Probabilistic Aspects of Progressive Damage in Composite Structures

by

Brandon M. Reynante

B.S., Mechanical Engineering
University of California, San Diego (2009)

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY September 2011

© Massachusetts Institute of Technology 2011. All rights reserved.
Probabilistic Aspects of Progressive Damage in Composite Structures

by

Brandon M. Reynante

Submitted to the Department of Aeronautics and Astronautics on July 18, 2011, in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics

Abstract

The effects and importance of incorporating probabilistic aspects of the progression of damage in the analysis of composite structures are assessed. Two specific cases of graphite/epoxy with centrally-located holes are considered: a [0/90/±45]laminate subjected to equal biaxial tension, and a [±15/0]laminate subjected to uniaxial tension. The variation in the basic composite material strength values are used with the maximum stress criteria to assess the probability of occurrence of various damage modes by evaluating the cumulative distribution functions of the corresponding material strength parameters at various locations in the structure for a given state of damage and applied load. An average stress approach is used in assessing the occurrence of both in-plane damage and delamination. Two-dimensional and three-dimensional finite element models are used to obtain stress fields. In-plane damage is simulated by setting in-plane elastic constants of damaged elements effectively to zero. Out-of-plane damage is simulated by setting out-of-plane elastic constants of elements in adjacent plies effectively to zero. The results demonstrate that consideration of probabilistic characteristics of damage progression allows for the possibility of many different damage progression scenarios for a single laminate configuration, including the possibility of damage initiation and propagation via different damage modes (both in-plane and out-of-plane, as well as coupling between the two) and in numerous different geometric locations. Four key items that affect the probabilistic progressive damage behavior of the structure are identified: the particular details of the material strength distributions, the redistribution of stresses caused by the occurrence of damage, the damage history of the laminate including the modes and locations of all previous damage, and the nonlinear nature of failure probability as a function of stress. Recommendations are given to address some key issues in expanding the work. These include nonsymmetry of damage initiation and progression, use of strain energy release rate in assessing delamination, use of various material property degradation models to simulate damage, and consideration of probabilistic aspects of other material properties.

Thesis Supervisor: Paul A. Lagacé
Title: Professor of Aeronautics and Astronautics and of Engineering Systems, Massachusetts Institute of Technology
Acknowledgments

First and foremost, I must give infinite thanks to my wife, Erin. I would not have made it through this adventure without her (exceptional) patience and support. I owe you big time! I must also thank Mr. Chips for all the unconditional love, cuteness, and silly antics. You two give me cause to smile every single day.

Also, I owe many thanks to my family. They have always supported and encouraged my endeavors, and for that I am grateful. Special thanks to my father, Don, for sparking my initial interests in engineering and aerospace, and for exposing me to the awesomeness of MIT.

Thank you to everyone at Alpha STAR for two enjoyable summers in Long Beach. I learned much about PFA (from the engineers) and golfing (from Kyle). The support for this research was greatly appreciated. I must also acknowledge the financial support of the National Science Foundation.

One of the primary reasons I chose to come to MIT for grad school in the first place was the great vibe I got from all the TELAMS students, and my first impressions have held true throughout my entire two years here. They have been wonderful lab mates, always willing to discuss research or help out with classwork, as well as wonderful friends. Thanks for introducing me to the wonders of sake bombing and for allowing (even encouraging) me to bring Mr. Chips to the office.

Last but not least is my advisor, Paul Lagacé. He has been much more than just a thesis supervisor — he has been a mentor. Under his guidance, my skills as an engineer, researcher, and technical writer have improved enormously. I have greatly appreciated the opportunity to learn from him, and I can say that thanks to him, I am proud of this thesis.
Foreword

This work was performed in the Technology Laboratory for Advanced Materials and Structures (TELAMS) of the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology. This work was sponsored by the Alpha STAR Corporation under Award Number Agmt Dtd 6/30/09.
# Table of Contents

1 Introduction 21

2 Previous Work 27

2.1 Progressive Failure Analysis 27 
   2.1.1 Structural Modeling 29
   2.1.2 Failure Theories 31
   2.1.3 Material Property Degradation Models 34
   2.1.4 Current Status 36

2.2 Probabilistic Modeling and Analysis of Composites 37 
   2.2.1 Mechanical Behavior 37
   2.2.2 Damage and Failure Behavior 39

3 Objective and Overall Approach 43

3.1 Objective 43

3.2 Overview of Approach 44

4 Approach Details 49

4.1 Probabilistic Progressive Failure Analysis 49 
   4.1.1 Probabilistic Failure Criteria 50
   4.1.2 Material Property Degradation Model 54
   4.1.3 Analysis Procedure 56

4.2 Deterministic Progressive Failure Analysis 57

4.3 Description of Particular Cases 60 
   4.3.1 Material Properties 61
   4.3.2 Geometrical and Laminate Configuration 66
5 Finite Element Modeling

5.1 Mesh Generation Considerations .......................... 73
5.2 Two-Dimensional Finite Element Model ..................... 76
  5.2.1 Description .............................................. 76
  5.2.2 Validation ............................................... 83
5.3 Three-Dimensional Finite Element Model ................... 90
  5.3.1 Description .............................................. 90
  5.3.2 Validation ............................................... 97

6 Quasi-Three-Dimensional Analysis .......................... 111

6.1 Deterministic Progressive Failure Analysis Results ........ 112
6.2 Probabilistic Damage Initiation Results .................... 119
6.3 Probabilistic Damage Propagation Results ................... 128
  6.3.1 Case 1: Straight Propagation Along the 90° Ply Midline ... 135
  6.3.2 Case 2: Turn Propagation Along the 90° Ply Midline ...... 139
  6.3.3 Case 3: Straight Propagation in the +45° Ply ............... 153
  6.3.4 Case 4: Straight Propagation Along the 90° Ply Midline at a
               Lower Applied Far-Field Stress ......................... 169
6.4 Discussion .................................................. 171

7 Three-Dimensional Analysis ................................ 191

7.1 Symmetries in the Stress Fields ............................ 191
7.2 Considerations for Presentation of Probabilistic Damage Results ... 202
7.3 Probabilistic Damage Initiation Results .................... 207
7.4 Probabilistic Damage Propagation Results ................... 219
  7.4.1 Case 1: Damage Initiation via the Transverse Tension (Y^T)
         Damage Mode .......................................... 221
  7.4.2 Case 2: Damage Initiation via the Interlaminar Longitudinal
         Shear (Z^{SI}) Damage Mode .............................. 228
7.5 Discussion .................................................. 247

8 Conclusions and Recommendations ............................ 257

Appendix A: Codes for Assessing Failure Probabilities......... 273
### List of Figures

1.1 Diagram of the ‘building block approach’ used in the design and certification of composite structures [6]. ........................................... 23

1.2 Diagram of the progressive failure analysis method for composites. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26

4.1 Flowchart of the quasi-three-dimensional probabilistic progressive failure analysis procedure. .................................................. 58

4.2 Flowchart of the three-dimensional probabilistic progressive failure analysis procedure. ...................................................... 59

4.3 Geometry and loading conditions of the plate investigated in the current work. ................................................................. 68

5.1 Schematic of the symmetry conditions that exist in the quasi-three-dimensional model. ......................................................... 78

5.2 Schematic of the boundary conditions applied to the two-dimensional finite element model. ..................................................... 79

5.3 Image of the mesh scheme in the two-dimensional finite element model. 81

5.4 Image of the different mesh regions in the two-dimensional finite element model. .............................................................. 82

5.5 Location of the element considered in the stress convergence study of the two-dimensional finite element model. .................. 85

5.6 Predicted average values of stress, \( \overline{\sigma}_{11} \), normalized by the applied far-field stress, \( \sigma_o \), in the element on the hole boundary for various meshes. 86

5.7 Predicted ultimate strength as a function of the total number of elements for various two-dimensional meshes. .................. 89
5.8 Comparison of the predicted tangential stress distribution along the $x_1$-axis (normalized by the far-field applied stress) with the analytical solution for an infinite width isotropic plate containing a circular hole subject to equal biaxial tension. ........................................... 91

5.9 Schematic of the boundary conditions applied to the three-dimensional finite element model. ......................................................... 94

5.10 Diagram of one-quarter of the in-plane mesh scheme of the three-dimensional finite element model. ........................................... 95

5.11 Distinct regions of one-quarter of the in-plane, three-dimensional finite element mesh scheme. ..................................................... 96

5.12 Zoomed in view of one-quarter of the finer region (Region 1) of the in-plane mesh scheme for the three-dimensional finite element model. 98

5.13 Displacement of a node at the hole boundary and laminate midplane used to check three-dimensional mesh convergence. ........ 101

5.14 Normalized interlaminar normal stress versus through-thickness location for various levels of through-thickness mesh refinement. 103

5.15 Predicted values of applied far-field stress at which delamination initiation occurs versus average linear dimension of elements at the hole boundary of a $[0/+45/90/-45]_s$ AS4/3501-6 laminate with a 0.378-inch diameter hole loaded in uniaxial tension. ..................................................... 107

5.16 Comparison of the predicted tangential stress distribution along the $x_2$-axis (normalized by the far-field applied stress) with the analytical solution for an infinite width isotropic plate containing a circular hole subject to uniaxial tension. ........................................... 109

6.1 Number of failed points of consideration as a function of applied stress in the deterministic analysis. ................................. 116

6.2 First equilibrium stage of the deterministic analysis. .................. 117

6.3 Second equilibrium stage of the deterministic analysis. ............... 118

6.4 Fifth equilibrium stage of the deterministic analysis. ................... 120

6.5 Sixth equilibrium stage of the deterministic analysis. ................. 121
6.6 Twenty-fifth equilibrium stage of the deterministic analysis. . . . . 122
6.7 Thirtieth (and final) equilibrium stage of the deterministic analysis. . 123
6.8 Failure probabilities in the undamaged quasi-three-dimensional model at an applied stress of 20.3 ksi. . . . . . . . . . . . . . . . . . . 125
6.9 Failure probabilities in the undamaged quasi-three-dimensional model at an applied stress of 29.3 ksi. . . . . . . . . . . . . . . . . . . 127
6.10 Diagram of the damage progression sequence investigated in Case 1. . 131
6.11 Diagram of the damage progression sequence investigated in Case 2. . 133
6.12 Diagram of the damage progression sequence investigated in Case 3. . 134
6.13 Failure probabilities of the points of consideration following the first damage event of Case 1 at an applied stress of 29.3 ksi. . . . . . . . 140
6.14 Failure probabilities of the points of consideration following the second damage event of Case 1 at an applied stress of 29.3 ksi. . . . . . . . 141
6.15 Failure probabilities of the points of consideration following the third damage event of Case 1 at an applied stress of 29.3 ksi. . . . . . . . 142
6.16 Failure probabilities of the points of consideration following the fourth damage event of Case 1 at an applied stress of 29.3 ksi. . . . . . . . 143
6.17 Failure probabilities of the points of consideration following the fifth damage event of Case 1 at an applied stress of 29.3 ksi. . . . . . . . 144
6.18 Changes in failure probabilities following damage event 1 of Case 1. . 145
6.19 Changes in failure probabilities following damage event 2 of Case 1. . 146
6.20 Changes in failure probabilities following damage event 3 of Case 1. . 147
6.21 Changes in failure probabilities following damage event 4 of Case 1. . 148
6.22 Changes in failure probabilities following damage event 5 of Case 1. . 149
6.23 Failure probabilities of the points of consideration following the fifth damage event of Case 2 at an applied stress of 29.3 ksi. . . . . . . . 154
6.24 Failure probabilities of the points of consideration following the sixth damage event of Case 2 at an applied stress of 29.3 ksi. . . . . . . . 155

6.25 Changes in failure probabilities following damage event 5 of Case 2. . 156

6.26 Changes in failure probabilities following damage event 6 of Case 2. . 157

6.27 Failure probabilities of the points of consideration following the first damage event of Case 3 at an applied stress of 29.3 ksi. . . . . . . . 161

6.28 Failure probabilities of the points of consideration following the second damage event of Case 3 at an applied stress of 29.3 ksi. . . . . . . . 162

6.29 Failure probabilities of the points of consideration following the third damage event of Case 3 at an applied stress of 29.3 ksi. . . . . . . . 163

6.30 Failure probabilities of the points of consideration following the fourth damage event of Case 3 at an applied stress of 29.3 ksi. . . . . . . . 164

6.31 Changes in failure probabilities following damage event 1 of Case 3. . 165

6.32 Changes in failure probabilities following damage event 2 of Case 3. . 166

6.33 Changes in failure probabilities following damage event 3 of Case 3. . 167

6.34 Changes in failure probabilities following damage event 4 of Case 3. . 168

6.35 Failure probabilities of the points of consideration in the undamaged model at an applied stress of 27.5 ksi. . . . . . . . . . . . . . . . . 170

6.36 Failure probabilities of the points of consideration following the first damage event of Case 4 at an applied stress of 27.5 ksi. . . . . . . . 174

6.37 Failure probabilities of the points of consideration following the second damage event of Case 4 at an applied stress of 27.5 ksi. . . . . . . . 175

6.38 Failure probabilities of the points of consideration following the third damage event of Case 4 at an applied stress of 27.5 ksi. . . . . . . . 176

6.39 Failure probabilities of the points of consideration following the fourth damage event of Case 4 at an applied stress of 27.5 ksi. . . . . . . . 177

6.40 Changes in failure probabilities following damage event 1 of Case 4. . 178

6.41 Changes in failure probabilities following damage event 2 of Case 4. . 179
6.42 Changes in failure probabilities following damage event 3 of Case 4. 180
6.43 Changes in failure probabilities following damage event 4 of Case 4. 181

7.1 Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_0$, in the $+15^\circ$ ply. 193
7.2 Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_0$, in the $-15^\circ$ ply. 194
7.3 Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_0$, in the $0^\circ$ ply. 195
7.4 Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_0$, at the $+15^\circ/-15^\circ$ ply interface. 196
7.5 Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_0$, at the $-15^\circ/0^\circ$ ply interface. 197
7.6 Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_0$, at the $0^\circ/0^\circ$ ply interface. 198
7.7 Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_0$, at the $+15^\circ/-15^\circ$ ply interface. 199
7.8 Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_0$, at the $-15^\circ/0^\circ$ ply interface. 200
7.9 Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_0$, at the $0^\circ/0^\circ$ ply interface. 201
7.10 Regions of the in-plane three-dimensional mesh (reproduced from Section 5.1). 204
7.11 Illustration of "super" elements within Region 1 used in displaying some of the results. 205
7.12 In-plane location of the subset of elements within Region 1 for which some results are displayed in the three-dimensional analysis work. 206
7.13 In-plane damage initiation probabilities at an applied stress of 32.5 ksi. 209
7.14 In-plane damage initiation probabilities at an applied stress of 32.9 ksi. 210
7.15 In-plane damage initiation probabilities at an applied stress of 36.4 ksi. 211

7.16 Interlaminar damage initiation probabilities at an applied stress of 36.4 ksi. ........................................ 212

7.17 Overall locations, using “super” element presentation, of all in-plane and out-of-plane damage initiation probabilities at an applied stress of 37.4 ksi. .................................................. 215

7.18 In-plane damage initiation probabilities at an applied stress of 37.4 ksi. 216

7.19 Interlaminar damage initiation probabilities at an applied stress of 37.4 ksi. ........................................ 217

7.20 Overall locations, using “super” element presentation, of all damage initiation probabilities using alternate interlaminar shear strength parameters. .................................................. 218

7.21 Location of in-plane damage simulated in the -15° ply in Case 1. . . . 222

7.22 In-plane damage probabilities following the occurrence of in-plane damage of Case 1 at an applied far-field stress of 37.4 ksi. .................. 224

7.23 Interlaminar damage probabilities following the occurrence of in-plane damage of Case 1 at an applied far-field stress of 37.4 ksi. .................. 225

7.24 Changes in the in-plane failure probabilities following the occurrence of in-plane damage in Case 1 at an applied far-field stress of 37.4 ksi. 226

7.25 Changes in the interlaminar failure probabilities following the occurrence of in-plane damage in Case 1 at an applied far-field stress of 37.4 ksi. .................................................. 227

7.26 Location of interlaminar damage simulated at +15°/-15° ply interface in Case 2. ........................................ 229

7.27 In-plane damage probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi. .................. 230

7.28 Interlaminar damage probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi. .................. 231

7.29 Changes in the in-plane failure probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi. .................................................. 233
7.30 Changes in the interlaminar failure probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi. ............................................................... 234

7.31 In-plane location considered in the three-dimensional substructure model. 236

7.32 In-plane mesh scheme of the three-dimensional substructure model. . 237

7.33 Schematic of the linear multi-point constraint mesh refinement method used in the three-dimensional substructure model. ................. 239

7.34 In-plane and interlaminar damage probabilities following the interlaminar damage simulated in Case 2 using the substructure model at an applied far-field stress of 37.4 ksi. .......................... 241

7.35 Isostress contours of in-plane shear stress, \( \sigma_{12} \), normalized by far-field applied stress, \( \sigma_o \), in the -15° ply for the original model and the substructure model. .................................................. 242

7.36 Isostress contours of interlaminar longitudinal shear stress, \( \sigma_{13} \), normalized by far-field applied stress, \( \sigma_o \), at the +15°/-15° ply interface for the original model and the substructure model. ............... 244

7.37 Isostress contours of interlaminar longitudinal shear stress, \( \sigma_{13} \), normalized by far-field applied stress, \( \sigma_o \), at the -15°/0° ply interface for the original model and the substructure model. ................. 245

7.38 Isostress contours interlaminar normal stress, \( \sigma_{33} \), normalized by far-field applied stress, \( \sigma_o \), at the -15°/0° ply interface for the original model and the substructure model. ....................... 246
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Deterministic material property data for AS4/3501-6 graphite/epoxy</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>Probabilistic strength data for AS4/3501-6 graphite/epoxy</td>
<td>63</td>
</tr>
<tr>
<td>4.3</td>
<td>Calculated stress concentration factors for $\sigma_{11}$ at the edges of the laminated plates under investigation</td>
<td>71</td>
</tr>
<tr>
<td>5.1</td>
<td>Normalized average values of stress, $\bar{\sigma}_{11}/\sigma_o$, for five different two-dimensional finite element meshes</td>
<td>87</td>
</tr>
<tr>
<td>6.1</td>
<td>Summary of the results of the deterministic progressive failure analysis</td>
<td>114</td>
</tr>
<tr>
<td>6.2</td>
<td>Number of points of consideration with a significant probability of failure following each damage event of Case 1 at an applied stress of 29.3 ksi</td>
<td>137</td>
</tr>
<tr>
<td>6.3</td>
<td>Number of points of consideration with a significant change in probability of failure following each damage event of Case 1 at an applied stress of 29.3 ksi</td>
<td>138</td>
</tr>
<tr>
<td>6.4</td>
<td>Number of points of consideration with a significant probability of failure following each damage event of Case 2 at an applied stress of 29.3 ksi</td>
<td>151</td>
</tr>
<tr>
<td>6.5</td>
<td>Number of points of consideration with a significant change in probability of failure following each damage event of Case 2 at an applied stress of 29.3 ksi</td>
<td>152</td>
</tr>
<tr>
<td>6.6</td>
<td>Number of points of consideration with a significant probability of failure following each damage event of Case 3 at an applied stress of 29.3 ksi</td>
<td>159</td>
</tr>
</tbody>
</table>
6.7 Number of points of consideration with a significant change in probability of failure following each damage event of Case 3 at an applied stress of 29.3 ksi................................................................. 160

6.8 Number of points of consideration with a significant probability of failure following each damage event of Case 4 at an applied stress of 27.5 ksi................................................................. 172

6.9 Number of points of consideration with a significant change in probability of failure following each damage event of Case 4 at an applied stress of 27.5 ksi................................................................. 173
# Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLPT</td>
<td>Classical Laminated Plate Theory</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of plate hole</td>
</tr>
<tr>
<td>$f_{T_i}(\sigma_i)$</td>
<td>probability density function of strength parameter $T_i$ as a function of stress component $\sigma_i$ $(i = 1, 2, 3, 4, 5, 6, 7, 8, 9)$</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FORM</td>
<td>First Order Reliability Method</td>
</tr>
<tr>
<td>FPI</td>
<td>Fast Probability Integration</td>
</tr>
<tr>
<td>$E_{ii}$</td>
<td>Young's modulus in the $i$-direction $(i = 1, 2, 3)$</td>
</tr>
<tr>
<td>$E_{ii}^*$</td>
<td>effective modulus of a composite laminate in the $i$-direction $(i = 1, 2)$</td>
</tr>
<tr>
<td>$G_{ij}$</td>
<td>shear modulus in the $i$-$j$ plane $(i, j = 1, 2, 3)$</td>
</tr>
<tr>
<td>$G_{12}^*$</td>
<td>effective in-plane shear modulus of a composite laminate</td>
</tr>
<tr>
<td>$L$</td>
<td>plate dimension in 1-direction corresponding to length</td>
</tr>
<tr>
<td>$P$</td>
<td>applied load</td>
</tr>
<tr>
<td>PFA</td>
<td>Progressive Failure Analysis</td>
</tr>
<tr>
<td>$R$</td>
<td>radius of plate hole</td>
</tr>
<tr>
<td>RVE</td>
<td>representative volume element</td>
</tr>
<tr>
<td>$s_i$</td>
<td>standard deviation of the normal (Gaussian) probability distribution for strength parameter $T_i$ $(i = 1, 2, 3, 4, 5, 6, 7, 8, 9)$</td>
</tr>
</tbody>
</table>
$S$  in-plane shear strength/damage mode  
$t_{pty}$  ply thickness  
$T_i$  strength parameter defined for the probabilistic failure criteria ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9$)  
$u_i$  displacement in the $i$-direction ($i = 1, 2, 3$)  
$W$  plate dimension in 2-direction corresponding to width  
WWFE  World Wide Failure Exercise  
$x$  distance along the $x_1$-direction from the center of the plate hole  
$x_1$  direction equivalent to the 1-direction  
$x_2$  direction equivalent to the 2-direction  
$x_3$  direction equivalent to the 3-direction  
$X^C$  in-plane longitudinal compressive strength/damage mode  
$X^T$  in-plane longitudinal tensile strength/damage mode  
$y$  distance along the $x_2$-direction from the center of the plate hole  
$Y^C$  in-plane transverse compressive strength/damage mode  
$Y^T$  in-plane transverse tensile strength/damage mode  
$z$  distance along the $x_3$-direction from the origin  
$Z^C$  interlaminar normal compressive strength/damage mode  
$Z^T$  interlaminar normal tensile strength/damage mode  
$Z^S$  interlaminar shear strength  
$Z^{S1}$  interlaminar longitudinal shear (1-3 plane) strength/damage mode  
$Z^{S2}$  interlaminar transverse shear (2-3 plane) strength/damage mode  
$\alpha_i$  scale factor of the Weibull probability distribution for strength parameter $T_i$ ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9$)  
$\beta_i$  shape factor of the Weibull probability distribution for strength parameter $T_i$ ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9$)
\( \lambda \) biaxiality ratio of the specimen loading condition

\( \mu_i \) mean of the normal (Gaussian) probability distribution for strength parameter \( T_i \) (\( i = 1, 2, 3, 4, 5, 6, 7, 8, 9 \))

\( \nu_{ij} \) Poisson's ratio relating stress in the \( i \)-direction to strain in the \( j \)-direction (\( i, j = 1, 2, 3 \))

\( \nu_{12}^* \) effective major Poisson's ratio of a composite laminate

\( \Phi_i \) rotation about the \( i \)-axis (\( i = 1, 2, 3 \))

\( \phi \) angular component of the polar coordinate system originating at the center of the plate hole

\( \sigma_i \) stress component defined for the probabilistic failure criteria

\( \sigma_{ij} \) stress tensor component in local material coordinates (\( i, j = 1, 2, 3 \))

\( \bar{\sigma}_{ij} \) averaged stress tensor component in local material coordinates (\( i, j = 1, 2, 3 \))

\( \sigma_o \) far-field applied stress

\( \sigma_{\phi\phi} \) tangential stress component in polar coordinate system

\( \theta \) lamination angle
Chapter 1

Introduction

The use of laminated composite materials in many structural applications has increased significantly over the last few decades, especially within the aerospace and automotive industries. This is primarily due to their high specific strength and stiffness, excellent corrosion- and fatigue-resistance, and the ability to tailor their properties to meet specific design requirements. These advantageous properties enable laminated composite structures to possess improved characteristics over structures that utilize traditional single-phase materials, such as metals [1]. For example, the Boeing 777 passenger aircraft contains approximately 12% composites by weight, and their recent 787 passenger aircraft contains approximately 50% composites by weight [2]. This use of composites in aircraft has led to significant weight savings that allows for reduced operating costs and decreased environmental impact over fully metallic aircraft. However, the potential benefits of composites over traditional materials have not been fully realized because relatively high safety factors are required to ensure the design of safe structures. This is largely attributable to the fact that the failure behavior of composites is complex and poorly understood (e.g. [3-5]). This complexity is not only due to the heterogeneous composition and orthotropic nature of composites, but it is also a result of the fact that composite material behavior is inherently probabilistic [6].
The properties of advanced laminated composites are dependent upon a multitude of primary variables that are statistical in nature, including manufacturing process variables (such as fiber and void volume ratio, fiber misalignment, and ply thickness) and material property variables (such as elastic constants and strength parameters) [7, 8]. There is sufficient variability, particularly with regard to strength, that composite behavior cannot be adequately quantified deterministically. The observed variability in composite strength is accompanied by, and in part is a result of, variability in the possible damage progression paths and failure modes that can occur in laminates with the same nominal configuration and materials [9]. Thus, the use of traditional deterministic methods in the design of composite structures may lead to either over-conservative or under-conservative designs [10]. This offsets the potential benefits of composite laminates and often results in the use of large knockdown factors. This leads to a substantial weight increase without a quantifiable increase in structural reliability [11]. Thus, there is a need for probabilistic models, particularly pertaining to damage and failure, which can account for the probabilistic behavior of composites.

The current methodology used for designing composite structures so as to not fail is known as the ‘building block approach’ [12]. This is an empirically based method. This process, illustrated in Figure 1.1, involves the experimental determination of allowable parameters (e.g. stresses) for the most strength-critical failure mode at one structural scale and then proceeds by transferring these allowables to higher structural scales under the assumption that the same damage/failure mode is manifested throughout, continuing until a full component design is obtained. The allowables are based on statistical characterization of composite behavior and their use is meant to ensure that the final design results in a safe structure, in that it involves choosing design values that minimize the probability of structural failure due to material variability. Current standards specify that the strength of critical components must be ensured with 99 percent probability and a 95 percent confidence level, which is known as the A-basis allowable value, and the strength for redundant structures must be ensured with 90 percent probability and a 95 percent confidence level, which is known as the B-basis allowable value [13]. Although this method acknowledges the
Figure 1.1 Diagram of the 'building block approach' used in the design and certification of composite structures [6].
underlying variability in material strength, it is still essentially deterministic because once the allowable strength value has been determined, it is then used as a deterministic value throughout the remainder of the design process, and only one failure mode is considered. Furthermore, determination of the variability in strength requires extensive experimental testing.

There are five basic in-plane loading conditions for a unidirectional ply: 1) longitudinal tension; 2) longitudinal compression; 3) transverse tension; 4) transverse compression; and 5) in-plane shear. Standards set by MIL-HDBK-17 (now CMH-17) for determining allowables require that the basic values for these conditions be based on tests of at least thirty specimens from at least five batches of a material per each loading direction [6]. Thus, a large number of specimens are required just to characterize the basic unidirectional ply. As the design process continues to higher structural scales, further testing is required. If it is determined, during the process, that a change is necessary at a lower structural scale, then the entire process must restart at the appropriate lower level, and none of the experimental data obtained during the previous design cycle may be applicable, leading to even greater costs. In order to carry the design process to completion, extremely time consuming and expensive experimental testing is required. Thus, accurate and reliable predictive tools are becoming more important in order to reduce the costs associated with an extensive testing program.

A very powerful predictive tool that is used for designing to prevent failure in composite structures is progressive failure analysis (PFA) [14]. Progressive failure analysis is a computational damage progression method aimed at predicting the multiple complex failure mechanisms in composite structures by combining finite element structural analysis with failure theories and material property/stiffness degradation models. Such analysis is used to predict the initiation and propagation of damage throughout a finite element (FE) model. Failure of a laminate consisting of multiple plies is a gradual process. The failure process begins with the initiation of damage at some location. This causes a redistribution of stresses within the laminate, and is followed by an interactive and progressive failure process that leads to ultimate
failure [15]. A diagram of the progressive failure analysis method for composites is shown in Figure 1.2. Despite the advantages offered by progressive failure analysis methods, they are still limited by the fact that they are deterministic and only offer one predicted failure progression and strength value for a given specimen.

The primary objective of this research is to investigate the role of probabilistic characteristics of progressive damage in the failure of composite structures and to illustrate the importance of incorporating probabilistic aspects of progressive damage into predictive models of composite failure behavior. This is accomplished through the use of a progressive failure analysis method by identifying the various possible failure sequences that can occur in a composite structure when probabilistic aspects are considered, and by investigating the overall ramifications on the prediction of laminate behavior. It is intended that this work will improve understanding of the complex failure processes in composite laminates, particularly with regard to the associated probabilistic behavior, and will lead to improved computational prediction capabilities.

The work is organized in this document as follows. A review of previous research related to this investigation is presented in Chapter 2. This includes traditional deterministic progressive failure analysis as well as probabilistic modeling and analysis of composites. The overall objectives of this work and the accompanying approach used to achieve such are described in Chapter 3, and the specific approach details are given in Chapter 4. The Finite Element (FE) modeling aspects of this work are presented in detail in Chapter 5. Results from the analysis work and the accompanying discussions are provided in Chapters 6 and 7. Finally, conclusions drawn from this work and recommendations for future work are presented in Chapter 8.
Validated computational model of structure (e.g. FEM) / Prediction of failure initiation sites and loads / Prediction of progressive failure patterns, sizes, and final failure loads

Material properties
Boundary conditions
Applied loads

Failure theories

Material property degradation models

Figure 1.2 Diagram of the progressive failure analysis method for composites.
Chapter 2

Previous Work

The present work is concerned with probabilistic progressive failure analysis of composite structures. Previous work regarding progressive failure analysis of composite structures is reviewed in Section 2.1, specifically focusing on the three main aspects of progressive failure analysis: structural modeling, failure theories, and material property degradation models. Section 2.1 ends with an assessment of the current status of progressive failure analysis. Only those works regarding deterministic progressive failure analysis are detailed in the first section. In Section 2.2, previous research pertaining to the probabilistic analysis of composites is reviewed, specifically focusing on work regarding the modeling and analysis of mechanical behavior as well as damage and failure behavior. The limited work pertaining to progressive failure analysis in a probabilistic framework is reviewed in Section 2.2.

2.1 Progressive Failure Analysis

As described briefly in Chapter 1, progressive failure analysis is a computational method used to simulate the initiation and propagation of damage in composite structures. There are three main components that constitute a progressive failure analysis. The first component is a computational model of the structure, such as a finite element
model, that includes definition of the material properties, boundary conditions, and applied loads. The second component is a failure theory, which is used to determine the applied loads at which damage occurs, and the locations thereof, by using some set of criteria to evaluate whether certain selected parameters (e.g. stresses and/or strains) have exceeded a predefined value indicating failure. The third component is a material property degradation model. This simulates the occurrence of damage within the structure by degrading certain material properties (typically stiffness) at the site of predicted damage.

The analysis is an iterative process involving four main steps. In Step 1, the model of the structure is evaluated at some applied load or displacement. The applied load/displacement is chosen to be small enough such that overstepping of the prediction of the increment in damage is avoided. In Step 2, the failure theory is applied to determine whether any damage is predicted. If any damage is predicted, then the material property degradation model is applied in Step 3 to degrade the material properties at the damage sites. The model is then re-evaluated to identify any further predicted damage. Once no further damage is predicted, Step 4 is achieved. Each time that Step 4 is reached, this is called an “equilibrium stage”, since the analysis has reached a point at which no further damage is predicted for a given applied load/displacement. The applied load (or displacement) is subsequently increased by an incremental amount. This incremental increase is chosen such that it does not greatly exceed the value for which the next damage event is predicted. The overall set of Steps 1 through 4 is repeated until ultimate failure is predicted. This occurs when an equilibrium stage can no longer be achieved.

