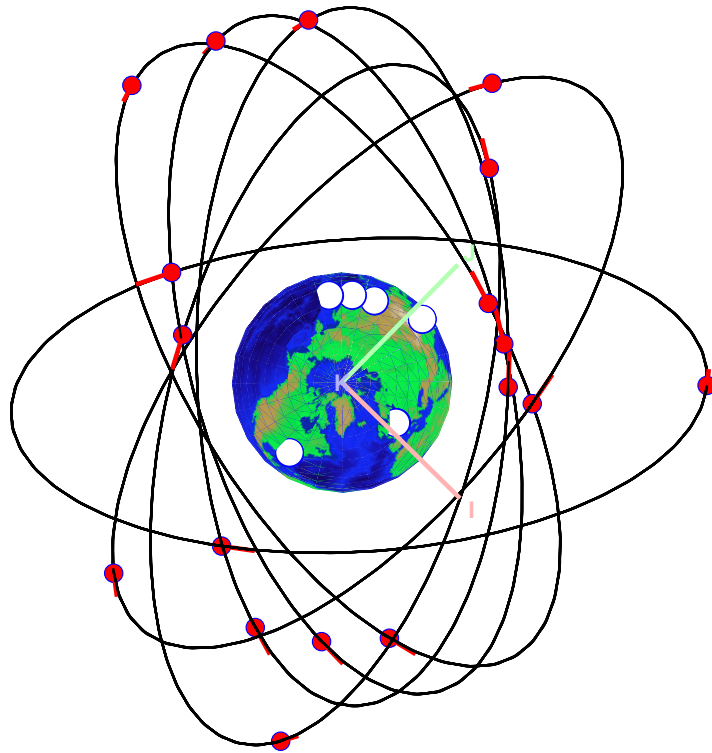# 16.888/ESD.77 2003 - A6

Optimization of a Hybrid Satellite System Constellation

Serena Chan   Ayanna Samuels   Nirav Shah
Jennifer Underwood
Massachusetts Institute of Technology, Cambridge, MA 02139

16.888/ESD.77 Multidisciplinary System Design Optimization
12 May 2003
Cambridge, MA

Optimization of a Hybrid Satellite System Constellation

Serena Chan[*]   Ayanna Samuels[†]   Nirav Shah[‡]   Jennifer Underwood[§]

Massachusetts Institute of Technology, Cambridge, MA 02139

Previous research in satellite constellation designs has focused on Low Earth Orbit (LEO) communication systems that service a global market of uniform demand. This paper addresses the challenge of designing a hybrid satellite system constellation (e.g., LEO satellites with elliptical orbiting satellites) to meet the demands of the emerging satellite broadband market. The broadband market model that is developed highlights the highly non-uniform distribution of traffic that future satellite systems must handle. The approach to the problem and the simulation design are discussed. The problem statement is formulated as an optimization problem. Results from techniques such as Sequential Quadratic Programming (SQP) and Simulated Annealing (SA) are provided and analyzed. A Pareto front of the set of optimal hybrid satellite system designs that solve the formulated optimization problem is provided. The presented design architecture allows for a phased satellite deployment strategy: a LEO backbone satellite constellation to capture a fraction of the total market demand with additional elliptical satellites to service areas of high and growing demand.

Introduction

SATELLITE constellation design is a complex process that requires evaluation of many issues and analysis of orbit characteristics. The fundamental rationale for using multiple satellites is to provide greater Earth coverage. A single satellite cannot provide as many observations nor as frequent communications as multiple satellites can. Unfortunately, there are no absolute rules nor a standard policy for designing a constellation. The complexity of the problem demands that in choosing a preliminary design, alternate designs must be carefully assessed and reasons for the final choices must be documented.

A discussion of the key issues and orbit characteristics necessary for the constellation design process is provided by Wertz and Larson (1999).[1] Use of circular polar orbit constellations have been shown by Rider (1985) to be efficient for providing highly redundant Earth coverage.[2] Adams and Rider (1987) extended that work to analyze circular polar constellations providing single or multiple coverage above a specified latitude.[3] Lang and Adams (1998) and Turner (2002) highlighted utilizing Walker patterns as another candidate constellation for providing continuous Earth coverage.[4,5] Numerous circular polar and Walker constellations are examined by de Weck and Chang (2002) in their development of a methodology for examining the architecture trade space for Low Earth Orbit (LEO) personal communication systems.[6]

Problem Motivation

There are two underlying assumptions common to previous research in satellite constellation design. First, all satellites within a constellation use circular orbits and the same altitudes. Alternate designs with varying satellite altitudes should be considered. Second, a uniform distribution of market demand for satellite communications is assumed. This assumption poorly models the amount of traffic seen on the satellites given the non-uniform distribution of the population on the Earth and their desire for or ability to access satellite communications. This paper relaxes these two common assumptions and explores the design of a hybrid satellite system constellation to meet the emerging satellite broadband market. The hybrid satellite system consists of a fleet of LEO satellites that serve as a backbone to provide global coverage that supports a small user base and a number of elliptical satellites the high demand concentrated in certain areas of the globe. A hybrid constellation allows system deployment to occur gradually. The first phase consists of deploying the LEO backbone. Subsequent phases involve deploying elliptical satellites. This can be an economically feasible strategy that prevents under-utilization of the space system.

In the following sections, the problem is defined and several approaches for multidisciplinary system design optimization are utilized to determine the final choice of a low-cost hybrid satellite constellation with enough

---

[*]Research Assistant, Laboratory for Information and Decision Systems (LIDS)

[†]Research Assistant, Space Systems Laboratory (SSL)

[‡]Research Assistant, Lean Aerospace Initiative (LAI)

[§]Student

capacity to meet the needs of the emerging satellite broadband market. Results from Sequential Quadratic Programming (SQP) and Simulated Annealing (SA) are provided and analyzed. A Pareto front of the set of optimal hybrid satellite system designs that solve the formulated optimization problem is given.

a) Gross National Product (GNP) adjusted by Purchasing Power Parity (PPP).

Problem Definition and Approach

The purpose of this paper is to present the design and optimization of a hybrid satellite system constellation. The systems architecture problem is rationalized and quantified by selecting appropriate design variables, objective functions and constraints. Methods and tools in multidisciplinary design optimization for systems are implemented and discussed. The traditional numerical optimization algorithm selected is sequential quadratic programming (SQP). The heuristic optimization technique selected is Simulated Annealing (SA). Interpretation of analysis and optimization results, including sensitivity analyses and an exploration of performance and cost tradeoffs are also provided.

