Numerical simulation of wave-induced fluid flow in digitized porous media using a coupled model with Level Set method

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Abstract

In this paper, we develop a unified mathematical model with velocity as the primary variable for modeling the interaction between incompressible fluid and compressible solid.   We numerically solve this problem by combining the finite different method with the level set method, which is able to easily capture the dynamic change of the interface between fluid and solid.   Additionally, the zero contour of a level set function can represent the interface or boundary smoothly; this smoothness can be taken into account in numerical modeling so as to avoid the staircase boundary in the traditional finite difference method.   A modified projection method is developed for solving fluid and solid sub-domain simultaneously.   This method is then applied on a digitized 2D section of Berea sandstone.   To obtain a level set function of the 2D section, we applied the region-based active contour method to segment the micro-tomography data.

1.   Introduction

Seismic propagating in porous and permeable media usually produces different particle motion in fluid and solid, the relative motion of which gives rise to the seismic attenuation.   At very high frequencies, motions in fluid consist of vibrations around equilibrium, while at low frequencies fluid flow takes place in pores due to the local pressure gradient.   The wave-induced fluid flow at different scales – "macroscopic," "mesoscopic," and "microscopic" – plays an important role in attenuating seismic wave energy (Pride et al., 2004).   For the sake of simplified geometry of porous media, theoretical models can be developed to understand the effect of wave-induced fluid flow on velocity and attenuation (Dvorkin and Nur, 1993; Dvorkin, et al., 1995). However, complex pore structure of real rocks requires numerical simulation on digitized rocks in microscale with coupling fluid and solid interactions.

In the work presented in this paper, we developed a unified mathematical model to govern the fluid and solid systems, which is able to take into account the interaction between incompressible fluid and compressible solid. The solid allows small strain deformations but can rotate and translate in the fluid domain (i.e. fluid containing solid particles). Instead of using the initial particle method, we use a level set method to capture the evolution of the boundary between fluid and solid. We only need to use the physical velocity in the fluid and solid domain to evolve a level set function. To avoid the diffusion problem coming with the re-initialization of level set function, we also use the Fast Match Method to reconstruct a level set.

In the paper, we first introduce the unified mathematical formulation that governs the fluid and solid systems in the Eulerian frame. Then we describe the details about the modified projection method we use in our modeling. We then apply this method on a digitized 2D section of Berea sandstone. The region-based active contour method is used in constructing a level set function of the 2D section from micro-tomography data. Finally, we will conclude with a summary of the research.

2. Mathematical Formulation

In a domain Ω consisting of fluid Ω_f and solid Ω_s sub-domains, energy due to any perturbation within this domain will proliferate in the form of wave propagation in solid and fluid flow in fluid, the governing equations of which are the wave equations and Navier-Stokes equations, respectively. To couple these two systems together, we need to find a set of unified equations to describe them. In this work, we assume the solid is isotropic and linear elastic and that it undergoes small strain deformation but is able to rotate and translate. We also assume that the fluid is incompressible linear viscous, or Newtonian. All formulas are described in Eulerian coordinates.

Governing equations in fluid and solid sub-domains obey the same momentum equation, but different constitutive laws will be applied. Therefore, in the coupled system we can unify these two systems with the equation

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho(\phi)}\nabla P + \frac{1}{\rho(\phi)}\nabla \cdot \tau + f , \qquad (1)$$

where v, P and τ are the velocity field, pressure, and stress in fluid and solid sub-domains.

In equation (1), based on the δ-function formulation method, we smear out the density and stress profiles across the boundary between fluid and solid with equations

$$\rho(\phi) = \rho_f + (\rho_s - \rho_f)H(\phi) \qquad (2)$$

$$\tau(\phi) = \tau_f^{vis} + (\tau_s^{dev} - \tau_f^{vis})H(\phi) \qquad (3)$$

$$H(\phi) = \begin{cases} 1 & \phi \geq 0 \\ 0 & \phi < 0 \end{cases} \tag{4}$$

where ρ_f and ρ_s are the densities of fluid and solid sub-domains, and τ_f^{vis} and τ_s^{dev} are the viscous stress in the fluid sub-domain and deviatoric stress in solid sub-domain, respectively.

