A REINFORCED CONCRETE STRUCTURE WITH CABLE SUPPORTED ROOF

by

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B. Arch. University of Southern California, 1957

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ABSTRACT

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Submitted in partial fulfillment of the requirements for the degree of Master in Architecture in the Department of Architecture on July 27, 1962.

The thesis presents the design and analysis of a reinforced concrete structure requiring a large interior clearspan, wherein the roof is supported by an autoclastic or doubly curved network of cables.

The focus of the investigation is directed at a generalized solution stressing an architectural synthesis which reflects the geometric, structural, and constructive character of the diverse elements. Assumed programmatic requirements for the structure are minimized, with emphasis being placed on the creation of a unified architectural system rather than the solving of specific functional requirements.
July 27, 1962

Pietro Belluschi, Dean
School of Architecture and Planning
Massachusetts Institute of Technology
Cambridge, 39, Massachusetts

Dear Dean Belluschi,

In partial fulfillment of the requirements for the degree of Master in Architecture, I hereby submit this thesis entitled, "A Reinforced Concrete Structure with Cable Supported Roof".

Respectfully,

Richard L. Martin
ACKNOWLEDGMENTS:

I am grateful to the following persons for their advice and assistance in the preparation of this thesis:

Professor Giulio Pizzetti
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Mr. A. J. Harris

Professor Eduardo F. Catalano Thesis Advisor

Turin, Italy
Cambridge, Massachusetts
London, England
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I. INTRODUCTION

A. Objectives

1. To design and analyze a reinforced concrete structure requiring a large interior clear span, wherein the roof is supported by an autoclastic or doubly curved network of cables.

2. To formulate the design program for the structure so that the architectural condition created will have a broad application of use.

3. To direct the design toward a solution stressing the resolution of those problems which would prevail in the structure regardless of specific function.

4. To direct the design of the structure toward an architectural entity which reflects its geometric, structural and constructive characteristics.

5. To investigate the structural and physical properties of the materials to be employed, and to determine the technological potential of the building industry with respect to the fabrication and erection of the structure.

6. To establish as much as possible within the generalized program of requirements the pertinent environmental considerations, whether of spatial or functional nature, and to direct the design of the structure so that these needs are satisfied by the inherent or organic qualities of the structure rather than as applications of non-structural nature.
A. Objectives (continued)

7. To equate the structure's relative economy and efficiency with the resulting architectural quality rather than with considerations which are solely economic.

B. Exposition

The thesis is generalized to a theoretical condition rather than one approximating a more comprehensive exploration of a complete architectural service. This approach is taken because it is felt that by minimizing the problems encountered in the solving of specific functional requirements, the purpose implied in the general thesis title "The Use of Reinforced Concrete" may best be exploited.

C. Approach

With respect to each of the following categories, a survey of known applications and techniques will be undertaken so as to best determine what is feasible and in turn facilitate the formulation of a constructive proposal.

1. The examination of basic dimension considerations with respect to determining a general architectural condition.

2. The determination of applicable structural and constructive techniques, including a survey of structural and physical properties of materials to be employed.

3. The determination of required service functions to
be provided with their means of incorporation into the structure.

4. The determination of the roof surface based on span, height, spatial, and structural considerations pertinent to the selected architectural condition.

5. The determination of a unified system of structure for roof system and supporting elements with the geometric determination of slopes and angles.

6. The determination of a sequence of construction.

7. The establishment of basic structure theory and approximate structural analyses.

8. Record findings and design proposal in written and graphic form.
II. RESEARCH

A. History

1. Development of Cable Roof Structures

The use of suspension cables as a primary means of support in roof structures owes its genesis to the suspension bridge which has prevailed in crude forms since the most primitive beginnings of civilization. The modern suspension bridge had its origin with the discovery of steel in the 1880's, and since that time has experienced a continuous development wherein spans reached 4200 feet in the 1930's with the construction of the Golden Gate Bridge in San Francisco. Nothing technically stands in the way of a 10,000 ft. span.

Cable supported roofs are of relatively recent origin, the earliest use being tent structures of various forms, while what was probably the first substantial construction occurred in the locomotive pavilion at the Chicago World's Fair of 1933. Since these beginnings, the utilization of cable roofs in a variety of forms has become more prevalent, but such use remains a basically new and incompletely understood or exploited engineering principle.

The engineering attraction of such a roof is derived from the fact that the most economical means known for spanning large distances is the steel cable. This is due to the physical fact that steel wire or cable in tension is four to six times stronger than steel in any other form. It is this fact which makes possible theoretical bridge spans far in excess of those presently constructed, and creates much of the appeal of cable
supported roof structures. In fact, the chief difficulties in suspension structures of any type lie not in spanning capacity, but rather with the problems of vibrations, flutter and uplift due to wind or unequal live loading.

Though these problems exist in suspension bridges, they become even more critical when suspension principles are employed in roof structures. This occurs because the added necessities of breadth and weathertight enclosure require that a greater degree of stability be maintained. This, coupled with the fact that the large tensile stresses developed in the cables must ultimately be resisted by some means has often resulted in excessively large edge beams, added roof weight, and secondary tie members, all of which can compromise structural clarity and economy. In effect, the method of delivering the cable stresses to the ground, and the means by which the problems of flutter, vibration and uplift are controlled become of primary consideration in the design or study of cable structures.

Of the cable roof structures that have been constructed or proposed to date, there emerge three distinct types or categories and a number of other projects or buildings which are derivations or combinations of the three. The latter are in some cases noteworthy as structures, but since none is unique in principle, they tend to be less important in a survey of suspension roof structures.

Of the basic types, each is distinguished by the geometry of its respective cable network, but the means in which its
supporting structure performs its function, and to a lesser degree in the manner in which the tendency to flutter or vibrate is resisted.