Previous works pertaining to the main aspects of progressive failure analysis are subsequently described in further detail. Specifically, reviews of structural modeling in the context of progressive failure analysis, failure theories, and material property degradation models are presented in separate subsections. Following these subsections, an assessment of the current state of progressive failure analysis is given.
2.1.1 Structural Modeling

The primary method for modeling composite structures in the context of progressive failure analysis is the finite element method. In this method, a computational model of a structure is discretized into a finite number of elements in order to approximate its behavior. Specific details of the method are not reviewed here, but an overview of the general finite element method can be found in various texts (e.g. Zienkiewicz [16]), and an earlier overview of finite element analysis in composite structures is given by Ochoa and Reddy [17]. For progressive failure analysis, structural modeling is typically done at the mesoscale, where a laminate is considered to consist of homogeneous plies. Explicit modeling of the entire microstructure is computationally expensive, so the behavior of the homogenized fiber and matrix structure is considered. The capabilities of modern commercially available finite element software programs enable the efficient and accurate simulation of structure geometry, boundary conditions, loads, and stress-strain response in composite structures (e.g. [17, 18]).

There are three main approaches to modeling composite structures for progressive failure analysis using the finite element method. The first approach, referred to herein as quasi-three-dimensional, employs a two-dimensional finite element model of the structure to reduce computational costs. Since only a single layer of elements is used to model the laminate, the material is modeled as a homogenization of the individual ply properties via Classical Laminated Plate Theory (CLPT). The laminate stresses are then evaluated on a ply-by-ply basis via CLPT. This method ignores out-of-plane stresses and has been used to investigate the progressive failure of laminates due to in-plane damage modes (e.g. [19-22]). Higher order plate theories have been used in place of CLPT in an attempt to model interlaminar damage while maintaining computational efficiency, but this approach has achieved little success due to the insufficient accuracy of the interlaminar stress calculations [23, 24]. The second approach, referred to as three-dimensional, utilizes a full three-dimensional model of the structure in order to accurately model interlaminar stresses and attempt to predict
delamination damage (e.g. [25-29]). The third approach, referred to in the literature as global-local, models certain features with three-dimensional elements for accuracy, and models less critical parts of the structure with two-dimensional elements for efficiency (e.g. [30-32]). However, the accuracy of the analysis depends on the size of the region modeled by three-dimensional elements [31].

The results of progressive failure analyses, namely the predicted ultimate strength value and damage progression sequence, have been shown to be sensitive to numerous structural modeling details. Specifically, the predicted results are dependent on the modeling approach used (e.g. quasi-three-dimensional versus three-dimensional), the level of mesh refinement, the boundary conditions, and the load increment. It has been demonstrated that use of a three-dimensional finite element model will predict different damage progression results than if a two-dimensional model is used, even with all other parameters being identical, including the condition that only in-plane damage modes are considered for both models [29]. Due to the complex three-dimensional stress state that exists and the associated possibility of delamination, it is a necessary requirement to use a full three-dimensional finite element model in order to yield accurate predictions of the behavior of composite structures [28].

The level of mesh refinement has a significant affect on the predicted damage initiation and final failure loads as well as on the predicted damage patterns. Increasing mesh refinement leads to lower predicted damage initiation and final failure values [22, 32]. This is due to the fact that within a finer mesh, each element will have a higher average stress than larger elements in a corresponding coarser mesh, so failure will be predicted at lower applied loads. However, use of a suitably refined mesh for two-dimensional models can eliminate the dependence of ultimate strength on mesh refinement, as the results will eventually converge [22]. The level of mesh refinement can also change the predicted locations of damage initiation and the subsequent damage propagation sequence, as has been shown for three-dimensional analysis of woven composites [33].

The types and locations of the boundary conditions can also affect the predicted failure results. In a general finite element analysis, it is important to ensure that
proper boundary conditions are specified in the appropriate locations in order to be able to obtain accurate results. This is even more important in progressive failure analysis of composite laminates because the occurrence of damage results in unloading of the damaged plies, so the boundary conditions may have an unwanted effect following the initiation of damage. For example, it is not accurate to use a uniform force to apply loading in a three-dimensional model of a laminate with plies of different orientations [28]. Furthermore, attempts to model the constraints due to the loading grips of a simple tension test have demonstrated that the insufficient understanding of the displacement boundary conditions at the grips can have significant effect on the predicted results [28]. In that work, it was found that the predicted failure load of a three-dimensional model of a laminate subject to uniaxial tension decreases significantly when the constraint on the displacement perpendicular to the loading direction in the plane of the laminate is applied at the center of the loading faces as opposed to being applied at the edges of the loading faces.

Lastly, parametric studies of the load step increment indicate that a variety of damage patterns can be obtained depending on the size of the increment [22, 32]. Typically, a larger load increment will result in a prediction with more elements failing per iteration due to overstepping of the onset of damage in several elements. However, the desire to minimize the computational time required for the analysis must be balanced against the ability to avoid overstepping any damage prediction. Thus, the size of the load increment must be chosen to suit the particular needs of the user. Additionally, the size of the load increment will likely need to vary throughout the course of the analysis, so it is important to choose an appropriate increment size at each iteration.

2.1.2 Failure Theories

The failure behavior of composite materials is extremely complex. There are many possible failure modes, such as matrix cracking, fiber breakage, fiber debonding, and delamination. Furthermore, multiple failure modes can be manifested in a single
structure. There are numerous failure theories described in the literature that attempt to predict the failure of composite structures, but a comprehensive review is not performed here, as many extensive reviews can be found in the literature (e.g. [34-36]). Instead, a general overview of failure theories is given, focusing specifically on the current state of understanding of composite failure and the application of failure theories in the context of progressive failure analysis.

Failure theories can be divided into two main categories. In the first category are mode-dependent failure theories. These failure theories directly identify the failure mode when predicting the occurrence of failure. The maximum stress [37], maximum strain [38], and Hashin [39] failure criteria fall under this category. In the second category are mode-independent failure theories. These failure theories predict the occurrence of failure, but do not directly identify the failure mode. The Tsai-Wu [40] failure criterion falls into this category.

In the World Wide Failure Exercise (WWFE), conducted by Hinton, et al., the current state of knowledge in this regard was assessed by comparing the predictive capabilities of many of the leading composite failure theories [3-5]. It was determined that no single theory was able to accurately predict all of the quantitative and qualitative aspects of failure for even simple test cases, and that there still exist many areas of potential improvement. In many of the test cases examined, the failure theories not only predicted significantly different final failure strengths (with differences of up to 970 percent) but also different failure modes. The large differences observed in the predictions of laminate failure by various theories are attributed to different methods of modeling the progressive failure process, the empirical nature of some theories, the nonlinear behavior of matrix-dominated laminates, the inclusion or exclusion of residual stresses due to curing in the analysis, and the definition of ultimate laminate failure [3-5, 34-36]. In fact, there is no consensus within the composites research community as to what actually constitutes the final failure of a laminate, as various researchers have considered final failure to be when the maximum load is attained, when first fiber failure is predicted, and when last ply failure is predicted [4, 14].

The maximum stress, maximum strain, Tsai-Wu, and Hashin failure theories are
still widely used in progressive failure analysis despite their limitations. This is because they are simple and easy to implement into a progressive failure analysis framework [14]. This is partially attributable to the fact that they are all applied at the ply level. Various researchers have also used micromechanical failure theories (e.g. [41, 42]). However, constituent-level modeling requires experimental data on fiber and matrix properties that is often difficult to obtain. This furthermore introduces an additional set of calculations that may lead to further inaccuracies in the predicted lamina properties [43]. In terms of failure theories utilized to predict delamination, there are two main approaches. The first approach is to use strength-based failure criteria, such as the Quadratic Delamination Criterion [44]. These criteria compare the local stress state with corresponding strength parameters. The second approach is to use fracture mechanics, where delamination criteria are formulated in terms of strain energy release rates (e.g. [45]), in order to analyze delamination initiation and growth. Many analyses combine stress-based and fracture-mechanics-based methods (e.g. [46]).

It has been demonstrated that the choice of failure theory significantly affects the results of progressive failure analysis (e.g. [28]). Since mode-independent failure criteria do not incorporate information on failure modes, it is difficult to interpret the results of progressive failure analyses that utilize such theories. Therefore, most progressive failure analyses utilize mode-dependent failure criteria since they provide a more rational basis for ensuing property degradation [47]. The World Wide Failure Exercise specifically identified the need for theories that are simple to implement in design situations, which is a primary application of progressive failure analysis. Furthermore, it was noted that the brittle nature of composite failure requires use of probabilistic failure theories for accurate prediction, but that no researchers came forward to address this in the World Wide Failure Exercise [48]. Thus, successful progressive failure analysis will require better understanding of failure in general as well as the development of simple, probabilistically-based mode-dependent failure criteria.
2.1.3 Material Property Degradation Models

Similar to the case of failure theories, there are numerous material property degradation models in the literature, as noted in the review by Garnich and Akula [47]. All degradation models fall into three main categories. The first category is sudden degradation models (e.g. [20-29]), which are widely used in progressive failure analysis due to their ease of implementation. In these models, the stiffness properties of the damaged material are instantaneously reduced to some fraction of the undamaged properties. These models allow multiple instances of damage at a single point, with each occurrence resulting in the degradation of different elastic constants. The second category is gradual degradation models (e.g. [49, 50]), which are generally thought to be more representative of reality but are utilized less frequently in progressive failure analysis due to their increased complexity and computational cost. In these models, the properties of the damaged material are gradually reduced. The value of the property being reduced is some function (typically an exponential decay) of an evolving field variable (e.g. strain). The third category of material property degradation models is constant stress models. In these models, such as the Hahn-Tsai model [51], the properties are degraded such that the material will continue to support its load at initial failure but it cannot sustain additional load. Most degradation models are applied at the ply-level, but some constituent-level models have also been used [52].

Models for simulating delamination damage are often distinct from those for in-plane damage. For delamination in finite element models with no explicit modeling of the resin-rich interply layers, two different techniques have been employed. The first technique is to degrade the out-of-plane properties of the two elements in adjacent plies whose shared face is the delaminated interface [25]. This effectively prohibits the transfer of stresses between the plies at the site of delamination. The second technique is to explicitly model the separation of ply layers by creating a new free surface at the location of predicted delamination [26, 46]. However, explicit modeling of layer separation typically requires creating a new finite element mesh that contains the free surface and/or the use of special elements surrounding the delaminated interface,
thus making it complex and computationally expensive.

Sudden degradation models are the most widely used type of material property degradation model employed in progressive failure analysis [47]. Implementation of sudden degradation models involves stiffness reduction while maintaining a linear-elastic material model. The degraded elastic constants should obey certain constraints on their relationships to each other [1, 53]. For example, the stiffness and compliance matrices must be positive definite, so the following stability requirements must be met:

\[
E_{11}, E_{22}, E_{33}, G_{12}, G_{13}, G_{23} > 0 \tag{2.1}
\]

\[
|\nu_{12}| < \sqrt{\frac{E_{11}}{E_{22}}}, \quad |\nu_{13}| < \sqrt{\frac{E_{11}}{E_{33}}}, \quad |\nu_{23}| < \sqrt{\frac{E_{22}}{E_{33}}} \tag{2.2}
\]

\[
1 - \nu_{12} \nu_{21} - \nu_{23} \nu_{32} - \nu_{31} \nu_{13} - 2 \nu_{21} \nu_{32} \nu_{13} > 0 \tag{2.3}
\]

The stiffness properties of the damaged material are typically reduced to either zero or to a small fraction of the original stiffness value, on the order of 0.0001% to 20%. However, it has been demonstrated that the predicted ultimate strength is very sensitive to the value of the degradation factor, which is the value (less than one) by which the elastic constants are multiplied in order to reduce their properties. In general, the predicted ultimate strength value decreases as the degradation factor decreases [22, 28]. The type of degradation model used can also have a significant affect on the predicted results. For example, an independent degradation model, in which each stress contributes only toward degradation of the corresponding stiffness property, will result in higher predicted ultimate strength than an interactive degradation model that couples multiple stiffness properties and causes them to be reduced simultaneously [28].
2.1.4 Current Status

Progressive failure analysis has been established as one of the most powerful methods for simulating the damage and failure behavior of composite structures. Comparisons with experimental results have given some confidence to using progressive failure analysis as a design tool. A number of researchers have obtained predictions of ultimate failure strength and damage progression that have been fairly consistent with experimental observations (e.g. [20-23, 26-28]). However, this was only achieved by performing extensive parametric studies and adjusting numerous parameter values to identify the combinations that produced the best agreement with experiments. Many of the choices were made more or less arbitrarily, and it has been shown that different combinations can give rise to identical results [28]. These facts hint at the limitations of progressive failure analysis as a predictive tool, since it is not true understanding of failure behavior that has led to agreement with experiments but rather careful empirical fitting.

Commercially available software programs exist for performing automated progressive failure analysis. An example of such is the GENOA program developed by Alpha STAR Corporation [54]. These programs typically couple with commercial finite element codes to obtain accurate stress-strain distributions within the structure being modeled, and then apply the failure theories and material property degradation models specified by the user. Since there is a noted lack of understanding of composite failure and a plethora of available failure theories, many programs simply allow multiple failure theories to be applied concurrently during each iteration of the analysis and then choose the theory that first predicts damage. This is done in an attempt to achieve a conservative result of the final failure strength, as the commercial progressive failure analysis codes are primarily utilized for design purposes. The development of commercial programs has greatly improved the ease and efficiency of performing progressive failure analysis, but there is still much that needs to be understood in terms of composite failure behavior before accurate predictions can be made with confidence.
2.2 Probabilistic Modeling and Analysis of Composites

The inherent probabilistic nature of composites requires that probabilistic methods be used to model and analyze their behavior. Initial research in this area focused on probabilistic modeling and analysis of mechanical behavior, such as stiffness and strength properties. More recently, attention has been directed towards probabilistic analysis of damage and failure behavior in composite structures.

2.2.1 Mechanical Behavior

Composite materials tend to be brittle, and it is well known that the strength of brittle materials is probabilistic. The strength of brittle materials can be described by the theory developed by Weibull [55], which is often called Weakest Link Theory. The main assumption of this theory is that the material can be considered as a series of small volumes, each with a different strength according to a probabilistic distribution, and the failure of any one of these leads to the failure of the whole component. Experimental investigations have verified that the tensile strength of individual advanced fibers is a monotonically decreasing function of fiber length that can be well described by Weakest Link Theory (e.g. [56-58]). The Weakest Link Theory was first applied to describe the strength of fiber bundles by Daniels [59], which provides a link between the probabilistic theory of brittle materials and that of fiber-reinforced composites. Increasing interest in fiber-reinforced composites led to the development of increasingly complex analytical and numerical models that extended the fiber bundle theory to incorporate the effects of parallel fibers embedded in a matrix [60-69]. These models give good qualitative predictions when compared with experiments, but the results are quantitatively inaccurate and often only give bounds on the strength. This inaccuracy has been attributed to model simplifications (e.g. fiber arrangement and diameter assumptions), difficulties in estimating the various parameters involved (e.g. stress concentration factors), and the fact that the theory does not consider other
sources of variations besides material flaws [70]. Furthermore, these models only consider a single layer of fibers or a representative volume element of a composite, and their application to larger structural scales is difficult.

Since the design and analysis of composites is often done using the assumption that the composite plies can be represented as a homogeneous material, mesoscale and macroscale models of the probabilistic properties of composites were developed. Models based on CLPT and Monte Carlo simulation to predict the probabilistic properties (e.g. stiffness and strength) of an entire laminate from given probability distributions of the input parameters have been developed (e.g. [71-73]). By performing CLPT calculations numerous times, where each time the input parameters are sampled randomly from their distributions, the models are able to generate statistical distributions of laminate response. This method is simple to implement and can incorporate variations in material properties and manufacturing parameters. However, the Monte Carlo simulation can be computationally intensive. A similar approach was developed to calculate probabilistic laminate response from ply property distributions using a Taylor Series expansion method called Fast Probability Integration (FPI) that is more efficient, but less accurate, than the Monte Carlo method [7]. Additionally, micromechanical models have been developed that allow the calculation of lamina material property distributions from probabilistic distributions of the constituents (e.g. [7, 74, 75]). However, these models (e.g. [7, 71-75]) assume that the probabilistic variation in material property values is homogeneous throughout the structure. This assumption is referred to, henceforth, as “spatially averaged variability”, indicating that the variability due to material characteristics is assumed to be the same at all locations within the material and thus within any structure made of that material.

In order to account for the known spatial variability of material properties at the ply level, the concept of Weakest Link Theory was extended to characterize the random spatially varying distributions (called random fields) of the in-plane strength properties of composite laminates in two dimensions using a numerical Monte Carlo simulation coupled with CLPT [76, 77]. This consisted of considering each ply to be composed of elements, and then randomly selecting a strength value for each
in-plane location by sampling from the probability distributions for each strength parameter. Building upon the concept of random fields, three-dimensional probabilistic finite element methods have been developed that use random fields to model multiple probabilistic quantities (e.g. stiffness, strength, loading, and geometry) while allowing more accurate analysis of laminates (e.g. [78-82]). However, the random field techniques are very computationally expensive to apply to full three-dimensional models. Additionally, these models cannot be completely verified because there is a lack of experimental data on the spatial variability of material properties [8].

The various methods used to model and analyze the probabilistic mechanical behavior of composite structures have progressed to allow modeling of probabilistic behavior that is both space- and time-dependent. These methods established the basis for allowing the analysis of probabilistic damage and failure behavior in composite structures. A review of previous work on such analysis is given in the following section.

2.2.2 Damage and Failure Behavior

In all of the previous work surveyed regarding the probabilistic analysis of composite failure, the analysis methods are formulated in the context of structural reliability. The goal of structural reliability is to quantify the failure probability of a structure in order to ensure that the structure will not fail. The general concept is to first calculate the probability of failure of each material location. Once the failure probabilities of individual material locations have been calculated, the overall probability of failure of the entire structure is calculated.

A variety of analytical and numerical methods have been developed for evaluating the probability of failure of individual material locations. For simple cases, analytical solutions can be found (e.g. [83, 84, 85]). For more complex cases, there are three primary classes of numerical methods used to obtain approximate solutions. The first of these methods is Monte Carlo simulation [86]. Monte Carlo simulation is extremely flexible and robust and has been applied in a wide variety of cases (e.g. [69, 79-
However, such simulation is computationally expensive, often requiring hundreds or even thousands of realizations to reach a converged solution. The second class of methods is Limit State Approximation, which includes approaches such as the First-Order Reliability Method (FORM) [90]. Their main advantage is that they offer significantly reduced computational cost compared to Monte Carlo simulation. However, they can become extremely complex with increasing number of random variables. Limit State Approximation methods have seen significant application in the last fifteen years (e.g. [91-94]). The third class of methods is Response Surface Approximation (e.g. [7, 73, 95, 96]). This method is typically more efficient than Monte Carlo simulation, but it only offers limited information about a single response parameter.

Calculating the probability of failure of the entire structure is very difficult if there are a large number of material locations in the structural model. There are a few methods for evaluating the overall structural failure probability. The first category of methods is based on Weakest Link Theory and includes both analytical and numerical methods (e.g. [76, 83, 97]). These models can only provide lower bounds on the probability of failure. Limit State Approximation methods can also be used to evaluate the system probability of failure (e.g. [91-93]), but similar to the weakest link methods, they can only provide lower bounds on the probability of failure. The only method able to give an approximation of the actual failure probability of the structure is Monte Carlo simulation. Thus, it has been widely employed (e.g. [71, 72, 77, 79-81, 88, 89]) despite the fact that it can be computationally expensive.

It was found that most of the analysis work in the literature surveyed was lacking in at least one of two key areas. Specifically, much of the work utilized spatially averaged variability and/or did not consider progressive failure. The majority of work concerned with probabilistic analysis of the damage and failure of composite structures has utilized spatially averaged variability of the material properties (e.g. [7, 83-85, 87-89, 91, 94, 95]). This is inaccurate, and the importance of including spatial variability has been demonstrated through work showing that the probability of failure in a simple case can increase significantly just by considering the spatial
variability of the tensile modulus along a single direction [98]. Of the works that do incorporate spatial variability, most consider the predicted onset of damage in a ply to be either the failure strength of that entire ply (e.g. [76]) or the ultimate failure of the entire laminate (e.g. [77-81, 92, 93, 96, 97]). There were only a few works surveyed that considered both spatially varying material properties and progressive failure [92, 93, 99]. Of these, the Limit State Approximation approach was used in two cases so they were only able to obtain lower bounds on the failure probability [92, 93], and the other only considered a single realization of spatially varying fiber strengths that was then analyzed by deterministic progressive failure analysis [99].

Perhaps even more important to note is the fact that all analysis methods that were surveyed are formulated in the context of structural reliability. That is, the primary objective is to allow the ability to design a structure that will not fail. The basic result is a probability of failure of the structure, or a probabilistic distribution of ultimate failure strengths for that structure. This reliability approach is fundamentally different from a so-called “probabilistic failure analysis” approach, which seeks to answer the question of what happens when damage/failure actually does occur [100]. This approach requires the assessment of response quantities conditioned on the occurrence of a damage event. The probabilistic failure analysis approach aims to investigate the probable scenarios that can occur following the initiation of damage. Such an approach is important for composite structures due to the progressive nature of their failure. Without a probabilistic failure analysis approach, it is not possible to investigate the mechanisms that lead to competing failure paths and the various possible damage modes and progressions that can manifest in a composite structure. Thus, there is a need for building fundamental knowledge concerning the probabilistic progressive failure of composite structures.
Chapter 3

Objective and Overall Approach

3.1 Objective

The goal of the present work is to begin building fundamental knowledge of the role of probabilistic aspects of progressive damage in the failure of composite structures. The aim is not to achieve quantitatively accurate predictions of the probabilistic failure behavior, but rather to assess the effects and obtain a qualitative sense of the importance of considering probabilistic characteristics of progressive damage in a failure analysis. Realistic quantitative values are still used for all parameters in order to enable physically and realistically meaningful understanding of the desired items. Of particular interest is the interaction of the different damage modes and the various possible damage progression sequences that may occur, as this may provide a better understanding of the mechanisms that lead to competing failure paths and the resulting performance of composite structures. The initial work begins with an investigation of how predictions of damage initiation are affected by including probabilistic aspects. This includes consideration of the different possible locations where damage initiation may occur, as well as the different modes by which damage may initiate. This is of key importance because subsequent damage progression may change depending on the type and location of damage initiation. Consideration is then given to the potential ways that damage may propagate following initial damage, and to
how the occurrence of damage affects the likelihood of other damage events, since the particular manifestation of damage is likely to be a significant factor in the overall failure behavior and ultimate strength of composite structures.

Since the present work is a first order investigation into the probabilistic aspects of damage in composite structures, many details of the work were chosen due to their simplicity of implementation and ease of understanding of the results while still providing the ability to gain useful knowledge. Only a few particular illustrative cases of structural configuration and damage progression are considered in this work. Various simplifying assumptions are also employed in order to reduce the scope and complexity of the problem. These simplifying assumptions are described in the following section. Furthermore, other details of the approach and the particular problems investigated are chosen based on their simplicity and ease of use, as described in Chapter 4. Therefore, the specific objective of this work is to begin building a base of fundamental knowledge of the effects of probabilistic characteristics of damage initiation and propagation by considering a few simple illustrative cases under some simplifying assumptions.

3.2 Overview of Approach

A computational progressive failure analysis method that incorporates probabilistic aspects of damage is used in this work to gain a better understanding of the probabilistic failure behavior of composite structures. As described in Chapter 2, a progressive failure analysis utilizes a finite element model, failure theories, and a material property degradation model in order to simulate the progression of damage within a composite structure. This work starts with a base progressive failure analysis framework, and then considers the occurrence of damage to be probabilistic.

For this work, only the material strength values are considered to be probabilistic. All other parameters are considered as deterministic. Strength parameters have been shown experimentally to have significantly larger variability than elastic constants [8]. Thus, a model that only considers strength probabilistically allows for simpler
evaluation of the effect of probabilistic aspects on overall composite behavior, while still giving significant consideration to those probabilistic aspects. Furthermore, the present work is primarily concerned with the strength of composite structures, so the material strength values are the base items to be considered as probabilistic in a first order investigation. The probabilistic aspects are incorporated into the analysis by considering that for each damage mode, a probabilistic distribution of failure exists. This probabilistic distribution is based on the statistical variation in the basic composite material strength values, which can be determined through experiments. For the present work, the material strength parameters for each location and failure mode are assumed to be independent random variables in order to reduce the complexity of the probabilistic calculations.

A quasi-three-dimensional analysis is used in the initial work. In this, a two-dimensional model is used for the finite element analysis and the stresses within each element are analyzed on a ply-by-ply basis via Classical Laminated Plate Theory. Each ply within the same in-plane location (i.e. within the same element) represents a distinct “point of consideration.” A quasi-three-dimensional analysis is used for the initial work as it allows investigation of the in-plane failure of each ply separately, and it is simpler to implement than a full three-dimensional analysis. By only considering in-plane damage modes, the possibility of delamination is ignored. Thus, a major damage mode is neglected. However, the reduced complexity is a good starting point since it allows easier understanding of the effects of including probabilistic characteristics of damage. Following the initial quasi-three-dimensional analysis work, full three-dimensional analysis is performed. A three-dimensional finite element model with each ply modeled explicitly is employed in the analysis. This allows for the modeling of interlaminar stresses and delamination damage. This is a necessary requirement for accurate prediction of the behavior of composite structures. For the three-dimensional analysis work, both intralaminar and interlaminar damage modes are considered, thus encompassing all possible damage modes. A macroscopic approach is used in modeling each individual ply as a homogeneous orthotropic continuum in both cases. Further details of the finite element models are given in Chapter
An average stress approach is employed in order to assess the various failure probabilities. In such an approach, the failure criteria are applied to the average stresses over some region. For the quasi-three-dimensional analysis case, the averaging region is the area of each point of consideration. For the three-dimensional case, the averaging region for the in-plane failure criteria is the volume of each element, and the averaging region for the interlaminar failure criteria is the in-plane area of the interface between elements in adjacent plies. These specific averaging regions are used because it simplifies the analysis, since the average stresses for each point of consideration, element, and element interface are directly obtained (or can be easily calculated) from finite element output results. The failure criteria and methods of obtaining the averaged stress values are described in Section 4.1. In Chapter 5, details regarding the choice of appropriate size for the averaging regions are described.

Within the probabilistic framework utilized the present work, the failure analysis involves the assessment of failure probabilities conditional on a given damage event. Furthermore, since each element (or interface between elements in adjacent plies) can fail via multiple different modes independently, the probability of failure is determined for each failure mode separately. Thus, given some state of damage and far-field applied stress, the probability of failure of each remaining element (or interface between elements in adjacent plies) via each damage mode can be determined. Once the failure probabilities have been determined for a given damage state and far-field applied stress, the failure analysis advances through either an increase in the far-field applied stress or the occurrence of additional damage. Since there may be multiple possible locations and modes of damage that may occur at any given stage of the analysis, it is up to the analyst to decide which damage, if any, to simulate. The choice may be made based on the relative probabilities of failure of each possible damage event, or it may be based on some other consideration. By re-starting the analysis from the beginning and choosing a different sequence of damage events, the different possible failure paths can be enumerated. Therefore, this framework provides the ability to qualitatively and quantitatively assess the effect of any particular damage event, and
it allows for exploration of different possible damage and failure scenarios.

For both of the analysis cases (quasi-three-dimensional and three-dimensional), only symmetric damage is considered. This means that the occurrence of any particular damage event is assumed to also occur simultaneously in all symmetrically equivalent locations within the structure. Under this assumption, only a portion of the structure is considered in the analysis work by utilizing any symmetry of the structural configuration and stress fields within the laminate. In reality, damage does not typically occur symmetrically. However, symmetric damage is assumed in this work in order to reduce the scope of the problems investigated, since the current work is only a first order investigation into probabilistic damage behavior. The consideration of symmetric damage still allows the ability to investigate the effects of probabilistic characteristics on the damage and failure behavior of composite structures. Relaxing this damage symmetry constraint will likely lead to even greater variability in overall behavior. This includes lack of symmetry in resultant stress and strain fields as damage initiates and propagates.

Based on the research approach as outlined, the following key questions were posed in order to guide this work. The first deals with the manners by which the failure of one element (or point of consideration) affects the probability of failure of the surrounding elements (or points of consideration). The second deals with the effects of allowing the failure strength of that point of consideration to take on different values within its probabilistic range. The results of the present work are largely focused on answering these two questions.
Chapter 4

Approach Details

In this chapter, the specific details used in the implementation of the approach as outlined in Chapter 3 are described. The details of the probabilistic progressive failure analysis, including the probabilistic failure criteria, the material property degradation model, and the analysis procedure are described in Section 4.1. A description of the quasi-three-dimensional deterministic progressive failure analysis used for comparison purposes is provided in Section 4.2. Details of the particular problems addressed in the current work, such as the material properties utilized and the geometrical and laminate configurations that are investigated, are provided in Section 4.3.

4.1 Probabilistic Progressive Failure Analysis

Details concerning the probabilistic progressive failure analysis method used in the current work are presented in this section. There are three main components involved in a progressive failure analysis: a computational model of the structure, a failure theory, and a material property degradation model. Finite element analysis for modeling of the structure is performed using Abaqus version 6.8 [101]. Details related to the finite element analysis are provided in Chapter 5. The failure criteria, which are probabilistic, are described in Section 4.1.1. The material property degradation
model is presented in Section 4.1.2. A detailed description of the analysis procedure utilized for implementation of the probabilistic progressive failure analysis is given in Section 4.1.3.

4.1.1 Probabilistic Failure Criteria

This work utilizes probabilistic failure criteria to predict the probability of failure of each element (in the three-dimensional analysis work) or point of consideration (in the quasi-three-dimensional analysis work), as defined in Section 3.2, within the finite element model. The probabilistic failure criteria are based on the maximum stress failure criteria [37], which are simple mode-dependent criteria. The mode-dependent nature of the criteria means that the mode of failure is directly indicated when failure is predicted. This is particularly important for progressive failure analysis as it allows for explicit understanding of how each individual damage mode manifests and interacts with other damage modes throughout the failure process. Furthermore, these criteria provide the ability to handle each stress component separately, thus providing ease in both implementation into an analysis and understanding of the analysis results.

The maximum stress failure criteria are inequality conditions, with failure predicted to occur when any one of the stresses in principal material coordinates exceeds the respective strength value. For tensile stresses, the maximum stress failure criteria can be expressed as:

\[
\begin{align*}
\sigma_{11} & \geq X^T \\
\sigma_{22} & \geq Y^T \\
\sigma_{33} & \geq Z^T
\end{align*}
\]  

where \(\sigma_{11}, \sigma_{22},\) and \(\sigma_{33}\) are tensile components of the stress tensor, \(X^T\) is the in-plane longitudinal tensile strength, \(Y^T\) is the in-plane transverse tensile strength, and \(Z^T\) is the interlaminar normal tensile strength. For compressive stresses, the maximum
stress failure criteria can be expressed as:

\[
\begin{align*}
|\sigma_{11}| & \geq X^C \\
|\sigma_{22}| & \geq Y^C \\
|\sigma_{33}| & \geq Z^C
\end{align*}
\]  

(4.1d) (4.1e) (4.1f)

where \(\sigma_{11}, \sigma_{22},\) and \(\sigma_{33}\) are compressive components of the stress tensor, \(X^C\) is the in-plane longitudinal compressive strength, \(Y^C\) is the in-plane transverse compressive strength, and \(Z^C\) is the interlaminar normal compressive strength. For shear stresses, the maximum stress failure criteria can be expressed as:

\[
\begin{align*}
|\sigma_{12}| & \geq S \\
|\sigma_{13}| & \geq Z^{S1} \\
|\sigma_{23}| & \geq Z^{S2}
\end{align*}
\]  

(4.1g) (4.1h) (4.1i)

where \(\sigma_{12}, \sigma_{13},\) and \(\sigma_{23}\) are shear components of the stress tensor, \(S\) is the in-plane shear strength, \(Z^{S1}\) is the interlaminar longitudinal shear (1-3 plane) strength, and \(Z^{S2}\) is the interlaminar transverse shear (2-3 plane) strength.

In employing the failure criteria in the current work, the need to represent material strength as a probabilistic distribution as opposed to a single-valued function is recognized. As mentioned in Chapter 3, it is assumed that the strength parameters of each element (or point of consideration) are independent random variables. This simplifies the calculation of the probability of failure so that an analytical method can be used. The random variables representing the material strength parameters are defined as follows for ease:
\[ T_1 = X^T \] \hspace{1cm} (4.2a)
\[ T_2 = Y^T \] \hspace{1cm} (4.2b)
\[ T_3 = Z^T \] \hspace{1cm} (4.2c)
\[ T_4 = X^C \] \hspace{1cm} (4.2d)
\[ T_5 = Y^C \] \hspace{1cm} (4.2e)
\[ T_6 = Z^C \] \hspace{1cm} (4.2f)
\[ T_7 = S \] \hspace{1cm} (4.2g)
\[ T_8 = Z^{S1} \] \hspace{1cm} (4.2h)
\[ T_9 = Z^{S2} \] \hspace{1cm} (4.2i)

where each \( T_i \) is a continuous random variable.