In equation (1) through (4), ϕ is a level set function defined in space and time as

$$\phi(x,t) \begin{cases} > 0 & \mathbf{x} \in \Omega_s \\ = 0 & \mathbf{x} \in \Omega_s \bigcap \Omega_f \\ < 0 & \mathbf{x} \in \Omega_f \end{cases} \tag{5}$$

and also a distance function defined as

$$|\phi| = 1. \tag{6}$$

As we see from (5), the zero contour of a level set function $\phi(x,t)$ represents the interface between fluid and solid sub-domains as shown in Figure 1. This interface can move as the whole level set function evolves with the velocity field

$$\frac{\partial \phi}{\partial t} + v \cdot \nabla \phi = 0. \tag{7}$$

3. Computational Method

We used the finite difference method with a level set method throughout our modeling work. Noticing that the unified equation (1) is in the same form of the Navier-Stokes equation, we utilized the projection method (used popularly in computational fluid dynamics) to solve this coupled system, which we call the modified projection method in this paper. On the spatial discretization, we employ a staggered-grid finite difference scheme with pressure P, normal stress components, a level set function and parameters for material properties in the cell center, and the velocities placed on the cell interfaces. The procedure is as follows:

(1) Treat the nonlinear term

We use the 5th order HJ-WENO for spatial discretization and the 3rd order TVD Runge-Kutta for time stepping. The intermediate velocity field v^* is obtained by velocity field v^n at time step n.

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$$\frac{v^* - v^n}{\Delta t} = -\left(v^n \cdot \nabla\right)v^n . \tag{8}$$

(2) Calculate the viscous term

The stress tensor in (1) is a generalized term, which refers to the actual viscous stress in the fluid sub-domain and the deviatoric stress in the solid sub-domain. It is straightforward to calculate the viscous stress in fluid, but requires solving a first-order differential equation to obtain the deviatoric stress in solid. By using an explicit scheme, we can obtain the intermediate velocity field v^{**} by

$$\frac{v^{**} - v^*}{\Delta t} = \frac{1}{\rho(\phi^n)}\nabla \cdot \tau^n + f^n . \tag{9}$$

(3) Correct the pressure term

In the projection method, the key procedure is to project pressure back to ensure that fluid is incompressible by solving a Poisson equation about pressure P. The enforcement of incompressibility should not be applied on the velocity field in solid, since $\nabla \cdot v^{n+1} \neq 0$ in the solid sub-domain, which requires special treatment to ensure the compressibility in solid when solving the Poisson equation.

$$\nabla \cdot \left(\frac{1}{\rho(\phi^n)}\nabla P^{n+1}\right) = \frac{1}{\Delta t}\left(\nabla \cdot v^{**} - \nabla \cdot v^{n+1}H(\phi^n)\right) . \tag{10}$$

The $\nabla \cdot v^{n+1}$ in the solid sub-domain can be predicted first by the velocity-stress scheme of the wave equation. After solving this equation, we obtain the pressure P^{n+1} in the whole domain.

(4) Update the velocity field

At the final step of the modified projection method, we can obtain the final velocity field v^{n+1} at time step n+1 by solving the equation

$$\frac{v^{n+1} - v^{**}}{\Delta t} = -\frac{1}{\rho(\phi^n)}\nabla P^{n+1} , \tag{11}$$

(5) Evolve a level set function

At end of this calculation, we use the Fast Matching Method (Adalsteinsson and Sethian, 1999) first to construct the extension velocity field with a level set function ϕ^n. Then we use the extension velocity field v_e^{n+1} to evolve a level set function to obtain ϕ^{n+1} by solving (7). In this way, we do not need to re-initialize a level set function at every time step so as to avoid the diffusion problem with re-initialization. Additionally we use the 5th order HJ-WENO for spatial discretization and the 3rd order TVD Runge-Kutta for time stepping.

4. Two Examples

4.1   Seismic wave propagation with modified projection method

For demonstrating the capability of the modified projection method on modeling seismic wave propagation, we chose a model occupied solely by elastic media. As shown in Figure 2a, a point explosive source is excited at (2, 2) m, and signals are received at (4, 2) m. From Figure 2b, we see that the result from the modified projection method matches that from the traditional standard-staggered-grid method.