The three types are:

a. Circular form with cables suspended radially from a central tension ring in midair to an exterior compression ring supported by a wall or columns.

b. A doubly curved saddle made up of two networks of cables of opposite curvature, the thrust of which is resisted by edge beams or arches supported on columns.

c. A singly curved network of cables spanning in one direction and slung between edge beams at each end of the cables.

A more detailed examination of an example or examples of each basic type as well as a partial list of constructed cable supported roof structures follows this section.

Beyond this elementary evaluation and classification of structural type, it should be noted that there exists in project or proposal form a number of diverse application for cables in general structural use. These are of general interest in the area of cable application, but are less pertinent to this thesis. Published applications of this nature, where possible, have been noted in the bibliography.
2. List of Constructed Examples

State Fair Arena  
Raleigh, North Carolina  
Matthew Nowicki designer  
William Henley Deitrick architect  
Fred Severud engineer

Congress Hall  
Berlin, Germany  
Hugh Stubbins architect  
Fred Severud engineer

David S. Ingalls Hockey Rink  
Yale University  
New Haven, Connecticut  
Eero Saarinen architect  
Fred Severud engineer

Dulles Airport Terminal  
Maryland  
Eero Saarinen architect  
Ammann & Whitney engineer

Utica Auditorium  
Utica, New York  
Gehron and Sletzer architects  
Lev Zetlin engineer

Pan American Airways Jet Hangar  
New York International Airport  
Port of New York Authority architects  
Ammann and Whitney engineer

United States Pavilion  
1958 Brussels World's Fair  
Edward D. Stone architect  
engineer

Corning Glass Cafeteria

Fred Severud architect  
engineer

Calo Food Market  
San Francisco, California  
Frank Wynkoop architect  
Dudley Wynkoop engineer

Municipal Stadium  
Montevideo, Uruguay  
Kondino, Viera and Miller architects  
Preload Co. Inc. engineer
French Pavilion
Zagreb, Yugoslavia 1935 Exposition

Bernard Lafaille architect

Health and Physical Education Building
Central Washington College
Ellensburg, Washington

Ralph H. Burkhard architect
Anderson, Birkeland & Anderson, engineers

Exposition Building
Century Twenty One Exposition
Seattle, Washington

Paul Thiry architect

Locomotive Pavilion
Chicago World’s Fair 1933

Marie Thumas Pavilion
Brussels Fair 1958

Baucher, Blondel, & Filippone—architects
Rene Sarger engineer
B. Theory

The general forms of equilibrium of suspension cables as well as the formulae for a parabola of equilibrium represent engineering information which falls outside the knowledge of most architects due to its rarity of application. Therefore basic pertinent information is included in this section as a brief survey of the geometric and engineering considerations employed in cable design. Where engineering formulae of more specialized nature are utilized in the analysis, they are explained in the calculations. Formulae utilized in the analysis of concrete structural members are explained where necessary in the calculations.

1. General Forms of Equilibrium of Suspension Cables.

a. Catenary of Equilibrium

A cable loaded uniformly along its length will take the form of a catenary. This applies also to the case of a cable supporting its own weight only.
b. **Parabola of Equilibrium**

A cable loaded uniformly along its chord length will take the form of a parabola.

c. **Funicular Polygon of Equilibrium**

A cable loaded with a system of concentrated loads will take the form of a funicular polygon.

2. **General Formulae for a Parabola of Equilibrium**

   a. When a cable supports a load that is uniform per unit length of the cable itself, such as its own weight, it takes the form of a catenary. Under closely spaced applied
loads of uniform magnitude the curve through the points of inflection of a funicular polygon will closely approach that of a catenary. In either case unless the sag of the cable is large in proportion to its length, the shape taken may often be assumed to be parabolic, the analysis being thus greatly simplified.

Structural calculations for cables included herein will utilize the cable formulae for a parabola of equilibrium and the following general formulae will apply:

**Horizontal Component of Cable Stress**  \( (H) \)

\[
H = \frac{w L^2}{8h} \quad \text{WHERE "W" = UNIT LOAD/FT.}
\]

**Maximum Stress in Cable**  \( (\tau_{MAX}) \)

\[
\tau_{MAX} = H \left( 1 + 16 \frac{\Theta^2}{L} \right)^{1/2} \quad \text{WHERE} \quad \Theta = \frac{h}{L}
\]

**Shape of Cable**

\[
y_c = \frac{4h}{L^2} x_c^2 \quad \text{ASSUME} \ x_c \text{ VALUES, AND SOLVE FOR VARIOUS} \ y_c \text{ VALUES.}
\]
### C. Application

The following general uses have been considered in determining possible application and size of structure.

1. Transportation Terminal
2. Airplane Hangar
3. Indoor Sports Enclosure
   a. gymnasium-basketball
   b. swimming pool
   c. tennis court
4. Exposition Pavilion or Arena
5. Retail Sales
   a. Supermarket
   b. Junior Department Store
   c. Discount Department Store

### D. Economy

No effort has been made to establish the relative economy of the structure due to the complexity of such an analysis. From empirical observation it is possible to sur-
mise that the cost of the structure relative to other more conventional means of spanning the space enclosed would not be competitive until spans were great. It is generally conceded that suspension structures will begin to compete on a cost basis with other structural methods when spans reach approximately three hundred feet. It is probable that this figure can be lowered if economy is of prime importance. This would be particularly true if full advantage is taken of the rapidity of erection possible in cable construction.

In the development of this problem it has been assumed that the resulting structure would be one in which low cost was not of prime importance, and that quality within some higher limit would permit the utilization of the structure employed in the final solution.

III. DESIGN CRITERIA

A. Building Type

The building is to be a high quality free standing structure with large interior clear space offering a high degree of flexibility, with the general requirement of a dynamic interior space and strong exterior form. Contemplated uses would be those of transportation terminal, indoor sports enclosure, and exposition pavilion or arena.

