based on the availability of the booking class requested on that flight leg, irrespective of the ultimate destination or itinerary of the request. As presented by Belobaba [4], on a multi-leg itinerary

booking request, seat availability is determined by evaluating the following equation:

$$BL_{ik} = \text{Min}[BL_{il}, \forall l \in i] \quad (2.1)$$

which states that the booking limit for any multi-leg itinerary,  $i$ , in booking/fare class,  $k$ , will be equal to the minimum booking limit for that fare class over all legs,  $l$ , traversed by itinerary,  $i$ . Recall that in a leg-based booking class approach, booking limits are determined to maximize revenue on a single flight leg only and that the booking classes represent an aggregation of demand that is not itinerary specific. This means that even though different itineraries represented by these multi-leg flights contribute different revenue values to the network, revenue is maximized solely on the flight leg level without regard to its impact on network revenue. If the airline's route structure consists of mainly point-to-point, non-stop flights serving distinct origin-destination markets, as illustrated in Figure 2.1, the use of a flight leg approach to seat inventory control is sufficient.

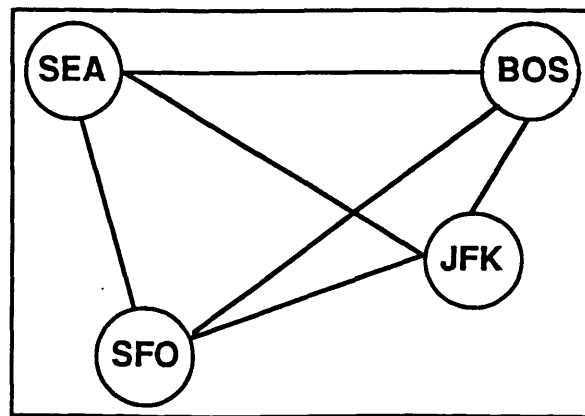
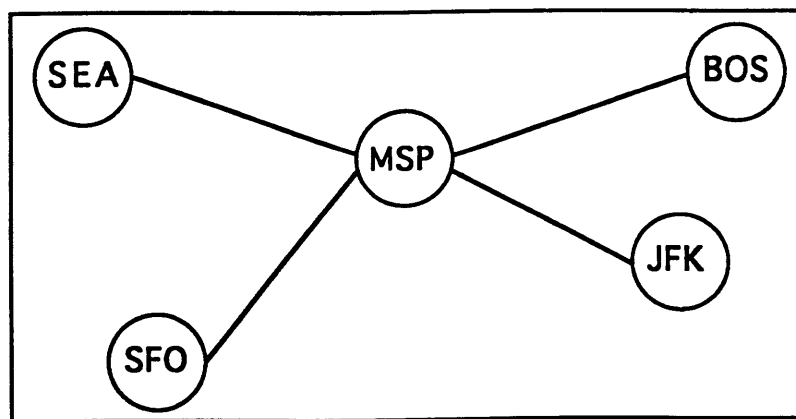


Figure 2.1 - Example of a Point to Point Route Structure

However, most airlines, especially in the United States, have adopted a hub and spoke route structure. The philosophy behind such a complex network is that it improves the operational reach of a carrier into a greater number of markets without expending a large amount of additional resources. As shown in Figure 2.2, instead of point-to-point, OD specific routes, a carrier transports passengers to an intermediate location, the hub, and transfers passengers to connecting flights to their final destinations. Thus on any one leg into a hub, demand for that flight leg could consist of many different OD itineraries. These different itineraries contribute varying amounts of revenue to the network, a fact that is not currently recognized by leg-based booking class seat inventory control methods. In a 'typical' hub complex with 30 legs in and 30 legs out with a total of 60 legs, there are 960 possible OD itineraries and by assuming 7 fare classes, 6,720 OD fare class combinations [5]. With each leg serving a possible 217 possible OD fare class combinations, a hub-and-spoke network illustrates the need to develop a seat inventory control approach capable of differentiating among the different ODF revenue contributions to the network.



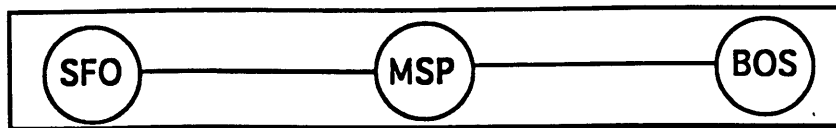
*Figure 2.2- Example of a Hub and Spoke Route Structure*

While there are great complexities introduced to the seat inventory control problem by the hub-and-spoke route structure, the ramification of the lack of differentiation between the revenue contributions of a local and a multi-leg itinerary that share a common flight leg is that current leg control does not maximize network revenues. Moreover, the implications of these observations strongly suggest that the seat inventory control problem is a network problem and that airlines need not only consider the management of flight leg seat inventories but the control of traffic flow as well.

### **2.3 Why Origin-Destination Seat Inventory Control?**

The switch to a hub and spoke route structure has had a significant impact on passenger travel. As passengers are routed through connecting hubs, the attributes of each flight leg and the associated control philosophy change quite dramatically. In a connecting hub environment, it is no longer sufficient to control inventories based solely at the flight leg level. Even though revenues may be maximized at the leg level, there may be significant revenue loss experienced on the network level.

Upon examination of a typical multiple-leg flight embedded within a hub network, the mix of passengers on any one flight leg may consist of varying amounts local and connecting passengers. A seat inventory control system must be able to maximize and differentiate among the revenue contributions of both types of passengers to the whole network and not only across the traversed flight leg. For the simple example shown in Figure 2.3, the complexity involved in optimally allocating seats to a multiple-leg scenario can be illustrated.



*Figure 2.3 - Example of a Multiple Leg Flight*

Consider a one stop flight from San Francisco through Minneapolis to Boston. On this two leg flight, there are three possible itinerary combinations: SFO to MSP, SFO to BOS and MSP to BOS. Assume only one fare product/class is available for sale in each itinerary; for the two short haul itineraries, a seat will be worth \$70 and on the long haul itinerary, a seat will be worth \$100. If seat inventory is controlled on a single flight leg level, a plausible revenue maximizing solution might be to protect as many seat as possible in a 'greedy' fashion for the long haul passenger. Since the flight legs are managed independently, a leg-based inventory control approach might recognize that long-haul fares contribute the greatest amount of revenue to the individual flight leg. Therefore, it is possible for a large number of seats on legs SFO-MSP and MSP-BOS allocated to SFO-BOS passengers. If demand for both local itineraries were extremely low or non-existent and long haul demand is high, the maximum revenue for this particular flight would probably be achieved. However, if local demand on both of the local itineraries were high, it would be more logical to take two local passengers over a long haul passenger. The sum of revenues contributed by two local leg passengers are usually greater than that of a single long haul passenger.

There are a number of different demand scenarios among the three itineraries. However, the important point is to recognize that different passenger itineraries contribute differently to total network revenue in a multiple leg scenario. A seat inventory control approach must be able to sort

out the different demand levels associated with each OD itinerary and assign the appropriate number of seats to each itinerary in a network revenue maximizing manner. Moreover, if there are multiple fare classes available for sale to each itinerary, the inventory control problem becomes substantially more complicated. Not only must the methodology control the different flight itineraries, it must now control the different fare products associated with each itinerary simultaneously.

Normal hub operations can consist of hundreds of origin-destination possibilities. Depending on the number of fare classes available, the number of possible origin-destination fare (ODF) combinations can reach into the thousands, with hundreds of possible ODF combinations on each flight leg, each with varying levels of attractiveness. This explosion in the number of control elements necessitates a very sophisticated seat inventory control process, with a level of control beyond that of simple flight leg booking class control. The ideal optimization techniques must take into account the interactions between flight legs in the total network, and at the finest level make decisions for possibly thousands of unique OD and fare options on each departure. As a major airline could have over 2000 departures daily with bookings being accepted a year in advance, one can easily comprehend the complexity and magnitude of the seat inventory control problem; all of which requires the use of an OD seat inventory control methodology.



## **2.4 Mathematical Approaches to OD Seat Inventory Control**

Mathematical formulations to the network or OD seat inventory control problem vary from the simple to the complex. Although most OD seat allocation algorithms do address the basic components of fare class mix and itinerary control intrinsic with the network seat inventory allocation problem; the scope of control achieved, the ease of implementation and the resulting impacts on revenue differ significantly between each formulation.

### **2.4.1 Network Approaches to OD Control**

One approach to the OD seat allocation problem take the form of mathematical linear programming formulations. These formulations usually work under an assumption of either a deterministic or a probabilistic demand environment. The assumption of a deterministic environment, although somewhat unrealistic, implies that all demand within the environment is known with certainty. Thus, demand for each fare class is known and can be forecast precisely well in advance of the time frame over which the demand materializes. While no assumptions are made to the relative order in which passengers book in different fare classes, an assumption of independence between booking classes is made.

A basic mathematical formulation for the deterministic seat inventory control problem as a linear program formulated is shown below. The objective function is to maximize total network revenues subject to capacity

constraints and forecast demands as follows:

$$\begin{aligned}
 \text{Max} \quad & \sum_i \sum_k F_{ik} S_{ik} \\
 \text{s.t.} \quad & \sum_i \sum_k S_{ik} \leq A_l \quad \forall i \in l, \forall l \\
 & S_{ik} \leq D_{ik} \quad \forall (i,k) \text{ pairs}
 \end{aligned} \tag{2.2}$$

Each decision variable,  $S_{ik}$ , represents the optimal allocation of seats to an OD itinerary,  $i$ , and a fare class,  $k$ . The revenue contribution associated with each decision variable is the full fare value associated with that OD fare class combination,  $F_{ik}$ . Two non-trivial constraint sets are also used for this formulation of the network problem. The first set of constraints prevents the total number of seats allocated to all ODF combinations on any particular leg,  $l$ , from exceeding the seat availability or capacity of aircraft assigned to that leg. The second set of constraints prevents the number of seats allocated to each ODF from exceeding the corresponding demand forecast for that ODF.

A probabilistic formulation of the network seat inventory problem is a model that better captures the true nature of passenger demand. Using the idea of probability distributions and expected marginal revenues, which will be discussed in Chapter 3, these mathematical formulations can account for the uncertainties associated with demand forecasts. Formulation of the probabilistic linear program differs from the deterministic one in that there is a separate, binary decision variable for each seat being allocated to an ODF. This implies that the number of decision variables just increased by a factor equal to the capacity of the aircraft on each leg. Several formulations of probabilistic linear programs as applied in leg-based OD control heuristics was developed and tested by Williamson in her doctoral dissertation [6].

While both basic network approaches to the OD problem seem to be a viable method to optimal seat allocation solutions and revenue maximization, there are many practical and theoretical shortcomings in using them. Section 2.4.2 of this Chapter is devoted to detailing some of the obstacles in utilizing a network optimization approach and it highlights the need for alternative methods to full network optimization techniques.

## **2.4.2 Obstacles to Network Optimization**

At first glance, the network seat inventory control problem would seem to benefit the most from a network optimization solution. However, there are certain inherent theoretical and practical features of the problem that stand to limit the effectiveness of these types of formulations. In most cases, network formulations do not represent the "real" seat inventory problem accurately.

One of the most important data inputs to any inventory control methodology is demand. Network optimization approaches require the forecast of demand for hundreds, sometimes thousands of individual ODF combinations. In most instances the forecast for such demand on each leg involve very small, highly variable numbers that are subject to large errors. Methods to forecast such demand accurately have, to date, not been fully developed by airlines, although some airlines have begun work on an ODF database. Even so, the computer databases required to store the massive amount of historical ODF data may not be worth the effort or investment.

Other obstacles include the fact that many mathematical programming formulations assume demand to be deterministic. As stated by Williamson

[6], passenger demand is in actuality highly probabilistic. There always exists a level of uncertainty to the level of demand for a future flight, itinerary or booking class. Furthermore, demand frequently experiences systematic fluctuations due to holidays, seasonal changes and even by day of week. While the stochastic nature of demand can be modeled and represented with a statistical probability distribution, the fact remains that there is some level of unpredictability associated with future demand that is not addressed in the most basic of network optimization formulations.

Demand in the travel industry is also by nature extremely dynamic. As time passes, the demand for any one ODF can change unpredictably. The effects of these demand fluctuations can have a tremendous impact on the optimal seat allocation solutions. From any one day to the next, the total number of bookings can change for a flight, affecting not only the solution for the OD in question but its fellow dependent ODs throughout the network. The dynamic characteristic of demand cannot be attributed only to passenger behavior, but airlines' frequent schedule and price changes effect demand as well. In order to take into account the dynamic stochastic nature of demand in a network formulation, dynamic programming methods could be applied. However, the increase in decision variables and computation time may prevent the speed and efficiency required of processing seat inventories within the reservations system.

Another constraint imposed on the network approach is that the recommended seat allocations must be integral numbers. Obviously, seats cannot be allocated or sold in fractions. The implication this constraint has on the mathematical formulation is an increase in the data processing time required to develop an optimal integer solution. Consequently, the increase

in extra processing time coupled with the dynamic nature of demand could have a substantial impact on the ability of the reservations system to make revisions on seat allocations interactively.

Moreover, airlines prefer the use of a nested inventory structure to minimize the possibilities of neglecting higher revenue collecting opportunities. Unfortunately, virtually all traditional network optimization techniques generate solutions that are consistent with a partitioned inventory structure. Distinct solutions are generally not optimal seat allocations for a nested inventory structure. In fact, the use of distinct allocations to control bookings have been shown to lead to negative revenue impacts.

The use of a nested structure in a network environment in conjunction with distinct allocations serves to further complicate the use network approaches. On a single flight leg, the hierarchical order in which nesting takes place is simple, with the highest fare at the top and the lowest at the bottom. However, in a network environment, the highest fare ODF on that leg may not necessarily be the greatest contributor of revenue to the entire system. As detailed in Section 2.3, depending on the demand of local passengers over the legs traversed by the multi-leg ODF, the more desirable passenger may be the local itinerary. Thus, the question is raised as to how different ODFs sharing the same seats on any particular flight leg be ranked in a hierarchical nesting structure by their relative contributions to the network as a whole. Trying to determine optimal seat allocations for a nested structure on a single flight leg is a difficult task in itself without considering network implications.

Practicality considerations involved in solving for optimal integer seat allocations in a probabilistic, dynamic, nested environment makes it difficult to be routinely utilized in an interactive system. While the solutions obtained from a network approach is optimal, optimality does not ensure network revenue maximization especially without a proper control structure. Optimization and control must be an integrated process. It is the main objective of this research to present alternative methods of OD seat inventory control that addresses the different complexities of the OD problem, but without the mathematical rigors of network approaches

### **2.4.3 Leg-Based OD Control**

In theory, revenues cannot be truly maximized without optimizing over an entire network of connecting flight legs and its individual ODF combinations. It is important to recognize that leg-based fare class control achieves results that represent sub-optimal solutions on the network level. Furthermore, full network optimization approaches present several practical implementation problems as was discussed in Section 2.4.2. Even though the interactions of passenger flows between connecting legs are taken into account, the formulations necessary for network optimization become very large, particularly when the probabilistic nature of demand is incorporated. Airlines are seeking benefits which can be achieved today from an OD control methodology, and thus, the current interest throughout the industry is focused on incorporating network effects into the simpler leg-based control environment [7].

As the name implies, "leg-based Origin-Destination control" is a method that takes into account information about passenger demand and traffic flows while optimization and control remains at the flight leg level. In this research, heuristic approaches to leg-based OD control are developed through examination of two distinct components of seat inventory control: the control structure and the optimization method.

First of all, changes to the current leg-based fare class control structure are examined. In maximizing network revenue, different OD contributions to total network revenue must be identified and placed into equally valued control "buckets" irrespective of fare type. In fare class control, ODFs with varying revenue contributions are usually aggregated together into control buckets that are associated with the fare type of the OD on that leg. This level of control results in a distorted view of the value of different ODs on the network level. In a leg-based OD control method, a first step is to allow for the differentiation of revenue contributions to the network by aggregating similarly fare valued ODFs into the same control buckets. By using "virtual nesting" or "stratified bucketing" control structures, which will be presented in the next chapters, ODF combinations are grouped together into control buckets that are representative of their revenue contributions to the entire network. This type of aggregation will allow the optimization routines to better identify the ODFs which are more valuable to the network and to allocate/protect seats accordingly. Hence, by allowing for the differentiation of ODF contributions to the network and utilizing leg-based optimization techniques such as EMSR, an OD control algorithm is developed without the need to venture from a leg-based control structure or the use of complex network optimization techniques.

The optimization component is also another area of development in leg-based OD seat inventory control. While the control structure has allowed for one level of OD control, there is still room to further differentiate OD revenue contributions to the network through the use of displacement cost logic embedded within the optimization methods. The addition of displacement cost is motivated by the need to further account for network effects through altering the leg-based revenue values associated with each connecting ODF. Since displacement cost represents the potential lost revenue associated with displacing a local leg passenger in favor of a connecting passenger, the lost revenue potential should be reflected in the revenue contribution levels of multi-leg itineraries. By subtracting the displacement cost from the fare values of multi-leg itineraries, the optimization method will be better able to determine the revenue potential of allocating seats to a connecting versus a local itinerary. The addition of displacement cost is a key factor in being able to better differentiate the potential revenue contributions of each ODF on a network level. Moreover, it improves the ability of any seat inventory control approach to control the flow of traffic throughout the system in an effort to maximize network revenues.

While the development of leg-based network seat inventory control approaches is of current interest in the airline industry, the development of seat inventory control approaches, in general, has a solid historical basis. The next chapter will describe some of the major research developments in the area of seat inventory control that have contributed to today's level of technology.



# Chapter 3

## Review of Past Research

Prior to the early 1970's, work in the area of revenue management focused on the development of sophisticated overbooking techniques. These techniques were used to ensure that the maximum number of passengers are carried per flight. Since then, the upsurge in development of revenue or seat inventory control techniques has been driven by deregulation of the North American airline industry. Increased price competition and the resulting proliferation of discount fares have forced airline planners to determine new ways of optimally allocating seats among the various groups of fare products.

Past research on the problem of optimal seat allocations has historically tended to fall into three areas of development. First, the problem was attacked utilizing mathematical programming and/or network models. These models represent the most theoretical approaches to the seat allocation problem. While the resulting solutions are "optimal", they are usually discrete and difficult to implement (Mayer [8]; Glover et al. [9]; Wollmer [10,11,12]). The second categorization centers around the development of expected marginal revenue analysis techniques. These methods are utilized in many heuristic approaches to the problem and include many highly restrictive assumptions in the formulation. Solutions often include

assumptions of a single flight leg, independent demands, no cancellations, and lower classes book first. Even though the resulting solution is optimal within its own environment, sub-optimality usually persist from the context of the overall network problem (Littlewood [13], Bhatia and Parekh [14]; Richter [15]; Belobaba [2]; Brumelle and McGill [16], Curry [3]). Lastly, development has sought to focus around techniques that merges both the above categorizations. The desired result is to capture some level of network effects while at the same time addressing implementation issues through mating leg-based optimization techniques with certain network models. Leg-based origin-destination control techniques recognizes the need to control seats beyond a single leg environment, while yet utilizing single-leg optimization and/or mathematical programming techniques in the process (Williamson [6,17], Smith and Penn [18], Belobaba [4,5,7]). The following is an overview of some of the developmental work in these areas that has marked the pathway to current revenue control practices.

In 1972, Kenneth Littlewood [13] of BOAC proposed a seat allocation methodology that is based on the determination of the expected marginal revenue concept. The expected marginal revenue of any particular seat in question is the probability of selling the seat multiplied by the related average revenue fare value for that class of service. Recognizing that the stochastic nature of demand can be modeled as a probability distribution; the probability of selling,  $S$ , number of seats is based on the probability of having  $S$  or more requests for seats,  $r$ , in any particular class of service or  $P[r \geq S]$ . Mathematically, this probability is:

$$P[r \geq S] = \int_S^{\infty} p(r) dr \quad (3.1)$$

With the probability of selling the Sth seat known,  $P[r \geq S]$  or  $P(S)$ , the expected marginal revenue for that seat is:

$$EMR(S) = f * P(S) \quad (3.2)$$

where  $f$  is the average fare level for the related fare class.

In a simple two class, single flight leg problem, Littlewood [6] contends that revenue would be maximized, assuming that lower classes book first, if lower fare passenger bookings are accepted as long as the revenue contribution from

$$f_2 \geq EMR(S_1) = f_1 * P_1(S_1) \quad (3.3)$$

the lower fare passengers,  $f_2$ , always exceeds the expected marginal revenue from the higher fare passengers. Shown in Equation 3.3, at a point where the probability of selling all remaining seats to higher fare passengers equal the ratio of lower fare to higher fare,  $f_2/f_1$ , no additional seats will be made available to lower fare passengers.

Similar expressions to Littlewood were formulated by Bhatia and Parekh [14] of TWA in 1973 and Richter [15] of Lufthansa in 1982, although they both utilized different approaches. Bhatia and Parekh approached their formulation by equating the ratio of higher to lower fares,  $f_2/f_1$ , to an integral of the distribution of higher fare class demand,  $f_1(x_1)$ , as follows:

$$\frac{f_2}{f_1} = \int_{C-S_2}^{\infty} f_1(x_1) dx_1 \quad (3.4)$$

where  $C$  represents the capacity of the aircraft and  $S_2$  is the optimal allocation of seats to lower fare passengers. When the integral, representing the probability of higher fare seats exceeding its current allocation, equal the ratio

of fares, a condition of revenue indifference is achieved and there is an optimal allocation of seats to lower fare passengers.

Richter, on the other hand, defines his point of indifference by examining the changes in expected revenue when additional seats are offered to lower class passengers. Termed the differential revenue method, the differential revenue is defined to be the difference between the displacement cost of removing a seat from the higher classes and the revenue gain from allocating that seat to a lower class. Furthermore, under the assumption of independent demands, the differential revenue can be rigorously determined through a simple probability analysis:

$$DR = f_2 * P_2(S_2) - f_1 * P_2(S_2) * P_1(C - S_2 + 1) \quad (3.5)$$

whereby,  $P_2(S_2)$ , represents the probability of selling the seat to a lower class passenger, and  $P_1(C - S_2 + 1)$ , represents the probability of sale to a higher fare passenger. As allocation of seats to lower fare passengers increases, the value of DR approaches zero at which a situation of revenue indifference is achieved and the optimal allocation of seats to the lower fare class is determined. An important observation made by Richter is to note that the optimal allocation of seats is influenced only by the distribution of the higher yield passengers, though the lower yield demand does influence the total expected revenue of the flight.

