Numerical Balancing in a Humidification
Dehumidification Desalination System

by

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Submitted to the Department of Mechanical Engineering
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Submitted to the Department of Mechanical Engineering on August 19, 2011, in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

Abstract

This thesis details research on the thermal and concentration balancing of a humidification dehumidification desalination system. The system operates similarly to the natural rain cycle. Seawater is heated, sprayed into an airstream to increase the air’s humidity, then pure water is condensed out of the same stream in a separate unit. These systems are typically inefficient due to entropy generation caused by mismatch between the temperature and humidity profiles in both the humidifier and dehumidifier components. Numerical models are developed for several different systems, and it is shown that for a given system with fixed inputs, entropy generation is minimized by way of balancing; i.e., the extraction and reinjection of the water or air streams within the humidifier and dehumidifier to equalize the capacity rates of the streams. Several modifications to existing baseline cycles are made to reach cases of minimum entropy generation. In these cases, the performance of the system is dramatically improved and the amount of energy needed to drive the system is reduced. For both on and off-design models, the addition of multiple extractions markedly improves the performance as compared to a baseline case with no extractions.

Thesis Supervisor: John H. Lienhard V
Title: Collins Professor of Mechanical Engineering
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**Nomenclature**

**Acronyms**

HDH  Humidification Dehumidification  
HME  Heat and Mass Exchanger  
ME  Multi-extraction  
MED  Mutiple Effect Distillation  
MSF  Multi-Stage Flash  
RO  Reverse Osmosis  
TTD  Terminal Temperature Difference  

**Dimensionless Quantities**

GOR  Gained Output Ratio \([\dot{m}h_f / \dot{Q}]\)  
HCR  Heat Capacity Rate ratio \([C_c / C_h]\)  
NTU  Number of Transfer Units \([UA / C_{min}]\)  
Pr  Prandtl number \([c_p \mu / k]\)  
Re  Reynolds number \([\rho v D_h / \mu]\)  
RR  Recovery Ratio \([\dot{m}_{pw} / \dot{m}_w]\)  
Sc  Schmidt number \([\mu / \rho - D_G]\)  

**Greek Symbols**

\(\alpha\)  heat transfer coefficient \([W/m^2\cdot K]\)  
\(\beta\)  mass transfer coefficient \([kg/m^2\cdot s]\)  
\(\Delta\)  change or difference \([-]\)  
\(\epsilon\)  effectiveness \([-]\)
\( \mu \) viscosity [\( \text{kg/m-s} \)]

\( \omega \) humidity ratio [\( \text{kg water vapor/kg dry air} \)]

\( \phi \) relative humidity [-]

\( \rho \) density [\( \text{kg/m}^3 \)]

**Roman Symbols**

\( \bar{z} \) dimensionless component length [-]

\( \dot{H} \) total enthalpy rate [kW]

\( \dot{m} \) mass flow rate [kg/s]

\( \dot{Q} \) heat transfer rate [kW]

\( \dot{S}_{gen} \) entropy generation rate [kW/K]

\( A \) area [m\(^2\)]

\( a \) specific surface area [\( \text{m}^2/\text{m}^3 \)]

\( C \) heat capacity rate [kW/K]

\( c_p \) specific heat capacity at constant pressure [kJ/kg-K]

\( C_f \) friction coefficient [-]

\( D \) diameter [m]

\( D_p \) nominal diameter of packing [m]

\( D_G \) mass diffusivity in gas [m\(^2\)/s]

\( D_h \) hydraulic diameter [m\(^2\)]

\( G \) mass flux of gas stream [kg/m\(^2\)-s]

\( h \) specific enthalpy [kJ/kg]

\( h_{fg} \) specific enthalpy of vaporization [kJ/kg]

\( k \) thermal conductivity [W/m-K]

\( L \) mass flux of liquid stream [kg/m\(^2\)-s]

\( m \) mass fraction [-]

\( m_r \) mass flow rate ratio, \( \dot{m}_w/\dot{m}_{da} \) [-]

\( N \) number [-]
\( P \) absolute pressure [kPa]
\( s \) specific entropy [kJ/kg-K]
\( T \) temperature [°C]
\( t \) thickness [m]
\( U \) overall heat transfer coefficient [W/m\(^2\)-K]
\( V \) volume [m\(^3\)]
\( Var \) normalized variance [-]
\( z \) height [m]

**Subscripts**
- \( a \) moist air
- \( aw \) wall, air-side
- \( c \) cold stream
- \( d \) distillate
- \( D_i \) dehumidifier \( i \)
- \( da \) dry air
- \( dp \) dew point
- \( ext \) extraction
- \( G \) gas
- \( h \) hot stream
- \( H_i \) humidifier \( i \)
- \( i \) inlet
- \( inj \) injection
- \( lm \) log mean
- \( o \) outlet
- \( pw \) product (pure) water
- \( s \) surface
- \( sat \) saturated condition
ss supersaturated
v vapor
w water
wb wet bulb
ww wall, water-side
int interface
max maximum
min minimum

**Superscripts**

$a$ evaluated at bulk air temperature

$ideal$ ideal condition

$w$ evaluated at bulk water temperature
Chapter 1

Introduction

As the world population continues to expand and nations become wealthier and more productive, demand for fresh water has grown. Residential, industrial, and agricultural processes all consume vast quantities of water; with many of the world’s fresh water resources already tapped, alternate solutions to acquiring this basic necessity are under development. Desalination technology has existed for many years, from the humble beginnings of small solar stills [1] to today’s colossal thermal desalination plants in the Middle East and high-tech reverse osmosis plants all over the world.

Desalination is a broad term which refers to several technologies and processes that remove salts from water, making it potable for human use. Any type of salt water can be desalinated, from highly saline seawater to slightly brackish inland water, although in general the greater the salt content, the more energy required to remove it. Desalination is an energy intensive process and requires considerable capital expenditure, making it far costlier than extracting fresh water from lakes, rivers, or groundwater. Even so, use of desalination processes has skyrocketed over the last several decades as water shortages in developing arid regions have driven demand for massive quantities of fresh water. The International Desalination Association (IDA) estimates that there are now over 15,000 contracted desalination plants with a capacity of 71.7 billion
liters of drinkable water per day [2]. The online capacity is over 65.2 billion liters per day, indicating that an additional 6.5 billion liters per day of future capacity is under construction. The majority of plants in the Middle East employ large thermally powered processes such as multi-stage flash (MSF) and multi-effect distillation (MED) plants. These plants are typically constructed alongside power plants to enable the coproduction of electricity and water. The plants consume considerable amounts of natural gas or oil to create steam which is used to generate electricity and drive the water/salt separation process. In locations such as the United States, where fossil fuels and construction of large plants are costlier, smaller, electrically driven reverse osmosis (RO) desalination plants have been the systems of choice. However, there are a variety of markets where neither of these technologies fully meets the needs of the customer. Many new technologies are emerging from universities and corporations to meet customers’ demands for lower cost, less energy intensive, easier to use technologies. Humidification dehumidification (HDH) desalination is one such potential technology and it is the focus of this study.

1.1 Humidification Dehumidification Desalination Systems

Humidification dehumidification systems have been employed for small scale desalination processes when large scale thermal systems such MSF and MED are unsuitable for the application due to cost and size, or where there is unsuitable electrical infrastructure to run RO. The HDH technology is a natural evolution from the solar still [3], and to date most systems have had a similar or slightly higher effectiveness of water production [4]. This, in turn, means water produced by HDH has often proved expensive [5, 6] due to the low efficiency of the separation process. Renewed efforts to increase this effectiveness are ongoing [7, 8, 9, 10], and this work also analyzes a
method to improve the efficiency of such devices. The thermodynamic cycle utilized in HDH technology is analogous to the natural rain cycle; just as in nature, water vapor is created by the evaporation of liquid water and then recondensed. Therefore, there are two major components of the HDH system: the humidifier and dehumidifier. In this research, a direct contact counterflow humidifier is employed to humidify the carrier gas stream. Next, an indirect contact counterflow dehumidifier recondenses liquid water out of the humid air. In between the two components a third component, a heater which may heat either the air or water stream, is added to drive the process. Figure 1-1 [11] illustrates a representative HDH system configuration.

![Schematic diagram of an air heated HDH system](image)

Figure 1-1: Schematic diagram of an air heated HDH system

The process involves the separation of pure water from a liquid mixture, typically
sea or brackish water. A humidifier is employed to evaporate water into a carrier gas. Any salt water not evaporated is rejected as brine. The moisture content of the carrier gas is increased, and it is then passed to a dehumidifier. In the dehumidifier, water is condensed out of the moist carrier gas and removed from the system. Heat or work must be input between or within any of the components in the air or water loop to drive the cycle. In summary, salt water and heat are input into the system (as well as some small amount of electric work to power any fans and pumps, if used) while the outputs are pure water and concentrated salt brine.

1.1.1 Performance Metrics of HDH Systems

The following parameters are typically used to describe the performance of an HDH system.

The figure of merit that defines energy performance for HDH and other thermal desalination systems is called the Gained Output Ratio (GOR). This parameter is a dimensionless number that measures the effectiveness of water production and is directly related to the amount of heat recovered within the system. Thus, a higher GOR corresponds to a more efficient system.

\[
\text{GOR} = \frac{\dot{m}_{pw} h_{fg}}{\dot{Q}_{in}}
\]  

(1.1)

where \( \dot{m}_{pw} \) is the flow rate of the product water stream, \( h_{fg} \) is the latent heat of evaporation of the water, and \( \dot{Q}_{in} \) is the energy input into the system. This figure of merit compares the amount of energy required to run the cycle with the amount of energy required to vaporize the product water.

Another frequently measured metric is the recovery ratio (RR). The recovery ratio is the fraction of product water to saline water input into the system:
A high recovery ratio is also desirable as more water is recovered from the inlet stream, requiring a smaller flow (thus, a smaller pump) of inlet water.

1.2 A Review of Heat and Mass Exchangers

The HDH system consists of two main components: a humidifier and a dehumidifier. These components are both heat and mass exchangers (HMEs), as within both components heat is transferred between the air and water streams in order to warm or cool the flow, and mass is transferred by way of water vapor diffusing in or out of the air stream. In order to describe key operating parameters of these systems, terminology must be developed to describe a few key phenomena. Section 1.2.1 formulates terminology for heat exchangers while Section 1.2.2 extends these concepts to heat and mass exchangers.

1.2.1 Heat Exchangers

While the humidifier and dehumidifier utilized in HDH systems are heat and mass exchangers, it is useful to review the concepts behind a simple heat exchanger as many of the definitions used in this research are derived from heat exchanger concepts. Several methods are available to analyze a heat exchanger, but one commonly employed method in two-stream counterflow design is the effectiveness method \cite{12, 13}. This approach is often used when the inlet or outlet temperatures must be calculated for given mass flow rates and exchanger design. In the effectiveness method, exchanger effectiveness may be characterized as the ratio of heat transferred to the ideal amount of heat transfer which could take place in an infinitely long exchanger. The actual amount of heat transferred between the two streams of the exchanger, assuming fixed
specific heats, is given by the first law of thermodynamics

\[ \dot{Q} = C_h(T_{h,in} - T_{h,out}) = C_c(T_{c,in} - T_{c,out}) \]  \hspace{1cm} (1.3)

where \( C_h = (m_c p)_h \) and \( C_c = (m_c p)_c \) are the capacity rates of the hot and cold streams respectively.

The temperature difference between the two streams acts as a driving force to propel heat from one stream to another. Any heat lost by one stream is picked up by the other. It is assumed that in an ideal heat exchanger, the ideal amount of heat may be transferred when one stream gives up all available heat to the opposed stream. Thus, the system must be limited by the stream with the lowest thermal capacity rate, \( C_{\text{min}} \). This stream, when all available heat is exchanged, changes temperature from the inlet temperature of one stream to that of the other. Therefore, the maximum amount of heat transferred is given by

\[ \dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,in} - T_{c,in}) \]  \hspace{1cm} (1.4)

where \( C_{\text{min}} \) is the capacity rate of the stream with the smallest thermal capacity.

Effectiveness, \( \epsilon \), then may be described as

\[ \epsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} \]  \hspace{1cm} (1.5)

where the value of \( \dot{Q} \) may be found from either the cold stream or hot stream, as both are equal. The value of effectiveness is a useful way to describe the performance of a heat exchanger. For a given effectiveness, mass flow rates, inlet hot stream temperature, and inlet cold stream temperature, the outlet conditions may be found via Equations 1.3, 1.4 and 1.5.

In order to optimize the performance of a heat exchanger, irreversibilities must be minimized. If entropy generation due to irreversibilities is minimized within the
component, a higher component efficiency may be realized. From the control volume form of the second law as applied to the heat exchanger, entropy generation may be calculated in terms of the inlet and outlet states

\[ \dot{S}_{\text{gen}} = \dot{m}_c (s_{\text{out}} - s_{\text{in}})_c + \dot{m}_h (s_{\text{out}} - s_{\text{in}})_h \]

\[ = \dot{m}_c c_p,c \ln \left( \frac{T_{c,\text{out}}}{T_{c,\text{in}}} \right) + \dot{m}_h c_p,h \ln \left( \frac{T_{h,\text{out}}}{T_{h,\text{in}}} \right). \]  

(1.6)

It may be found that \( \dot{S}_{\text{gen}} \) is minimized when the heat capacity rates are equal [14]. Thus, one further definition is useful in this analysis: the heat capacity rate ratio is defined as [15]

\[ \text{HCR} = \frac{C_c}{C_h} = \frac{(\dot{m}c_p)_c}{(\dot{m}c_p)_h}. \]  

(1.7)

For any value of effectiveness and for fixed inlet temperatures, when HCR is equal to unity, the non-dimensionalized entropy generation reaches a minimum [14, 16]. Consequently, this condition is said to be “balanced”. The term balanced implies that the system is thermally equalized and the change in temperature of the hot stream is equal to that of the cold. Both streams have a linear temperature variation with equal slope such that the \( \Delta T \) between the two is constant. As the capacity rates change and HCR strays further from 1, entropy generation increases and the performance of the heat exchanger suffers. Previously attainable outlet temperatures are no longer realistic because the capacity rates have changed. Thus, a balanced heat exchanger optimizes the performance of the system.

### 1.2.2 Heat and Mass Exchangers

In a heat and mass exchanger, such as either the humidifier or dehumidifier in an HDH desalination cycle, mass is exchanged via evaporation or condensation of water,
respectively. Whereas the heat exchanger transfers heat by a temperature gradient between the two streams, the driving force of a heat and mass exchanger is both the temperature gradient and the concentration gradient. In the HDH system, the concentration of water vapor in the air stream drives the mass transfer between the liquid and gaseous streams. It is useful to characterize a heat and mass exchanger in similar terms to the heat exchanger, but some modifications must be applied to the equations introduced in Section 1.2.1 to account for mass transfer. While Equation 1.5 defines effectiveness as the ratio of heat transfer to ideal heat transfer, the analogous equation in a heat and mass exchanger is the ratio of the actual change in total enthalpy rate of either stream to the maximum (or ideal) change in total enthalpy rate. Therefore, effectiveness is given as

$$
\epsilon = \frac{\Delta H}{\Delta H_{\text{max}}} \quad (1.8)
$$

where $\Delta H = \dot{m}_{w,i}h_{w,i} - \dot{m}_{w,o}h_{w,o}$ for the water stream or $\Delta H = \dot{m}_{da,i}h_{a,i} - \dot{m}_{da,o}h_{a,o}$ for the air stream. Additionally, $\Delta H_{\text{max}} = \dot{m}_{w,i}h_{w,i} - (\dot{m}_{w,o}h_{w,o})^{\text{ideal}}$ or $\Delta H_{\text{max}} = \dot{m}_{da,i}h_{a,i} - (\dot{m}_{da,o}h_{a,o})^{\text{ideal}}$ for the water and air streams respectively.

