Measurement of the $B_s^{0}$ Lifetime in Fully and Partially Reconstructed $B_s^{0} \rightarrow D_s^{-}(\pi^{-})X$ Decays in $p\bar{p}$ Collisions at $\sqrt{s}=1.96\text{TeV}$

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272001-2
In the spectator model of heavy hadron decay, the lifetimes of all $b$ hadrons are equal, independent of the flavor of the lighter quarks bound to the $b$ quark. Using the heavy-quark expansion [1,2] in the calculation of the width, spectator quark interactions enter in higher order ($\Lambda_{QCD}/m_b$)$^3$ terms where $m_b$ is the mass of the $b$ quark and $\Lambda_{QCD}$ is the energy scale of the QCD interactions within the hadron. This leads to the lifetime hierarchy $\tau(B^0_s) < \tau(B^0)$, $\tau(B^{+})$. Theoretical results predict $\tau(B^+)/\tau(B^0) = 1.06 \pm 0.02$ and $\tau(B^0_s)/\tau(B^0) = 1.00 \pm 0.01$ [3,4]. The world averages for the corresponding experimental numbers are $1.071 \pm 0.009$ and $0.965 \pm 0.017$, respectively [5]. The precision of our knowledge of the $B^0_s$ lifetime is much less than for the $B^0$ and $B^+$ lifetimes, and therefore, a more precise measurement would be useful, both in general and for comparison with theoretical calculations. Such a measurement is especially warranted since the agreement on the lifetime ratio between theory and experiment is only fair.

In this Letter, we present a measurement of the $B^0_s$ lifetime in fully and partially reconstructed $B^0 \rightarrow D_s^{-}(\phi \pi^-)X$ decays in 1.3 fb$^{-1}$ collected in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV by the CDF II detector at the Fermilab Tevatron. We measure $\tau(B^0_s) = 1.518 \pm 0.041 \text{(stat)} \pm 0.027 \text{(syst)}$ ps. The ratio of this result and the world average $B^0$ lifetime yields $\tau(B^0_s)/\tau(B^0) = 0.99 \pm 0.03$, which is in agreement with recent theoretical predictions.

February 2002 and November 2006. This sample yields more than 1100 fully reconstructed (FR) $B^0 \rightarrow D_s^- \pi^+$ candidates with $D_s^- \rightarrow \phi \pi^-$ and $\phi \rightarrow K^+ K^-$ after online and off-line selection [6]. In addition, the sample reconstructed as $B^0 \rightarrow D_s^- \pi^+$ includes partially reconstructed (PR) $B^0$ candidates that are used in this lifetime measurement and more than double the number of $B^0$ candidates available for analysis. One such PR decay is $B^0 \rightarrow D_s^+ \rho^+$ with $\rho^+ \rightarrow \pi^+ \pi^0$, where the $\pi^0$ is not reconstructed. The inclusion of PR decays introduces an uncertainty in the momentum measurement of a given candidate. However, a correction to the proper decay time has been estimated, and the total uncertainty on the lifetime measurement is improved by the use of the PR final states.

The CDF II detector is described in detail in Ref. [7]. The detector elements relevant for this analysis are the silicon vertex detectors [8–10] and the central drift chamber (COT) [11]. The silicon detectors consist of 7 or 8 layers of microstrip silicon sensors covering the pseudorapidity [12] range $|\eta| < 2.0$. The COT is an open cell drift chamber covering $|\eta| < 1.0$. Both the COT and silicon vertex detectors are immersed in a uniform 1.4 T axial magnetic field with the field axis parallel to the proton beam.