While the above methodologies have been shown to lead to optimal allocations for single flight leg, two fare class scenarios; their extension to a multi-leg, multiple fare class environment is a non-trivial task. In particular, the inclusion of the interactions between multiple probability demand distributions provide for a difficult transition from an optimal two class to a

general multi-class solution. Belobaba [2] in 1987 proposed a solution to the multiple fare class problem on a single flight leg through what is defined as the Expected Marginal Seat Revenue (EMSR) method. In this method, the number of seats to be allocated to any lower class in question, or booking limit, is determined from the sum of seats protected for higher classes with respect to that lower class. The protection level for any higher class,  $i$ , over a lower class,  $j$ ,  $S_j^i$ , is determined through the expected marginal revenue approach discussed by Littlewood [13]. Consequently, the protection level for the highest fare class,  $\Pi_1$ , is simply,  $S_2^1$ , such that:

$$EMR(S_2^1) = f_1 * P_1(S_2^1) = f_2 \quad (3.6)$$

Furthermore the protection level for the two highest fare classes,  $\Pi_2$ , is defined as the sum of the individual protections  $S_3^1$  and  $S_3^2$ , as determined from the relationship in Equation 3.6. In general, the total protection level for the  $n-1$  highest fare class is determined by the combination of the  $n-1$  individual protection levels:

$$\Pi_{n-1} = \sum_{i=1}^{n-1} S_n^i \quad (3.7)$$

Therefore the booking limits,  $BL_i$ , for any fare class,  $i$ , are determined from the capacity of the aircraft,  $C$ , subtracted by the total number of seats protected for higher fare classes,  $\Pi_{n-1}$ , as follows:

$$BL_i = C - \Pi_{i-1} \quad (3.8)$$

While the EMSR method does obtain optimal seat allocations for any pair of fare classes taken in isolation, it disregards that fact that fare classes are nested sequentially within each other and therefore interrelated. Since the

EMSR method does not take into consideration the joint probability distribution of demand for each fare class, while it provides for easy implementation, it does not produce optimal booking limits for multiple nested fare classes.

In 1991, Belobaba [2] modified the EMSR heuristic to generate joint protection levels for higher fare classes relative to lower fare classes. This "EMSRb" methodology, utilizes an approach similar to that of the EMSR method, with the exception that probability densities of higher classes are combined to determine joint protection levels from the lower class. In the general case, for class n, the probability distribution of classes 1 to n are combined as follows:

$$\bar{X}_{1,n} = \sum_{i=1}^n \bar{X}_i \quad (3.9)$$

$$\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^n \hat{\sigma}_i^2} \quad (3.10)$$

$$f_{1,n} = \frac{\sum_{i=1}^n f_i \bar{X}_i}{\bar{X}_{1,n}} \quad (3.11)$$

Using the combined probability distribution and the average fare value for the n classes involved, the joint protection level,  $\Pi_n$ , is:

$$EMSR_{1,n}(\Pi_n) = P_{1,n}(S) * f_{1,n} = f_{n+1} \quad (3.12)$$

where the booking limit for class n+1 is determined as follows:

$$BL_{n+1} = C - \Pi_n \quad (3.13)$$

While the approach of the EMSRb method is similar to that of the EMSR method, the inclusion of joint densities have allowed simulation results from previous studies to show positive revenue gains above that of EMSR in a variety of scenarios. In addition to the positive revenue impacts, EMSRb requires 1/3 fewer computations than that of EMSR for 6 nested fare classes.

The search for optimal booking limits in a multiple nested fare class environment on a single flight leg found independent solutions from Brumelle and McGill [16], Wollmer [11] and Curry [3]. Curry addresses the problem utilizing the optimal booking limit (OBL) approach. Assuming a continuous distribution and utilizing convolution integrals, an expression for optimal booking limits is determined:

$$f_{i+1} = f_i \int_{\Pi_i - \Pi_{i-1}}^{\infty} p_i(r_i) dr_i + \int_0^{\Pi_i - \Pi_{i-1}} dr_i p_i(r_i) SL_{i-1}(\Pi_{i-2}, \Pi_i - r_i) \quad (3.14)$$

where function  $SL_i$ , represents the combined expected revenue function from  $i$  fare classes. The expression, which is similar to Littlewood's rule for two fare classes (Equation 3.3) when  $i=1$ , is recursively solved for joint protection levels for each nested fare class. The final optimal booking limits for each fare class can be determined as before by subtracting the joint protection levels from the capacity of the aircraft.

Wollmer [11], on the other hand, approached the problem by not assuming continuous demand distributions but rather focusing on known discrete demand distributions and deriving optimal booking limits through the use of a convolution sum, rather than an integral. In the paper, Wollmer follows through on a tedious formulation of the solution in addition to providing an algorithm of applying the formulation in determining the

optimal seat allocations and the optimal expected revenues. Furthermore, application of Wollmer's formulation in booking simulations produced results that, while optimal, were within 1% of Belobaba's non-optimal EMSR heuristic results under a variety of cases tested.

Although some have focused their research on optimal allocations within a multi-class environment on a single flight leg, others sought to incorporate the interactions between multiple flight legs through the control of origin-destination itineraries within their methodologies. The use of mathematical programming and network flow techniques have been the most common optimization framework for optimal solutions to OD control problems. As exemplified by Buhr [19] of Lufthansa, the optimal allocation of seats between 2 flight legs (3 airports) with one fare class was considered. In very much the same way the expected marginal revenue of an individual seat is determined in Littlewood, the expected marginal revenue for a seat in any particular OD itinerary,  $S_{OD}$ , is defined to be:

$$EMR_{OD}(S_{OD}) = f_{OD} * P_{OD}(S_{OD}) \quad (3.15)$$

In the formula,  $f_{OD}$  is the average fare of the OD itinerary and  $P_{OD}$  is the probability of selling the  $S_{OD}$ th seat. Assuming that the demand for each particular OD itinerary is independent, the total revenue for a two leg flight between three points is maximized, according to Buhr, by minimizing the following relationship:

$$\Delta EMR = |EMR_{13}(S_{13}) - [EMR_{12}(S_{12}) + EMR_{23}(S_{23})]| \quad (3.16)$$

By adjusting the values of  $S_{ij}$  in an iterative procedure described by Buhr, the optimal seat allocation or sales limits for each flight segment can be found.



Further evidence of the effectiveness of this technique can be found in Buhr's paper through the results of specific case examples performed using actual Lufthansa flight data. While no specific algorithm was described by Buhr for a multi-leg, multi-class scenario; suggestions were made as to the inclusion of such a scenario into the above methodology.

In 1983, Wang [20] at Cathay Pacific provided a first attempt at a feasible solution to the multi-class, multi-leg seat allocation problem. By extending Buhr's formulation to include a multiple class structure, Wang was able to optimally allocate seats to each OD and fare class (ODF) by maximizing the estimated marginal revenue of each seat in the ODF combination across the multi-leg flight path. In this method, each seat of the aircraft is assigned to the ODF combination that will provide the greatest expected marginal return on revenue. One by one, the seats are allocated in a greedy fashion to the highest expected marginal revenue combination across the flight path. At the point when the aircraft is filled, the process is complete. Even though the solution is an optimal one, the feasibility of such a methodology in a large network structure is highly suspect due to computational difficulties.

In formulating the ODF seat inventory problem as a large network flow model, Glover et al. [9] was able identify the optimal ODF seat allocation or passenger mix across all points in the flight path. The author proceeded with solving the problem by formulating a minimum cost/maximum profit network flow model with special side constraints. Each node on the network represented origin/destination point and were connected by two sets of arcs. The forward arcs represent the number of passengers traveling on that flight segment, while the reverse arcs represent the individual flight

itineraries/fares of the passengers. Limitations were set on the forward arc by the authorized capacity of the assigned aircraft on the flight segment, in addition to limits set on the reverse arcs by the deterministic ODF demand estimates. Subject to these constraints, the objective function of the network flow problem is solved by maximizing the flow through the arcs with the highest fare itineraries. Once again, while the solution is optimal, the resulting set of discrete seat allocations provide for implementation difficulties in a nested environment. Additionally, the dubious assumption of deterministic demand makes the model highly impractical and inapplicable in any real-world setting.

Wollmer [10, 12] of McDonnell Douglas proposed additional optimization techniques for the multi-leg, multi-class problem by modeling it as a large, yet simple, network flow problem. In the paper, Wollmer formulated models and their associated solution algorithms for a variety of scenarios. These included the single flight leg, two flight leg and hub scenarios, in addition to extending these models to a general N leg case. In the general case, the solution began with the development of a linear program utilizing binary decision variables,  $x_{ij}$ , to represent a seat on a flight leg and ODF combination. A single program constraint was set up such that the sum of seats allocated to a particular flight leg is limited to the capacity of the aircraft assigned to that leg. The resulting LP formulation is written as follows:

$$\text{Max } Z_i(n_1, n_2, \dots, n_N) = \sum m_j(k)x_{jk} \quad (3.17)$$

such that

$$\sum_{j \in B(h)} x_{jk} = n_h \text{ where } h = 1, \dots, N \quad (3.18)$$

$$0 \leq x_{jk} \leq 1, j \neq i$$

In maximizing the objective function,  $Z(n)$ , the author seeks to maximize the expected revenue if all  $n$  seats, on all respective flight legs considered, were protected for higher classes based on the marginal revenue approach, as illustrated by Littlewood. By combining mathematical programming with marginal revenue optimization techniques, Wollmer was able to produce large solution sets of optimal, although discrete (non-nested), seat allocations to multiple itineraries within the network.

From the standpoint of implementation, the network optimization techniques reviewed so far, in general, do not allow for an easily transition from current yield management systems. In addition to requiring large databases, most of the above techniques are mathematically complicated to solve and produce large solution sets of small partitioned seat allocations. As illustrated by Williamson [17] and Belobaba [7], contrary to popular belief, the use of optimal partitioned network solutions can in fact lead to decreases in total revenues. Therefore, the need for optimization approaches that incorporates not only a multiple-leg, multiple-class environment but a nested seat inventory structure has led to the development of solution techniques from D'Sylva [21], Curry [22] and Williamson [6].

Curry's [22] approach to the network level problem consists of employing mathematical programming in conjunction with the expected marginal revenue approach. Even though this combination approach is not unique, as previously shown by Wollmer, the solution does incorporate a nested fare class structure. In a methodology similar to that suggested by

Buhr [19] for a multi-leg multi-class problem, a two step approach is utilized; however, the steps are done jointly by Curry rather than in succession. First off, distinct itinerary allocations are obtained from the combined optimal expected revenue function of the nested fares classes of the OD. In conjunction with the mathematical programming solution, nested fare class allocations are determined within each OD based on Curry's OBL methodology. In other words, in this method, each OD within the network is first allocated the optimal number of seats based on the highest marginal revenue contribution. Individual fare class nesting is then performed based on the number of total seats allocated to the particular OD. While Curry addresses the problems associated with discrete seat allocations by nesting fare classes within each OD, the true benefits of CRS nesting are not reaped, as inventory allocations between ODs are not shared. This inability to share inventory between ODs make the revenue impacts of Curry's solution "sub-optimal". However, unlike previous mathematical programming approaches, advances are made towards a network wide shared inventory structure.

In her Ph.D. thesis, Williamson [6] recognized the practical problems associated with large scale network optimization. Besides implementation difficulties into today's revenue management systems, direct application of the partitioned optimal network ODF seat allocations have been shown to lead to poor revenue performance. Williamson proposed a number of leg-based approaches to solving the network seat inventory control problem, in addition to other nesting heuristics based on network solutions. In the bid price approach, once again, mathematical programming solutions are utilized in conjunction with fare class control techniques. The bid price is in actuality

the shadow price associated with the capacity constraint on a leg, and can in fact be interpreted as the network revenue contribution of the last seat on that leg. A seat request for an ODF is granted as long as the fare contribution is greater than the sum of the bid prices over all legs traversed. Williamson claims that due to the requirement of frequent re-optimization and the risk of over-selling undesirable ODFs; the use of this approach, while effective, is limited in practice.

In addition to different fare class nesting heuristics based on bid prices, fares and shadow prices of demand, Williamson identifies a myriad of leg-based OD control methodologies. "Virtual Nesting", as described by Williamson, is one of the most common methodologies associated with leg-based approaches. While virtual nesting is not an optimization technique, it was developed by American Airlines [18] as an inventory control structure for providing limited ODF availability control at the flight leg level. When used in conjunction with leg-based optimization techniques such as OBL or EMSR, significant incremental revenue gains can be achieved. Williamson provides a detailed analysis of the utilization of virtual nesting in its purest form, in addition to extensions of virtual nesting that incorporate network revenue effects into the nesting structure. Other approaches proposed by Williamson include a leg-based bid price approach, which is similar to the network approach, but without the mathematical complexities of a full network optimization. An extensive amount of simulation results using the proposed methods, in the form of multi-leg and hub scenarios, are also provided by Williamson. Since one of the major objectives of this piece of research is to further develop methods of leg-based origin and destination control, more

detailed discussion of the basic control and optimization framework will be presented in the following chapters.

A review of the current literature indicates the need for more sophisticated inventory control methods, especially when airlines continue to expand their route structures. The general consensus in today's airline industry is that Origin-Destination control will be the next step in improving current seat allocation methods. While past research have provided optimal solutions based on network optimization techniques, they have failed to examine the feasibility of implementation into current reservation control systems. In the following chapters, alternative methods for Origin-Destination control that address the issues raised by the OD seat inventory control problem will be presented.

# Chapter 4

## The OD Seat Inventory Control Structure

### 4.1 Defining the Control Structure

The purpose of seat inventory control is to determine the right mix of seats available at different fares on a flight leg in order to maximize revenue. To accomplish this task, Figure 4.1 as presented by Belobaba [7] shows that the main functional components of seat inventory control are integrated together in a cyclic process, similar to that of a large feedback loop. In this process, historical demand is updated with input from the control mechanism, new

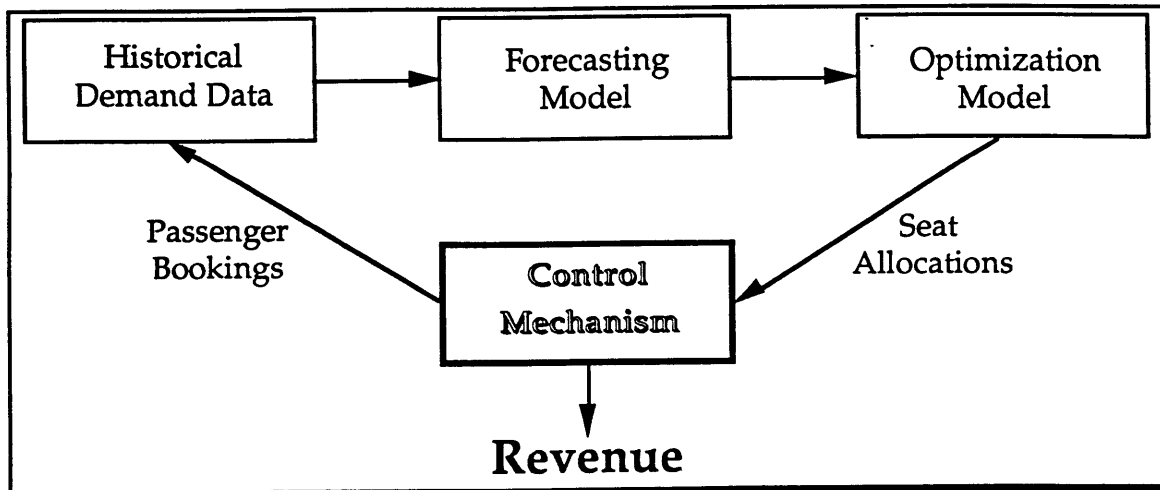


Figure 4.1 - Flow Chart of the Seat Inventory Control Process

forecasts are generated and the optimization model feeds new seat allocations back to the control mechanism. The process continues in a regulated cycle for as many as 15 to 20 times for each flight on every flight leg in the network, beginning as early as 360 days from departure.

Historical demand data is collected to aid in the forecasts of demand for future flights through the use of the forecasting model. The type of data collected is dependent on the capabilities of the CRS system and the data requirements of the forecasting model. Currently, many airlines collect demand data on each flight leg by fare class, although the capability exists to collect segment demand data by itinerary and fare type for multiple leg flights. An optimization model such as the EMSR heuristic or the OBL approach discussed in Chapter 3 then uses the demand forecast to determine "optimal" seat allocations. It is important that the optimization approach utilized by the seat inventory control method matches the inventory control structure. That is, the EMSR heuristic produces seat allocations in a form that is suitable for nested inventory structures whereas different optimization models are required for partitioned or hybrid structures [7]. The last step in the cyclic process involves the component of interest in this section of the research, the control mechanism and in particular the inventory control structure used in the development of leg-based origin-destination/network control algorithms.

The primary function of the control mechanism is to implement the seat allocation recommendations provided by the optimization method by converting them into a form compatible with the inventory control structure utilized by the airline reservations system. This implies that in a nested booking class inventory structure, the control mechanism is responsible for



converting the seat protection levels obtained from the optimization method into nested booking limits for each booking class within the inventory structure. Additionally, the control mechanism is the outlet through which the seat inventory control algorithm communicates its seat allocations to the user end of the computer reservations control system, namely the travel agent and, in turn, the air-travel consumer. In most systems, the control mechanism resides in the computer reservations system (CRS) and thus shares system capabilities that are dictated by the CRS.

The importance of the type of control structure housed by the control mechanism must not be overlooked. In terms of fare-type and traffic flow control, the inventory structure utilized will dictate the level of control that can be achieved by the inventory control algorithm. For instance, in a leg-based approach, the definition of the control buckets determines how the aggregated demand for a group of fare products is viewed by the forecasting and optimization method. Each control bucket in a particular inventory structure thus defines the level at which the forecasting of future demand is conducted. Consequently, the inventory control structure dictates the environment in which the seat inventory control algorithm resides and works.

Beginning with a discussion of leg-based fare class nesting, the next few sections will describe several inventory control structures that are utilized by airlines in the United States. While fare class nesting is the most common control structure used today, it does preclude the control of OD traffic flows. Increasingly, "stratified bucketing" and "virtual nesting" are becoming a popular alternative inventory control structure, since itinerary control can be

accomplished, while yet remaining in a leg-based control and optimization environment.

#### 4.1.1 Fare Class Nesting

Leg-based fare class nesting is an inventory control structure that does not incorporate itinerary/OD control into the seat inventory control methodology. Even so, it is important to gain an understanding of this most common form of seat inventory control to apply its lessons to more sophisticated methodologies. As previously mentioned, fare classes are used to represent groups of fare products within the control method. That is, a Y fare class would commonly represent a non-restricted full fare ticket, while a Q fare class may represent a 14 day advance purchase non-refundable discount ticket. The classifications associated with each fare product may differ among airlines and the number of fare classes may also vary, but in all cases, fare classes are representations of product types, not necessarily total fare value.

In a leg-based nested fare class seat inventory control approach, demand for different fare products are aggregated and classified by their associated fare classes irrespective of flight itinerary or fare value. Historical data is collected and forecasting is also conducted on the flight leg, fare class level. Relying on leg-based optimization techniques such as EMSR or OBL, protection levels are recommended to the control mechanism for each fare class based on a demand forecast for the fare products on each flight leg independently. In a nested environment, protection levels represent the number of seats that should be allocated exclusively to a given fare class and is not shared with lower fare classes in the hierarchy. On the flight leg level,

it has been shown that the control of bookings through a nested structure of inventory classes generate higher expected revenue than a partitioned, or distinct, structure under the same conditions. Logic also dictates that a higher valued passenger booking request should never be refused when seats originally allocated to lower valued passengers remain available -- a criterion that is met using a nested inventory structure.

The control mechanism then determines booking limits for each fare class by subtracting the number of seats protected for higher fare classes from the capacity of the aircraft or the remaining available seats left on that flight. Table 4.1 illustrates the booking limits for an example 7 class nested fare

*Table 4.1 - Example of Nested Fare Classes*

Fare Class	Fare Product	Protection Level	Booking Limit
Y	Full Fare	25	200
B	Discount One-Way	35	175
M	7 Day Non-Refund.	45	140
Q	14 Day Non-Refund.	35	95
H	21 Day Non-Refund.	15	65
K	"Sale" Fares	15	50
L	Special Promotions	0	35

structure with an aircraft capacity of 200. Booking requests are then evaluated based on the availability of seats or the booking limit of the fare class in question. On a multi-leg itinerary request, the lowest booking limit for a particular fare class over all legs traversed is used to evaluate the booking request. If seats are available, a booking request is accepted and booking limits are decremented from the appropriate fare classes.

Even though effective leg-based fare class inventory control approaches have been shown to bring incremental revenue gains over that of no inventory control, it is limited by its inability to control itinerary flows over multiple legs within a network. These limitations are predominantly a function of its control philosophy of aggregating by fare products rather than by revenue values; resulting in potentially "sub-optimal" seat allocations on the network level. As shown in Figure 2.3 of Chapter 2, the limitations of a leg-based nested fare class control structure can be especially pronounced when utilized in a multi-leg multi-fare class environment. Section 4.3 will show supporting empirical evidence that the lack of itinerary control does indeed lead to poorer results, particularly when compared to leg-based control structures that attempt to differentiate among the revenue contribution of different passenger itineraries within a hub and spoke route structure.

#### **4.1.2 Stratified Bucketing**

A problem with aggregating ODF itineraries on a flight leg into fare class buckets is that there often can be a significant amount of overlap in revenue values between fare classes due to the lack of revenue differentiation between passenger itineraries. Consider a single fare class, Y, on any one flight leg in a hub and spoke structure. Based on the ODF combinations that traverse over the leg, Y-class can easily consist of demand ranging in value from \$300 to \$800. If the demand for the lowest OD fare (\$300) is much greater than the highest OD fare (\$800), the weighted average fare for Y class may in some instances be less than the weighted average fare for the next lowest class. When the weighted average fares are not decreasing with respect to the fare class hierarchy, leg-based fare class optimizations are not very effective

since seats are no longer protected for high revenue fare classes from that of lower revenue fare classes. Recognizing the need to properly take into account the different values to the network of passenger itineraries, "stratified bucketing" was developed as a simple OD control structure that can be implemented as an extension to the concepts of current leg-based fare class control approaches.

The conceptual basis of "stratified bucketing" centers around the need to differentiate between passenger itinerary revenue contributions to the network within a leg-based control environment. Stratified bucketing accomplishes this first by abandoning the concept of equating booking class to fare product and redefines each booking class to manage bookings by specific value to the network, regardless of the fare type or itinerary. In this case, the value of an itinerary is defined by its total itinerary ticket fare value. Through the redefinition of fare classes to represent a specific range of revenue values, demand is aggregated into control buckets based on its revenue contribution to the network. Specific booking requests are then evaluated by the network solely on the basis of its revenue contribution to the network.