The ideal values of enthalpy, $h_{w,o}^{\text{ideal}}$ and $h_{a,o}^{\text{ideal}}$, are the values of enthalpy of water at the air inlet temperature and saturated air at the inlet water temperature respectively (where $P$ is taken at the actual pressure conditions of the stream).

The change of terms from $\dot{Q}$ to $\dot{H}$ is driven by the necessity to include the humidity of the air in the amount of energy transferred. Neglecting pressure variation, the enthalpy of moist air is a function of temperature, pressure, and humidity ratio, such that

$$
dh_a = c_{p,a}dT + \left(\frac{\partial h_a}{\partial \omega}\right)_{P,T} d\omega. \quad (1.9)
$$

It is clear that in the case of the heat exchanger, $dh_a$ reduces to $c_{p,a}dT$ because $d\omega = 0$. 

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Thus, for the heat exchanger, the effectiveness equation reduces back to Equation 1.5 which is a typical definition of effectiveness for a two stream heat exchanger [17]:

$$
\epsilon = \frac{\Delta \dot{H}}{\Delta \dot{H}_{\text{max}}} = \frac{\Delta (\dot{m}_a h_a)}{\Delta (\dot{m}_a h_a)_{\text{max}}} = \frac{\Delta (\dot{m}_a c_{p,a} T)}{\Delta (\dot{m}_a c_{p,a} T)_{\text{max}}} = \frac{\dot{Q}}{\dot{Q}_{\text{max}}}. \tag{1.10}
$$

But in heat and mass exchangers, the enthalpy rate must be employed to include the impact of humidity. Humidity can vary significantly within a single heat and mass exchanger, and this must be taken into account when determining the total energy transfer between the streams.

While $\dot{S}_{\text{gen}}$ in a heat and mass exchanger may be again expressed by Equation 1.6 (as the entropy of moist air term includes the effects of humidity), HCR must undergo a modification to make it usable. From Equation 1.7, HCR for a heat exchanger was defined as

$$
\text{HCR} = \frac{C_c}{C_h}.
$$

But this equation may be rephrased as [15]

$$
\text{HCR} = \frac{\Delta \dot{H}_{\text{max,c}}}{\Delta \dot{H}_{\text{max,h}}} \tag{1.11}
$$

because the maximum temperature difference in the terms $\Delta \dot{H}_{\text{max,c}}$ and $\Delta \dot{H}_{\text{max,h}}$ is the same: namely, $T_{h,i} - T_{c,i}$. In the case of heat transfer only, Equation 1.11 reduces to Equation 1.7. In a heat and mass exchanger, the impact of mass exchange does not allow an identical $\Delta T$. Therefore, Equation 1.11 is employed for a heat and mass exchanger.

Similarly to the case of a simple heat exchanger (Section 1.2.1), a heat and mass exchanger is considered balanced and entropy generation is minimized when HCR = 1 [14]. This case demonstrates an optimal point in which performance of a given system is dramatically improved, lowering the energy requirements to run the cycle.
1.3 Concept of Balancing

As noted in Section 1.2.1, a heat exchanger is thermally balanced when the heat capacity ratio, HCR, is equal to one. While this is a well known concept for heat exchangers, it has been recently applied to heat and mass exchangers [14] as well. To completely balance a heat and mass exchanger, the driving force of energy transfer, namely a combination of temperature and mass concentration difference, must be balanced. A balanced HDH system maintains a constant driving force by maintaining a constant difference in temperature and mass concentration between the air and water streams of both the humidifier and the dehumidifier. In the case of HDH desalination, the mass concentration difference is a function of the humidity of the bulk air stream and of the interfacial region between air and water.

In order to balance the humidifier and dehumidifier, the air and water streams of each must transfer heat between the two while the driving force within the system is kept constant. In such a case, the temperature and mass concentration profiles of the two streams along the length of the humidifier and dehumidifier are parallel. In an unbalanced case, the heat flow from one stream to the other is far from ideal because the heat capacity of one stream is dissimilar to the other. If one stream is unable to transfer enough heat to the other, the temperature gap between the streams widens and the system becomes less efficient. Conversely, when the heat capacity rate between the streams is equivalent, the heat flow between the streams is optimized, temperature and concentration divergences are eliminated, and efficiency increases. In the following sections, a simple analysis of an unbalanced and a balanced HDH system is presented.

1.3.1 Imbalanced HDH System

A schematic diagram of a simple HDH desalination system is presented in Figure 1-2. An unoptimized system consists of three components: the humidifier, dehumidifier,
and a heater. The system in Figure 1-2 is an air heated, closed air open water system (CAOW), though there are many permutations of such systems [7]. Cool seawater enters at the bottom of the dehumidifier and is heated by the warm air entering at the top in a counterflow configuration. As the air cools, pure water condenses out of the stream and exits the component. The heated water enters the top of the humidifier which warms and humidifies the incoming air from the bottom of the device. The water cools, and any water that is not evaporated exits the component as brine. The warm air is further heated in the heater between the air outlet of the humidifier and inlet of the dehumidifier.

Figure 1-2: Simple schematic of an air-heated CAOW HDH system

A system such as the one displayed in Figure 1-2 may be optimized for components of given effectiveness by varying such parameters as the inlet temperatures and mass flow rates. However, it is not possible to balance both the humidifier and dehumidifier
simultaneously in this configuration. A single component may be balanced such that HCR = 1, but this will typically leave the other component unbalanced. Figures 1-3a and 1-3b illustrate the temperature profiles of an example case where the components are unbalanced. As water enters the dehumidifier at 30°C, it is gradually heated by the air. The temperature profiles are not parallel because the capacity rates of the streams are not matched. Therefore, water is not sufficiently heated in 1-3a and exits the component at a relatively low temperature of 50°C. Similarly, the profiles are not matched in the humidifier shown in 1-3b. Additionally, due to the low inlet water temperature, the air in the humidifier does not reach a high temperature at the outlet. This result lowers performance for two reasons: first, even if the outlet air is saturated, the absolute humidity in the air is relatively low compared to a case where the air leaves at a much higher temperature. This results in less vapor entering the dehumidifier so less product water can be expected. Second, much more energy much be added to the system by way of the heater to raise the temperature of the air before it enters the system. As the performance metric, GOR, is a function of both the flow rate of product water as well as the input heat, this imbalance directly (and negatively) impacts the value of GOR.

As discussed in Section 1.2.2, the humidifier and dehumidifier in an HDH cycle
are heat and mass exchangers. Therefore, unlike a heat exchanger, it is not only the
temperature profiles that need to be balanced, but the concentration profiles as well.

Figure 1-4 illustrates the corresponding humidity profile derived from Figure 1-3a.
It is clear that at the component is very unbalanced as the profiles are not parallel,
particularly at the hotter end of the equipment where they diverge considerably.

![Figure 1-4: Unbalanced dehumidifier humidity profile](image)

### 1.3.2 Balanced HDH Systems

It has been proposed that to balance the humidifier and dehumidifier fluid from either
the air or water stream may be extracted from one one component and injected into
the other \([14, 18]\). An extraction may be placed at any height along the length of
the component and does not necessarily need to be injected at the same height, or
temperature (though injecting at the same temperature is favorable as it circumvents
mixing losses), in the other. By changing the heat capacity rate of the streams at any
point inside the component, the temperature and concentration gradients of the two
streams may be adjusted, forcing their profiles closer to each other in order to reduce
entropy generation at a given height within the component. For example, extracting
water from the dehumidifier per Figure 1-5a will adjust the temperature profile of
the water stream and bring it closer to a parallel arrangement with the air stream.
temperature profile. Additionally, the extraction adjusts the concentration profiles, though as seen in Figure 1-5b, the impact here is smaller. Nonetheless, this single extraction point reduces the entropy generation in the component. If this combination of extraction and injection is performed multiple times, in an arrangement known as multi-extraction (ME), the difference between the two streams in both components is reduced along the entire length of the device as seen in Figures 1-6a and 1-6b.

In the fully balanced system, the maximum water temperature (before heating) has been increased from 50 to 85°C by reducing the terminal temperature difference in the dehumidifier [19]. It is possible to increase the temperature range with a single or small number of extraction points, and such balancing would improve GOR. However, this method will not have a value of GOR which matches an equivalent MED system because there is a finite stream to stream temperature difference to drive the rate processes. With a single extraction, the slope of the temperature profile within a component can be improved to better match the adjacent stream, but there will still be a large divergence of the two streams away from the system inlet, outlet, and extraction point. Only when there are multiple extractions can the stream profiles be forced together along the entire length of the component causing the mismatch between the streams to reach a minimum. In this fully balanced case, the system
has a similar temperature range to a comparable MED system and, importantly, has reduced entropy generation due to small and constant mismatch between the profiles of each stream.

Additionally, the concentration profiles are considerably more parallel in the fully balanced case. Per Figure 1-7, the interface humidity closely matches the humidity in the bulk airflow. However, it is important to note that even though the temperature profile is fully balanced, the concentration profile is not: there is still variance in the profile, particularly at high temperatures. This result comes from the nonlinear relationship between temperature and humidity. As temperature increases, humidity
increases exponentially. In fact, Thiel et. al [20] show that while the ideal case is one where the variance in both driving forces is equal to zero, it is not possible to achieve this state. If one driving force variance is brought to zero, the other will have a finite, non-zero value. Thus, in Chapters 2 and 3, when cycles are created to minimize entropy generation via extraction, the variance does not become zero in both driving forces.

1.4 Comparison to MED

In this section, HDH technology will be put side by side with a Multiple Effect Distillation (MED) system. A useful comparison of thermal performance is drawn in order to illustrate the aims of thermal balancing delineated in Section 1.3.

MED consists of several consecutive chambers (effects) in which seawater is vaporized and subsequently cooled to form pure liquid water per Figure 1-8. In a single effect system, the input heat required to vaporize the water is equivalent to the latent heat of evaporation, \( h_{fg} \), for the mass of water transformed from a liquid to gaseous state. All of the heat input into the system is used once to evaporate the water and there is no heat recovery. This cycle corresponds to a Gained Output Ratio (GOR) of about 1. \(^1\)

To improve GOR, heat can be recycled from this first effect to evaporate an additional mass of water in subsequent effects. In a typical arrangement like Figure 1-8, no additional heat supplements the downstream effects, and the heat from the previous effect is recovered in the condenser to power the boiler of the next effect. Thus, each effect will be at a lower temperature than the preceding, and therefore the pressure must be reduced at each stage in order to induce vaporization. Ideally, this process could be repeated in an infinite number of effects, but in application the

\(^1\)Recall per Section 1.1.1, GOR is the figure of merit that describes HDH as well as other thermal desalination systems and will be utilized throughout this paper to evaluate the performance of various system configurations.
number of effects is limited by the boiling point elevation of evaporating seawater, thermal losses, and the size of the equipment; therefore, there is a finite temperature drop from each stage to the next. If the system is sized to produce an equivalent mass of water in each stage, the GOR is approximately equal to the number of stages. For instance, for the system mapped in Figure 1-9, ideally the GOR is about 13.

Figure 1-9: Temperature and pressure per stage in an example MED plant

For a comparable HDH system (Figure 1-2), the cycle is not broken up into separate effects because the evaporation and condensation do not occur at a constant
temperature. Using the same boundary conditions as with the HDH system above described in Section 1.3, the MED system initially has a much higher value of GOR than the unbalanced HDH system. Wherein the MED system, both the water and air temperature ran the full range of 90 °C to 30 °C, in the HDH dehumidifier the water stream is heated linearly and cannot maintain the same ΔT to the air stream throughout the length of the dehumidifier. The temperature range for the water is about \( \frac{1}{3} \) to \( \frac{1}{4} \) of the MED system. The GOR sees a similar reduction down to a value of approximately 3-4.

Only when the HDH system is balanced does the GOR approach the value of the similar MED system. In Figure 1-6, the GOR of the HDH system approaches 11 compared to the MED GOR of 13. In this case, balancing has improved GOR dramatically by optimizing the heat flux between the streams of both humidifier and dehumidifier and minimizing entropy generation. For an HDH system to be on par with a similar MED system, it is clear that both the humidifier and dehumidifier must be thermally balanced.
Chapter 2

On-Design Humidification

Dehumidification Model

In order to evaluate the impact of multi-extraction (ME) on the HDH system, two thermodynamic cycle models have been created. These models are considered to be on-design: they are “black-box” models that do not evaluate transport properties. They are evaluated thermodynamically for feasibility. In Chapter 3, off-design models are explored which simulate real systems utilizing transport properties. In this chapter, first, a model that treats the humidifier and dehumidifier as distinct and separate units was designed to evaluate the impact of ME on each component’s total entropy production. Minimizing entropy production in each unit should in turn increase total system performance when the humidifier and dehumidifier are considered in relation to the total desalination cycle. In Section 2.4, the two separate units are tied together with the addition of the heater to produce the complete cycle. In this case, the entire system is included in the model and the thermal performance, in terms of GOR, may be determined.
2.1 Individual Humidifier and Dehumidifier

Before creating a full system cycle, the humidifier and dehumidifier are analyzed as individual units. While system parameters such as GOR and RR cannot be determined in this initial model (as there is no full system to evaluate), from Narayan et al. [14] it is clear that entropy generation within the component directly impacts performance. In this study, entropy generation will be evaluated for individual components undergoing a single extraction or injection. The following assumptions are made for these models:

- The cycle operates under steady state conditions.
- Pumping and blowing power is negligible.
- There are no pressure losses.
- All components are adiabatic with respect to their surroundings, i.e., heat loss is negligible.
- Kinetic and potential energy terms are not included in the energy balance.

2.2 Humidifier

In the first pass, the humidifier is “split” into two separate subcomponents per Figure 2-1. This method allows for a single extraction or injection stream to be inserted between the two subcomponents. Each subcomponent may be analyzed individually, or the entire humidifier may be scrutinized. With the water and air inlet states specified (temperature, mass flow rate, humidity in the air stream) as well as the extraction rate and the total component effectiveness, the only unknowns in the system are the outlet temperatures and outlet mass flow rate of water (dry air mass flow rate is assumed to be constant). These three unknowns are solved via three equations: a first law energy balance, a mass balance on water, and the effectiveness equation. Additionally, another first law control volume is drawn around the injection
site to determine the state of water entering humidifier 2. Finally, the second law is used as a check to ensure no components are in violation of entropy production.