A data sample enriched in hadronic $B$ decays is selected with a three-level trigger system that searches for tracks displaced from the primary vertex [13]. At level 1, patterns of hits in the COT are identified as tracks by the extremely fast tracker (XFT) [14]. At level 2, the silicon vertex trigger [15] associates a set of silicon hits with the XFT tracks and improves track measurement precision. The trigger requires each event to contain a pair of charged particle tracks, each having transverse momentum $p_T \geq 2$ GeV/c and transverse impact parameter $d_0$ in the range $d_0 \in [120 \mu m, 1 \ mm]$, where $d_0$ is defined as the distance of closest approach between the particle trajectory and the beam line, measured in the transverse plane. The opening angle between the tracks’ trajectories ($\Delta \phi$ in the plane transverse to the beam) must be between $2^\circ$ and $90^\circ$, and their intersection must be at least 200 $\mu m$ from the interaction point, as measured in the plane transverse to the beam direction. At level 3, track reconstruction is performed entirely in software, with the full precision of the tracking system available, and the level 1 and 2 requirements are confirmed. These trigger requirements preferentially select events containing long-lived particles and sculpt the proper time distribution of the particles that are accepted for analysis. As the background rate of this trigger requires prescaling at higher instantaneous luminosities, CDF also employs two more restrictive triggers that require the tracks in the trigger pair to have opposite charges, individual $p_T \geq 2(2.5)$ GeV/c, and the scalar sum $p_T \geq 5.5(6.5)$ GeV/c.

We reconstruct $B^0 \rightarrow D_s^- \pi^+$ candidates (where $B^0$ and $D_s^-$ imply $B^0_s$ candidates and $D_s^-$ candidates) by first identifying $D_s^- \rightarrow \phi(K^- K^+)\pi^-$ from tracks with $p_T > 350$ MeV/c using the invariant mass requirements $|m(K^- K^+)| \approx 1020.5 < 7.5$ MeV/c$^2$ and $|m(K^- K^+ \pi^-) - 1968.3| < 20$ MeV/c$^2$. The $D_s^-$ daughter tracks must satisfy a three-dimensional vertex fit. We then combine each $D_s^-$ with a positively charged track with $p_T > 1.0$ GeV/c to form a $B^0 \rightarrow D_s^- \pi^+$ candidate and require the pair to satisfy an additional three-dimensional vertex fit. We do not constrain the mass of the $\phi$ or $D_s^-$ in this fit. The decay length of the $B^0_s$ is measured with respect to the event’s primary vertex and must satisfy requirements on the following quantities: the decay length of the $B^0_s$ projected along the transverse momentum, $L_{xy}(B^0_s) > 450 \mu m$, and its significance, $L_{xy}(B^0_s)/\sigma_{L_{xy}}(B^0_s) > 5$; the transverse distance between the $B^0_s$ and $D_s^-$ decay points is greater than 0; the transverse impact parameter of the $B^0_s$, $|d_0(B^0_s)| < 60 \mu m$; and the significance of the longitudinal impact parameter, $|z_0(B^0_s)/\sigma_{z_0}(B^0_s)| < 3$. Both fits for the $B^0_s$ and $D_s^-$ vertices must have reasonable goodness-of-fit values when considering only the track parameters measured in the transverse plane.

To further separate $B^0_s$ mesons from backgrounds with similar topologies, we require the transverse momentum of the $B^0_s$, $p_T(B^0_s) > 5.5$ GeV/c, and the angular separation between the $D_s^-$ and the $\pi$ from the $B^0_s$, $\Delta R(D_s^-, \pi) = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} < 1.5$. We require the isolation of the $B^0_s$ to be greater than 0.5, defined as $p_T (B^0_s) \times \sigma(B^0_s)$ divided by the scalar sum of the transverse momenta of all the tracks in a cone of $\Delta R < 1$ around the $B^0_s$. We tighten the requirement on the mass of the $D_s^-$ ($m - 1968.3 < 12$ MeV/c$^2$) and veto $D_s^-$ candidates consistent with $D_s^+ \rightarrow D^0 \pi^- \pi^0$. The $D^+$ veto is accomplished by taking the $D_s^-$ daughter tracks ($K^- K^+ \pi^-$), assigning the pion mass to the negative kaon, and requiring $\Delta m = m(K^- K^+ \pi^-) - m(K^- K^+) > 180$ MeV/c$^2$. We also require that the decay contain two reconstructed tracks satisfying the level 2 trigger requirements.