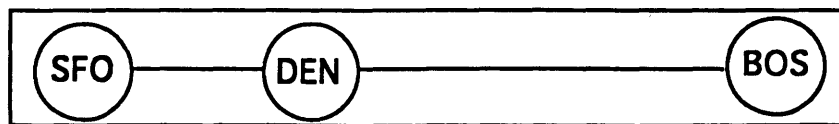
In order to convert a fare class structure into a stratified control structure, two basic changes to the control concept are made. First the fare classes have to be redefined to represent a range of revenue values. Even though the class letter classifications are still maintained, the control buckets no longer represent fare products, as illustrated in Table 4.2 for a 7 booking class structure. Secondly, the different OD fares would have to be re-filed into the new control buckets by the reservations system. This would entail sending

Table 4.2 - Example of Stratified Bucket Definitions

Letter Classification	Stratified Bucket Revenue Ranges
Y	\$650 +
B	\$550 - \$649
M	\$450 - \$549
Q	\$350 - \$449
H	\$250 - \$349
K	\$150 - \$249
L	\$0 - \$149

the fare changes through the Air Tariff Publishing Company (ATPCO) and actually displaying fare products in control buckets that are not associated with their traditional fare class definitions. That is, on certain travel itineraries, the highest priced unrestricted fare would not be a Y class fare, but in a stratified structure may show up as an H class fare. This departure from traditional fare class definitions may pose as a initial confusion factor to users unfamiliar with a stratified control structure. However, with a little training, much of the cost associated with the confusion factor can be minimized. In Example 4.1, an illustration of converting from a fare class to a stratified bucketing control structure for a multiple-leg flight is made.

Note that through a rather simplistic change in control philosophy, a form of minimal OD control has been established within an existing leg-based inventory control algorithm without the need to venture into network



Multi-Leg Flight from San Francisco through Denver to Boston

### Conventional Fare Class Bucketing

	Definition	Short Haul SFO - DEN	Long Haul DEN - BOS	Connection SFO - BOS
Y	Full Fare	\$467	\$648	\$724
B	Discount One-Way	\$259	\$440	\$467
M	7 Day Non-Refund.	\$204	\$324	\$357
Q	14 Day Non-Refund.	\$184	\$302	\$269
H	21 Day Non-Refund.	\$164	\$257	\$251
K	"Sale" Fares	\$140	\$179	\$199
L	Special Promotions	\$110	\$149	\$179

### Stratified Bucketing

	Revenue Range	Short Haul	Long Haul	Connection
Y	\$650 +			Full-Fare
B	\$550 - \$649		Full-Fare	
M	\$450 - \$549	Full-Fare		One-Way
Q	\$350 - \$449		One-Way	7 Day
H	\$250 - \$349	One-Way	7/14/21 Day	14/21 Day
K	\$150 - \$249	7/14 /21 Day	Sale	Sale
L	\$0 - \$149	Sale/Special	Special	Special

*Example 4.1 - Conversion from Conventional Fare Class Bucketing to Stratified Bucketing.*

optimization techniques. Not only will this approach improve operational performance network wide, as will be presented in Section 4.3, stratified bucketing also provides substantial incremental revenue gains when compared to traditional leg-based fare class seat inventory control methods.

Brunger [23] of Continental Airlines, who coined the term "stratified bucketing", stressed that stratified bucketing is a control structure that can

bring about incremental gains in revenue for airlines that fit specific demand profiles. He additionally pointed out that the financial investment required would be minimal and that no structural changes are needed to implement this method into airlines that already utilize a leg-based fare class seat inventory control approach. The bottom line is that stratified bucketing can provide a level of itinerary control to current fare class control methods and is suitable for airlines that lack the resources to develop an entirely new OD seat inventory control algorithm.

### **4.1.3 Virtual Nesting**

Similar to stratified bucketing, "virtual nesting" establishes a control structure hierarchy that is based on some measure of network contribution or value associated with each specific OD fare combination. However, unlike stratified bucketing, virtual nesting is not confined by the control structure previously configured for conventional fare class control methods; i.e. 7 booking classes equals 7 stratified control buckets. As its name implies, the users of a virtual nesting control structure will be hidden from any changes to the seat inventory control method as far as booking class definitions and fares are concerned. That is a Y fare would still be defined as a full unrestricted fare in a virtual structure for all ODF markets, alleviating the initial confusion factor found in stratified bucketing. Even so, in many respects, stratified bucketing is considered to be a limited form of virtual nesting.

As previously discussed in Chapter 2, American Airlines [18] developed the virtual nesting concept as a means of achieving some level of control over passenger itineraries without resorting to complex network



optimization techniques. Each virtual control bucket on a flight leg represents a range of actual fares or revenue contributions, thereby allowing ODFs with similar revenue values to be aggregated together for optimization and control purposes. As in stratified bucketing, ODFs are aggregated into their associated virtual control buckets based on their actual itinerary fares. Once each ODF is "mapped" to a virtual control bucket, demand forecasts generation and optimization of seat allocations take place within each virtual class on each flight leg, in a process similar to conventional leg-based fare class inventory control algorithms. Using leg-based optimizations techniques such as EMSR nested booking class heuristic or the OBL algorithm, nested booking limits can be determined for each virtual class on a flight leg departure. Thus when a booking request is made by the user, the passenger OD itinerary "points" to its related virtual class table and bookings are accepted or denied based on the availability of the virtual class to which the ODF is mapped.

The virtual nesting approach is illustrated in Example 4.2 using the two flight leg network used in Example 4.1. In this example, fares for the San Francisco through Denver to Boston flight are listed in the OD market revenue tables. However, control and optimization actually takes place at the level of the virtual buckets to which each ODF is mapped. Consequently, each virtual inventory bucket is defined over a range of revenue values or fares, with virtual bucket V1 defined over the revenue range of \$700 and up,

OD Market Revenue Tables

<u>SFO - DEN</u>	
Class	Fare
Y	\$467
B	\$259
M	\$204
Q	\$184

<u>DEN - BOS</u>	
Class	Fare
Y	\$648
B	\$440
M	\$324
Q	\$302

<u>SFO - BOS</u>	
Class	Fare
Y	\$724
B	\$467
M	\$357
Q	\$269

Network Virtual Mapping Table

Virtual Class	Virtual Range	Mapping of ODFs
V1	700 +	SFOBOS Y
V2	550 - 699	DENBOS Y
V3	450 - 549	SFODEN Y/SFOBOS B
V4	400 - 449	DENBOS B
V5	350 - 399	SFOBOS M
V6	300 - 349	DENBOS M/DENBOS Q
V7	260 - 299	SFOBOS Q
V8	200 - 259	SFODEN B/SFODEN M
V9	150 - 199	SFODEN Q
V10	0 - 149	

*Example 4.2 - Mapping Two-Leg ODFs into Their Respective Virtual Buckets as Defined by a Revenue Range*

virtual bucket V2 over the range \$550 to \$699 and so on. Based on the total ticket value of the fares, for a Y class (full fare, refundable) ticket on the SFO to BOS OD valued at \$724; it is mapped into virtual bucket V1 and resides in this bucket for purposes of forecasting, optimization and availability. Note that in this case, the Y class demand for the SFO to DEN leg is mapped into a lower control bucket, whereas in a leg-based fare class control structure all Y class demand would be aggregated into the same control bucket. This process

continues in the same manner for all other OD fare combinations in the network.

Since similarly valued ODFs are mapped into their respective virtual buckets, the problem associated with overlapping average weighted fares is now eliminated. However, a problem associated with utilizing the full itinerary fares as a basis for mapping ODF itineraries on each flight leg into virtual buckets is that when combined with a leg-based optimization techniques such as EMSR or OBL the resulting seat inventory control algorithm tends to be "greedy"; with priority given to long haul, higher revenue itineraries over short haul, lower revenue itineraries. Depending on the demand level of the ODs, always giving priority to long haul passengers can result in negative revenue impacts. This is true in cases where long haul ODFs may displace combinations of short haul or local traffic which have a greater combined total revenue value contribution to the network. While the "greedy" approach still generates positive revenue impacts, the addition of displacement costs to the optimization model is a method proposed to alleviate the greediness effect of pure virtual nesting. In this research, leg-based heuristic methods to network inventory control that take account of displacement costs are examined in Chapter 5.

In addition to issues concerning the greediness of pure virtual nesting, several specific characteristics of a virtual nesting control structure are also at issue and are detailed in Section 4.3. These characteristics include the methods in which revenue ranges are determined for each virtual bucket; the number of virtual buckets to used; and the system level of the virtual mappings tables -- should they be created on a network level or on a flight leg

level. Simulation results will be used to illustrate the role these parameters play in the performance of a virtual nesting inventory control algorithm.

## **4.2 Optimization/Booking Process Simulation**

In this research, a number of new OD control algorithms are developed and tested. One of the most effective ways to estimate the revenue impacts of these control processes is to simulate the booking process of an airline and the way in which the control methodology will affect the acceptance and/or rejection of booking requests. A simulation is a mathematical representation of the way a process functions in a controlled environment. In the case of seat inventory control design, a simulation will set the stage in which to evaluate the effectiveness of the different inventory control approaches.

A major motivation behind development of new seat inventory control methods is to capture incremental revenue, as such, one of the most effective measures of initial performance is to determine the increase in total revenue a control process will provide an airline. In particular, the results obtained from the simulation will help to quantify the expected revenue gain relative to that of current practices in seat inventory control. While the exact level of incremental revenue obtained is difficult to pinpoint due to the inability of the simulation to accurately represent all facets of the "real world" environment, it is still important to estimate these measures as a basis for judging the effectiveness of the seat inventory control method and to help justify the large investment required in redesigning inventory structures and reservations control systems.

A major advantage of using a simulation in this manner is that an environment in which factors that influence the amount of revenue collected such as pricing strategies, the economy and reservation control policies can be controlled and their impacts separated from the inventory control methodology. Additionally, identical demand patterns can be generated for each inventory control process evaluated, aiding in the accuracy of comparing different methods. At the same time, a methodology can also be tested over a range of demand scenarios in order to evaluate the robustness and effectiveness of the control structure under varying demand patterns. In effect, the different demand patterns could be interpreted as representations of time of day, time of week and even different flight departure demand variations. Consequently, a simulation provides an environment which is impossible to emulate in real-time experimentation, that will provide a good estimation of the relative levels of revenue and the potential benefits that can be obtained from a newly developed seat inventory control method, without a large investment in risk and costs.

The simulation routine used in the evaluation process for this research was initially developed by Williamson [6] at the Massachusetts Institute of Technology, Flight Transportation Laboratory, and since has been expanded to incorporate the algorithms formulated for this research. An integrated optimization and booking routine that simulates a real-time booking process for a set of interrelated flight departures was programmed in conjunction with assumed airline reservations control practices to estimate the performance of different inventory control algorithms. An actual connecting hub-and-spoke network for a major airline was modeled as the operating environment in which to test the different inventory control approaches.

This connecting bank consists of 15 flight legs into the hub and 15 flight legs out for a total of 30 flight legs. A variety of flight legs are represented from long-haul international routes to short haul domestic routes, providing a good mix of ODF traffic across the network. Demand on each leg was also varied with demand factors ranging in value from 0.42 to 1.17 on the inbound legs and from 0.48 to 1.31 on the outbound legs, creating a relatively balanced network. Of the 240 possible OD combinations, actual airline demand data was collected for 197 pairs. With 7 different fare classes offered for each OD market, there are a total of 1379 different OD fare combinations possible. The variety of OD markets used in addition to varying levels of demand and capacities on each flight represent a highly realistic operating environment in which to evaluate the network inventory control algorithms.

In addition to the setup of the network operating environment, other inputs required by the simulation include the aircraft or cabin capacity on each flight leg, the fare products offered in each OD market, the number of booking iterations and the fares and incremental demand densities for each booking period between revision points. To reflect the dynamic nature of the actual booking process, scheduled revision points are incorporated into the booking/optimization simulation at which seat allocations and booking limits can be updated. Using the inputted incremental demand densities, booking requests are generated in between revision points thus allowing for adjustments to be made to seat availability at each revision point on the basis of current bookings on hand and bookings to come. The number of revision points (frequency) is determined by the airline and rests on the availability of incremental booking information; a constraint of the airline's forecasting

capabilities. In this research, 10 revision points are utilized, resulting in 10 booking periods before each flight departure.

Once all the necessary input data is collected, the simulation runs in a fashion similar to that of a typical seat inventory control approach illustrated in Figure 4.1. Based on demand forecasts of future flight departures, booking limits are first calculated using a specific optimization technique. Specifically, the EMSRb heuristic developed by Belobaba is used as the base optimization technique for all simulation runs in this research. Deriving a probability distribution from the mean and standard deviation of the demand forecast for the booking period, demand for each ODF is randomly generated in a poisson process. The types of booking requests generated are influenced by the historical booking curves of each fare product incorporated into the simulation's incremental demand data. These booking curves are highly representative of passenger booking behavior for a flight departure. For example, demand for lower fare types tends to be higher at the earlier stages of the booking process versus demand levels for higher fares towards the end of the booking process. In any event, given that seats are available, demand is accepted and booked one request at a time, followed by the decrementing of the appropriate booking limits. At the point where all demand is satisfied relative to seat availability for the current booking period, the booking limits are re-optimized and the booking process repeats itself for subsequent booking periods.

To obtain a statistically significant sample of results, the entire booking and revision process for a single network of departures is repeated 100 times. It is important to keep in mind that the intent of this simulation is not to accurately model every aspect of the "real-world" environment, instead it is

to provide a realistic yet sterile environment to compare the relative effectiveness of one control method over another under identical circumstances. For a more detailed discussion of the simulation and some of the modeling assumptions made, refer to Williamson's [6] doctoral dissertation.

### **4.3 Simulation Results of Different OD Control Structures**

As mentioned in the previous section, the primary objective in developing new methods for controlling seats on a flight leg is the potential for increasing revenue. In this section, the control structures described in the previous sections will be teamed with a leg-based optimization technique (EMSRb heuristic) and subjected to the optimization/booking simulator under a variety of demand scenarios within the hub-and-spoke environment. In this manner, the revenue impacts of the different inventory control approaches can be compared to each other and in particular to current leg-based fare class control methods prevalent in inventory control approaches used today.

In all cases examined, five different demand scenarios are used, differentiated on the basis of average percentage of local OD traffic on each leg across the network. Local OD traffic is defined as passengers who travel over only one flight leg of the network from origin to destination, whereas connecting passengers are those who travel on two or more flight legs in the network. In these scenarios, average local demand ranges from 26% to 47% over the entire network. Additionally, within each local demand scenario, global demand adjusters were also used to vary the demand factor (i.e. ratio of



demand to capacity). In the base case of 30.5% average local traffic demand, which corresponds to the actual demand data and the historical scheduled flight capacities obtained for this simulation from an actual airline hub, the demand factors were adjusted to represent a range from 0.66 to 1.42. In other scenarios, such as the one with an average local demand of 47%, the average leg demand factor may range from 0.81 to 1.73, representing a much more heavily loaded network. By comparing the revenue impacts across different local demand patterns, in conjunction with the varying demand factors within each scenario, a good representation of the capabilities of the tested control methods can be analyzed in relation to its ability to generate incremental revenue.

In the analysis of the simulation results, a leg-based fare class control structure that utilizes an EMSRb optimization heuristic is set as the base case. As mentioned, the utility of a new OD control method is dependent on its ability to produce revenue gains above that of current seat inventory control approaches. The leg-based fare class control algorithm is recognized as a standard seat inventory control method that is currently being used by many airlines worldwide. Therefore, the fare class approach is used as a standard to judge the capabilities of newly developed OD control algorithms.

Applying the stratified bucketing inventory control approach to the simulated hub-and-spoke network environment, the booking simulation yielded positive revenue impacts across all demand scenarios. Figure 4.2 illustrates some of the demand scenarios tested. Assuming that the base inventory control approach is that of a leg-based fare class control structure utilizing an EMSRb optimization heuristic, as much as 7.75% can be realized

at an extremely high demand factor of 1.73, for an average local demand of 47%. More realistically, depending on the average local demand

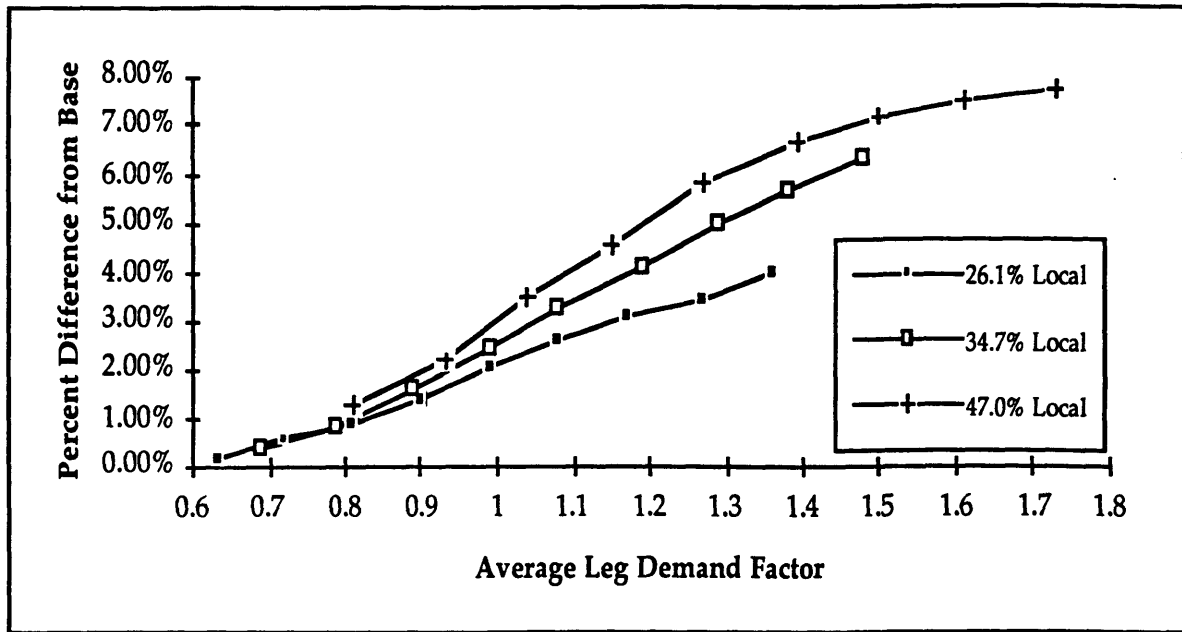


Figure 4.2 - Revenue Impacts of the Stratified Bucketing Inventory Control Algorithm

in the network, a revenue impact of 1% to 4% can be obtained corresponding to an average leg demand factor of 0.70 to 1.10.

Further analysis of the results reveals a clear relationship between the average percentages of local traffic traversing across the network and the marginal revenue impacts achieved. At lower demand factors of 0.65 to 1.0, revenue impacts in comparison to their respective base cases differed by only slight margins of less than 1%, while larger differences begin to emerge at demand factors greater than 1.0. The simulation results reveal that revenue impacts differ as much as 3% between local demand scenarios and 6% from the base case at a demand factor of 1.2. Additionally, the rate at which incremental revenue growth occurs also differ significantly among the different scenarios. Examining the slopes of the curves in Figure 4.2, note

that revenue growth begin with similar levels at the lower demand factors, however, as the demand factor grew, scenarios with higher average

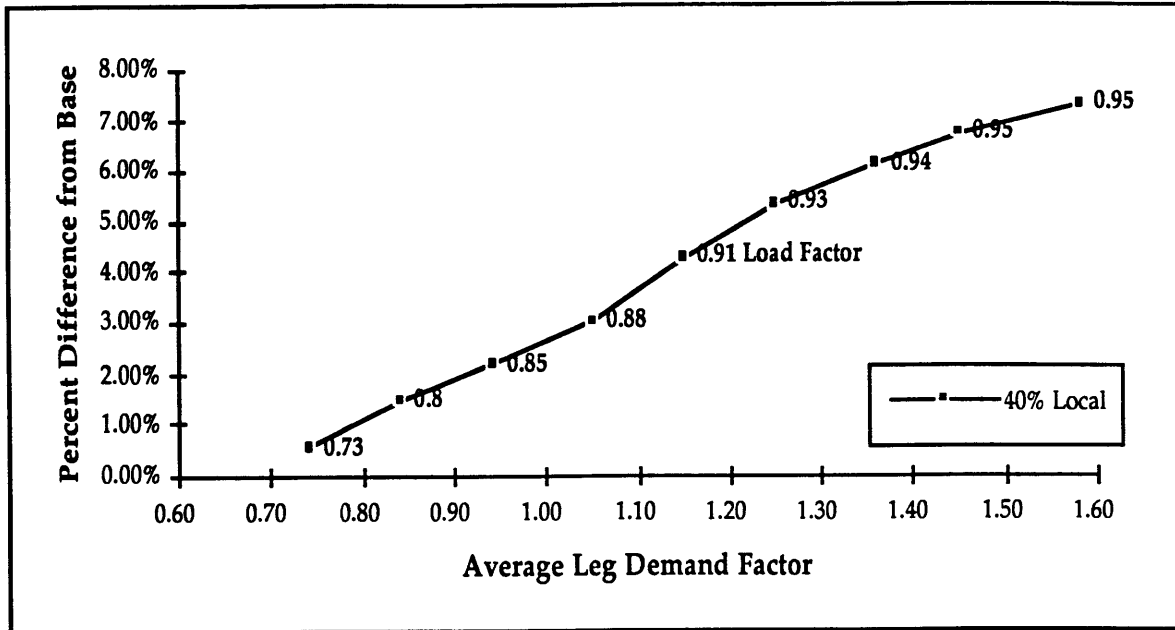


Figure 4.3 - Average Hub Load Factors Achieved by a Stratified Bucketing Control Algorithm

local demands tends to achieve a higher growth rate. Although, as exemplified by the 47% local demand scenario, there does exist a level at which incremental revenue growth begins to flatten out, beginning around a demand factor of 1.40. Realistically, most airlines would be more concerned about the revenue impacts of their control methods at demand factors ranging from 0.7 to 1.1.