4.2   A falling elastic ball in water tank

In this case, an elastic ball falls in water tank under gravitational force. We use this to demonstrate (1) the coupling between the fluid and solid sub-domains; (2) the capability of a level set method on capturing the evolution and deformation of boundary implicitly. Snapshots at several time steps are shown in Figure 3.

5. Application on Berea sandstone

5.1   Level set construction

The purpose of developing this coupled method is to simulate wave-induced fluid flow in digitized porous media. Here we are using a 2D section of Berea sandstone. First we need to construct a level set function of this section with zero contours representing the boundary between grains and pores. We worked from the micro-tomography data after diffusion processing. As shown in Figure 4, we cut a 2D section of Berea sandstone from a larger one. Processed by traditional segmentation method, we obtain a segmented data with a staircase boundary as shown in Figure 4c. Using the region-based active contour method (Chan and Vese, 2001) and this segmented data, we can obtain a level set function (Figure 5a) with smooth zero contours describing the boundary between grain and pores, as shown in Figure 5b.

5.2　2D numerical result

Due to the smooth boundary described by a level set function and smear boundary condition used in computation, we can capture the smoothness of the boundary in the numerical modeling. This method is more optimal than using segmented data with a staircase, which causes numerical noise and inaccuracy when resolution is too low to capture the high curved boundary.

In the model, the solid is taken to be quartz ($\mu = 44$ GPa, $\lambda = 7.6$ GPa, $\rho_s = 2650$ kg/m^3), and the pore space is saturated with water ($\eta = 10^{-3}$ Pa.s, $\rho_f = 1000$ kg/m^3). Since the size of the digitized rock is small, we simulate an incoming compressional plan wave with source frequency of 20 MHz. Figure 6 shows the snapshots of velocity fields, in which blue arrows represent the velocity vectors in solid and red arrows are those in fluid. We can clearly see the interaction between fluid and solid. Fluid flow takes place within a single pore, and it moves back and forth between two pore spaces connected by a small channel.

6.　Conclusions

In this paper, to simulate the wave-induced fluid flow in porous media, we first develop a coupled method with a unified mathematical model to govern the seismic wave equations and Navier-Stokes equations. Then a modified projection method is used as a general solver to handle this fluid and solid interaction problem. We incorporate the idea of level set into this modeling, and we use the zero contour of a level set function to describe the boundary between grains and pores in order to represent the boundary smoothly. Compared to tradition segmentation data with a staircase boundary, the smoothness of the boundary can be taken into account in the numerical modeling, which is one advantage of using a level set method. Finally, we applied this coupled method on a 2D section of Berea sandstone to show its effectiveness, showing that this is a promising method for modeling propagation and flow in multiphase media.

Acknowledgments

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Dvorkin, J., R. Nolen-Hoeksema, and A. Nur, 1995, *The squirt-flow mechanism: Macroscopic description*, Geophysics, 59 (3), 428-438.