![Diagram of two sub-component humidifier submodel](image)

Figure 2-1: Two sub-component humidifier submodel

### 2.2.1 Governing Equations

The following are governing equations for the humidifier. Shown below are equations for the *entire* humidifier (both components and injection). The same equations are utilized for the individual subcomponents as well, with the appropriate inlet and outlet states for each.
Mass Balance

The dry mass flow of air is constant through the humidifier, thus each state has an identical mass flow rate (in kg-dry air/s). The mass flow rate of water (in kg/s) entering the system is a fixed, known value, but the subsequent states are not identical to the first. Water evaporates as it passes through each humidifier stage, reducing the flow rate downstream. This loss of water is taken into account in Equations 2.2 and 2.4 by the \( \dot{m}_w \) terms where \( \omega \) is the humidity ratio of the air stream, in kg-water/kg-dry air. Additionally, per Equation 2.3, the injection point adds (or removes) extra water into the stream.

\[
\dot{m}_{da,H2,i} = \dot{m}_{da,H2,o} = \dot{m}_{da,H1,i} = \dot{m}_{da,H1,o} \tag{2.1}
\]
\[
\dot{m}_{w,H1,o} = \dot{m}_{w,H1,i} - (\dot{m}_{da,H1,o}\omega_{a,H1,o} - \dot{m}_{da,H1,i}\omega_{a,H1,i}) \tag{2.2}
\]
\[
\dot{m}_{w,H2,i} = \dot{m}_{w,H1,o} + \dot{m}_{w,inj} \tag{2.3}
\]
\[
\dot{m}_{w,H2,o} = \dot{m}_{w,H2,i} - (\dot{m}_{da,H2,o}\omega_{a,H2,o} - \dot{m}_{da,H2,i}\omega_{a,H2,i}) \tag{2.4}
\]

First Law Energy Balance

A control volume is drawn around the full system (both components). Energy enters the system at the air and water inlets as well as the injection site. Energy exits the control volume at the air and water outlets. Thermophysical pure water properties \(^1\), such as enthalpy, \( h_w \), come from the property correlation of Harr, Gallagher and Kell [22]. Thermophysical properties of moist air are derived from Hyland and Wexler [23] (similar to ASHRAE’s in [24]) and treat humid air as a binary mixture of dry air and

\(^{1}\)All water streams in this paper are considered to be pure water for ease of calculation. Seawater properties such as enthalpy and entropy vary by under 10% from that of pure water for the temperature ranges evaluated in this paper [21].

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water vapor. Specifically, \( h_a = h_{da} + \omega h_v \). 

\[
0 = \dot{m}_{w,H_1,i}h_{w,H_1,i} - \dot{m}_{w,H_2,o}h_{w,H_2,o} + \dot{m}_{da,H_2,i}h_{a,H_2,i} - \dot{m}_{da,H_1,o}h_{a,H_1,o} + \dot{m}_{w,inj}h_{w,inj}
\] (2.5)

A separate control volume is drawn around the injection site in order to correctly assess the temperature of the water immediately downstream of the injection.

\[
\dot{m}_{w,H_1,o}h_{w,H_1,o} + \dot{m}_{w,inj}h_{w,inj} = \dot{m}_{w,H_2,i}h_{w,H_2,i} \quad \text{(Injection site)}
\] (2.6)

**Second Law**

The second law is calculated for the same control volume as in the first law. The entropy generation in the humidifier, \( \dot{S}_{gen,H} \), is evaluated to ensure it is greater than zero and the second law has not been violated for a given set of input conditions. Entropy values, \( s \), are again derived from Harr, Gallagher and Kell [22] and Hyland and Wexler [23]. Like with enthalpy, entropy for moist air is described as a mixture of dry air and water vapor: \( s_a = s_{da} + \omega s_v \).

\[
\dot{S}_{gen,H} = \dot{m}_{w,H_2,o}s_{w,H_2,o} - \dot{m}_{w,H_1,i}s_{w,H_1,i} + \dot{m}_{da,H_1,o}s_{a,H_1,o} - \dot{m}_{da,H_2,i}s_{a,H_2,i} - \dot{m}_{w,in}s_{w,inj}
\] (2.7)

**Effectiveness**

Per Section 1.2.2, the effectiveness is the final equation to solve the unknown outlet states of the system. Effectiveness compares the actual versus ideal energy transfer for each stream. The ideal values of enthalpy, \( h_{w,H,o}^{ideal} \) and \( h_{a,H,o}^{ideal} \), are the values of enthalpy of water at the air inlet temperature and saturated air at the inlet water temperature respectively. The total humidifier effectiveness, \( \epsilon_H \), is defined as the
maximum of the effectiveness of the air and water streams.

\[
\dot{Q}_{w,H} = \dot{m}_{w,H,i} h_{w,H,i} - \dot{m}_{w,H,o} h_{w,H,o} + \dot{m}_{w,inj} h_{w,inj}
\]  
(2.8)

\[
\dot{Q}_{\text{ideal},w,H} = \dot{m}_{w,H,i} h_{w,H,i} - \dot{m}_{w,H,o} h_{w,H,o} + \dot{m}_{w,inj} h_{w,inj}
\]  
(2.9)

\[
\dot{Q}_{a,H} = \dot{m}_{da,H,i} h_{a,H,i} - \dot{m}_{da,H,o} h_{a,H,o}
\]  
(2.10)

\[
\dot{Q}_{\text{ideal},a,H} = \dot{m}_{da,H,i} h_{a,H,i} - \dot{m}_{da,H,o} h_{a,H,o}
\]  
(2.11)

\[
\epsilon_{w,H} = \frac{\dot{Q}_{w,H}}{\dot{Q}_{\text{ideal},w,H}}
\]  
(2.12)

\[
\epsilon_{da,H} = \frac{\dot{Q}_{a,H}}{\dot{Q}_{\text{ideal},a,H}}
\]  
(2.13)

\[
\epsilon_H = \max(\epsilon_{w,H}, \epsilon_{a,H})
\]  
(2.14)

### 2.2.2 Results

A comprehensive study was previously performed which demonstrated that the non-dimensional entropy production is minimized and that a heat and mass exchanger reaches a balanced state when HCR = 1 for a component of fixed energy effectiveness [15]. In the case of analyzing the individual humidifier and dehumidifier, it was found that this statement holds true for cases when an extraction or an injection is applied to the component. At this state, heat flow between the air and water stream is optimized and entropy reaches a minimum for the given boundary conditions. With no extraction, a certain mass flow rate ratio will balance each component separately minimizing the entropy generation for either the humidifier or dehumidifier. However, these components do not exhibit optimized temperature profiles thus entropy generation may be reduced by an appropriate extraction or injection.

Figure 2-2 illustrates the impact of extracting or injecting into the water stream when total effectiveness is held at $\epsilon_H = 0.8$. The curves are generated by varying the mass flow rate ratio while keeping all other inputs constant. It is clear that, as within
the non-extraction cases, when flow is injected the normalized entropy generation is still minimized in the balanced condition, i.e. HCR = 1. Additionally, it may be seen that the $\dot{S}_{gen}$ vs. HCR curve shifts for a given injection or extraction rate such that the normalized entropy generation may be altered. In this case, injecting flow reduces the total normalized entropy generation at its minimum point and along the curve. The following Table 2.1 illustrates the results from the run with the smallest entropy generation in Figure 2-2 (namely, 60% injection rate at HCR = 1). The specified input values (boundary conditions) are in bold while the resultant values are in plain text.

Figure 2-2: Total normalized entropy generation, humidifier submodel; $\epsilon_H = 0.8$
<table>
<thead>
<tr>
<th>State</th>
<th>Mass Flow [kg/s]</th>
<th>Temperature [°C]</th>
<th>Relative Humidity [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W,H1,i</td>
<td>1.00</td>
<td>55.0</td>
<td>–</td>
</tr>
<tr>
<td>W,H1,o</td>
<td>0.979</td>
<td>41.2</td>
<td>–</td>
</tr>
<tr>
<td>INJ</td>
<td>0.600</td>
<td>46.8</td>
<td>–</td>
</tr>
<tr>
<td>W,H2,i</td>
<td>1.579</td>
<td>43.3</td>
<td>–</td>
</tr>
<tr>
<td>W,H2,o</td>
<td>1.568</td>
<td>38.6</td>
<td>–</td>
</tr>
<tr>
<td>DA,H1,o</td>
<td>0.524</td>
<td>52.3</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,H1,i</td>
<td>0.524</td>
<td>42.8</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,H2,o</td>
<td>0.524</td>
<td>42.8</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,H2,i</td>
<td>0.524</td>
<td>35.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2.1: Minimum entropy generation states from humidifier submodel

2.3 Dehumidifier

Like the humidifier, the dehumidifier is “split” into two separate subcomponents per Figure 2-3. Again, the water and air inlet states are specified (temperature, mass flow rate, humidity in the air stream) as well as the extraction rate, the only unknowns in the system are the outlet temperatures and outlet mass flow rate of product water (dry air mass flow and inlet water flow rates are assumed to be constant). These three unknowns are solved via three equations: a first law energy balance, a mass balance, and the effectiveness equation. The second law is used as a check to ensure no components are in violation.

2.3.1 Governing Equations

The following are governing equations for the dehumidifier. Shown below are equations for the entire dehumidifier (both components and extraction). The same equations are utilized for the individual subcomponents as well, with the appropriate inlet and outlet states for each.

Mass Balance

The dry mass flow of air is constant through the dehumidifier, thus each state has an
identical mass flow rate. Unlike the humidifier where water evaporates from the water to air stream, here the water stream is only in indirect contact with the air stream so it does not experience a change in mass flow rate within a component. However, the extraction point removes (or adds) extra water into the stream. A final difference between the humidifier and dehumidifier: in the case of dehumidification, additional equations (2.19 and 2.20) are required to calculate the generation of product water. Here, the product water mass flow rate is found from the difference between inlet and
exit humidity in the air stream multiplied by the mass flow rate of the same.

\[
\dot{m}_{da,D1,i} = \dot{m}_{da,D1,o} = \dot{m}_{da,D2,i} = \dot{m}_{da,D2,o} \quad (2.15)
\]

\[
\dot{m}_{w,D2,i} = \dot{m}_{w,D2,o} \quad (2.16)
\]

\[
\dot{m}_{w,D1,i} = \dot{m}_{w,D2,o} - \dot{m}_{w,ext} \quad (2.17)
\]

\[
\dot{m}_{w,D1,o} = \dot{m}_{w,D1,i} \quad (2.18)
\]

\[
\dot{m}_{pw,D1} = \dot{m}_{da,D1,i}\omega_{a,D1,i} - \dot{m}_{da,D1,o}\omega_{a,D1,o} \quad (2.19)
\]

\[
\dot{m}_{pw,D2} = \dot{m}_{da,D2,i}\omega_{a,D2,i} - \dot{m}_{da,D2,o}\omega_{a,D2,o} \quad (2.20)
\]

**First Law Energy Balance**

A control volume is drawn around the full system (both components). Energy enters the system at the air and water inlets. Energy exits the control volume at the air and water outlets, the product water outlets, and the extraction site.

\[
0 = \dot{m}_{w,D2,i}h_{w,D2,i} - \dot{m}_{w,D1,o}h_{w,D1,o} + \dot{m}_{da,D1,i}h_{a,D1,i} - \dot{m}_{da,D2,o}h_{a,D2,o} \\
- \dot{m}_{pw,D1}h_{pw,D1} - \dot{m}_{pw,D2}h_{pw,D2} - \dot{m}_{w,ext}h_{w,ext} \quad (2.21)
\]

The enthalpy of the pure water streams leaving each dehumidifier subcomponent is evaluated by a polynomial function created by K. Mistry [25] which calculates the bulk temperature of the product stream as a function of air inlet and outlet wet bulb temperatures:

\[
T_{pw,D1} = 0.0051918T_{wb,a,D1,i}^2 + 0.0027692T_{wb,a,D1,o}^2 - 0.007417T_{wb,a,D1,i}T_{wb,a,D1,o} - 0.41913T_{wb,a,D1,i} + 1.0511T_{wb,a,D1,o} + 61.6186 \quad (2.22)
\]
\[ T_{pw,D2} = 0.0051918T_{wb,a,D2,i}^2 + 0.0027692T_{wb,a,D2,o}^2 - 0.007417T_{wb,a,D2,i}T_{wb,a,D2,o} \]
\[ - 0.41913T_{wb,a,D2,i} + 1.0511T_{wb,a,D2,o} + 61.6186 \]

(2.23)

These equations for temperature assume a continuous removal of condensate from the condensing surface.

**Second Law**

The second law is calculated for the same control volume as in the first law. The entropy generation in the dehumidifier, \( \dot{S}_{gen,D} \), is evaluated to ensure it is greater than zero and that the second law has not been violated for a given set of input conditions.

\[ \dot{S}_{gen,D} = \dot{m}_{w,D1,o}s_{w,D1,o} - \dot{m}_{w,D2,i}s_{w,D2,i} + \dot{m}_{da,D2,o}s_{a,D2,o} - \dot{m}_{da,D1,i}s_{a,D1,i} \]
\[ + \dot{m}_{pw,D1}s_{pw,D1} + \dot{m}_{pw,D2}s_{pw,D2} + \dot{m}_{w,ext}s_{w,ext} \]

(2.24)

**Effectiveness**

Per Section 1.2.2, the effectiveness is the final equation to solve the unknown outlet states of the system. Effectiveness compares the actual versus ideal energy transfer for each stream. The ideal values of enthalpy, \( h_{w,D,o}^{ideal} \) and \( h_{da,D,o}^{ideal} \), are the values of enthalpy of water at the air inlet temperature and saturated air at the inlet water temperature respectively. The total dehumidifier effectiveness, \( \epsilon_D \), is defined as the
maximum of the effectiveness of the air and water streams.

\[
\dot{Q}_{w,D} = \dot{m}_{w,D,i} h_{w,D,i} - \dot{m}_{w,D,o} h_{w,D,o} - \dot{m}_{w,ext} h_{w,ext} \tag{2.25}
\]

\[
\dot{Q}_{\text{ideal},w,D} = \dot{m}_{w,D,i} h_{w,D,i} - \dot{m}_{w,D,o} h_{\text{ideal},w,D,o} - \dot{m}_{w,ext} h_{w,ext} \tag{2.26}
\]

\[
\dot{Q}_{a,D} = \dot{m}_{a,D,i} h_{a,D,i} - \dot{m}_{a,D,o} h_{a,D,o} - \dot{m}_{pw,D1} h_{pw,D1} - \dot{m}_{pw,D2} h_{pw,D2} \tag{2.27}
\]

\[
\dot{Q}_{\text{ideal},a,D} = \dot{m}_{a,D,i} h_{a,D,i} - \dot{m}_{a,D,o} h_{\text{ideal},a,D,o} - \dot{m}_{pw,D1} h_{pw,D1} - \dot{m}_{pw,D2} h_{pw,D2} \tag{2.28}
\]

\[
\epsilon_{w,D} = \frac{\dot{Q}_{w,D}}{\dot{Q}_{\text{ideal},w,D}} \tag{2.29}
\]

\[
\epsilon_{a,D} = \frac{\dot{Q}_{a,D}}{\dot{Q}_{\text{ideal},a,D}} \tag{2.30}
\]

\[
\epsilon_D = \max(\epsilon_{w,D}, \epsilon_{a,D}) \tag{2.31}
\]

2.3.2 Results

Figure 2-4 illustrates the impact of extracting or injecting into the water stream when total effectiveness is held at \(\epsilon_D = 0.8\). Again, as within the non-extraction cases, when flow is extracted the normalized entropy generation is still minimized in the balanced condition, i.e., \(HCR = 1\). Additionally, it may be seen that the \(\dot{S}_{\text{gen}}\) vs. \(HCR\) curve shifts for a given injection or extraction rate such that the normalized entropy generation may be altered. In this case, extracting flow reduces the total normalized entropy generation at its minimum point and along the curve. The following Table 2.2 illustrates the results from the run with the smallest entropy generation in Figure 2-4 (namely, 60% extraction rate at \(HCR = 1\)). The specified input values (boundary conditions) are in bold while the resultant values are in plain text.