The systems architecture problem can be stated as follows: "How should the satellite constellation orbit, altitude, and elevation angle selections be sized for a given market demand distribution, satellite transmit power, antenna size, channel bandwidth, satellite lifetime, and number of satellites, such that user data rates are maximized, while minimizing cost and maximizing system throughput?"

For this problem, the LEO backbone satellites and the elliptical orbit satellites are constrained to be uniform (i.e., the same spacecraft design) but the two sets may not necessarily be the same. In order to optimize the hybrid satellite space system to non-uniformly distributed demand, the satellite broadband market must first be identified. The satellite broadband market data is collected from Kashitani (2002).[7] The market demand is determined by using the Gross National Product (GNP) adjusted by Purchasing Power Parity (PPP) (i.e., converting GNP to international dollars), shown in Figure 1(a), and using the worldwide population distribution, shown in Figure 1(b). The international dollar is equivalent to the United States dollar. Figure 1(a) and Figure 1(b) are then combined to create the market demand for satellite broadband services. The market demand is illustrated in Figure 1(c).

b) Global Population Distribution.

c) Market Demand Distribution.

Fig. 1   Global Distribution Maps.[7]

Problem Formulations

The single objective problem is to minimize the lifecycle cost (LCC) of the entire hybrid satellite constellation architecture. The LEO backbone constellation optimization is derivative of the research done by de Weck and Chang (2002). The elliptical system is constrained to meet at least 90% of market demand and must have a strictly positive total lifecycle cost. These constraints are satisfied by meeting the following requirements:

- A satellites must service 100% of the users in its footprint

- The satellite system must service at least 90% of the demand at any given point in time

- Data rate provided by the satellites must be greater than or equal to the date rate demanded

- All satellites must be deployable with current launch vehicles.

Table 1 LEO Backbone Constellation Design Vector.

Symbol	Variable	unit
C	Constellation type (Polar or Walker)	[-]
h	Orbital altitude	[km]
emin	Minimum elevation angle	[deg]
Pt	Satellite transmitter power	[W]
DA	Satellite antenna diameter (parabolic antenna)	[m]
MA	Multiple access (TDMA, FDMA, CDMA, MFTD, MFCD)	[-]
ISL	Intersatellite links (1 = yes, 0 = no)	[-]

Table 2 Elliptical Constellation Design Vector.

| Symbol	Variable	unit
T	Orbital period	[day]
e	Eccentricity	[-]
NP	Number of planes	[-]
Pt	Satellite transmitter power	[W]
DA	Satellite antenna diameter (parabolic antenna)	[m]

Design Vectors

Design vectors embody the architectural design choices. Table 1 and Table 2 summarize the design variables used for the LEO backbone optimization and elliptical constellation optimization, respectively.

Model Decomposition

The optimization of a hybrid satellite system can be decomposed into two separate optimization problems. The total lifecycle cost of the entire system is the sum of the total lifecycle cost of the LEO backbone constellation and the total lifecycle cost of the elliptical constellation. Because the design of the LEO backbone constellation is based on research done by de Weck and Chang (2002), only slight modification of their simulation code is necessary to address the broadband market demand (e.g., number of subscribers, required data rate per user, average monthly usage, etc.). The LEO backbone is assumed to service 240,000 users and the satellite market is assumed to demand a data rate of 1 Mbps per user. The remainder of this section discusses the development and validation of the simulation code necessary for optimizing an elliptical constellation to service the remaining market demand.

The elliptical satellite constellation problem is decomposed into "black boxes," or modules, based on
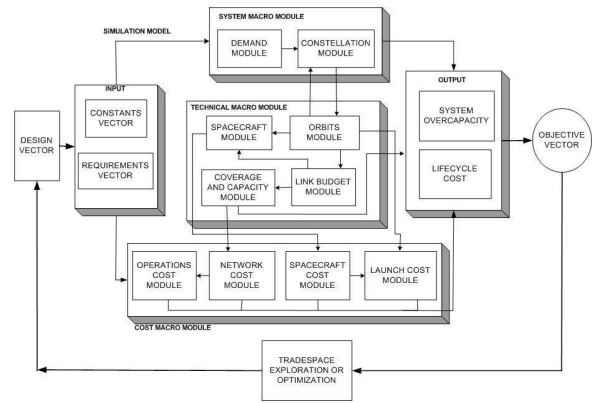
Fig. 2 Hybrid Constellation Simulation Block Diagram.
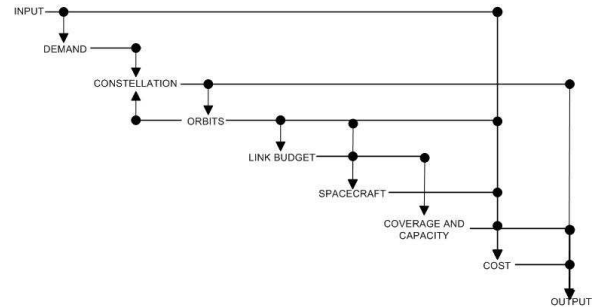
Fig. 3 N^2 Diagram of Simulation Architecture.

disciplinary tradition and degree of coupling of governing equations. A module in multidisciplinary system design optimization is defined as a finite group of tightly coupled mathematical relationships where some variables represent independent inputs while others are dependent outputs.[11] Figure 2 illustrates the block diagram of the various simulation modules used for this paper.

To develop and organize interface information, an $N \times N$ matrix is used. Each module within the simulation architecture is placed along the diagonal. Figure 3 provides a visual representation of the flow information through the simulation architecture. This technique is used to identify critical modules that have many inputs and outputs. Attempts have been made to minimize the number of feedback loops between modules. The fidelity of critical modules are then thoroughly tested and verified.

Simulation Design

Design of Experiments (DOE)

A design of experiments (DOE) provides a systematic method to sample the design space of potential solutions. It is a technique often used to set up a formal optimization problem. Key drivers among potential design variables, appropriate design variable

ranges and achievable objective function values can all be identified.

In the design space of the elliptical constellation, there are 5^4 possible combinations of design variables to explore (i.e., 5 factors, each with 4 levels). An orthogonal array is selected for the design space exploration. Orthogonal arrays have a balancing property, i.e., for any pair of columns, all combinations of factor levels occur and they occur an equal number of times. Results are provided in Table 3. Lifecycle cost is only calculated when the perigee of the satellite constellation is not too low (i.e., they must enter the upper atmosphere). Feasible solutions are defined to be servicing 90% of the market demand at any given time.