The simulated data samples used in this analysis consist of single $b$ hadrons generated by BGENERATOR [16,17] with $p_T$ spectra consistent with next-to-leading-order QCD and decayed with EVTGEN [18]. Full detector and trigger simulations are performed. The simulated $B$ candidates are reconstructed with the same procedure and the same selection as the data candidates. We reweight the simulated sample to match the data distributions for $p_T(B)$ and trigger mixture.

The lifetime of the $B^0_s$ meson is determined from two sequential fits. The first is a fit to the invariant mass distribution of candidates reconstructed as $D_s^- \pi^+$ and is used to determine the fractions of the total number of events found in the various decay modes. These fractions are fixed inputs to the second fit, which is a fit to the proper decay time distribution of the candidates. The uncertainties on the fractions returned by the mass fit are treated as sources of systematic uncertainty.
The mass fit is an unbinned maximum likelihood fit to the invariant mass of the candidate reconstructed as $D_s^+ \pi^+$ with $m_{B_s}^{\text{rec}} \in [4.85, 6.45] \text{ GeV}/c^2$. The mass fit components can be characterized as coming from one of three possible sources: single $b$ hadrons, real-$D_s^+$ track background, and fake-$D_s^+$ track background. The mass probability distribution functions (PDFs) for single $b$ hadrons were obtained from simulation, with an additional small shift and resolution smearing to bring the simulated $B_s^0$ peak central value and width into agreement with data. The single-$b$ modes were separated in the fit as follows: $B_s^0 \rightarrow D_s^+ \pi^-(n\gamma)$, $B_s^0 \rightarrow D_s^+ K^+$, $B_s^0 \rightarrow D_s^+ \rho^+$, $B_s^0 \rightarrow D_s^+ \pi^+$, $B_s^0 \rightarrow D_s^- \pi^+$, $B_s^0 \rightarrow D_s^- \pi^+$, $B_s^0 \rightarrow D_s^- \pi^+$, and $B_s^0 \rightarrow D_s^- \pi^+$. The $D_s^+ K^+ / D_s^- \pi^+$ ratio was constrained to the results of Ref. [19].

Real-$D_s^+$ track backgrounds consist of a real $D_s^+$, produced promptly or from a $b$-hadron decay, plus an additional track produced in the event. The mass PDF for these events is obtained from an auxiliary fit to the wrong-sign sample, which consists of data events reconstructed as $D_s^+ \pi^-$ and sideband subtracted in the $D_s^+$ mass. The mass PDF for the fake-$D_s^+$ + track background is obtained from an auxiliary fit to the $D_s^+$ sidebands. The results of the mass fit are shown in Fig. 1 with various modes combined for plotting only. The real-$D_s^+$ + track background and fake-$D_s^+$ + track background components are drawn together as the “combinatorial background.”

For the lifetime fit, the variable of interest is the proper decay time, defined as $ct = (L_{xy}(B_s^0)m_{B_s}^{\text{rec}}) / p_T(B_s^0)$. The reconstructed mass $m_{B_s}^{\text{rec}}$ is used instead of the world average $B_s^0$ mass. A salient feature of this analysis is the treatment of partially reconstructed $B_s^0$ mesons as signal events that contribute to the lifetime measurement. Since in the partially reconstructed cases $L_{xy}(B_s^0)$, $m_{B_s}^{\text{rec}}$, and $p_T(B_s^0)$ are extracted from candidates that are missing particles after reconstruction or have the wrong mass assignment for a daughter particle, a multiplicative correction factor $K$ to the decay time is needed. $K$ is defined as $K = [p_T(B_s^0)m_{B_s}^{\text{true}}]/[p_T(B_s^0)m_{B_s}^{\text{rec}} \cos \theta_{\text{FR}}]$ where $\theta_{\text{FR}}$ is the angle in the $x$-$y$ plane between the true momentum of the $B_s^0$ and the momentum of the partially reconstructed $B_s^0$. Because the ratio $m_{B_s}^{\text{rec}} / p_T(B_s^0)$ is numerically very close to the ratio $m_{B_s}^{\text{true}} / p_T(B_s^0)$, this choice of $ct$ definition forces the K factor distributions to be centered near $K = 1$ with widths of a few percent. The K factor distributions are determined with simulation.

