A Monte-Carlo Pricing Model for Commercial Mortgage-Backed Securities

by

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Bachelor Of Science
Trent University
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Submitted to the Department of Urban Studies and Planning in Partial Fulfillment of the Requirements for the Degree of

Master of Science
in Real Estate Development
at the

Massachusetts Institute of Technology

September 1995

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ABSTRACT

The rather large differences in yield spreads between fixed rate commercial mortgage-backed securities (CMBS) and similarly rated investment grade corporate bonds lead one to question the true risk facing the rated CMBS investor. In an effort to quantify the risks inherent in these securities, a monte-carlo pricing model is developed that endogenizes several factors affecting the underlying mortgages, and incorporates conditions related to security design. The model and subsequent results are not only consistent with numerous market pricing intricacies, but also lend original insight into the effects of diverse marketplace parameters. As an example, the model’s analysis of mortgage extension presents a valid explanation for a portion of these excess spreads.

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Yield spreads on fixed rate commercial mortgage-backed securities (CMBS) exceed similarly rated investment grade corporate bonds at all rating levels. At non-investment grade levels for example, the spread differences have exceeded three hundred basis points. Although spreads have tightened somewhat over the past eighteen months, substantial return differences still exist across all rating levels. These differences immediately lead one to question the true risk facing the rated CMBS investor.

Under the assumptions of an efficient market, any discrepancy between equally rated securities that are equivalent in all other respects would be eliminated through arbitrage competition. Although it is plausible that investor and rating agency unfamiliarity with these securities may account for a portion of these differences, the theory of a competitive securities market suggests a deeper explanation.

In assessing the true risk exposure of a CMBS, numerous uncertainties must be properly addressed. In particular, the inherent risks in the underlying collateral are compounded by complex issues surrounding borrower behavior. In addition, due to the youthfulness and unique design of this type of security, the response nuances of commercial mortgage partitioning are not well understood. Although, the security derives its value from a pool of whole loans, varying levels of claim priority facing the distinct investment classes (tranches) are likely to impart effects that differ from those of a whole loan viewed separately.

The focus of this thesis is to determine the direct valuation effects of these inherent risk factors. By developing a CMBS pricing model that endogenizes several factors affecting
the underlying collateral, the effects of a changing environment are examined under varying security designs.

The result is not only a model that is consistent with numerous market pricing intricacies, but one that also lends original insight into the effects of diverse marketplace parameters. As an example, the model’s analysis of the untested area of mortgage extension presents a valid explanation for a portion of the excess spreads currently viewed in the market.

In order to provide insight into the model’s framework, it is helpful to further detail the historical aspects related to the development of CMBS. The dramatic growth in this market stems from the unique combination of factors present in the marketplace in the early 1990’s. With the understanding that CMBS are derivative securities, the proper place to begin this review lies within the area of commercial whole loan investment.

1.1 Investment Background

1.1.1 Commercial Whole Loans

Historically, commercial mortgages had offered greater yields than treasury securities and investment grade corporate bonds. Institutions were attracted to these yields and allocated increasing amounts of capital to this investment class. In fact, outstanding commercial mortgage debt grew steadily from an early 1982 amount of $425 Billion, to a mid 1991 peak of $1.08 Trillion. This dramatic increase is evident in Figure 1-1 which shows year end Total Mortgage Debt Outstanding through the years of 1985-1989.
The combination of i) preferential tax treatment for real estate, ii) deregulation of the financial services industry and, iii) a prosperous economy, fostered the 1980's boom in real estate. This resulted in a situation in which loans were being underwritten on the assumption of continually increasing prices (Quigg, 1993). As a result, this market grew to be larger than the municipal bond market and only 20% less in size than the corporate bond market.

As we are all well aware, the real estate market has undergone profound change since the turn of the decade. The tax reform act of 1986 removed almost all of the attractive tax
incentives for owning real estate. At the same time, the overabundant availability of debt capital resulted in overbuilding, and produced a drastic oversupply in the market. The combined action of this increase in supply coupled with the lowered demand decreased prices, and brought the commercial real estate market into collapse. To make matters worse, prior enactment of lenient regulatory action was reversed and tighter restrictions were implemented.