The results in Table 3 are subsequently used to calculate effects. The effect of a factor is the change in the response as the level of the factor is changed. The overall mean response, approximately 2661.2 [$M], is the average of the lifecycle cost of each experiment. The average over all experiments for a given factor and level is the average of the lifecycle costs of experiments with that factor and level. The effect of each specified factor and level is defined as the difference between the overall mean response and the average over all experiments. The effects are all summarized in Table 4.

From this analysis, it can be seen that the factor of Antenna Power (DA) has the largest effect at a power level of 10000 [W]. The experiment with the lowest lifecycle cost is recommended to be the initial starting point for solving the optimization problem. Thus, the design vector is: $x_0 = [\frac{1}{6}, 0.6, 4, 500, 3]^T$. It should be noted that this design has a perigee that is too low, therefore the optimization problems that follow uses a different design vector as the initial starting point.

Simulation Design

For computational efficiency, the resolution of the satellite broadband market demand map is reduced to square grids of 15° latitude by 15° longitude, as shown in Figure 4. The demand distribution map is also normalized such that the sum of all elements equals one. The areas demanding the greatest satellite broadband services are located in the United States, Europe, and East Asia.
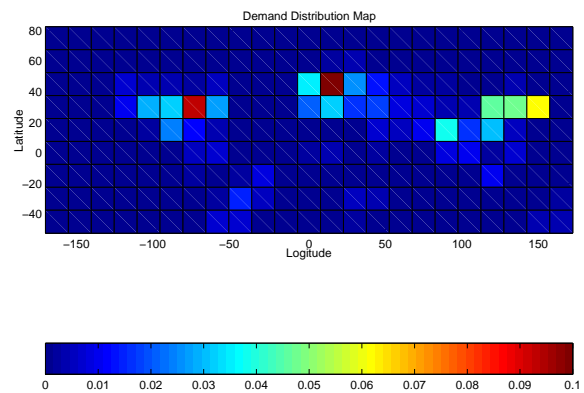
Several simplifying assumptions are included in the elliptical constellation simulation. The inclination angles of the elliptical satellites are fixed at 63.4°. At this inclination, the perigee will not rotate thus both apogee and perigee can be maintained over fixed latitudes. The elliptical satellites are also designed to use a repeating ground track to ensure specified longitudes are visited regularly.

Code Validation and Benchmarking

The satellite system Ellipso is used to benchmark the written simulation routines. The Ellipso satellite constellation consists of 24 satellites placed in 4 elliptical orbital planes phased 90° apart. The doc-

Table 4 Elliptical Constellation DOE Effects.

Factor	Level	Units	Average over all Experiments	Effect
T	4	[hours]	2453.9	-207.3
T	6	[hours]	2821.0	159.8
T	12	[hours]	2792.3	131.13
T	24	[hours]	2565.7	-95.5
e	0	[-]	2715.0	53.8
e	0.2	[-]	2647.2	-13.98
e	0.4	[-]	2879.1	217.93
e	0.6	[-]	2145.65	-515.55
NP	1	[-]	2399.2	-262
NP	2	[-]	2624.4	-36.85
NP	3	[-]	3378.8	717.57
NP	4	[-]	2342.1	-319.13
Pt	500	[W]	1685.4	-975.78
Pt	1000	[W]	1811.7	-849.5
Pt	5000	[W]	3193.2	532.03
Pt	10000	[W]	4102.3	1441.1
DA	1.5	[m]	2977.0	315.8
DA	2.0	[m]	2686.3	25.15
DA	2.5	[m]	2827.4	166.25
DA	3.0	[m]	2090.2	-571.0

Table 3 Elliptical Constellation Design of Experiments.

Experiment Number	Period (T) [hours]	Eccentricity (e)	Number of Planes (NP)	Antenna Power (Pt) [W]	Antenna Diameter (DA) [m]	Lifecycle Cost [$M]	Feasible Solution?
1	4	0	1	500	1.5	1558.6255	No
2	4	0.2	2	1000	2	1842.9555	Yes
3	4	0.4	3	5000	2.5	3960.0706	Yes
4	4	0.6	4	10000	3	Perigee too low	No
5	6	0	2	5000	3	2901.7951	Yes
6	6	0.2	1	10000	2.5	3752.2352	No
7	6	0.4	4	500	2	1809.0035	Yes
8	6	0.6	3	1000	1.5	Perigee too low	No
9	12	0	3	10000	2	4490.2813	Yes
10	12	0.2	4	5000	1.5	3307.9225	Yes
11	12	0.4	1	1000	3	1682.8762	Yes
12	12	0.6	2	500	2.5	1688.1829	Yes
13	24	0	4	1000	2.5	1909.2554	Yes
14	24	0.2	3	500	3	1685.8701	Yes
15	24	0.4	2	10000	1.5	4064.5362	Yes
16	24	0.6	1	5000	2	2603.0889	Yes

Fig. 4 Market Demand Map Used in Simulation.

umentation filed with the Federal Communications Commission (FCC) in 1990 by Ellipso, Inc. provided the data necessary for benchmarking. Variables and parameters that are required by the simulation but are not readily available in the documentation required best-guess assumptions to be made. Several of the simulation routines required validation that could not be benchmarked with the Ellipso data. A brief discussion of all of the important simulation modules and of their validation and Ellipso benchmarking is provided in the following subsections.

Orbit and Constellation Calculations

The orbit calculation simulation code (*compute_orbit*) generates a time series of the satellite orbit in inertial-earth centered Cartesian and spherical coordinates. The function takes in the following inputs: the apogee altitude [km], the perigee altitude [km], the inclination angle [deg], the right ascension of the ascending node (RAAN), the argument of perigee and the true anomaly at epoch. The following outputs are returned: a vector of Cartesian coordinate positions, a vector of Cartesian coordinate velocities, a vector of longitudes, a vector of latitude and a vector of radius. Each element in the vectors represents a time step, measured in seconds. Orbits are propagated for 1 day. Because ground tracks repeat, coverage results are repeatable from day to day.