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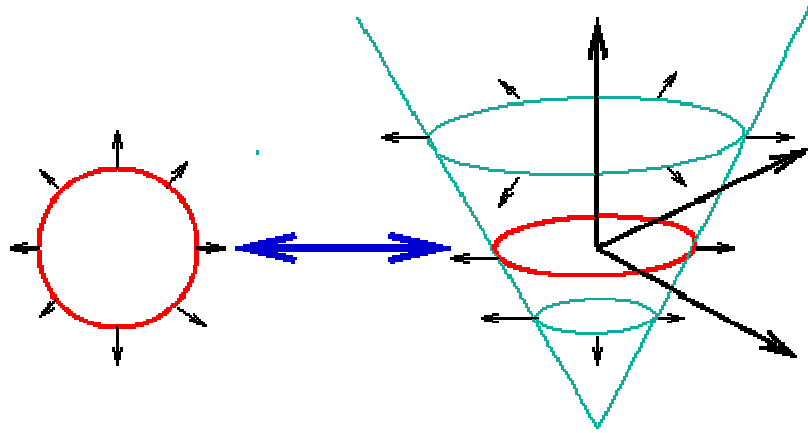
Figure 1: Definition interface with zero contour of level set function. The red circle at left is the interface or boundary of a 2D object defined by zero contour of level set function. The green curves define the high dimension level set function. (J.A. Sethian, UCB)
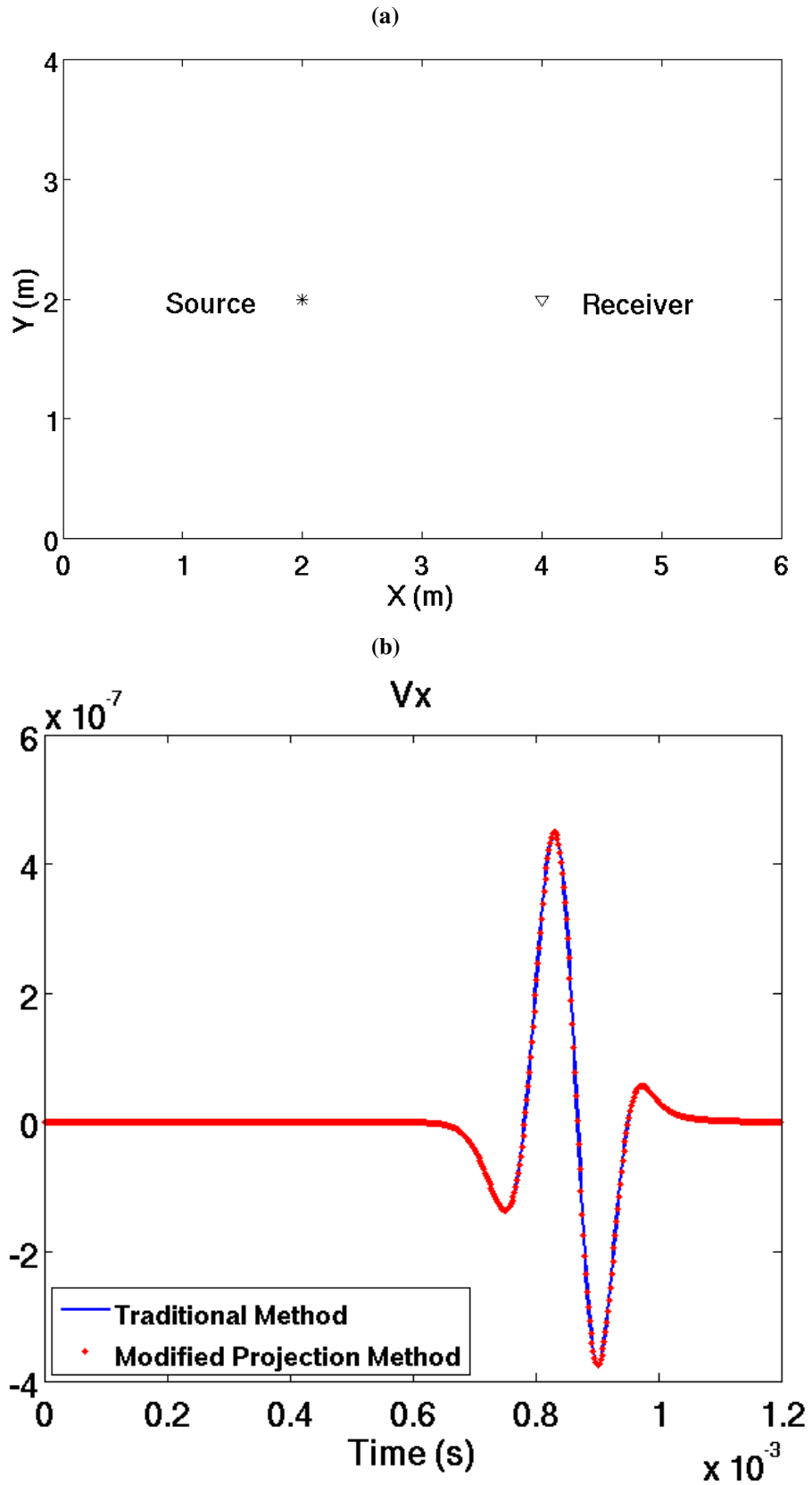
(a)