As mentioned in Section 3.2, an average stress approach is used in the present work. Thus, the probabilistic failure criteria are applied to averaged stress values. For tensile stresses, the averaged stress components are defined as follows for ease:

\[ \sigma_1 = \bar{\sigma}_{11} \] \hspace{1cm} (4.3a)
\[ \sigma_2 = \bar{\sigma}_{22} \] \hspace{1cm} (4.3b)
\[ \sigma_3 = \bar{\sigma}_{33} \] \hspace{1cm} (4.3c)

where \( \bar{\sigma}_{11}, \bar{\sigma}_{22}, \) and \( \bar{\sigma}_{33} \) are averaged tensile components of the stress tensor. For compressive stresses, the averaged stress components are defined as:

\[ \sigma_4 = |\bar{\sigma}_{11}| \] \hspace{1cm} (4.3d)
\[ \sigma_5 = |\bar{\sigma}_{22}| \] \hspace{1cm} (4.3e)
\[ \sigma_6 = |\bar{\sigma}_{33}| \] \hspace{1cm} (4.3f)

where \( \bar{\sigma}_{11}, \bar{\sigma}_{22}, \) and \( \bar{\sigma}_{33} \) are averaged compressive components of the stress tensor.
For shear stresses, the averaged stress components are defined as:

\[ \sigma_7 = |\bar{\sigma}_{12}| \]  
\[ \sigma_8 = |\bar{\sigma}_{13}| \]  
\[ \sigma_9 = |\bar{\sigma}_{23}| \]  

(4.3g)  
(4.3h)  
(4.3i)

where \( \bar{\sigma}_{12} \), \( \bar{\sigma}_{13} \), and \( \bar{\sigma}_{23} \) are averaged shear components of the stress tensor.

For each basic damage mode, the probability of failure is calculated using the cumulative distribution function that corresponds to the material strength for that damage mode. The probabilistic failure criteria can thus be expressed in equation form as:

\[ Pr(T_i < \sigma_i) = \int_0^{\sigma_i} f_{T_i}(\sigma_i) \, d\sigma_i \]  

(4.4)

where the left hand side of Equation (4.4) represents the probability that the material strength for a particular damage mode, \( T_i \), takes on a value less than or equal to the averaged stress component corresponding to that damage mode, \( \sigma_i \), and the right hand side of Equation (4.4) is the cumulative distribution function of that material strength parameter defined in terms of its probability density function, denoted as \( f_{T_i}(\sigma_i) \).

The probability density function describes the relative likelihood of the material strength to be at a certain stress value, \( \sigma_i \). The material strength distributions are obtained through experimental testing. Two common probability distributions used in this work are the normal (Gaussian) distribution and the Weibull distribution. The probability density function for the normal distribution is:

\[ f_{T_i}(\sigma_i) = \frac{1}{\sqrt{2\pi s_i^2}} e^{-\frac{(\sigma_i - \mu_i)^2}{2s_i^2}} \]  

(4.5)
where $s_i$ is the standard deviation of strength parameter $T_i$, and $\mu_i$ is the mean of strength parameter $T_i$. The probability density function of the Weibull distribution is:

$$f_{T_i}(\sigma_i) = \begin{cases} \frac{\beta_i}{\alpha_i} \left( \frac{\sigma_i}{\alpha_i} \right)^{\beta_i - 1} e^{-\left( \frac{\sigma_i}{\alpha_i} \right)^{\beta_i}} & \text{if } \sigma_i \geq 0 \\ 0 & \text{if } \sigma_i < 0 \end{cases}$$

(4.6)

where $\alpha_i$ is the scale factor of strength parameter $T_i$, and $\beta_i$ is the shape factor of strength parameter $T_i$.

The probability distributions for each material strength parameter are taken from experimental data reported in the literature. Details of the material properties used in this work are presented in Section 4.3.1. For in-plane damage modes, the appropriate failure criteria are applied to the average stresses in each element in order to assess the probability of failure of that element. For the interlaminar damage modes, the appropriate failure criteria are applied to the average stresses over the shared interface between elements in adjacent plies in order to assess the probability of delamination damage between those elements. These probabilistic failure criteria provide the ability to determine the probability of failure of each element via each damage mode conditioned on a given state of damage and applied stress.

### 4.1.2 Material Property Degradation Model

Damage in the finite element model is simulated through the use of a material property degradation model. This is a mathematical model that defines the residual properties of the damaged material. As is typical for material property degradation models, the stiffness properties of damaged elements are degraded. There is identification of in-plane and out-of-plane damage modes in this work, so different material property degradation models are employed for these two distinct types of damage. Sudden degradation models are used to simulate both in-plane and out-of-plane damage since such models are simple to implement.
The proposed degradation model for predicted in-plane damage is the so-called “total-ply discount method” [103], in which all in-plane elastic moduli of the damaged element are instantaneously reduced to zero. This degradation model was chosen because it is simple to implement. The in-plane material property degradation model expressed in equation form is given as:

\[ E_{11} = E_{22} = G_{12} = 0 \]  \hspace{1cm} (4.7a)

and, in order to assure compatibility:

\[ \nu_{12} = \nu_{13} = \nu_{23} = 0 \]  \hspace{1cm} (4.7b)

The material property degradation model for predicted delamination damage follows the one used by Lee [25]. If damage is predicted at the interface of two elements, the out-of-plane elastic constants in those two elements are instantaneously set to zero so that they do not carry any out-of-plane load. This prohibits the transfer of stresses between the two plies adjacent to the delaminated interface, thus simulating delamination damage. The delamination degradation model is expressed in equation form as:

\[ E_{33} = G_{13} = G_{23} = 0 \]  \hspace{1cm} (4.8a)

and, in order to assure compatibility:

\[ \nu_{23} = \nu_{13} = 0 \]  \hspace{1cm} (4.8b)

In the actual implementation of the degradation models, very small nonzero values (on the order of \(10^{-4}\)) are used for the degraded stiffness values in order to avoid numerical convergence issues that may arise.
4.1.3 Analysis Procedure

The analysis procedure used in this work allows for the exploration of various possible damage initiation and progression sequences. The procedure consists of three main steps: finite element stress analysis, application of the failure criteria, and degradation of the material properties at the damaged locations. Abaqus finite element software is used to obtain in-plane stress values for each element within the finite element model. Since delamination occurs at ply interfaces, the interlaminar stresses in the three-dimensional case are obtained at the nodes of each element using the "LOCATION=NODES" option for the results output in Abaqus and are then averaged over the shared face between two elements in adjacent plies. The average stresses over the ply interface are used because an average stress approach is more accurate for the prediction of delamination than a point stress approach due to the existence of a stress singularity between plies at free edges [44]. These stress values are obtained for the shared face of the two elements in adjacent plies because the finite element model does not contain separate resin-rich interply regions from which to obtain interlaminar stresses, as described in Chapter 5. All stress values are then input into a MATLAB [102] code that applies the appropriate failure criteria in order to determine the corresponding failure probabilities for each mode of damage at each point of consideration using the probabilistic failure criteria described in Section 4.1.1. It is sufficient to only perform the finite element analysis at one prescribed load in the initial step of the work. No material nonlinearity is employed in this work prior to initial damage, so the MATLAB code can be used to multiply the element stresses to determine the stress state corresponding to a higher applied load and then determine the corresponding failure probabilities.

The results from application of the probabilistic failure criteria are then displayed spatially. For each ply and ply interface in the structure, the element failure probabilities are indicated for each damage mode. For display purposes, the failure probabilities are discretized into pre-defined intervals that are chosen based on the level of detail desired in the analysis. Only those failure probabilities that are above a certain
level, referred to in this work as the “significance level”, are displayed. This provides the ability to view the locations and modes of all probabilistically significant damage that is predicted. One of the elements with a significant probability of failure is then chosen to be failed in order to advance the damage progression process. Once this element is chosen, the material property degradation model is implemented manually within the finite element model by assigning the degraded material properties to the damaged element. The updated finite element model is then reanalyzed and the process repeats. Different possible damage progression sequences can be investigated by restarting the analysis from the beginning and choosing different elements to fail. A diagram of this overall probabilistic damage initiation and propagation analysis procedure for the quasi-three-dimensional case is shown in Figure 4.1. The procedure for the three-dimensional case is shown in Figure 4.2.

4.2 Deterministic Progressive Failure Analysis

The results from the quasi-three-dimensional probabilistic progressive failure analysis are compared to results from a deterministic progressive failure analysis performed using the GENOA commercial software suite developed by Alpha STAR Corporation. The results from the deterministic analysis provide a baseline comparison for the probabilistic results. A full three-dimensional deterministic progressive failure analysis was not performed because GENOA does not provide the capability to identify and simulate delamination damage in the manner that is used in the current work. Thus, probabilistic results would not provide information that can be directly compared.

A static analysis was performed using the same two-dimensional finite element model as employed in the probabilistic progressive failure analysis. Mean values of all material strength properties were used for the deterministic analysis. Maximum stress failure criteria, as given in Equation 4.1, were utilized to predict the occurrence of in-plane damage. A slightly different material property degradation model than that used in the probabilistic analysis is employed in the deterministic analysis because
Create 2-D FE mesh
Define loads and boundary conditions
Define ply elastic constants
Define laminate layup

Perform finite element analysis of model using Abaqus
Extract $\sigma_{11}$, $\sigma_{22}$, and $\sigma_{12}$ stress components for each point of consideration and input values into MATLAB code
MATLAB code applies failure criteria to predict the probability of failure of each point of consideration for each damage mode
For each point of consideration and damage mode, MATLAB code calculates the change in failure probability from the previous state
Display results spatially on a ply-by-ply basis
Choose a point of consideration to fail and degrade the material properties according to the material property degradation model

End analysis

Final Failure?

Figure 4.1 Flowchart of the quasi-three-dimensional probabilistic progressive failure analysis procedure.
Figure 4.2 Flowchart of the three-dimensional probabilistic progressive failure analysis procedure.
GENOA only permits use of a degradation model that degrades stiffness properties in the direction of the triggered damage mode. The material property degradation model for the deterministic analysis operates by instantaneously setting the elastic modulus to nearly zero (on the order of $10^{-4}$) in the direction of the triggered failure mode. It is expected that this change in degradation model will not significantly affect the predicted results, but it is acknowledged that this may cause additional differences between the deterministic and probabilistic cases.

As noted in Chapter 2, the progressive failure analysis results are dependent on the size of the load step increment. An iterative algorithm that automatically decreases the load increment if the damage predicted at a particular step of the analysis exceeds a pre-defined allowable amount is used in GENOA. The amount of damage allowed to occur per iteration, typically defined as number of elements allowed to fail, is defined by the user. For this work, the maximum number of elements allowed to fail is set to one in order to track individual element failures, as is done in the probabilistic analysis. A range of load increments is specified by the user, and GENOA will start with the maximum increment and successively decrease the increment as needed. For this work, the allowable load increment was set to be between 1.25 pounds and 125 pounds since this should provide a small enough increment to avoid overstepping while still providing a large enough increment to allow efficient analysis.

4.3 Description of Particular Cases

The probabilistic progressive failure analysis method described in Section 4.1 is applicable to any general composite structure. However, only a few particular illustrative examples are considered in the present work. Details of the specific material properties and geometrical and laminate configurations that are considered in the current work are presented herein.
4.3.1 Material Properties

The composite material considered was chosen based on the following considerations: one, the material should be a typical material that has been used; two, the experimental material property data should be well documented; and three, the material strength values should be characterized probabilistically. Based on these considerations, a Hercules AS4/3501-6 graphite/epoxy unidirectional tape material was chosen for use in this work. All material property data was gathered from experimental results reported in the available literature.

A complete list of the ply elastic constants is reported in Table 4.1. As previously noted, all elastic constants are assumed to be deterministic for the current work. The in-plane elastic constant data \((E_{11}, E_{22}, G_{12}, \text{ and } \nu_{12})\) and out-of-plane Poisson's ratio \((\nu_{23})\) values were gathered from existing literature. The ply is assumed to be transversely orthotropic, which gives that \(\nu_{12} = \nu_{13}, E_{33} = E_{22}, \text{ and } G_{13} = G_{12}.\) This also allows calculation of \(G_{23}\) from \(E_{22}\) and \(\nu_{23}\) as follows:

\[
G_{23} = \frac{E_{22}}{2(1 + \nu_{23})}
\]

The material strength values are considered to be probabilistic. Thus, complete probabilistic parameters, including mean, coefficient of variation, and distribution type, are reported. As a consequence of the probabilistic modeling used in this analysis, the probability of occurrence for any damage event can never reach 100 percent, even when the material is subject to stresses far exceeding its realistic capabilities. Therefore, the maximum strength values of the material were assumed to be the highest strengths recorded in the data utilized. All strength parameters are given in Table 4.2.

Longitudinal and transverse tensile properties \((E_{11}, E_{22}, X^T, Y^T)\) were taken from MIL-HDBK-17 (now CMH-17) [6]. The tensile properties were obtained using the ASTM D 3039-76 standard [104]. Longitudinal tensile data was obtained for 21 specimens from 7 batches and transverse tensile data was obtained for 6 specimens from 2 batches. The as-measured values are reported here.
Table 4.1  Deterministic material property data for AS4/3501-6 graphite/epoxy

<table>
<thead>
<tr>
<th>Property</th>
<th>Type [unit]</th>
<th>Case</th>
<th>Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus [Msi]</td>
<td>$E_{11}$</td>
<td></td>
<td>19.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_{22}$</td>
<td></td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_{33}$</td>
<td></td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>$\nu_{12}$</td>
<td></td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_{13}$</td>
<td></td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_{23}$</td>
<td></td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Shear Modulus [Msi]</td>
<td>$G_{12}$</td>
<td></td>
<td>0.946</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G_{13}$</td>
<td></td>
<td>0.946</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G_{23}$</td>
<td></td>
<td>0.521</td>
<td></td>
</tr>
<tr>
<td>Ply Thickness [in]</td>
<td>$t_{ply}$</td>
<td></td>
<td>0.0055</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2 Probabilistic strength data for AS4/3501-6 graphite/epoxy

<table>
<thead>
<tr>
<th>Strength Parameter</th>
<th>Mean [ksi]</th>
<th>Coefficient of Variation</th>
<th>α [ksi]</th>
<th>β</th>
<th>Maximum* [ksi]</th>
<th>Distribution Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>XT</td>
<td>295</td>
<td>5.05%</td>
<td>302</td>
<td>20.3</td>
<td>326</td>
<td>Weibull</td>
</tr>
<tr>
<td>XC</td>
<td>214</td>
<td>13.5%</td>
<td>-</td>
<td>-</td>
<td>260</td>
<td>Normal</td>
</tr>
<tr>
<td>YT</td>
<td>7.8</td>
<td>12.1%</td>
<td>-</td>
<td>-</td>
<td>9.5</td>
<td>Normal</td>
</tr>
<tr>
<td>YC</td>
<td>30</td>
<td>7.3%</td>
<td>-</td>
<td>-</td>
<td>40.4</td>
<td>Normal</td>
</tr>
<tr>
<td>S</td>
<td>13.9</td>
<td>5.70%</td>
<td>-</td>
<td>-</td>
<td>15.3</td>
<td>Normal</td>
</tr>
<tr>
<td>ZT</td>
<td>6.2</td>
<td>15.6%</td>
<td>-</td>
<td>-</td>
<td>10</td>
<td>Normal</td>
</tr>
<tr>
<td>ZS</td>
<td>4.7</td>
<td>6.5%</td>
<td>-</td>
<td>-</td>
<td>5.2</td>
<td>Normal</td>
</tr>
</tbody>
</table>

*Note: The maximum strength of the material is assumed to be the highest strength recorded in the literature, thus representing 100% probability of failure.
sive strength ($X^C$) properties were also taken from MIL-HDB-17/CMH-17 [6]. The SACMA SRM 1-88 standard [105] was used to obtain the compressive strength data for 26 specimens from 7 batches. The as-measured values are reported here. Values for the mean and coefficient of variation for the transverse compressive strength ($Y^C$) were taken from work by Shokrieh and Lessard [106]. The results were obtained using the ASTM D 3410-87 standard [107] to test 5 specimens. They did not provide a maximum test value, so the maximum in-plane transverse compressive strength value is instead taken from AS4/3502 data reported in MIL-HDBK-17/CMH-17 [6]. This result was obtained using the D 695M-96 standard [108] to test 5 specimens from 5 batches. The major in-plane Poisson’s ratio ($\nu_{12}$) and out-of-plane Poisson’s ratio ($\nu_{23}$) values were taken from data reported in MIL-HDBK-17/CMH-17 [6]. The major Poisson’s ratio was obtained by performing uniaxial tension tests using the ASTM D 3039-76 standard [104], and the out-of-plane Poisson's ratio value was obtained using the double notch test method as described in the ASTM D 2733-70 and ASTM D 3846-93 standards [109, 110, respectively]. Swanson et al. compared the torsion tube and Iosipescu methods for determining in-plane shear ($S$) properties [111]. Excellent agreement was found between the results for the two methods, with initial modulus values and strength values differing by 1% and 5%, respectively. The values from the torsion tube results, which were obtained from 12 specimens, are reported herein.

Statistically characterized data for interlaminar strength values based on reliable test methods is very limited. Some of the literature surveyed contained unsuitable results due to scale effects and/or poor manufacturing (e.g. [112]). Furthermore, it has been observed that interlaminar material properties are dependent on the orientation of the surrounding plies [113, 114]. Thus, the effect of ply orientation may require consideration when conducting an in-depth three-dimensional analysis. However, for a first-order investigation, it is assumed that the interlaminar material properties are independent of the orientations of the surrounding plies. No out-of-plane compressive strength ($Z^C$) data for AS4/3501-6 was found in the literature surveyed. However, the interlaminar compressive strength of laminated composites is typically much larger.
than the interlaminar tensile strength, and normal compressive stresses generally do not contribute to delamination initiation and propagation as much as normal tensile stresses [115]. Thus, for an initial investigation, failure via interlaminar compressive stresses is ignored.

Interlaminar tensile strength ($Z^T$) data was taken from work by Lagace and Weems [116] that contributed to the development of the ASTM D 7291 [117] standard for measuring through-thickness flatwise tensile properties. A total of twenty-three flatwise tension specimens were tested. The method seemed to yield a true measure of the normal tensile strength, and the through-the-thickness tensile strength was found to be independent of layup. No maximum value was reported, so the value for maximum interlaminar tensile strength was taken from work by Shivakumar et al. [118]. In that work, an L-shaped curved beam specimen was used to obtain results for laminates with varying numbers of plies. It was found that the thickest laminates (32 plies) suffered from poor manufacturing, so the maximum value selected was that from the 16-ply specimens, which displayed good manufacturing results. Furthermore, the mean value of the 16-ply specimens is similar to the value reported by Lagace and Weems, thus giving confidence that the chosen maximum strength is suitable.

The interlaminar shear strength is assumed to be the same along the longitudinal ($Z^{S_1}$) and transverse ($Z^{S_2}$) directions. Thus, only a single parameter for interlaminar shear strength ($Z^S$) is reported herein. Traditional methods of measuring interlaminar shear strength in composite materials include the short beam shear test (ASTM D 2344 [119]), the double notch shear test (ASTM D 2733 [109] and ASTM D 3846 [110]), and the Iosipescu V-notched beam test (ASTM D 5379 [120]). It has been recognized that short beam shear tests provide only an estimate of the apparent interlaminar shear strength (e.g. [121]). The double notch shear test also has complications in that the measured interlaminar shear strength is sensitive to various parameters, such as the notch space to thickness ratio [122], notch depth [123], length between notches [124], and specimen thickness [124]. The Iosipescu method, on the other hand, has been verified as a reliable method for obtaining the interlaminar shear
strength of composite laminates [125]. Therefore, the interlaminar shear strength ($Z_S$) parameters were obtained from work that utilized the Iosipescu method to test six uni-directional laminate specimens [126].

Reported fiber volume fractions for all specimens found in the referenced literature ranged from 58 to 65 percent, and ply thicknesses ranged from 0.0048 to 0.0058 inches. However, a nominal fiber volume fraction of 60 percent and ply thickness of 0.0055 inches is assumed throughout, since only variations in strength values are to be considered. By using the as-measured strength data, the variations in ply thickness and fiber volume fraction are essentially captured in the reported strength values.

### 4.3.2 Geometrical and Laminate Configuration

The quasi-three-dimensional analysis focuses on a quasi-isotropic [0/90/±45], laminate. This layup was chosen because it is a typical configuration and, if probabilistic aspects are considered in the analysis, it may give rise to significantly different final failure results from a given initial probabilistic damage prediction due to the possibility of overlapping probabilistic failure distributions of points of consideration in different plies. A [±15/0], laminate was investigated via three-dimensional analysis, since this laminate is known to delaminate [44].

The configuration considered is a rectangular plate with a single centrally-located circular hole. In the quasi-three-dimensional analysis of the [0/90/±45], laminate, equal biaxial tension loads are applied along the $0^\circ$ and $90^\circ$ fiber directions. This loading condition has a biaxiality ratio, defined by the variable $\lambda$, equal to one. In the three-dimensional analysis of the [±15/0], laminate, a uniaxial tension load is applied along the $0^\circ$ fiber direction. This loading condition has a biaxiality ratio of $\lambda$ equal to zero. The overall geometry was chosen for two particular reasons. First, high gradient stress fields identify a location where damage is most likely to occur, and therefore observations can be focused. Second, such high gradient stress fields, such as regions near a hole, are more likely to excite different modes and locations of failure when accounting for probabilistic aspects. This would allow for distinct damage initiation
events to be possible and for these aspects to be more easily assessed. Furthermore, the open hole tension strength of a composite has been widely studied, and it provides an important baseline for analysis since it is one of the simplest tests combining the interaction between geometrical parameters and material properties. Nevertheless, the results are difficult to accurately predict.

A hole diameter, \(D\), of 0.25 inches was chosen, since this is a typical size in composite considerations. The model width-to-diameter and length-to-diameter ratios were both chosen to be ten in order to allow the in-plane stress concentrations to reach close to one at the model edges, with consideration given to the effect of increasing orthotropy due to damage. A model length, \(L\), and width, \(W\), of 2.5 inches were thus chosen. The overall geometrical and loading configuration is shown in Figure 4.3. The origin of the coordinate system is defined to be at the center of the hole.

The stress concentrations for both laminates were calculated analytically using the equations for the stress distribution near a circular hole in a composite orthotropic plate under biaxial in-plane loading as derived by Lekhnitskii [127]. The in-plane stress components in rectangular coordinates are given by:

\[
\sigma_{11} = P + \text{Real} \left\{ \frac{P}{(\xi_1 - \xi_2)} \left[ \frac{(\lambda \xi_2 - i)\xi_1^2}{(1 + i\xi_1)} \left( 1 - \frac{z_1}{\sqrt{z_1^2 - R^2(1 + \xi_1^2)}} \right) \right] \right\} \\
\quad + \text{Real} \left\{ \frac{P}{(\xi_1 - \xi_2)} \left[ \frac{(-\lambda \xi_1 + i)\xi_2^2}{(1 + i\xi_2)} \left( 1 - \frac{z_2}{\sqrt{z_2^2 - R^2(1 + \xi_2^2)}} \right) \right] \right\} 
\]  

(4.10)

\[
\sigma_{22} = \lambda P + \text{Real} \left\{ \frac{P}{(\xi_1 - \xi_2)} \left[ \frac{(\lambda \xi_2 - i)\xi_1^2}{(1 + i\xi_1)} \left( 1 - \frac{z_1}{\sqrt{z_1^2 - R^2(1 + \xi_1^2)}} \right) \right] \right\} \\
\quad + \text{Real} \left\{ \frac{P}{(\xi_1 - \xi_2)} \left[ \frac{(-\lambda \xi_1 + i)\xi_2^2}{(1 + i\xi_2)} \left( 1 - \frac{z_2}{\sqrt{z_2^2 - R^2(1 + \xi_2^2)}} \right) \right] \right\} 
\]  

(4.11)

where \(P\) is the applied force, \(\lambda\) is the biaxiality ratio, and \(R\) is the radius of the hole. The parameters \(z_1\) and \(z_2\) are defined as:
Figure 4.3  Geometry and loading conditions of the plate investigated in the current work.
\[ z_1 = x + \xi_1 y \]  

(4.12a)

\[ z_2 = x + \xi_2 y \]  

(4.12b)

where \( x \) is the distance along the \( x_1 \)-direction from the center of the hole, and \( y \) is the distance along the \( x_2 \)-direction from the center of the hole. The values for \( \xi_1 \) and \( \xi_2 \) are calculated as:

\[
\xi_1 = \begin{cases} 
  i \left( \sqrt{\frac{\beta_0 - \alpha_0}{2}} + \sqrt{\frac{\alpha_0 + \beta_0}{2}} \right) & \text{if } \alpha_0 \leq \beta_0, \\
  \sqrt{\frac{\alpha_0 - \beta_0}{2}} + i \sqrt{\frac{\alpha_0 + \beta_0}{2}} & \text{if } \alpha_0 \geq \beta_0,
\end{cases}
\]

(4.13)

\[
\xi_2 = \begin{cases} 
  i \left( -\sqrt{\frac{\beta_0 - \alpha_0}{2}} + \sqrt{\frac{\alpha_0 + \beta_0}{2}} \right) & \text{if } \alpha_0 \leq \beta_0, \\
  -\sqrt{\frac{\alpha_0 - \beta_0}{2}} + i \sqrt{\frac{\alpha_0 + \beta_0}{2}} & \text{if } \alpha_0 \geq \beta_0,
\end{cases}
\]

(4.14)

where \( \alpha_0 \) and \( \beta_0 \) are two parameters based on the elastic properties of the laminate and are given by:

\[
\alpha_0 = \sqrt{\frac{E_{11}}{E_{22}}} 
\]

(4.15)

\[
\beta_0 = \left[ \frac{E_{11}}{2G_{12}} - \nu_{12}^* \right]
\]

(4.16)

where \( E_{11}^* \) is the effective longitudinal modulus of the laminate, \( E_{22}^* \) is the effective transverse modulus of the laminate, \( G_{12}^* \) is the effective shear modulus of the laminate, and \( \nu_{12}^* \) is the effective major Poisson's ratio of the laminate. These effective
elastic constant values were calculated using Classical Laminated Plate Theory. The calculated stress concentration factors for $\sigma_{11}$ at the edges of the plate for the two laminate and loading cases are presented in Table 4.3.
Table 4.3 Calculated stress concentration factors for $\sigma_{11}$ at the edges of the laminated plates under investigation

<table>
<thead>
<tr>
<th>Laminate</th>
<th>Biaxiality Ratio, $\lambda$</th>
<th>$\left(x_1, x_2\right)$ Coordinates [inches]</th>
<th>Stress Concentration Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\pm15/0]_s$</td>
<td>0</td>
<td>(0, 1.25)</td>
<td>1.0051</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.25, 0)</td>
<td>0.92</td>
</tr>
<tr>
<td>$[0/90/\pm45]_s$</td>
<td>1</td>
<td>(0, 1.25)</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.25, 0)</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Chapter 5

Finite Element Modeling

In this chapter, the details of the finite element modeling are described. All modeling was performed using Abaqus version 6.8 [101]. This is a commercially available finite element code. The choice of finite element mesh is based on consideration of numerous items. These considerations, specifically in the context of probabilistic progressive failure analysis, are described in Section 5.1. In the present work, analysis is performed using both two-dimensional and three-dimensional finite element models. A description and validation of the two-dimensional finite element model, used for the overall quasi-three-dimensional analysis, is presented in Section 5.2. This is followed by a description and validation of the three-dimensional finite element model in Section 5.3.

5.1 Mesh Generation Considerations

Consideration of four main items is key in the choice of element size for the probabilistic progressive failure analysis methodology utilized in this work. The first item is accurate prediction of stress fields within the structure, in which case a smaller element size is preferable. The second item is convergence of laminate ultimate strength predicted from a progressive failure analysis, since it has been reported in the liter-
ature that this value can be sensitive to the level of mesh refinement, as described in Section 2.1.1. The third item of consideration is statistical homogeneity, since the element size should be large enough that the assumption of homogeneous material properties within the element is valid, in which case an element size that is sufficiently large based on the physical realities of the system is needed. Finally, the fourth item of consideration is computational efficiency, in which case the use of fewer elements is preferable.

The accurate prediction of structural response in a finite element model requires the use of a suitably refined model. There are two main procedures for increasing the refinement of a finite element model [16]. The first is the h-refinement method, where increasingly smaller element sizes are used to improve response predictions. The second is the p-refinement method, where increasingly higher-order interpolation functions are used to reach a satisfactory result. In the current work, the h-refinement method is used for model refinement. Typically, a converged value can be reached at which point any further increase in mesh refinement leads to only a small change in the predicted results, usually on the order of a few percent. Thus, mesh convergence studies were performed for the two-dimensional and three-dimensional models. These are described in Sections 5.2.2 and 5.3.2, respectively. However, it has been shown that an interlaminar stress singularity exists at the interface of two plies at a free edge [128]. The existence of a stress singularity prohibits the ability to obtain a converged stress solution for the three-dimensional model. Since stress-based failure criteria, including a distance over which stresses are averaged, are used to predict delamination damage in this work, it is important to choose a mesh refinement level that will yield stress predictions that can give rise to realistic damage predictions. Therefore, the choice of in-plane mesh refinement level in the three-dimensional finite element model was based on a comparison of the predicted delamination initiation stress with experimental results drawn from the literature.

The second important consideration for mesh generation is the predicted ultimate strength obtained from a progressive failure analysis. It has been reported in the literature that the predicted ultimate strength is sensitive to mesh refinement, but that it
can reach a converged value. A mesh convergence study was therefore performed using deterministic progressive failure analysis for the two-dimensional finite element model. The GENOA software package from Alpha STAR Corporation [54] was employed for this purpose. As noted in Chapter 4, a three-dimensional progressive failure analysis using the procedure adopted in the current work is not possible using this software. Thus, a convergence study of predicted ultimate strength was not performed for the three-dimensional finite element model. However, the two-dimensional mesh that is converged based on the ultimate strength predicted by a progressive failure analysis is used as a minimum bound on the in-plane mesh refinement for the three-dimensional model.

The third main item of consideration for mesh generation is statistical homogeneity, in which case the notion of a representative volume element (RVE) is important. A representative volume element is usually regarded as a volume of heterogeneous material that is sufficiently large to be statistically representative of the composite material [129]. For determination of elastic properties, the random distribution of fibers and other microstructural heterogeneities in realistic composite materials requires that the RVE must include a large number of such microheterogeneities [130] and also that the average fiber volume fraction of the RVE must be equal to that of the whole composite. For damage and failure analysis, the representative volume element used should be on the order of physical failure length, accounting for the assumption of homogeneous material properties throughout the RVE. Therefore, it must be large enough to include a sufficient number of microvoids and microcracks [131]. Since the progressive failure analysis methodology adopted in the present work assigns material property values and assesses the occurrence of damage at the lengthscale of a single element, the element size is thus identical to the RVE size. Therefore, the choice of element size in the present work is partially based on experimental measurements of random fiber distributions reported in the literature as well as consideration of relevant damage lengthscales. The fiber volume fraction in a ply of HTA/6376 unidirectional prepreg composite (with a nominal fiber volume fraction of 62% and fiber diameter of approximately 7μm) was found to vary by 10% to 20% over distances
on the order of 0.1 mm to 0.5 mm (0.004 inches to 0.02 inches) [132]. Since the AS4/3501-6 material properties utilized in the current work assume a similar fiber volume fraction and fiber diameter as the HTA/6376 material, a minimum bound of 0.004 inches (0.1 mm) for each linear dimension was set on the choice of representative volume element in an attempt to maintain a sufficient level of statistical homogeneity. An RVE with this linear dimension encompasses many fibers and is large enough to include numerous microcracks, thus satisfying the desired conditions.

The fourth main item of consideration in mesh generation is the computational efficiency. The structural modeling and progressive failure analysis procedures require increased computational resources with increasing mesh refinement, and often the level of mesh refinement may become very computationally expensive. Thus, the mesh refinement level was chosen such that it was the minimum level of refinement necessary to satisfy the other three mesh generation considerations.

5.2 Two-Dimensional Finite Element Model

A two-dimensional finite element model was used for stress analysis in the quasi-three-dimensional case of the [0/90/±45]s laminate subjected to equal biaxial loading. In this section, the specific details of the two-dimensional finite element modeling are provided. A description of the finite element mesh is given in Section 5.2.1, with validation of the model presented in Section 5.2.2.