Figure 4.3 continues illustrating the revenue impacts of a stratified bucketing inventory control algorithm in addition to the average network load factors achieved at each demand factor. The demand factor is defined to be the ratio of demand to capacity. To obtain an average demand factor for a

hub-and-spoke network, the following formulation was used:

$$\text{Average DF} = \frac{\sum_{ODF} (Dem_{odf} \times Distance_{odf})}{ASM} = \frac{RPM \text{ Demand}}{ASM} \quad (4.1)$$

In equation 4.1, the demand factor for the network is determined by dividing the total revenue passenger miles of demand (RPM Demand) by available seat miles (ASM). RPM Demand is found by summing the product of each forecasted ODF demand by the distance of the OD itinerary. On the other hand, the load factor is defined to be the ratio of passengers carried (load) to capacity. Once again, in the case of determining an average network-wide load factor, the following formulation based on a weighted average of passengers carried was used:

$$\text{Average LF} = \frac{\sum_{ODF} (Pax_{odf} \times Distance_{odf})}{ASM} = \frac{RPM}{ASM} \quad (4.2)$$

In this case, RPM is determined by summing the product of actual passengers carried by the distance of the OD itinerary. In every case, as illustrated in Figure 4.3, the demand factor exceeds the average network load factor indicating that there is some level of unsatisfied demand left in the network. Ideally, average network load factors should be at 1.0 implying that every seat available is filled, although in reality, average network load factors achieved today is currently around 0.63 in the U.S.

Implementing a stratified bucketing control structure has been empirically shown to provide significant revenue impacts ranging from 1% to 4% for realistic demand scenarios with hub load factors ranging from 0.63 to 0.93. These revenue gains are a result of redefining the control buckets

used in the inventory control structure with no other changes made to either the optimization technique or to the seat inventory control process shown in Figure 4.1. In many respects, stratified bucketing is a form of virtual nesting utilizing control buckets that are defined by revenue ranges rather than fare types. As previously stated, the application of stratified or virtual nesting control structures without accounting for displacement costs usually results in "greedy" inventory control algorithms whereby long-haul higher revenue itineraries are always given higher priority over short-haul or local itineraries. The following paragraphs will describe the revenue impacts achieved by utilizing a virtual nesting control structure in a "greedy" seat inventory control algorithm.

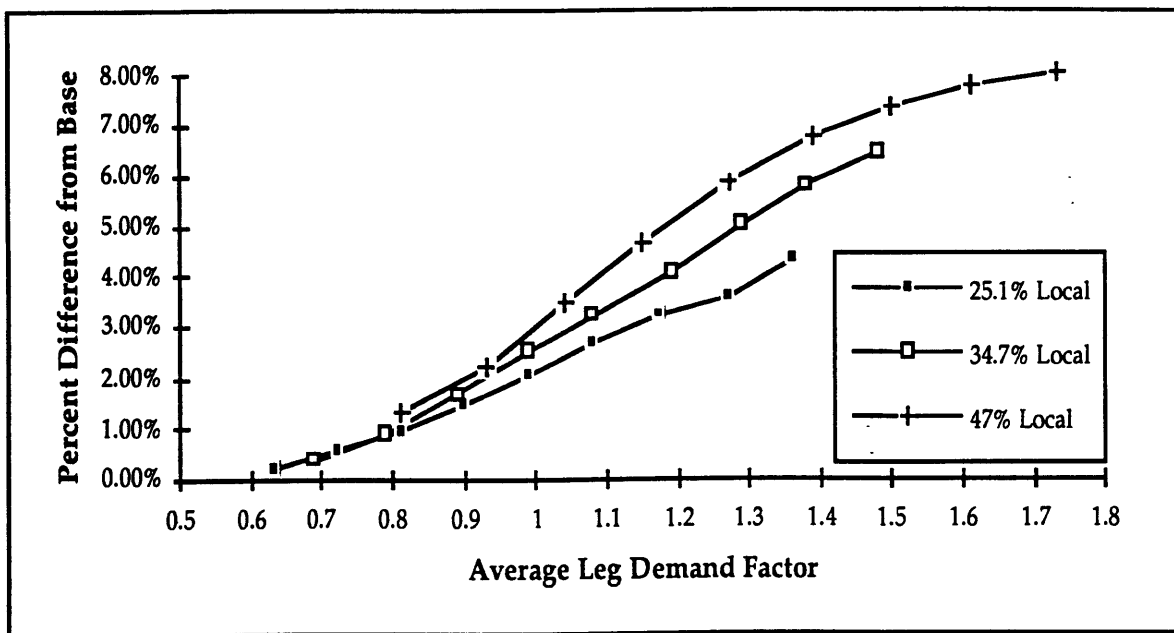


Figure 4.4 - Revenue Impacts of the "Greedy" Network Demand Virtual Nesting Algorithm

In testing the "greedy" approach in the simulation, revenue impacts as shown in Figure 4.4 were recorded. Depending on the average demand factor of the network and the average proportion of local traffic, revenue impacts range anywhere from 0.2% to 8% above that of leg-based fare class control

approaches. Take, for example, a scenario with an average local demand of 40% on the network, at realistic demand factors of 0.70 to 1.10, the incremental revenue impacts obtained range from 0.6% to 4% with average network load factors ranging from 0.72 to 0.92.

Up to this point, the "greedy" algorithm utilized a network-wide or network virtual mapping table for the purposes of ODF revenue differentiation and control. In the context of the simulation, it means that a single virtual table with its virtual buckets defined over a range of predetermined revenue values are used to map every ODF over the entire hub network into their appropriate control buckets. Thus if a BOS-SFO Y class ODF was mapped into virtual bucket V1, it remains in bucket V1 over all legs traversed in the network. While there are no recognizable negative impacts from using a network-wide virtual table, the possibility exists to take ODF revenue differentiation one step further with the development of flight leg specific virtual tables.

In this approach, each flight leg will have a virtual mapping table specific to only that leg, whereby all ODFs that traverse the leg will be mapped to a virtual control bucket as defined by leg specific revenue ranges. In the case of some multi-leg OD itineraries, an ODF combination may reside in virtual bucket V1 on the first flight leg, but bucket V2 on the second flight leg. Table 4.3 shows that the extra segmentation and differentiation of revenue values is important in determining which itinerary combinations are of greater value to the network, in turn resulting in higher revenue impacts.

In Table 4.3, leg-based virtual tables were incorporated into the "greedy" algorithm's virtual nesting control structure. Depending on the

demand profiles, the revenue impacts are significant and range from 0.2% to 8.23% above that of the base case. On average, leg specific virtual range definitions provided an additional 0.12% of incremental revenue above that of network virtual tables. Although a statistically significant increase in incremental revenue was recorded, the value of switching to leg-specific virtual tables must be weighted against the increase investment required in storing and processing the additional information.

*Table 4.3 - Percentage Revenue Impacts of a Network versus a Leg Specific Virtual Table*

Avg. Local Dem.	Virt. Table	<u>Average Demand Factor</u>								
		0.63	0.72	0.81	0.90	0.99	1.08	1.17	1.27	1.36
27%	Network	0.18	0.60	0.96	1.47	2.08	2.67	3.23	3.63	4.30
	Leg	0.18	0.61	0.97	1.52	2.19	2.80	3.28	3.62	4.52

Avg. Local Dem.	Virt. Table	<u>Average Demand Factor</u>								
		0.81	0.93	1.04	1.15	1.27	1.39	1.50	1.61	1.73
47%	Network	1.31	2.24	3.53	4.69	5.88	6.78	7.37	7.80	8.06
	Leg	1.33	2.29	3.61	4.75	5.97	6.90	7.57	8.06	8.23

The development of leg-specific virtual tables is a logical means to prioritize the ODF combinations at a leg level and to add an additional level of flow control into the process. Since traffic mix and fares vary significantly by leg, it seems that by redefining virtual ranges for each leg, ODFs considered more valuable to the network will be better differentiated and accordingly prioritized on each leg. For example, multi-leg ODFs are assigned booking limits equal to the minimum booking limit over all legs traversed. Using a network virtual table, a BOS-DFW-SFO itinerary is assigned to virtual bucket V2 and remains in that same bucket over all legs. However, in a leg based

virtual table, each leg is better able to prioritize the ODFs traversing the leg through development of its own virtual table. Thus, if there is greater demand for higher fares on the DFW-SFO leg, the ODF may be assigned to V3 on that leg and V2 on the BOS-DFW leg where the ODF is perceived to be more valuable. This extra level of differentiation may account for the slight improvement in revenue performance over a network-wide virtual mapping table.

In developing a virtual table, it is also important to examine the methods by which virtual bucket revenue ranges are defined. In this research, two heuristic forms of defining virtual ranges were examined. The first heuristic entails equal assignment of forecasted demand over all virtual buckets. Simply, total demand in the network or on a leg is divided equally into the number of virtual control buckets used in a hierarchical order. By ranking all demand in the network in a hierarchical fashion based on its revenue contribution (fare), virtual ranges are defined by the highest fare and the lowest fares represented in the equal demand slice assigned to the virtual control bucket.

The second heuristic is a value based or revenue based approach which defines virtual ranges on the basis of equalizing total expected revenue contributions from each virtual control bucket. The expected total revenue to be collected in the network is first determined and divided by the total number of virtual control buckets. Demand is then allocated to each control bucket in a hierarchical fashion until the expected revenue contribution by the bucket equals the revenue level predetermined in the first step. The virtual ranges are then defined by the highest and lowest fare level represented in each virtual bucket.



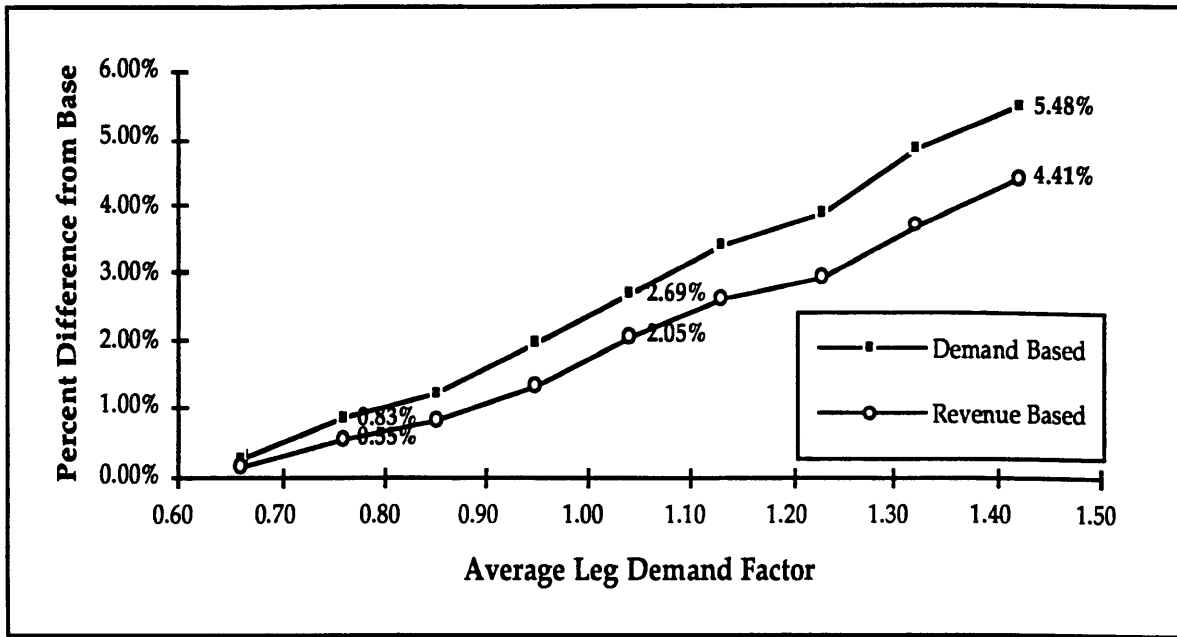


Figure 4.5 - Network Demand Based versus Network Revenue Based Virtual Range Definition Heuristic

Figure 4.5 shows the results of a simulation run based on an average local demand of 30.5% utilizing a network-wide virtual mapping table and Figure 4.6 illustrates results from a leg-based virtual mapping table. It is evident in both cases that a demand based virtual range definition approach performs better in comparison to a value based definition. This result is true of all five scenarios tested, over all demand factors. On average, the demand based approach produced revenue impacts anywhere from 0.3% to 1.5% above that of a revenue based approach when compared to leg-based fare class control.

The consistency of the results strongly supports the notion that a higher level of inventory control is achieved when equally dividing the inventory units, in this case forecasted demand, into each control bucket. A revenue based approach tends to assign a greater amount of demand to the

lower virtual control buckets since each unit of demand contributes a lower amount of revenue to the network than those from higher buckets. This "bottom-heavy" assignment may result in a greater number of seats being

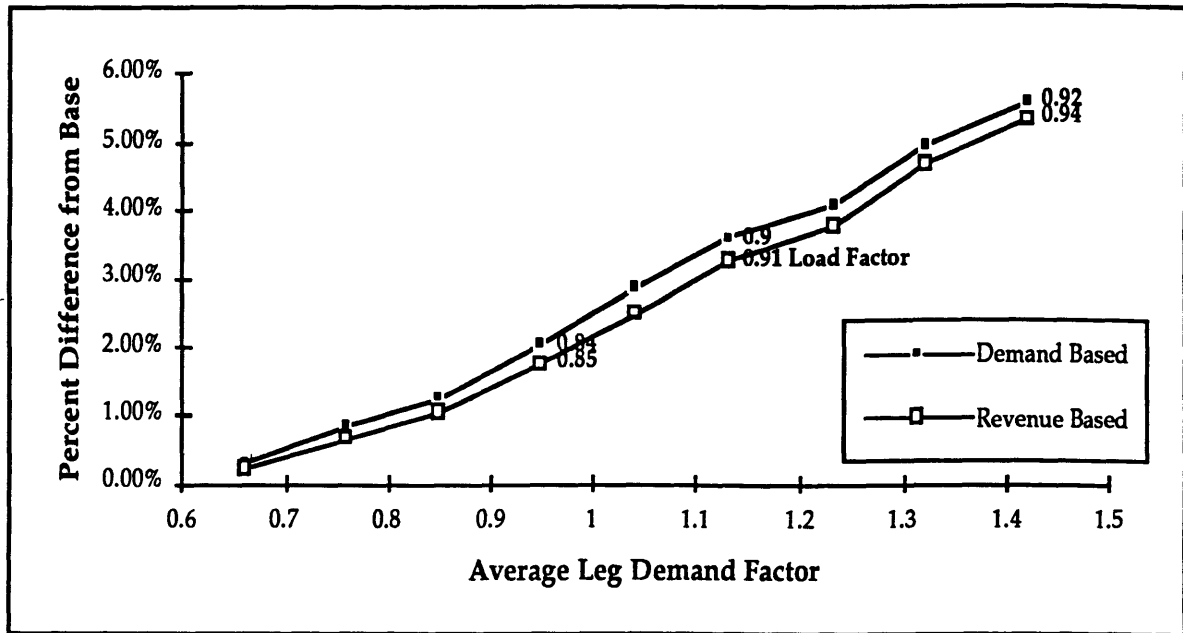


Figure 4.6 - Leg Demand Based versus Leg Revenue Based Virtual Range Definition Heuristic

protected for the lower virtual buckets, allowing for higher booking limits. In fact, in most scenarios examined, the network load factors of a revenue based approach were 1 to 2 percentage points higher than a demand based approach indicating that there is less control over seat allocations to lower fare products thus resulting in more demand being accepted. Even so, while a revenue based approach may accept more passenger bookings, the types of bookings accepted are obviously contributing a lesser total amount of revenue to the network.

In general, a major advantage of having an well defined virtual table, whether it is leg specific or network-wide, is the potential to have mapping tables that are sensitive to a whole range of factors that cause demand

variations such as seasonality, schedule variations, pricing schemes, etc. Although, in all the aforementioned simulation results, all virtual tables have been formed on an unrealistic level. That is, all virtual tables, whether on a leg or network level, or demand or revenue based, have been formed with "perfect" knowledge of the demand forecast which makeup each demand factor and average local demand scenario. In the 'real-world' environment, this is not always the case. Therefore, it is important to examine the robustness of virtual tables and in general the virtual nesting control structure in its response to varying levels of demand not coincident with the demand forecast used in its inception.

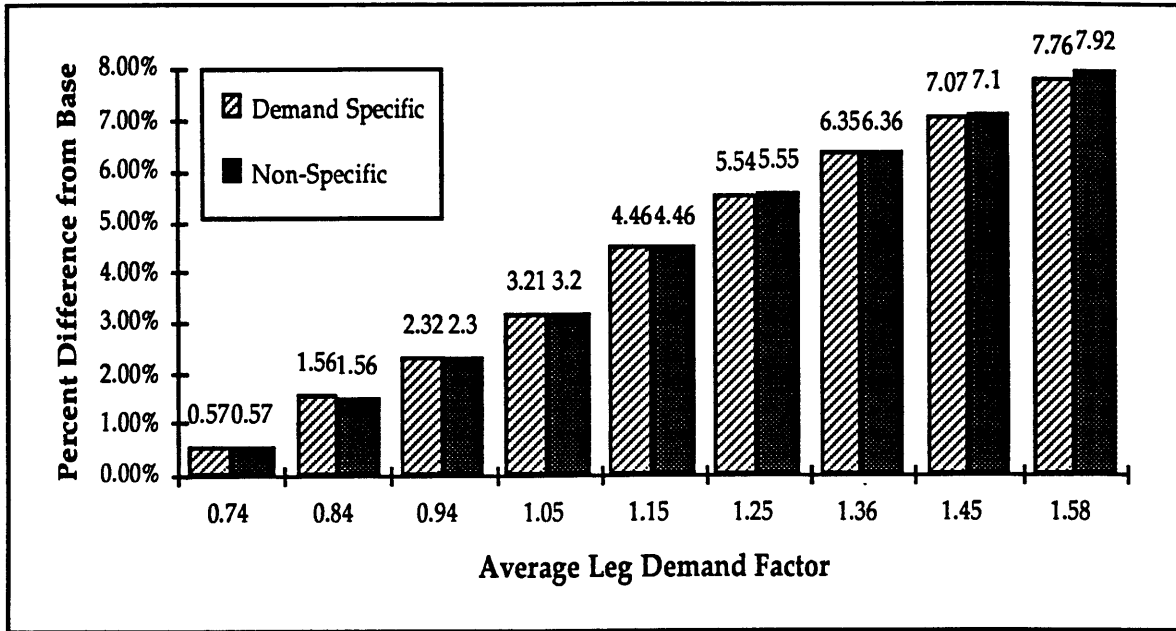


Figure 4.7 - Evaluation of Robustness of Virtual Nesting Control Structure

To test the robustness of virtual tables, in particular the virtual range definitions, two sets of virtual tables were constructed and tested. One table's virtual range definitions were formed on flight departure specific demand forecast, implying that the virtual table is fully aware of the demand characteristics of the set of flight departures. Non-flight specific demand

information was used to form the other table. That is a "generic" demand forecast was used to developed the virtual range definitions that does not necessary correspond to a specific flight departure. In "real-world" terms, this test will determine the revenue impacts of developing virtual tables that account for the seasonality, time of day variations in demand for a set of flight departures versus one that does not.

Figure 4.7 shows results of a simulation run based on a average local demand scenario of 40%. In the figure, a comparison is made between virtual mapping tables that have been formed with flight departure specific demand forecasts and mapping tables that are formed without the use these demand forecasts. From these results, it is empirically shown that the virtual nesting control structure is extremely tolerant of variations in demand. In the realistic demand factor ranges of 0.7 to 1.1 there is almost no significant difference between the revenue impacts realized by both cases. Simulation runs of other scenarios also result in the lack of significant differences in revenue impacts between the two cases. These results suggest that "perfect" virtual tables and therefore "perfect" virtual revenue range definitions are not necessary to realize the revenue impacts of seat inventory control methods that use a virtual nesting control structure. In any case, even without developing "perfect" virtual tables, flight-leg specific virtual tables have been shown to provide for the highest revenue impact when it comes down to developing a "greedy" OD inventory control algorithm.

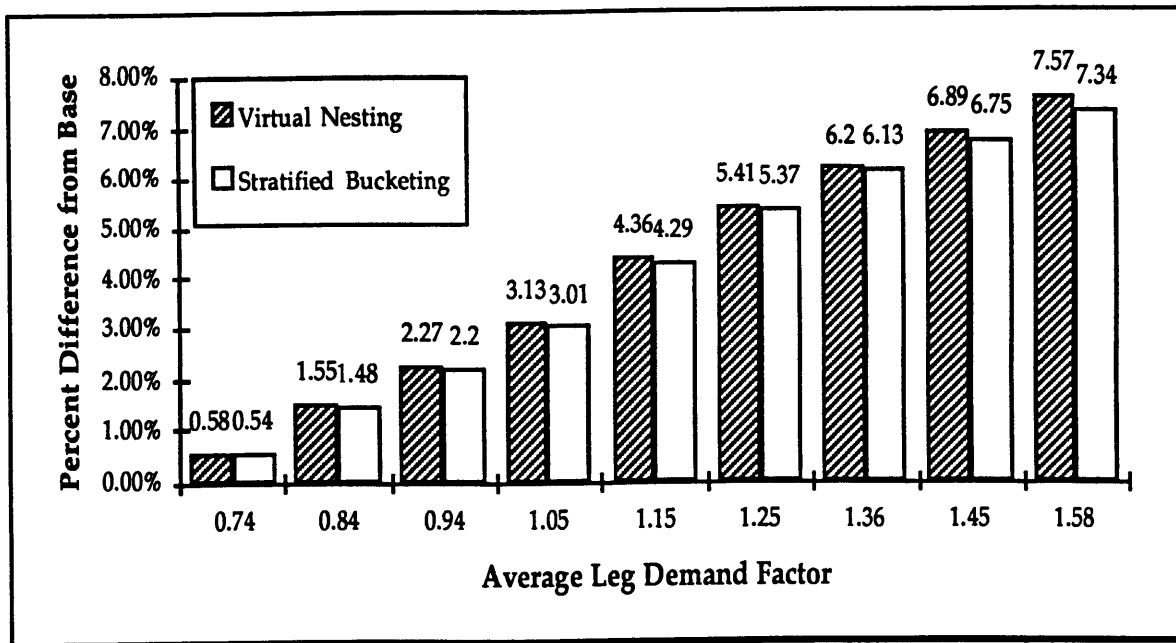


Figure 4.8 - Revenue Impacts of a Stratified Bucketing versus a Network Virtual Nesting Control Algorithm

In comparing the revenue impacts between a virtual nesting control algorithm and a stratified bucketing control algorithm, it is important to state that in many respects, the stratified bucketing control algorithm used in this research can be considered to be a network based virtual nesting algorithm utilizing 7 virtual or stratified booking classes instead of 12. In Figure 4.8, the stratified bucketing inventory control algorithm with 7 control buckets was compared to a "greedy" algorithm that utilizes a virtual control structure with 12 virtual buckets within a network based virtual mapping table.