The lifetime of the $B_s^0$ meson is determined from an unbinned likelihood fit to the $B_s^0$ candidates with invariant masses in the range $[5.00, 5.45] \text{ GeV}/c^2$. There are three main types of lifetime fit components that will be described in the following paragraphs: fully reconstructed $B_s^0$, partially reconstructed $B_s^0$, and backgrounds. The treatment of each component depends on its decay structure and whether it can provide information about the $B_s^0$ lifetime.

Fully reconstructed modes where all of the $B_s^0$ daughter particles are included with the correct mass assignment in the construction of the $B_s^0$ candidate are the first type of lifetime fit component. The only FR mode in this analysis is the $D_s^+ \pi^+$. The core functional form of the FR PDF is an exponential with decay constant $c\tau(B_s^0)$ convoluted with a Gaussian resolution function with width $\sigma$: $P_{\text{FR}}(ct) = \left[ \frac{1}{c \tau} e^{-ct/(c\tau)} \Theta_{c \tau} \left[ \frac{1}{\sqrt{2\pi}\sigma} e^{-(ct-c\tau)^2/2\sigma^2} \right] \right]_{\text{eff}}(ct)$.

A multiplicative “efficiency curve” accounts for the trigger and analysis selection criteria: $\text{eff}(ct) = \sum_{i=1}^3 N_i (ct - \beta_i)^2 e^{-ct/(c\tau_i)} \Theta(\beta_i)$.

The shape parameters ($\sigma$, $\beta_i$, $N_i$, and $\tau_i$) of the PDF are determined in a fit to a simulated $B_s^0$ sample where the lifetime used for generation is known. All the parameters for the PDF are then fixed and only $\tau(B_s^0)$ is varied in the final fit to the data. As we depend on the simulation of the displaced-track trigger, we use a data sample of $J/\psi \rightarrow \mu^+ \mu^-$ decays collected with a dimuon trigger to assess the accuracy of this assumption and assign a “trigger simulation” systematic uncertainty based on these studies. The partially reconstructed, PHOTOS-modeled $D_s^+ \pi(n\gamma)$ decays [20] are combined with the FR $D_s^+ \pi$, as the momentum carried by the photon is small and their lifetime distribution is extremely close to the FR one. This simplification is considered as a possible source of systematic uncertainty.

Partially reconstructed modes either neglect $B_s^0$ daughter particles in the construction of the $B_s^0$ candidate or assign them an incorrect mass. $B_s^0 \rightarrow D_s^+ K^+, D_s^+ \rho^+, D_s^+ \pi^+$, and other decay modes partially reconstructed under the $D_s^+ \pi^+$ hypothesis can also contribute to the $B_s^0$ lifetime.

FIG. 1 (color online). Mass distribution for candidates reconstructed as $B_s^0 \rightarrow D_s^+ \pi^+$ with fit projections overlaid.
The analysis procedure was tested extensively on three control samples: \(B^0 \rightarrow D_s^- \pi^+\) with \(D^- \rightarrow K^- \pi^- \pi^-\), \(B^0 \rightarrow D_s^+ \pi^-\) with \(D^+ \rightarrow D^0 \pi^-\) and \(D^0 \rightarrow K^+ \pi^-\), and \(B^+ \rightarrow D_s^0 \pi^+\) with \(D^0 \rightarrow K^+ \pi^-\) before performing the \(B^0\) fits. Furthermore, the lifetime fit of the \(B^0\) was performed using a blind approach, i.e., by determining the statistical and systematic uncertainties without knowledge of the fit result itself. Good agreement with the world average values [5] of the \(B^0\) and \(B^+\) lifetimes was found.