2.4 Combined System Model

Thus far, both the humidifier and dehumidifier have had fixed inlet conditions and a fixed component energy effectiveness. With these boundary conditions, it is clear
that the system entropy generation is minimized at HCR = 1 and improved by way of flow extractions and injections. The subsequent stage of modeling considers the full system: a cycle including a linked humidifier, dehumidifier, and heater. In such a cycle, certain inputs from the previous model are no longer fixed, such as the humidifier air and water inlet temperatures, as they are derived from the respective dehumidifier outlets. It was determined that holding the full component effectivenesses, $\epsilon_H$ and $\epsilon_D$, constant gave ambiguous results. This is a consequence of the definition of effectiveness. When effectiveness is fixed but an extraction is added to the component, the extraction has a much larger effect on the inlet and outlet streams than it physically should due to its large impact in the effectiveness equation. If effectiveness is defined for the subcomponents, the extraction stream does not show up in the equation. Thus, the individual components have a fixed effectiveness with no influence from the extraction, but the full humidifier or dehumidifier is influenced by
<table>
<thead>
<tr>
<th>State</th>
<th>Mass Flow [kg/s]</th>
<th>Temperature [°C]</th>
<th>Relative Humidity [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>W,D1,i</td>
<td>0.400</td>
<td>46.8</td>
<td>–</td>
</tr>
<tr>
<td>W,D1,o</td>
<td>0.400</td>
<td>56.9</td>
<td>–</td>
</tr>
<tr>
<td>EXT</td>
<td>0.600</td>
<td>46.8</td>
<td>–</td>
</tr>
<tr>
<td>W,D2,i</td>
<td>1.000</td>
<td>30.0</td>
<td>–</td>
</tr>
<tr>
<td>W,D2,o</td>
<td>1.000</td>
<td>46.8</td>
<td>–</td>
</tr>
<tr>
<td>PW,D1</td>
<td>0.007</td>
<td>68.6</td>
<td>–</td>
</tr>
<tr>
<td>PW,D2</td>
<td>0.028</td>
<td>57.0</td>
<td>–</td>
</tr>
<tr>
<td>DA,D1,i</td>
<td>0.169</td>
<td>70.0</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,D1,o</td>
<td>0.169</td>
<td>67.3</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,D2,i</td>
<td>0.169</td>
<td>67.3</td>
<td>1.0</td>
</tr>
<tr>
<td>DA,D2,o</td>
<td>0.169</td>
<td>46.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2.2: Minimum entropy generation states from dehumidifier submodel

the extraction and its effectiveness changes accordingly. In the following model then, \( \epsilon_{H1} \), \( \epsilon_{H2} \), \( \epsilon_{D1} \), and \( \epsilon_{D2} \) are held constant while \( \epsilon_H \) and \( \epsilon_D \) are allowed to float. Because of this change, the extraction temperature is no longer fixed but is determined by the effectiveness of the dehumidifiers 1 and 2. In the humidifier, the injection stream is no longer the same temperature as the location where it is injected; rather the downstream temperature from the injection point is solely determined by the energy balance at the injection site.

### 2.4.1 Zero Extraction Model

The model (Figure 2-5) is completed by linking the humidifier air outlet to the heater inlet, the heater outlet to the dehumidifier air inlet, and the dehumidifier water outlet to the humidifier water inlet.

Running the full model with zero extraction yields a measure of GOR with respect to mass flow rate ratio. With no extractions, the GOR in this system is low due to the excess entropy generation caused by non-ideal humidifier and dehumidifier temperature profiles. As illustrated in Figure 2-6, GOR peaks near 2.75, while from the review of HDH technology in Section 1.4, one may expect a GOR of up to 7-8 for
a fully balanced case. Input conditions for this case are illustrated in Table 2.3.

An important feature of this model to take into consideration is the fact that it is effectively two pairs of two components (humidifier and dehumidifier) each with an effectiveness of 0.8 per subcomponent. This is markedly different that a single component with $\epsilon = 0.8$ as energy will be transferred in the first component and then again in the second. For this reason, GOR in this two subcomponent baseline will necessarily be higher than in the single component case. The impact of this byproduct of the model is discussed further in Section 2.4.5 and in Appendix A.
2.4.2 Single Extraction Model

In order to improve the system efficiency, a single extraction is made per Figure 2-7. Water is extracted from the dehumidifier and injected into the humidifier. The extraction site is taken as a point between the two dehumidifier subcomponents and is tied to a point between the two humidifier subcomponents. In this model, these points are dictated by the assigned effectivenesses of each subcomponent. For example, the extraction point will be at a different temperature when each subcomponent is at \( \epsilon = 0.8 \) than when they are at \( \epsilon = 0.9 \). At certain values of mass flow rate ratio\(^2\) (\(m_r\)), this single extraction/injection pair serves to reduce the system total entropy.

\(^2\)The mass flow rate ratio, \(m_r\), is evaluated as the mass flow rate of water entering the dehumidifier at its coldest point (in this case, entering Dehumidifier 2) divided by the mass flow rate of air entering the dehumidifier at its hottest point (in this case, from the heater into Dehumidifier 1).
Extraction rates were increased from 0 to 60% of the inlet water flow as seen in Figure 2-8. Input conditions were held to the same as the zero extraction model. It is clear that for $m_r < 1.5$, extracting water has no significant impact on the system or can even prove detrimental. Note that there is a portion of the curve that is marked by a dashed line. These values of extraction produce a cycle that violates the second law, so the system is unable to operate using those boundary conditions. However, for $m_r > 1.5$, GOR increases with increasing extraction flow. Additionally, as seen in Figure 2-8, higher levels of GOR correspond to reduced system entropy generation. In the water extraction case, at high levels of $m_r$, and at higher extraction rates, the terminal temperature difference at the hot end of the dehumidifier decreases dramatically (29.5 to 13.2 °C) while the cool end is relatively unchanged (1.2 to 1.5°C). Therefore, the balancing of the dehumidifier is greatly improved and entropy production decreases. The balancing on the humidifier worsens (0.7 to 14.0 °C at the hot end and 6.5 to 4.7 °C at the cool end) but the increase in entropy generation of the humidifier is smaller than that of the dehumidifier. This consequence also impacts the water production rate, by enabling a larger $\Delta \omega$ across the dehumidifier. The total product water generated increases, which in turn increases the GOR. Additionally, due to the balancing, the entropy generation in the heater is reduced because the input temperature rises while the output remains fixed. Thus, a reduced heat input is required and GOR increases. Conversely, for low values of $m_r$, with increasing extraction the entropy generation gain in the humidifier outweighs the reduction in the dehumidifier and the heater also sees an increase as opposed to a decrease. For low $m_r$, GOR is not improved via extraction.

Also of note is the importance of HCR. For $m_r = 2$, as extraction increases, GOR increases until it peaks at 2.27 at an water extraction of 68%. When the

---

3Extractions will be expressed as % of circuit flow. For water extractions, this value is the percentage of inlet water flow rate. For air, it is the percentage of air flow entering the top of the dehumidifier.
extraction rate is zero, HCR for both dehumidifier subcomponents is far from unity, while the humidifier subcomponents are both slightly below 1. But with increasing extraction, \( \text{HCR}_{D_1} \) and \( \text{HCR}_{D_2} \) move towards 1. Dehumidifier 1 has the highest entropy production rate of all four components. It is seen that when \( \text{HCR}_{D_1} = 1 \) at an extraction of 68\%, entropy production for this component is minimized. A summary of entropy production values is provided in Table 2.4. It is clear that the largest entropy producing component, Dehumidifier 1, has entropy production minimized at \( \text{HCR} = 1 \). GOR is also maximized at this operating point as seen in Figure 2-9 (where GOR and \( \text{HCR} = 1 \) are marked with dotted lines). The other components reach \( \text{HCR} = 1 \) at different extraction values but it clearly Dehumidifer 1 that is driving performance. For each component, however, the respective \( \dot{S}_{gen} \) is minimized at \( \text{HCR} = 1 \).

Alternatively, a single air extraction can be made as illustrated in Figure 2-10. It is interesting to note while the water extraction case showed improved performance for \( m_r > 1.5 \), the air extraction case demonstrates an opposite effect. In Figure 2-11 it
Figure 2-8: Performance of a two subcomponent air heated cycle with water extraction
Figure 2-9: Subcomponent HCR versus extraction rate for an air heated water extraction cycle

is clear that for small values of $m_r$, there is an extraction value that optimizes GOR. As extraction rate is increased, the terminal temperature difference in the dehumidifier improves, reducing entropy generation. The humidifier sees increased entropy generation, but again the effect on the dehumidifier overshadows it. The heater also has reduced entropy generation. Again, GOR increases as required heat input falls and the product water flow rate increases. For larger values of $m_r$, dehumidifier performance worsens while the humidifier improves, and the heater worsens as well. Overall, there is a net entropy increase: more heat is required to run the system and less water is produced; therefore, GOR is reduced. Unlike water extraction, air extraction can cause GOR to hit a peak such that additional extraction is detrimental to performance. Additionally, with air extraction, values of GOR significantly higher than the peak GOR at zero extraction are achievable: for instance, at zero extraction
Table 2.4: Subcomponent entropy production versus extraction rate for air heated water extraction cycle

<table>
<thead>
<tr>
<th>Extraction % $m_w$ kg/s</th>
<th>$S_{gen,D}$</th>
<th>$S_{gen,D1}$</th>
<th>$S_{gen,D2}$</th>
<th>$S_{gen,H}$</th>
<th>$S_{gen,H1}$</th>
<th>$S_{gen,H2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.72×10^{-3}</td>
<td>2.71×10^{-3}</td>
<td>1.22×10^{-4}</td>
<td>7.46×10^{-5}</td>
<td>9.08×10^{-6}</td>
<td>6.86×10^{-5}</td>
</tr>
<tr>
<td>20</td>
<td>1.93×10^{-3}</td>
<td>1.99×10^{-3}</td>
<td>1.53×10^{-4}</td>
<td>1.35×10^{-4}</td>
<td>9.33×10^{-5}</td>
<td>4.44×10^{-5}</td>
</tr>
<tr>
<td>40</td>
<td>1.22×10^{-3}</td>
<td>1.33×10^{-3}</td>
<td>1.64×10^{-4}</td>
<td>3.56×10^{-4}</td>
<td>3.83×10^{-4}</td>
<td>3.76×10^{-5}</td>
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<tr>
<td>60</td>
<td>3.12×10^{-4}</td>
<td>1.49×10^{-4}</td>
<td>1.71×10^{-4}</td>
<td>9.11×10^{-4}</td>
<td>1.42×10^{-3}</td>
<td>3.31×10^{-5}</td>
</tr>
<tr>
<td>80</td>
<td>3.48×10^{-4}</td>
<td>7.40×10^{-4}</td>
<td>2.62×10^{-4}</td>
<td>1.04×10^{-3}</td>
<td>2.88×10^{-3}</td>
<td>2.57×10^{-5}</td>
</tr>
</tbody>
</table>

The GOR peaked at 2.75, but at a $m_r$ of 1.25, GOR peaks at 3.4 with an extraction rate of 30% of total airflow. Reducing $m_r$ further results in states for which GOR appears to have grown even larger, but in actuality the 2nd law has been violated in one or more components in the dehumidifier. Thus, these are impossible states and the maximum GOR appears to occur closer to $m_r = 1.25$.

Figure 2-10: Air-heated CAOW HDH system with single air extraction

Additionally, the same tests were run for an equivalent water-heated cycle with the same top temperature and effectivenesses. As seen in Figures 2-12 and 2-13, the same impact due to the mass flow rate ratio is observed. In brief, for the given
Figure 2-11: Performance of a two subcomponent air heated cycle with air extraction

(a) GOR vs. extraction rate

(b) \(\dot{S}_{\text{gen}}\) vs. extraction rate
system values of $m_r < 1.5$, air should be extracted instead of water to improve system performance while water should be extracted for $m_r > 1.5$. In each case, a reduction of entropy in the dehumidifier improves GOR by reducing required heat input and increasing water production.

Figure 2-12: GOR vs. extraction rate for a two subcomponent water heated cycle with water extraction

Figure 2-13: GOR vs. extraction rate for a two subcomponent water heated cycle with air extraction

2.4.3 Dual Extraction Model

The concept of extraction is now carried further by creating a two extraction model. On the basis that the air-heated, air extraction CAOW model produced the highest GOR for a single extraction, this model was extended to a dual extraction case. In
<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{a,D1,i}$</td>
<td>70 °C</td>
</tr>
<tr>
<td>$T_{w,D3,i}$</td>
<td>30 °C</td>
</tr>
<tr>
<td>$\epsilon_{D1}, \epsilon_{D2}, \epsilon_{D3}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\epsilon_{H1}, \epsilon_{H2}, \epsilon_{H3}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$P$</td>
<td>101.325 kPa</td>
</tr>
</tbody>
</table>

Table 2.5: Two extraction model input conditions for Figure 2-15

Figure 2-14, the dual extraction model is illustrated. There are now three subcomponents each for the humidifier and dehumidifier, and 2 extraction streams between them. There are a greater number of variables to control in the case of two extractions. The boundary conditions are controlled similarly to the previous 1 extraction model:

An $m_r$ of 1.5 produced the highest values of GOR. Additionally, it was determined that the highest GOR occurred when the extraction streams were counter to
one another. Namely, the stream toward the top (hot end) of the device was extracted
“forward”, from dehumidifier to humidifier, while the stream toward the bottom was
extracted in “reverse”, from humidifier to dehumidifier. This configuration creates
a loop of opposing flow within the main loop. Extracting flow in the same direc-
tion did little to improve GOR while extracting in opposition enhanced performance
considerably as seen in Figure 2-15.

2.4.4 Multiple Extractions: Validating Multi-Extraction

Ideally, the process of multi-extraction should be able to fully balance a humidi-
 fier and dehumidifier operating in an HDH cycle [7]. To verify this hypothesis, the
process illustrated in the previous sections was expanded to create a humidifier and
dehumidifier consisting of $N + 1$ subcomponents and $N$ extractions. In this study,
all variables were kept constant whilst increasing the number of extractions. With
each additional extraction, an additional subcomponent was created in the model
with a fixed effectiveness of $\epsilon_i = 0.8$. While this procedure does have the effect of
decreasing the relative “size” of each subcomponent (discussed in Section 2.4.5), it
does allow for a very large number of extractions/injections to be analyzed. In Figure
2-16, from one to five extractions are made for three different mass flow rate ratios.
It is important to note that regardless of the number of extractions, the total amount
of flow extracted is identical in each run. For example, when $N = 1$, 10% of the
flow may be extracted from a single point in the dehumidifier. When $N = 2$, 5% is
extracted from two points, totaling 10% flow extracted from the dehumidifier. This
procedure continues for higher values of $N$, such that the same physical amount of
flow is extracted from the dehumidifier in each case. In Figure 2-16, several different
extracted flow schemes are evaluated for different values of $m_r$. Values of $m_r$ were
selected to be on either side of and including $m_r = 1.5$ as this value was optimal in
the dual extraction case presented in Section 2.4.3.
Figure 2-15: Performance of a three subcomponent air heated cycle with air extraction

(a) GOR vs. extraction rate

(b) $\dot{S}_{gen}$ vs. extraction rate
It is clear that there are values of extraction rate and $m_r$, which are ideal for multi-extraction. In Figure 2-16b GOR has a near linear relationship with the number of extractions. Overall, an increased number of extractions does appear to improve GOR, but with diminishing returns as higher values of $N$. This result reflects the hypothesis that each additional extraction will reduce stream to stream variation, and an increasingly large number of extractions will closer resemble the ideal case. However, in a few cases of low airflow extracted (2-16a), it appears that additional extractions actually serve to worsen system performance.
Figure 2-16: *N* Air Extractions

(a) *N* Air Extractions, 10% Flow

(b) *N* Air Extractions, 20% Flow

(c) *N* Air Extractions, 50% Flow

Figure 2-16: *N* Air Extractions
2.4.5 Validating On-Design Extraction Model

While the scenarios illustrated in Sections 2.4.2, 2.4.3, and 2.4.4 do show high values of GOR for various extraction cases, they do not directly show that for a given system, adding an extraction improves GOR. This is a consequence of the method of construction of the on-design model. By assigning a fixed effectiveness to each subcomponent, the total equivalent effectiveness for the humidifier or dehumidifier changes based on the number of subcomponents assigned to it. For further detail on this phenomenon, refer to Appendix A.