The constellation calculation simulation code (*compute_constellation*) calls *compute_orbit* to propagate the entire constellation of orbits. Repeating ground tracks are created for the orbiting satellites based on the design variable for the orbital period. The number of orbital planes is another design variable input. Satellite planes are priority allocated to regions of highest demand (i.e., the first satellite plane is used to serve the location with the highest user demand, the second satellite plane is used to serve the next highest demand location, etc.). The *nosatscalculator* function calculates the number of satellites needed within each satellite plane to provide a continuous street of coverage within a 15° band. A continuous street of coverage within the 15° band helps to ensure that the constraint of servicing 90% of market demand at any point in time is met. The number of satellites within each plane is also evenly spaced apart in time in order to provide continuous access and coverage to the users. This *nosatsclculator* computation is done at perigee because a spacecraft provides the least amount of Earth coverage at perigee. If continuous coverage is achieved at perigee, it will guarantee coverage overlaps for the remainder of the satellite orbit with the greatest amount of overlap at apogee.

To validate the above functions, the plotting routines (*plot_orbit* and *plot_constellation*) were used to visually check that the correct orbits are calculated.

Link Budget Calculation

The inputs to the link budget calculation routine are: the altitude [km], the minimum elevation angle [deg], the satellite transmit power [W], the aperture diameter of the satellite antenna [m], the Multiple Access Scheme [-], the downlink frequency [GHz], the Bit Error Rate [-], the gain of the receiving antenna [dB], the illumination efficiency [-], the modulation scheme used [-], and the link margin [dB]. In this routine, the number of downlink channels (NDC) per satellite is arbitrarily assumed to be 1000. The data rate per satellite is then returned as the output.

Spacecraft Module

The inputs to the spacecraft module are: the altitude [km], the satellite transmit power [W], the antenna aperture [m], and the number of ISL links per satellite [-]. The spacecraft module returns payload mass and spacecraft mass as outputs. For an elliptical orbiting satellite, the calculation is taken at apogee because it will provide the worst case output.

Lifecycle Cost Calculation

Lastly, the lifecycle cost calculation module is benchmarked with the Ellipso satellite constellation. First, launch costs for an elliptical satellite is determined by the cost of launching the system into its perigee altitude and the cost of a kick motor for orbital maneuvering. The inputs for calculating launch costs are: the perigee altitude [km], the eccentricity [-] of the orbit, the wet mass of the satellite [kg], the volume of the satellite [m^3], the number of satellites launched [-], the number of planes [-], and the inclination of the orbits [deg]. To compute the lifecycle cost of the elliptical satellite constellation, the inputs are: the number of satellites launched [-], the number of planes [-], the altitude of the orbit [km], the wet mass of the satellite [kg], the dry mass of the satellite [-], the antenna aperture [m], the transmitter power [W], the total power of the satellite [W], the launchcost [$], the number of ground stations [-], the number of ISL's [-], and the lifetime of the satellites [years]. Again, the altitude is taken to be the perigee of the elliptical orbit. This module also includes an estimated cost required for radiation hardening.

Table 5 summarizes the results of benchmarking the elliptical constellation simulation code with values taken from the FCC filing for the Ellipso satellite constellation system.

Optimization Results

Single Objective Optimization

The single objective problem is to minimize the life-cycle cost of the elliptical constellation system subject to the constraints listed in the Problem Formulations section. The design vector is:

Orbital period (T)	[days]
Eccentricity (e)	[-]
Number of places (NP)	[-]
Transmitter power (Pt)	[W]
Antenna diameter (DA)	[m]

Gradient-Based Optimization: SQP

The gradient-based algorithm most appropriate to solve the single objective optimization problem at hand is Sequential Quadratic Programming (SQP). This algorithm is a non-linear technique that has a strong theoretical basis and is widely used for many engineering applications. The integer design variable for the number of planes did not cause any problems in the use of SQP.

The initial design vector used here is: $x_0 = [0.5, 0.01, 4, 4000, 3]^T$ and the corresponding value of the objective, J, is 6280.5999 [$M]. The optimization routine utilizing SQP converged to a solution with the following design vector: $x^* = [0.7, 0, 4, 3999.7, 1.76]^T$. The corresponding value of the objective, J^*, is 6187.8559 [$M]. Because the simulation runs are extremely time intensive, alternate design vectors used

Table 5   Benchmarking Results Using Ellipso Values.

System Module		Ellipso	Simulation	Units
Link Budget	Antenna Gain	12	11.93	[dBi]
	EIRP	27	24.93	[dBW]
	Data Rate	2.2	1.08	[Mbps]
Spacecraft	Sat Mass	68	98.68	[Kg]
	Sat Volume	0.0008	0.810	m³
Lifecycle Cost		249.6	290.9	[YR2002 $M]

as the initial starting point are not explored.

The limitation to using SQP is that it requires continuous design variables. However, the orbital period is a discrete design variable whose value must be $\frac{1}{n}$ where n is a positive integer. This constraint is due to the fact that the simulation only propagates orbits for one day. In order to assess the performance of x^*, which has a period of 0.7 [day], the orbit needs to be propagated until it repeats. Therefore, x^* cannot be verified as the optimal solution.

Sensitivity Analysis

Sensitivity analysis is conducted to understand which design variables, constraints and parameters are important drivers for the optimum solution x^*. The question to be examined is: "How does the objective function change as the elements of the design vector x change?". Using the x^* and J^* above, the raw sensitives are calculated to be the following:

$$\nabla J = \begin{bmatrix} \frac{\partial J}{\partial T} \\ \frac{\partial J}{\partial e} \\ \frac{\partial J}{\partial NP} \\ \frac{\partial J}{\partial Pt} \\ \frac{\partial J}{\partial DA} \end{bmatrix} = \begin{bmatrix} -102.1317 \\ 114.5666 \\ 204.0848 \\ 0.3328 \\ 40.5873 \end{bmatrix}$$

In order to analyze which design variables are the major drivers, the above raw sensitivities are normalized. The normalized sensitivities are calculated in the following manner:

$$\nabla J_{normalized} = \frac{x^*}{J(x^*)} \nabla J = \begin{bmatrix} 0.0116 \\ 0 \\ 0.1319 \\ 0.0002 \\ 0.0118 \end{bmatrix}$$

From the normalized sensitivity values, it is clear that the number of planes (NP) is the main driver in the optimization problem. This result does not match the earlier conclusion that antenna diameter is the main driver. This discrepancy is due to the improved fidelity of the simulation code at this point in time. The result of the number of planes (NP) being the main driver is reasonable due to the fact that several satellites exist within each satellite plane and the costs of launching and maintaining a satellite over its lifetime can be very expensive.