(b)

Figure 2: Comparison of the modified projection method with the tradition method. (a) A simple elastic model; (b) the signals received.
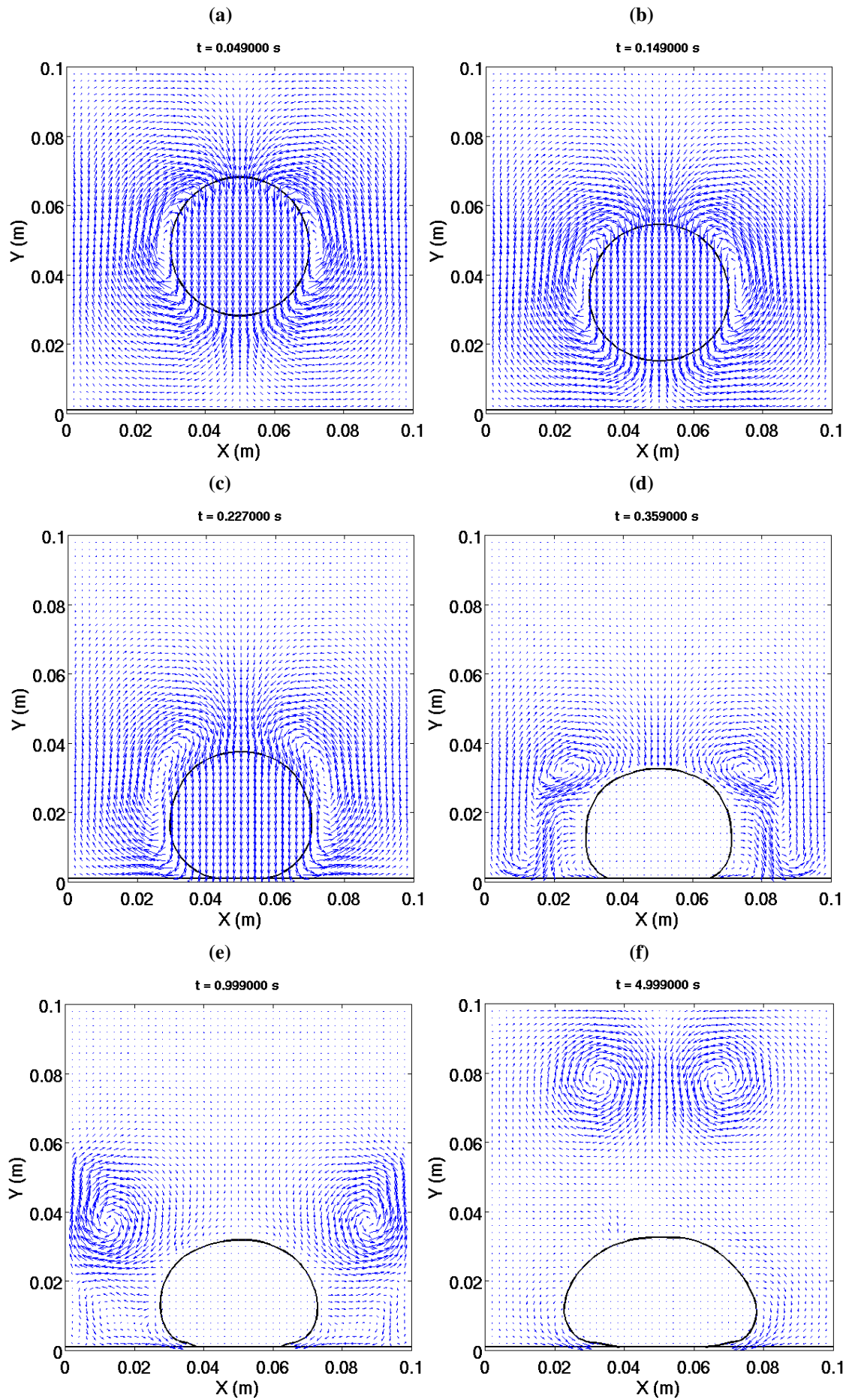
(a)