5.2.1 Description

The two-dimensional finite element model is composed of four-node linear quadrilateral shell elements, denoted as "S4" elements in Abaqus. These are isoparametric displacement-based shell elements with six degrees-of-freedom per node. In the quasi-three-dimensional analysis, the laminate material properties are homogenized using CLPT to allow use of a two-dimensional finite element model. Furthermore, only symmetric damage scenarios are considered in the current work, as described in Chapter
3. Thus, the quasi-three-dimensional analysis model can take advantage of the symmetries of the problem. There are three symmetry conditions that exist in this case. These are shown in Figure 5.1. In this figure, the $x_1$-axis is parallel to the $0^\circ$ fiber direction in the plane of the laminate, the $x_2$-axis is parallel to the $90^\circ$ fiber direction in the plane of the laminate, and the $x_3$-axis denotes the through-thickness direction. The first symmetry is about a mirror plane perpendicular to the $x_1$-axis in the center of the model. Second, there is a mirror plane of symmetry perpendicular to the $x_2$-axis in the center of the model. Third, there is a mirror plane of symmetry about the laminate midplane because a symmetric layup is used. Thus, only one-quarter of the plate geometry and half the plies need to be considered in the analysis. This reduces the overall problem by a factor of eight.

Utilizing the symmetry boundary conditions, only one-quarter of the in-plane geometry is modeled. The boundary conditions that are applied to the model are shown in Figure 5.2. The planes of mirror symmetry that are perpendicular to the $x_1$-axis and $x_2$-axis are implemented in the finite element model using symmetry boundary conditions, which in this case are specified using the “XSYMM” and “YSYMM” commands in Abaqus to constrain the nodes along the $x_2$-axis and $x_1$-axis, respectively. These are displacement boundary conditions that enforce the following constraints:

\[ XSYMM : u_1, \Phi_2, \Phi_3 = 0 \]  
\[ YSYMM : u_2, \Phi_1, \Phi_3 = 0 \]

where $u_1$ and $u_2$ denote displacement in the $x_1$-direction and $x_2$-direction, respectively, and $\Phi_1$, $\Phi_2$, and $\Phi_3$ denote rotation about the $x_1$-axis, $x_2$-axis, and $x_3$-axis, respectively. Distributed tractions are applied along the loading edges of the model, and the hole edge is stress-free.

The two-dimensional finite element mesh was created to avoid any large angular or aspect ratio distortions of the elements. Based on the considerations for element size as described, it was decided that the mesh would contain two main regions. Finer elements are used in the vicinity of the hole to capture the stress gradients. Coarser
Figure 5.1 Schematic of the symmetry conditions that exist in the quasi-three-dimensional model.
Figure 5.2  Schematic of the boundary conditions applied to the two-dimensional finite element model.
elements are employed in the remaining regions of the model for computational efficiency. It is desired that the region of finer elements be large enough to encompass most of the region with steep stress gradients, and also to be able to capture the damage progression that has been experimentally observed to occur near a hole boundary. For a uniaxial loading condition, the stress concentration along a line perpendicular to the loading direction at the laminate midline begins to level out to a value of about 1.1 at a distance-to-diameter ratio of 1, which for this case corresponds to 0.25 inches from the hole edge. Furthermore, this distance is large enough to capture considerable damage progression. Thus, the region of finer elements was chosen to extend one hole-diameter (0.25 inches) from the hole edge.

The overall one-quarter model comprises a total of 208 elements. A diagram of the model is shown in Figure 5.3. The choice of mesh scheme was partially based on the results of two convergence studies, which are presented in the following subsection. As noted, there are two main regions in the mesh. One of these main regions is further divided into three sub-regions. These regions are labeled in Figure 5.4. The finer region, labeled as “1” in Figure 5.4, was chosen to be square in order to avoid excessive element distortions at the boundary where the mesh transitions to the coarser region. The finer region mesh scheme is symmetric about the line at 45° to the model midline. The finer region consists of twelve elements in the radial direction (normal to the hole) and twelve elements in the tangential direction. The smallest elements, which are located at the hole boundary adjacent to the x1-axis and x2-axis, have a linear dimension of approximately 0.021 inches. This element size satisfies the minimum bound set to maintain statistical homogeneity, as defined in Section 5.1.

The coarser mesh region consists of three subregions. Two of the subregions, labeled as “2a” and “2b” in Figure 5.4, are identical in size and mesh scheme. Each of these sub-regions is a rectangular arrangement of rectangular elements, with six elements along one direction and four elements along the other direction. These sub-regions are directly adjacent to the two edges of the finer region. The longer edge of subregion “2a” is parallel to the x2-direction, and the longer edge of subregion “2b” is parallel to the x1-direction. The final subregion of the coarser region is labeled as
Figure 5.3  Image of the mesh scheme in the two-dimensional finite element model.
Figure 5.4 Image of the different mesh regions in the two-dimensional finite element model.
"2c" in Figure 5.4. This subregion covers the remainder of the model with square and rectangular elements, with a mesh scheme of four elements in the $x_1$-direction and four elements in the $x_2$-direction. It is finally noted that the entire mesh is symmetric about the line at $45^\circ$ to the model midline as illustrated in Figure 5.4.

### 5.2.2 Validation

The two-dimensional finite element model was validated using three separate metrics. The first validation performed was a mesh convergence study of predicted average stress in the $x_1$-direction in the $0^\circ$ ply, $\sigma_{11}$, of an element at the hole boundary. This single ply within an element is referred to as a "point of consideration" (previously defined in Chapter 3) in the current work. This measure of convergence was utilized to verify that accurate predictions of average stresses for each point of consideration would be obtained for comparison with the chosen failure criteria. The second validation performed was a convergence study of predicted ultimate strength from a deterministic progressive failure analysis, since this value is sensitive to the level of mesh refinement. Finally, the chosen mesh was used for the case of an isotropic material to predict the tangential stress distribution along a line perpendicular to one of the loading directions at the laminate midline. The finite element results were then compared to the analytical plane stress elasticity solution for an isotropic plate of infinite width with a circular hole subject to equal biaxial tension in order to verify the overall accuracy of the model.

To check convergence of the mesh using the stress metric for the composite laminate, five different meshes were created. Each mesh had an increased level of refinement over the previous mesh. Mesh refinement was performed only along the radial direction (normal to the hole) in the finer mesh region, so that the coarser mesh region is identical in all five models. Refinement was performed along the radial direction using the h-refinement method, as described in Section 5.1. The finer region of the first mesh has three elements in the radial direction, giving a total of 100 elements in the model. The smallest elements along the hole boundary have a linear dimension of
0.083 inches. The second mesh has six elements in the radial direction, giving a total of 136 elements in the model. The smallest elements along the hole boundary have a linear dimension of 0.042 inches. The third mesh has twelve elements in the radial direction, giving a total of 208 elements in the model. The smallest elements along the hole boundary have a linear dimension of 0.021 inches. The fourth mesh has eighteen elements along the radial direction, giving 280 total elements in the model. The smallest elements along the hole boundary have a linear dimension of 0.014 inches. Finally, the fifth mesh has twenty-four elements in the radial direction, giving a total of 352 elements in the model. The smallest elements along the hole boundary have a linear dimension of 0.010 inches.

For the convergence study, the model was subjected to an equal-biaxial tensile load. The average value of the stress, \( \bar{\sigma}_{11} \), was taken from the 0\(^\circ\) ply of the element on the hole boundary adjacent to the \( x_1 \)-axis, as shown in Figure 5.5. These average stress values, normalized by the far-field applied stress (\( \sigma_0 \)), are plotted as a function of the number of elements in the mesh in Figure 5.6. The mesh with 352 elements only yields a 0.4% increase in normalized average stress compared to the mesh with 280 elements. This was therefore considered to be converged. The normalized average stress values for each mesh are given in Table 5.1, along with the percent change from the immediately coarser mesh and the percent difference from the converged value considered to be 2.26 as obtained from the model with 352 elements. The chosen mesh of 208 elements results in a 0.8% difference from the converged mesh.

The second validation procedure performed is a mesh convergence study based on the predicted ultimate strength from a deterministic progressive failure analysis. The GENOA software program from Alpha STAR Corporation was used to perform the progressive failure analyses. Four of the meshes used in the stress convergence study were used in this study. These are the meshes with 100, 136, 208, and 280 total elements. Furthermore, since the predicted ultimate strength from a progressive failure analysis is known to be dependent on the load increment level, two different load increment scenarios were considered in order to ensure mesh convergence was independent of load increment. However, GENOA does not allow the user to specify
Figure 5.5  Location of the element considered in the stress convergence study of the two-dimensional finite element model.
Figure 5.6 Predicted average values of stress, $\bar{\sigma}_{11}$, normalized by the applied far-field stress, $\sigma_0$, in the element on the hole boundary for various meshes.
Table 5.1 Normalized average values of stress, $\overline{\sigma}_{11}/\sigma_o$, for five different two-dimensional finite element meshes

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>Normalized Average Stress, $\overline{\sigma}_{11}/\sigma_o$</th>
<th>% Change from Coarser Mesh</th>
<th>% Difference from Converged Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2.10</td>
<td>-</td>
<td>7.1</td>
</tr>
<tr>
<td>136</td>
<td>2.18</td>
<td>3.8</td>
<td>3.5</td>
</tr>
<tr>
<td>208</td>
<td>2.24</td>
<td>2.8</td>
<td>0.8</td>
</tr>
<tr>
<td>280</td>
<td>2.25</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>352</td>
<td>2.26</td>
<td>0.4</td>
<td>0</td>
</tr>
</tbody>
</table>
a single load step increment. Instead, it requires the user to define a range of possible load increments along with the maximum number of elements allowed to damage per iteration. The GENOA code starts each new step of the progressive failure analysis with the maximum defined load increment and continually decreases it until the number of elements with predicted damage is not more than the maximum number allowed to be damaged. Therefore, it was decided to keep the load increment range the same for both cases, but make the maximum allowed number of elements different for the two scenarios. Thus, the load increment range was defined to be 1.25 pounds to 125 pounds for both cases. The maximum number of elements allowed to damage per iteration was set to one element for the first case and 4% of the total number of elements in the model for the second case. This yields two failure strength values for each of the four meshes.

The resulting ultimate failure strengths of the meshes with 100, 136, 208, and 280 elements are shown as a function of total number of elements in Figure 5.7 for each load increment scenario. For both load increment scenarios, the mesh with 208 elements gives the exact same ultimate strength value as the mesh with 280 elements. Thus, the mesh with 208 elements is considered converged based on this criterion.

Using the mesh with 208 elements, a final model validation was performed by comparing the predicted tangential stress distribution along the \( x_1 \)-axis with the analytical plane stress solution for an isotropic plate of infinite width containing a circular hole subject to equal biaxial tension. This provides an assessment of the overall accuracy of the model. An equal biaxial load of 125 pounds was applied. The values of stress, \( \sigma_{22} \), were obtained as averaged at the nodes for all nodes along the \( x_1 \)-axis, since this stress component is equivalent to the tangential stress component along the \( x_1 \)-axis. The plane stress analytical solution for the tangential stress distribution along the \( x_1 \)-axis (normalized by the applied far-field stress) in an isotropic plate of infinite width containing a circular hole subject to equal biaxial tension is given in polar coordinates by [133]:
Figure 5.7  Predicted ultimate strength as a function of the total number of elements for various two-dimensional meshes.
\[
\frac{\sigma_{\phi\phi}}{\sigma_o} = 1 + \left(\frac{R}{x}\right)^2
\]

where \(\sigma_{\phi\phi}\) is the tangential stress, \(\sigma_o\) is the far-field applied stress, \(R\) is the hole radius, and \(x\) is the distance from the center of the hole along the \(x_1\)-direction. For the case along the \(x_1\)-axis, \(\phi\) is equal to zero. Thus, \(\sigma_{\phi\phi}\) is equal to \(\sigma_{22}\). The predicted stress distribution as a function of normalized distance from the hole edge obtained from the finite element model is plotted along with the theoretical solution as a function of the distance-to-hole radius ratio \((x/R)\) in Figure 5.8. It is seen that the finite element model is in good agreement with the analytical solution.

5.3 Three-Dimensional Finite Element Model

The details of the three-dimensional finite element model used in the analysis of the uniaxially loaded \([\pm 15/0]\) laminate are provided in this section. A description of the finite element model is given in Section 5.3.1. Validation of the model is presented in Section 5.3.2.

5.3.1 Description

The three-dimensional finite element model employed in this work is composed of eight-node linear solid brick elements. These are denoted as "C3D8" elements in Abaqus. These are displacement-based isoparametric elements with three degrees-of-freedom per node. Each ply is modeled as a separate layer and it is assumed that the plies are perfectly bonded. As described in Section 4.3.1, the interlaminar strength properties used in this work are based on measurements of an entire laminated composite system. These strength values ignore the local details of the resin-rich interply regions. Thus, the resin-rich interply regions are not explicitly modeled in the current work.

There is one symmetry condition that can be assumed in this case. This symmetry condition is a mirror plane perpendicular to the \(x_3\)-axis at the laminate midplane.
Figure 5.8  Comparison of the predicted tangential stress distribution along the $x_1$-axis (normalized by the far-field applied stress) with the analytical solution for an infinite width isotropic plate containing a circular hole subject to equal biaxial tension.
This symmetry can be assumed because the in-plane stresses and interlaminar normal stresses are identical across the mirror plane, and the interlaminar shear stresses are equal in magnitude but opposite in sign, which give identical predictions of damage based on the methodology employed in the present work. The other symmetries that are present in the quasi-three-dimensional case are no longer applicable in the three-dimensional model due to the fact that all plies are now modeled explicitly instead of homogenized. Therefore, the presence of the angle plies eliminates any mirror symmetries about planes perpendicular to the $x_1$-axis and $x_2$-axis. Furthermore, the interlaminar stress distributions cannot be assumed to be symmetric about these planes because the mismatch between coefficients of mutual influence in adjacent plies, which gives rise to interlaminar stresses [134], is different at each ply interface. Thus, it cannot be assumed that the interlaminar stresses between plies in one quadrant are identical to those in another quadrant of a different ply interface, as was done in assuming identical stress states in different quadrants of the $+45^\circ$ and $-45^\circ$ plies of the quasi-three-dimensional model. Utilizing the symmetry about a mirror plane perpendicular to the $x_3$-axis at the laminate midplane, only the top three plies of the laminate are modeled for the entire in-plane geometry.

The boundary conditions applied to the model are shown in Figure 5.9. The mirror plane symmetry condition at the laminate midplane was implemented in Abaqus by applying the “ZSYM” symmetry boundary condition to all nodes along the laminate midplane. This is a displacement boundary condition that enforces the following constraints:

$$ZSYM : u_3, \Phi_1, \Phi_2 = 0$$  \hspace{1cm} (5.4)

where $u_3$ denotes displacement in the $x_3$-direction, and $\Phi_1$ and $\Phi_2$ denote rotation about the $x_1$-axis and $x_2$-axis, respectively. The uniaxial tensile load is applied along the $x_1$-direction of the plate parallel to the $0^\circ$ fiber direction. Application of a uniform external force to simulate loading, as is done in the two-dimensional model, is not appropriate for three-dimensional models of composite laminates with layers of
different orientations. This is especially true for progressive failure analysis, since the development of damage leads to unloading of the damaged plies. Therefore, the uniaxial tensile load is simulated by constraining one of the edges of the plate that is perpendicular to the $x_1$-axis to have zero displacement in the $x_1$-direction, and prescribing an axial extension along the opposite edge. In order to calculate the applied load associated with a given applied axial displacement, the reaction forces from the nodes on the loading face are obtained from Abaqus and summed. In order to prevent rigid body translation of the plate along the transverse direction without disrupting Poisson’s effects of the material, the nodes along a line parallel to the $x_3$-axis at the center of the loading faces are constrained to have zero displacement in the $x_2$-direction. The location of these constrained nodes are indicated by the dashed line in Figure 5.9. The hole edge, both outer edges parallel to the $x_1$-$x_3$-plane, and the top surface of the model parallel to the $x_1$-$x_2$-plane are left to be stress-free.

The three-dimensional finite element mesh was created to avoid any large angular or aspect ratio distortions of the elements. The overall model comprises a total of 30,144 elements. A diagram of one-quarter of the in-plane mesh scheme is shown in Figure 5.10. The choice of mesh scheme was partially based on the results of two convergence studies, which are presented in the following subsection. One convergence study involved refinement of the mesh in the through-thickness direction, and the other convergence study involved refinement of the in-plane mesh. Each ply is modeled by one element in the through-thickness direction. The in-plane mesh scheme was chosen to be similar to the two-dimensional model. A diagram showing the distinct regions in one-quarter of the in-plane mesh is shown in Figure 5.11. This one-quarter in-plane mesh contains two main regions as per the two-dimensional model. The first is a finer region near the vicinity of the hole to capture the stress gradients, labeled as “1” in Figure 5.11. This inner region is surrounded by a coarser region elsewhere to improve computational efficiency. The coarser mesh region is composed of three subregions, labeled as “2a,” “2b,” and “2c” in Figure 5.11.

The finer region is a square section that extends one hole-diameter (0.25 inches) from the edge of the hole. A zoomed-in view of one-quarter of the finer region of
Figure 5.9  Schematic of the boundary conditions applied to the three-dimensional finite element model.
Figure 5.10 Diagram of one-quarter of the in-plane mesh scheme of the three-dimensional finite element model.
Figure 5.11  Distinct regions of one-quarter of the in-plane, three-dimensional finite element mesh scheme.
the in-plane mesh scheme is shown in Figure 5.12. The finer region was chosen to be square in order to avoid excessive element distortions at the boundary where the mesh transitions to the coarser region. The finer region mesh scheme is symmetric about the line at 45° to the model midline. The finer region consists of forty-eight elements in the radial direction (normal to the hole) and forty-eight elements in the tangential direction. The smallest elements, which are located at the hole boundary adjacent to the x1-axis and x2-axis, have a linear dimension of approximately 0.0052 inches (0.1321 mm). This element size satisfies the minimum bound set to maintain statistical homogeneity, as defined in Section 5.1.

The coarser mesh region consists of three subregions. Two of the sections, labeled as “2a” and “2b” in Figure 5.11, are identical in size and mesh scheme. Each of these regions is a rectangular arrangement of rectangular elements with twenty-four elements along one direction and four elements along the other direction. These regions are directly adjacent to the two edges of the finer region. The longer edge of subregion “2a” is parallel to the x2-direction, and the longer edge of subregion “2b” is parallel to the x1-direction. The final subregion of the coarser region is labeled as “2c” in Figure 5.4. This subregion covers the remainder of the model with square and rectangular elements, with a mesh scheme of four elements in the x1-direction and four elements in the x2-direction. It is finally noted that the entire mesh is symmetric about the line at 45° to the model midline as illustrated in Figure 5.11, as in the two-dimensional model.

5.3.2 Validation

Two methods were used to validate the three-dimensional finite element model. The first method was to perform mesh convergence studies. Three such studies were conducted, with refinement of the through-thickness mesh considered in one case and refinement of the in-plane mesh considered in the other two. Due to the existence of an interlaminar stress singularity between plies at the hole edge, the predicted interlaminar stress values at the edge of the hole will keep increasing with increas-
Figure 5.12  Zoomed in view of one-quarter of the finer region (Region 1) of the in-plane mesh scheme for the three-dimensional finite element model.
ing in-plane mesh refinement. Therefore, it is not possible to use an in-plane mesh convergence study that only considers convergence of interlaminar stresses to identify the proper element size that should be used for prediction of delamination. Instead, convergence of the in-plane mesh was evaluated by two different methods. In the first in-plane mesh convergence study, convergence in the displacement of a node at the hole boundary was considered, since this is a typical method used to measure overall mesh convergence that does not rely on predicted stress values. However, the present work assesses the occurrence of delamination damage based on predicted interlaminar stress values. Therefore, as described in Section 4.1.3, an average stress approach, rather than a point stress approach, is used to predict delamination. In an average stress approach, the interlaminar stresses are averaged over some distance, and this average stress value is used to predict delamination. In order to use an average stress approach, an appropriate averaging distance must be identified (the present work uses in-plane element dimensions for the averaging distance of interlaminar stresses, as mentioned in Section 3.2). Thus, the second in-plane mesh validation method compared results of predicted delamination initiation stress with experimental values for various levels of in-plane mesh refinement in order to find a suitable element size. The final validation method was a comparison of the predicted stress distribution for the case of an isotropic material with the analytical plane stress solution for an isotropic plate of infinite width containing a circular hole subject to uniaxial tension. This was done in order to verify the overall accuracy of the model compared to a known theoretical solution.

In the first in-plane mesh convergence study, the through-thickness mesh was kept constant at one element per ply while the refinement of the finer region of the in-plane mesh was varied. Refinement was performed using the h-refinement method, as described in Section 5.1. Three different in-plane meshes were investigated. For the first case, the finer region of one-quarter of the first mesh contained twenty-four elements in the radial direction (normal to the hole) and twenty-four elements in the tangential direction. Regions “2a” and “2b” (as defined in Figure 5.11) were composed of twelve elements along one direction and four elements along the other
direction. This resulted in 2,752 elements per ply, giving a total of 8,256 elements in the model. The finer region of the second mesh contained forty-eight elements in the radial direction and forty-eight elements in the tangential direction. Regions “2a” and “2b” were composed of twenty-four elements along one direction and four elements along the other direction. This resulted in 10,048 elements per ply, giving a total of 30,144 elements in the model. Finally, the finer region of the third mesh contained ninety-six elements in the radial direction and ninety-six elements in the tangential direction. Regions “2a” and “2b” were composed of forty-eight elements along one direction and four elements along the other direction. This resulted in 38,464 elements per ply, giving a total of 115,392 elements in the model. In all of these models, the number of elements in Region 2a and Region 2b increases along their shorter direction. This is necessary to allow these regions to match the mesh scheme of Region 1. No refinement was performed along the longer directions of Regions 2a and 2b because these regions are far enough away from the hole, and thus the region of steep stress gradients, that no further refinement is needed. Region 2c is not refined in any of the models because it is also far enough from the hole that further refinement is not needed, and it is not affected by changes in the Region 1 mesh scheme.

In this first in-plane mesh convergence study, the $u_1$-displacement of a node at the hole boundary on the $x_1$-axis and laminate midplane was used to check overall convergence of the mesh. This displacement is plotted as a function of total number of elements in the model in Figure 5.13. Using this measure as a criterion for convergence, it can be seen that the mesh with 30,144 elements has reached a converged value. Therefore, it is expected that all model behavior is converged except for the interlaminar stresses at the ply interfaces on the free edges.

For the convergence study of the through-thickness mesh, the in-plane mesh scheme was kept constant and specified to be the in-plane mesh scheme found to be converged in the previously described in-plane mesh convergence study. This mesh had 10,048 in-plane elements. In the through-thickness direction, two levels of mesh refinement were investigated. The first mesh scheme has one element per ply
Figure 5.13  Displacement of a node at the hole boundary and laminate midplane used to check three-dimensional mesh convergence.
in the through-thickness direction, referred to herein as a “single layer” refinement, giving a total of 30,144 elements in the model. The second mesh scheme has two elements per ply in the through-thickness direction, referred to herein as a “double layer” refinement, giving a total of 60,288 elements in the model.

Since delamination damage in this work is predicted based on average interlaminar stress values at the ply interfaces, the interlaminar normal stress at the interface between elements in adjacent plies was used to check convergence. This stress is calculated by averaging the nodal stresses (obtained from Abaqus using the “LOCATION=NODES” option for the results output) from the four shared nodes that comprise the ply interface of the two elements. For this convergence study, the average interlaminar normal stress was obtained for the ply interfaces of the elements at the hole edge adjacent to the $x_1$-axis at a location of 0° around the hole boundary. This corresponds to the same element location as that used in the stress convergence study for the two-dimensional finite element model, as shown in Figure 5.5.

The average interlaminar normal stress values ($\sigma_{33}$) are normalized by the applied far-field stress ($\sigma_0$) and plotted as a function of distance along the $x_3$-axis ($z$) normalized by the ply thickness ($t_{\text{ply}}$) for the two mesh schemes in Figure 5.14. It can be seen that the normalized interlaminar normal stress at the ply interfaces is very similar for both the single layer and double layer refinement schemes. Based on this result, it was decided that one element per ply in the through-thickness direction is sufficient for the purposes of this work. This finding agrees with other work pertaining to delamination prediction using finite element modeling found in the literature [135].

The third and final mesh convergence study was the second convergence study to consider in-plane mesh refinement. As previously noted, an interlaminar stress singularity exists at the ply interfaces on the hole boundary. Therefore, increasing levels of in-plane mesh refinement only lead to increasing stress levels at the hole boundary and will not yield a converged value. Therefore, an average stress approach is used to predict delamination. In this approach, the average stresses over some distance are used to assess the occurrence of delamination. Furthermore, it has been conjectured
Figure 5.14 Normalized interlaminar normal stress versus through-thickness location for various levels of through-thickness mesh refinement.
that the averaging distance is a material property [44]. Thus, a mesh validation was performed by considering three finite element models with various levels of in-plane mesh refinement and comparing the predicted delamination initiation stress to an experimental value to determine the size of the average linear dimension of the elements around the hole boundary that best match experimental results of a laminate with the same material properties. The experimental result was obtained from Reference [136], in which a [(0/+45/90/-45)]₄ AS4/3501-6 laminate with a 0.378-inch diameter hole was tested in uniaxial tension. The applied stress at which delamination initiated was reported as 36 ksi (248 MPa).

A finite element model of this laminate was created. The overall dimensions of the model are the same as those in Figure 5.10, except that the hole radius is 0.189 inches instead of 0.125 inches. The overall mesh scheme comprised the same regions as those identified in Figure 5.11, with the edge of Region 1 still extending 0.375 inches from the origin along the x₁-axis and x₂-axis. Thus, Region 1 is smaller in this model. For all models, one element per ply was used in the through-thickness direction as determined via the through-thickness mesh convergence study. Three different levels of in-plane mesh refinement were investigated. Refinement of the in-plane mesh was performed uniformly throughout the finer mesh region, with each increasingly finer mesh produced by cutting the elements in half along both the longitudinal and transverse directions. For the first case, the finer region of one-quarter of the first mesh contains ten elements in the radial direction (normal to the hole) and ten elements in the tangential direction. Regions “2a” and “2b” (as defined in Figure 5.11) are composed of five elements along one direction and four elements along the other direction. This gives 624 elements per ply. The first mesh has elements around the hole boundary with an average linear dimension of 0.026 inches (0.67 mm). One-quarter of the finer region of the second mesh contains twenty elements in the radial direction and twenty elements in the tangential direction. Regions “2a” and “2b” were composed of ten elements along one direction and four elements along the other direction. This resulted in 1,984 elements per ply. The second mesh has elements around the hole boundary with an average linear dimension of 0.013 inches (0.335
Finally, one-quarter of the finer region of the third mesh contains forty elements in the radial direction and forty elements in the tangential direction. Regions “2a” and “2b” were composed of twenty elements along one direction and four elements along the other direction. This resulted in 7,104 elements per ply. The third mesh has elements around the hole boundary with an average linear dimension of 0.0066 inches (0.167 mm). This mesh has an element size at the hole boundary that is the same as the actual mesh chosen for the probabilistic progressive failure analysis aspects of this work, as shown in Figures 5.10 and 5.11. In all of these models, the number of elements in Region 2a and Region 2b increases along their shorter direction. This is necessary to allow these regions to match the mesh scheme of Region 1. No refinement was performed along the longer directions of Regions 2a and 2b because these regions are far enough away from the hole, and thus the region of steep stress gradients, that no further refinement is needed. Region 2c is not refined in any of the models because it is also far enough from the hole that further refinement is not needed, and it is not affected by changes in the Region 1 mesh scheme.

The predicted delamination initiation stress for each model was determined using the Quadratic Delamination Criterion [44] under the assumption that the interlaminar normal compressive stresses could be ignored (as noted in Section 4.3.1). As mentioned in Section 3.2, the averaging length for the interlaminar stresses was chosen to be equal to the in-plane element length. The experimental results indicated that the delamination initiated at the boundary of the hole [136]. Therefore, only the elements along the boundary of the hole were considered in the prediction of delamination initiation. For each set of four nodes that comprise the shared interphase between elements in adjacent plies, the nodal stresses were obtained using the "LOCATION=NODES" option in the results output. The Quadratic Delamination Criterion was then applied using a two-step process. In the first step, the value for each interlaminar stress component was found at the middle of the element edge on the hole boundary by interpolating between the two nodes on the hole boundary. Similarly, the value for each interlaminar stress component was found at the middle of the element edge opposite the hole boundary by interpolating between the two nodes.
not on the hole boundary. In the second step, the Quadratic Delamination Criterion was applied along the centerline of each element in the radial direction (normal to the hole). It was assumed that the stress distribution along the centerline in each element varied linearly between the two values calculated in the first step, since linear elements were used in the analysis model. The averaging distance used was the length of the element centerline in the radial direction (normal to the hole). This value was calculated by interpolating the element length along the radial direction (normal to the hole boundary). The interlaminar strength values used are the mean values reported in Section 4.3.1.

The results are shown in Figure 5.15. This is a plot of the applied far-field stress at which delamination initiation is predicted to occur as a function of the average linear dimension of the elements at the hole boundary. The experimental result of 36 ksi is also shown [136]. The mesh with an average linear element dimension at the hole boundary of 0.0066 inches (0.167 mm) gives a predicted stress for delamination initiation of 41.0 ksi (283 MPa). This is in good agreement with the experimental result of 36 ksi (248 MPa). The average linear dimension of the elements along the hole boundary matches extremely well with the value of 0.178 mm found for the averaging length of AS1/3501-6 [44], thus lending further confidence in the result found here. Thus, the original model with a mesh scheme of 10,048 in-plane elements (30,144 total elements) is considered to have the appropriate element size to predict delamination initiation.

The final validation was performed on the chosen mesh with 30,144 total elements by comparing the predicted tangential stress along a line perpendicular to the loading direction (normalized by the far-field applied stress) with the analytical plane stress solution for an isotropic plate of infinite width containing a circular hole and subject to uniaxial tension. The plane stress analytical solution for the tangential stress distribution perpendicular to the loading direction (normalized by the far-field applied stress) of an isotropic plate of infinite width containing a circular hole subject to uniaxial tension is given in polar coordinates by [137]:

106
Figure 5.15 Predicted values of applied far-field stress at which delamination initiation occurs versus average linear dimension of elements at the hole boundary of a $(0/45-45/90)$$_4$ AS4/3501-6 laminate with a 0.378-inch diameter hole loaded in uniaxial tension.
\[ \frac{\sigma_{\phi\phi}}{\sigma_o} = 1 + \frac{1}{2} \left( \frac{R}{y} \right)^2 + \frac{3}{2} \left( \frac{R}{y} \right)^4 \]  

(5.5)

where \( \sigma_{\phi\phi} \) is the tangential stress, \( \sigma_o \) is the far-field stress, \( R \) is the hole radius, and \( y \) is the distance from the center of the hole along the \( x_2 \)-direction. For the case along the \( x_2 \)-axis, \( \phi \) is equal to 90°. This, \( \sigma_{\phi\phi} \) is equal to \( \sigma_{11} \). The predicted stress, normalized by the applied far-field stress, at the midplane along a line perpendicular to the loading direction as a function of the distance-to-hole radius ratio \( (y/R) \) is shown in Figure 5.16, along with the plane stress elasticity solution. Only the nodal values from every fourth node are plotted in order to maintain clarity. The finite element predictions match the analytical solution very well.
Figure 5.16  Comparison of the predicted tangential stress distribution along the $x_2$-axis (normalized by the far-field applied stress) with the analytical solution for an infinite width isotropic plate containing a circular hole subject to uniaxial tension.
Chapter 6

Quasi-Three-Dimensional Analysis

In this chapter, the results from and discussion concerning the quasi-three-dimensional analysis work are presented. As mentioned in Chapter 4, the quasi-three-dimensional analysis focuses on the specific case of a quasi-isotropic \([0/90/\pm45]_s\) AS4/3501-6 laminated plate containing a centrally located circular hole subject to an equal biaxial tensile load. The results from the deterministic progressive failure analysis are given in Section 6.1. The probabilistic damage initiation results are given in Section 6.2. The probabilistic damage propagation results for several propagation scenarios are presented in Section 6.3. Finally, a discussion of the results is given in Section 6.4.