The sole active constraint at x^* exists at the lowest numerical bound for the eccentricity (i.e., an eccentricity value of 0). Because the active constraint is

bounded by the laws of the universe (i.e., eccentricity cannot be less than 0), adjusting the constraint to improve the design does not appear to be possible. All the simulation outputs seen to date had resulted in eccentricity values between 0 and 0.01.

Parameter Sensitivity Analysis

Sensitivity analysis can also be conducted on fixed parameters of the optimization problem. The two fixed parameters chosen are: (1) the user data rate and (2) the number of subscribers. To calculate the sensitivity of the objective value with respect to the fixed parameters, the method of finite differencing is used. It is defined as:

$$\frac{\Delta J}{\Delta p} = \frac{J(p^0 + \Delta p) - J(p^0)}{\Delta p} \qquad (1)$$

At the optimal point $x*$, the data rate (R) per user is 1000 [kbps]. The lifecycle cost for the elliptical constellation is 2008.7703 [$M]. When the data rate (R) is changed to 1010 [kbps], the optimal design vector becomes: $x^* = [0.7477, 3.9948, 3999.7, 1.7616]^T$. The lifecycle cost of this design is 2003.8840 [$M]. The step size of 10 [kbps] is chosen in an effort to keep the step size as, small as possible without becoming so small as to be meaningless. As $\Delta p \to 0$ the derivative approaches the exact derivative. The sensitivity using Equation 1 is thus computed in the following manner:

$$
\begin{aligned}
\frac{\Delta J}{\Delta p} &= \frac{J(p^0 + \Delta p) - J(p^0)}{\Delta p} \\
&= \frac{2003.8840 \text{ [\$M]} - 2008.7703 \text{ [\$M]}}{10} \\
&= -0.48863
\end{aligned}
$$

This result indicates that as the data rate is increased by 10 kbps, the total cost actually decreases. Changing the parameter step size could change this result depending on the topology of the design space around the point of interest. This sensitivity result seems to be counter-intuitive as it shows that an increase in the data rate per person results in a decrease in infrastructure cost. This may indicate that the parameter step size should be adjusted. However, it is more likely that the small change in the cost falls within three sigma of the regression of the cost model. Therefore, the observed change in cost may not be statistically significant.

At the optimal point $x*$, the LEO backbone constellation serves 240,000 users (i.e., 1000 users within each map grid). The lifecycle cost of the elliptical constellation is 2008.7703 [$M]. When the LEO backbone constellation serves 1010 users within each map grid, the optimal design vector becomes: $x^* =$

$[0.7480, 0, 3.9948, 3999.7, 1.7602]^T$. The lifecycle cost of this design is 2003.7966 [$M]. Once again, the step size of 10 users was chosen in an effort to keep the step size as small as possible without becoming so small as to be meaningless. The sensitivity is thus computed as follows:

$$
\begin{aligned}
\frac{\Delta J}{\Delta p} &= \frac{J(p^0 + \Delta p) - J(p^0)}{\Delta p} \\
&= \frac{2003.7966 \text{ [\$M]} - 2008.7703 \text{ [\$M]}}{10} \\
&= -0.49737
\end{aligned}
$$

This result indicates that as the number of subscribers to the LEO backbone constellation is increased by 10, the total cost decreases. Changing the parameter step size could change this result depending on the topology of the design space around the point of interest. This sensitivity makes sense given that the number of subscribers to be serviced by the elliptical constellation has decreased. Once again, the small change in the cost falls within three sigma of the regression of the cost model. Therefore, the observed change in cost may not be statistically significant.

Heuristic-Based Optimization: SA

The heuristic technique chosen for the single objective optimization is Simulated Annealing (SA). The SA technique is not computationally complex, thus it is simple to implement.

In order to optimize the system under the Simulated Annealing algorithm, certain decisions such as the nature of the cooling schedule, the degrees of freedom, and the initial temperature for the algorithm must be made. For this SA optimization, the initial cooling schedule is in the form of a geometric progression with a 5% decrease in the temperature per iteration. The system is given two degrees of freedom. Jilla (2002) has determined that two degrees of freedom has provided the best SA results for optimization problems.[10] The initial temperature is set to the initial lifecycle cost of the entire system which is equal to $6278.7 [$M], corresponding to the following design vector: $x = [0.5, 0.01, 4, 4000, 3]^T$. This initial design variable and corresponding total lifecycle cost are equivalent to the initial design vector and objective value that are used as inputs in the gradient search optimization.

The SA algorithm can successfully optimize the system at hand. In comparison to the SQP technique which produces a solution with a total lifecycle cost of 6,187 [$M] million dollars, the SA algorithm produces a total lifecycle system cost of 5,389 [$M] million. However, because SA is a non-deterministic algorithm, one cannot be certain that an optimum returned by

the algorithm is the true global optimum. The simulated annealing algorithm is not structured to check that an optimum obtained is a true global optimum. Many SA simulation runs should be conducted to find solutions that converge. The SA algorithm can also be varied in terms of its parameters (e.g., cooling schedule, degrees of freedom, etc.). Table 6 summaries the various experiments conducted by tuning the SA algorithm and their subsequent results. It appears that the best tuning parameter setting for this optimization problem involves a geometric progression cooling schedule with a 5% decrease per iteration, an initial temperature equal to the initial lifecycle cost of the combined hybrid system and the design variable set to $x = [0.5, 0.01, 4, 4000, 3]^T$.

Multi-Objective Optimization

The multi-objective optimization problem undertaken consists of two objectives. The two objectives are formulated as:

1. LCC (lifecycle cost) = Minimize total lifecycle cost of the elliptic constellation [$B] (Note: The LEO backbone constellation is optimized separately and it is fixed across different elliptic options.)

2. AOC (average excess capacity) = Maximize unused capacity of the system [channels] (for possible future growth)

Table 6 Simulated Annealing Tuning.