Based on a local demand scenario of 40%, it is evident in Figure 4.8 that a stratified bucketing approach does not perform as well as an OD heuristic that employs a virtual nesting control structure. A reason might be that in stratified bucketing, a form of "adaptive" virtual tables cannot be implemented into the control algorithm. This is due to the fact that any shifts in OD fare bucket assignments have to be filed with the ATPCO,

making constant changes inconvenient and slow. On the other hand, besides satisfying forecasting and data integrity requirements, the assignment and reassignment of ODFs to different virtual buckets in virtual nesting can be done efficiently and is invisible to the CRS users. Therefore, a major advantage of a virtual control structure is the ability to differentiate potential revenue contributions of an ODF to the network at a level greater than that expected from a stratified bucketing approach. However, implementing a virtual control structure into an existing leg-based seat inventory control method requires a physical change of the current inventory control structure which is unnecessary in a switch to a stratified bucketing approach. Consequently, the resulting positive revenue impact differences between the two approaches to OD control should be weighed against the financial and operational investment necessary to switch to one method or the other.

*Table 4.4 - Revenue Impacts for a Range of Different Number of Control Buckets*

# of Control Buckets**	% Difference from Leg-Based Fare Class Heuristic		
	DF* = 0.62	DF = 0.88	DF = 1.32
8	0.26	1.80	5.44
12	0.27	1.98	5.49
16	0.28	1.96	5.23
20	0.29	1.73	4.82

\* DF = Demand Factor

\*\* Average Local Demand scenario of 30.5%

Since the only difference between the two algorithms presented in Figure 4.8 was essentially the number of control buckets used, an empirical study of the revenue impact the number of control buckets has on an inventory control methodology was conducted and the results are presented in Table 4.4. Based on a "greedy" approach that uses a network based virtual

mapping table, the simulation results in Table 4.4 indicate a slight advantage for algorithms using 12 control buckets to provide the greatest expected revenue gains in the demand scenarios tested. While there does not exist a solid theoretical foundation to support the claim, empirical results do indicate that the number of virtual buckets does play a significant role in determining the level of revenue gains that can be expected. Whether it is an insufficient level of revenue differentiation or too much differentiation resulting in a "small numbers" problem, the number of control buckets to use should not be considered trivial in the design of an OD control methodology. The differences recorded in the comparison of a stratified bucketing approach and a "greedy" approach could be accounted for by the differences in the number of control buckets utilized.

#### **4.4 Summary of Simulation Findings**

Figure 4.9 illustrates the maximum expected simulated revenue impacts of the most effective leg-based OD seat inventory control algorithms for a realistic range of demand factors ranging from 0.7 to 1.1. Recall that the revenue impacts observed from the simulation runs are a result of changes exclusively made to the inventory control structure and does not include any changes to the optimization or forecasting models used in the process. Clearly the use of a virtual nesting control structure which incorporates leg virtual tables that are formulated using a demand based virtual range definition provide the highest possible expected revenue impact. The average network load factors achieved under all inventory control algorithms are not significantly different, differing by only 1 or 2 percentage points.

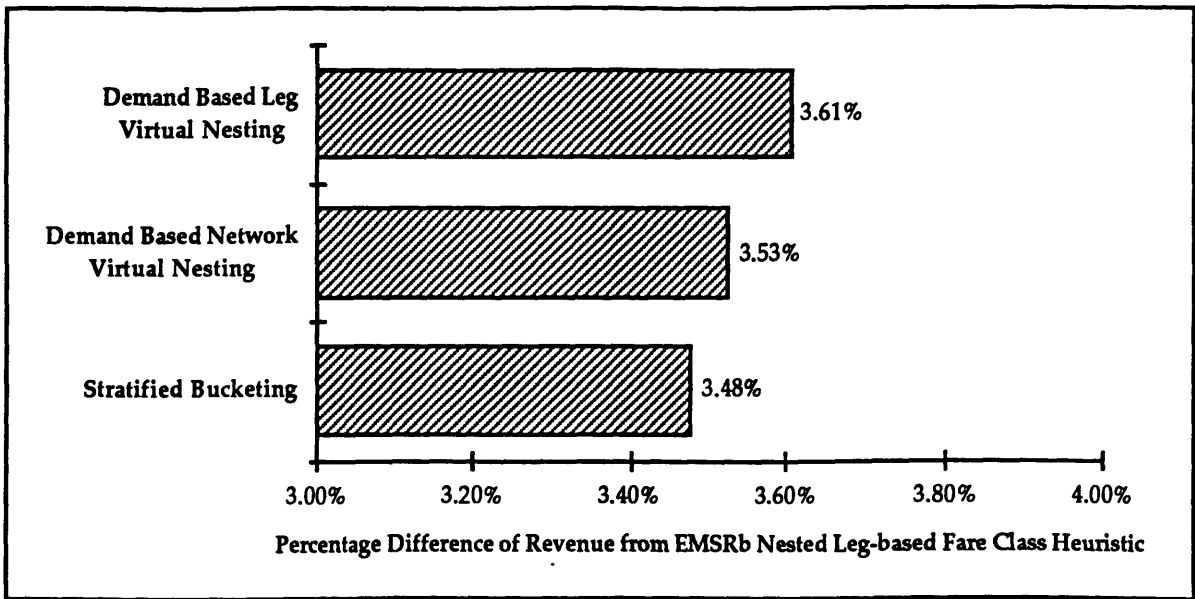


Figure 4.9 - Maximum Expected Revenue Impacts for Several OD Control Algorithms

While simulations are great tools for determining the effectiveness of a newly developed inventory control algorithm, one has to be careful in accepting the resulting revenue impacts as fact. Even though simulated revenue impacts of inventory control algorithms developed with a stratified bucketing or virtual nesting control structure have resulted in maximum expected incremental gains of 3.5% above that of leg-based fare class nesting methods, these simulations were performed under somewhat unrealistic conditions. Average network demand factors in the simulation are constant throughout any one simulated run and can be adjusted to feed unrealistically high demand factors to the control algorithm. In the "real world", average network demand factors rarely exceed 1.0 and more importantly tend to fluctuate by time of day, time of month and even by flight departure/leg. Thus, the consistency of demands encountered in a simulated environment usually encourages the overstatement of revenue impacts provided by a inventory control approach. However, this does not necessarily invalidate



the effectiveness of a control process, but rather paints a more optimistic picture of the resulting revenue impacts.

In previous empirical observations made by Belobaba [24], fare class nesting has been claimed to provide an additional 2 to 6% increase in revenues above that of no inventory control. In this research, while simulation results have tended to show average gains in excess of 3% above fare class nesting for a demand factor range of 0.7 to 1.1; a more realistic estimate is between 1 to 2% when the type of uncertain environment the control process will be in is taken into consideration. However, the level of revenue gains encountered is highly dependent on a number of different factors, some of which are uncontrollable. The definition of the virtual tables in particular the number of virtual classes and the value ranges; the proportion of local traffic on each leg; in addition to the amount of short to long leg connections encountered will clearly dictate the amount of revenue the network can expect to collect. Even so, it is empirically evident that the adaptation of some form of itinerary control is a beneficial next step in improving the revenue performances of current seat inventory control approaches.

## Chapter 5

# Leg-Based Displacement Cost Heuristics for O-D Inventory Control

### 5.1 Local Displacement Cost

As motivated in Chapter 4, OD control approaches based on a "virtual nesting" or "stratified bucketing" control structure without the application of displacement cost logic represent "greedy" algorithms for revenue maximization. In these algorithms, higher revenue long-haul multi-leg itineraries are always given higher priority over lower revenue short-haul or local leg itineraries. While a primary advantage of a virtual nesting or stratified bucketing control structure is its simplicity and an improved level of performance over leg-based fare class nesting, always prioritizing long-haul multi-leg bookings may not necessarily achieve network revenue maximization. In a network only constrained by a few high demand flight legs, giving priority to the long haul passenger may be a revenue maximizing approach to inventory control. However, as demand for each flight leg increases across a network, continued utilization of a "greedy" algorithm can result in negative revenue impacts as long-haul multi-leg traffic begin to displace short-haul and local traffic combinations of greater total revenue.

In the case of a hub-and-spoke network, a connecting passenger generally produces less revenue than the sum of local markets into and out of the hub. That is, if a flight path through a hub consisting of local ODs contributing \$300 each to the network is constrained on the inbound and outbound legs, the network revenue maximizing decision would be to accept two local bookings contributing a total of \$600 to the network rather than a connecting booking contributing only \$400. To alleviate the "greediness" of virtual nesting or stratified bucketing and to incorporate the revenue impacts of displacing local traffic into the OD control algorithms developed in Chapter 4, both American [18] and United [25] have proposed seat inventory control methodologies that take into account local displacement cost. Local displacement cost represent the lost revenue to the network of displacing local passengers in favor of connecting passengers on any one flight leg.

Implementing local displacement logic implies that the value of each multi-leg itinerary to the network is equal to the total fare of the itinerary adjusted to account for the expected displacement of local passengers on upline and downline legs. By penalizing the fare value of multi-leg itineraries, the risk of displacing higher valued local passengers is incorporated into the control method. Furthermore, the expected displacement cost on each leg of a multi-leg itinerary is determined based on a forecast of the demand and fares for the different types of traffic flow across each flight leg. Thus, the value of multi-leg itineraries is dependent on the proportion of local traffic demand that exists and the associated local revenue contribution to the network. As local traffic increases, the value of connecting markets to the network is reduced accordingly. While local

displacement cost logic does account for a significant portion of displacement risk, it does ignore the potential costs associated with displacing passengers from other constrained connecting itineraries by concentrating only on local passenger displacement.

In the development of leg-based inventory control algorithms, a problem exists as to the procedure by which displacement costs can be determined for each connecting itinerary without the use of network optimization tools. Leg-based OD control methodologies cannot rely on network shadow prices and data is available only on a flight leg level with control, optimization and forecasting accomplished by stratified or virtual nesting based inventory control algorithms. The objective of this chapter is to develop and present simulation results of alternative leg-based OD control algorithms that take into account local displacement costs without using complicated computational approaches or information about individual ODFs.

### **5.1.1 Approximations to Local Displacement Cost**

One estimate of the displacement cost associated with a flight leg,  $j$ , as proposed by Williamson [6] is the expected marginal revenue of the last available seat on the flight leg,  $EMR_j(C_j)$ , where  $C$  is the available capacity. The value of a multi-leg itinerary to a given flight leg, or the "network revenue value" is determined by adjusting the total itinerary ticket fare value,  $f_{odf}$ , by the  $EMR(C)$  value of all other legs traversed by the flight. Thus, the "network revenue value" on flight leg  $i$  of an ODF itinerary,  $NV_{i,odf}$ , is

defined as:

$$NV_{i,odf} = f_{odf} - \sum_{j \neq i} EMR(C_j) \quad (5.1)$$

for all flight legs  $j$  over which the ODF traverses, where  $j \neq i$ . In a two leg connecting itinerary, the network revenue value to leg 1 of an ODF which traverses legs 1 and 2 can be approximated as [5]:

$$NV_{1,odf} = f_{odf} - EMR(C_2) \quad (5.2)$$

where the  $EMR(C)$  value can be determined directly from the expected marginal revenue curves of the associated booking classes on that leg. Recall that the EMR value for seat  $C$  is defined by the following equation:

$$EMR(C) = \overline{P}(C) \times \overline{REV} \quad (5.3)$$

where  $P(C)$  represents the probability of selling seat  $C$  and  $REV$  is the mean fare value of all ODFs that traverse the leg.

However, Belobaba [24] identified a problem with using the full EMR value taken from their respective EMR curves. The fact is that the  $EMR(C)$  value contains aggregated information about total fare value and demand of seat  $C$  to the leg, which is not necessarily representative of local displacement cost. As evident in the formulation of the EMR value, the related components are an aggregation of fare values and demand levels taken from all ODF itineraries that traverse the leg and does not specifically represent inputs from only local itineraries. Therefore, the approximation of local displacement cost based purely on the  $EMR(C)$  value would tend to overestimate the downline local displacement cost.

In order to better approximate the local displacement cost value, it would be necessary to determine the portion of the EMR(C) value that is directly related to local traffic demand. Breaking down the individual components that makeup the EMR(C) value in Equation 5.4:

$$EMR(C) = \bar{P}(C) \times \overline{REV}$$

$$EMR(C) = \bar{P}(C) \times \left[ \frac{Pax_{loc} Fare_{loc} + Pax_{cnx} Fare_{cnx}}{Pax_{total}} \right] \quad (5.4)$$

the mean fare value component, REV, has been separated into local,  $Pax_{loc}Fare_{loc}$ , and connecting,  $Pax_{cnx}Fare_{cnx}$ , passenger revenue contribution components and divided by the total expected passenger demand,  $Pax_{total}$ . This first step in developing a theoretical local displacement cost formula is used to establish the mean fare value of only local traffic demand. By multiplying the REV component in Equation 5.4 by an adjustment factor,  $REVCO_{loc}$ , a mean local fare value variable,  $REV_{loc}$ , is determined as follows:

$$\overline{REV}_{loc} = \left[ \frac{Pax_{loc} Fare_{loc} + Pax_{cnx} Fare_{cnx}}{Pax_{total}} \right] \times REVCO_{loc} \quad (5.5)$$

where,

$$REVCO_{loc} = \left[ \frac{Pax_{loc} Fare_{loc}}{Pax_{loc} Fare_{loc} + Pax_{cnx} Fare_{cnx}} \right] = \left[ \frac{TotRev_{loc}}{TotRev} \right] \quad (5.6)$$

which defines  $REVCO_{loc}$  as the proportion of total expected revenue,  $TotRev$ , attributable to local passenger revenue contributions,  $TotRev_{loc}$ .

With the average local fare value established, it is also important to estimate the probability that the last seat is sold to a local passenger on the flight leg.  $P_{LOC}$  represents the probability that the last seat is sold to a local

passenger and is formulated as the product of the probability of selling the last seat,  $P(C)$ , and the probability of a local passenger booking request occurring,  $P(loc)$ , as follows:

$$\bar{P}_{LOC} = \bar{P}(C) \times \bar{P}(loc) \quad (5.7)$$

To complete the formulation of the local displacement cost,  $DISP_{loc}$ , the  $EMR(C)$  value is then multiplied together with the displacement coefficients,  $P_{LOC}$  and  $REVCO_{loc}$ , resulting in an approximation for the down-line local displacement cost on a flight leg as shown in Equation 5.8.

$$DISP_{loc} = EMR(C) \times \bar{P}_{LOC} \times REVCO_{loc} \quad (5.8)$$

The approximation of local displacement cost formulated in Equation 5.8 is a more realistic estimate of the value than using the entire  $EMR(C)$  value as previously proposed. In Equation 5.8, the local contributions of revenue and demand are accounted for exclusively while the revenue contributions from connecting itineraries are removed, thereby resulting in a better approximation of the local displacement cost. The network revenue value to leg 1 of a two leg ODF itinerary can now be formulated as follows:

$$\begin{aligned} NV_{1,odf} &= f_{odf} - DISP_{loc2} \\ NV_{1,odf} &= f_{odf} - [EMR_2(C) \times \bar{P}_{LOC} \times REVCO_{loc}] \end{aligned} \quad (5.9)$$

Similarly, Equation 5.9 can be applied to all multi-leg ODFs for all flight legs in the network to determine the network revenue value of each ODF over each leg traversed.

The formulation of a plausible local displacement cost approximation sets the stage for developing OD control algorithms that utilize local

displacement cost logic in its optimization and control process. The next few sections of this Chapter will describe several applications of local displacement cost logic in developing new OD control algorithms, beginning with a discussion of "static" and "real-time" applications of local displacement cost.

## 5.2 Static and Real-Time Applications of Local Displacement Cost

In this research, the incorporation of local displacement cost into a leg-base OD control methodology will be examined from two perspectives. The first being a "static" approach and the second, a "real-time" or dynamic application of local displacement cost. The main function of local displacement cost is to include the potential of displacing higher revenue local itineraries by determining the value of connecting itineraries to the network. In a "static" displacement cost OD control algorithm that utilizes a virtual nesting control structure and an EMSRb optimization method, the value to the network of each connecting ODF combination is represented by the network revenue value found according to Equation 5.9.

The "static" displacement cost inventory control algorithm begins with an initial mapping all ODF combinations into their respective virtual control buckets, as determined by the demand-based network virtual nesting control structure presented in Chapter 4. EMR curves for each flight-leg in the network are then calculated based on the aggregation of ODF demand from each booking class. The next step entails calculating local displacement cost,  $DISP_{loc}$ , on each flight leg in the network by taking the EMR of the last available seat and multiplying by the displacement coefficients,  $P_{LOC}$  and



$REVCO_{loc}$ . The network revenue value,  $NV_{i,odf}$ , of each connecting ODF that traverses over leg,  $i$ , is then calculated through the adjustment of itinerary fare values by the local displacement cost as shown in Equation 5.9. Each connecting ODF can then be re-mapped or reassigned into a new virtual control bucket based on the maximum network revenue value,  $NV_{i,odf}$ , obtained over all legs,  $i$ , instead of its itinerary fare value. The use of the maximum network revenue value is explained by the fact that if only one leg of a two leg flight is highly constrained, the airline should still accept the two leg connecting passenger over a single local passenger. In most cases, the connecting itinerary will provide a higher revenue contribution than a single local itinerary.

The logic behind the "static" displacement cost algorithm rests on the notion that if the risk of displacement is high due to highly constrained flights, the network revenue value of the connecting ODF will be sufficiently reduced, resulting in the re-mapping of the ODF into a lower virtual control bucket, and in turn, reducing its booking availability. On the other hand, if the demand conditions are low, connecting itineraries will not be penalized as severely and usually no re-mapping of the connecting ODF is required. Logically, local ODFs are not affected by local displacement costs and are not re-mapped into new control buckets. While the displacement cost applications introduced in this section requires the use of the maximum network revenue value of a multi-leg ODF as the value basis to remap the ODF, other seat inventory control algorithms will be introduced in later sections that do not require the use of this criterion.

The "static" nature of the above local displacement cost algorithm is explained by the fact that the local displacement costs are fixed for all flight

departures on a flight leg, irrespective of day or time of departure. In the "static" algorithm, local displacement costs are only calculated periodically and reassignment of ODFs to new control buckets thereby occurs periodically. While re-mapping of ODFs to new virtual control buckets should ideally be done for each future flight departure/date or even dynamically after each booking period, the need to maintain the consistency of historical data by virtual buckets prevents frequent reassignment of ODF demand [24]. Therefore, after each re-mapping procedure, ODFs usually remain fixed or "static" in their respective virtual control buckets for extended periods of time until the next scheduled update of displacement cost values.

Consider the two leg flight network presented in Example 4.2 in which the fare for a SFO-DEN Y fare ticket is \$467, a SFO-BOS Y fare ticket is \$724 and a DEN-BOS Y fare ticket is \$648. Assume that the local displacement cost approximated for each flight leg is as follows:

$$\begin{aligned}DISP_{loc}(SFO - DEN) &= \$150 \\DISP_{loc}(DEN - BOS) &= \$225\end{aligned}$$

Based on these local displacement cost values, the cost of displacing a local passenger by accepting a connecting SFO-BOS passenger is estimated at \$150 on the SFO-DEN leg and \$225 on the DEN-BOS leg. Thus the network revenue value of the connecting itinerary, SFO-BOS, is estimated at \$499 on the first leg and \$574 on the second leg, respectively. Based on the network "static" displacement cost algorithm, the maximum network revenue of \$574 is utilized to reassign the ODF to a new virtual control bucket. Referring to the network virtual table shown in Example 4.2, note that the SFO-BOS Y fare will no longer be mapped into virtual bucket V1, instead it will be re-mapped into a lower virtual bucket V2 for the purposes of optimization and control.

The local itineraries will remain in the initially assigned buckets of V3 for the SFO-DEN Y fare and V2 for the DEN-BOS Y fare. In this example, due to the significant impacts of local displacement costs requiring the re-mapping of the SFO-BOS Y fare, this ODF is no longer considered more valuable to the network than a local DEN-BOS Y fare mapped into the same virtual bucket.

Upon completion of the re-mapping procedure, the rest of the network "static" displacement cost OD control algorithm continues in a fashion similar to that of leg-based booking class control methods introduced in Chapter 4. Optimization, forecasting and historical data collection are accomplished on an individual flight leg level for each virtual control bucket. ODF's are then controlled according to the booking limits of their respective "newly" assigned virtual control buckets on each flight leg, as is done in other leg-based booking class approaches. By re-mapping the ODFs based on their network revenue values, a better "picture" of the value of multi-leg ODFs to the network is drawn, allowing for the optimization method to be better able to allocate seats accordingly. Re-mapping based on network revenue value also alleviates some of the "greediness" of OD control algorithms introduced earlier. As shown in the previous example, if local itineraries prove to be more valuable to the system as represented by higher local displacement cost figures, a multi-leg itinerary may not always be given higher booking priority.

Note that the application of "static" displacement cost approaches are not limited only to virtual nesting control structures but can be applied to stratified bucketing as well. As previously mentioned, in many respects, stratified bucketing can be considered a limited form of virtual nesting and shares many of the same benefits. However, a stratified bucketing control structure requires re-filing of fares through ATPCO at every instance an ODF

is moved from one control bucket to another. This inconvenient procedure can limit the effectiveness of re-mapping an entire network of multi-leg ODFs into new control buckets based on their network revenue values. Even with infrequent re-mappings as is the case in a "static" displacement cost inventory control algorithm, the additional effort required to re-file the fares may overshadow the benefits of applying "static" displacement cost logic in a stratified bucketing structure.

The development of a "real-time" displacement cost application provides an alternative approach to applying local displacement cost to control algorithms which cannot conveniently re-map their ODF combinations. Especially with the advent of seamless CRS availability communication, booking requests can now be evaluated by the selling airline on a real-time basis, thus allowing for "real-time" local displacement cost calculations to be made before a booking request is evaluated [5].