The on-design model may still be validated, however, via a different method which forces a two subcomponent model to mirror a single component model. Consider the system presented in Figure 2-17a. In this case, the humidifier and dehumidifier are each a component and each carries an effectiveness of 0.8. In previous models, adding another subcomponent of 0.8 effectiveness increased the total effectiveness to greater than 0.8. However, as illustrated in Figure 2-17b, a 2-subcomponent model may be created to have a total combined effectiveness of 0.8. There exists a large number of combinations of two subcomponents to realize this model, but just one possibility is shown. When no extractions or injections are present, both models in Figure 2-17 are equivalent. They share the same input and output conditions and have identical values of performance. For example, with an inlet water temperature of 30°C, a maximum air temperature of 70°C, a mass flow rate ratio of 0.85, both systems have a GOR of 1.67. At this stage, it is certain that the two models are equivalent. Now, extraction may be turned on to change the system performance. An air extraction is added from the dehumidifier to the humidifier in Figure 2-17b. In Figure 2-18 it is clear that in this case, GOR increases up until extraction reaches approximately 60% of the upstream airflow. GOR increases to nearly 2 before dropping rapidly at higher extraction rates. Thus, we are assured that extraction can significantly improve performance of a system. And unlike the models presented in previous sections, it
is clear that the extraction model (two subcomponents) is equivalent to the baseline model with the exception of the extraction and injection.

Additionally Figure 2-18 illustrates the impact of extraction rate on effectiveness. Though both components started at an effectiveness of 0.8, as the rate of extraction increases, the overall humidifier effectiveness drops while the overall dehumidifier effectiveness approaches 1. This is due to the definition of effectiveness: it must include extractions and injections. At high extraction or injection rates, it rapidly diverges from its initial value when no extractions or injections were present. This model presents the best approximation of a validation of the on-design experiments, but it is still clear that an off-design model will be needed to resolve uncertainties uncovered in the on-design studies.

Figure 2-17: On-design validation of one versus two subcomponent models

(a) Single component, Total $\epsilon = 0.8$

(b) Two subcomponents, Total $\epsilon = 0.8$
Figure 2-18: Impact of extraction on the two subcomponent HDH system model; initially $\epsilon_D = \epsilon_H = 0.8$

### 2.4.6 Impact of Effectiveness on Extraction

Thus far, all models in this section have had a fixed subcomponent effectiveness of $\epsilon = 0.8$. This value was chosen as an achievable target value for actual hardware. Of interest, however, is the effect of extractions when the hardware has a different value of effectiveness. Interesting questions could be posed: Does improving the hardware quality affect the impact of extractions? Does GOR significantly improve with higher quality hardware and does this improvement outweigh the cost of larger and mostly expensive components? To assess these questions, cycles from Section 2.4.2 were rerun with higher values of subcomponent effectiveness. All other inputs equal, effectiveness was increased from $\epsilon = 0.8$ to $\epsilon = 0.9$.

From Figure 2-19, it is clear that varying the subcomponent effectiveness has a large affect on GOR. But even more significantly it becomes apparent that the
Figure 2-19: Performance of single extraction cycles at subcomponent effectiveness $\epsilon_{all} = 0.9$

Increase in GOR due to a single extraction is much more significant at the higher value of effectiveness. For example, compare Figure 2-19a against Figure 2-12: in both cases, GOR is most improved by extraction at $m_r = 5$ and the peak GOR occurs with an extraction of 40% of the inlet water flow rate. But for the lower effectiveness case, GOR increases from 1.15 to 1.66, an increase of 44%. When effectiveness is increased, GOR instead is boosted from 1.44 to 2.96: a two-fold increase. The same phenomena is observed for the water heated air extraction cycle, particularly for $m_r = 1$ and $m_r = 2$. Small increases in GOR when effectiveness = 0.8 are translated into relatively large gains (up to 100%) for effectiveness = 0.9. Further, this effect is repeated in
the air heated, air extraction cycle as seen in Figure 2-19d. In the air heated cycle, however, the majority of the cycle operating conditions violate second law so these states are not physically achievable. Again, output conditions that violate second law are marked by a dashed line. This analysis may be continued for even higher values of effectiveness, but the majority of these models violate the second law so the very high GOR cases are not achievable. However, with some manipulation, some cases can avoid a second law violation by lowering the effectiveness on the violating component. For instance, in Figure 2-19d, the GOR of 7.7 is not achievable due to a second law violation in Dehumidifier 1. Lowering $\epsilon_{D1}$ to 0.74 while leaving all other effectiveness at 0.9 creates a cycle where there are no second law violations even for large extractions. An air extraction of 36% of the flow rate reaches a GOR of 5.19. This performance is significantly better than the baseline for this case (without extraction, this array of components can reach a peak GOR of 4.12). In further research, the optimization of the individual component effectivenesses to achieve high GOR cycles that do no violate second law would be a valuable study.

It is clear that extracting in a case where the component effectivenesses are larger has a larger and positive impact on GOR. High effectiveness corresponds to smaller terminal temperature differences (TTDs) as well as a smaller stream to stream temperature variation along the length of the components. Thus, the benefit of extracting is greater for higher quality equipment that has smaller values of TTD.

### 2.4.7 Avoiding Humidifier Temperature Crosses

At this time, it should be noted that the second law control volume violations are not the only possible thermodynamic errors that can occur with on-design system model. In Section 2.4.6 many of the high GOR cases were the result of small terminal temperature differences (TTDs). At small TTDs, the temperature profiles of the two streams get considerably closer to one another. In a heat and mass exchanger, this
can be an issue as the temperature profiles may be non-linear due to evaporation or condensation of water. Typically in the dehumidifier the pinch point (i.e. the smallest $\Delta T$ between the streams at a given location) occurs at one of the terminal locations because of the shape of the air temperature profile. In the humidifier, however, the pinch point can occur somewhere internal to the component. If the humidifier has a small TTD, the pinch may be even smaller. In the models illustrated so far, only the inlet and outlet temperatures are calculated. Thus, it is conceivable for models that exhibit small TTDs, the pinch point could go to zero or even negative. A negative pinch would imply a temperature cross; this is another violation of the second law but does appear in the control volume approach. Therefore, full temperature profiles were created for the humidifiers studied in Chapter 2 to locate possible temperature crosses.

To create the temperature profiles, the inlet and outlet enthalpies for both streams were used as end points. Then, the humidifier was divided into 100 nodes, and assuming a constant $\Delta h_a$ and $\Delta h_w$ step between each node, the enthalpies were plotted for each node within the humidifier. Assuming a constant pressure and that the air stream remained fully saturated, the temperatures for each node could be determined from the respective enthalpy. This method generated full temperature profiles for the air and water stream of the humidifier. For example, in Figure 2-19d the GOR for $m_r = 1.75$ peaks just above 7. The humidifier temperature profiles for this case are plotted in Figures 2-20a and 2-20b. It is clear that though the profile for Humidifier 2 does approach a pinch of zero, the pinch remains positive (about 0.19 °C). Therefore, there is no temperature cross and because this run did not violate control volume formulation of the second law, the cycle is valid. Again, the double check for temperature crosses is another method to verify the thermodynamic feasibility of the cycle. All previous humidifier models in this chapter were also examined and did not show a temperature cross. The small TTD/high GOR cycle, of all the cycles
examined, was the closest to a zero pinch.

(a) Humidifier 1 temperature profile  
(b) Humidifier 2 temperature profile

Figure 2-20: Temperature profiles of humidifiers for air-heated air extraction cycle with GOR=7.2

2.4.8 High GOR On-Design Cases

In a separate study, it proved useful to determine the highest performance cases to establish a goal for systems with extraction. Effectiveness was set equal to 1 for both the total humidifier and dehumidifier. These parameters simulate a cycle that has a components of infinite area. For most mass flow rate ratios, either the humidifier or dehumidifier shows negative entropy generation, but for $2.10 < m_r < 2.15$, both components show positive entropy generation in the control volume assessment. Table 2.6 shows some of the key operating parameters for such a cycle.

This simulation yielded extremely high performance by utilizing components of infinite area. However, as discussed in Section 2.4.7, this system has a temperature cross. With a TTD of zero, there is a clear temperature cross per Figure 2-21. Thus, this design is not feasible. While the control volume first and second law are met, the design of the counterflow heat and mass exchanger cannot meet the requirements of such a system. Different hardware would need to be employed to build this cycle. The off-design studies in Chapter 3 demonstrate actual high performance systems. It
Table 2.6: Input and output values for air-heated HDH cycle, $\epsilon_D = \epsilon_H = 1$

will be shown that achieving such a high effectiveness would require extremely large components. Yet, extraction has been shown to increase overall component effectiveness, so one goal of multi-extraction is then to achieve a high effectiveness while still utilizing smaller equipment. This concept will be explored further in Chapter 3.

2.5 Summary of On-Design HDH System

Though the studies in this chapter do not lead to a generalized description of extraction in an HDH system, several key findings were made that elucidate key aspects of extraction:

- First, it was confirmed that driving the system toward $HCR = 1$ either by way of adjusting the mass flow rate ratio, $m_r$, or via extractions significantly reduces entropy generation, $\dot{S}_{\text{gen}}$, and increases GOR.

  - When the system is split into multiple subcomponents, adjusting the extraction rate between the components such that $HCR = 1$ for the components with the largest $\dot{S}_{\text{gen}}$ is the best method to improve performance.
  
  - Optimally, adjusting extractions and injections to force $HCR = 1$ for all subcomponents should yield an exceptionally high performance system.
Next, for all single extraction systems, performance was optimized by extracting and injecting from the stream with the maximum capacity rate.

- For example, when \( m_r \) is large and the water stream has the maximum capacity rate, a larger performance gain was realized by extracting water.
- Conversely, a system with a small \( m_r \) with air at the maximum capacity rate had improved performance when air was extracted.

Finally, it was found that in the majority of cases, multiple extractions improved performance more so than a single extraction.

- This statement is particularly true for systems with low TTD (and high effectiveness).
- While multiple extractions produce higher performance, it was also determined that the first extraction typically resulted in the highest performance gain, while subsequent extractions yielded diminishing returns. In designing an HDH system with multiple extractions, it is clear that there will be a trade off between the cost and the performance gain of the \( N \)th extraction.
Chapter 3

Off-Design Humidification
Dehumidification Model

In Chapter 2, an on-design HDH system model was evaluated and it was found that systems with multiple extractions and injections exhibit high performance, but it proved difficult to adequately compare these systems to a fixed baseline. Upon further investigation, it was clear that multi-extraction on-design studies were not tied to a fixed system “size”, rather the system size would float with the number of extractions. So while all models evaluated are physically feasible designs that may be realized in a laboratory or production, the validation of multi-extraction to a baseline system was inadequate in the on-design case. An off-design system model is assessed in this chapter to better understand the impact of multi-extraction. Again, an off-design model incorporates transport processes in addition to the thermodynamic evaluation to more accurately simulate a real, physical system.

While the on-design system models enabled the assessment of several input and output states (temperatures, humidity, mass flow rates, etc.) they do not allow for the examination of a fixed size component along its complete profile. In off-design analysis, it is possible to fix the physical size of the system and then evaluate the state
of the air and water streams at certain fixed locations. In this finite difference model, high resolution of the temperature and humidity profiles is achieved by choosing a large number of nodes in the finite difference discretization. From this analysis, a detailed map of temperature and humidity within the humidifier and dehumidifier may be established. Additionally, it is also possible to determine at what relative height within the humidifier water is evaporated from the air, and where it is condensed in the dehumidifier. This detailed data enables a full evaluation of humidifier and dehumidifier performance as well as combined system performance. It also ensures that there are no temperature crosses or other second law violations at any point within the system. The next sections detail the underlying assumptions and mathematics in the finite difference humidifier and dehumidifier models.

3.1 Finite Difference Humidifier Model

The humidifier selected for this model is a packed bed humidifier. Younis [26] and Ben-Amara [27] utilized the same type of humidifier in their respective HDH systems. The finite difference humidifier model was constructed from cooling tower analysis performed by Kloppers [28], Klimanek and Białecki [29], and Onda [30]. It is a 1D counterflow cooling tower model that is formulated by a set of differential equations. The differential equations are reproduced numerically in Engineering Equation Solver (EES) [31] to illustrate the state of the air or water stream at a specified height within the humidifier.

3.1.1 Control Volume Analysis

Figure 3-1a, adopted from Kloppers[28], illustrates the control volume in the fill of a counterflow wet-cooling tower. Figure 3-1b shows the air-side control volume.

From Figure 3-1a, we define a mass balance equation where the change in humidity
is directly proportional to the change in water mass flow

\[ \frac{d\dot{m}_w}{dz} = \dot{m}_{da} \frac{d\omega}{dz} \]  \hspace{1cm} (3.1)

and an energy balance equation relating the transfer of energy from the water stream to the air

\[ h_w \frac{d\dot{m}_w}{dz} + \dot{m}_w \frac{dh_w}{dz} = \dot{m}_{da} \frac{dh_a}{dz} \]  \hspace{1cm} (3.2)
From Figure 3-1b the mass transfer equation may be expressed as

\[
\frac{d\dot{m}_w}{dz} = \beta (\omega_{sat}^w - \omega) \frac{dA}{dz} \tag{3.3}
\]

where \(\beta\) is the mass transfer coefficient in kg/m²-s, \(\omega_{sat}^w\) is the humidity ratio at saturation evaluated at the local water liquid surface temperature (assumed to be equal to the bulk water temperature) and where

\[
dA = aA_z dz. \tag{3.4}
\]

\(a\) is the fill packing density (in \(\frac{m^2}{m^3}\)) and \(A_z\) is the cross-sectional area, perpendicular to \(z\). Also from Figure 3-1b, the energy equation may be expressed as the latent and sensible heat transfer from the water to the air:

\[
\dot{m}_d a \frac{dh_a}{dz} = [h_v \beta (\omega_{sat}^w - \omega) + \alpha (T_w - T_a)] aA_z \tag{3.5}
\]

where \(h_v\) is the enthalpy of the water vapor (at the bulk water temperature) transferred from water to air stream and \(\alpha\) is the heat transfer coefficient in W/m²-K.