Nature of Tuning Implemented	J* [$M]	x* [T, e, NP ,Pt, DA]^T	Improvement from optimal SA cost of 5389 [$M]?
1. Geometric progression cooling schedule with a 15% decrease per iteration	$5753.4 (50 runs)	$[1/7, 0.01, 2, 2918.23, 2.33]^T$	No, optimal cost increased by $364 million dollars
2. Geometric progression cooling schedule with a 25% decrease per iteration	$5427.9 (50 runs)	$[1/7, 0.01, 3, 1581.72, 2.23]^T$	No, optimal cost increased by $39 million dollars
3. Stepwise reduction cooling schedule with a 25% reduction per iteration	$6278.7 (50 runs)	$[1/2, 0.01, 4, 4000, 3]^T$	No, optimal cost and design vector remained the values they were before optimization
4. Geometric progression cooling schedule with a 15% decrease per iteration but with the added constraint that the result of each iteration has to be better than the one preceding it.	$5800.1 (41 runs)	$[1/2, 0.01, 3, 3256.08, 2.17]^T$	No, optimal cost increased by $411 million dollars
5. Initial Temperature is doubled (i.e., initial temperature changed from 6278.7 [$M] to 12557.4 [$M]	$6278.7 (50 runs)	$[1/2, 0.01, 4 , 4000, 3]^T$	No, optimal cost and design vector remained the values they were before optimization
6. Initial Temperature is halved. (i.e., initial temp changed from 6278.7 [$M] to 3139.4 [$M]	$5622.7 (50 runs)	$[1/2, 0.01, 2, 3658.08, 2.3]^T$	No, optimal cost increased by $234 million dollars
7. Initial design vector is altered such that $x_0 = [1, 0, 3, 3000, 3]^T$	$5719.1 (50 runs)	$[1, 0, 3, 3000, 3]^T$	No, optimal cost increased by $330 million dollars
8. Initial design vector was altered such that $x_0 = [0.25, 0.5, 5, 3000, 3]^T$	Failed to find a feasible solution	---	----

These two objectives are mutually opposing. Maximizing the unused capacity of the satellite system will necessarily require either an increase in the number of satellites in the constellation or an increased capacity per satellite, both of which increase cost. The objective of minimizing the total lifecycle cost will attempt to drive down the number and cost of satellites, opposing the drive for excess capacity.

Furthermore, defining the second objective requires some care. It is not simply the total system capacity less the number of users (channels) served. There are design solutions that can place satellites in orbits where there is a lack of users, thus leading the simulation to conclude that there is a lot of "excess" capacity. This is clearly not the meaning of excess capacity that is useful for this optimization problem. Therefore, an alternative definition is required:

```
If percent of the market served > minimum market share
     Excess capacity = Total system capacity − Market served
Else
     Excess capacity = 0
End
```

Because the market served varies over time, the second objective is defined as the average excess capacity (AOC).

Weighted Sum Approach

Initially, the system was optimized for both objectives with the weighted sum method embedded into the SA simulation code. Repeatable results are again difficult to achieve because of the stochastic nature of the SA code. Furthermore, nearly a hundred (if not more) objective evaluations are required per point on the Pareto front. This leads to more than a 1000 objective evaluations in order to find a reasonable number of points on the Pareto front. A much more informative full factorial evaluation of the trade space can be done for the same computational expense.

Pareto Optimality Results

Table 7 summarizes the factor levels used for the full factorial evaluation of the tradespace. Several pareto fronts are estimated and shown. Figure 5 shows the optimization results and a Pareto front generated without any requirement on demand satisfaction. The results seen seem paradoxical at first because the definition of AOC requires demand to be satisfied (denoted as a success in the Figure 5). The unexpected trend can be revealed upon closer examination of the results, as shown in Figure 6. Figures 6 (a)-(e) reveal the same figure as Figure 5 with the addition that each subfigure is color coded by the value of the indicated design variables.

Table 7   Factor Levels For Full Factorial Evaluation of Tradespace.

| Factor	Levels	Units
T	1, 1/2, 1/3, 1/4, 1/5	[days]
e	0.001, 0.1, 0.3, 0.5	[-]
NP	2, 3, 4, 6	[-]
Pt	1, 2, 4, 6	[kW]
DA	1.5, 2, 2.5, 3	[m]
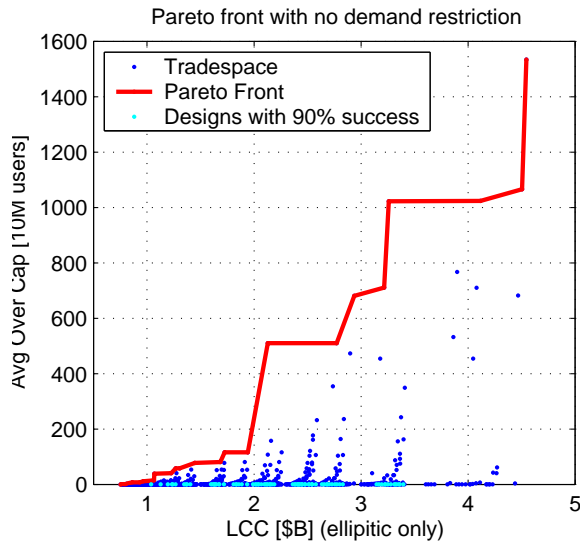
Fig. 5   Initial Pareto Front.
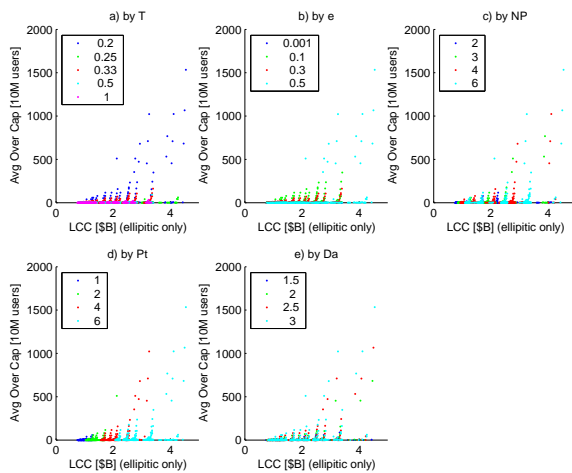
Fig. 6   Initial Tradespace Colored by Design Variables.