Many of the results with information regarding damage are presented spatially in order to indicate the locations of potential and existing damage. The existence of the centrally located circular hole in the finite element model used in this work creates a region of high stress gradient that allows attention to be focused on a small region surrounding the hole. Therefore, only the finer region (as defined in Section 5.2.1) of the finite element model is presented for each ply in the laminate considered. For the present work, it is found that this is sufficient to capture all the relevant information from both the deterministic and probabilistic analyses. Furthermore, since the finite element model used in the quasi-three-dimensional analysis work only represents one quarter of the actual laminate under investigation, an identical state of damage is
assumed in the other quadrants of the laminate. The only difference is that in some quadrants, the damage state shown here in the $+45^\circ$ ply is actually occurring in the $-45^\circ$ ply using suitable assumptions of symmetry.

6.1 Deterministic Progressive Failure Analysis Results

In this section, the results from the deterministic progressive failure analysis are presented. The analysis was performed using the GENOA software code developed by Alpha STAR Corporation [54]. All details of the analysis setup and procedure are described in Chapter 4. The analysis begins from an undamaged model and then simulates the entire predicted damage progression process from damage initiation all the way through to final failure. All of the results presented herein focus on the equilibrium stages of the analysis, which are the stages of the analysis at which no further damage is predicted at a given applied stress. Furthermore, only the equilibrium stages that result in the occurrence of further damage are presented. This is done because it provides the ability to show all damage predicted at a given applied stress while reducing the number of figures needed to fully convey the information regarding the state of damage within the model. Selected damage results are presented spatially. Each ply of the finite element model is displayed at a chosen applied stress for each damage mode that is predicted. Each ply within each element is referred to as a “point of consideration” in the present work, so each ply in the results figures is divided into numerous “points of consideration” according to the mesh scheme. For each point of consideration, existing damage from previous equilibrium stages and newly predicted damage are denoted separately. Additionally, the applied stress, the fiber angle direction, the damage mode being presented, and a scale bar are indicated in each figure.

The predicted damage initiation is considered to be the first equilibrium stage for the purposes of this work, since all previous equilibrium stages are reached without the
occurrence of any damage. There are a total of thirty equilibrium stages containing damage reached during the analysis. All points of consideration in the deterministic progressive failure analysis are predicted to fail via the transverse tension \((Y^T)\) failure mode. No other failure mode is manifested throughout the entire damage progression process, so all damage results presented in this section refer only to the transverse tensile failure mode. Due to the nature of the structural configuration and loading, there is actually another sort of symmetry that exists in the model. As can be seen in subsequent figures, there is a line of symmetry parallel to the \(+45^\circ\) fiber direction and located \(+45^\circ\) around the hole boundary. This line defines a plane of mirror symmetry such that the damage in the \(+45^\circ\) ply is symmetric about the line in the plane of the laminate. Furthermore, any damage predicted in the \(0^\circ\) ply is mirrored across this line of symmetry into the \(90^\circ\) ply.

An overview of the predicted damage progression is provided in Table 6.1. The applied stress, number of newly failed points of consideration, and number of total failed points of consideration in the model are given for each equilibrium stage. Specifying the plies that are damaged at each stage indicates the general locations of predicted damage. Listing the number of newly failed points of consideration at each stage indicates the extent of the predicted damage. Due to the existence of the symmetry plane parallel to the \(+45^\circ\) fiber direction described in the previous paragraph, the damage predicted in the \(0^\circ\) and \(90^\circ\) plies are grouped together in a single column in Table 6.1. The number listed in this column is the total number of newly predicted points of consideration in both the \(0^\circ\) and \(90^\circ\) plies.

The total number of failed points of consideration in the model is plotted as a function of applied far-field stress in Figure 6.1. This gives an indication of the damage growth rate during the failure process. For the majority of the damage progression process, no more than four points of consideration fail per equilibrium stage. As the ultimate strength of the laminate is neared, the damage growth rate speeds up considerably. For example, sixty-six points of consideration fail over a 10.4 ksi increase in stress between damage initiation at 24.0 ksi and the twenty-fourth equilibrium stage at 34.4 ksi. The number of failed points of consideration then more

113
Table 6.1 Summary of the results of the deterministic progressive failure analysis

<table>
<thead>
<tr>
<th>Equilibrium Stage</th>
<th>Stress [ksi]</th>
<th>Number of Newly Failed Points of Consideration</th>
<th>Total Number of Failed Points of Consideration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0°/90° plies</td>
<td>+45° ply</td>
</tr>
<tr>
<td>1</td>
<td>24.0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>24.7</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>25.6</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>25.9</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>26.1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>27.4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>27.7</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>28.0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>28.3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>29.0</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>29.3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>30.1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>30.4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>31.3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>31.5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>31.8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>17</td>
<td>32.1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>32.4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>19</td>
<td>32.7</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>33.0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>33.2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>33.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>33.8</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>34.4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>34.7</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>26</td>
<td>35.2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>27</td>
<td>35.5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>28</td>
<td>35.8</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>29</td>
<td>36.1</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>30</td>
<td>36.4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

114
than doubles with only a 2.0 ksi increase in applied stress.

Damage initiation is predicted to occur simultaneously in the 0° and 90° plies at an applied far-field stress of 24.0 ksi. The locations of the failed points of consideration are shown in Figure 6.2. The predicted damage initiation consists of two points of consideration failing at the boundary of the hole. The failed point of consideration in the 0° ply is located at 0° around the hole boundary. The failed point of consideration in the 90° ply is located at 90° around the hole boundary. The second equilibrium stage is reached after damage is predicted in the +45° ply at an applied stress of 24.7 ksi. The locations of predicted damage at the second equilibrium stage are shown in Figure 6.3. Again, the damage consists of two points of consideration predicted to fail along the boundary of the hole. The two failed points of consideration are located at +45° around the hole boundary. The second damage event occurs at an applied stress that is only 0.9 ksi higher than the stress at which damage initiation is predicted, thus indicating the occurrence of near simultaneous damage initiation in the 0°, 90°, and +45° plies.

The damage continues to progress throughout the 0°, 90°, and +45° plies as the applied stress increases. Each point of consideration failure is adjacent to existing failed points of consideration. No damage is ever predicted to occur in the -45° ply in this quarter model. The damage tends to propagate in two main directions. The first main direction of propagation is tangentially along the hole boundary. The second main direction of propagation is normal to the hole boundary, with the damage extending towards the plate edges. Typically, the direction of damage propagation alternates between these two main directions at each subsequent equilibrium stage. As an example, consider the fifth and sixth equilibrium stages. The predicted damage at the fifth equilibrium stage is shown in Figure 6.4. At this stage, the damage has propagated along a direction normal to the boundary of the hole in the 0° and 90° plies. This is indicated by noting that the newly failed points of consideration are located in the second row of points of consideration away from the boundary of the hole, while the existing damage was predicted to occur in the row of points of consideration directly adjacent to the boundary of the hole. Then, in the sixth
Figure 6.1  Number of failed points of consideration as a function of applied stress in the deterministic analysis.
Applied Far-Field Stress = 24.0 ksi

Fiber Direction

Damage Mode: Y

0° Ply

90° Ply

+45° Ply

-45° Ply

Damage Prediction

- Newly Predicted Damage
- Existing Damage
- No Damage

Figure 6.2 First equilibrium stage of the deterministic analysis.
Applied Far-Field Stress = 24.7 ksi

Fiber Direction

+0° 0°

Damage Mode: \( Y^T \)

Damage Prediction

- Newly Predicted Damage
- Existing Damage
- No Damage

Figure 6.3 Second equilibrium stage of the deterministic analysis.
equilibrium stage, shown in Figure 6.5, the damage propagates tangentially along the hole in the first row of points of consideration adjacent to the boundary of the hole in the $0^\circ$ and $90^\circ$ plies.

As seen in Table 6.1, the occurrence of damage at each subsequent equilibrium stage often alternates between the $0^\circ$ and $90^\circ$ plies and the $+45^\circ$ ply. This general trend of alternating damage locations continues until the twenty-fifth equilibrium stage. After the twenty-fifth equilibrium stage, all subsequent equilibrium stages include damage in each of the $0^\circ$, $90^\circ$, and $+45^\circ$ plies. Furthermore, from the twenty-fifth equilibrium stage and up to, but not including, the final equilibrium stage, the number of points of consideration predicted to fail is substantially more than in the previous equilibrium stages. The maximum number of points of consideration predicted to fail in a single equilibrium stage is not more than four in all previous stages, but twenty-six points of consideration are predicted to fail at the twenty-fifth equilibrium stage, as shown in Figure 6.6. At this equilibrium stage, the damage propagates simultaneously along both of its two main propagation directions.

Ultimate failure is predicted to occur at an applied stress of 36.4 ksi. This value is more than 50% higher than the stress at which damage initiation is predicted to occur, thus illustrating the extensive residual strength capability of the laminate after initial damage. The predicted damage at the thirtieth and final equilibrium stage is shown in Figure 6.7. In each of the $0^\circ$, $90^\circ$, and $+45^\circ$ plies, the damage has reached the edges of the finer mesh region that are farthest from the hole. For each of these plies, the damage path is mainly oriented along a line normal to the boundary of the hole. There are also a few small branches of damage that extend perpendicular to the main damage paths in each of the three damaged plies. Following the final equilibrium stage, catastrophic failure of the laminate occurs.

6.2 Probabilistic Damage Initiation Results

In this section, the results from the probabilistic damage initiation analysis work
Applied Far-Field Stress = 26.1 ksi

Damage Prediction
- Newly Predicted Damage
- Existing Damage
- No Damage

Figure 6.4 Fifth equilibrium stage of the deterministic analysis.
Applied Far-Field Stress = 27.4 ksi

Fiber Direction

Damage Mode: YT

Damage Prediction
- Newly Predicted Damage
- Existing Damage
- No Damage

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.5  Sixth equilibrium stage of the deterministic analysis.
Applied Far-Field Stress = 34.7 ksi

Fiber Direction

Damage Mode: YT

Damage Prediction

- Newly Predicted Damage
- Existing Damage
- No Damage

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.6  Twenty-fifth equilibrium stage of the deterministic analysis.
Figure 6.7  Thirtieth (and final) equilibrium stage of the deterministic analysis.
are presented. The damage initiation analysis focuses on the undamaged model and indicates the locations and damage modes of possible damage initiation. Spatial presentation is the main method of showing the results. Each ply of the finite element model is displayed at a chosen applied far-field stress for each damage mode that has a predicted probability of failure greater than 10% for at least one point of consideration in the model. Each 10% range of failure probabilities for the points of consideration is represented by a distinct symbol. The failure probabilities are discretized into intervals of 10% in order to balance the increased detail of using smaller probability intervals with the accompanying increase in computational cost. Each point of consideration in the figure is then assigned a symbol based on the interval of probabilities that its probability of failure falls into at the given applied stress. The value for 100% probability of failure is also explicitly indicated. Additionally, the applied stress, the fiber angle direction, the damage mode being presented, and a scale bar are indicated in each figure.

At an applied stress of 20.3 ksi, which is the applied stress at which damage first becomes probabilistically significant, there are two points of consideration that exhibit a probability of failure above 10% via the transverse tension ($Y^T$) damage mode. The transverse tension damage mode is the only mode that is manifested at this applied stress. The location of these points of consideration can be seen in Figure 6.8. They are both located at the boundary of the hole. One point of consideration is in the $0^\circ$ ply at an angle of $0^\circ$ around the hole boundary, and the other point of consideration with a significant probability of failure is in the $90^\circ$ ply at an angle of $90^\circ$ around the hole boundary. The locations and failure modes of these points of consideration correspond with the locations and failure modes of the first failed points of consideration predicted in the deterministic progressive failure analysis.

At an applied stress of 29.3 ksi, the first points of consideration reach 100% probability of failure via the transverse tension ($Y^T$) damage mode. Any increase in applied stress would require these points of consideration to fail based on the approach details as described in Chapter 4, so the damage initiation analysis cannot be taken any further. The transverse tension damage mode is the only mode that is manifested
Applied Far-Field Stress = 20.3 ksi

<table>
<thead>
<tr>
<th>Fiber Direction</th>
<th>Damage Mode: $Y^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0°</td>
<td>0°</td>
</tr>
<tr>
<td>0.1 inch</td>
<td></td>
</tr>
</tbody>
</table>

Probability of Failure

- 100%
- 90 ≤ $x$ < 100%
- 80 ≤ $x$ < 90%
- 70 ≤ $x$ < 80%
- 60 ≤ $x$ < 70%
- 50 ≤ $x$ < 60%
- 40 ≤ $x$ < 50%
- 30 ≤ $x$ < 40%
- 20 ≤ $x$ < 30%
- 10 ≤ $x$ < 20%
- 0 ≤ $x$ < 10%

Figure 6.8 Failure probabilities in the undamaged quasi-three-dimensional model at an applied stress of 20.3 ksi.
at this applied stress. There are a total of sixty-eight points of consideration with a significant probability of failure via the transverse tension damage mode at this applied stress.

The first points of consideration to reach 100% probability of failure are the same two points of consideration that first exhibited greater than 10% probability of failure. All points of consideration with a significant probability of failure at this applied stress of 29.3 ksi are shown in Figure 6.9. Thirty-two of the points of consideration are located in the +45° ply and thirty-six of the points of consideration are located in the 0° and 90° plies. The points of consideration are located in three distinct groups. The first group is located in the 0° ply adjacent to the boundary of the hole at 0° around the hole boundary. The second group is located in the 90° ply adjacent to the boundary of the hole at 90° around the hole boundary. Finally, the third group is located in the +45° ply adjacent to the boundary of the hole at 45° around the hole boundary.

The 0° and 90° plies each contain at least one point of consideration in every failure probability interval. Similarly, the +45° ply contains points of consideration that span the range of failure probability intervals. The maximum failure probability in the +45° ply is greater than 90%. Several points of consideration in the +45° ply have a failure probability greater than 90%, and all of these points of consideration are adjacent to the boundary of the hole near a location that is 45° around the hole boundary. In all of the plies with the potential for damage initiation, the failure probabilities decrease with increasing distance away from the boundary of the hole and increasing tangential distance along the hole boundary. In general, the failure probabilities decrease more quickly along the direction normal to the boundary of the hole as opposed to the tangential direction along the boundary of the hole.

As a means to check the choice of 10% as the value required for the failure probability to be considered significant, which is referred to hereafter as the "significance level", the number of points of consideration in the undamaged model with significant failure probability at an applied stress of 29.3 ksi are found for significance levels of 10%, 5%, and 1%. Reducing the significance level from 10% to 5% increases the
Figure 6.9  Failure probabilities in the undamaged quasi-three-dimensional model at an applied stress of 29.3 ksi.
number of points of consideration with a significant probability of failure from 68 to 90. This adds only one additional layer of points of consideration with a significant probability of failure around the three main groups, so the number of possible damage initiation locations does not increase much. However, reducing the significance level to 1% increases the number of points of consideration with a significant failure probability to 206. The possible damage initiation locations cover most of the finer mesh region, but a significance level of 1% represents nearly three standard deviations from the mean. Therefore, it was determined that using a significance level of 5% would not significantly affect the results or provide much additional insight compared to a significance level of 10%, and that use of a 1% significance level would increase the scope beyond that desired for the current first order analysis.

6.3 Probabilistic Damage Propagation Results

The results of the probabilistic damage propagation analysis work are presented in this section. There are numerous possible damage progression sequences that can be investigated using the framework outlined in the present work. An exhaustive investigation of all possible damage scenarios would be an extremely time consuming task that is well beyond the scope of the present work. Therefore, only a few illustrative cases are investigated in order to ensure the tractability of the work while still providing sufficient information to achieve the objectives as described in Chapter 3. Furthermore, the present work only serves as a first-order investigation into the probabilistic aspects of damage, so none of the cases investigated herein are taken to final failure. However, the results obtained from these carefully selected and limited propagation cases are still able to provide insight into the probabilistic behavior of damage in composites.

The results are presented using two primary methods. The first method is identical to the method used in Section 6.2, in which the failure probabilities for each point of consideration are displayed using 10% intervals of probability. Each 10% interval is denoted by a distinct symbol. Furthermore, there is also a distinct symbol for
100% probability of failure. Since this section is focused on damage propagation, there is a need to indicate that certain points of consideration have failed. This is accomplished by removing any failed points of consideration from the ply figure. It should be noted that these points of consideration are not actually removed, since they are still present in the finite element model. These points of consideration are not shown simply to indicate that their properties have been fully degraded according to the material property degradation model as described in Chapter 4. Each failure of a point of consideration is referred to herein as a “damage event”.

The second method for presenting results is similar to the first method and focuses on spatial presentation. However, the second method indicates the change in failure probability of each point of consideration following some damage event. The change in failure probability is determined by subtracting the failure probability value of the point of consideration before the damage event from the failure probability value after the damage event. Changes in magnitude less than 1% are not indicated. This gives a resulting value expressed in terms of change in (percent) probability of failure. This indicates whether the failure probability of the point of consideration increases (\(|\text{positive value}| \geq 1\%)\), decreases (\(|\text{negative value}| \geq -1\%)\), or stays the same (\(|\text{value} - 1\%)\) as a result of the occurrence of that particular damage event. Such a method provides direct information about the effects of damage in a probabilistic framework. In the presentation of the results, both the magnitude and direction (increase or decrease) of the change in failure probability are indicated by using distinct symbols for each probability interval and each direction of change. The changes in failure probability are generally found to be less than 10% in the cases considered in the present work. Therefore, the intervals indicating the changes in failure probability are no longer discretized into equal values of 10%. Instead, the intervals begin smaller and gradually increase. The smallest interval represents changes greater than or equal to 1% and less than 5%, while the largest interval includes any change in failure probability with a magnitude greater than or equal to 50% and less than 100%. The change in failure probability cannot be greater than 100%.

There are four distinct damage propagation cases investigated in the present work.
These four cases were chosen in order to explore distinctly different damage initiation and propagation scenarios. Three of the cases are simulated to occur at the same applied stress but follow different damage paths. Two of these three cases have damage in the same ply but follow different paths. The third of these cases has damage simulated in a different ply than the other two. The fourth case follows a path identical to the first case, but damage is simulated to progress at a different value of applied far-field stress. This allows comparison between the results of different damage progression scenarios in order to identify the effects of different damage paths and different applied stress stress values at which a given path can occur. For all four cases, damage is initiated by failing the point of consideration with the highest or nearly highest probability of failure, and subsequent damage builds off of the initial damage in a compatible mode.

The first damage propagation case considered is a single line of transverse tensile damage that initiates at the boundary of the hole in the 90° ply at 90° around the hole boundary and then propagates along the ply midline parallel to the 90° fiber direction. The entire propagation sequence is simulated to occur at an applied stress of 29.3 ksi. This scenario is chosen for two main reasons. First, it allows for exploration of a possible damage progression sequence. The first point of consideration chosen to fail has a failure probability between 90% and 100%, and all subsequent points of consideration that are chosen to fail have failure probabilities greater than 10%. Second, it provides the ability to separate the effects of damage from the effects of increasing the applied stress. The failure probability of a point of consideration is dependent on both the state of damage within the model and the applied stress. Therefore, keeping the applied stress constant allows investigation of the changes in probability of failure solely due to the occurrence of damage. A total of five points of consideration are failed along this path. The sixth point of consideration along the path does not have a significant probability of failure, so the propagation ends after the fifth damage event. A diagram of the propagation path is shown in Figure 6.10. The sequence of damage events is denoted by labeling each point of consideration that fails with the number that corresponds to the damage event at which it fails.
Figure 6.10  Diagram of the damage progression sequence investigated in Case 1.
This propagation path represents a likely possible path, since all of the points of consideration failed during the propagation have a significant probability of failure. Furthermore, all damage events occur via the transverse tension damage mode, so it represents a compatible damage progression sequence in the sense that the same damage mode is propagated. Due to the symmetry noted in Section 6.1, this propagation is identical to the case of propagation along the $0^\circ$ ply midline parallel to the $0^\circ$ fiber direction. The results from Case 1 are presented in Section 6.3.1.

The second case explores a single line of transverse tensile damage propagation that initially follows a path identical to Case 1 for the first four damage events, but then turns to follow a path perpendicular to the initial crack growth direction. A diagram of the damage progression sequence is given in Figure 6.11. This case was chosen because it was found from the Case 1 results that the perpendicular propagation path increases in likelihood following the fourth damage event. This propagation is simulated to occur at a constant applied stress value of 29.3 ksi for the same reasons as was done in Case 1. There are two points of consideration along this perpendicular propagation path that have a significant probability of failure via the transverse tension damage mode. These two points of consideration are failed, after which there are no further points of consideration along the crack growth direction that have a significant probability of failure. Therefore, the propagation analysis ends after failure of the sixth point of consideration. The results from Case 2 are presented in Section 6.3.2.

In the third case, shown in Figure 6.12, damage propagation is simulated to occur via the transverse tensile damage mode in the $+45^\circ$ ply in order to explore possible damage progression in a different ply. All damage events are simulated to occur at a constant applied far-field stress of 29.3 ksi in order to provide a direct comparison with the results of Case 1 and Case 2. The damage is simulated to initiate in a point of consideration at the boundary of the hole at a location of approximately $45^\circ$ around the hole. The damage then propagates along a path that is parallel to the $+45^\circ$ fiber direction. This represents a likely damage path, since the initial point of consideration to fail has a failure probability in the 90% to 100% interval, and all
Figure 6.11  Diagram of the damage progression sequence investigated in Case 2.
Figure 6.12  Diagram of the damage progression sequence investigated in Case 3.
subsequent failures occur for points of consideration with a significant probability of failure. There are four possible damage events under these conditions. The results from Case 3 are presented in Section 6.3.3.

For the fourth and final case, the effect of allowing the points of consideration to fail at lower failure probabilities is investigated. This essentially considers a particular manifestation of the laminate that has a lower transverse tensile strength than in the previous cases. Damage propagation is simulated along the 90° ply midline in the same sequence as Case 1, except that all damage events are simulated to occur at a constant applied far-field stress of 27.5 ksi. Thus, all points of consideration in the model have a lower failure probability than in Case 1. This allows investigation of the effects of allowing the damage propagation to occur with lower failure probabilities. Since the points of consideration fail at lower failure probabilities in this particular manifestation, the strength of these points of consideration can thus be considered to be lower than in Case 1. The results from Case 4 are presented in Section 6.3.4.

6.3.1 Case 1: Straight Propagation Along the 90° Ply Midline

In this section, the results from the first damage propagation case are presented. There are a total of five damage events that are possible along the Case 1 propagation path. Once the fifth point of consideration along the propagation path fails, the probability of failure of the next point of consideration along the path is below 10%. Thus, it is considered to not have a significant probability of failure, so the propagation analysis ends at this stage. The number of points of consideration in the model with a significant probability of failure via each damage mode is presented in Table 6.2 following each of the five damage events. Only the transverse tension damage mode is manifested throughout the entire propagation process. The number of points of consideration slowly decreases over the course of the damage propagation. In the undamaged model, there were sixty-eight points of consideration with a significant probability of failure. Following all five damage events, the number of points of
consideration with significant probability of failure decreases to sixty-one.

The number of points of consideration that exhibit a significant change in probability of failure via each damage mode is presented in Table 6.3 for each damage event. All points of consideration that exhibit a significant change in probability of failure do so solely via the transverse tension damage mode. Following the first damage event, four points of consideration exhibit a significant increase in probability of failure and ten points of consideration exhibit a significant decrease in probability of failure. As the damage progresses along the next two damage events, the numbers of points of consideration that exhibit a significant change, either an increase or decrease in failure probability, decreases. However, following the fourth and fifth damage events, the number of points of consideration that exhibit a significant change, either an increase or a decrease in failure probability, begins increasing. Following the fifth damage event, five points of consideration experience a significant increase in failure probability and eight points of consideration exhibit a significant decrease in probability of failure.

Spatial presentations of the failure probabilities of the points of consideration are shown in Figures 6.13 through 6.17 for each of the five damage events. The failure probabilities of each point of consideration are indicated for each stage of the damage propagation. The first damage event involves failure of the point of consideration adjacent to the boundary of the hole. This point of consideration has a 100% probability of failure, so it is a very likely location for damage initiation. The second point of consideration to fail has a failure probability between 80% and 90% following the first damage event. The third point of consideration to fail has a failure probability between 50% and 60%, and the fourth point of consideration to fail has a probability of failure between 20% and 30%. Following the fourth damage event, there are two additional points of consideration along the propagation path that have a significant probability of failure. However, following failure of the fifth point of consideration at a probability of failure between 10% and 20%, the next point of consideration along the propagation path no longer has a significant probability of failure. Therefore, as noted previously, the damage propagation analysis ceases at
Table 6.2  Number of points of consideration with a significant probability of failure following each damage event of Case 1 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.3  Number of points of consideration with a significant change in probability of failure following each damage event of Case 1 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Change in Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
The changes in failure probabilities following each of the five damage events are shown in Figures 6.18 through 6.22. Since all of the points of consideration only experience significant changes via the transverse tension damage mode, only one figure per damage event is needed to show all of the relevant changes in failure probabilities. For all of the figures, any point of consideration with a change in failure probability that is less than 1% in magnitude is indicated by leaving the point of consideration unmarked.

Following the first two damage events, shown in Figures 6.18 and 6.19 respectively, the 90° and +45° plies both exhibit significant changes in failure probability. In both of these cases, the likelihood of continued propagation along the 90° ply midline increases, while propagation perpendicular to the crack growth direction at the crack tip decreases in likelihood. There are also points of consideration in the +45° ply near the same in-plane location as the crack tip that experience a significant change in failure probability. Following the third damage event, shown in Figure 6.20, no points of consideration in the 90° ply exhibit a significant change in failure probability. However, the point of consideration in the +45° ply at the same in-plane location as the crack tip experiences an increase of failure probability. Following the fourth damage event, the likelihood of propagation along the 90° ply midline decreases, while the likelihood of propagation perpendicular to the growth direction at the crack tip increases. Again, the point of consideration in the +45° ply at the same in-plane location as the crack tip experiences an increase of failure probability. These trends observed following the fourth damage event also hold after the fifth point of consideration fails. Additionally, the point of configuration in the -45° ply at the same in-plane location as the crack tip also experiences an increase in failure probability.

6.3.2 Case 2: Turn Propagation Along the 90° Ply Midline

In this section, the results from the second probabilistic damage propagation case are presented. As seen in Case 1, there is an increase in the likelihood of damage
Applied Far-Field Stress = 29.3 ksi

<table>
<thead>
<tr>
<th>Fiber Direction</th>
<th>Damage Mode: YT</th>
<th>Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0.1 inch

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.13  Failure probabilities of the points of consideration following the first damage event of Case 1 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Fiber Direction

+0°  0°

0.1 inch

Damage Mode: YT

Probability of Failure

- 100%
- 90 ≤ x < 100%
- 80 ≤ x < 90%
- 70 ≤ x < 80%
- 60 ≤ x < 70%
- 50 ≤ x < 60%
- 40 ≤ x < 50%
- 30 ≤ x < 40%
- 20 ≤ x < 30%
- 10 ≤ x < 20%
- 0 ≤ x < 10%

Figure 6.14 Failure probabilities of the points of consideration following the second damage event of Case 1 at an applied stress of 29.3 ksi.
Figure 6.15  Failure probabilities of the points of consideration following the third damage event of Case 1 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Probability of Failure

Fiber Direction

Damage Mode: $Y^T$

![Diagram showing failure probabilities for different fiber directions](image)

Figure 6.16 Failure probabilities of the points of consideration following the fourth damage event of Case 1 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Probability of Failure

- 100%
- 90 ≤ x < 100%
- 80 ≤ x < 90%
- 70 ≤ x < 80%
- 60 ≤ x < 70%
- 50 ≤ x < 60%
- 40 ≤ x < 50%
- 30 ≤ x < 40%
- 20 ≤ x < 30%
- 10 ≤ x < 20%
- 0 ≤ x < 10%

Damage Mode: \( Y^T \)

Figure 6.17 Failure probabilities of the points of consideration following the fifth damage event of Case 1 at an applied stress of 29.3 ksi.
Figure 6.18 Changes in failure probabilities following damage event 1 of Case 1.
Change in Probability of Failure

Applied Far-Field Stress = 29.3 ksi

Damage Mode: YT

Fiber Direction

+0°  0°

0.1 inch

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.19  Changes in failure probabilities following damage event 2 of Case 1.
Applied Far-Field Stress = 29.3 ksi

Fiber Direction

Damage Mode: YT

0.1 inch

<table>
<thead>
<tr>
<th>Change in Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase (%)</td>
</tr>
<tr>
<td>□ 50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>□ 40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>□ 30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>□ 20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>□ 10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>□ 5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>□ 1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Figure 6.20 Changes in failure probabilities following damage event 3 of Case 1.
Applied Far-Field Stress = 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Mode: ( Y^T )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0° Ply</th>
<th>90° Ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>+45° Ply</td>
<td>-45° Ply</td>
</tr>
</tbody>
</table>

### Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Figure 6.21 Changes in failure probabilities following damage event 4 of Case 1.
Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 \leq x &lt; 100$</td>
<td>$50 \leq x &lt; 100$</td>
</tr>
<tr>
<td>$40 \leq x &lt; 50$</td>
<td>$40 \leq x &lt; 50$</td>
</tr>
<tr>
<td>$30 \leq x &lt; 40$</td>
<td>$30 \leq x &lt; 40$</td>
</tr>
<tr>
<td>$20 \leq x &lt; 30$</td>
<td>$20 \leq x &lt; 30$</td>
</tr>
<tr>
<td>$10 \leq x &lt; 20$</td>
<td>$10 \leq x &lt; 20$</td>
</tr>
<tr>
<td>$5 \leq x &lt; 10$</td>
<td>$5 \leq x &lt; 10$</td>
</tr>
<tr>
<td>$1 \leq x &lt; 5$</td>
<td>$1 \leq x &lt; 5$</td>
</tr>
</tbody>
</table>

Fiber Direction

Damage Mode: $Y_T$

Applied Far-Field Stress = 29.3 ksi

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.22 Changes in failure probabilities following damage event 5 of Case 1.
propagation perpendicular to the initial crack growth direction following the fourth damage event. From Figure 6.16, it is seen that there are two points of consideration along this perpendicular propagation path that have a significant probability of failure via the transverse tension damage mode. These two points of consideration are failed, after which there are no further elements along the crack growth direction that have a significant probability. Therefore, the propagation analysis for Case 2 ends after the failure of the sixth point of consideration.

The numbers of points of consideration with a significant probability of failure are presented in Table 6.4 for each of the six damage events. Only the transverse tension ($Y^T$) damage mode has a significant probability of occurring for all of the damage events. The number of points of consideration with a significant probability of failure decreases by one point of consideration after each damage event except for the final damage event, after which the number of points of consideration with significant failure probability does not change.

The numbers of points of consideration that exhibit a significant change in probability of failure following each damage event are presented in Table 6.5. For the first four damage events, the results are identical to those from the first case, given in Table 6.3. In Case 1, the number of points of consideration that exhibit a significant change in failure probability, either an increase or a decrease, increases after the third damage event. This trend also holds in Case 2. Following the fifth damage event, there are seven points of consideration that experience a significant increase in failure probability and five points of consideration that experience a significant decrease in failure probability. After the sixth damage event, the number of points of consideration that exhibit a significant increase in failure probability increases to ten, and the number of points of consideration that exhibit a significant decrease also increases to ten.

The failure probabilities following the fifth and sixth damage events are shown in Figures 6.23 and 6.24, respectively. The first four damage events in this propagation are identical to Case 1, so spatial results of the failure probabilities for those damage events are not repeated here. In Figure 6.23, it can be seen that the point
Table 6.4  Number of points of consideration with a significant probability of failure following each damage event of Case 2 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.5  Number of points of consideration with a significant change in probability of failure following each damage event of Case 2 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Change in Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>
of consideration in the $+45^\circ$ ply at the same in-plane location as the crack tip now has a significant probability of failure. This is also seen following the sixth damage event, as shown in Figure 6.24, in which the point of consideration in the $+45^\circ$ ply at the same in-plane location as the crack tip exhibits a failure probability between 30% and 40%. Additionally, two points of consideration in the $+45^\circ$ ply that are in the same in-plane locations as the two previously failed points of consideration now have a significant probability of failure as well.

The changes in failure probabilities following the fifth and sixth damage events are shown in Figures 6.25 and 6.26, respectively. In Figure 6.25, it is seen that continued propagation along the perpendicular crack growth direction in the $90^\circ$ ply increases in likelihood following the fifth damage event, while the probability of propagation along the ply midline decreases. The failure probabilities of multiple points of consideration in the $+45^\circ$ ply near the in-plane location of the crack experience increases, with the failure probability of the point of consideration in the same in-plane location as the crack tip increasing by 10% to 20%. Furthermore, there is a point of consideration in the $-45^\circ$ ply in the same in-plane location as the fourth failed point of consideration that also exhibits an increase in failure probability.