Kloppers [28] demonstrates through manipulation of Equations 3.1, 3.2, 3.3, and 3.5 that the following differential equations may be obtained.

\[
\frac{dh_a}{dz} = \frac{\beta aA_z}{\dot{m}_da} [Le_f(h_{a, sat}^w - h_a) + (1 - Le_f)h_v(\omega_{sat}^w - \omega)] \tag{3.6}
\]

\[
\frac{d\omega}{dz} = \frac{\beta aA_z}{\dot{m}_da}(\omega_{sat}^w - \omega) \tag{3.7}
\]

\[
\frac{dT_w}{dz} = \frac{1}{m_r} \left( \frac{1}{c_{p,w}} \frac{dh_a}{dz} - T_w \frac{d\omega}{dz} \right) \tag{3.8}
\]

\(Le_f\), the Lewis factor, is an representation of of the relative rate of heat and mass transfer in the system.

80
\[ Le_f = \frac{\alpha}{c_{p,a} \beta} \] 

and is calculated for each node in the discretization. The Lewis factor is calculated numerically per the empirical relation developed by Bosnjakovic [32] for air-water systems:

\[ Le_f = 0.866^{2/3} \frac{\omega_{sat} + 0.622}{\omega + 0.622} - 1 \ln \frac{\omega_{sat} + 0.622}{\omega + 0.622}. \] 

These equations specify the change in air enthalpy, air humidity, and water temperature per unit height and may be employed to describe the humidifier system along with equations of state and boundary conditions. The only missing information is the value of the mass transfer coefficient. Previous authors [28, 29] have employed a Merkel number [33] approach to determine the mass transfer coefficient. This present model, however, was built with the intention of being divided up into different sections with different mass flow rates due to numerous extractions and injections. Merkel assumes a constant mass flow rate of water through the humidifier (which even under normal conditions is not precisely accurate due to the evaporation of water into the air stream). In the presence of of injections and extractions, any degree of accuracy vanishes because at any given point the mass flow rate may change dramatically. An alternate method of determining the mass transfer coefficients will be required for a humidifier with multi-extraction.

### 3.1.2 Determination of the Mass Transfer Coefficient

Onda [30] created a correlation for mass transfer coefficients in gas absorption, desorption, and vaporization that covers a variety of packing materials in cooling towers. Onda’s correlation uses the geometric property of the packing, flow rates and thermophysical properties of the gas and liquid to determine the mass transfer coefficient.
as follows:

\[
\beta = \rho_v C \text{Re}_{GA}^{0.7} \text{Sc}_G^{\frac{1}{3}} (ad_p)^{-2} a_D \]

(3.11)

where

\[
\text{Re}_{GA} = \frac{G}{\mu_G} \quad (3.12)
\]

\[
\text{Sc}_G = \frac{\mu_G}{\rho_G D_G} \quad (3.13)
\]

\[
D_G = 1.87 \times 10^{-10} \left( \frac{T_a + 273.15}{P} \right)^{2.072} \frac{P}{101.325} \quad (3.14)
\]

\(d_p\) is the diameter of a packing bead, \(C\) is a geometric coefficient, 5.23 for \(d_p > 0.015\) or 2 for \(d_p < 0.015\), and \(D_G\) is the diffusion coefficient from water to air (with \(T_a\) in °C Celsius and \(P\) in kPa). Using this correlation enables a determination of the mass transfer coefficient at any height within the humidifier. If directly upstream or downstream of an extraction, the change in mass flow affects the mass transfer coefficient. Thus, a different mass transfer coefficient is found at every node within the discretization of the component.

### 3.1.3 Solution of Humidifier Model

Utilizing the set of differential equations describing the humidifier, Equations 3.6, 3.7, 3.8, as well as the Onda correlation, Equation 3.11, a finite difference model of the humidifier may be constructed. Necessary inputs include inlet water and air temperatures, inlet air humidity, mass flow rates of air and water, humidifier height and cross-sectional area, packing nominal diameter, and fill specific surface area. To solve the model, the humidifier must be divided into a number of discrete nodes and the outlets of each node tied to the inlets of the adjacent node as shown in Figure 3-2. With this input criteria, the model is fully defined (\(N\) equations, \(N\) unknowns)
and may be solved. This model is a two point boundary value problem where some, but not all, conditions are known at one point, and some conditions are known at the other. In this case, water temperature and mass flow is known at the top of the device while air temperature, mass flow, and humidity is known at the bottom. These types of problems are typically solved with an iterative approach. Whereas other authors typically solve this system of equations with an iterative mathematical model [34], here, given appropriate guess values, the EES software [31] automatically iterates until it finds the solution. Figure 3-3 illustrates a temperature profile for a humidifier solved with the inputs listed in Table 3.1.
### Table 3.1: Humidifier model inputs

<table>
<thead>
<tr>
<th>State Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet air temperature $T_{a,i}$</td>
<td>$30 , ^\circ C$</td>
</tr>
<tr>
<td>Inlet water temperature $T_{w,i}$</td>
<td>$60 , ^\circ C$</td>
</tr>
<tr>
<td>System pressure $P$</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>Water mass flow $\dot{m}_w$</td>
<td>0.09 kg/s</td>
</tr>
<tr>
<td>Air mass flow $\dot{m}_{da}$</td>
<td>0.03 kg/s</td>
</tr>
<tr>
<td>Inlet air humidity $\phi$</td>
<td>1</td>
</tr>
<tr>
<td>Component height $L$</td>
<td>2 m</td>
</tr>
<tr>
<td>Cross-sectional area $A_z$</td>
<td>0.5 m²</td>
</tr>
<tr>
<td>Fill density $a$</td>
<td>157 m²/m³</td>
</tr>
<tr>
<td>Packing diameter $d_p$</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Number of nodes $N$</td>
<td>20</td>
</tr>
</tbody>
</table>

3.1.4 **Humidifier Performance Evaluation**

In Sections 2.2 and 2.3, the humidifier and dehumidifier performance was evaluated on an entropy generation basis. In order to determine whether or not the off-design component models behaved similarly to their on-design equivalents, several analyses were performed. First, normalized $\dot{S}_{gen}$ versus HCR was plotted to uncover the operating point which produces the minimum entropy generation. For the physical system described in Section 3.1.3, it was found that, like in the on-design model, when HCR = 1, normalized entropy generation is minimized. This result is plotted in Figure 3-4. Also plotted is the component effectiveness, described by Equation 1.8. It is seen that for a component of fixed area, at the same time entropy generation is minimized, effectiveness is minimized as well. The validity of this result becomes apparent in a comparison to the effectiveness-NTU method of heat exchanger analysis [35]. For a fixed sized (or fixed NTU), when $\frac{C_{\text{min}}}{C_{\text{max}}}$ (equivalent to the heat and mass exchanger heat capacity ratio or HCR) nears zero, the stream to stream driving force is greatly increased and the component has a very high duty. A great deal of heat is moved, and effectiveness is high. However, the driving force is unbalanced (large on one end of the device, small on the other) and therefore the system contains significant irreversibilities and $\dot{S}_{gen}$ is high. For the same size or NTU, when $\frac{C_{\text{min}}}{C_{\text{max}}}$
approaches 1, the variation is driving force is minimized. This corresponds to a small amount of irreversibility and lower entropy generation. However, by minimizing the variation is driving force, the driving force itself becomes smaller and the duty of the component is reduced. Thus, for the same component, there is less heat transfer and the effectiveness of the device drops. To increase effectiveness in this case, bigger hardware must be employed (larger NTU) to reach the larger duty. Thus, high effectiveness hardware that has minimal entropy generation must be quite large.

Relatedly, it bears mentioning that when the size of the humidifier is increased significantly (from $H = 1$ to $H = 3.7$ and from $A_z = 0.5$ to $A_z = 1.5$), the effectiveness is increased by over 10% to 0.8 when HCR = 1. This is the typical effectiveness used in the on-design studies in Chapter 2. In this case, the normalized entropy production drops to 0.000241. This result differs from the on-design case presented in Figure 2-2 by only 2%. The normalized entropy generation minimized at 0.000236 for the on-
design model with zero extractions. In Figure 3-4, the entropy production was about three times larger at HCR = 1. Thus, to achieve the approximately $2/3$ reduction in entropy production, the humidifier size was increased by about a factor of $11x$, a considerably larger and more expensive piece of equipment.

### 3.1.5 Validation of Humidifier Model

The humidifier model was validated by comparing analytical results from the model to experimental data gathered by Sharqawy and Husain [36, 37]. The humidifier code was modified to match conditions given by Sharqawy and Husain. It was altered to match the geometry used in the experimental studies and was given different flow rates and temperatures. Namely, the humidifier size was reduced to 0.6 m tall by 0.15 wide by 0.15 long with a specific fill area of $110 \text{ m}^2/\text{m}^3$. Though the data covers both fresh and salt water runs, only runs where fresh water was used were compared.
to minimize errors due to salinity. Initially, the experimental data did not match the model data. This mismatch was caused by the Onda correlation generating very low (0.001 kg/m²-s) mass transfer coefficients. When the Onda correlation was replaced by a Merkel approach, mass transfer coefficients increased dramatically (to around 0.02-0.03 kg/m²-s) and the data correlated well as illustrated in Table 3.2. Error was calculated as

$$\text{Error} = \frac{\Delta T_{\text{model}} - \Delta T_{\text{exp}}}{\Delta T_{\text{exp}}}$$ (3.15)

where $\Delta T$ is the change between inlet and outlet temperature. Error was found to be small: typically under ±5% and no more than ±10%.

It was clear that the Onda correlation did not match experimental data for the range of inputs employed by Sharqawy and Husain. These boundary conditions were, however, considerably different than those employed in the humidifier model. Particularly, temperatures were very low and the size of the humidifier was quite small. To ensure that Onda’s correlation was valid for the conditions being examined in the HDH system, another comparison was made.

Onda’s correlation is a general correlation, and per Section 3.1.1, it is clear that the value of $\beta$, the mass transfer coefficient, is strongly linked to the change in temperature and humidity for a given cell. As such, a simple test was run to determine the accuracy of $\beta$ and ensure the correlation’s validity for the humidifier model. The model was run at several values of mass flow ratio for a given set of input conditions and the value of $\beta$ computed for each case. This data was in turn compared to known

<table>
<thead>
<tr>
<th>$m_r$</th>
<th>Sharqawy [36, 37]</th>
<th>Present Work</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{a,o}$ °C</td>
<td>$T_{wb,o}$ °C</td>
<td>$T_{w,o}$ °C</td>
</tr>
<tr>
<td>1.0</td>
<td>25.8</td>
<td>23.7</td>
<td>25.5</td>
</tr>
<tr>
<td>1.5</td>
<td>26.9</td>
<td>24.0</td>
<td>27.1</td>
</tr>
<tr>
<td>2.0</td>
<td>27.6</td>
<td>24.5</td>
<td>28.2</td>
</tr>
</tbody>
</table>

Table 3.2: Humidifier model validation
values of $\beta$: data from performance curves of a Brentwood CF-1900SB/MA packing for counterflow cooling towers [38]. The performance curve gives a single value of $\beta$ for the full cooling tower. This value was found to be consistently between the minimum and maximum values computed along the length of the humidifier in the model. Note that in the model, a new mass transfer coefficient is found for each cell based on the local air and water properties. While the Brentwood data tended toward the lower range of mass transfer coefficients found in the model (see Figure 3-5), it was reasonably close to the average value for mass flow rate ratios in the system operating range (about 1-3). Thus, the mass transfer coefficient values are accurate for the humidifier model and model output is considered valid.

![Figure 3-5: Comparison of actual and calculated mass transfer coefficients](image)

3.2 Finite Difference Dehumidifier Model

The finite difference dehumidifier was constructed as a tube in tube model [20]. From literature, this is similar in scope to the finned tube used by Al-Hallaj et al. [39].
This is not a compact dehumidifier design such as the finned tubes utilized by El-Agouz et al. [40] or the flat-plate exchanger employed by Müller-Holst et al. [41]. It was chosen as a simple model where extraction points would be easy to locate on a single length of tube. It is a 1D counterflow, non-contacting dehumidifier model that is formulated by a set of differential equations. The differential equations are reproduced numerically in Engineering Equation Solver (EES) [31] to illustrate the state of the air or water stream at a specified height within the dehumidifier.

### 3.2.1 Control Volume Analysis

Figure 3-6 illustrates the control volume in the counterflow dehumidifier. From Figure 3-6, the mass balance may be described by

\[ \dot{m}_{da} \frac{d\omega}{dz} = \dot{m}_d \]  \hspace{1cm} (3.16)

while the energy balance is given by

\[ h_w \frac{d\dot{m}_w}{dz} + \dot{m}_w \frac{dh_w}{dz} = \dot{m}_{da} \frac{dh_a}{dz} + d\dot{m}_d h_d. \]  \hspace{1cm} (3.17)
The heat and mass transfer equations for the tube in tube dehumidifier can be derived from a simple resistor network realization. Per Figure 3-7, a series of equations derived from these resistances is utilized to determine temperatures. Resistances emerge from the air to film convective resistance (which includes a parallel current for latent heat moved through mass transfer), a film conductive resistance, a wall conductive resistance, and a wall to water convective resistance. Additionally illustrated in Figure 3-7 is the mass transfer resistance from film to bulk air stream.

Figure 3-7: Resistance network model for dehumidifier

\[
\frac{d\dot{Q}}{dA_s} = h_{c,a}(T_a - T_i) + \frac{\dot{m}_d}{A_s} h_{fg} \tag{3.18}
\]

\[
\frac{d\dot{Q}}{dA_s} = h_{c,w}(T_{ww} - T_w) \tag{3.19}
\]

\[
\frac{d\dot{Q}}{dA_s} = 2k_{wall}\D_1 \ln \left( \frac{D_2}{D_1} \right) (T_{aw} - T_{ww}) \tag{3.20}
\]

\[
\frac{d\dot{Q}}{dA_s} = 2k_{film}\D_2 \ln \left( \frac{D_3}{D_2} \right) (T_{int} - T_{aw}) \tag{3.21}
\]

\[
\frac{d\dot{m}_d}{dA_s} = \beta(x_a - x_{int}) \tag{3.22}
\]

It should be noted Equation 3.22 is only active for when the air side wall temperature,
$T_{aw}$, is lower than the dewpoint temperature of the bulk airstream, $T_{dp}$. When $T_{aw} > T_{dp}$ no condensation can occur and the flow rate of distillate, $m_d$, is zero.

Heat transfer coefficients are calculated assuming fully developed internal flow within pipes or annuli. A new heat transfer coefficient is found for each cell based on local fluid properties. The coefficient is given by

$$\text{Nu}_D = \frac{\alpha D_h}{k_f}$$

where Nu is the Nusselt number, $D_h$ the hydraulic diameter of the pipe, $\alpha$ is the heat transfer coefficient, and $k_f$ the thermal conductivity of the fluid exchanging heat to the surface. In turn, the Nusselt number is calculated via the Dittus-Boelter equation [42], a good approximation for the dehumidifier in which the bulk fluid temperature is close to the heat transfer surface temperature and the Reynolds number is within 10,000 to 120,000:

$$\text{Nu}_D = 0.023Re^{\frac{4}{5}}Pr^n$$

where $n = 0.4$ for heating of the fluid (in this case, water) and $n = 0.3$ for cooling (air).