Note that all points with AOC > 500 have two design variable values in common, namely a period of $\frac{1}{5}$ of a day and an eccentricity of 0.5. The small period (relative to the other design choices implies that a shorter semi-major axis (14,420 [km]) and a high eccentricity lead to a low perigee (871 [km]). Given that the number of spacecraft in a given plane is determined by computing the minimum needed for continuous coverage at perigee, designs with low perigees tend to have a large number of spacecraft. Coverage overlaps are desirable as it allows load balancing of demand across multiple spacecraft leading to lower power (and cost) solutions. Combined with a multi-kilowatt transmitter power, a very large system capacity will result. With a period of $\frac{1}{5}$ of a day and an eccentricity of 0.5 there are 7 spacecrafts per plane. Pareto optimal designs have 3-6 planes leading to a maximum of 42 spacecrafts in the elliptical constellation.
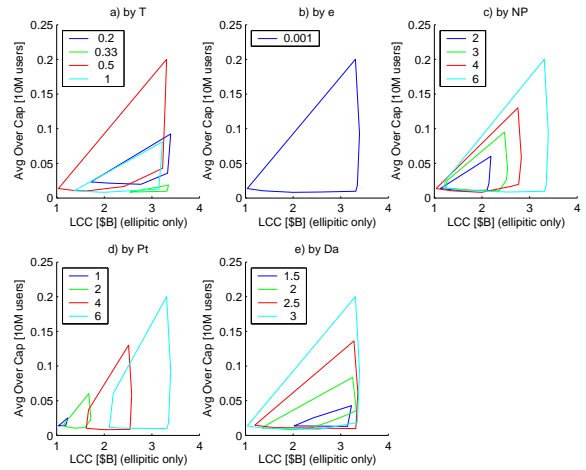
Number of spacecraft alone is not sufficient to explain the large overcapacity seen in Figure 5. As the orbital period is $\frac{1}{5}$ of a day, the ground track repeats every fifth orbit. Thus, each spacecraft will visit more of the Earth each day than in designs that use longer periods (i.e., orbital period $\rightarrow 1$). Clearly, alternate schemes for determining the number of satellites per plane would lead to different results and therefore this initial Pareto front is largely an artifact of the choice of algorithm rather than a fundamental limit of spacecraft constellation design.

Designs that meet the demand satisfaction constraint formulated in the section on gradient based optimization will now be examined. These are the points in Figure 5 labeled as "Designs with 90% success." These solutions are re-plotted in Figure 7. Note the much smaller values of AOC. This indicates that the available satellite capacity is being used more efficiently, however there is little room to increase capacity to meet a growing market. Figure 8 uses the same data in Figure 7 with the addition that each subfigure is color coded by the value of the indicated design variables.
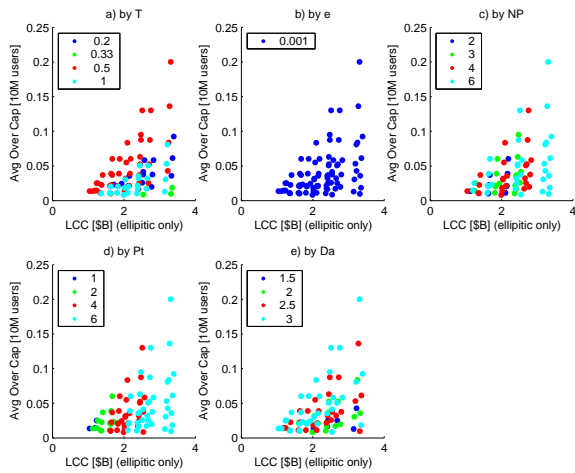
Once again, the dominant characteristics of designs on the Pareto front are eccentricity and orbital period. A period of $\frac{1}{2}$ a day and a (nearly) circular orbit seem to be the best design. Beyond these design variables, the choice of transmitter power tends to determine where on the Pareto front a design lies, with number of planes and the antenna diameter having secondary effects. These trends can be better visualized through a novel visualization of the trade space developed for this optimization problem. For each design choice, the convex hull of the colored points in Figure 8 is drawn in Figure 9. Each convex polygon identifies the smallest convex region in which all designs that include a
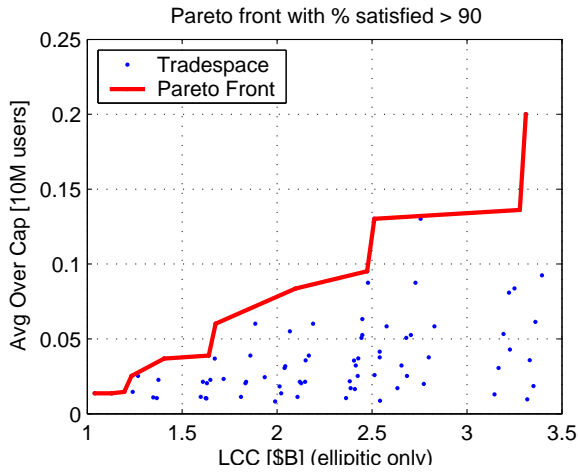
Fig. 7 Pareto Front with Requirement that 90% of entire broadband market is met 90% of the time.

Fig. 8 Constrained Trade Space Colored by Design Variable Choices.
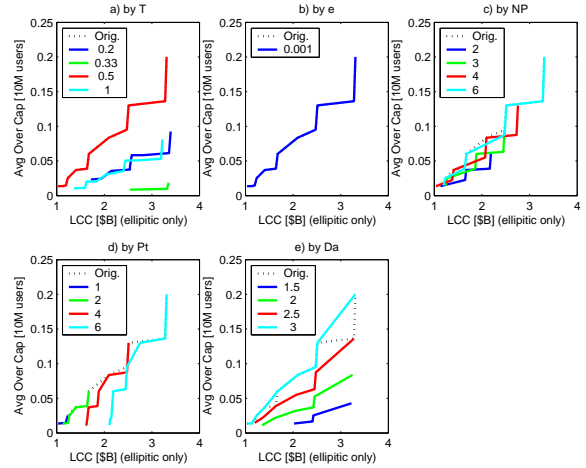

Fig. 9 Convex Hull-based Visualization.

Fig. 10 Pareto Fronts given each fixed Design Variable.

particular design decision fall in the trade space.

With the trade space visualized in Figure 9, the impact of each design choice on subsequent design freedom is made apparent to the designer. By intersecting corresponding polygons, the designer can visualize the impact of more complex design choices, e.g. a low power, high gain system vs. a high power, low gain system. Another useful visualization is a plot of the Pareto front given one fixed design variable while the others are allowed to vary, as shown in Figure 10.

One of the most powerful features of Figure 10 is its ability to identify the design variable that prevents improved performance at various points along the Pareto front. From 1 to 1.5 [$B] the system is power limited; then from 1.5 to 2.5 [$B] the system is gain (antenna diameter) limited; and finally from 2.5 to 3 [$B] the system is limited by the number of planes. These results provide a roadmap to allocate design resources given an estimated lifecycle budget. For example, if a

budget of 1.75 [$B] is available, then attention should be focused on obtaining more power as opposed to antenna gain. Admittedly, in most circumstances the dominance of particular design variables will not be quite so obvious, the tools presented here can be used by a designer to gain fresh insight into the workings of the system design tradespace.