t = 0.049000 s

(b)

t = 0.149000 s

(c)

t = 0.227000 s

(d)

t = 0.359000 s

(e)

t = 0.999000 s

(f)

t = 4.999000 s

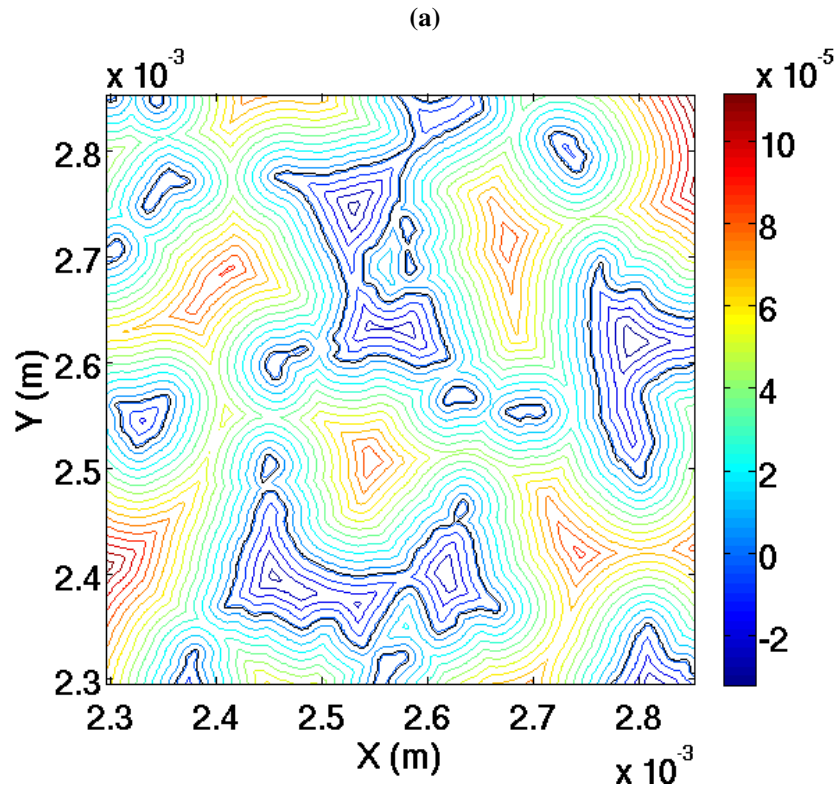Figure 3: Snapshots for a falling elastic ball in a water tank.
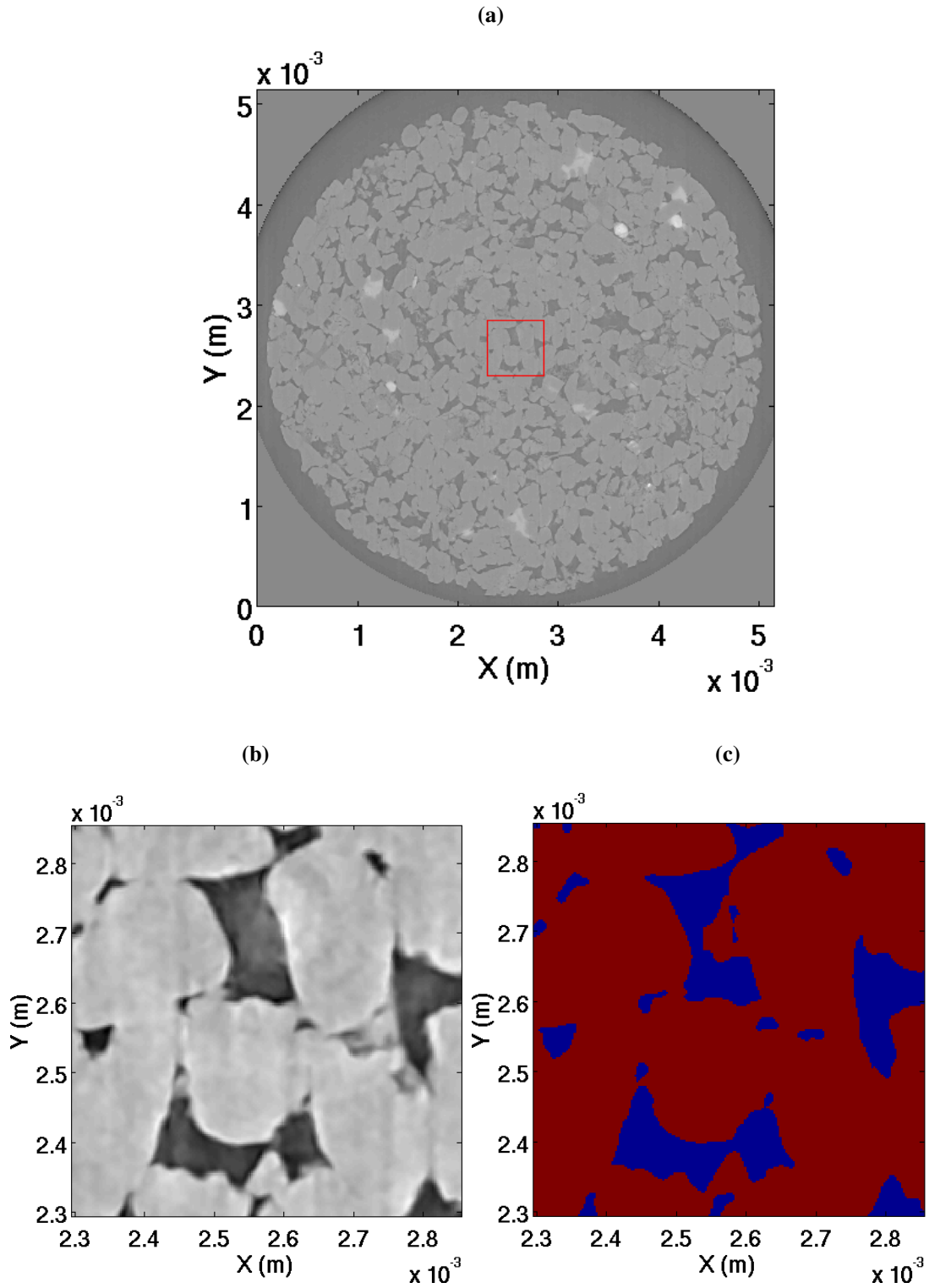