B. Building Size

From the studies of possible application, with respect to ground plan size, interior volume enclosed, and exterior overhang, the following basic dimensional considerations are established:
1. An enclosed clear space of approximately 30,000 sq. ft.
2. A rectangular plan with dimensions of approximately 150 x 200 ft. at the floor plane.
3. Overhang to be closely tied to the geometric, structural, and spatial characteristics of the system. As such, desirable dimensions are difficult to predetermine. They should, however, reflect in their generosity the interior spatial character and the overall quality of the building.

C. Quality of Space

In light of contemplated uses, spatial considerations would reflect primarily the basic requirement of a large clear space of dynamic character. The end result of these considerations would be a product of the cable network geometry selected, the spatial characteristics of the general structural system, and the articulation of its parts. Other objective considerations such as natural and artificial light, acoustics etc. are discussed under separate sections.

D. Structural Considerations

Beyond the applicable general requirements outlined under building type, certain general considerations apply to the formulation of the structural system.

1. Structural and modular order offering the following advantages:
   a. Similarity of parts
   b. Repetition of components
   c. Ease of fabrication and erection
   d. Mass production of shop fabrication
   e. Use of advanced concrete technological levels.
E. Functional Considerations

1. Lighting

Since no specific use is advanced for the structure, the lighting requirements must of necessity remain general in nature.

   a. Natural Light

      No specific requirements to be established. The following general conditions to be pursued:

      1. No natural light to be provided through roof surface.

      2. Possibility of large glass area to be maintained throughout wall surface.

      3. Any sun or glare control other than that afforded by structural overhang will not be considered.

   b. Artificial Light

      No specific requirements for artificial illumination are to be established. The following general conditions will be pursued:

      1. Accommodations for a general illumination system should be incorporated within the structure.

      2. Such a system should offer a flexibility which would accommodate the lighting required for the contemplated uses of the structure.

2. Acoustics

   A proper acoustic environment for the general program requirements will depend upon the following considerations:
a. Reverberation Time

1. With a fixed volume, required reverberation time can be attained by:

   a. Variation of sound absorptive material at roof surface.

   b. Use of distributed sound amplification system.

b. Background Noise Criteria

1. For the contemplated uses, an ambient background noise level of between 30 and 40 decibels is indicated. This would be provided by the air handling system.

c. Sound Distribution

1. For the contemplated uses, and in light of the relatively large volume of the space, a distributed system of sound amplification would be indicated. Such a system would incorporate a high degree of absorption on ceiling and if possible on walls, with the amplification source directed downward upon the inhabitants of the building.

2. In the event that the space were to serve a function requiring presence - i.e. performance of some sort - it would be necessary to employ a more elaborate system incorporating time delay for proper acoustic conditions.

d. Sound Absorption

Primary sound absorption will be necessary in varying degrees at ceiling level depending upon use. Major source of noise at floor level would be produced by persons in the building. This sound will be attenuated in a horizontal direction by the presence of the persons creating the sound.
3. Mechanical Equipment and Services

All mechanical equipment will wherever possible be placed underground or at some distance from the structure.

a. All air handling duct work wherever possible to be incorporated within the structural or floor system.

b. A forced air system is contemplated which would require supply at floor level with return air accommodations at ceiling level - preferably at the high points of the ceiling plane. Exhaust system to be underground at some distance from the building.

c. Waste, soil, and water supply systems to be underground and as such not a part of the design program.

4. Thermal Insulation

a. Where possible to be incorporated into the structural system.

5. Roofing

a. High quality roof in conjunction with overall character of the structure.

IV. PRELIMINARY PROPOSAL

The proposal consists primarily of the study of a cable roof network supporting concrete panels.

A. Dimensional Characteristics

A cable network plan projection of approximately 150 x 200 feet.
B. Structural Characteristics

1. Cable Network

The cable network consists of two families of cables perpendicular and at opposite curvatures to one another producing a doubly curved surface whose separate parts are essentially square,

a. Suspension cables hang between two equal parallel lines 200 ft. apart and at equal distance above the ground plane and describe 22 equal divisions in the lines of 6.318 ft. each. Cable sags are determined by a parabola at the centerlines and perpendicular to the suspension cables. The parabolas' low points are at the outside cables with its high point at the centerline of the central suspension cable.

b. The stressing cables are perpendicular to and at opposite curvatures to the suspension cables and are spaced at intervals that vary from 6.783 ft. at the centerline suspension cable to 6.896 ft. at the outside cables.

2. Concrete Roof Panels

Roof panels are solid three inch thick slabs spanning two ways between canted integral edge beams. The edge beams provide stiffness to the panel during fabrication and erection, and become a forming expedient for the concrete poured around the cables. The lattice formed by these edge members, the cables, and the concrete pour serves to distribute unequal live loads across the cable network, and provide rigidity to resist the forces of uplift.

C. Functional Characteristics

1. Lighting

The possibility of lighting conduit runs is
provided by the articulated joint between panels.

2. Acoustics

The possibility of applying absorptive material within the coffered portion of the roof panel is provided.

D. Criticism

1. Building Size

Depending upon method of support, cable network may dictate enclosed building area under that determined in design criteria.

2. Structural Characteristics

a. Cable Network

1. Spanning cables in longitudinal rather than transverse direction are inefficient. If possible spanning cables should be in transverse direction with stressing cables in longitudinal direction.

2. Transverse curvature is insufficient to provide resistance to flutter and uplift.

3. Transverse curvature at network extremities is insufficient to apply prestress to suspension cables.

b. Concrete Roof Panels

1. Solid concrete panels have inefficient span to weight ratio and as such will reflect in total cable area required for assumed roof area.