Following the sixth damage event, the likelihood of continued propagation along the perpendicular crack growth direction increases. There are several points of consideration in the $+45^\circ$ ply that also experience a significant increase in failure probability. Additionally, the point of consideration in the $+45^\circ$ ply in the same in-plane location as the crack tip exhibits an increase in failure probability of 20% to 30%. This is a higher change than that observed in Case 1.

### 6.3.3 Case 3: Straight Propagation in the $+45^\circ$ Ply

In this section, the results from the third damage propagation case are presented. The numbers of points of consideration that have a significant probability of failure following each damage event are shown in Table 6.6. Similar to all previous cases, the transverse tension ($Y_T$) damage mode is the only mode with a significant probability
Applied Far-Field Stress = 29.3 ksi

Probabilty of Failure

Fiber Direction
+0° 0°

Damage Mode: YT

0.1 inch

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.23 Failure probabilities of the points of consideration following the fifth damage event of Case 2 at an applied stress of 29.3 ksi.
Figure 6.24 Failure probabilities of the points of consideration following the sixth damage event of Case 2 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Fiber Direction

Damage Mode: Y

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.25 Changes in failure probabilities following damage event 5 of Case 2.
Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Applied Far-Field Stress = 29.3 ksi

Damage Mode: YT

Figure 6.26  Changes in failure probabilities following damage event 6 of Case 2.
of occurrence. Following the first damage event, there are sixty-seven points of consideration with a significant probability of failure, and this number decreases by one or two points of consideration after each subsequent damage event, finally reaching a value of sixty-three following the fourth damage event.

In Table 6.7, the numbers of points of consideration that exhibit a significant change in failure probability following the four damage events are given. Similar to all previous cases, the transverse tension \((Y^T)\) damage mode is the only mode that experiences a significant change in probability of occurrence. The number of points of consideration that experience a significant change in failure probability decrease following the second and third damage events. However, this number increases from one point of consideration that exhibit a significant increase in failure probability to two points of consideration following the fourth damage event, and the number of points of consideration that exhibit a significant decrease in failure probability also increases from two following the fourth damage event to eight following the fifth damage event. A similar trend is observed in all previous cases as well.

The failure probability results are presented spatially for the four damage events in Figures 6.27 through 6.30. The failure probabilities for the undamaged model are displayed in Figure 6.9, so the results are not repeated here. After the fourth damage event, the next point of consideration along the crack growth direction does not have a significant probability of failure, as shown in Figure 6.30. Thus, the damage propagation ends after the fourth damage event.

The changes in failure probabilities are shown in Figures 6.31 through 6.34. Following the first damage event, the likelihood of continued propagation along the crack growth direction increases between 5% to 10%, as shown in Figure 6.31. There is also an increase in the likelihood of propagation along two paths that branch out perpendicular from the crack growth direction. Furthermore, the likelihood of propagation tangentially along the hole boundary decreases. After the second damage event, shown in Figure 6.32, the likelihood of continued propagation along the crack growth direction increases, while the likelihood of propagation perpendicular to the crack growth direction decreases. Furthermore, the point of consideration in the 0° ply at
Table 6.6  Number of points of consideration with a significant probability of failure following each damage event of Case 3 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>$X^T$</th>
<th>$X^C$</th>
<th>$Y^T$</th>
<th>$Y^C$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>67</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>66</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>65</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>63</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.7  Number of points of consideration with a significant change in probability of failure following each damage event of Case 3 at an applied stress of 29.3 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Change in Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td></td>
<td>$+$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 6.27  Failure probabilities of the points of consideration following the first damage event of Case 3 at an applied stress of 29.3 ksi.
Figure 6.28  Failure probabilities of the points of consideration following the second damage event of Case 3 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Probability of Failure

Fiber Direction  
+0°  0°  0.1 inch

Damage Mode: YT

- 100%
- 90 ≤ x < 100%
- 80 ≤ x < 90%
- 70 ≤ x < 80%
- 60 ≤ x < 70%

- 50 ≤ x < 60%
- 40 ≤ x < 50%
- 30 ≤ x < 40%
- 20 ≤ x < 30%
- 10 ≤ x < 20%
- 0 ≤ x < 10%

Figure 6.29  Failure probabilities of the points of consideration following the third damage event of Case 3 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Probability of Failure

Fiber Direction

Damage Mode: Y^T

- 0°
- +θ°

0.1 inch

- 100%
- 90 ≤ x < 100%
- 80 ≤ x < 90%
- 70 ≤ x < 80%
- 60 ≤ x < 70%
- 50 ≤ x < 60%
- 40 ≤ x < 50%
- 30 ≤ x < 40%
- 20 ≤ x < 30%
- 10 ≤ x < 20%
- 0 ≤ x < 10%

Figure 6.30  Failure probabilities of the points of consideration following the fourth damage event of Case 3 at an applied stress of 29.3 ksi.
Applied Far-Field Stress = 29.3 ksi

Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Damage Mode: YT

Figure 6.31  Changes in failure probabilities following damage event 1 of Case 3.
Applied Far-Field Stress = 29.3 ksi

Fiber Direction

+0°  0°

0.1 inch

Damage Mode: YT

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Figure 6.32  Changes in failure probabilities following damage event 2 of Case 3.
Applied Far-Field Stress = 29.3 ksi

Fiber Direction

Damage Mode: YT

0.1 inch

Change in Probability of Failure

Increase (%) Decrease (%)

- $50 \leq x < 100$
- $40 \leq x < 50$
- $30 \leq x < 40$
- $20 \leq x < 30$
- $10 \leq x < 20$
- $5 \leq x < 10$
- $1 \leq x < 5$

50 40 30 20 10 5 1

Figure 6.33 Changes in failure probabilities following damage event 3 of Case 3.
Applied Far-Field Stress = 29.3 ksi

Damage Mode: YT

Figure 6.34 Changes in failure probabilities following damage event 4 of Case 3.
the same in-plane location as the crack tip has an increase in probability of failure. Following the third damage event, the likelihood of continued propagation along the crack growth direction does not exhibit a significant change. However, the point of consideration in the $0^\circ$ ply at the same in-plane location as the crack tip exhibits a significant increase in failure probability. Finally, after the fourth damage event, the likelihood of continued propagation along the crack growth direction does not exhibit a significant change, but the point of consideration in the $0^\circ$ ply at the same in-plane location as the crack tip exhibits a significant increase in failure probability.

6.3.4 Case 4: Straight Propagation Along the $90^\circ$ Ply Midline at a Lower Applied Far-Field Stress

In this section, the results from the fourth damage propagation case are presented. The failure probabilities are displayed for the undamaged model in Figure 6.35. The overall distribution of points of consideration with a significant probability of failure is similar to Case 1, except that there are fewer points of consideration and they all have lower failure probabilities. This is due to the fact that the damage propagation analysis is performed at a lower applied far-field stress value than in Case 1. The points of consideration with the highest failure probability are in the interval of 80% to 90%. There are four points of consideration with a significant probability of failure along the $90^\circ$ ply midline. This holds throughout the damage propagation process, so there are a total of four possible damage events.

The numbers of points of consideration with a significant probability of failure following each damage event are presented in Table 6.8. It can be seen that the number of points of consideration with a significant probability of failure is less than in Case 1. This is due to the fact that the lower applied stress results in lower stress values within each point of consideration. The numbers of points of consideration that exhibit a significant change in failure probability following each damage event are presented in Table 6.9. After each of the first three damage events, the number of points of consideration that exhibit either a significant increase or decrease in failure
Figure 6.35  Failure probabilities of the points of consideration in the undamaged model at an applied stress of 27.5 ksi.
probability decreases. However, after the fourth damage event, the number of points of consideration that experience either a significant increase or decrease in failure probability increases. This same trend is observed in all three of the other cases.

The failure probabilities of the points of consideration following the four damage events are shown spatially in Figures 6.36 through 6.39. The four points of consideration fail at probabilities of 80% to 90% for the first damage event, 60% to 70% for the second damage event, 30% to 40% for the third damage event, and 10% to 20% for the fourth damage event. Following the fourth damage event, shown in Figure 6.39, the next point of consideration allowing the crack growth direction does not have a significant probability of failure, so the propagation ceases.

The changes in failure probabilities of the points of consideration following the four damage events are displayed in Figures 6.40 through 6.43. There are two main items to note here. The first item is that following the fourth damage event, the likelihood of propagation from the crack tip along a path perpendicular to the crack growth direction does not increase. This is distinctly different from the results of Case 1, in which the likelihood of a perpendicular propagation path increased following the fourth damage event, as shown in Figure 6.21. The second main item is that following the fourth damage event, there is an increase in the probability of failure of the point of consideration in the +45° ply at the same in-plane location as the crack tip. This is the same as in Case 1.

6.4 Discussion

In this section, a discussion of the quasi-three-dimensional analysis results is presented. The main goal of the present work is to assess the effects and importance of incorporating probabilistic aspects of damage into a progressive failure analysis. Therefore, this goal is addressed in the present section using the results obtained from the deterministic and probabilistic analysis cases, as presented in the previous sections of this chapter. The results confirm the expectation that consideration
Table 6.8  Number of points of consideration with a significant probability of failure following each damage event of Case 4 at an applied stress of 27.5 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X^T$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.9  Number of points of consideration with a significant change in probability of failure following each damage event of Case 4 at an applied stress of 27.5 ksi

<table>
<thead>
<tr>
<th>Damage Event</th>
<th>Number of Points of Consideration with a Significant Change in Probability of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X^T )  ( X^C ) ( Y^T ) ( Y^C ) ( S )</td>
</tr>
<tr>
<td>1</td>
<td>0 0 2 8 0 0</td>
</tr>
<tr>
<td>2</td>
<td>0 0 2 2 0 0</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>4</td>
<td>0 0 1 2 0 0</td>
</tr>
</tbody>
</table>
Figure 6.36  Failure probabilities of the points of consideration following the first damage event of Case 4 at an applied stress of 27.5 ksi.
Applied Far-Field Stress = 27.5 ksi

Probability of Failure

- 100%
- 90 ≤ x < 100%
- 80 ≤ x < 90%
- 70 ≤ x < 80%
- 60 ≤ x < 70%
- 50 ≤ x < 60%
- 40 ≤ x < 50%
- 30 ≤ x < 40%
- 20 ≤ x < 30%
- 10 ≤ x < 20%
- 0 ≤ x < 10%

Figure 6.37 Failure probabilities of the points of consideration following the second damage event of Case 4 at an applied stress of 27.5 ksi.
Applied Far-Field Stress = 27.5 ksi

Probability of Failure

<table>
<thead>
<tr>
<th>Damage Mode: $Y^T$</th>
<th>100%</th>
<th>90 ≤ x &lt; 100%</th>
<th>80 ≤ x &lt; 90%</th>
<th>70 ≤ x &lt; 80%</th>
<th>60 ≤ x &lt; 70%</th>
<th>50 ≤ x &lt; 60%</th>
<th>40 ≤ x &lt; 50%</th>
<th>30 ≤ x &lt; 40%</th>
<th>20 ≤ x &lt; 30%</th>
<th>10 ≤ x &lt; 20%</th>
<th>0 ≤ x &lt; 10%</th>
</tr>
</thead>
</table>

Fiber Direction

+0° 0° 0.1 inch

0° Ply

90° Ply

+45° Ply

-45° Ply

Figure 6.38 Failure probabilities of the points of consideration following the third damage event of Case 4 at an applied stress of 27.5 ksi.
Figure 6.39  Failure probabilities of the points of consideration following the fourth damage event of Case 4 at an applied stress of 27.5 ksi.
Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Figure 6.40 Changes in failure probabilities following damage event 1 of Case 4.
Applied Far-Field Stress = 27.5 ksi

Fiber Direction

Damage Mode: $Y^T$

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ≤ x &lt; 100</td>
<td>50 ≤ x &lt; 100</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>40 ≤ x &lt; 50</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>30 ≤ x &lt; 40</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>20 ≤ x &lt; 30</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>10 ≤ x &lt; 20</td>
</tr>
<tr>
<td>5 ≤ x &lt; 10</td>
<td>5 ≤ x &lt; 10</td>
</tr>
<tr>
<td>1 ≤ x &lt; 5</td>
<td>1 ≤ x &lt; 5</td>
</tr>
</tbody>
</table>

Figure 6.41  Changes in failure probabilities following damage event 2 of Case 4.
Applied Far-Field Stress = 27.5 ksi

Fiber Direction

Damage Mode: $Y^T$

Change in Probability of Failure

<table>
<thead>
<tr>
<th>Increase (%)</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 \leq x &lt; 100$</td>
<td>$50 \leq x &lt; 100$</td>
</tr>
<tr>
<td>$40 \leq x &lt; 50$</td>
<td>$40 \leq x &lt; 50$</td>
</tr>
<tr>
<td>$30 \leq x &lt; 40$</td>
<td>$30 \leq x &lt; 40$</td>
</tr>
<tr>
<td>$20 \leq x &lt; 30$</td>
<td>$20 \leq x &lt; 30$</td>
</tr>
<tr>
<td>$10 \leq x &lt; 20$</td>
<td>$10 \leq x &lt; 20$</td>
</tr>
<tr>
<td>$5 \leq x &lt; 10$</td>
<td>$5 \leq x &lt; 10$</td>
</tr>
<tr>
<td>$1 \leq x &lt; 5$</td>
<td>$1 \leq x &lt; 5$</td>
</tr>
</tbody>
</table>

Figure 6.42 Changes in failure probabilities following damage event 3 of Case 4.
Figure 6.43  Changes in failure probabilities following damage event 4 of Case 4.
of probabilistic aspects of damage allows for the possibility of many different damage progression scenarios for a single laminate configuration. The single prediction obtained using a deterministic progressive failure analysis is unable to capture the range of possible damage propagation paths and laminate strength values. For example, by not including probabilistic characteristics into the prediction of damage initiation strength, a deterministic analysis may lead to errors as large as 20% for the case investigated. A deterministic progressive failure analysis also does not allow for the possibility of non-symmetric damage progression. The results of the present work show that incorporation of probabilistic aspects of damage allows for the possibility of non-symmetric damage, and that the damage path becomes increasingly likely to be non-symmetric as further damage occurs due to local redistribution of stresses caused by the occurrence of damage. This redistribution of stresses can lead to significant increases and decreases in the failure probabilities of the remaining points of consideration. Furthermore, the results show that the magnitude of change in the probability of failure of a remaining point of consideration following the occurrence of damage depends on the stress level of that remaining point of consideration when the damage occurs. This can be attributed to the fact that the probability of failure of any given point of consideration is a nonlinear function of stress. Finally, the results indicate the importance of considering lengthscale issues in a probabilistic progressive failure analysis, particularly in regards to element size. These conclusions are discussed in more detail throughout the remainder of this section.

A main conclusion that can be drawn from the results of the present work is that the incorporation of probabilistic aspects of damage allows for many possible damage progression scenarios that are not allowed in the deterministic case. In the deterministic analysis, there is only a single damage progression pattern that is possible. All damage is predicted to occur via the transverse tension damage mode in this case, which can be attributed to the low transverse tensile strength of the material and the nature of the ply layup and loading condition. Since the laminate is subject to an equal biaxial loading condition, the 0° and 90° plies of the quasi-isotropic layup configuration experience significant tensile stresses. When coupled with the low
transverse tensile strength, this is likely a major reason that the laminate fails entirely via the transverse tension damage mode. The layup and loading condition also contribute to the observed symmetry about a plane along the $+45^\circ$ fiber direction, as described in Section 6.1. Furthermore, the configuration of the layup and the loading condition also cause the observed similarities in the damage growth rates of the $0^\circ$ and $90^\circ$ plies and the $+45^\circ$ ply, which is apparent from the fact that incremental damage alternates between these plies, as listed in Table 6.1.

By including probabilistic aspects in determining the location and mode of damage initiation, it is seen that there are numerous additional possibilities when compared to the deterministic case. For the particular case investigated in the present work, there are only two points of consideration predicted to initially fail in the deterministic case. When probabilistic aspects are included, there are a total of sixty-eight points of consideration with a significant probability of failure at the maximum applied far-field stress for damage initiation, as shown in Section 6.2. Additionally, in the deterministic case, the initial point of consideration failures are predicted to occur at the boundary of the hole in the $0^\circ$ and $90^\circ$ plies at locations of $0^\circ$ and $90^\circ$ around the hole boundary, respectively. This is due to the fact that the stress concentration is highest at the boundary of the hole at those specific locations in the plies. However, when probabilistic aspects are included, several other points of consideration have failure probabilities in the range of 90% to 100% and are thus also likely to be the first points of consideration to fail. Many possible damage initiation locations are not even on the boundary of the hole.

Furthermore, there is a range of possible applied stress values at which damage initiation can occur when probabilistic aspects are incorporated. In the deterministic analysis, damage initiation is predicted to occur at 24.0 ksi. The predicted stress corresponding to damage initiation in the probabilistic case ranges from a minimum value of 20.3 ksi to a maximum value of 29.3 ksi. The minimum value is about 15% lower than the value predicted by the deterministic case, and the maximum value is about 22% higher. Thus, there may be a significant error between the deterministic analysis results and the actual damage initiation strength depending on the partic-
ular manifestation of the laminate. The range of possible damage locations, modes, and applied stress values presented in Section 6.2 are obtained using a probabilistic significance level of 10%. It is expected that lowering the significance level will serve to increase the number of points of consideration with a significant probability of failure, and thus extend the range of possible damage initiation strength values. This will allow further damage propagation possibilities as well. This highlights the importance in including probabilistic aspects of damage when considering prediction of damage initiation.

The importance of including probabilistic characteristics into a progressive failure analysis is also significant once damage propagation begins. For example, the initiation and propagation of damage in the deterministic case is predicted to occur with nearly equal amounts of damage in the $0^\circ$, $90^\circ$ and $+45^\circ$ plies. However, when probabilistic aspects are included, it is almost equally likely for the damage to initiate and propagate exclusively in either the $90^\circ$ ply (and thus the $0^\circ$ ply due to the observed symmetry) or the $+45^\circ$ ply, as demonstrated in Sections 6.3.1 and 6.3.3, respectively. Additionally, damage propagation within a single ply can follow different paths that each have a similar probability of occurrence. For example, in Case 1 the damage propagates entirely along the $90^\circ$ ply midline, while in Case 2 the damage initially propagates along the $90^\circ$ ply midline but then turns to follow a path perpendicular to the original crack growth direction. Both of these paths have a similar probability of occurring, but are distinctly different.

Another main finding from the present work is that the redistribution of stresses caused by the occurrence of damage can have significant effects on probabilistic failure behavior. For example, the redistribution of stresses due to damage can cause significant increases and decreases in the failure probabilities of the remaining points of consideration. The redistribution of stresses occurs both in the same ply as the damage and also in other plies. For example, in many instances there is an increased likelihood of failure of the points of consideration in other plies at the same in-plane location as the crack tip. The failed point of consideration cannot carry any load, so the load path must avoid the damaged region. If one of the plies at a material location
fails, then the overall stiffness of the element decreases, thus redirecting some of the 
load to the surrounding elements. However, the whole element still carries some load, 
so the load is also partially redirected to other plies in the same in-plane location due 
to the use of homogenized material properties in the finite element model. Since one 
of the points of consideration has failed, that load is distributed among the remain-
ing points of consideration within that element. The 90\degree ply is aligned with one of 
the loading directions, and thus carried the majority of the load in that direction. 
When the 90\degree ply sustained damage, as in the first two cases explored, a significant 
amount of the load needed to be redistributed to the other plies in the same in-plane 
location. This caused some of the remaining points of consideration to experience 
a significant change in probability of failure. This result points to the possibility of 
some significant interlaminar effects, since the load not only redistributes around the 
damaged region in the plane of the ply, but also in other plies. Interlaminar effects 
are therefore considered explicitly in Chapter 7.

The redistribution of stresses due to damage can have a far-reaching and large 
magnitude effect. Failure of a single point of consideration primarily causes localized 
changes, but it can also affect points of consideration that are a significant distance 
away. The failure probabilities of points of consideration directly adjacent to or within 
a few element lengths of the damage location typically experience the majority of 
changes. However, there are also instances in which points of consideration that are 
relatively far from the damage location are affected. For example, in Figure 6.21 it is 
seen that failure of an element on the 90\degree ply midline causes a significant change in 
an element located in the 0\degree ply at 0\degree around the hole boundary. Failure of a single 
point of consideration can also cause large changes in the failure probabilities of other 
points of consideration. For example, the sixth damage event in Case 2 causes one 
point of consideration to experience a 20\% to 30\% change in failure probability. This 
indicates that failure of a single point of consideration can significantly influence the 
subsequent damage progression that may occur.

The redistribution of stresses can also increase the probability of non-symmetric 
damage propagation. As mentioned previously, incorporation of probabilistic char-
acteristics of damage allows for non-symmetric damage propagation. The occurrence of damage in these cases causes non-symmetric redistribution of stresses. This can be observed by noting that the points of consideration exhibiting significant changes in failure probability due to propagation in the 90° ply are very different from the points of consideration that experience a significant change in failure probability due to propagation in the +45° ply. Therefore, the redistribution of stresses increases the likelihood of further non-symmetric damage propagation. It is expected that this will affect all subsequent damage propagation and may lead to very different final failure results.

Stress redistribution can cause significant changes in the failure probability of selected elements, partially due to the fact that the failure probability of a point of consideration is a nonlinear function of the stress state in that point of consideration. The nonlinear nature of the failure probability has important implications in assessing the effects of incorporating probabilistic aspects of damage into a progressive failure analysis. The cumulative distribution function that defines the probability of failure for each damage mode of each point of consideration is a nonlinear function of stress. Near the tails of the distribution, the probability of failure increases very slowly with increasing stress. However, as the probability of failure nears the mean value, the probability of failure can increase rapidly given only a small change in the relevant stress of the point of consideration.

Due to the nonlinear nature of the probability of failure as a function of stress, use of probability of failure provides a normalization that is better able to quantify the effect of damage events than simply using changes in stress as a metric. For example, consider the failure of some point of consideration that causes another point of consideration to experience a 100% increase in stress. If the remaining point of consideration is at a very low stress level with very low probability of failure before the damage occurs, then a 100% increase in stress may not be significant in terms of change in probability of failure. However, if the remaining point of consideration is near its failure stress when the damage occurs, and thus already has a high probability of failure, then a 100% increase in stress level will likely lead to failure of the remaining
point of consideration. Thus, the significance of the change in stress state is a function of both the stress state of the point of consideration before the damage event, and also the amount of change in stress that occurs, as these are related to probability of failure. Probabilistic methods take both of these values into account and provide a normalization of the effects of damage by providing values in terms of likelihood of failure. This enables direct comparison between effects observed for different damage modes as well.

One of the implications of the nonlinear behavior of the failure probability is shown by noting that in all cases investigated, it is seen that as damage initially progresses, the total number of points of consideration that experience a significant change in probability of failure decreases. However, once a certain number of damage events have occurred (typically three in the cases investigated in the present work), the total number of points of consideration with a significant change in failure probability begins monotonically increasing. This may be attributed to the fact that as each point of consideration fails, the remaining points of consideration must carry increasingly more load. As a result, many of the points of consideration experience a change in stress due to the redistribution of stresses caused by damage, as described previously. Some of the points of consideration experience a change in failure probability that bring them closer to their mean value, so further changes in stress in the same direction cause increasingly larger changes in failure probability. Once some critical number of damage events is reached, the increase in stress from subsequent damage events is sufficient to cause significant changes in the probability of failure of the remaining points of consideration.

The nonlinear nature of the failure probability also has a significant implication when considering the particular strength manifestation of a point of consideration. If a particular manifestation of a point of consideration has low failure strength, then the surrounding points of consideration will be at lower stresses when the point of consideration fails. Conversely, a particular manifestation of a point of consideration with high failure strength will result in the surrounding points of consideration being at a higher stress state when the point of consideration fails. Depending on how
near to the mean failure probability value each surrounding point of consideration is when damage occurs, the redistribution of stresses caused by the damage event may cause differing magnitudes of change in failure probability of those surrounding points of consideration. As a result, different damage propagation paths may or may not experience significant changes in their probability of occurrence. This is shown in the results by comparing Case 1 and Case 4. In Case 1, the points of consideration fail at higher stress values, and there is a damage path perpendicular to the crack growth direction that increases in likelihood. However, if the points of consideration fail at lower values, as in Case 4, this path does not increase in likelihood. This shows that the likelihood of a particular damage path occurring depends not only on the mode and location of previous damage, but also on the applied stress values at which the previous damage occurs.

Although not specifically considered herein, these results suggest the importance of considering the probabilistic dependency/independency of strength parameters in the same point of consideration and also in points of consideration at different geometric locations. The present work assumes that the material strength parameters for each point of consideration are independent, uncorrelated random variables. This means that the strength of one point of consideration in a particular manifestation is completely unrelated to the strength of the other points of consideration. However, if a strength parameter in a particular manifestation of the laminate is low in one point of consideration, then it may also be possible that the other strength modes of that point of consideration have a decreased strength as well. Furthermore, it is also possible that the strength parameters of directly adjacent points of consideration may also be low (e.g. a void may extend across multiple points of consideration). Since the strength values of the points of consideration for a particular manifestation can influence subsequent damage propagation, a more detailed analysis may need to take into account possible dependencies of various strength parameters. This will significantly increase the complexity of the analysis, but it may be an important item to include given that the damage progression path is dependent on the particular manifestation of strength values of the points of consideration.
A final item to note is the importance of lengthscale issues in a probabilistic progressive failure analysis. After a certain amount of propagation, for both the $+45^\circ$ and $90^\circ$ ply propagation cases, the likelihood of increased propagation along the crack growth direction drops to zero or begins decreasing. This seems to occur near the location where the stress concentration due to the presence of the hole drops to a low value. Thus, the global stress concentration due to the hole results in the points of consideration far away not having a significant probability, and this outweighs the local stress concentration due to the occurrence of damage. This points to a mesh size dependence issue. When the stresses are averaged over the element size, the stress gradient due to the failed point of consideration has a smaller effect farther from the global high stress gradient field. Furthermore, the lengthscale over which the stresses are averaged to determine the occurrence of damage is too large to capture the steep stress gradient caused by the failed point of consideration.

Mesh dependence has been observed for deterministic progressive failure analysis, as noted in Chapter 2. However, the mesh size dependence issue seems to be more important when probabilistic aspects are taken into account due to the nonlinear nature of the failure probability as a function of stress as well as the possibility for non-symmetric damage to be simulated. In deterministic progressive failure analysis, the predicted strength values will eventually converge and the same overall damage patterns will develop. In contrast, the probabilistic propagation may have different paths depending on the element size. For example, damage may be able to extend farther along a particular path, causing additional non-symmetric redistribution of stresses. Depending on the initial damage location and propagation path, any subsequent damage propagation will be different. For example, consider the straight-line propagation cases in the $90^\circ$ and $+45^\circ$ plies, the results of which are presented in Cases 1 and 3, respectively. In each case, most of the increased likelihood occurs in the same ply and same general region as damage that has already occurred. Therefore, lengthscale issues must be carefully addressed in future work on this topic.

There are many limitations imposed by the assumptions of the current work. One limitation is the bounds set on the probabilities investigated. The use of 10% as the
significance level and the definition of 100% probability of failure to be the maximum recorded experimental strength value are both used in order to limit the scope of the work. However, it is very possible that the material strength parameters may actually be able to exceed the 100% limit set in the present work, and it is also possible that a particular manifestation of the laminate may contain regions of weak material where damage may initiate at a level below the 10% used herein. The possibility of a particular manifestation of the laminate containing regions of weak and strong material also requires consideration of correlated strength parameters. Furthermore, all damage propagation investigations were conducted at a constant applied far-field stress. This reduces the complexity of the analysis and enables the ability to focus on effects caused solely by damage. However, the full probabilistic behavior of failure will require investigation of damage progression that includes increasing applied stresses. One of the most limiting assumptions, however, may be that of symmetric damage within each quadrant of the laminate. It is demonstrated that damage could propagate non-symmetrically within a quadrant, and that this may lead to further non-symmetric damage behavior. Furthermore, it is seen that the likelihood of the various subsequent damage propagation events is highly dependent on the previous damage that has occurred. Therefore, in order to provide a more realistic assessment of the importance of probabilistic aspects of damage, fully non-symmetric damage progression will need to be considered.
Chapter 7

Three-Dimensional Analysis

In this chapter, the results from and discussion concerning the three-dimensional analysis work are presented. As described in Chapter 4, the specific case of a \([\pm 15/0]\), AS4/3501-6 laminated plate containing a centrally located circular hole subject to a unidirectional tensile load is the focus of the three-dimensional analysis. Observed symmetries in the model stress fields, which were used to simplify the analysis work, are presented in Section 7.1. Considerations for the presentation of the probabilistic damage results are described in Section 7.2. The probabilistic damage initiation results are given in Section 7.3. The probabilistic damage propagation results for various propagation scenarios are presented in Section 7.4. Finally, a discussion of the results is given in Section 7.5.

7.1 Symmetries in the Stress Fields

Since the present work is a first-order investigation into the probabilistic aspects of damage, available symmetries in the stress fields of the starting undamaged configuration are used in order to simplify the analysis work. This is also used in propagating damage such that symmetry of damage follows symmetries in stresses. This is specifically described in Section 7.3. As mentioned in Section 5.3, there is an assumed
plane of symmetry about the laminate midplane. Thus, only the top three plies of the laminate are modeled.

The in-plane stress fields in the model exhibit “2-fold rotational symmetry” in the $x_1$-$x_2$-plane about the $x_3$-axis. A figure is said to have “n-fold rotational symmetry” about an axis of rotation if it is symmetrical about a rotation of $360^\circ/n$ about that axis [138]. This means that stress results are the same for locations in the $x_1$-$x_2$-plane at the same radial distance from the hole center in two angular locations: $\phi$ and $(\phi + 180^\circ)$. Thus, stress fields can be rotated by $180^\circ$ and will appear the same. This is illustrated by considering isotress contours of in-plane longitudinal stress, $\sigma_{11}$, normalized by the far-field applied stress in each ply, as shown in Figures 7.1 through 7.3. These stress contours were generated by plotting the centroidal stress value in each element. This value represents the average stress within an element and is used since these are the values used in assessing the occurrence of damage. It can be seen that rotating the stress field by $180^\circ$ within the $x_1$-$x_2$-plane provides an identical result. All other in-plane stress components display the same 2-fold rotational symmetry as well.

The interlaminar normal stresses, $\sigma_{33}$, also exhibit 2-fold rotational symmetry about the center of the hole in the $x_1$-$x_2$-plane. This is shown by plotting isostress contours of interlaminar normal stresses, $\sigma_{33}$, normalized by the far-field applied stress for each ply in Figures 7.4 through 7.6. The stress contours were generated by plotting the nodal stresses at each ply interface. These values are used in the calculation of the averaged stress value employed to assess the occurrence of damage. The interlaminar shear stresses do not directly exhibit 2-fold rotational symmetry about the center of the hole in the $x_1$-$x_2$-plane. Instead, it is found that rotating the interlaminar shear stress fields by $180^\circ$ gives identical stress contour shapes, but that the sign of the stresses are reversed. This is shown by plotting the interlaminar longitudinal shear (1-3 plane) stress, $\sigma_{13}$, normalized by the far-field applied stress for each ply in Figures 7.7 through 7.9. The stress contours were generated by plotting the nodal stresses at each ply interface. The current work utilizes the absolute value of the interlaminar stresses in assessing the occurrence of damage. Thus, in terms of the
Figure 7.1 Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_o$, in the $+15^\circ$ ply.
Figure 7.2 Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_o$, in the -15° ply.
Figure 7.3  Isostress contours for in-plane longitudinal stress, $\sigma_{11}$, normalized by far-field applied stress, $\sigma_o$, in the $0^\circ$ ply.
Figure 7.4  Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_o$, at the $+15^\circ/-15^\circ$ ply interface.
Figure 7.5  Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_o$, at the -15°/0° ply interface.
Figure 7.6  Isostress contours for interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_o$, at the $0^\circ/0^\circ$ ply interface.
Figure 7.7 Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_o$, at the $+15^\circ/-15^\circ$ ply interface.
Figure 7.8  Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_o$, at the $-15^\circ/0^\circ$ ply interface.
Figure 7.9  Isostress contours for interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_o$, at the $0^\circ/0^\circ$ ply interface.
damage analysis employed in the current work, the absolute value of the interlaminar shear stresses also exhibit 2-fold rotational symmetry about the center of the hole in the $x_1$-$x_2$-plane.

In order to simplify the analysis work, all damage is simulated so as to also exhibit the 2-fold rotational symmetry. Thus, the damage results are the same for locations in the $x_1$-$x_2$-plane at the same radial distance from the hole center in two angular locations: $\phi$ and $(\phi + 180^\circ)$. Introducing the same symmetry of damage in such a symmetric stress field yields the same symmetry of stresses following the occurrence of damage. This eliminates the complexity of having to consider non-symmetric damage, and it also simplifies the presentation of the results, as described in Section 7.2.