The mass transfer coefficient is then calculated from the heat transfer coefficient employing the Chilton-Colburn J-factor analogy [42].

$$\frac{\alpha}{\rho V c_p} Pr = \frac{C_f}{2} = \frac{\beta}{V} Sc$$

### 3.2.2 Solution of Dehumidifier Model

Utilizing the set of differential equations describing the dehumidifier, Equations 3.16 through 3.25, a finite difference model of the dehumidifier may be constructed. Necessary inputs include inlet water and air temperatures, inlet air humidity, mass flow rates of air and water, dehumidifier height and cross sectional area. Additionally, the
dehumidifier must be divided into a number of discrete inspection sections and the outlets of each cell tied to the inlets of the adjacent cell per Figure 3-8. Here, the model is fully defined (N equations, N unknowns) and may be solved. This model is a two point boundary problem where some conditions are known at one side of the dehumidifier while some conditions are only known on the other side. In this case, water temperature and mass flow is known at the bottom of the device while air temperature, mass flow, and humidity is known at the top. These types of problems are typically solved with an iterative approach. Here, given appropriate guess values, the EES software [31] automatically iterates until it finds the solution. Figure 3-9 illustrates temperature and humidity profiles for a dehumidifier solved with the inputs listed in Table 3.3.

Figure 3-8: Linking of cells in the dehumidifier model
<table>
<thead>
<tr>
<th>State</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet water temperature</td>
<td>$T_{w,i}$</td>
<td>30 °C</td>
</tr>
<tr>
<td>Inlet air temperature</td>
<td>$T_{a,i}$</td>
<td>60 °C</td>
</tr>
<tr>
<td>System pressure</td>
<td>$P$</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>Water mass flow</td>
<td>$\dot{m}_w$</td>
<td>0.081 kg/s</td>
</tr>
<tr>
<td>Air mass flow</td>
<td>$\dot{m}_{da}$</td>
<td>0.03 kg/s</td>
</tr>
<tr>
<td>Inlet air humidity</td>
<td>$\phi$</td>
<td>1</td>
</tr>
<tr>
<td>Component length</td>
<td>$L$</td>
<td>50 m</td>
</tr>
<tr>
<td>Water pipe cross-sectional area</td>
<td>$A_w$</td>
<td>0.00011 m²</td>
</tr>
<tr>
<td>Air pipe cross-sectional area</td>
<td>$A_a$</td>
<td>0.0019 m²</td>
</tr>
<tr>
<td>Total heat transfer area</td>
<td>$A_s$</td>
<td>1.875 m²</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>$N$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.3: Dehumidifier model inputs

3.2.3 Dehumidifier Performance Evaluation

Similarly to Section 3.1.3, normalized $\dot{S}_{gen}$ versus HCR was plotted to determine the operating point which produces the minimum entropy generation. For the physical system described in Section 3.2.2, it was found that, like in the on-design model, when HCR = 1, normalized entropy generation is minimized. This result is plotted in Figure 3-10. Also plotted is the component effectiveness, described by Equation 1.8. It is seen that for a component of fixed area, at the same time entropy generation is minimized, effectiveness is minimized as well. As in Section 3.1.4, this result is validated by the effectiveness-NTU analysis of a similar heat exchanger. Again it is seen that to increase effectiveness larger hardware must be employed (larger NTU) to reach a larger duty. Thus, high effectiveness hardware that has minimal entropy generation will be quite large.

When the model is altered to operate at the same inlet conditions as the off design dehumidifier, the normalized entropy production is 0.00464 at HCR = 1 and $\epsilon_D = 0.8$. This result is very similar to the on-design case presented in Figure 2-4 where the normalized entropy generation minimized at 0.00491 for the case with zero extractions.
3.2.4 Dehumidifier Validation

For the humidifier, cooling tower experimental data may be leveraged to validate the code. For the dehumidifier, however, due to the relatively simplistic and inefficient concentric tube design, a literature review yielded no solid experimental data that matches the conditions utilized in this model. Again, the concentric tube dehumidifier was chosen to simplify the model, particularly with regards to adding extractions at known locations. To validate the model, a few limiting cases were analyzed to compare the model against known points typically found in literature. For instance, a run was performed where the inlet air humidity was brought to zero. With completely dry air as one stream, and water as the other, the problem reduces to a simple heat exchanger. No condensation forms in this run, so there is no mass transfer. A run was evaluated at the conditions in Table 3.4.

This run yielded an effectiveness of $\epsilon_D = 0.755$ and a $\frac{C_{\text{min}}}{C_{\text{max}}}$ or HCR of 1. Per a standard effectiveness-NTU model of a counterflow, concentric tube heat exchanger [35], this effectiveness should yield NTU as
Thus, \( NTU = 3.08 \). To validate the model then, NTU was reproduced analytically. First, the log mean temperature was calculated from inlet and outlet conditions of the air and water stream:

\[
\Delta T_{lm} = \frac{\Delta T_{hot} - \Delta T_{cold}}{\ln \frac{\Delta T_{hot}}{\Delta T_{cold}}} \quad (3.27)
\]

\[
\Delta T_{lm} = \frac{(70.0 - 59.1) - (39.8 - 30.0)}{\ln \frac{70.0-59.1}{59.9-30.0}} \quad (3.28)
\]

\[
\Delta T_{lm} = 10.3^\circ C \quad (3.29)
\]

The log mean temperature was used to find \( U \), the overall heat transfer coefficient
<table>
<thead>
<tr>
<th>State Variable</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet water temperature</td>
<td>$T_{w,i}$</td>
<td>30 $^\circ$C</td>
</tr>
<tr>
<td>Inlet air temperature</td>
<td>$T_{a,i}$</td>
<td>70 $^\circ$C</td>
</tr>
<tr>
<td>System pressure</td>
<td>$P$</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>Water mass flow</td>
<td>$\dot{m}_w$</td>
<td>0.0125 kg/s</td>
</tr>
<tr>
<td>Air mass flow</td>
<td>$\dot{m}_{da}$</td>
<td>0.05 kg/s</td>
</tr>
<tr>
<td>Inlet air humidity</td>
<td>$\phi$</td>
<td>0.0</td>
</tr>
<tr>
<td>Component length</td>
<td>$L$</td>
<td>100 m</td>
</tr>
<tr>
<td>Water pipe cross-sectional area</td>
<td>$A_w$</td>
<td>0.00011 m$^2$</td>
</tr>
<tr>
<td>Air pipe cross-sectional area</td>
<td>$A_a$</td>
<td>0.0019 m$^2$</td>
</tr>
<tr>
<td>Total heat transfer area</td>
<td>$A_s$</td>
<td>3.75 m$^2$</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>$N$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.4: Dehumidifier validation inputs - zero humidity case

for the dehumidifier

$$U = \frac{Q}{\Delta T_{lm} A_s}$$  \hspace{1cm} (3.30)

$$U = \frac{1521 \text{ W}}{(3.75 \text{ m}^2)(10.3 \text{ $^\circ$C})}$$ \hspace{1cm} (3.31)

$$U = 39.2 \text{ W/m}^2 - \text{K}$$ \hspace{1cm} (3.32)

which was then used to calculate NTU:

$$\text{NTU} = \frac{UA}{C_{\text{min}}}$$ \hspace{1cm} (3.33)

$$\text{NTU} = \frac{(39.2 \text{ W/m}^2 - \text{K})(3.75 \text{ m}^2)}{(50.3 \text{ W/K})}$$ \hspace{1cm} (3.34)

$$\text{NTU} = 2.96$$ \hspace{1cm} (3.35)

This value of NTU agrees with the original calculated value to within 4%. This result, then, supports the validity of the dehumidifier model. Another run was performed where $\dot{m}_{da} \gg \dot{m}_w$. In this case, the temperature of the wall between the streams stays constant at the air side temperature. The system reduces to a single stream exchanger where
<table>
<thead>
<tr>
<th>State Variable</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet water temperature</td>
<td>$T_{w,i}$</td>
<td>40 °C</td>
</tr>
<tr>
<td>Inlet air temperature</td>
<td>$T_{a,i}$</td>
<td>60 °C</td>
</tr>
<tr>
<td>System pressure</td>
<td>$P$</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>Water mass flow</td>
<td>$\dot{m}_w$</td>
<td>0.05 kg/s</td>
</tr>
<tr>
<td>Air mass flow</td>
<td>$\dot{m}_{da}$</td>
<td>0.50 kg/s</td>
</tr>
<tr>
<td>Inlet air humidity</td>
<td>$\phi$</td>
<td>1.0</td>
</tr>
<tr>
<td>Component length</td>
<td>$L$</td>
<td>50 m</td>
</tr>
<tr>
<td>Water pipe cross-sectional area</td>
<td>$A_w$</td>
<td>0.00011 m$^2$</td>
</tr>
<tr>
<td>Air pipe cross-sectional area</td>
<td>$A_a$</td>
<td>0.0019 m$^2$</td>
</tr>
<tr>
<td>Total heat transfer area</td>
<td>$A_s$</td>
<td>1.875 m$^2$</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>$N$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.5: Dehumidifier validation inputs - single exchanger case

$$
\epsilon = 1 - e^{-NTU}
$$

(3.36)

Using the parameters found in Table 3.5, this run yielded an effectiveness of $\epsilon_D = 0.9485$ and per Equation 3.36, $NTU = 2.967$. A repeat of the mathematics laid out in Equations 3.27, 3.30, and 3.33 yielded a log mean temperature of $T_{lm} = 6.32$, an overall heat transfer coefficient of $U = 334.8$, and a calculated $NTU = 3.002$. The error between estimated and calculated $NTU$ was then under 1.2%. Again, this limiting case lends credibility to the validity of the model.

### 3.3 Full Cycle Model

A full HDH system is constructed by using the humidifier model from Section 3.1 and the dehumidifier model from Section 3.2. Some input variables are removed and the outputs of the two models are tied together. For instance, as the cycle is a closed air open water cycle (CAOW), the air outlet of the dehumidifier is linked to the air inlet of the humidifier. Thus, $h_{D,a,o} = h_{H,a,i}$, $\dot{m}_{D,da,o} = \dot{m}_{H,da,i}$, and $\omega_{D,a,o} = \omega_{H,a,i}$. 

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### Table 3.6: High GOR water-heated HDH cycle component data

<table>
<thead>
<tr>
<th>Component</th>
<th>Heat Transfer Area [m²]</th>
<th>Length/Height [m]</th>
<th>Cross Sectional Area [m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dehumidifier</td>
<td>4.88</td>
<td>130</td>
<td>0.0019</td>
</tr>
<tr>
<td>Humidifier</td>
<td>455</td>
<td>2.9</td>
<td>1</td>
</tr>
</tbody>
</table>

3.3.1 Water-Heated System Model

In constructing a water-heated model, the inlet water temperature to the humidifier is specified, as is the cold water inlet to the dehumidifier. Thus, the dehumidifier air outlet is linked to the humidifier air inlet. The water streams are not linked; the water temperature into the humidifier is specified and the dehumidifier water outlet temperature is allowed to float. The energy input into the cycle is determined by

\[
\dot{Q}_{in} = \dot{m}_w(h_{H,w,i} - h_{D,w,o}).
\]  

Mistry et al. [43] described a high performance water heated HDH cycle utilizing very high values of effectiveness. For instance, a cycle with the dehumidifier effectiveness, \(\epsilon_D = 0.96\) and humidifier effectiveness, \(\epsilon_H = 0.92\) at an optimal mass flow rate ratio had a GOR peaking near 4.9. This system was based on an on-design, fully thermodynamic model, so component sizes were not known or given. As this information is necessary in building a system, the off-design model developed in this section was used to replicate the cycle and determine the feasibility of building such a system. Representative values for the system are shown in Table 3.6. The performance of the system matches very closely with the model given by Mistry [43] as seen in Table 3.7. GOR and entropy generation values are very similar. The only key difference between the two models are the values of effectiveness which were higher in Mistry’s model.

In Table 3.6 it became apparent that achieving a high GOR would nominally require a system of considerable size, particularly with regards to the humidifier. The
Table 3.7: Comparison of high GOR water-heated HDH cycles

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mistry On Design [43]</th>
<th>Off Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOR</td>
<td>4.87</td>
<td>4.86</td>
</tr>
<tr>
<td>TTD</td>
<td>2.0</td>
<td>2.2</td>
</tr>
<tr>
<td>$S_{gen}$ [kW/K]</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>$m_r$</td>
<td>2.44</td>
<td>2.44</td>
</tr>
<tr>
<td>$\epsilon_D$</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>0.93</td>
<td>0.85</td>
</tr>
<tr>
<td>$T_{min}$ [°C]</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>$T_{max}$ [°C]</td>
<td>58.6</td>
<td>58.6</td>
</tr>
</tbody>
</table>

humidifier used to obtain these cycle parameters was 2.9 m in height. To assess the impact of different component sizes on GOR, an additional series of runs was performed while varying humidifier and dehumidifier size. The results are plotted in Figure 3-11.

In these runs, the peak GOR was obtained by varying the mass flow rate ratio, $m_r$, only. Peak GOR typically occurred for $1.9 < m_r < 2.3$. From the figure, it becomes apparent that high GOR is a very strong function of component size. However, both components are important factors in GOR. For a smaller humidifier of volume 1.7 m$^3$ (cross sectional area = 1 m$^2$, height = 1.7 m) GOR ranged from about 2.1 to 3.6 by significantly varying the size of the dehumidifier. Based on the curvature of the data, it does not appear that the GOR will be able to surpass much more than 4 even if an extremely large dehumidifier is employed. These diminishing returns on component size were also shown, in terms of water production, by Nawayseh et al. [44]. For a large humidifier of volume 3.5 m$^3$, GOR had a much larger range: from 2.7 to 6.9. The presence of the larger humidifier enabled a large dehumidifier to have more impact on GOR. Doubling the size of the dehumidifier from 1.5 m$^2$ to 3.0 m$^2$ nearly doubles GOR. Thus, it is clear that larger components can yield very high performance and reduce the energy requirements of the HDH system considerably. However, it should be noted that in this series of runs, water production rate varied from 0.0016 kg/s
Figure 3-11: Peak GOR versus component size

to 0.0027 kg/s at a GOR of 2.1 and 6.9 respectively. So while the GOR more than tripled, water production rate increased by only about 70%. Therefore, this increase in component size is doing more to reduce $Q_{in}$ than increasing $m_d$. In Figure 3-12, water production is plotted against total humidifier and dehumidifier volume. It appears that water production rate follows a logarithmic function. For small systems, increasing size has a large impact on the water production rate, but larger systems get significantly diminishing returns. Increasing component size decreases the pinch at the hot side of the dehumidifier (thereby reducing the $\Delta T$ the water must go through in the heater, and correspondingly reducing $Q_{in}$) but is not reducing the pinch at the cold side as much (therefore $\omega$ at the cold side of the device is not as low and the $\Delta \omega$ through the dehumidifier is not as large). Clearly, system size has a very strong relationship with the amount of energy needed to run the cycle, but a weaker response in terms of water production. In designing an HDH system, there is a considerable
trade off between overall performance and the amount of water production.