3. Functional Characteristics

a. Thermal Insulation

No provisions are made for the incorporation of thermal insulation into the roof panels.

b. Mechanical

No provisions are made for the incorporation of mechanical services into the structure.
V. THE FINAL PROPOSAL

A. General Description

The building is a free-standing clearspan structure wherein the roof is constructed of precast concrete slabs supported by a doubly curved network of steel cables. The cables deliver the roof loads to the ground through a system of reinforced concrete edge beams and supporting members.

B. Dimensional Characteristics

1. Floor area at ground plane is determined by a rectangle 140 ft. by 224 for an enclosed area of 31360 sq. ft.
2. Ceiling heights and corresponding volume vary with the geometry of the roof structure.

C. Structural System

1. Cable Network

The cable network consists of two families of cables perpendicular and at opposite curvature to one another. The geometric breakup produces a doubly curved surface whose parts are sufficiently uniform to permit a repetition of similar building components and a resulting ease of fabrication and erection.

a. The suspension family of "I Cables" are suspended at nine foot intervals from two parallel lines 185 feet apart and at a height of 54 feet above the finish floor line. These cables vary in sag from 37 ft. at the outside cables to 18 ft. at the centerline cables. The sags of the intermediate cables are determined by a parabola at the center-
line and perpendicular to the suspension cables, with a chord length of 207 feet and a rise of 19 feet.

b. The stressing family of "II Cables" are perpendicular and at opposite curvature to the suspension cables and are spaced at equal intervals which scale approximately 9 feet 6 inches at their ends and 9 feet 0 inches at their centerline.

c. One family by being essentially perpendicular and at opposite curvature to the other family, exerts a prestress to its opposite when it is tensioned. An initial prestress is given to the suspension cables by the roof dead load, while tension introduced into the stressing cables by means of jacks provides additional load to the suspension cables to assure their continued prestress throughout the live load cycle. Further, by tensioning the stressing cables a sufficient amount, it is possible to assure that they will continue to act beyond the maximum live load deflection of the suspension cables.

2. Roof Panels

a. The roof panels are prefabricated flat slab units of a sufficiently small number of different sizes to permit efficient shop fabrication and assure visual uniformity. Slight differences will be taken up in the joints that occur between panels.
b. Since the amount of steel cable utilized in the cable network will be almost directly proportional to the span to weight efficiency of the roof panels, these panels have been designed with a light weight foamed poly-urethane core which reduces slab weight while retaining sufficient material at the extreme fiber to cover the reinforcing steel and assure slab action. This design has the advantage of producing a greater spanning capacity for weight of panel than a solid slab, while at the same time incorporating thermal insulation into the panel unit.

c. The flat two-way portion of the roof panel is surrounded by an inclined integrally poured edge beam which serves to stiffen the panel during fabrication and erection, and when suspended from the cable network, becomes a forming expedient for concrete poured around the cables. The lattice thus formed becomes a diaphragm for distributing unequal loads in the completed structure.

3. Roof Supporting System

The roof surface of cable network and concrete slabs is supported by a composite system which consists of alternately similar pairs of structural elements surrounding the perimeter of the rectilinear plan.
a. Major Support Structures

All forces, both vertical and horizontal, generated within the total structure are ultimately resisted by a system of regularly spaced column members which occur on alternate long sides of the rectilinear plan. These members perform their combination of structural requirements in the following manner.

1. Six columns of translational shape spaced at 45 ft. o.c. along each of the two longitudinal elevations ultimately carry all forces produced in the structure to the ground by moment and compressive resistance.

2. The six columns on each major elevation translates in form upward, increasing in width, and outward from the building centerline in a cantilevered, fan shaped, doubly curved surface. The resulting shape gives added column resistance to twisting, and serves to shorten the span between columns.

3. The remaining 18 ft. between columns is spanned by an integral concrete beam in what is now the same plane as the translational column. This beam carries the relatively small live and
dead loads that occur above it in the supporting structure, and serves to tie the columns together so that twisting stresses introduced by unequal forces delivered in each column panel by the cables are more equally distributed to the six columns. In addition, the beam ties the six columns together into a rigid frame so that wind leads are not taken solely by the first column adjacent to the wind exposed side.

4. Directly above the previously mentioned beam, and translating from it and the columns along the length of each long elevation, is a horizontal concrete section or beam which resists the cable tension between the columns. Where this horizontal beam continues across the top of the column, it serves as a transitional element to deliver the cable forces that occur within the column width at the roof line intersection.

b. Suspended Side Trusses

Between the two major supporting columns systems at the extremities of the suspension cables on the two minor sides of the plan rectangle is a suspended truss of precast concrete
elements. The truss elements are suspended from cables in a manner similar to the roof panels. After joints between the elements are grouted, the truss is post tensioned with separate wires placed for that purpose. When completed, the trusses resist the stressing cable tensions which are introduced by jacks positioned between the separate truss elements before grouting. The trusses deliver the tension forces thus resisted to the major supporting structure from which they are suspended.

D. Functional Characteristics

1. Lighting
   a. Natural Light

1. No provisions are made for the utilization of skylighting. The possibility of adding skylights exists, as the overall negative curvature of the roof presents no drainage problems, while the modular breakup of the cable network and prefabricated concrete panels would permit the introduction of specially constructed skylight panels.

2. Extensive vertical open area throughout the structure permits total utilization of glass in the two minor elevations with approximately 65% glass area possible on major elevations. The effective control of these
large glass areas is accomplished in part by overhangs of approximately 15 ft. on minor elevations and 30 ft. on major elevations. The relative effectiveness of this overhang varies with the geometry of the structural form.