7.2 Considerations for Presentation of Probabilistic Damage Results

Similar to Chapter 6, many of the results with information regarding damage are presented spatially in order to indicate the location of potential and existing damage. Presentation of the results can be simplified by considering symmetries of damage in the model. Since only the top three plies of the laminate are modeled (as described in Section 5.3), all damage results are presented only for the top three plies and the corresponding ply interfaces. As described in Section 7.1, damage is simulated so as to display 2-fold rotational symmetry about the center of the hole. Therefore, presentation of the results can be accomplished via presentation of only one in-plane half of the laminate. It was decided to focus attention on the half of the laminate that is to the left of the $x_1$-$x_3$-plane (values of $\phi$ equal to $0^\circ$ to $180^\circ$). Thus, for all results presented in this chapter, it should be noted that an identical state of damage exists in the other in-plane half of the laminate, as well in the other three plies and ply interfaces, using the appropriate symmetries as described. Furthermore, the existence of the centrally located circular hole in the finite element model used in this work creates a region of high stress gradients that allows attention to be focused on the
"finer" region of elements surrounding the hole. One-quarter of the in-plane mesh is presented in Figure 7.10 (reproduced from Section 5.1) in order to show the "finer" region of elements. This is labeled as region "1" in the figure.

The damage results are presented using two different methods. In the first method, one-half of the "finer" region of each ply and ply interface of the finite element model is presented by showing each group of sixteen elements (in quadrilateral arrangements with four elements to a side) as a single "super" element. This is illustrated in Figure 7.11, which shows the "finer" region of the mesh along with a single "super" element outlined and shaded. These "super" elements have the same dimensions and configuration as the elements in the "finer" region of the two-dimensional finite element model, which is shown in Section 5.2. For each ply and ply interface, the individual damage modes are not specified. Instead, the result from the element with the largest magnitude in each "super" element is indicated. This method is used because the elements near the hole in the three-dimensional finite element model are very small, with the smallest elements having a linear dimension of 0.0052 inches. This is on the order of a ply thickness. This first presentation method provides the ability to view the overall locations of the relevant results for each ply and ply interface in a single figure. Since the plies and ply interfaces are displayed separately, it provides the ability to distinguish between in-plane and out-of-plane damage modes.

The second method of presenting results uses the actual element sizes in the finite element model and distinguishes between distinct damage modes in each ply. This provides the ability to identify the exact location, damage mode, and magnitude of each result. It was determined that all relevant information regarding damage from the probabilistic analysis work performed herein can be captured by a subset of elements of Region 1 located near the edge of the hole at an angle of 90° around the hole boundary. The in-plane location of this subset region is outlined in bold in Figure 7.12. This region contains three rows of elements in the radial direction (normal to the hole), seventeen elements in the clockwise tangential direction from the 90° line, equal to an angle of 31.9°, and eighteen elements in the counterclockwise tangential direction from the 90° line, equal to an angle of 33.8° (each element width
Figure 7.10  Regions of the in-plane three-dimensional mesh (reproduced from Section 5.1).
Fiber Direction

$+\theta^\circ$ $0^\circ$

Thicker lines denote edges of "super" elements

Figure 7.11 Illustration of "super" elements within Region 1 used in displaying some of the results.
Figure 7.12  In-plane location of the subset of elements within Region 1 for which some results are displayed in the three-dimensional analysis work.
in the tangential direction spans slightly less than 2° around the hole boundary). The region is this exact size because it captures all damage predicted in the present work and no more, thus showing all results without showing regions with no predicted damage. There is a lack of symmetry about the 90° line because it is not a line of symmetry. In-plane and out-of-plane damage modes are presented in separate figures. All in-plane damage modes are indicated for each ply, and all out-of-plane damage modes are indicated for each ply interface. However, some elements and element interfaces may exhibit a significant probability of failure for multiple damage modes. In such cases, only the mode with the highest probability of failure is indicated.

In presenting the failure probabilities of each element or element interface, the values are discretized into intervals in order to simplify the presentation of results and increase clarity, as in the presentation of the quasi-three-dimensional results. In those quasi-three-dimensional analysis results, presented in Chapter 6, the failure probabilities were discretized into intervals of 10%. However, in the three-dimensional work, it was found that using an interval of 10% reduced the clarity of the presentation of the results since many more elements need to be shown in order to capture all predicted damage with failure probability above 10%. Thus, the failure probabilities in the three-dimensional analysis results are discretized into intervals of 25% in order to balance the increased detail of using smaller probability intervals with the accompanying decrease in result clarity. This improves presentation clarity while still providing ample information regarding the probabilistic aspects of damage.

7.3 Probabilistic Damage Initiation Results

In this section, the results from the probabilistic damage initiation analysis work are presented. The damage initiation analysis involves the undamaged model and indicates the locations and damage modes of possible damage initiation. Spatial presentation is the main method of showing the results. The results are displayed at a chosen applied stress for each damage mode that has a predicted probability of failure greater than 25% for at least one element (or element interface) in the model.
Each range of failure probabilities equal to 25\% for each mode is represented by a distinct symbol. Each element or element interface in the figure is then assigned a symbol based on the appropriate interval of the probability of failure associated with its results at the given applied stress. The value for 100\% probability of failure is also explicitly indicated. Additionally, the applied stress, the fiber angle direction, the damage mode being presented (when applicable), and a scale bar are indicated in each figure.

At a far-field applied stress of 32.5 ksi, the first element reaches a probability of failure above 25\% via the in-plane shear (S) damage mode. The location of the element, shown in Figure 7.13, is on the edge of the hole in the 0° ply at an angle of 105° around the hole boundary.

Another damage mode with probability greater than 25\% becomes possible at an applied far-field stress of 32.9 ksi, as an element first exhibits a probability of failure via the transverse tension (Y^T) damage mode. This element is located on the edge of the hole in the -15° ply at an angle of 116° around the hole boundary, as shown in Figure 7.14. At this applied stress, there are now three elements in the 0° ply with a probability of failure above 25\% via the in-plane shear (S) damage mode. These two additional elements are on each side of the original element. The original element continues to have the maximum probability of failure via the in-plane shear (S) mode with a failure probability of 34\% still being in the range of 25\% to 50\%.

A third damage mode, the interlaminar longitudinal shear (Z^{S1}) mode, first becomes possible with a probability greater than 25\% at an applied far-field stress of 36.4 ksi. The potential delamination region is located on the edge of the hole at the +15°/-15° ply interface at an angle of 91° around the hole boundary. At this applied stress, there are sixteen elements with a probability of failure above 25\% via the in-plane shear (S) damage mode and eleven elements with a probability of failure above 25\% via the transverse tension (Y^T) damage mode. The locations and failure probability intervals of these elements at an applied stress of 36.4 ksi are shown for in-plane damage modes and out-of-plane damage modes, respectively, in Figures 7.15 and Figure 7.16. All elements with predicted failure probability above 25\% are
Figure 7.13  In-plane damage initiation probabilities at an applied stress of 32.5 ksi.
Figure 7.14  In-plane damage initiation probabilities at an applied stress of 32.9 ksi.
In-plane damage initiation probabilities at an applied stress of 36.4 ksi.

Figure 7.15
Figure 7.16  Interlaminar damage initiation probabilities at an applied stress of 36.4 ksi.
located at the boundary of the hole. The $+15^\circ$ ply contains four elements with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and four elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage mode. The $-15^\circ$ ply contains one element with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and seven elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage mode. The $0^\circ$ ply contains eleven elements with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and does not contain any elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage mode. The element with the maximum probability of failure is via the in-plane shear ($S$) mode and has a failure probability of 92% within the range of 75% to 100%. The element with the maximum probability of failure via the transverse tension ($Y^T$) mode has a failure probability of 55% within the range of 50% to 75%.

The first element to reach 100% probability of failure does so via the in-plane shear ($S$) mode at an applied far-field stress of 37.4 ksi. The locations of all damage initiation possibilities with a probability of failure above 25% are shown in Figure 7.17 using the "super" element presentation method, as described in Section 7.2. The specific in-plane and interlaminar failure probabilities for each mode of the actual elements and element interfaces are shown in Figures 7.18 and 7.19, respectively. There are a total of twenty-two elements with a probability of failure greater than 25% via the in-plane shear ($S$) damage mode. There are a total of thirteen elements with a probability of failure greater than 25% via the transverse tension ($Y^T$) damage mode at an applied stress of 37.4 ksi, with the maximum probability of failure for this mode being 64% in the element located on the edge of the hole in the $-15^\circ$ ply at an angle of $116^\circ$ around the hole boundary. The $+15^\circ$ ply contains six elements with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and five elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage mode. The $-15^\circ$ ply contains four elements with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and eight elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage.
mode. The 0° ply contains twelve elements with a probability of failure above 25% via the in-plane shear ($S$) damage mode, and does not contain any elements with a probability of failure above 25% via the in-plane transverse tension ($Y^T$) damage mode. There are a total of four element interfaces with a probability of failure via the interlaminar longitudinal shear ($Z^{S_1}$) damage mode at a far-field applied stress of 37.4 ksi. All of these elements are at the boundary of the hole at the $+15^\circ/-15^\circ$ ply interface. The element with the maximum probability of failure for this mode has a failure probability of 40% and is located on the edge of the hole at the $+15^\circ/-15^\circ$ ply interface at an angle of 91° around the hole boundary.

The sensitivity of the probabilistic damage results to the experimental strength data was also investigated. The probabilistic interlaminar shear strength parameters found in the literature displayed a large variation depending on test method. As described in Section 4.3.1, the accuracy and usefulness of some test methods are questionable. However, even accepted test methods can give rise to a large variation in interlaminar shear strength results depending on ply orientation [113,114]. For example, a mean value of 13.1 ksi and a coefficient of variation of 1.9% has been reported for the interlaminar shear strength of AS4/3501-6 cross-ply specimens using the Iosipescu test method on six replicates [126]. This is in contrast to the mean value of 4.7 ksi and coefficient of variation of 6.5% reported in the same work based on tests of six unidirectional laminate specimens. This latter value was chosen for the current work, and the corresponding results are presented previously. In order to investigate the effect of using different experimental data, the mean of 13.1 ksi and coefficient of variation of 1.9% for the interlaminar shear strength are used and the probabilistic damage initiation analysis is re-performed.

The potential damage locations using these other interlaminar shear strength distribution values, with a mean value of 13.1 ksi and a coefficient of variation of 1.9%, are shown using the “super” elements in Figure 7.20 for a far-field applied stress of 37.4 ksi, since this represents the stress at which all possible damage initiation possibilities can be identified. It can be seen that the interlaminar longitudinal shear ($Z^{S_1}$) damage mode no longer has a probability of occurrence greater than 25% at
Figure 7.17  Overall locations, using "super" element presentation, of all in-plane and out-of-plane damage initiation probabilities at an applied stress of 37.4 ksi.
Figure 7.18  In-plane damage initiation probabilities at an applied stress of 37.4 ksi.
Figure 7.19  Interlaminar damage initiation probabilities at an applied stress of 37.4 ksi.
Figure 7.20 Overall locations, using “super” element presentation, of all damage initiation probabilities using alternate interlaminar shear strength parameters.
this applied stress. All other in-plane damage initiation results remain the same as in Figures 7.18 and 7.19. The damage initiation analysis cannot go any further, as defined in the approach, since an element has reached 100% probability of failure via the in-plane shear (S) mode. Thus, delamination is no longer predicted to be probabilistically possible for damage initiation at this applied far-field stress.

In order to determine the far-field applied stress required for the interlaminar longitudinal shear \( (Z^{S1}) \) damage mode to have a probability of occurrence greater than 25%, the maximum bound on material strength set to be 100% probability of failure is ignored. By doing so, it is found that the far-field applied stress must go to 104.7 ksi before the interlaminar longitudinal shear \( (Z^{S1}) \) mode has a probability of occurrence greater than 25%. Thus, a far-field applied stress of 2.8 times greater is required for delamination to have a probability of occurrence greater than 25% when using the alternate interlaminar shear strength values as compared to the original values.

### 7.4 Probabilistic Damage Propagation Results

In this section, the results from two different damage propagation scenarios are presented. The three-dimensional analysis work provides the ability to distinguish between in-plane and out-of-plane damage modes. Therefore, in one propagation scenario, the occurrence of an in-plane damage mode is considered, while in the other scenario the occurrence of an out-of-plane damage mode is considered. In the first scenario, referred to as “Case 1,” damage via the transverse tension \( (Y^T) \) damage mode is simulated. The results from Case 1 are presented in Section 7.4.1. In the second scenario, referred to as “Case 2,” damage via the interlaminar longitudinal shear \( (Z^{S1}) \) damage mode is considered. The results from Case 2 are presented in Section 7.4.2. These two damage modes were chosen for investigation because it has been experimentally observed in open-hole tension specimens that matrix tensile cracks and delaminations can grow together during the damage evolution process [9]. In each of the scenarios, failure of only a single element or element interface is simulated. This is
done in order to limit the scope of the current work while still providing a comparison of damage propagation via in-plane and out-of-plane damage modes. This is assessed by considering the probabilities of failure of elements, including the changes thereof, after failure of this first element. Both damage propagation cases are simulated to occur at an applied far-field stress of 37.4 ksi, since this represents the stress at which damage initiation must occur, due to the material strength limits set to be 100% probability of failure, as described in Section 4.3.1.

In this section, two different types of results are presented. The first type of result presented is similar to what was presented in Section 7.3, where the location, mode, and failure probability of possible damage are given for the actual elements and element interfaces within the subset region outlined in bold in Figure 7.12. The results are displayed at a chosen applied stress for each damage mode that has a predicted probability of failure greater than 25% for at least one element (or element interface) in the model based on the failure criteria and stress averaging method described in Chapter 4. Each range of failure probabilities of 25% for each mode is represented by a distinct symbol. Since this section is focused on damage propagation, there is a need to indicate the failed element or element interface. This is accomplished by removing the failed element or element interface from the figure. It should be noted that the element (or element interface) is not actually removed, since it is still present in the finite element model. The element (or element interface) is not shown simply to indicate that the corresponding properties have been fully degraded according to the material property degradation model as described in Chapter 4. Each failure of an element or element interface is referred to herein as a “damage event”.

The second type of result indicates the change in the failure probability of each damage mode for each element (or element interface) following some damage event at the same far-field applied stress. The change in failure probability is determined by subtracting the failure probability value of each damage mode for each element (or element interface) before the damage event from the failure probability after the damage event. Changes in magnitude less than 1% are not indicated. This gives a resulting value expressed in terms of change in the percent probability of failure. This
indicates whether the failure probability increases ($|\text{positive value}| \geq 1\%$), decreases ($|\text{negative value}| \geq -1\%$), or stays the same ($|\text{value}| <1\%$). In presenting the results, the failure mode and the magnitude and direction (increase or decrease) of the change in failure probability are indicated by using distinct symbols for each mode, probability interval, and direction of change. There are three intervals of changes in magnitude. The first interval represents changes greater than or equal to $1\%$ and less than $10\%$. The second interval represents changes greater than or equal to $10\%$ and less than $50\%$. The third interval represents changes greater than or equal to $50\%$ and less than $100\%$. The change in failure probability cannot be greater than $100\%$.

### 7.4.1 Case 1: Damage Initiation via the Transverse Tension ($Y^T$) Damage Mode

Failure of a single element via the transverse tension ($Y^T$) damage mode is simulated in the first damage propagation case. The element that is chosen to fail is shown in Figure 7.21. It is located on the edge of the hole in the $-15^\circ$ ply at an angle of $116^\circ$ around the hole boundary. This element was chosen to fail because it has the highest probability of failure, equal to $64\%$, via the transverse tension ($Y^T$) damage mode in the undamaged model at an applied stress of $37.4$ ksi. No other damage is simulated for Case 1.

The failure probabilities, for values greater than $25\%$, of the remaining elements and element interfaces following the damage event at an applied far-field stress of $37.4$ ksi are shown in Figures 7.22 and 7.23, respectively. All plies contain elements with a probability of failure greater than $25\%$ via the in-plane shear ($S$) damage mode. The $+15^\circ$ and $-15^\circ$ plies both contain elements with a probability of failure above $25\%$ via the transverse tension ($Y^T$) damage mode. The locations and modes of possible damage are very similar to those of the undamaged model, shown in Figure 7.18, except that one of the elements in the $-15^\circ$ near the failed element with a probability of failure greater than $25\%$ via the transverse tension ($Y^T$) damage mode in the undamaged case no longer has a probability of failure above $25\%$ following the
Figure 7.21  Location of in-plane damage simulated in the -15° ply in Case 1.
simulated damage. No other in-plane damage modes have a probability of occurrence greater than 25%. The interlaminar longitudinal shear ($Z^{51}$) damage mode is the only out-of-plane damage mode with a probability of occurrence above 25%. There are four element interfaces with a probability of failure greater than 25% via the interlaminar longitudinal shear ($Z^{51}$) mode at the +15°/-15° ply interface. These element interfaces are located on the edge of the hole between 89° and 95° around the hole boundary. These predicted interlaminar damage probabilities are the same as the predicted results in the undamaged case.

The predicted changes in failure probabilities of the remaining elements and element interfaces following the damage event are shown in Figures 7.24 and 7.25, respectively. The -15° and 0° plies experience changes in failure probability greater than 1% via the transverse tensile ($Y^T$) and in-plane shear ($S$) modes. No other in-plane damage modes exhibit changes in failure probability beyond 1%. There are seventeen elements that exhibit a decrease in probability of failure, and three elements that exhibit an increase in probability of failure in the -15° and 0° plies. The in-plane locations of the elements that exhibit a change in failure probability are near to the site of simulated damage initiation. The three elements that experience an increase in probability of failure are located in an in-plane location directly adjacent to the simulated damage. There is an increased likelihood of continued damage propagation in the -15° ply via the transverse tension ($Y^T$) mode along the radial direction adjacent to the site of damage. The only interlaminar damage mode that experiences a change in failure probability is the interlaminar longitudinal shear ($Z^{51}$) mode, as shown in Figure 7.25. There are eight element interfaces at the +15°/-15° ply interface that experience an increase in probability of interlaminar longitudinal shear ($Z^{51}$) failure, indicating an increase in the likelihood of delamination at this interface. These element interfaces are located on the edge of the hole at an angle of 90° around the hole boundary. No other element interfaces exhibit a change in failure probability above 1%.
Figure 7.22 In-plane damage probabilities following the occurrence of in-plane damage of Case 1 at an applied far-field stress of 37.4 ksi.
Figure 7.23  Interlaminar damage probabilities following the occurrence of in-plane damage of Case 1 at an applied far-field stress of 37.4 ksi.
Figure 7.24 Changes in the in-plane failure probabilities following the occurrence of in-plane damage in Case 1 at an applied far-field stress of 37.4 ksi.
Figure 7.25  Changes in the interlaminar failure probabilities following the occurrence of in-plane damage in Case 1 at an applied far-field stress of 37.4 ksi.
7.4.2 Case 2: Damage Initiation via the Interlaminar Longitudinal Shear ($Z^{S_1}$) Damage Mode

Out-of-plane failure (delamination) of a single element interface via the interlaminar longitudinal shear ($Z^{S_1}$) damage mode at a far-field applied stress of 37.4 ksi is simulated in the second damage propagation case. The element interface that is chosen to fail is shown in Figure 7.26. It is located on the edge of the hole at the $+15^\circ/-15^\circ$ ply interface at an angle of $91^\circ$ around the hole boundary. This element interface was chosen to fail because it has the highest probability of failure via the interlaminar longitudinal shear ($Z^{S_1}$) damage mode in the undamaged model, with a probability of failure of 40%. No other damage is simulated for Case 2.

The failure probabilities, for values greater than 25%, of the remaining elements and element interfaces following this damage event are shown in Figures 7.27 and 7.28, respectively. All plies contain elements with a probability of failure greater than 25% via the in-plane shear ($S$) damage mode. The $+15^\circ$ and $-15^\circ$ plies both contain elements with a probability of failure above 25% via the transverse tension ($Y^T$) damage mode. The locations and modes of possible damage are very similar to those of the undamaged model, shown in Figure 7.18. No other in-plane damage modes have a probability of failure above 25%. There are no element interfaces with out-of-plane damage modes with probability of failure greater than 25% following the simulated damage. This is a marked change from the results for the undamaged case shown in Figure 7.19, where there are four element interfaces with a probability of failure above 25%. If the maximum material strength limit (as described in Section 4.3.1) is ignored, then the far-field applied stress must go to 38.5 ksi in order for the interlaminar longitudinal shear ($Z^{S_1}$) damage mode to have a probability of occurrence above 25% of an element interface located on the edge of the hole at the $+15^\circ/-15^\circ$ ply interfaces at an angle of $89^\circ$ around the hole boundary.

The predicted changes in failure probabilities of the remaining elements and element interfaces following the damage event are shown in Figures 7.29 and 7.30, respectively. There are a total fifteen elements that exhibit a decrease in the prob-
Figure 7.26  Location of interlaminar damage simulated at $+15^\circ/-15^\circ$ ply interface in Case 2.
Figure 7.27  In-plane damage probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi.
Figure 7.28  Interlaminar damage probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi.
ability of failure via the in-plane shear (S) damage mode. These decreases occur in each of the plies, and all decreases in failure probability are in the range of 1% to 10%. There is one element that experiences an increase in probability of failure via the in-plane shear (S) damage mode. It is located in the +15° ply at an in-plane location that is on the edge of the hole at an angle of 4° clockwise along the hole boundary from the simulated interlaminar damage. No other in-plane damage modes exhibit a change in probability of failure greater than 25%.

The +15°/-15° ply interface contains five element interfaces that exhibit a decrease in the probability of interlaminar longitudinal shear (Z_{S1}) damage following the damage event. These element interfaces are located at the edge of the hole directly adjacent to the failed interface. This ply interface also contains four element interfaces with an increase in failure probability following the simulated damage. These element interfaces are located on the edge of the hole, adjacent to the element interfaces that exhibit a decrease in failure probability. Therefore, there is a predicted increase in likelihood of delamination between the +15° and -15° plies, but not directly adjacent to the initial delamination. There is a single element interface that experiences an increase in failure probability via the interlaminar normal tensile (Z_T) damage mode. This interface is located at the edge of the hole at the -15°/0° ply interface at an in-plane location that is directly adjacent to the failed interface. No other interlaminar damage modes exhibit a change in probability of failure greater than 1%.

As noted, a marked result is that delamination is no longer predicted to have a probability of occurrence greater than 25% following the simulated longitudinal interlaminar shear failure. There are four element interfaces with a probability of failure greater than 25% in the undamaged model, but following failure of one of those interfaces via the longitudinal interlaminar shear (Z_{S1}) damage mode, the other three interfaces no longer have a probability of failure above 25%, as shown in Figure 7.28. This could be due to mesh size issues, since the interlaminar stress gradients are likely to be extremely steep around the damaged region, thus causing the calculated averaged stress values around the failed region to be low. Therefore, a substructure model was created in order to investigate the effects of using a more refined mesh.
<table>
<thead>
<tr>
<th>Change in Probability of $Y^T$ Failure</th>
<th>Change in Probability of $S$ Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Increase</strong></td>
<td><strong>Increase</strong></td>
</tr>
<tr>
<td>- $50 \leq x &lt; 100%$</td>
<td>- $50 \leq x &lt; 100%$</td>
</tr>
<tr>
<td>- $10 \leq x &lt; 50%$</td>
<td>- $10 \leq x &lt; 50%$</td>
</tr>
<tr>
<td>- $1 \leq x &lt; 10%$</td>
<td>- $1 \leq x &lt; 10%$</td>
</tr>
<tr>
<td><strong>Decrease</strong></td>
<td><strong>Decrease</strong></td>
</tr>
<tr>
<td>- $50 \leq x &lt; 100%$</td>
<td>- $50 \leq x &lt; 100%$</td>
</tr>
<tr>
<td>- $10 \leq x &lt; 50%$</td>
<td>- $10 \leq x &lt; 50%$</td>
</tr>
<tr>
<td>- $1 \leq x &lt; 10%$</td>
<td>- $1 \leq x &lt; 10%$</td>
</tr>
</tbody>
</table>

**Applied Far-Field Stress = 37.4 ksi**

Figure 7.29  Changes in the in-plane failure probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi.
Figure 7.30 Changes in the interlaminar failure probabilities following the occurrence of interlaminar damage in Case 2 at an applied far-field stress of 37.4 ksi.
around the failed element interface region to capture the steeper stress gradients while still keeping the averaging region the same.

The substructure model represents a small section of the full finite element model. All symmetries used in the full three-dimensional model are also used in the substructure model. The substructure lies within the “finer” region of elements (one-quarter of which is labeled as region “1” in Figure 7.10). The in-plane region considered via this substructure is outlined in bold in Figure 7.31. The substructure model considers seven of the original mesh elements along the tangential direction and four of the original mesh elements along the radial direction. All three plies are modeled. The substructure region was chosen to include the location of the failed $+15^\circ/-15^\circ$ ply interface, as well as some additional area in order to allow transition from the “finer” region mesh of the full three-dimensional model (Region 1 in Figure 7.10) to a more refined mesh surrounding the region of delamination damage.

The in-plane mesh scheme of the substructure is shown in Figure 7.32. The mesh at the outer edges of the model exactly match the “finer” region mesh of the full three-dimensional model. Thus, the bold outline in Figure 7.32 is the same bold outline in Figure 7.31. This is done in order to allow use of nodal displacement results from the full three-dimensional model to define the appropriate boundary conditions on the substructure model. The boundary conditions on the outlined bold edge of the substructure model are obtained by performing an analysis with the full three-dimensional model, extracting the nodal displacement results from the nodes corresponding to the bold region in Figure 7.31, and then applying them as displacement boundary conditions to the bold outlined edges in Figure 7.32. The other boundary conditions on the substructure model are the same as for the full three-dimensional model, as described in Section 5.3.

Linear multi-point constraints are used for mesh refinement near the failed region. This is a typical refinement method for linear displacement-based elements [101]. This refinement scheme is illustrated in Figure 7.33. In this refinement method, “irregular” nodes are introduced into the mesh at locations where a transition to a more refined mesh is desired. A nodal point is termed “regular” if it acts as a common nodal point.
Figure 7.31 In-plane location considered in the three-dimensional substructure model.
Figure 7.32  In-plane mesh scheme of the three-dimensional substructure model.
for each of the neighboring elements (labeled as "1" and "2" in Figure 7.33); otherwise it is referred to as an "irregular" nodal point (labeled by the number "3" in figure 7.33) [139]. The irregular node is introduced where the edges of a refined element (labeled as “B” and “C” in Figure 7.33) meet at the midsides of a coarser one (labeled as “A” in Figure 7.33). Typically, a “one-irregular” (or “one-level”) rule is used, in which only one irregular node is allowed on the edge or face of an element [139,140]. The displacements of the “irregular” nodes are defined using multi-point constraints, meaning that the solution at an irregular node is obtained by interpolating from the adjacent regular nodes forming the edge or face of the coarser element.

The region corresponding to the in-plane location of initial delamination damage is shaded in Figure 7.32. These element interfaces are failed in the substructure model. The linear dimension of the smallest elements is 0.00065 inches (16.5 microns). This is equivalent to approximately two AS4 fiber diameters, which are on the order of 7.1 microns in diameter [141]. Since the present work assumes homogeneity of the material, the size of the smallest elements violates the minimum bound of 0.004 inches (0.1 mm) imposed as a means of maintaining a sufficient level of statistical homogeneity as described in Section 5.1. However, this substructure model is merely used as a means to highlight issues with the full three-dimensional model, so the minimum bound is ignored. Furthermore, since the averaging length is kept the same as the previous work, this does not directly affect the strength distribution. There are six layers of these small elements surrounding the failed region in order to allow sufficient ability to capture the details of the local stress fields near the damaged region. One element per ply in the through-the-thickness direction is used, as is done in the full three-dimensional model. The smallest elements have an aspect ratio of 8, which is within the Abaqus recommended range, and all elements are considered to be well-shaped (meaning that the angles between two consecutive edges in an element are close to $\pi/2$ [142]) according to the Abaqus recommendations. This helps to ensure that the obtained solution is of sufficiently high quality.

The elements in the substructure model are assigned material properties according to those given in Table 4.1. The initial delamination damage is defined by degrading
Figure 7.33  Schematic of the linear multi-point constraint mesh refinement method used in the three-dimensional substructure model.
the out-of-plane material properties (according to the material property degradation model described in Chapter 4) of the elements in the +15° and -15° plies that are located within the failed region (as indicated in Figure 7.27). This prevents the failed elements from carrying any out-of-plane loads.

The probabilistic damage results are shown in Figure 7.34. Each ply and ply interface of the substructure are shown in this figure, where each region assigned a probability of failure corresponds to the region over which the stresses were averaged. This is equal to the in-plane area of an element interface in the original model, which is a quadrilateral area with a linear dimension of 0.0052 inches. It is seen that damage is predicted to be probabilistically significant via the in-plane shear (S) and interlaminar longitudinal shear (Z) damage modes. Thus, use of a more refined mesh with the same averaging region yields predicted delamination damage with a probability of failure greater than 25% at the +15°/-15° ply interface following the initial delamination damage. This was not predicted with the original mesh. Furthermore, the in-plane shear (S) damage predicted in the -15° ply did not have a probability of failure greater than 25% in the original three-dimensional model. Thus, it is seen that mesh refinement, with no change to the averaging length, can change the predicted probabilistic damage results.

In order to gain a better understanding of how use of a refined mesh can affect the predicted damage probabilities, a few predicted stress fields from the substructure model are compared with those from the original model. A comparison between the in-plane shear stresses in the -15° ply predicted by the substructure model and those predicted by the original model are given in Figure 7.35. The region considered by the substructure model is outlined in bold and the in-plane location of the simulated delamination at the +15°/-15° interface is shaded. Isostress contours of in-plane shear stress, \( \sigma_{12} \), normalized by the far-field applied stress, \( \sigma_o \), are plotted. The stress contours were generated using linear interpolation of the element centroidal stress values. It can be seen that the shapes of the stress contours change slightly. Also, a higher magnitude of maximum stress is predicted by the substructure model as compared to the original model. Thus, use of a refined mesh around the simulated
Figure 7.34  In-plane and interlaminar damage probabilities following the interlaminar damage simulated in Case 2 using the substructure model at an applied far-field stress of 37.4 ksi.
Figure 7.35  Isostress contours of in-plane shear stress, $\sigma_{12}$, normalized by far-field applied stress, $\sigma_o$, in the $-15^\circ$ ply for the original model and the substructure model.
delamination damage region leads to changes in the predicted in-plane stresses.

A comparison between the interlaminar longitudinal shear stresses at the $+15^\circ/-15^\circ$ ply interface predicted by the substructure model and those predicted by the original model is given in Figure 7.36. The region considered by the substructure model is outlined in bold and the in-plane location of the simulated delamination at the $+15^\circ/-15^\circ$ interface is shaded. Isostress contours of the interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by the far-field applied stress, $\sigma_o$, are plotted. The stress contours were generated using linear interpolation of the nodal stress values. The general shapes of the stress contours are similar, but the stresses predicted using the substructure model have a higher gradient near the hole edge and delamination region. Thus, use of a refined mesh around the simulated delamination also leads to changes in the predicted interlaminar stresses at the same ply interface as the simulated damage.

Finally, predicted interlaminar longitudinal shear stress, $\sigma_{13}$, and interlaminar normal stress, $\sigma_{33}$, results at the $-15^\circ/0^\circ$ ply interface for the substructure model and original model are compared in Figures 7.37 and 7.38, respectively. The region considered by the substructure model is outlined in bold and the in-plane location of the simulated delamination at the $+15^\circ/-15^\circ$ interface is shaded. Isostress contours of the stresses normalized by the far-field applied stress, $\sigma_o$, are plotted. The stress contours were generated using linear interpolation of the nodal stress values. It can be seen that the stress shapes change significantly with use of the refined mesh in the substructure model. Furthermore, higher stress concentrations are predicted in the substructure model that are not predicted with the original model. These regions of high stress concentration are located within, and adjacent to, the same in-plane location as the simulated delamination damage. This is particularly significant in the case of the interlaminar normal stress, where the magnitude of the maximum isostress contour is 2.67 times higher in the substructure model than in the original model. Thus, use of a refined mesh leads to significant changes in the predicted interlaminar stresses, especially the interlaminar normal stress, at the ply interface directly below the damaged ply interface.
Figure 7.36 Isostress contours of interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_a$, at the $+15^\circ/-15^\circ$ ply interface for the original model and the substructure model.
Figure 7.37 Isostress contours of interlaminar longitudinal shear stress, $\sigma_{13}$, normalized by far-field applied stress, $\sigma_o$, at the $-15^\circ/0^\circ$ ply interface for the original model and the substructure model.
Figure 7.38  Isostress contours interlaminar normal stress, $\sigma_{33}$, normalized by far-field applied stress, $\sigma_0$, at the $-15^\circ/0^\circ$ ply interface for the original model and the substructure model.