![Figure 3-12: Water production rate versus system size for various humidifier and dehumidifer sizes](image)

3.3.2 Air-Heated System Model

In the case of an air-heated cycle, the dehumidifier air inlet temperature is specified, as is the cold water inlet temperature. In this case, the air stream is not linked, but the water stream (dehumidifier outlet to humidifier inlet) is linked. The energy input is specified by

$$\dot{Q}_{in} = \dot{m}_{da}(h_{D,da,i} - h_{H,da,o}).$$

(3.38)

While it was possible to emulate Mistry’s water heated cycle, the air heated cycle produced considerably more difficulty. For the air heated cycle, the model was unable to replicate the high GOR cycle. This result was largely due to convergence issues in the model when utilizing very large components (to achieve high values of
effectiveness). The high levels of effectiveness were unable to be obtained utilizing the current model. However, an air heated cycle was run that resulted in a GOR of 3.14 per Table 3.8.

### 3.4 Extractions in Off-Design Model

Like in the on-design model, it is possible to improve system efficiency via extractions and injections. Using the high performance water heated cycle discussed in Section 3.3.1 as a baseline, a single water extraction is made. Water is extracted from the dehumidifier at the local bulk water temperature of the node outlet. It is then injected into the humidifier at the same temperature to minimize entropy generation from mixing losses. This is modeled by removing water downstream of the outlet of a node, and upstream of the next node. To ensure temperatures match at both the extraction and injection sites, the nodes are iterated upon until the temperatures match. Several extractions were made along the length of the dehumidifier to compare the impact of extracting at a different location. Because the extraction and injection temperatures were required to match, it was not possible to extract at the very bottom of the dehumidifier as this water was cooler than water at any location in the humidifier. It is possible, however, to extract near the top and middle of the component at higher water temperatures. A compilation of extraction data is illustrated in Table 3.9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mistry On Design[43]</th>
<th>Off Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOR</td>
<td>7.76</td>
<td>3.14</td>
</tr>
<tr>
<td>TTD</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>$S_{gen}$ [kW/K]</td>
<td>0.15</td>
<td>0.38</td>
</tr>
<tr>
<td>$m_r$</td>
<td>2.12</td>
<td>2.10</td>
</tr>
<tr>
<td>$\epsilon_D$</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>0.96</td>
<td>0.87</td>
</tr>
<tr>
<td>$T_{min}$ [°C]</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>$T_{max}$ [°C]</td>
<td>90.0</td>
<td>90.0</td>
</tr>
</tbody>
</table>

Table 3.8: Comparison of two different air-heated HDH cycles
was seen that extracting water at relatively high temperature from the dehumidifier (extraction location at approximately 80% dehumidifier height) and injecting it very close to the top of the humidifier (at the same temperature and therefore enthalpy) produced a reasonably large increase in performance. Like with many of the on-design cycles, it was also apparent that for extracting at a fixed location, there is often an ideal extraction rate that achieved the highest GOR. In Figure 3-13, GOR was improved to 5.4 from the baseline of 4.9 via a 40% extraction in water flow (.029 kg/s) at approximately 50 °C.

Also plotted in Figure 3-13 is the normalized stream to stream variance. An important advantage of this off-design study over the on-design studies is that full temperature and humidity profiles may be developed for the entire length of the humidifier and dehumidifier. Tondeur et al. [45] proposed that the optimal method of reducing entropy production within a contacting or separation device was to “equipartition” the entropy generation such that entropy production is evenly distributed along the length of the device. The fundamental motivation here is that in equipartition, all entropy producing driving forces are uniform (specifically, the variance is zero). Tondeur and Thiel et al. [45, 20], who recently extended this concept to HDH dehumidifiers, show that when the variance reaches zero, entropy production is minimized. For both humidifiers and dehumidifiers, the driving forces in question are
heat and mass transfer. Carrington and Sun [46] illustrate the same by showing that entropy generation is a product of temperature gradients, concentration gradients, and a coupled term encompassing both. Thiel [20] illustrated that it is not possible to achieve simultaneous uniform driving forces in both heat and mass transfer due to the exponential dependency of humidity on temperature. However, it is possible to drive one or both forces close to a uniform state and reduce entropy production. The normalized stream to stream variance depicted in Figure 3-13 illustrates this analysis. GOR is maximized when the heat and mass transfer variances are minimized. The normalized variance in humidity (mass fraction of water) is defined as

$$Var_m = \sum_{i=1}^{n} \frac{(\Delta m - \overline{\Delta m})^2}{n}$$  (3.39)
where
\[
\Delta m = \frac{m_a - m_i}{\text{Average}(m_a - m_i)} \quad (3.40)
\]
and normalized variance in temperature is defined as
\[
\text{Var}_T = \frac{\sum_{i=1}^{n} (\Delta T - \Delta \overline{T})^2}{n} \quad (3.41)
\]
where
\[
\Delta T = \frac{(T_a - T_i)}{\text{Average}(T_a - T_i)} \quad (3.42)
\]
The variance is a measure of the uniformity of the stream to stream driving force. In the case of the dehumidifier shown in Figure 3-13, the variance in humidity compares the humidity of the bulk air stream to that of the air water interface of the condensate. The variance in temperature compares the bulk air temperature to that of the condensate interface. For the humidifier, the humidity variance comes from the difference between bulk air and humidity at the liquid/air interface on the fill. Temperature variance measures the bulk air temperature and interfacial temperature on the fill. Clearly the variance in the mass transfer is the larger of the two, though GOR appears to minimize where both variances reach a relative minimum in the dehumidifier. For the humidifier, increasing extraction appears to increase the variance in both \( m \) and \( T \). This is also reflected in the HCR, which grows from 1.15 to 1.34 at 40% extraction rate. Figure 3-13 confirms that, like the on-design studies, it is the dehumidifier that is driving the system performance more so than the humidifier. Again similar to on-design, by extracting and balancing the dehumidifier, the humidifier performance worsens. However, as seen in Figure 3-13, the variance in \( m \) and \( T \) for the humidifier is much smaller at low extraction rates and does not become larger than the dehumidifier until after the peak GOR value is reached. At the peak GOR, the dehumidifier has reached a peak performance while the humidifier performance worsens. To the right of peak GOR, the humidifier entropy generation begins to drive...
the system performance and GOR quickly drops in response.
Chapter 4

Conclusions

Thermodynamic system balancing by way of multi-extraction is a clear and effective method to increase the performance of an HDH system without necessarily changing equipment size. While it is true that increasing the equipment size (and therefore effectiveness) for the humidifier and dehumidifier of an HDH system is perhaps the most straightforward method of improving system performance, for an existing system, single or multiple fluid extractions can be made to increase performance (GOR) considerably.

Both on and off design studies did not generate many sweeping generalizations for multi-extraction, but many critical ideas were assessed and evaluated and highly useful results were obtained.

4.1 On-Design Lessons Learned

- First, it was confirmed that driving the system toward $HCR = 1$ either by way of adjusting the mass flow rate ratio, $m_r$, or via extractions significantly reduces entropy generation, $\dot{S}_{gen}$, and increases GOR.

  - When the system is split into multiple subcomponents, adjusting the extraction rate between the components such that $HCR = 1$ for the components with the largest $\dot{S}_{gen}$ is the best method to improve performance.
- Optimally, adjusting extractions and injections to force HCR = 1 for all subcomponents should yield an exceptionally high performance system.

- Next, for all single extraction systems, performance was optimized by extracting and injecting from the stream with the maximum capacity rate.

  - For example, when $m_r$ is large and the water stream has the maximum capacity rate, a larger performance gain was realized by extracting water.
  - Conversely, a system with a small $m_r$ with air at the maximum capacity rate had improved performance when air was extracted.

- Finally, it was found that in the majority of cases, multiple extractions improved performance moreso than a single extraction.

  - This statement is particularly true for systems with low TTD (and high effectiveness).
  - While multiple extractions produce higher performance, it was also determined that the first extraction typically resulted in the highest performance gain, while subsequent extractions yielded diminishing returns. In designing an HDH system with multiple extractions, it is clear that there will be a trade off between the cost and the performance gain of the $N$th extraction.

4.2 Off-Design Lessons Learned

- Like in heat exchangers, for a fixed area component, entropy minimization occurs at the same cycle condition as the minimum effectiveness. Therefore, to have a high performance system (both high effectiveness and low entropy generation) the equipment utilized must be large.

- To reduce the necessary size of equipment to achieve a high GOR system, extractions may be made. Adding extractions to the cycle can allow for a higher system performance without changing the equipment size. Alternatively, increasing both system size and the number of extractions can generate even higher GOR cycles.

- High GOR cycles (of up to 6-7) are possible in the water-heated configuration. These cycles require large equipment (3-4 m tall humidifier) and will not produce significantly more product water than a lower GOR case. However, the energy input requirement of these cycles drops dramatically (from 1.78 to 0.94 kW thermal for a GOR of 6.90 and 2.17 respectively). This sharp reduction in heat input could significantly rescale the size of a solar collector if the HDH is to be powered by solar thermal energy, or could reduce the amount of steam leeched.
from a power generation cycle to run the system. In designing an HDH system, cost of the solar collector needs to be weighed against costs of the humidifier and dehumidifier. It may be that an optimum point exists in terms of minimizing energy intensity or maximizing water production.

• GOR is directly and strongly related to both the humidifier and dehumidifier size. It is possible to increase the dehumidifier size, for instance, to improve GOR but as the size increases, there will be diminishing returns. Eventually there is no benefit from increasing dehumidifier size without increasing the humidifier size simultaneously.

• Extractions boost system performance by minimizing entropy generation. Entropy generation is minimized by minimizing the variation in stream to stream humidity and temperature profiles. While both temperature and humidity are important, it is the humidity that typically has the largest variation, often by a factor of 5-7x. Therefore, while both profiles should be made parallel via extractions for optimum performance, humidity is the more desirable target.

4.3 Future Work

While numerous on and off-design cycles were analyzed in this study, a more generalized relationship between performance, entropy generation, and extractions will be developed. Key principles and ideas regarding extraction were assessed, but a quick yet rigorous model must be created than can easily predict the optimal extraction locations and flow rates based on any input system or cycle. This thesis showed a significant amount of overlap between the on and off design models which validates the theory. So while a simple thermodynamic model (on-design) would be adequate to begin the optimization, a full off-design model with extractions is a more rigorous way of evaluating real hardware. As seen in the off-design studies, some cycles (particularly air-heated cycles) are considerably more difficult to reproduce with actual hardware. While thermodynamically feasible, some of these cycles would require significantly more advanced hardware to meet the required heat and mass transfer characteristics. High GOR HDH cycles with significantly smaller energy inputs have been identified, but with a more complete multi-extraction off-design model, it is
likely that even higher performance systems may be obtained.

There may be an optimum design for an HDH system based on dollars per kg of water or in terms of energy input. Increasing humidifier and dehumidifier size in order to boost system performance requires a smaller solar collector. However, the increase in GOR does not scale 1:1 with the amount of water production. If a system is being evaluated in terms of water production, there likely exists a point where increasing the system size becomes impractical because the additional amount of water produced is insufficient to justify the cost. Conversely, if the goal of the system is to minimize energy use per unit cost, the optimal system size would be different. Further thermo-economic studies are required to optimize the system on a given parameter such as production rate or energy use.

## 4.4 Final Remarks

While HDH systems have existed for many years, there is clearly more optimization required to bring them up to par with other desalination systems which has been studied, fine-tuned, and perfected over the last few decades. A basic HDH system has comparably low performance when placed side by side with a large MED or MSF plant. However, by way of these and other studies, it is likely that HDH systems may be dramatically improved upon in the years to come. Though HDH may not take the place of existing systems, there are many niche markets, particularly in the developing world, where a small scale HDH system may be soon become competitive with other desalination technologies.
Appendix A

Comparison of Zero Extraction Cycles

As discussed in Section 2.4.5, the method by which the on-design model was constructed introduces some complications. Specifying the total component effectiveness is not possible when each is built of two or more subcomponents because doing so does not generate enough equations and conditions to fully specify the system. Additionally, it was found that attempting to specify a total fixed effectiveness when extractions were involved was not a valid method of analyzing the system. As found in Chapter 3, the addition of an extraction/injection into the cycle necessarily changes the total effectiveness of each component. Leaving the total effectiveness fixed, then, produces a different cycle that cannot be directly compared with the previous. Instead, each subcomponent must have a fixed effectiveness specified. This method too has its own ramifications. Each extraction requires one additional subcomponent. With each additional subcomponent added in series, the overall effectiveness of the system increases. As effectiveness directly impacts GOR, these systems will have a higher GOR, even with the extractions turned off. Thus, as described in Section 2.4.5 each case needs to be handled individually when adding extractions to ensure
the move from zero to one to two extractions (etc.) does not alter the performance of the baseline until the extractions are actually turned on.

Mistry et al. [47] described several on-design models without extraction and also at a fixed effectiveness. It was found that when a single component cycle without extraction was run, the models produced for this thesis yielded results nearly identical to [47] in terms of GOR versus mass flow rate ratio, $m_r$. Figure A-1 illustrates a single component water heated and air heated cycle each with $\epsilon_D = \epsilon_H = 0.8$. The curve exhibits the same shape depicted by Mistry. Note that in the air heated cycle, Figure A-1b, there is a portion of the curve that is marked by a dashed line. These values of $m_r$ produce a cycle that violates the second law, so the system is unable to operate using those boundary conditions.

When models with additional subcomponents are generated, however, the baseline GOR vs. $m_r$ curve changes considerably. Figure A-2 demonstrates the impact of increasing the number of subcomponents in the cycle model. The three models in Figure A-2a share the same input conditions and are not subject to any extractions or injections; this is the base case. However, the models have either one, two, or three subcomponents in the humidifier and dehumidifier which considerably changes the shape of the GOR vs. $m_r$ curve. The shape if the curves corresponds to the
individual performance of each subcomponent. For instance, each inflection point on the GOR curves occur when one of the subcomponents’ HCR = 1. In the case of the single subcomponent model, the inflection at $m_r = .7$ corresponds to HCR$_D = 1$ while the inflection at $m_r = 1.7$ corresponds to HCR$_H = 1$. Similarly, for the three subcomponent model, the inflection at $m_r = .4$ reflects HCR$_D1 = 1$, the inflection at $m_r = 1.75$ reflects HCR$_D2 = 1$, and even the small, difficult to see inflection at $m_r = 1.9$ corresponds to HCR$_H2 = 1$. At these inflections, the minimum capacity rate changes from one stream to another and the slope of the curve is altered accordingly.

The greater the number of subcomponents in the device, the higher the initial GOR before extraction is added to the cycle. This is because each subcomponent has a fixed effectiveness (in this case, $\epsilon_{all} = 0.8$). Therefore, the first subcomponent operates at an 80% effectiveness and then feeds into a further component operating at 80% effectiveness, and so forth. This model of the system reduces the “size” of each sub-component further downstream. If the temperature drop through the first component is 15 °C, the next component may be 10, then 5, etc. because each component is 80% effective. This series of heat and mass exchangers totals to an equivalent effectiveness of greater than 80%. Even when there are no extractions, the initial performance of a three subcomponent system is far superior to a system with a single component humidifier and dehumidifier. While such a system is still physically conceivable, it is physically distinct from the previous system in terms of size and architecture. Therefore, this approach is dissimilar to a laboratory or production design which would have components of a fixed size, and then injections/extractions would be introduced to improve the otherwise fixed design. Specifically, the multi-subcomponent method effectively increases the size of the humidifier and dehumidifier such that each system can not be directly compared with one another if each individual component has the same effectiveness. Thus, an off-design analysis of the HDH system is considerably more feasible and is discussed in Chapter 3.
Figure A-2: Zero extraction comparison for a 1, 2, and 3 subcomponent air heated model
Bibliography