Figure 4: (a) Tomography data of a large 2D section of Berea sandstone; (b) tomography data of a small 2D section of Berea sandstone (the area marked by red box in (a)); (c) segmented data of the 2D section shown in (b) by the traditional segmentation method.
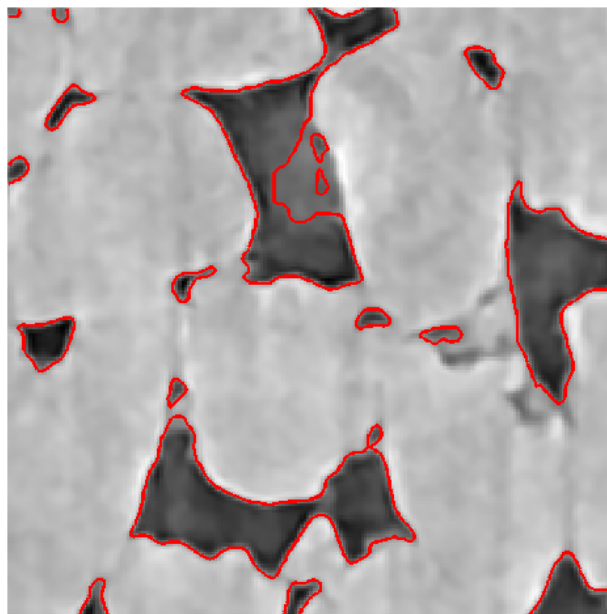
(a)