The lifetime of \(\tau(B^0) = 1.518 \pm 0.041\) (stat) ps is obtained from the full fit. The fit results are plotted in Fig. 2. The results of the fits performed separately in the FR mass region (1.456 ± 0.067 ps) and PR mass region (1.544 ± 0.051 ps) agree with each other at a level of 1.0\(\sigma\).

We use a Monte Carlo technique to assess the systematic uncertainties. For each source of systematic uncertainty, we generate 1000 simulated experiments with the number of events in each experiment Poisson distributed around the number of events in data. The simulated experiments are generated with a nonstandard lifetime fit configuration (where the PDFs or numbers of events in the various modes are modified to account for the systematic effect) and fit with the default configuration. The mean biases returned from the fits to the simulated experiments (\(\bar{\tau}_{\text{sim}} - \tau_{\text{gen}}\)) are used to set the size of the systematic uncertainties. We consider several sources of systematic uncertainty: combinatorial background fraction, modeling of backgrounds from single-b-hadron decays, effect of reweighting the full simulations to match the data, modeling of the trigger bias as a function of \(ct\), off-line–on-line impact parameter correlation, accuracy of the trigger simulation, and detector alignment. Table I contains the final list of systematic uncertainties for this measurement. The largest contribution comes from the uncertainty on the total amount of combinatorial background and the amount of promptly produced real-\(D_s^-\) background.

The displaced-track trigger, in addition to modifying the accepted decay length distribution from a simple exponential to the form in Eq. (1), alters the expected mixture of mass eigenstates \(B_{sL}\) and \(B_{sH}\) in the flavor-specific \(B^0 \rightarrow D_s^- \pi^+\) decay by preferentially selecting the longer-lived \(B_{sH}\). The size of the imbalance can be calculated using the parameters of the efficiency curve and the world average of \(1/\Gamma = 1.47\) ps [5]. Our result can be corrected back to a flavor-specific lifetime measurement with \(\delta \tau(B^0) = -0.11(\Delta \Gamma/\Gamma)^2\) ps. Given the world average \(\Delta \Gamma/\Gamma = 0.092_{-0.054}^{+0.051}\) [5], the correction would be smaller than our statistical and systematic uncertainties. Therefore, we do not correct the central value or assess an additional systematic uncertainty.

In summary we have measured the \(B^0\) lifetime using both fully reconstructed \(B^0 \rightarrow D_s^- (\phi \pi^-) \pi^+\) and partially reconstructed \(B^0 \rightarrow D_s^- (\phi \pi^-)X\) decay modes, in a sample

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<td>Background modeling and fractions</td>
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| Fixed single-
  b background \(ct\) | 0.003 |
| Reweighting for \(p_T\) and trigger | 0.012 |
| Lifetime contribution of \(D\pi\) radiative tail | 0.002 |
| Efficiency curve parametrization | 0.002 |
| Trigger simulation | 0.014 |
| Impact parameter correlation | 0.003 |
| Detector alignment | 0.003 |
| Total systematic uncertainty | 0.027 |

FIG. 2 (color online). Distribution of \(ct\) for candidates reconstructed as \(B^0 \rightarrow D_s^- (\phi \pi^-) \pi^+\) with fit projection overlaid.
with 1.3 fb$^{-1}$ of integrated luminosity. We measure $\tau(B^0_s) = 1.518 \pm 0.041({\text{stat}}) \pm 0.027({\text{syst}}) \text{ps}$, which is consistent with theoretical expectations $[3,4]$.

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References:

[6] Reference to the charge conjugate modes is implied throughout this Letter.
[12] CDF II uses a right-handed coordinate system with the origin at the center of the detector, in which the $z$ axis is along the proton direction, the $y$ axis points up, and $\phi$ are the polar and azimuthal angles, and $r$ is the radial distance in the $xy$ plane. The pseudorapidity $\eta$ is defined as $-\log \tan(\theta/2)$.