The "real-time" displacement cost algorithms developed in this research have been applied to a stratified bucketing control structure. "Real-time" displacement costs may also be applied to a virtual control structure with no significant difference from stratified bucketing expect for the number of control buckets utilized. As in "static" algorithms, "real-time" local displacement cost inventory control algorithms begin by initially mapping all ODFs into a "default" control bucket based on the ODF's full itinerary fare value. It is at the level of these control buckets that forecasts of passenger demands and "optimal" allocations of seats are generated at all points in the control algorithm. Using Equation 5.9, at the time a booking request for a multi-leg itinerary is received, the network revenue value of the connecting ODF is determined based on current seat availability and demands on each

flight leg traversed. As in the "static" case, the maximum network revenue value over all legs is used for the purposes of establishing the value of the connecting ODF to the network. Once the network revenue value is established, instead of re-mapping the ODF into a new control bucket, the "real-time" algorithm only uses the availability of the control bucket corresponding to the network revenue value of the multi-leg ODF. Thus, even though the ODF demand is not reassigned to a new control bucket, ODF seat availability is still potentially reduced based on the most current network revenue value approximated over all legs traversed. Therefore, if at the time of a booking request for a two leg itinerary, both legs traversed are highly constrained, the multi-leg itinerary will receive lower availability as determined from its network revenue value, and preference will be given to local passengers. Conversely, if demand for either legs are low or if there is an abundance of available capacity, the availability of the connecting ODF will most likely be unaffected due to insignificant local displacement cost values on one or both flight legs.

Take for example the two flight leg network described earlier. Using the network "static" local displacement cost algorithm, the connecting itinerary, SFO-BOS, was reassigned to virtual control bucket V2 from bucket V1. If a network "real-time" local displacement cost algorithm was used, at the time of the booking request, network revenue values would be updated with the most recent information available and a maximum network revenue value would be established. Assuming a value of \$574 is used, the algorithm would then seek out the booking class that includes the updated network revenue value within its revenue range definition and display the seat availability associated with the booking limit of that control bucket. If

seats are available, the booking request would be accepted, and booking limits are decremented from the "default" booking classes. This same procedure is accomplished at every multi-leg booking request, and the rest of the algorithm proceeds in the same manner as leg-based booking class control methods, except for the "real-time" evaluations of booking requests.

The advantage of utilizing a "real-time" local displacement cost algorithm is rooted in its ability to perform simple displacement cost calculations at the time a booking request is made based on the most current information available, and return a seat availability corresponding to the most up-to-date value to the network of the booking request. Unlike "static" algorithms that calculate network revenue values periodically based on historical demand and fare information, "real-time" approaches utilize relevant demand and fare information during the same period of time in which the request is made. Additionally, the flexibility of "real-time" local displacement cost logic allows it to be applied to a myriad of seat inventory control algorithms which are incompatible with local displacement cost applications that require the shifting or re-mapping of demand information between control buckets. Seat inventory control algorithms that utilize control structures such as fare class nesting or stratified bucketing may benefit from the incorporation of a non-remapping "real-time" local displacement cost application into their control process. The next section will present simulated revenue impacts of some leg-based OD control algorithms developed in this research that incorporate "static" and "real-time" local displacement cost logic in conjunction with a variety of control structures.

### 5.3 Simulation Results of "Static" Displacement Cost Heuristics

The optimization/booking simulator was used to evaluate the revenue benefits of several leg-based OD control algorithms that take into account local displacement costs. As in Chapter 4, these inventory control algorithms were subjected to several demand scenarios within a hub-and-spoke environment and the resulting revenue impacts are compared to an EMSRb leg-based fare class control algorithm.

The first leg-based displacement cost control algorithm examined is one based on the application of a network "static" displacement cost logic to an inventory control algorithm that utilizes a virtual nesting control structure and an EMSRb seat allocation heuristic. Recall that the basis of the "static" displacement cost approach involves the periodic calculation of

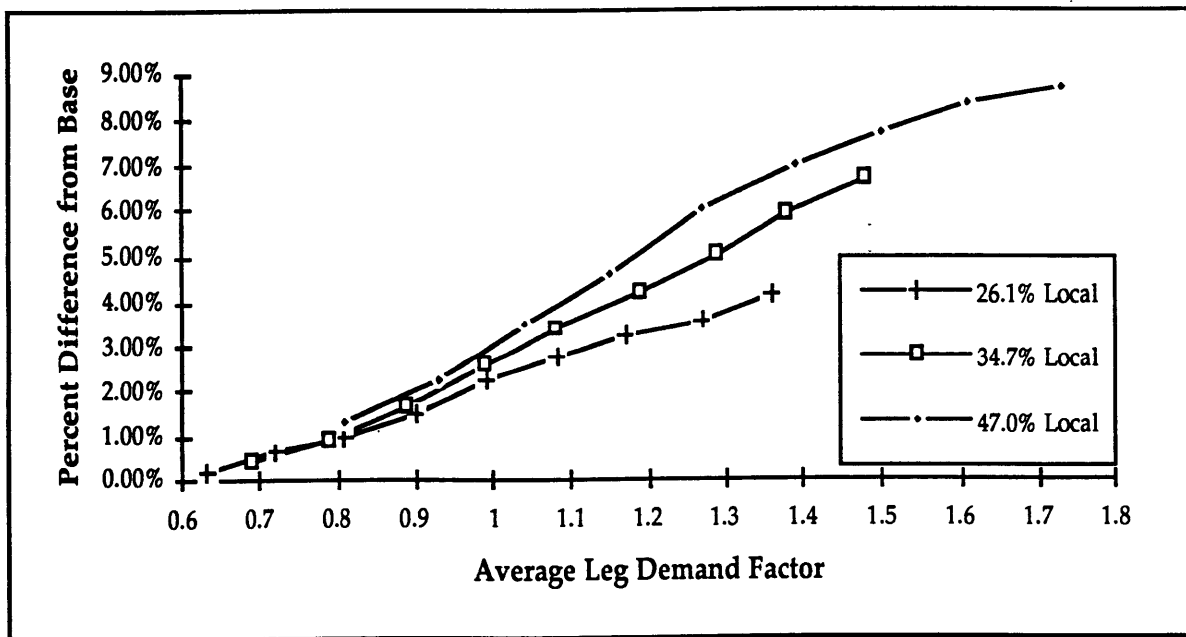


Figure 5.1- Revenue Impacts of the Virtual Network "Static" Displacement Cost Algorithm

network revenue values for each multi-leg ODF in the network, followed by the subsequent re-mapping of these ODFs into related control buckets. In Figure 5.1, the revenue impacts of a network "static" displacement cost algorithm is shown for a range of demand scenarios as compared to the base case. Once again, the base case represent a leg-based fare class nesting seat inventory control algorithm.

The incremental revenue gains over the leg-based fare class control approach shown in Figure 5.1 are significant and are consistent over all demand factors and average local demand scenarios. Simulation results show that the revenue impacts of a network "static" displacement cost algorithm range from a low of 0.2% to a high of 9% depending on the demand factor and the level of average local demand. Focusing in on a more realistic demand factor range of 0.7 to 1.1, revenue impacts varied from 0.6% to 5% with corresponding average hub loads factors of 0.71 to 0.89.

It is evident from examination of the simulation results that there is a clear relationship between demand factor and the magnitude of revenue impacts realized. Additionally, a similar relationship is found between the average level of local demand that exists in the network and expected revenue gains. As exemplified in Figure 5.1, at lower demand factors of 0.9, there was an insignificant difference in revenue impact recorded between the three average local demand scenarios examined. At a more constraining demand factor of 1.2, a revenue impact difference of more than 1.1% between the three local demand scenarios is recorded when compared to the base case. Obviously, an objective of displacement cost approaches is to be better able to handle local demands within constrained demand environments. The simulation results have shown empirically that the algorithm does recognize



to a certain extent the level of local demand present in the network as evident in the level of comparative revenue impacts achieved with respect to the average local demand present in the network.

To illustrate the effectiveness of a displacement cost algorithm's ability to alleviate some of the "greediness" effects of using a "greedy" algorithm in a virtual nesting control structure, Figure 5.2 draws up a comparison between the "greedy" inventory control algorithm presented earlier and a network "static" displacement cost algorithm. As found in Figure 5.2 for an average local demand scenario of 40%, the virtual network "static" displacement cost

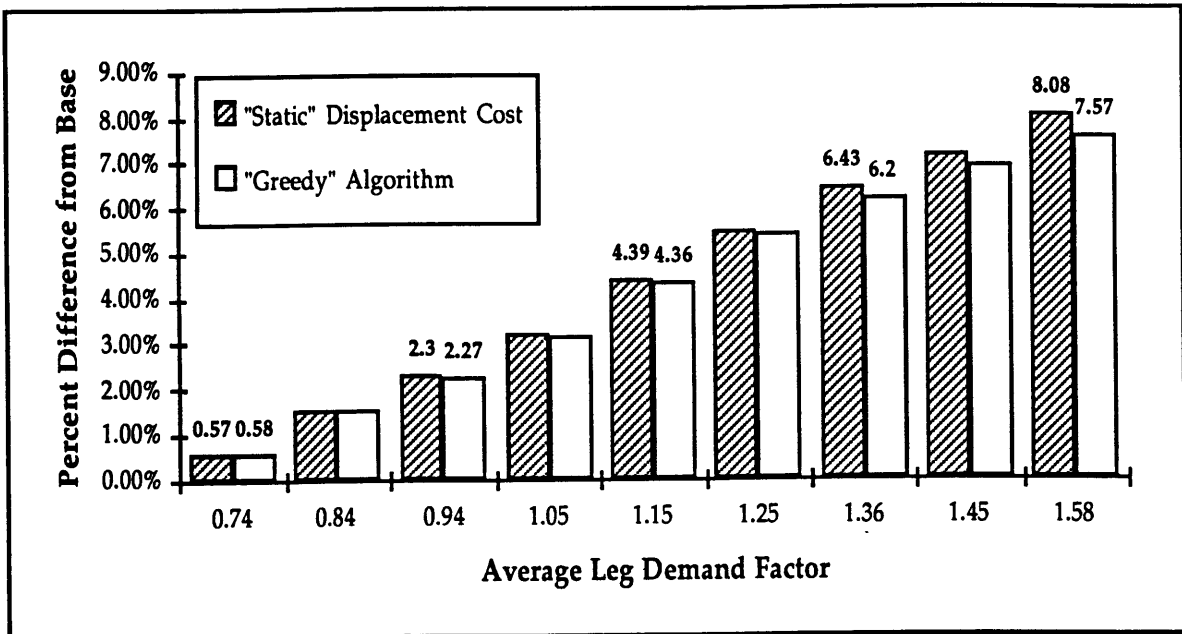


Figure 5.2 - Revenue Impacts of a "Greedy" versus a Virtual Network "Static" Displacement Cost Algorithm

algorithm recorded revenue impacts slightly above that of the "greedy" approach at the lower end of the demand factor range. As the demand factor climbs, the magnitude of the differences in revenue impacts recorded began to differ more substantially, reaffirming the notion that incorporating some form of displacement cost logic in the control methodology is beneficial to

better control of OD itineraries or traffic flow in the more constrained networks.

A second "static" displacement cost control algorithm developed in this research differs from the network "static" approach in the method by which ODFs are re-mapped into a control bucket. In a network "static" approach the maximum network revenue value over all legs traversed by a multi-leg ODF is used as the value basis for re-mapping the ODF. For a leg "static" displacement cost algorithm, in a similar process as described by Williamson's "Value Net of Opportunity Cost" approach introduced in her doctoral dissertation [6], each multi-leg ODF will now be re-mapped into the corresponding control bucket representative of the network revenue value calculated for the ODF on that leg. It is assumed that this additional level of OD revenue differentiation may provide a better picture of the value of a multi-leg ODF to the network.

Reconsider the example of the SFO-DEN-BOS flight where the network revenue value of a SFO-BOS connecting ODF has been calculated to be \$499 on the SFO-DEN leg and \$574 on the DEN-BOS leg. In the case of a network "static" displacement cost approach, the SFO-BOS would be re-mapped into a new control bucket based on the \$574 value. However, in a leg "static" displacement cost approach, the SFO-BOS would be remapped on the SFO-DEN leg into a control bucket that contains \$499 in its revenue range definition and remapped on the DEN-BOS leg in a corresponding control bucket that contained \$574 valued ODFs. A single multi-leg ODF may be mapped into different virtual control buckets on different flight legs with different booking limits; a procedure similar to using leg specific virtual mapping tables in the "greedy" control algorithm presented in Chapter 4.

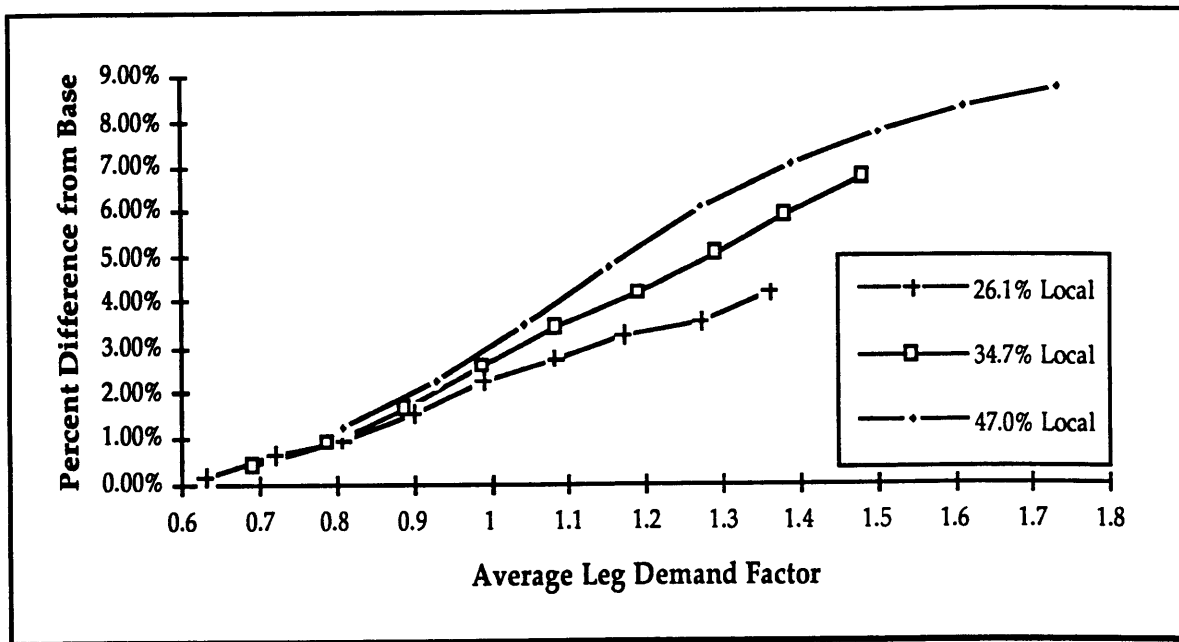


Figure 5.3 - Revenue Impacts of the Virtual Leg "Static" Displacement Cost Algorithm

Figure 5.3 illustrates the revenue impacts recorded from simulation of the virtual leg "static" displacement cost seat inventory control algorithm. Depending on the level of demand encountered, this control algorithm realized revenue impacts ranging from 0.2% to 9% above the base case. Additionally, load factors achieved ranged in value from 0.63 to 0.94 for a variety of demand factors ranging from 0.63 to 1.73. Taking for example a realistic average local leg demand of 35%, within the demand factor ranges of 0.7 to 1.1, incremental revenue gains realized ranged anywhere from 0.4% to 3% above that of a leg-based fare class nesting control algorithm.

Summarizing the effectiveness of the two "static" displacement cost approaches, Table 5.1 lists the comparative revenue impacts of both algorithms in addition to a "greedy" virtual nesting algorithm for a range of plausible demand levels. The revenue impacts recorded by both displacement cost simulations are highly dependent on the level of demand,

Table 5.1- Percentage Revenue Impacts of "Static" Displacement Cost and "Greedy" Algorithms

Avg. Local Dem.	Algorithm	<u>Average Demand Factor</u>									
		0.63	0.72	0.81	0.90	0.99	1.08	1.17	1.27	1.36	
27%	"Greedy"	0.18	0.60	0.96	1.47	2.08	2.67	3.23	3.63	4.30	
	Network	0.18	0.60	0.98	1.52	2.25	2.77	3.25	3.56	4.15	
	Leg	0.18	0.60	0.97	1.52	2.26	2.77	3.25	3.58	4.17	

Avg. Local Dem.	Algorithm	<u>Average Demand Factor</u>									
		0.81	0.93	1.04	1.15	1.27	1.39	1.50	1.61	1.73	
47%	"Greedy"	1.31	2.24	3.53	4.69	5.88	6.78	7.37	7.80	8.06	
	Network	1.32	2.23	3.55	4.65	6.05	7.01	7.74	8.33	8.72	
	Leg	1.26	2.22	3.53	4.78	6.13	7.05	7.74	8.29	8.68	

especially local demand on the network. At lower levels of local demand, the positive impacts of displacement cost algorithms are minimal and at points result in marginally lower revenues than the "greedy" approach. As local demand increased, the revenue gains achieved by displacement cost algorithms began to show substantial gains above that of the "greedy" algorithm, especially at the higher demand factor ranges.

It is important to reiterate that the use of a "static" displacement cost algorithm is not limited only to a virtual nesting control structure, but may be applied to a stratified bucketing control structure as well. While the re-mapping of ODFs in a stratified bucketing control methodology is inconvenient, it does not affect the operational applicability of "static" displacement cost logic. Recall that in this research, the stratified bucketing algorithms are in fact virtual nesting algorithms utilizing 7 control buckets

instead of 12. In Figure 5.4, a network "static" displacement cost algorithm was applied to a stratified bucketing control structure.

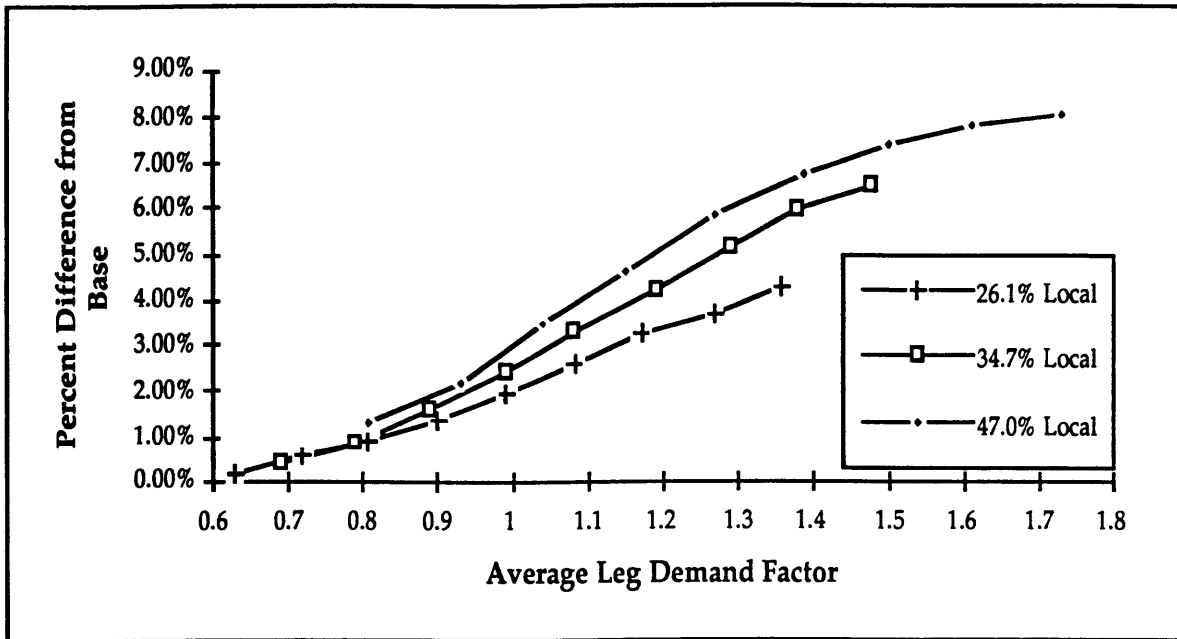


Figure 5.4 - Revenue Impacts of the Stratified Network "Static" Displacement Cost Algorithm

The revenue impacts achieved in the application of the stratified network "static" displacement cost algorithm are positive throughout all demand scenarios tested. Incremental revenue gains above that of the base case ranged from 0.2% to 8% with average hub load factors of 0.63 to 0.96, depending on the demand level. Within more realistic demand factors of 0.7 to 1.1, revenue impacts for an average local demand proportion of 35% ranged from 0.4% to 3%, with average hub load factors ranging from 0.68 to 0.89.

In comparison to the "greedy" stratified bucketing control algorithm, the trends in revenue impacts realized are similar to those found in comparisons made between the "greedy" virtual nesting control algorithm and the virtual network "static" displacement cost algorithm. Figure 5.5

shows that the difference in incremental gains are minimal at the lower ranges of the demand factors, and as expected, a larger impact appears as the demand factors begin to climb into the higher ranges. Unlike the virtual "static" displacement cost algorithm, the impact of local displacement cost logic applied to the stratified bucketing "greedy" algorithm results in at most a 0.2% increase in revenues above the "greedy" algorithm when compared to the base case.