(b)

Figure 5: (a) Constructed level set function using region-based active contour method and segment data as initial, with zero contours in black; (b) zero contour of the level set function (red line) overlapping on tomography data.

(a)

t = 7.360000e-08 s

(b)

t = 8.760000e-08 s

(c)

t = 1.196000e-07 s

(d)

t = 1.996000e-07 s

(e)

t = 2.996000e-07 s
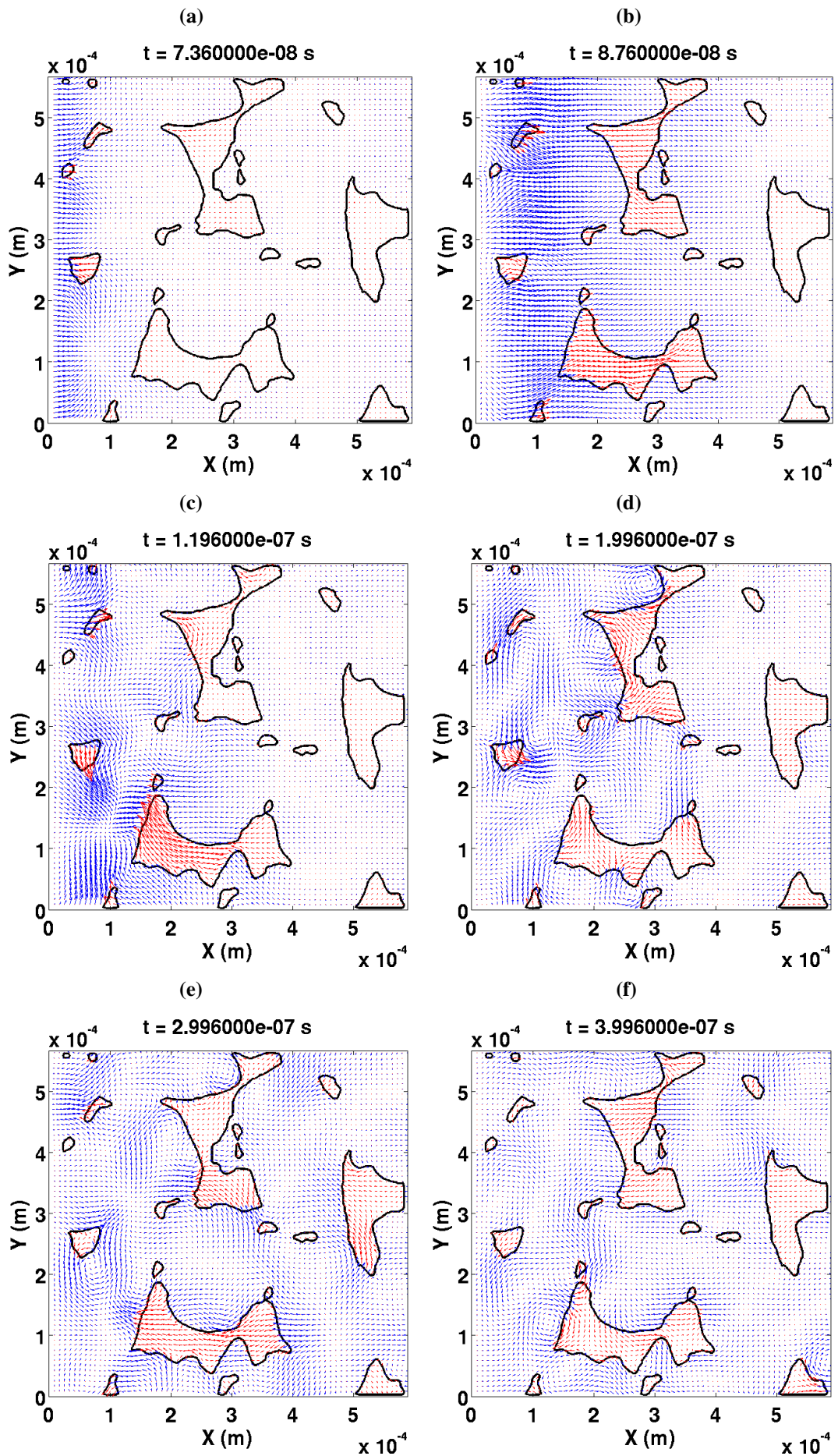
(f)

t = 3.996000e-07 s

Figure 6: Snapshots of a velocity field in the 2D section of Berea sandstone in an incoming compressional plan wave. Blue arrows show velocity vectors in solid, while red arrows show velocity vectors in fluid.