Once the downturn in the market took place, this overleveraged exposure to real estate risk resulted in miserable loan portfolio performance. Lenders were faced with tremendous losses. According to one study by Coopers & Lybrand, approximately 23% of the total commercial mortgages underwritten during the period between 1984 and 1990 were restructured or foreclosed (Childs, Ott & Riddiough, 1994). They estimate that this represents over $300 Billion of commercial loan investment. Several banks and insurance company loan portfolios experienced delinquency rates of thirty percent (30%). Certain mid-80's cohorts experienced ex post holding period returns that differed from contract rates by a reduction exceeding five percent (5%) (Ciochetti & Riddiough, 1994).

As a result of this experience, many lenders withdrew from the market. In addition, numerous others were forced out. The major reason for many Savings & Loan failures were these overwhelming losses brought on by commercial loan investment. Between the first quarter of 1988 and the year end of 1992, thrift holdings of commercial and multifamily mortgages were reduced by 55% and 35% respectively. Figure 1-2 clearly illustrates the withdrawal of mortgage capital by traditional lending sources, an important factor fostering the growth of the CMBS market.
Further fallout resulting from these blatant losses has come from increased regulatory pressure. Governing bodies have compelled institutions to write down or dispose of non-performing mortgages. The beginning of 1994 ushered in new, more stringent, risk-based capital requirements for life insurance companies. As a result, additional capital reserve provisions are placed upon the lender as commercial loans are now included in the riskiest investment category. The following section shall outline how these regulatory changes have become another key factor in the growth of the CMBS market.
1.1.2 Commercial Mortgage-Backed Securities

The interrelated and synergistic effects of, i) the distressed real estate market, ii) the poor performance of commercial loans, iii) the retrenchment of lenders, and iv) the subsequent regulatory reaction, led to several new marketplace requirements. First of all, a capital starved real estate market required re-supply. Secondly, “credit enhancement” of portfolios was needed in order to meet the new risk based capital rules, and finally, quick disposal of troubled assets was now imperative.

The requirement of supply side capital resulted from the drastically curtailed lending activity. As previously outlined, the retreat from the market of traditional lending sources was severe. Between the first quarter of 1990 and the second quarter of 1993, the share of major institution’s multifamily and other commercial loans dropped from 56% to 46%, and from 90% to 83% respectively. Although approximately one half of this $130 billion decline may be accounted for by an overall decline in the market, the balance none the less created an enormous credit vacuum.

Furthermore, regulatory changes resulting from the aforementioned risk based capital rules forced institutions to seek alternative investments. Table 1-A outlines the new requirements and illustrates the incentive provided to hold investment grade securities as opposed to whole loans.
Table 1-A
Risk Based Capital Requirements

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Capital Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Faith &amp; U.S. Gov’t. Credit: Treasuries, GNMA’s</td>
<td>0 %</td>
</tr>
<tr>
<td>NAIC 1 (AAA-A Sec.): Agency deb., corp.,</td>
<td>0.3 %</td>
</tr>
<tr>
<td>MBS, CMO, ABS</td>
<td></td>
</tr>
<tr>
<td>NAIC 2 (BBB Securities)</td>
<td>1 %</td>
</tr>
<tr>
<td>NAIC 3 (BB Securities)</td>
<td>4 %</td>
</tr>
<tr>
<td>NAIC 4 (B Securities)</td>
<td>9 %</td>
</tr>
<tr>
<td>NAIC 5 (CCC Securities)</td>
<td>20 %</td>
</tr>
<tr>
<td>NAIC 6 (Securities in Default)</td>
<td>30 %</td>
</tr>
<tr>
<td>Residential Mortgages (single &amp; multi-family)</td>
<td>0.5 % *</td>
</tr>
<tr>
<td>Farm Mortgages</td>
<td>3 % *</td>
</tr>
<tr>
<td>City Mortgages Insured or Guaranteed</td>
<td>0.1 % *</td>
</tr>
<tr>
<td>Other Mortgages (Including Commercial Mortgages)</td>
<td>3 % *</td>
</tr>
<tr>
<td>Preferred Stock</td>
<td>2.3 % - 30 %</td>
</tr>
<tr>
<td>Common Stock</td>
<td>30 %</td>
</tr>
<tr>
<td>Real Estate</td>
<td>10 %</td>
</tr>
</tbody>
</table>