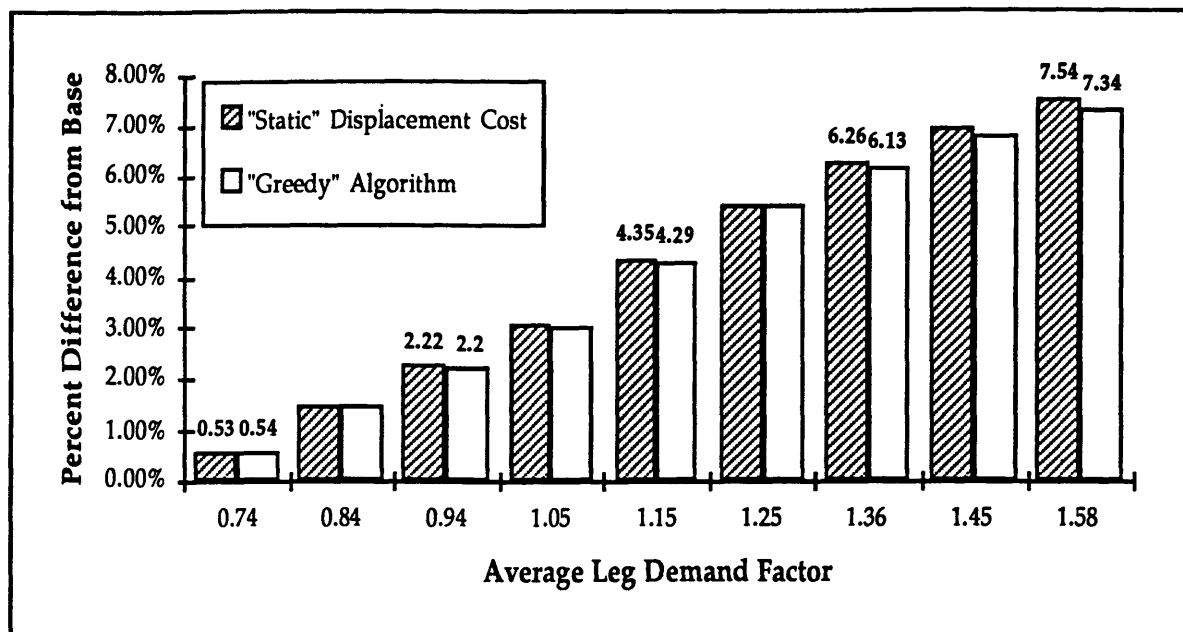


Figure 5.5 - Revenue Impacts of the "Greedy" versus the Stratified Network "Static" Displacement Cost Algorithm

However, is the relatively poorer revenue performance of the "static" displacement cost logic when used in conjunction with a stratified bucketing control structure a function of the displacement cost methodology or the control environment? In a comparison of revenue impacts made between a stratified bucketing based network "static" displacement cost algorithm and a virtual nesting based network "static" displacement cost algorithm shown in Figure 5.6, the results obtained are as expected, with the virtual nesting

algorithm realizing higher revenue impacts, especially at the lower demand factor ranges. It is possible that the differences in revenue impacts are not a function of the "static" displacement cost methodology, but a function of the control structure's ability to provide enough revenue differentiation between the ODFs. As discussed in Chapter 4, the only difference between stratified

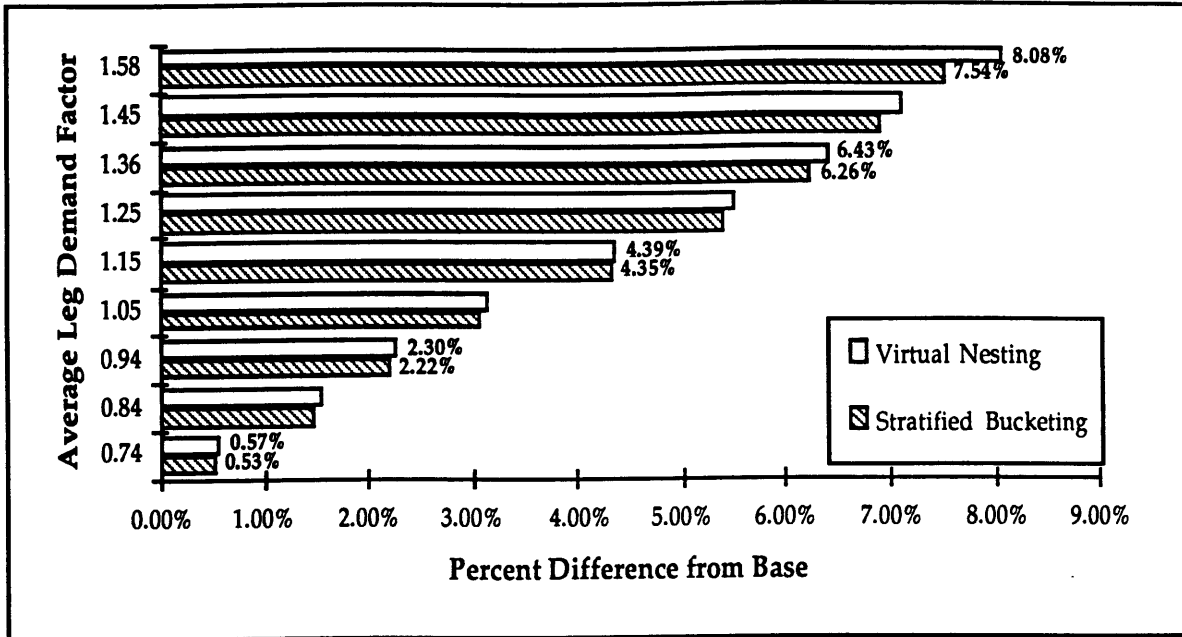


Figure 5.6 - Revenue Impacts of the Virtual "Static" versus the Stratified "Static" Displacement Cost Algorithm

bucketing and virtual nesting is the number of control buckets each control structure uses. In Table 4.4 of Chapter 4, simulation results have already shown that the number of control buckets does have a significant impact on the level of revenue impacts realized. Nonetheless, the revenue gains achieved by a stratified structure is still quite substantial, and more importantly, the cost of implementing a virtual nesting structure versus that of a stratified control structure must be weighted against the differences in revenue gains obtainable by each approach.

In any event, the simulation results shown up to this point represent the effectiveness of using "static" displacement cost logic in conjunction with either form of OD seat inventory control structure. While the revenue impacts realized do differ among the inventory control algorithms introduced so far, the incremental revenue benefits of using any of these algorithms have been empirically shown to be substantial and positive over all demand levels.

#### **5.4 Simulation Results of "Real-Time" Displacement Cost Heuristics**

The application of a "static" displacement cost algorithm in a stratified bucketing control structure has proven to be effective in producing substantial revenue gains above that of leg-based fare class nesting methods. However, "static" displacement cost algorithms require the periodic re-mapping of ODF combinations. While the re-mapping procedure should ideally occur as often as operationally possible to update the displacement cost values, especially in markets that experience much fare and demand fluctuations, the stratified bucketing control structure does not allow for convenient shifts in control bucket assignments. A "real-time" displacement cost algorithm was developed to take into account the operational inefficiency of constant re-mapping, by taking advantage of evolving CRS capabilities to perform "real-time" updates of displacement costs and seat availability. A "real-time" displacement cost algorithm requires no re-mapping of ODFs, maintaining the consistency of historical data collection, yet it still provides the benefits of incorporating local displacement cost information into a leg-base OD seat inventory control process.



In this research, several "real-time" displacement cost algorithms were developed that utilize an EMSRb optimization heuristic and a stratified bucketing control structure. In Figure 5.7, a comparison of the revenue impacts between the stratified network "static" and the stratified network "real-time" displacement cost algorithm is made. As there is no re-mapping of ODFs to different control buckets, "real-time" calculation of a multi-leg ODF's network revenue value is performed at the time of the booking request. Once again, the network definition of the algorithm refers to the use of the maximum network revenue value calculated for the ODF as the basis for determining from which control bucket to obtain seat availability information.

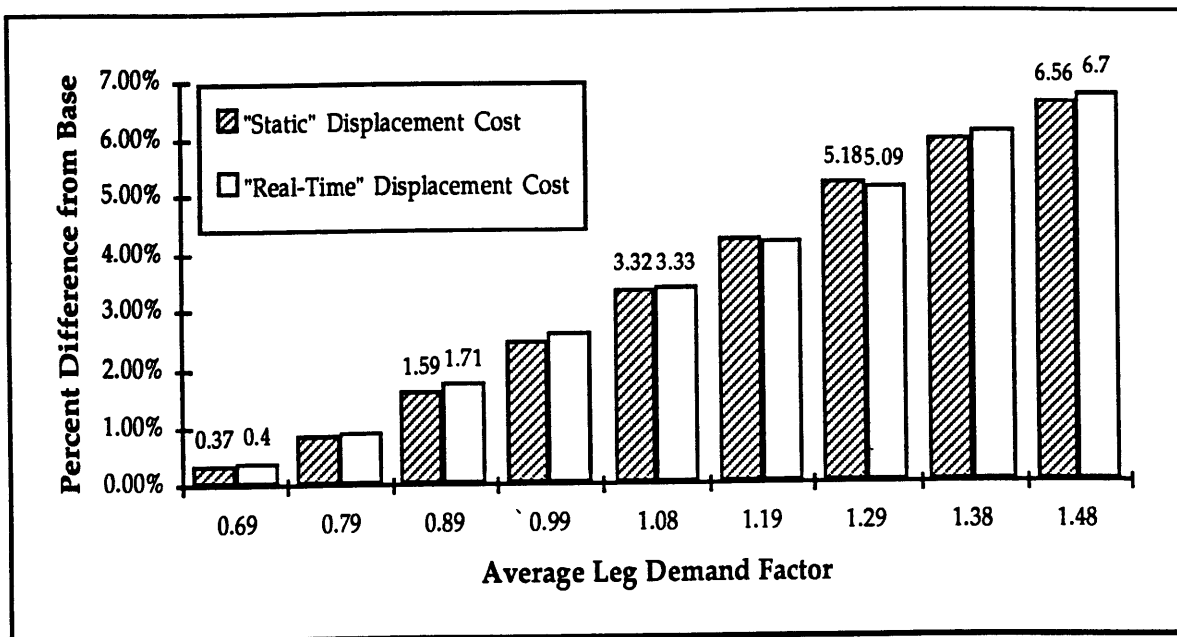


Figure 5.7 - Revenue Impacts of the Network "Static" versus the Network "Real-time" Inventory Control Algorithm

It would appear that the use of a "real-time" approach with real-time information does provide a significant but small improvement in revenue impacts over a "static" approach. In the 40% average local demand scenario

shown in Figure 5.7, at almost all demand factors, the "real-time" displacement cost algorithm realized slightly higher revenue than the network "static" displacement cost algorithm. Although the load factors achieved by the "static" approach was 1 to 2 percentage points higher, it is obvious that the "real-time" approach was better able to determine which bookings would be able to provide better total revenue contributions to the network. The level of incremental revenue benefits achieved indicate the feasibility of utilizing "real-time" displacement cost logic in cases where re-mapping of ODF demand is not a viable alternative to taking into account displacement risk.

A stratified leg "real-time" displacement cost algorithm was also developed. This algorithm is similar to that of the previously introduced virtual leg "static" displacement cost algorithm except for the fact that "real-time" displacement cost calculations are performed and no re-mapping of ODFs are required. Simulation results of this algorithm proved to be positive and ranged in revenue impacts, for a 34.7% average local demand scenario, from 0.4% to 7% with average load factors of 0.68 to 0.92. In comparison to a network "real-time" displacement cost algorithm shown in Figure 5.8, the revenue gains achieved were slightly lower, differing by as much as 0.2% from the base case for the same average local demand.

The persistence of this trend throughout all demand scenarios tested would tend to indicate that the application of a leg "real-time" displacement cost algorithm in conjunction with a stratified bucketing control structure may prove to be too restrictive on the flow of multi-leg ODFs in the network. Especially with the availability of more accurate displacement cost values in a "real-time" approach, it would seem that the need to further differentiate

connecting ODF seat availability on an individual leg basis is an additional level of control unnecessary for this type of seat inventory control approach.

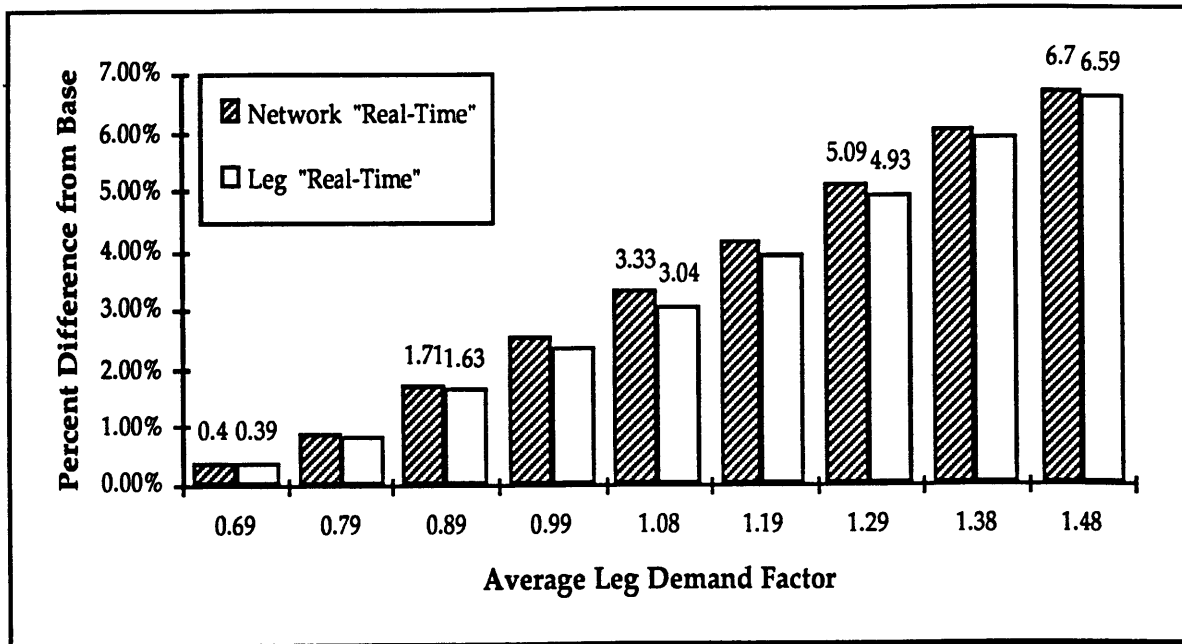


Figure 5.8 - Revenue Impacts of the Leg "Real-Time" versus the Network "Real-Time" Displacement Cost Algorithm

The positive revenue impacts realized by all "real-time" displacement cost algorithms developed serves testimony to the effectiveness of utilizing such an approach. Even though the above simulations are of leg-based "real-time" displacement cost algorithm based on a stratified bucketing control structure, there are no reasons not to apply a similar approach to a virtual nesting control structure. As mentioned, the only difference between a stratified bucketing control structure and the virtual nesting control structure utilized in this research is the number of control buckets utilized. It can be speculated that application of "real-time" displacement cost logic to a virtual nesting control structure will produce revenue impacts above that of "static" approaches in either control structures for reasons previously mentioned.

However, the use of "real-time" displacement cost algorithms are not without its drawbacks.

One possible algorithmic difficulty encountered is the risk of leaving empty seats protected for multi-leg ODFs during each booking period. As a tradeoff to re-mapping, ODFs are maintained in a "default" control bucket usually defined by the ODF's itinerary fare value. Consequently, as protected seats for lower control buckets are being shared with connecting ODFs assigned to higher buckets, the seats protected for the higher control buckets are not being shared with the lower ones in a "real-time" displacement cost concept. Thus, in a highly constrained flight leg, even though connecting ODFs are being penalized by seeking availability from lower control buckets, local ODFs are not implicitly provided any additional preference to seats initially protected for higher buckets.

These complications are due to the fact that all ODF combinations are maintained in their original control buckets, insulating the optimization heuristic from the potential displacement risk involved. Thereby, the protection of seats are still performed in a "greedy" fashion based on each control bucket's demand and revenue contribution potential. The resulting effect is a lower average load factor and lost of potential demand. Although these problems can be overcome with more frequent revisions of protection levels, the additional computations required may negate any performance advantages. Nonetheless, the application of leg-based displacement costs algorithms in either control environment has proven its effectiveness in capitalizing on its ability to alleviate some of the "greediness" effects of a virtual nesting or stratified bucketing control structure and to return a substantial level of incremental revenue gains.

## 5.5 Application of Displacement Cost to Fare-Class Nesting

The previous two sections have described the application of displacement cost logic to OD control structures, i.e. virtual nesting, stratified bucketing. While it has been motivated that displacement cost approaches were developed to alleviate the "greedy" effects of certain OD inventory control algorithms, local displacement cost logic can also be applied to fare class nesting control structures. Even though fare-class nesting has been identified as an ineffective inventory control structure in its ability to differentiate and maximize OD traffic flows and revenues, the addition of displacement cost logic to this structure may prove to be a beneficial and low cost first step towards improving the current control process.

The use of a displacement cost algorithm without the use of virtual or stratified control buckets has become a feasible control methodology with the advent of seamless CRS and "real-time" ODF seat availability evaluation [24]. In a process similar to that of the "real-time" displacement cost algorithms introduced previously, ODF assignments to fare classes are kept intact and no shifting/re-mapping of fare class assignments are made during the control process. At the time of a multi-leg booking request, the network revenue value of the request is calculated on a "real-time" basis utilizing the expected marginal revenue analysis displacement cost approximation. Once the displacement cost is determined and deducted from the itinerary fare value, seat availability is determined corresponding to the network revenue value of the ODF. However, unlike a stratified or virtual control structure with control buckets defined by revenue ranges, the average fare value of each fare

class is used as the upper bound in which to judge if the booking request's network revenue value corresponds to the value of the fare class on the flight leg.

As an example, consider a two flight leg network, with current fare class information for each of the flight legs listed below in Example 5.1. Assuming that the local displacement cost calculated at the time of a \$210 V class multi-leg booking request is as follows:

$$DISP_{loc}(LEG 1) = \$30$$

$$DISP_{loc}(LEG 2) = \$40$$

the corresponding network revenue value of the V class fare on leg 1 and leg 2 would be \$170 and \$180, respectively. Referring to average fare values in Example 5.1, the maximum seat availability for a network revenue value of \$170 on leg 1 corresponds to a V class availability of 45 seats, since it is less than the upper bound average fare value of \$180. On leg 2, the \$180 value would correspond to a seat availability of 10 seats. Therefore, the number of seats available for the two leg V class itinerary is equal to the minimum availability over both legs - 10 seats. In this example, the \$210 V class itinerary was worth more to the network on Leg 2 than the average fare value

Fare Class	LEG 1		LEG 2	
	Avg. Fare	Avail.	Avg. Fare	Avail.
Y	\$450	130	\$300	65
B	\$400	95	\$250	50
M	\$325	80	\$210	30
Q	\$280	59	\$180	10
V	\$180	45	\$145	0

*Example 5.1 - Example average fare values and seat availability for a two leg flight network.*

represented by that class. Consequently, the ODF specific booking request was given a higher availability than was shown by a basic leg-based fare class nesting algorithm. This improved inventory control process is defined as the leg fare class displacement cost algorithm.

It is hoped that incorporating displacement cost logic will improve the fare class nesting control algorithm's ability to evaluate the value of each ODF combination to the network and in turn provide significant revenue gains. In Figure 5.9, booking simulation results of the network fare class displacement cost algorithm are first presented. Unlike Example 5.1, the maximum network revenue value over both legs is used to decide which fare class to seek seat availability. Once again the maximum network revenue value is utilized to prevent the potential of rejecting a multi-leg booking request when only one leg of a two leg flight is highly constrained. Thus, in the previous example, the network revenue value of \$180 is used on both legs

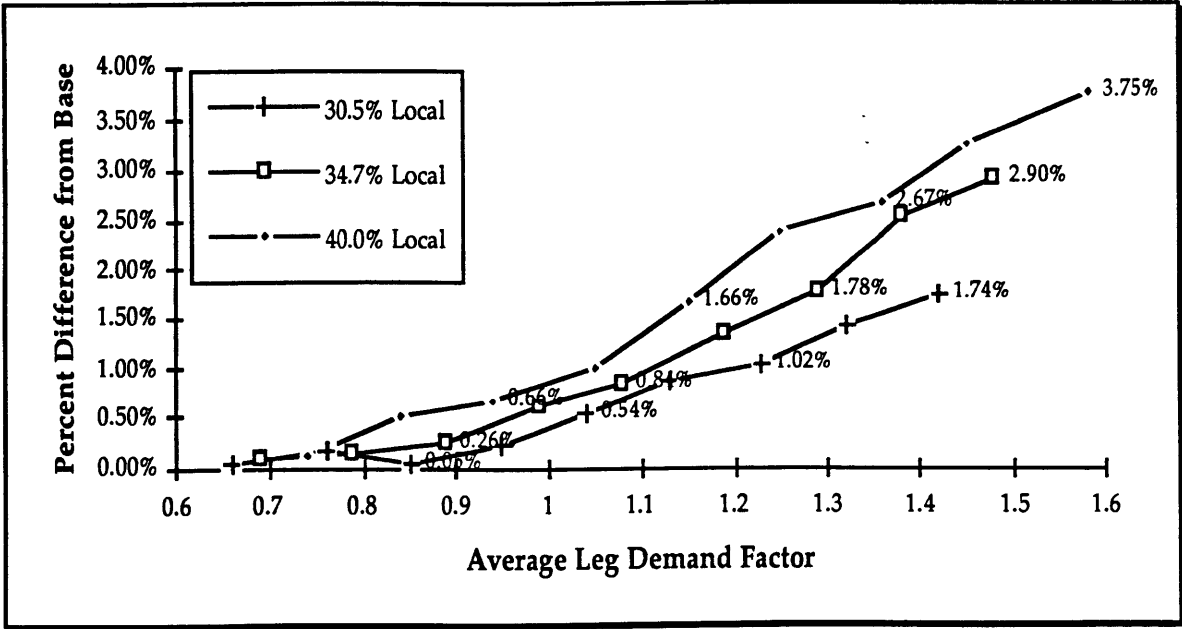


Figure 5.9 - Revenue Impacts of the Network Fare Class Displacement Cost Algorithm

to determine availability, instead of the network revenue values associated with each leg traversed.

The level of incremental gains over a fare class nesting approach realized by a network fare class displacement cost algorithm is quite substantial, ranging from 0.04% to 4% depending on the level of demand encountered. Load factors achieved were also good ranging from 0.66 to 0.91. On a more realistic demand factor range of 0.7 to 1.1, the level of revenue gains expected ranged from 0.1 to 1% for an average local demand of 34.7% and 0.5% to 2% for an average local demand of 47%. The trends illustrated by the algorithm are similar to those exhibited by other displacement cost algorithms with revenue impacts dependent on the level of local demand in addition to the average leg demand factor of the network.