246
7.5 Discussion

In this section, a discussion of the three-dimensional analysis results is presented. One major conclusion that can be drawn from the results presented in this chapter is that the incorporation of probabilistic aspects of damage into a three-dimensional progressive failure analysis framework allows for many possible damage initiation and propagation scenarios. Not only are there multiple possible locations for damage to initiate, as was also observed in the quasi-three-dimensional analysis work presented in Chapter 6, but use of a three-dimensional finite element model also yields the possibility of both in-plane and out-of-plane damage modes, as well as switching between them, for damage initiation and subsequent propagation. For the case of a [±15/0],\textsubscript{AS4/3501-6} laminated plate containing a centrally located circular hole subject to a unidirectional tensile load that was investigated in the present work using the full three-dimensional model, it was found that damage initiation was predicted to be possible via transverse tensile (\(Y^T\)), in-plane shear (\(S\)), and longitudinal interlaminar shear (\(Z^{s1}\)) damage modes. The locations of possible in-plane damage initiation span multiple in-plane locations of all plies of the laminate. Furthermore, coupling between in-plane and out-of-plane damage modes was observed, in which the occurrence of one type of damage mode affected the probability of occurrence of the other type of damage mode. The initiation of damage by a different mode or in a different ply or in-plane location may lead to different subsequent damage propagation possibilities, and thus different overall failure behavior and laminate performance.

Similar to the results presented in Chapter 6, there is a range of possible applied stress values at which damage initiation can occur when probabilistic aspects are incorporated into a three-dimensional progressive failure analysis. For the case considered, the in-plane shear (\(S\)) mode first has a probability of occurrence above 25\% at a far-field applied stress of 32.5 ksi, and the first element reaches 100\% probability of failure via the in-plane shear (\(S\)) damage mode at a far-field applied stress of 37.4 ksi. This is an increase in applied stress of 15\% between when damage initiation is first predicted to be possible and when it reaches the applied stress level at which
damage must initiate. Thus, there is a significant range of applied stress values over which damage can initiate. The range of possible damage locations, modes, and applied stress values presented in Section 7.3 are obtained using a probabilistic level of 25% or greater and an imposed value for 100% probability of failure based on the maximum recorded material strength value from limited experimental testing results found in the literature. It is expected that lowering the significance level of 25% and/or increasing the material strength value corresponding to 100% probability of failure will serve to increase the number of elements and element interfaces with a significant probability of failure, and thus extend the range of possible damage initiation modes and strength values. This also will allow further damage propagation possibilities. This highlights the importance in including probabilistic aspects of damage when considering prediction of damage initiation.

Probabilistic aspects of damage are also seen to be very important for prediction of damage propagation. Following the initiation of in-plane damage, it is seen that multiple damage modes, both in-plane and out-of-plane, still have probabilities of occurrence above 25%. Similarly, following the initiation of damage via delamination, there are still multiple in-plane damage modes with probability of occurrence greater than 25%. No out-of-plane damage modes are predicted with probability of occurrence greater than 25% following simulated delamination initiation in the full three-dimensional model. However, this may not be an accurate prediction of true physical behavior, but may be due to modeling issues. Use of the substructure model with a refined mesh, but identical averaging region, does predict further damage propagation with probability greater than 25% via the interlaminar longitudinal shear ($Z^{SI}$) damage mode. This highlights the importance of mesh size in performing three-dimensional progressive failure analysis within a probabilistic framework.

Mesh size is important in the case of the quasi-three-dimensional analysis, but it seems to be even more important in a three-dimensional framework due to the steep interlaminar stress gradients that arise at the ply interfaces of free edges and the edges of delaminated regions. Thus, elements of the appropriate size must be utilized in order to properly capture the interlaminar stress fields and the changes caused by the
occurrence of out-of-plane damage. This was shown via use of the substructure model in Section 7.4.2. The original model did not predict any out-of-plane damage to have a probability of occurrence above 25% following the simulated delamination damage at the $+15^\circ/-15^\circ$ ply interface. The substructure model employed a refined mesh (using elements with a linear dimension of 0.00065 inches, which is 8 times smaller than the original model elements) in order to capture the steep interlaminar stress gradients surrounding the damaged region, while still using the original averaging region for calculating the averaged interlaminar stress values used in assessing the probability of delamination. It was found that reducing the mesh size but retaining the original averaging length changed the predicted failure probabilities, since one element and one element interface were predicted to have failure probabilities greater than 25% in the substructure model. However, as noted in Section 7.2.2, the element size used in the substructure model is below the minimum bound chosen (as described in Section 4.1) to maintain a sufficient level of statistical homogeneity. The use of the original averaging length alleviates this issue somewhat in terms of the material strength values, since the strength distribution used is applied to the entire averaging region. However, a homogeneous orthotropic continuum is assumed for the finite element model. Since the elements in the substructure are on the order of two fiber diameters, this may not be a valid assumption in the substructure model. Future work considering element size for such work needs to take this into consideration, especially if probabilistic aspects of elastic material properties are also considered, since such an analysis must acknowledge that the distribution of fibers in a realistic composite is random, and thus a sufficiently large representative volume element must be employed.

Another main finding, that is also observed for the quasi-three-dimensional damage propagation cases in Chapter 6, is that the redistribution of stresses caused by the occurrence of damage can have significant effects on probabilistic failure behavior. In the three-dimensional analysis work, it is seen that complex stress redistribution occurs following the occurrence of both in-plane and out-of-plane damage. In the discussion of the quasi-three-dimensional analysis work presented in Section 6.4, it was
noted that the occurrence of in-plane damage caused in-plane stress redistribution and seemed to hint at out-of-plane effects as well. In the three-dimensional analysis work, the interlaminar effects are considered explicitly. It is seen that the occurrence of in-plane damage indeed causes significant changes in the probability of failure of both in-plane and out-of-plane damage modes due to a redistribution of the stresses. It is also seen that out-of-plane (delamination) damage causes significant changes in the failure of both in-plane and out-of-plane damage modes due to stress redistribution as well. Thus, in a three-dimensional analysis framework, the likelihood of a particular damage mode can be affected by the occurrence of damage by a distinctly different mode as a result of stress redistribution.

This redistribution of stresses due to damage can have effects both near the site of damage as well as significantly far from that site. Many of the changes in failure probabilities of remaining elements and element interfaces following the simulation of damage occur near the location of damage. For example, in the first damage propagation case involving simulated in-plane damage in the -15° ply, the surrounding elements directly adjacent to the failed element exhibit both increases and decreases in failure probability greater than 1%. Propagation along the radial direction (normal to the hole) increases in likelihood, while propagation tangentially around the hole decreases in likelihood. The occurrence of damage can also cause significant changes in the failure probabilities of elements at different in-plane locations and in different plies (or ply interfaces) that can be considered to be located relatively far from the site of damage. For example, in the second damage propagation case presented in Section 7.4.2, the occurrence of delamination located on the edge of the hole at the +15°/-15° ply interface at an angle of 90° around the hole boundary caused changes greater than 1% in the in-plane shear ($S$) failure probability of elements located at the edge of the hole in the 0° ply at an angle of 109° around the hole boundary. Thus, the occurrence of the delamination damage affected an element that is not even in one of the plies surrounding the delamination and that is at an in-plane distance of ten element widths.

One major issue that needs further consideration in future work is the accuracy
and availability of probabilistically characterized material strength data, particularly the interlaminar strength data. Reliable, probabilistically characterized experimental interlaminar strength data is difficult to find in the open literature. For example, the interlaminar shear ($Z_s$) strength data chosen for the current work is based on tests of only six specimens, and this was found to be some of the more reliable data, since some alternative test methods have been shown to yield questionable values. However, there is often more than one available set of probabilistically characterized material strength data, so the choice of what data to use is extremely important. It can make the difference between which damage modes are predicted to be probabilistically significant. For example, in Section 7.1, the results obtained using the originally chosen interlaminar shear ($Z_s$) strength distribution yield prediction of probabilistically significant damage initiation via an out-of-plane damage mode. However, using a different strength distribution obtained for the same material and test method results in no out-of-plane damage modes predicted to be probabilistically possible. However, this does not change the basic importance of probabilistic aspects of progressive damage demonstrated in the current work, only the specific details regarding when the various damage possibilities occur.

The availability of accurate material strength data also ties into the effect of ply orientation on material property values that is noted in Sections 4.3.1 and 7.3. As described in Section 4.3.1, the present work assumes that the interlaminar material strength properties are independent of the orientations of the surrounding plies. However, this does not reflect the physical realities of composite laminates. Significant variations in probabilistic interlaminar shear strength distributions for different ply orientations have been reported in the literature [126]. As shown in Subsection 7.3.1, the exact interlaminar shear strength distribution employed in probabilistic damage analysis can have significant effects on the predicted damage initiation possibilities. Using the alternate distribution, delamination is not predicted to have a probability of occurrence greater than 25% before in-plane damage must occur (based on the maximum material strength limit assumed in the present work). Furthermore, the predicted far-field applied stress at which the interlaminar longitudinal shear ($Z_{s1}$)
damage mode first exhibits a probability of occurrence above 25% is 2.9 times higher than the value predicted with the original interlaminar shear strength distribution. This is due to the fact that the alternate distribution has a mean value that is 2.8 times higher than the original distribution (13.1 ksi versus 4.7 ksi). The predicted far-field stress is more than 2.8 times higher due to the differences in coefficient of variation between the two distributions employed. The original interlaminar shear strength values chosen for the present work are based on unidirectional (0°) laminates and thus are closer to the [±15/0] s laminate investigated in the current work than the cross-ply specimens used to obtain the interlaminar shear properties that were used for comparison purposes in Subsection 7.3.1. Therefore, the original results for predicted damage initiation possibilities are likely more accurate. However, the fact that the probabilistic out-of-plane material strength distributions can vary depending on test method and ply orientation, as well as the general lack of available experimental data to choose from, suggests that further consideration should be given to this item.

Another possible limitation of the present work that needs further consideration in future work is the material property degradation model that is used for simulating delamination. In reality, delamination between plies causes those plies to be completely separated, with new free surfaces created in the delaminated region. This exact process is not modeled in the present work. Instead, delamination damage is simulated in the current work by preventing the elements in the plies surrounding the delaminated region from carrying out-of-plane loads. This is done by degrading the out-of-plane material properties of the elements in the two plies surrounding the delaminated region, as described in Section 4.1. Thus, if the out-of-plane properties of an element are degraded due to simulated delamination on one face of that element, then the predicted damage in the remaining elements and element interfaces may be unrealistically affected in two primary ways. First, the stress fields may redistribute unrealistically following the simulated damage due to the fact that an entire element no longer carries out-of-plane loads, not just the actual interface between elements in adjacent plies (since resin-rich interface regions are not explicitly modeled in this
work). Second, the interlaminar stresses used in assessing the occurrence of out-of-plane damage are averaged over the shared ply interface between elements in adjacent plies, so if the out-of-plane properties of an element has been degraded due to delamination at one ply interface, then that element will lower the calculated average stress values at the other ply interface because it does not carry any out-of-plane loads, and thus the interlaminar stress in that element are effectively zero.

A possible example of such unrealistic behavior can be seen by considering the second damage propagation case, investigated in Section 7.4.2. In this example, the elements in the +15° and -15° plies at the site of simulated delamination damage are degraded so that they cannot carry out-of-plane loads. It is seen that the interlaminar normal tensile ($Z_T$) damage mode at the -15°/0° ply interface interface at a location directly adjacent to the degraded element in the -15° ply exhibits a change in probability of occurrence greater than 1%. This may be due to the out-of-plane properties of the failed element in the -15° ply being degraded, thus preventing the element from carrying any out-of-plane stresses, so that these stresses are redistributed around this element into the neighboring elements and there is now a steep interlaminar normal stress gradient at the edge of the failed region. Thus, the element adjacent to the failed element may see an unrealistic increase in interlaminar stresses, leading to the predicted out-of-plane damage. Therefore, the method of simulating delamination damage may need consideration in future work.

A final limitation of the current work is the simulation of symmetric damage within the laminate under investigation. As described in Chapter 3, simplifying assumptions are employed in the current work in order to reduce the complexity of the analysis and limit the scope of the work. The assumption of symmetric damage is a main simplifying assumption in the present work. As described in Section 5.3, only half of the plies are modeled using the assumption of a plane of mirror symmetry about the laminate midline. In Section 7.1, stress contour results are presented in order to show the existence of a 2-fold rotational symmetry in the $x_1$-$x_2$-plane about the center of the hole. This symmetry was used in order to simplify the problem further, providing the ability to essentially only consider one-quarter of the laminate. Thus, all results
presented within the current chapter can be assumed to be identical in the other regions of the laminate (using the suitable assumptions of symmetry). However, as previously discussed in Section 6.4, the occurrence of damage within a probabilistic progressive failure analysis can be non-symmetric. It is expected that considering non-symmetric damage possibilities will greatly affect the predicted damage initiation and propagation possibilities. An additional item for consideration specific to the three-dimensional analysis work is related to the fact that interlaminar damage is explicitly modeled. Since it was shown that the occurrence of interlaminar damage can affect the probabilities of occurrence of in-plane damage modes, and vice versa, it seems likely that even further non-symmetric damage progression possibilities may result in a full three-dimensional analysis framework as compared to a quasi-three-dimensional analysis.

The three-dimensional analysis work has shown that, as expected, the incorporation of probabilistic characteristics of damage allows for the possibility of many different damage initiation and progression scenarios for a single laminate configuration. The use of a full three-dimensional model allows for the possibility of both in-plane and out-of-plane damage modes for damage initiation and propagation. The three-dimensional nature of the stress fields in the model leads to complex redistribution of stresses following the occurrence of damage. This causes significant increases and decreases in the failure probabilities of the remaining elements and element interfaces, and this redistribution causes the occurrence of damage via one damage mode to affect the likelihood of other damage modes and locations. Additionally, the results presented in this chapter illustrate the importance and difficulties in choosing the size of elements in a three-dimensional probabilistic failure analysis framework, since the finite element model must be able to sufficiently capture the steep interlaminar stress gradients that exist between plies at free edges and the edges of failed regions. Four main limitations are identified that need further consideration in future work. The first limitation is the questionable accuracy and limited availability of probabilistically-characterized interlaminar material strength data. The second limitation is the assumption that the interlaminar strength properties are independent of
the orientations of the surrounding plies. The third limitation is the possibly unrealistic nature of the particular method used to simulate delamination damage in the model. Finally, the fourth limitation is related to the assumptions of symmetric damage within the model. However, despite these limitations and needs for refinement in the details of the work, this work has achieved its main goal of demonstrating and assessing the importance, and accompanying effects, of incorporating probabilistic aspects of progressive damage into a progressive failure analysis framework.
Chapter 8

Conclusions and Recommendations

In this work, the effects and importance of considering probabilistic aspects of damage progression in composite structures are assessed. Various possible damage initiation and propagation sequences that may occur for the cases of a graphite/epoxy laminated plate with a central circular hole subjected to uniaxial or equal biaxial tension are examined, and the manners by which the occurrence of damage affect the subsequent probabilistic damage behavior of the structure are investigated. Consideration is also given to the interaction of different damage modes. Progressive failure analysis that incorporates probabilistic material strength parameters is utilized to conduct these investigations.

The conclusions drawn from this investigation are:

1. Consideration of probabilistic characteristics of damage progression allows for the possibility of many different damage progression scenarios for a single laminate configuration and loading, including the possibility of damage initiation and propagation via different damage modes (both in-plane and out-of-plane) and in numerous different geometric locations.

2. Consideration of probabilistic aspects reveals a coupling between in-plane and out-of-plane damage modes, in which damage initiation and subsequent propagation can switch between the two types of damage modes and the occurrence
of damage via one type of damage mode can affect the probability of occurrence of the other type of damage mode.

3. Values of applied loading where damage initiation can occur span a large range when probabilistic aspects are included, and such a range cannot be adequately captured using a deterministic progressive failure analysis methodology.

4. Incorporation of probabilistic aspects into a progressive failure analysis allows for the possibility of non-symmetric damage to occur, and the damage progression can become increasingly likely to be non-symmetric as further damage occurs due to the non-symmetric redistribution of stresses caused by the occurrence of such damage.

5. The redistribution of stresses caused by the occurrence of damage can lead to significant increases and decreases in the failure probabilities of remaining elements (or points of consideration) in a structure, both locally and in locations relatively far from the site of damage.

6. The magnitude of change in failure probability of an undamaged element (or point of consideration) following the occurrence of damage in the structure is nonlinearly dependent on the stress level at that location, since probability of failure is a nonlinear function of stress.

7. Damage can propagate along different paths in geometrically different locations and via different damage modes with nearly equal likelihood.

Based on the results of this investigation, recommendations for further research are:

1. Examine the effects of simulating the occurrence of non-symmetric damage. It was shown in this work that when probabilistic aspects are included, damage must not occur symmetrically. The occurrence of non-symmetric damage will result in non-symmetric stress fields within the laminate, thus allowing for even further non-symmetric damage possibilities.
2. Investigate the effects of increasing the applied far-field stress during the damage progression process. The present work primarily investigates the effects of damage occurring at a constant applied far-field stress, but it has demonstrated the importance of considering different applied far-field stress values.

3. Consider use of a material property degradation model to simulate interlaminar damage that is more physically representative of actual delamination damage, such as explicit modeling of the separation between delaminated plies, since the method employed in the present work yielded some questionable results regarding the redistribution of stresses following simulated delamination.

4. Carefully consider finite element mesh discretization, especially in regions near simulated damage, and thus high stress gradients, in three-dimensional finite element models. Changing mesh refinement, while keeping the stress averaging region the same, can affect the predicted probabilistic damage results, since smaller elements can more accurately capture the stress gradients surrounding the regions of damage.

5. Investigate the use of an approach based on strain energy release rate to assess delamination that considers probabilistic aspects of the strain energy release rates. The present work utilizes strength-based delamination criteria, but the general approach of the work is also applicable to delamination criteria that are formulated in terms of strain energy release rates.

6. Obtain higher quality experimental probabilistic material strength data, primarily for out-of-plane strength values, that is based on larger numbers of test specimens and that takes into account the effect of ply orientation on interlaminar shear strength in order to improve the quantitative accuracy of probabilistic progressive damage modeling. The present work identified a lack of available probabilistically characterized out-of-plane material strength data, and demonstrated that the particular choice of interlaminar material strength data can significantly affect the results of predicted probabilistic damage progression.

259
7. Consider lower failure probability levels (i.e. below 10% or 25%), as well as higher material ultimate strength values, in order to allow for exploration of extreme damage progression possibilities. It was shown in this work that considering lower failure probability levels allows for more damage possibilities.

8. Utilize structural models that consider probabilistic behavior of other properties, such as stiffness, in order to investigate the effects of considering other sources of variation in the laminate that may affect the probabilistic aspects of damage progression. The present work assumes that only the material strength values are probabilistic, since they have been shown experimentally to exhibit larger variation than elastic constants. However, the variability of other properties may be important to consider as well.

9. Consider dependencies between strength parameters of different damage modes and at different geometric locations. The present work assumes that all strength parameters are independent random variables, meaning that the particular strength value/distribution of an element (or point of consideration) is not related to the strength values of other damage modes or geometric locations. However, a physical structure may actually exhibit dependencies between material strength parameters.
REFERENCES


Appendix A

Codes for Assessing Failure Probabilities

The MATLAB codes used to assess the failure probabilities for each failure mode of each element (or point of consideration) in the finite element models are listed in this appendix. One main program, entitled `probabilistic_damage_analysis.m`, requires the user to input the stress data obtained from all finite element analyses associated with a particular damage progression sequence. It furthermore requires the user to define the number of plies in the laminate, the number of elements per ply, the original far-field applied stress value from the finite element analysis, the locations and order of each simulated damage event, the probabilistic significance levels, and the probabilistic material strength parameters (specifically the mean, standard deviation, and ultimate strength value). In the code given herein, all locations where a user must enter values are denoted with an asterisk. Two subroutines, entitled `arrange_failure_probability.m` and `arrange_failure_probability_TT.m`, were written to calculate the failure probabilities for each damage mode of each element for the in-plane and out-of-plane damage modes, respectively. The program named `failure_prob_change.m` is a subroutine used to calculate the changes in failure probabilities of each failure mode for each element (or point of consideration) from the previous damage event. All programs are written for MATLAB Release 14.
% probabilistic_damage_analysis.m

% This code is used to calculate the absolute failure probabilities
% and changes in probability of failure that each element exhibits
% after each damage event

close all;
clear;
clc;

%%
%%--Variables Required for User to Define--%%

%%--Specify 2D or 3D--%%
check = *;  % 0 for 2D, 1 for 3D

%%--Define Laminate Model Parameters--%%
ply_number = *;  % total # of plies
element_number = *;  % number of elements per ply
element_number_fine = *;  % number elements in 'finer' region

%%--Define Applied Far-Field Stress--%%
original_stress = *;  % stress during FE analysis in [ksi]
new_stress = *;  % new applied stress in [ksi] - enter as '#.#'

%%--Define In-plane Stress and Damage data--%%
load inplane_stress.mat  % load in-plane element stress data as
% inplane_stress.mat formatted as 3 columns:
% [sigma_11,sigma_22,sigma_12].
% Additional stress data obtained following damage events are loaded here as well.

element_stress = horzcat(inplane_stress,*); % arrange stresses

number_damage_events = *; % number of damage events
damage_event_list = [*,*]; % ordered list of damage events listed % as [element,ply #]

damage_events = damage_event_list(:,1) +
element_number_fine*(damage_event_list(:,2) - 1);

%%--Define Through-the-Thickness Stress and Damage Data--XX
if check == 1
    load interfacestress.mat % load out-of-plane stress data
    % saved as interfacestress.mat
    % formatted in three columns as:
    % [sigma_33,sigma_13,sigma_23].
    % Additional stress data obtained
    % following damage events can be
    % loaded here as well.

        element_stress_TT = horzcat(interfacestress,*);

        damage_event_list_TT = [*,*]; % ordered list of damage events % as [element,ply interface #]

        damage_events_TT = damage_event_list_TT(:,1) +
element_number_fine*(damage_event_list_TT(:,2) - 1);
end
%%--Define Probabilistic Significance Levels--%%

sig_level_prob_actual = *;  % percent failure probability
% considered

sig_level_prob_change = *;  % percent failure probability
% change considered

%%--Define Probabilistic Material Strength Parameters--%%

% Longitudinal tensile strength. Weibull distribution
Xt_scale = *;  % Weibull scale parameter in units of [ksi]
Xt_shape = *;  % Weibull shape parameter
Xt_max = *;  % Maximum material strength possible [ksi]

% Longitudinal compressive strength. Normal distribution
Xc_mean = *;  % Mean value given in units of [ksi]
Xc_stdev = *;  % Standard deviation given in units of [ksi]
Xc_max = *;  % Maximum material strength possible [ksi]

% Transverse tensile strength. Normal distribution
Yt_mean = *;  % Mean value given in units of [ksi]
Yt_stdev = *;  % Standard deviation in units of [ksi]
Yt_max = *;  % Maximum material strength possible [ksi]

% Transverse compressive strength. Normal distribution
Yc_mean = *;  % Mean value given in units of [ksi]
Yc_stdev = *;  % Standard deviation given in units of [ksi]
Yc_max = *;  % Maximum material strength possible [ksi]

% In-plane shear strength. Normal distribution
\textbf{S\_mean = \_*}; \% \text{ Mean value given in units of [ksi]}
\textbf{S\_stdev = \_*}; \% \text{ Standard deviation given in units of [ksi]}
\textbf{S\_max = \_*}; \% \text{ Maximum material strength possible [ksi]}

\% Normal tensile strength. Normal distribution
\textbf{Zt\_mean = \_*}; \% \text{ Mean value given in units of [ksi]}
\textbf{Zt\_stdev = \_*}; \% \text{ Standard deviation in units of [ksi]}
\textbf{Zt\_max = \_*}; \% \text{ Maximum material strength possible [ksi]}

\% Interlaminar shear strength. Normal distribution
\textbf{Zs\_mean = \_*}; \% \text{ Mean value given in units of [ksi]}
\textbf{Zs\_stdev = \_*}; \% \text{ Standard deviation in units of [ksi]}
\textbf{Zs\_max = \_*}; \% \text{ Maximum material strength possible [ksi]}

\textbf{material\_strength\_parameters = [Xt\_scale,Xt\_shape,Xt\_max,}
\textbf{Xc\_mean,Xc\_stdev,Xc\_max,Yt\_mean,Yt\_stdev,Yt\_max,}
\textbf{Yc\_mean,Yc\_stdev,Yc\_max,S\_mean,S\_stdev,S\_max,}
\textbf{Zt\_mean,Zt\_stdev,Zt\_max,Zs\_mean,Zs\_stdev,Zs\_max];}

\%
\%--No User Input Required Beyond this Point--\%

\%--Multiply Element Stresses to Desired Applied Stress--\%
\textbf{stress\_increase\_factor = new\_stress/original\_stress;}

\textbf{element\_stress = element\_stress.\*(stress\_increase\_factor*10^-3);}

\textbf{if check == 1}
\textbf{element\_stress\_TT = element\_stress\_TT.*}
(stress_increase_factor*10^-3);
end

%%
%This section calculates the probability of failure of each
%element for each damage mode. The calculations are
%performed using the CDF in order to find the probability
%of failure at the current applied load.

%%--In=Plane--%%
actual_failure_probabilities = zeros(element_number*ply_number,1);

for i = 1:(number_damage_events+1)
    damage_stress = horzcat(element_stress(:,1+3*(i-1)),
      element_stress(:,2+3*(i-1)),element_stress(:,3+3*(i-1)));
    arrange_failure_probability;
    actual_failure_probabilities =
      horzcat(actual_failure_probabilities,failure_probabilities);
end

%%--Through-the-Thickness--%%
if check == 1
    actual_failure_probabilities_TT =
      zeros(element_number*ply_number,1);

    for i = 1:(number_damage_events+1)
        damage_stress_TT =
          horzcat(element_stress_TT(:,1+3*(i-1)),
            element_stress_TT(:,2+3*(i-1)),
            element_stress_TT(:,3+3*(i-1)));
    end

278
arrange_failure_probability_TT;
actual_failure_probabilities_TT =
horzcat(actual_failure_probabilities_TT,
failure_probabilities_TT);
end

%%
%%--Calculate Changes in Failure Probability
%% Following each Damage Event--%%
failure_prob_change
% arrange_failure_probability.m
%
% Calculates probability of failure for each in-plane damage mode

failure_probabilities = zeros(element_number*ply_number,5);

for i = 1:element_number*ply_number
    if (damage_stress(i,1) > 0) && (damage_stress(i,1) < material_strength_parameters(3))
        failure_probabilities(i,1) = wblcdf(damage_stress(i,1),
                    material_strength_parameters(1),
                    material_strength_parameters(2))*100;
        failure_probabilities(i,2) = 0;
    elseif (damage_stress(i,1) > 0) && (damage_stress(i,1) >= material_strength_parameters(3))
        failure_probabilities(i,1) = 100;
        failure_probabilities(i,2) = 0;
    elseif (damage_stress(i,1) < 0) && (-damage_stress(i,1) < material_strength_parameters(6))
        failure_probabilities(i,1) = 0;
        failure_probabilities(i,2) = normcdf(-damage_stress(i,1),
                    material_strength_parameters(4),
                    material_strength_parameters(5))*100;
    elseif (damage_stress(i,1) < 0) && (-damage_stress(i,1) >= material_strength_parameters(6))
        failure_probabilities(i,1) = 0;
        failure_probabilities(i,2) = 100;
    end

    if (damage_stress(i,2) > 0) && (damage_stress(i,2) <
material_strength_parameters(9))
    failure_probabilities(i, 3) = normcdf(damage_stress(i, 2),
    material_strength_parameters(7),
    material_strength_parameters(8))*100;
    failure_probabilities(i, 4) = 0;
elseif (damage_stress(i, 2) > 0) && (damage_stress(i, 2) >=
    material_strength_parameters(9))
    failure_probabilities(i, 3) = 100;
    failure_probabilities(i, 4) = 0;
elseif (damage_stress(i, 2) < 0) && (-damage_stress(i, 2) <
    material_strength_parameters(12))
    failure_probabilities(i, 3) = 0;
    failure_probabilities(i, 4) = normcdf(-damage_stress(i, 2),
    material_strength_parameters(10),
    material_strength_parameters(11))*100;
elseif (damage_stress(i, 2) < 0) && (-damage_stress(i, 2) >=
    material_strength_parameters(12))
    failure_probabilities(i, 3) = 0;
    failure_probabilities(i, 4) = 100;
end

if (abs(damage_stress(i, 3)) < material_strength_parameters(15))
    failure_probabilities(i, 5) = normcdf(abs(damage_stress(i, 3)),
    material_strength_parameters(13),
    material_strength_parameters(14))*100;
elseif (abs(damage_stress(i, 3)) >=
    material_strength_parameters(15))
    failure_probabilities(i, 5) = 100;
end
end
% arrange_failure_probability_TT.m

% Calculates probability of failure for each interlaminar damage mode

failure_probabilities_TT = zeros(element_number*ply_number,3);

for i = 1:element_number*ply_number
    if (damage_stress_TT(i,1) > 0) && (damage_stress_TT(i,1) < material_strength_parameters(18))
        failure_probabilities_TT(i,1) = normcdf(damage_stress_TT(i,1),
                                               material_strength_parameters(16),
                                               material_strength_parameters(17))*100;
    elseif (damage_stress_TT(i,1) > 0) && (damage_stress_TT(i,1) >= material_strength_parameters(18))
        failure_probabilities_TT(i,1) = 100;
    elseif damage_stress_TT(i,1) < 0
        failure_probabilities_TT(i,1) = 0;
    end

    if abs(damage_stress_TT(i,2)) < material_strength_parameters(21)
        failure_probabilities_TT(i,2) = normcdf(abs(damage_stress_TT(i,2)),
                                               material_strength_parameters(19),
                                               material_strength_parameters(20))*100;
    elseif abs(damage_stress_TT(i,2)) >= material_strength_parameters(21)
        failure_probabilities_TT(i,2) = 100;

282
end

if abs(damage_stress_TT(i,3)) < material_strength_parameters(21)
    failure_probabilities_TT(i,3) = normcdf(abs(damage_stress_TT(i,3)),
                                                material_strength_parameters(19),
                                                material_strength_parameters(20))*100;
elseif abs(damage_stress_TT(i,3)) >= material_strength_parameters(21)
    failure_probabilities_TT(i,3) = 100;
end
end
Failure Prob Change

failure_prob_change.m

% calculates the change in failure probability following each damage event

%--In-Plane--%

change_damage = zeros(element_number*4, number_damage_events*5);

for i = 1:number_damage_events
    change_damage(:,1+5*(i-1)) = actual_failure_probabilities(:,6+5*(i-1)) - actual_failure_probabilities(:,1+5*(i-1));
    change_damage(:,2+5*(i-1)) = actual_failure_probabilities(:,7+5*(i-1)) - actual_failure_probabilities(:,2+5*(i-1));
    change_damage(:,3+5*(i-1)) = actual_failure_probabilities(:,8+5*(i-1)) - actual_failure_probabilities(:,3+5*(i-1));
    change_damage(:,4+5*(i-1)) = actual_failure_probabilities(:,9+5*(i-1)) - actual_failure_probabilities(:,4+5*(i-1));
    change_damage(:,5+5*(i-1)) = actual_failure_probabilities(:,10+5*(i-1)) - actual_failure_probabilities(:,5+5*(i-1));
end

%--Interlaminar--%

if check == 1
    change_damage_TT = zeros(element_number*4, number_damage_events*5);
end
number_damage_events*3);

for i = 1:number_damage_events
    change_damage_TT(:,1+3*(i-1)) =
    actual_failure_probabilities_TT(:,4+3*(i-1)) -
    actual_failure_probabilities_TT(:,1+3*(i-1));
    change_damage_TT(:,2+3*(i-1)) =
    actual_failure_probabilities_TT(:,5+3*(i-1)) -
    actual_failure_probabilities_TT(:,2+3*(i-1));
    change_damage_TT(:,3+3*(i-1)) =
    actual_failure_probabilities_TT(:,6+3*(i-1)) -
    actual_failure_probabilities_TT(:,3+3*(i-1));
end
end