The demand for further control of light admitted through glass area would vary with specific site conditions and functional requirements, and as such would require specialized solutions. It is possible that these would include the following:

a. Heat and glare reducing glass.

b. Louvers or deep window mullions incorporated into the fenestration.

c. The replacement of areas indicated herein as glass with opaque material.

b. Artificial Light

Provisions for the installation of artificial lighting, for general and specialized illumination are provided in the roof construction as an integral part of the structure. This is accomplished by providing a continuous slot below the joint between roof panels. This slot provides a semi-concealed two directional conduit chase which offers complete flexibility in the position and extent of lighting.
2. **Acoustics**

The following provisions or accommodations are made to assure a proper acoustic environment as outlined under design criteria section:

a. The doubly curved surface of the roof structure serves to break up sounds directed at the ceiling plane, avoiding flutter.

b. Coffered roof panels provide a ceiling configuration which permits the installation of sound absorbing material necessary for proper reverberation time without compromising structural and visual order.

c. Contemplated air handling equipment would be sufficient to provide desired ambient sound background of 30 to 40 decibels.

3. **Mechanical Equipment and Services**

a. **Mechanical Equipment**

   Heating, ventilating and air cooling equipment is to be housed separately or underground and as such is not a part of this problem. Accommodations for ductwork are provided in the following manner:

   1. Supply and exhaust ducts arrive underground or originate at basement level. Supply, return, and exhaust air registers are at floor level or above, as dictated by special functional requirements.
2. Exhaust and return air intakes occur at high points of the roof surface along the two major interior elevations. Ductwork for these services would be cast into the reinforced concrete columns and would reach mechanical equipment under ground. An estimate of total cross sectional area required for this function is 24 sq. ft. which would necessitate 2 sq. ft. of duct per column. It is assumed that the structural mass of the column is sufficient to accommodate this total area if it is in a sufficient number of increments.

b. Waste, Soil and Water Supply Systems

Systems will be underground and as such are not a part of this problem.

c. Electrical

Service to be supplied to floor slab underground, and to roof by means of conduit runs set in concrete columns.

4. Thermal Insulation

a. Thermal insulation for roof is incorporated into the precast slabs, and would be of lightweight foamed poly-urethane of 4 inch thickness.
5. Roofing

a. Proposed roofing is of sheet copper with standing seams and matching crimps at regular intervals following lines parallel to the stressing cables.

b. The geometry of the roof surface assures drainage to the minor elevation gutters which possess sufficient capacity to handle the run off delivered. Water from the gutter is delivered to concrete receptacles at both minor elevations by precast concrete down spouts of sufficient vertical depth to eliminate undue splashing.

E. Construction Sequence

It is assumed that prefabrication of structural elements where possible, can be scheduled so that components would be available when required in the construction sequence. A general order of construction of building components is as follows:

1. Site preparation, placement of service facilities, pouring of fittings, and other preliminary requirements not studied herein.

2. Pour major elevation structural supports, employing standardized forms in number consistent with economic considerations. Columns could be poured individually employing movable forms, or additional duplicate formwork can be constructed consistent with
economic considerations. Each column would incorporate the necessary ductwork, conduit runs and attachment accommodations for suspension cables.

3. Hang cables for suspended precast concrete side trusses.

4. Attach precast concrete truss members, place post tensioning wires and tensioning jacks for stressing cables and grout joints.

5. Post tension concrete side trusses, using temporary vertical tension members to assure truss remains in proper position.

6. Hang suspension cables employing falsework at centerline to assure proper sag.

7. Position stressing cables and stress to predetermined partial tension.

8. Place concrete roof slabs by means of crane. Mechanical attachment to be accomplished by workmen raised to cable network by crane in building interior.

9. Place sandbag overload on panels to simulate added dead load of concrete lattice to be poured.

10. Adjust tension in suspension and stressing cables to final predetermined stresses, and secure mechanical attachment of cables at crossing points.

11. Pour concrete lattice, removing overload on panels as pours are made.

12. Attach precast facia members.
13. Roof structure and make final check of roof position and cable tension.

14. Replace temporary tension restraint at centerline of side truss and replace with permanent mullions.
**VI STRUCTURAL ANALYSIS**

**A. CONCRETE ROOF PANELS**

![Diagram of 4" thick thermal insulation of foamed polyurethane](image)

**Assumed 9'-0" sq.**

(Actual dimensions vary slightly)

**Panel Section:** 1/2" x 1/0"

**TEXT:**

It is assumed that greatest stresses will occur in panel during fabrication and erection handling. Therefore depth of section is determined by thickness of desired thermal insulation and extreme fiber concrete necessary for steel coverage. Capacity of section as two-way slab with four edges discontinuous is checked in calculations. A 9'-0" clearspan is assumed to be more critical than the actual edge beam condition, and is assumed for ease of calculation.

\[
\begin{align*}
3000 \text{ psi Conc.} \times 45 &= 1350 = f_c \\
20,000 \text{ psi Steel} & \quad \text{LIVE LOAD 40 #/SQ.FT.} \\
& \quad \text{DEAD LOAD 45 #/SQ.FT.} \\
& \quad \text{TOTAL LOAD 85 #/SQ.FT.}
\end{align*}
\]

**Design Moment**

\[
M = \frac{wL^2}{20}
\]

\[
M = \frac{85 \times 64 \times 12}{20}
\]

\[
M = 3264 \text{ #}
\]

**Allowable Moment**

\[
M = 1061 \text{ psi} \times 12'' \times 1.5''
\]

\[
M = 19,098 \text{ #}
\]

**Steel Required**

\[
T_{max} = \frac{3264}{5.5} \quad \text{(couple arm)} = 595 \text{ #}
\]

As Req'd. = \frac{595}{20,000} = .0298 SQ. IN.

Use: 6x6 - 6/10 SQ. WELDED WIRE FABRIC = .058 SQ. IN.