Conclusions and Future Work

It has been shown that hybrid constellations combining a global backbone with additional rings added to cover high demand areas can provide a lower cost solution to the broadband communication constellation design problem. The distribution of demand for broadband services is highly asymmetric, with a few peaks of high demand separated by large areas with low or no demand at all. This asymmetry of demand has led to failure modes of traditional design approaches (that do not account for this asymmetry). In one case, capacity, which in aggregate could serve

the entire market, is uniformly distributed leading to excess capacity in low demand areas and a dearth of capacity in high demand, and therefore high revenue areas. This mismatch between demand and capacity makes such systems unprofitable. In the other case, large satellites (or a large number of small satellites) are used to ensure demand is met all over the earth. Such systems will have extremely high start-up costs that prevent them from ever getting off of the ground.

Hybrid constellations solve this problem by first providing a baseline service (via the backbone) to start the market and then supplementing the backbone with strategically placed spacecraft to serve high demand areas. Previous researchers have successfully used polar constellations to give higher fold coverage in high demand latitude bands. This paper extends that work by using elliptical orbits with repeating ground tracks to account for both longitudinal and latitudinal asymmetry in demand. The relative crudeness of the cost models used do not allow one to conclude that such a strategy is strictly better than the polar orbit-based strategies proposed by others, however hybrid designs do seem to show promise. The current models demonstrate that with the addition of only 2 planes of 3 satellites each at a lifecycle cost of less than two billion dollars, almost a million additional users can be served. Since the additional spacecraft are deployed in independent rings (i.e., one need not launch rings in pairs or triples), they can be deployed in a staged manner allowing the designer to react to observed demand instead of predicting future demand. Further research will be needed to verify and extend this conclusion. More generally the results presented are important to the practice of constellation design.

Through conditional Pareto analysis, whereby the Pareto optimal set of designs given a particular design choice are identified, it was determined that Pareto optimal solutions to the hybrid constellation design problem will have a period $\frac{1}{2}$ day and near zero eccentricity. These results hold irrespective of the other design choices. Further, transmitter power was found to be the key determiner of the location of a particular optimal design on the efficient front. The number of orbital planes and the antenna gain had lesser effects on Pareto optimality. From these analyses the designer would conclude that future detailed model design effort should be focused on designs with a period of $\frac{1}{2}$ day and zero eccentricity. Other orbits are clearly dominated. Since power is the key determiner of cost, improving the fidelity of the spacecraft module should also be a focus of subsequent design effort. Viewing the conditional Pareto fronts and the convex hulls of each design choice allows one to come to these conclusions. The usefulness of these techniques even with the crude models used is reflective of their applicability to the problem of conceptual design.

Major open issues to be addressed in the development of this research:

1. Coding for radiation shielding due to Van Allen Belts: Radiation shielding is necessary because a hybrid satellite system may choose orbits that pass through or near the Van Allen Belts. The current cost estimation used for satellite hardening is taken as 2-5% of the total satellite cost.[1] The simulation presented currently uses this metric because a more sophisticated relation between cost and satellite hardening is not readily available.

2. Accurate costs and a table of available motors need to be obtained for the apogee and Geo-Transfer Orbit kick motors used in the new launch cost module: For the values being inputted, the launch cost module works correctly, thus once accurate values are obtained the module will be suitably improved. However, for the optimal cost to be reasonable, the actual costs and a table of available motors will need to be added to the simulation.

3. Hand-off problem: The handoff problem is not addressed in the current simulation. Satellite handoff modeling is a key component of interconnected network satellite simulations. However, it is difficult to encode and simulate handoffs. For example, a handoff management strategy (e.g., "make before break" or "break before make") must be implemented with considerations to bandwidth allocation, Quality of Service provisioning and mobility management. Maintaining connectivity between two asymmetrical satellite constellations is difficult. With more time, it is certainly something that should be considered for a more refined system model simulation analysis.

4. Increase the fidelity of the simulation modules with less simplifying assumptions.

References

[1]Wertz, J. R. and Larson, W. J., *Space Mission Analysis and Design*, Microcosm Press & Kluwer Academic Publishers, El Segundo, California & Dordrecht, The Netherlands, 3rd ed., 1999.

[2]Rider, L., "Optimized Polar Orbit Constellations for Redundant Earth Coverage," *The Journal of the Astronautical Sciences*, Vol. 33, No. 2, April - June 1985, pp. 147 – 161.

[3]Adams, W. and Rider, L., "Circular Polar Constellations Providing Continuous Single or Multiple Coverage Above a Specified Latitude," *The Journal of the Astronautical Sciences*, Vol. 35, No. 2, April - June 1987, pp. 155 – 192.

[4]Lang, T. J. and Adams, W. S., "A Comparison of Satellite Constellations for Continuous Global Coverage," *Mission Design & Implementation of Satellite Constellations*, edited by J. C. van der Ha, International Astronautical Federation, The Netherlands, 1998, pp. 51 – 62.

[5]Turner, A. E., "Constellation Design Using Walker Patterns," No. 2002 - 4636, August 2002, In AIAA/AAS Astrodynamics Specialist Conference and Exhibit Proceedings.

[6]de Weck, O. L. and Chang, D., "Architecture Trade Methodology for LEO Personal Communication Systems," AIAA Paper 2002 - 1866, May 2002, In 20th AIAA International Communication Satellite Systems Conference and Exhibit Proceedings.

[7]Kashitani, T., "Development and Application of an Analysis Methodology for Satellite Broadband Network Architectures," No. 2002 - 2019, May 2002, In 20th AIAA International Communication Satellite Systems Conference and Exhibit Proceedings.

[8]Parker, M., *Broadband Satellite Constellation Design and Evaluation*, Master's thesis, Massachusetts Institute of Technology, Department of Aeronautics and Astronautics, May 2001.

[9]Ellipso, Inc., "Ellipso," *FCC filing*, 1990.

[10]Jilla, C., *A Multiobjective, Multidisciplinary Design Optimization Methodology for the Conceptual Design of Distributed Satellite Systems*, Ph.D. thesis, Massachusetts Institute of Technology, 2002.

[11]de Weck, O. and Willcox, K., "Lecture 4: Modeling and Simulation," *MSDO Lectures*, 2003.