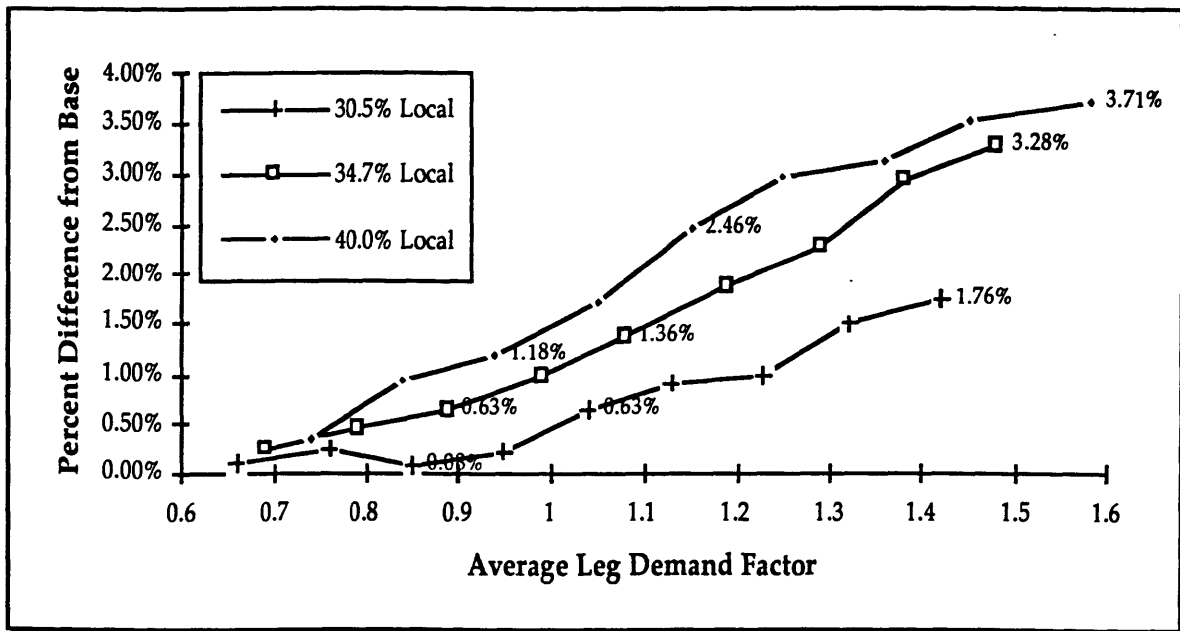


Figure 5.10 - Revenue Impacts of the Leg Fare Class Displacement Cost Algorithm



Figure 5.10 illustrates the simulation results of the leg fare class displacement cost algorithm presented in Example 5.1. Once again, significant incremental revenue gains above the base case are realized over all demand scenarios. Take for example an average local demand of 34.7%, for demand factors of 0.7 to 1.1, revenue impacts realized ranged from 0.3 to 1.4% with good average load factors of 0.68 to 0.87. Overall, expected incremental revenue gains ranged from 0.1% to 4% above that of the base case with average load factors of 0.66 to 0.90.

Recall that the network algorithm utilizes the maximum network revenue value over all legs traversed to determine which control bucket to display availability and the leg algorithm utilizes the individual network revenue values calculated on each leg traversed. In a comparison to the

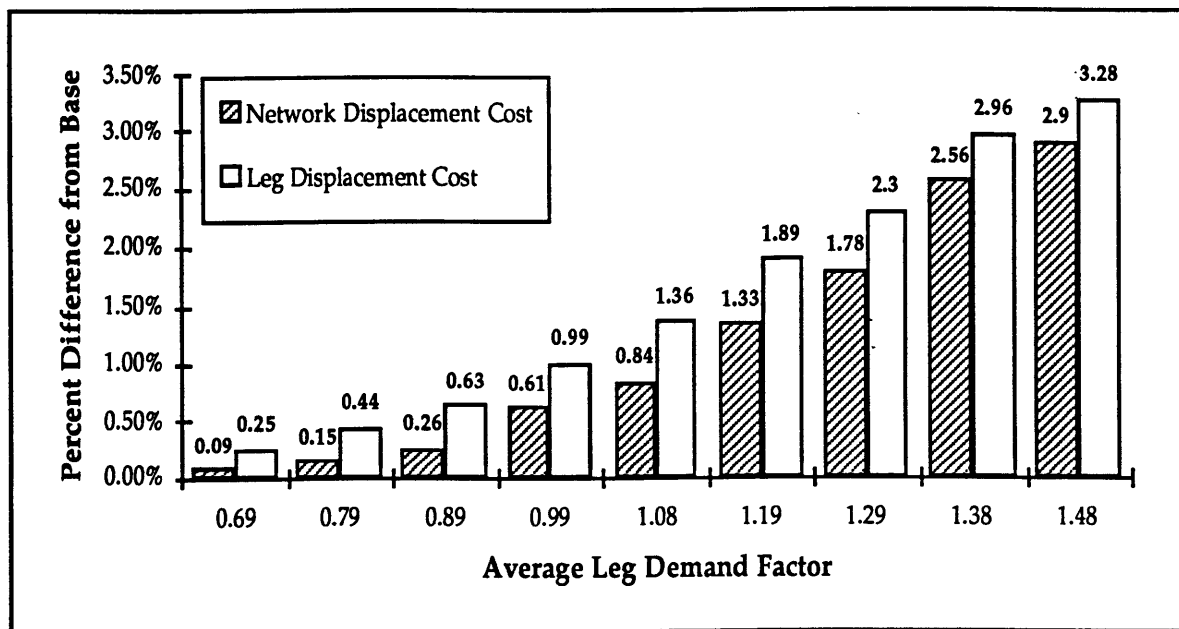


Figure 5.11 - Revenue Impacts of the Network versus the Leg Fare Class Displacement Cost Algorithm

network fare class displacement cost algorithm, Figure 5.11 shows that the leg fare class displacement cost out performs the former, based on revenue gains

over the base case in all levels of demand tested. In particular, the leg algorithm showed revenue gains over the network algorithm of over 0.4% in some cases. Although the average load factors achieved by the leg algorithm were on average 1 to 2 percentage points lower than the network algorithm, the extra revenue gains realized signify that the determination of ODF availability though utilizing the network revenue values associated with each leg is beneficial for improved revenue performance by a fare class displacement cost algorithm.

Even though the level of potential revenue benefits that can be realized from a fare class displacement cost algorithm is quite substantial when weighed against fact the no additional investment in developing an OD control structure is necessary. It is obvious from Figure 5.12, that a fare class displacement cost seat inventory control algorithm is not capable of out performing an OD seat inventory control algorithm that utilizes a virtual

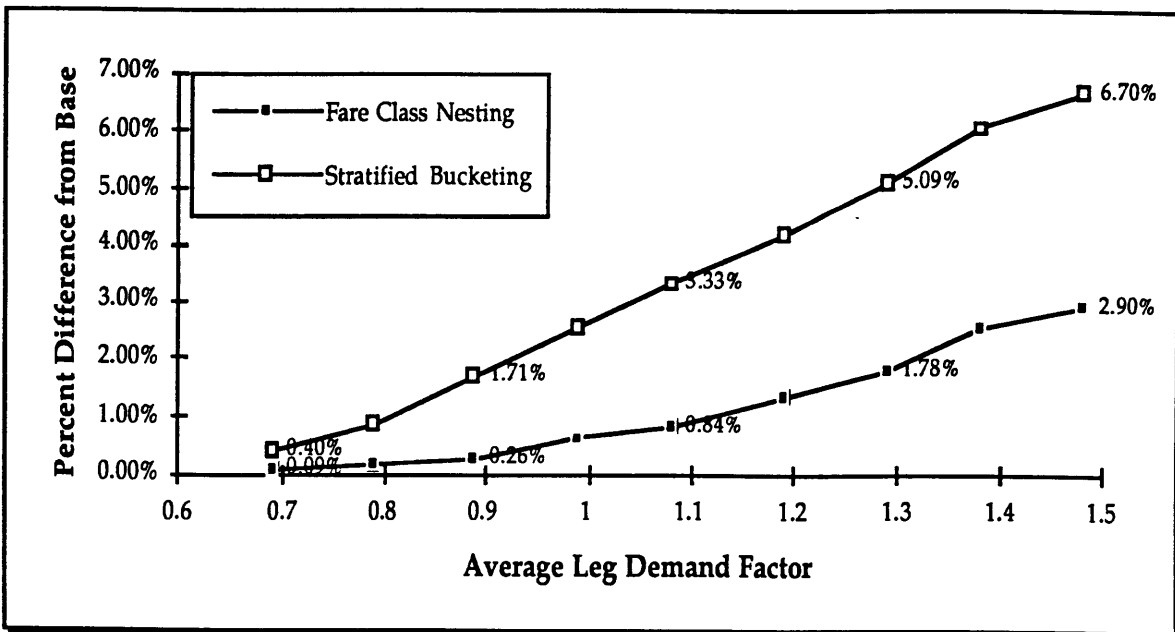


Figure 5.12 - Revenue Impacts of the Network Fare Class versus the Network "Real-Time" Displacement Cost Algorithm

nesting or stratified bucketing control structure. However, the positive revenue impacts from utilizing displacement cost logic on a non-OD based inventory control structure is a good indication of the potential benefits to be had from incorporating such a concept into any control process.

## 5.6 Summary of Simulation Results

It is not the intent of this research to make judgments on which OD control algorithm will prove to be the best method to implement. On the contrary, its intent is to present a range of possible alternative approaches to an OD control methodology in addition to providing empirical evidence as to its effectiveness. In this Chapter, a study of the effectiveness of several leg based inventory control algorithms that incorporated local displacement cost logic was undertaken through the analysis of simulation results. Based on a set of demand levels that is assumed representative of "real-world" demand environments, Figure 5.13 and 5.14 summarizes the revenue impacts realized by the displacement cost control algorithms introduced within this chapter for a demand factor range of 0.7 to 1.1.

Figure 5.13 represents the maximum expected revenue gains for OD control algorithms that have incorporated a "static" displacement cost methodology into the inventory control process. These algorithms have been developed with both a virtual nesting and a stratified bucketing control structure. As empirically shown, the leg "static" displacement cost methodology has realized the largest expected incremental revenue gains over the base case, although the other algorithms do perform as well in comparison. In some respects, the leg "static" algorithm can be expected to

perform the best as it utilizes the highest amount of available information in its decision process.

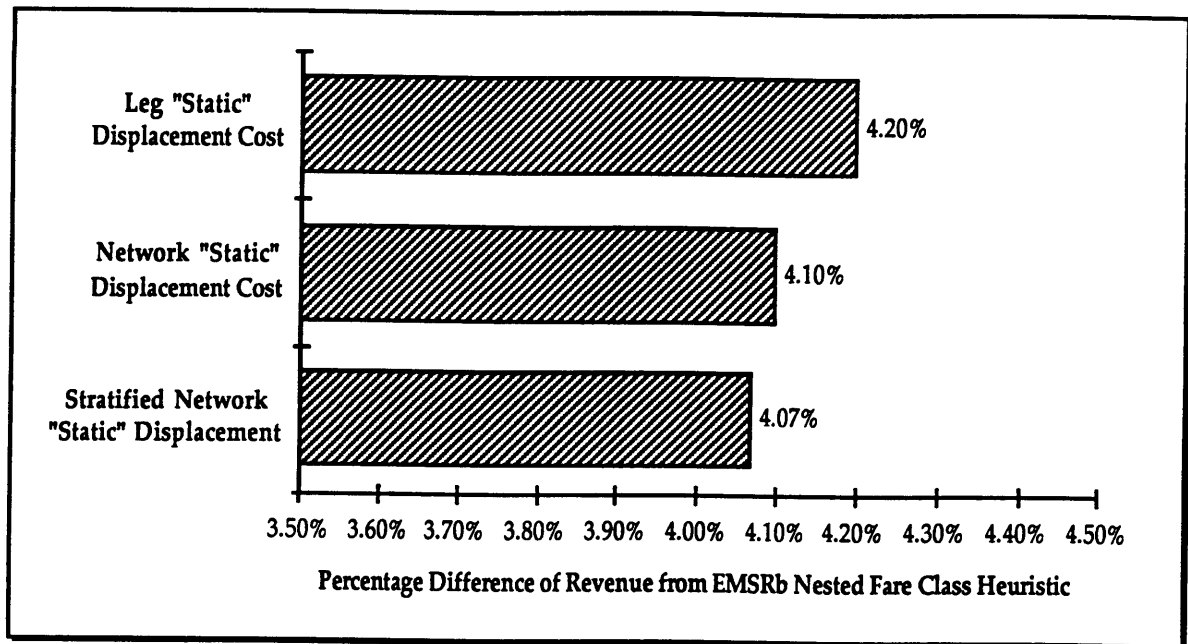


Figure 5.13 - Maximum Expected Revenue Impacts for Several "Static" Displacement Cost Algorithms

Figure 5.14 illustrates the maximum expected incremental revenue gains for a set of seat inventory control algorithms based on "real-time" displacement cost logic. As previously mentioned, the flexibility of a "real-time" approach has allowed the "real-time" algorithms to utilize control structures that are not compatible with frequent shifts in fare assignments. In this research a virtual nesting approach was not tested with a "real-time" displacement cost approach. Nevertheless, for our purpose, a stratified bucketing control structure is identical in design to that of a virtual nesting control structure, only differing by the number of control buckets used. Thus, the revenue impacts realized by a stratified inventory control method can be extended to that of a virtual nesting based algorithm.

In examining the simulation results of a "real-time" algorithm, unlike the "static" algorithms, the "real-time" algorithm that incorporates the greatest amount of available information into the control process did not provide the largest return on expected revenue. Additionally, the lower revenue impacts obtained from incorporating displacement cost logic into a fare-class nesting control structure serves to stress the importance of integrating the correct optimization and control elements in a seat inventory control algorithm. That is, even though displacement cost algorithms were developed to enhance the effectiveness of controlling inventories by evaluating the revenue contributions of bookings with respect to the network, without a proper control structure, even the effectiveness of the best displacement cost algorithms are severely restricted.

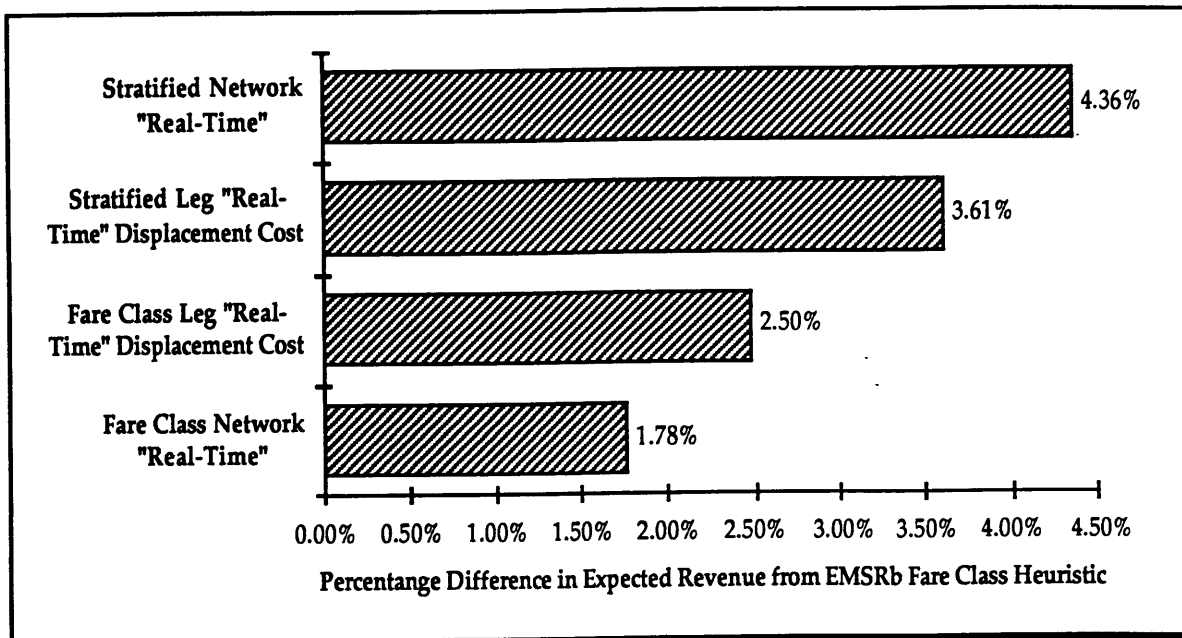


Figure 5.14 - Maximum Expected Revenue Impacts for Several "Real-Time" Displacement Cost Algorithms

In general, while the simulation results have shown average expected revenue gains ranging anywhere from 1 to 8%, taking into consideration the

kind of uncertain demand environments the control process works in, a more realistic expectation would range from 2 to 4% above that of the base case. Furthermore, the impact of these incremental revenue gains must be weighed against the financial and operational investment required for the development and implementation of new inventory control approaches. Consequently, the algorithms introduced in this chapter differ not only in the expected revenue impacts but also in the level of investment required in implementing such algorithms. Still the positive levels of revenue gains shown by incorporating different displacement cost logic to a variety of control structures have illustrated the potential range of revenue benefits that can be realized.

# Chapter 6

## Conclusions

### 6.1 Summary

The primary objective of this research was to develop leg-based inventory control approaches that address the problems associated with the network seat inventory control problem. By taking into account the interactions between flight legs and traffic flow across the network, two structural components of the basic seat inventory control approach were pinpointed as areas of interest. The OD control structure was first examined for its effectiveness in controlling seat inventories by revenue value rather than product type such that the potential for increased network revenue is realized. The second involves the incorporation of displacement cost logic into the optimization algorithm to improve the allocation of seats to different local and multi-leg OD fare itineraries across the network. By combining the different control structures and optimization algorithms presented in this research, a variety of leg-based network inventory control approaches were developed and evaluated within a simulated hub-and-spoke network.

A major reason behind the development of new approaches to manage and control seat inventories is to capture incremental revenue gains over

current leg-based seat inventory approaches. As such, the revenue performances of new approaches can be determined through the comparison of revenue impacts obtained versus an existing inventory control approach. Consequently, by utilizing the integrated optimization/booking simulator developed at MIT, a sterile environment was constructed, whereby the revenue impacts realized by the newly developed control algorithms with respect to a leg-based fare class seat inventory control algorithm can be compared.

Empirical results obtained through the use of the integrated optimization/booking simulation show that direct application of the OD control structures introduced in this research into a leg-based seat inventory control algorithms utilizing an EMSRb optimization heuristic can achieve incremental revenue gains ranging from 0.5% to 3.5% depending on the types of demands encountered. However, recall that the simulation while an effective tool in evaluating the relative performances of new algorithms, operates under unrealistic demand conditions. Passenger demand is by nature highly dynamic and stochastic, which is not taken into full consideration in the simulation. Taking this into account, the revenue impacts obtained by the simulation are usually a more optimistic representation of "real-world" results.

In this research, while the "greedy" algorithm utilizing a virtual nesting control structure has been shown to provide the largest revenue gains compared to all other "greedy" approaches utilizing some variation of virtual nesting or stratified bucketing, it is important to evaluate the significance of these empirical results relative to the additional investment required to implement such inventory control algorithms.



In incorporating displacement cost logic to alleviate the greediness of "greedy" seat inventory control algorithms, simulation results reveal revenue gains ranging from 1% to 4% above that of a leg-based fare class control algorithm. Naturally, the level of revenue gains realized is dependent on how displacement cost logic is applied by the inventory control algorithm. In this research, both static and dynamic applications of displacement cost within an OD control structure were considered. Furthermore, an experimental application of dynamic displacement cost logic was examined within a fare-class control structure. Revenue impacts for a fare-class based displacement cost algorithm ranged from 0.5% to 2.5% above a leg-based fare class nesting control algorithm that does not employ displacement cost logic.

The level of incremental revenue gains obtained from simulation results in this research corresponded well with a predicted upper bound approximation of 4% for leg-based network inventory control methods employed within a hub-and-spoke network [6]. While there is no doubt that effective control of ODF itineraries at the network level can provide significant revenue benefits, the utilization of full network optimization methods still must overcome several practical obstacles to proper implementation within existing reservations control systems. The development and implementation of leg-based heuristic control approaches, which incorporate information about passenger demands and traffic flows across the network, can result in incremental revenue gains of as much as 4% above current control approaches. These revenue gains can translate to a substantial increase in total airline revenues.

## 6.2 Future Research Directions

As the science of network seat inventory control is a relatively recent area of development, there are many opportunities to extend the research presented in this thesis to new areas of continued research. In particular, much of the conclusions reached about the effectiveness of the control approaches presented in this research are based on simulation results. The booking/optimization simulator used in this research disregards the effects of cancellations, no-shows, passenger upgrades and many other "real-world" factors in order to simplify the simulation and evaluation process. It is important to address the role each of these factors will play in altering the effectiveness of a seat inventory control approach before actual implementation into a new reservation control system.

In this research, many of the control algorithms introduced employ some form of virtual nesting or stratified bucketing as the basis for controlling seat inventory. While the basic application of these control structures, the number of control buckets, the definition of revenue ranges, and the level at which a virtual mapping table is applied were discussed in the thesis, there is still much that can be done to further the research in these areas. The number of control buckets used and methodologies developed to define revenue ranges in the control algorithms presented cannot be considered "optimal". Since the appropriate determination of these factors have been shown to have a significant effect on the performance of a seat inventory control algorithm, further work into the proper definition of these parameters should be examined.

The approximation of displacement cost and the application of displacement cost logic are necessary areas of continued research and development. In this research, an approximation to local displacement cost reflects the probability of selling a seat to a local passenger and the revenue contribution to the network of a local itinerary. However, the displacement of passengers on down-line flight legs does not only include local passengers, but other passengers connecting to the leg from other flight legs in the network. It is important to determine the effects of disregarding the displacement of other connecting passengers on the performance of control approaches that apply displacement cost logic. Furthermore, displacement cost is currently approximated on a flight leg basis, based on the aggregation of all ODF itinerary demand and fares on the flight leg. Would it be possible to break displacement cost approximations down to an OD level whereby each OD traversing the leg can have an associated displacement cost value? Or, have displacement cost values associated with each control bucket rather than the last seat on each flight leg? Or, have displacement cost values determined on some network level? Is the expected marginal revenue curve the best approximation of displacement cost or are there some other variations of the EMR curve? Obviously, there is much to be done in this area of research

Additional analysis with respect to the type of optimization heuristic utilized in each control algorithm would be interesting. In this research, the EMSRb optimization heuristic is utilized in every algorithm introduced. Consequently, is this optimization heuristic generally compatible with all control structures and displacement cost approaches discussed? Are there other optimization heuristics more compatible with certain control structures versus others? The proper integration of optimization and inventory control

components is important to the successful performance of any control algorithm. Thus it is just as important to examine the optimization heuristic itself in addition to any other components that makeup a leg-based network seat inventory control algorithm.

While the focus of this research has been on the development of leg-based approaches to network seat inventory control, a natural extension of this research is to move towards the development of full network seat inventory control methods. With respect to full network approaches, work on developing an effective and efficient mathematical algorithm is necessary. The development of an ODF database and its associated implementation issues would also be required, in addition to the potential effects on revenue of implementing network seat allocations in current control structures, and the communication difficulties associated with relaying ODF availability between CRS's. The financial investment required to develop these approaches and the flexibility built into future reservation control systems to adapt to changes in a dynamic airline industry should also be addressed for both leg-based and full network inventory control approaches.

An increasingly competitive airline industry has dictated the development of more sophisticated tools to manage and control seat inventories, not only as a means to remain competitive but as a means to better utilize limited resources. Depending on the nature of the competitive environment in which a particular carrier operates, its route structure, the size of the carrier, and the types of markets served can all affect the determination of which seat inventory control approach will best serve the carrier. However, in almost any environment, the utilization of a network approach to control seat inventories can provide a substantial return in terms

of increased revenues. As carriers develop route structures that include many multi-leg segments, the revenue benefits of a network control approach will become increasingly important.

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