**Section Stress Diagram**
VI. STRUCTURAL ANALYSIS (continued)

B. Cable Network

1. General Cable Information and Assumptions

a. The strength and physical characteristics of cable vary with the method of their fabrication and the materials employed. A more detailed study of the various types of cables available would be required to determine the best type of cable for this particular design.

b. In practice the load capacity is determined by test of particular cable size rather than applying allowable unit stress values to the cross section area. Cable makeup is such that this load capacity does not have a simple relationship to the amount of material in cross section.

c. For the purposes of design in this problem it is assumed that the cable used is factory prestretched galvanized wire bridge rope with the following properties:

1. \( E = 20,000,000 \) psi
2. Maximum breaking strength in tension = \( 250,000 \) psi
3. Design strength in tension = \( 125,000 \) psi

d. Sizes of cable determined in the calculations will remain in the form of area of effective steel required. In selecting cables from manufacturer's handbooks, the maximum tension...
value for the particular cable rather than the effective steel area required would be used.

2. General Prestressing Design Approach

Supporting cables must be designed for both live and dead loads. Under most circumstances, however, they will be subjected to dead loads only. Thus, this loading will dictate their normal state of repose unless the stressing cables are employed to introduce additional load.

Since it would be desirable for the structure to remain as close as possible in a constant shape; and since it would also be desirable to have the supporting cables under a downward or "tiedown" stress independent of the normal roof dead loads, a portion of the cable deflection which would result from live loads will be introduced by the stressing cables.

This prestress is possible since the stressing cables are essentially perpendicular to and at opposite curvature to the supporting cables, and thus when placed under tension will exert a downward component upon the supporting cables.

The total procedure by which the design of the cable network is accomplished includes both the sizing of the individual cable members and the balancing process which determines the final position of the overall network.
A detailed outline of this process, with the theory and formulae involved follows this section.
CABLE DEFLECTION THEORY

The upward deflection, $\Delta l$, which the suspension cables would have if they were free from the live loads and/or the counteraction of the stressing cables, will take place only for a certain proportion, $\Delta l/n$, due to the presence of the stressing cables. When the spanning cables attempt to lift from their designed position (with capacity for both live and dead load), the stressing cables will be loaded at crossing points by a certain load, $P$, which can be determined by trial and error. This is accomplished by guessing what might be the actual $\Delta l/n$, and checking if the "guess" is confirmed by the application of formula $\alpha$.

---

**Diagram Description:**
- Position of suspension cables with dead load only.
- Position of suspension cables with dead load and stressing cable load.
- Position of suspension cables with both live and dead loads. (In theory, load in stressing cable is zero at this point.)
- Position of supporting cables with load of suspension cables.
- Position of stressing cables with no load. (I.e., when suspension cables are subjected to both live and dead loads.)
- Normal attitude of stressing cables.
DERIVATION OF FORMULA $\alpha$

Balancing the elongation of the suspension cables with that of the stressing cables is accomplished by means of a formula referred to herein as Formula $\alpha$.

The formula is in actuality to different formulae for elongation equated one to the other. The two formulae are those of "Geometric Variation" and "Elastic Elongation" and are as follows:

**Geometric Variation**

$$\Delta l = \frac{8}{3}L \left( \frac{h_1^2}{L^2} - \frac{h_2^2}{L^2} \right) \quad \text{or} \quad \frac{8}{3L} \left( h_1^2 - h_2^2 \right)$$

where:
- $L$ = chord length
- $h_1$ = greater deflection
- $h_2$ = lesser deflection
- $\Delta l$ = change in actual cable length

**Elastic Elongation**

$$\Delta l = \frac{(T_1 - T_2)L}{E \alpha} \quad \text{or} \quad \frac{L^3}{6E \alpha} \left( \frac{F_1}{h_1} - \frac{F_2}{h_2} \right)$$

where:
- $T_1 = \frac{P_1 L^2}{\delta h_1}$
- $T_2 = \frac{P_2 L^2}{\delta h_2}$
- $\Delta l$ = change in actual cable length
- $P_1$ = greater unit load
- $P_2$ = lesser unit load
- $h_1$ = greater deflection
- $h_2$ = lesser deflection
- $E$ = modulus of elasticity
- $\alpha$ = cross sectional area of cable sf'l. (effective area)
DERIVATION OF FORMULA $\alpha$ (CONTINUED)

\[ \frac{8}{3L} (h_1^2 - h_2^2) = \frac{L^3}{8E\Omega} \left( \frac{P_i}{h_1} - \frac{P_2}{h_2} \right) \]

or

\[ h_1^2 - h_2^2 = \frac{3L^4}{64E\Omega} \left( \frac{P_i}{h_1} - \frac{P_2}{h_2} \right) \]

FOR FORMULA $\alpha$ SUBSTITUTING:

$P_i = P_2$ (INITIAL)

$P_2 = P_f$ (FINAL)

$h_1 = h_i$ (INITIAL)

$h_2 = h_f$ (FINAL)

\[ \therefore \text{FORMULA } \alpha = \]

\[ h_i^2 - h_f^2 = \frac{3L^4}{64E\Omega} \left( \frac{P_i}{h_i} - \frac{P_f}{h_f} \right) \]
CABLE NETWORK CALCULATIONS

Diagram of Centerline Cables

Unit Panel Loads:
- Live Load = 40 #/sq. ft.
- Dead Load = 63.4 #/sq. ft.
- Total Load = 930 #/ft.