* This percentage is multiplied by a factor ranging from 0.5 to 3.0, reflecting the individual company’s two-year past delinquency and foreclosure experience compared to the two-year industry average.

While the most significant capital factor requirement for investment grade securities is only 1 %, with AAA-A being only 0.3 %, commercial mortgages on the other hand require anywhere from 1.5 %, to a high of 9 %. The more highly rated tranches of CMBS fall under the securities guidelines and therefore enhance the credit of an institutional mortgage portfolio. This drastically reduces the capital reserve allocations required to hold such investments and provides a strong outlet for CMBS.
In addition, the pressure to dispose of non-performing and problem assets resulted in a huge over supply of mortgages on the sale block. More than $10 billion of book value was transferred by thirty two private institutions as of November 1993. It is believed that there was another $1.7 billion still in the pipeline at that time. Perhaps more importantly, RTC liquidations during this same timeframe totaled $13.4 billion. Such massive conveyances required a new, more efficient market.

The intermediary nature of the capital markets is well set up to handle these newly created market needs. Firstly, they provide an efficient market for capital re-supply. Secondly, they are a source for qualified credit enhancement for institutions familiar with the underlying collateral. Finally, securitized transactions may easily accommodate the swift transfer of assets that was required.

As a result, the CMBS market developed rapidly. The RTC provided the initial impetus for this growth with its willingness to take huge losses in exchange for rapid disposal of its acquired assets. This large transaction volume helped to develop the critical mass of infrastructure, and allowed investors to become more familiar with this new security without the perception of great risk.

Many have joined the ranks of the RTC in taking advantage of this vehicle, and total CMBS issuance has grown rapidly over the last several years. Since 1990 there have been more than $65 billion of CMBS securities supplied to the market. This dramatic growth is well reflected by Figure 1-3 which shows total CMBS issuance segregated into RTC and non-RTC components.
It is important to note that although the RTC was heavily responsible for the early rise of this market, RTC issuance has steadily declined from its peak in 1992. The RTC supplied the necessary initial stimulus and has left behind a stronger market. Although, RTC issuance will soon phase out entirely, the market has its own momentum and continued growth prospects are excellent.

While some estimates expect as much as 25% - 40% of the $290 billion multifamily mortgage market and 5% - 10% of the $700 billion comprising other commercial mortgages to become securitized, only 2.7% of non-residential commercial mortgages and 10.3% of multifamily mortgages were securitized by the end of the second quarter of 1993. Whereas Figure 1-4 shows the relatively small proportion of securitized...
commercial mortgages as of the first quarter of 1994, recently, some buyers of CMBS have estimated that the market will grow to encompass 20%-30% of the entire trillion dollar market.

![Commercial Mortgages Pie Chart]

Source: Morgan Stanley

Figure 1-4

Commercial Mortgage Securitization

The market obviously has tremendous expansion potential. With $160 billion of existing commercial mortgages maturing each year, this market could reach $18- $30 billion of annual issuance on turnover alone. Traditional lenders will continue to be encumbered by the regulatory burden as the new capital requirements have the support of bank depositors, insurance policy holders and the financial markets. Mortgagees may then resort more to conduit arrangements, a turnover process that will foster continued lending. Overall, the potential for close to $200 billion of outstanding CMBS may even be possible by the end of the decade (Quigg, 1993).
1.2 Pricing Considerations

The interplay of pricing between the private and capital markets will undoubtedly play a role in the growth of the CMBS market. It is therefore important to understand the similarities and differences between securities and their underlying whole loans in terms of risk.