Solve for Horizontal Component of Theoretical & Suspension Cable

\[ H = \frac{PL^2}{8h} = \frac{930 \times 185^2}{8 \times 18} = \frac{31,800,000}{144} \]

\[ H = 221,000 \text{ #} \]

Solve for Maximum Tension in Cable

\[ T_{\text{max}} = H \left(1 + 16 \theta^2 \right)^{1/2} \]

\[ \theta = \frac{h}{L} = \frac{18}{185} = 0.097 \]

\[ \theta^2 = 0.0094 \]

\[ 16\theta^2 = 1.15 \]

\[ T_{\text{max}} = 237,000 \text{ #} \ (\text{Max. Tension in C Suspension Cable}) \]

Solve for Required Effective Cable Area

\[ A = \frac{T_{\text{Actual}}}{T_{\text{Allowable}}} = \]

\[ = \frac{237,000}{125,000} \]

\[ A = 1.9 \text{ sq. inches} \]
SOLVE FOR SUSPENSION CABLE HEIGHT DUE TO REMOVAL OF LIVE LOAD

\[ h_u^2 - h_f^2 = \frac{3.14}{64E/2} \left( \frac{P_i}{h_u} - \frac{P_f}{h_f} \right) \] (FORMULA \( \alpha \))

\[ 18^2 - h_f^2 = \frac{3 \times 18^4}{64 \times 20 \times 10^6 \times 1.9} \left( \frac{180}{18} - \frac{570}{h_f} \right) \text{ or } 324 - h_f^2 = \frac{3509 \times 10^6}{2432 \times 10^6} \left( 51.7 - \frac{570}{h_f} \right) \]

\[ 324 - h_f^2 = 75 \cdot \frac{825}{h_f} \quad \text{or} \quad 249 - h_f^2 = \frac{825}{h_f} \]

\[ h_f^3 - 249 \cdot h_f = 825 \] EQUATION CAN BE SOLVED BY QUADRATICS, BUT IS MORE READILY DONE BY TRIAL & ERROR

TRY \( x = 7 \) FOR \( x^3 - 249x = 825 \)

\[ 4913 - 4270 = 643 \times 825 \] NO

TRY \( x = 17.2 \)

\[ 5088 - 4280 = 808 \approx 825 \]

\[ h_f = 17.2 \text{ FT.} \]

SOLVE FOR \( \Delta i \)

\[ \Delta i = h_i - h_f = 18 - 17.2 \]

\[ \Delta i = 0.8 \text{ FT.} \]

Since the stressing cables reintroduce a portion of the live load, it is possible to say that the "final position" will be somewhere between 0 and .8 ft. By trying various values between 0 and .8, it is possible to determine what the actual \( \Delta i \) would be for a given size of stressing cable capable of taking the loads applied upward by the suspension cables when they try to move from their "initial" to "final" position. Since there are a number of variables involved, this is necessarily a trial and error process. Therefore, start by trying \( \Delta i/2 \) and check what would be the shape of the stressing cables when the supporting cables lift and transfer their loads.

NOTE: It is necessary to retain the same size cable "2" selected for stressing cables at \( \Delta i/2 \) throughout the calculations. This results in excess load carrying capacity in stressing cables which will be utilized to give an additional stress to assure that stressing cables do not go slack under total live and dead loads.

TRY \( \Delta i/2 \)

\[ \Delta i/2 = \frac{0.8}{2} = 0.4 \]
SOLVE FOR $h_3$ (H FOR $\Delta x/2$ ASSUMPTION)

$$h_3 = h - \Delta x/2 = 18 - .4$$
$$h_3 = 17.6 \text{ FT}$$

SOLVE FOR $P_3$ (P FOR $\Delta x/2$ ASSUMPTION)

$$h_i^2 \cdot h_3 = \frac{3L^4}{64EI} \left( \frac{P_i}{h_i} - \frac{P_3}{h_3} \right) \text{ or } 324 - 310 = \frac{3 \times 185^4}{64 \times 20 \times 10^6 \times .19} \left( \frac{930 - P_3}{18 - 17.6} \right)$$
$$14 = 1.45 \left( \frac{51.6 - P_3}{17.6} \right) \text{ or } 14 = 75 - \frac{1.45P_3}{17.6} \text{ or } \frac{61 \times 17.6}{1.45} = P_3$$

$$P_3 = 740 \#$$

DETERMINE P FOR STRESSING CABLES

$$P_3 - P_f = 740 - 570$$
$$P_3 - P_f = 170 \# / \text{FT}$$

SIZE STRESSING CABLE @ Q.

$$H = \frac{P_L^2}{8h} = \frac{170 \times 207^2}{8 \times 19} = \frac{7,300,000}{152}$$
$$H = 48,000 \#$$
$$\Theta = \frac{h}{L} = \frac{19}{207}$$
$$\Theta = .092$$

$$T_{\text{MAX}} = H(1 + 16 \Theta^2)^{1/2} = 48,000 \times (1.064)$$
$$T_{\text{MAX}} = 50,600 \#$$

area of = $T_{\text{ACTUAL}}$ 

$T_{\text{ALLOWABLE}} = 50,600 \over 125,000$

$$\Omega = .41 \text{ SQ. IN. AREA REQUIRED FOR Q STRESSING CABLE}$$

CHECK $\Delta x/2$ ASSUMPTION

SOLVE FOR $h_f$

$$h_i^2 - h_f^2 = \frac{3L^4}{64EI} \left( \frac{P_i}{h_i} - \frac{P_f}{h_f} \right) \text{ WHERE } P_i = 0$$

$$19^2 - h_f^2 = \frac{3 \times 207^4}{64 \times 20 \times 10^6 \times .41} \left( 0 - \frac{170}{h_f} \right) \text{ OR } 361 - h_f^2 = 560 \times 10^6 \left( \frac{170}{h_f} \right)$$

$$361 - h_f^2 = 10.6 \left( \frac{170}{h_f} \right) \text{ OR } h_f^3 - 361h_f = 1800$$
SOLVE FOR \( h_f \) BY TRIAL AND ERROR
\[ x^3 - 361x = 1800 \]

TO CHECK, \( h_f \) WOULD EQUAL 19.4 \( (19 + .4) \) \( \therefore \rightarrow \)
TRY \( x = 19.4 \)

\[ 7301 - 7000 = 301 < 1800 \therefore \text{CHECK NOT GOOD} \]
AND STRESSING CABLE WILL DEFLECT GREATER THAN
.4 FT. UNDER 170#/FT. LOAD.

TRY \( x = 21.1 \)

\[ 9394 - 7620 = 1774 \geq 1800 \therefore \text{OK} \]

SAY \( h_f = 21.1 \) FT.

STRESSING CABLE WOULD DEFLECT 21.1 FT. UNDER 170#/FT. LOAD.

\[ \therefore \text{ASSUME DIFFERENT PROPORTION OF } \Delta \text{ THAT WILL PRODUCE A} \]
LOWER VALUE OF \( P_s - P_f \) AND CHECK AGAIN.

TRY \( 3/4 \Delta \text{I} \)

\[ 3/4 \times 18 = .6 \]

SOLVE FOR \( h_s \)

\[ h_s = h_i - 3/4 \Delta i = 18 - .6 \]
\[ h_s = 17.4 \]

SOLVE FOR \( P_s \) \( \text{FOR } 3/4 \Delta \text{I ASSUMPTION} \)

\[ h_i^2 - h_s^2 = \frac{3L^4}{64EI} \left( \frac{P_s}{h_i} - \frac{P_s}{h_s} \right) = 324 - 303 = 1.45 (17.4 - P_s) \]

\[ 7.5 = 75 - \frac{145P_s}{17.4} \text{ OR } 54 \times 17.4 = P_s \]

\[ P_s = 650\# \]

DETERMINE \( P \) FOR STRESSING CABLES

\[ P_s - P_f = 650 - 570 = 80 \# \text{/FT.} \]

CHECK \( 3/4 \Delta \text{I ASSUMPTION} \)

SOLVE FOR \( h_f \)

\[ h_i^2 - h_f^2 = \frac{3L^4}{64EI} \left( \frac{P_i}{h_i} - \frac{P_f}{h_f} \right) \]

\[ 361 - h_f^2 = 10.6 (-\frac{80}{h_f}) \text{ OR } h_f ^2 - 361 h_f = 848 \]

\[ x^3 - 361x = 848 \text{ WHERE } x = h_f \]
**SOLVE FOR h_f BY TRIAL AND ERROR**

\[ 5x - 361x = 848 \]

Where \( x = h_f \)

To check \( h_f \) would equal 19 + .67 or 19.67

**TRY \( x = 19.6 \)**

\[ 7600 - 7760 = 460 < 848 \]

10

Check not good and stressing cable will deflect more than .6 ft under 80#/ft load.

**TRY \( x = 20 \)**

\[ 8000 - 7250 = 750 < 848 \]

10

**TRY \( x = 20.1 \)**

\[ 8121 - 7250 = 871 \geq 848 \]

Ok

Say \( h_f = 20.1 \)

Stressing cable would deflect 20.1 ft under 80#/ft load. 

Assume a different proportion of \( \Delta A \) that will produce a lower value of \( P_3 - P_f \) and check again.

**TRY \( 9/6 \text{Δ}i \)**

\[ 5/6x + .8 = .67 \]

**SOLVE FOR \( h_3 \)**

\[ h_3 = h_i - 3/4 \text{Δ}i \] or \( 18 - .67 = 17.33 \)

**SOLVE FOR \( P_3 \) (FOR 9/6Δi ASSUMPTION)**

\[ h_i^2 - h_3^2 = \frac{3L^4}{64EI} \left( \frac{P_i - P_3}{h_i} \right) = 324 - 300 = 1.45(51.6 - P_3) \]

\[ 24 = 75 - 1.45P_3 \] or \( \frac{51 \times 17.33}{1.45} = P_3 \)

\[ P_3 = 610 \]  

**P FOR STRESSING CABLES**

\[ P_3 - P_f = 610 - 670 = 40 \] #/ft.,

**CHECK 9/6Δi ASSUMPTION**

**SOLVE FOR \( h_f \)**

\[ h_i^2 - h_f^2 = \frac{3L^4}{64EI} \left( \frac{P_i - P_f}{h_i} \right) = 361 - h_f^2 = 10.6 \left( \frac{-40}{h_f^2} \right) \]

\[ h_f^3 - 361h_f = 425 \]

or \( x^3 - 361x = 425 \)

Where \( x = h_f \)

**SOLVE FOR \( h_f \) BY TRIAL AND ERROR**

To check \( h_f \) would equal 19 + .67 or 19.67

**TRY 19.67**

\[ 7600 - 7710 = 500 \geq 425 \]

No stressing cable will deflect less than .67 under 40#/ft load.

**TRY 19.6**

\[ 7530 - 7070 = 960 \geq 425 \]

\( h_f = 19.6 \) ft.
From the previous calculations it is possible to say that the actual \( \Delta i \) proportion is between \( \frac{3}{4} \) and \( \frac{5}{6} \Delta i \), that the actual stressing cable load \( (P_i - P_f) \) is between 80 and 40#/ft, and that the actual \( h_f \) is between 20.1 and 19.6 ft.

**Assume values by interpolation and check**

\[
\begin{align*}
    h_f &= 19.8 \\
    P_i - P_f &= 58\# / ft \quad (P_f \text{ for stressing cables})
\end{align*}
\]

\[
\begin{align*}
    a^2 - h_f^2 &= \frac{3L^4}{64EI_2} \left( \frac{P_i - P_f}{h_i} \right) \\
    361 - 392 &= 10.6 \left( 1 - \frac{58}{19.8} \right) \quad \text{or} \quad -31 = -10.6 \times \frac{58}{19.8}
\end{align*}
\]

\[
31 = 31 \quad \therefore \text{Assumption OK}
\]

**Final cable system state of repose would be controlled by the centerline suspension and stressing cables (both theoretical in this case since network has an odd number of spaces in transverse and longitudinal directions), the suspension cables at the outside of the cable network (i.e., the line of the suspended side truss), and the two equal parallel lines from which the suspension cables hang. Calculations for suspension and stressing cables are figured by means of dimensions on diagram at beginning of cable network calculations, while the final network shape is controlled by the lines in the following diagram.**
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