Uncertainty of cash flows produced by the underlying mortgages is a major factor affecting the pricing of CMBS. Three important, yet unstable decision factors contributing to whole loan uncertainty are: i) default, ii) extension, and iii) prepayment. Due to the fact that many commercial mortgage contracts contain prepayment lockout or yield maintenance clauses, only the first two are considered in this study.

Commercial lending is typically done on a non-recourse basis. That is, the only source for recovery of principal is the property itself. In addition, unlike the single family residential market, there is no insurance available to cover default losses in the commercial mortgage market. Although such insurance has been attempted, the problem of adverse selection has been a major factor in its demise. As a result, the exposure resulting from borrower default is the major concern to the commercial mortgage backed securities investor.

The mortgagor has the option to relinquish his title to the property and walk away from the debt. The investor is then left with an asset which may or may not be as valuable as the note. Certainly, in most cases, the underlying collateral is not as valuable at this point, otherwise the borrower would not have foregone the equity.

The second uncertain risk of concern results from the decision to extend the term of a loan beyond its original maturity. Upon encountering a situation in which the borrower is
either unable or unwilling to retire the note, the investor may grant additional time for repayment of principal. This will certainly affect the timing of cash flows to the investor by delaying full repayment of the note. Even in a flat term structure, the potential negative yield impact may become significant as the result of increased exposure to default.

Typically, extended loans tend to be in financial distress. For example, the decision to extend is not likely to be enacted for the case in which it is certain that recovery from property price will cover the outstanding debt. As a result, extension usually involves the continued financing of a problem loan. A problem loan has an increased probability of default, and if property value continues to deteriorate, recovered principal could further diminish.

Under the unique design structures typical of commercial mortgage backed securities, there exist various levels of insulation against this overall exposure. Higher priority tranches are afforded “credit protection” by subordinate tranches, as these lower level tranches are first to absorb losses. Partitioning and prioritization of whole loan cash flows alters the return dynamics and each investment tranche behaves differently than the loan as a whole.

Altered by this type of structure, changes affecting the underlying collateral may not necessarily affect each security tranche in the same manner. For instance, Childs, Ott, and Riddiough (1995) indicated that the known benefits from loan diversification extended directly to the upper tranches, but that same diversification is actually detrimental to a first loss tranche. Consequently, the specifics of security design in terms of priority and tranche size will play an important role in determining required yield spreads. In order to gain further insight into the inherent pricing uncertainty, the following chapter outlines some of the research relevant to these pricing considerations.
2. Relevant Literature Review

2.1 General Profile

The research most relevant to this topic tends to originate from two key areas of debt related pricing inquiry: i) commercial whole loan pricing, and ii) commercial mortgage-backed security pricing. Due to the newness of the CMBS market, there has been little formal study covering this topic directly. Valuation of the derivative security of CMBS is certainly dependent upon proper pricing of its underlying asset composition of whole loans however, and direct CMBS pricing insight may therefore be gained from whole loan research.

The analysis techniques utilized for much of the recent pricing investigation generally fall into two categories: i) rational option pricing, and ii) monte-carlo simulation. The former quantifies the options embedded in mortgage contracts (put-default, call-prepayment) and utilizes sharp decision boundaries for borrower behavior. The latter on the other hand, incorporates uncertainty by placing probabilities upon borrower decisions and then generates numerous sample paths to realize value. Both models will be more thoroughly outlined in the next chapter.

Please note that this chapter is not intended as an exhaustive outline of research in these areas. Rather than review the entire body of influential work, several studies are highlighted instead, with the intention of giving the reader sufficient background to understand the evolution of more recent research surrounding contingent claims approaches, and their application for pricing CMBS. This approach should assist the reader to grasp the logic behind the methodology utilized in this paper.
The summary embodied by Table 2-A is a useful guideline for review of the significant factors of each outlined piece of work. The table summarizes each paper’s: i) area of research focus, ii) overall methodology as applicable to that of this paper, and iii) significant findings. The table is organized by progression through each general area of inquiry.
### Table 2-A

**LITERATURE REVIEW**

<table>
<thead>
<tr>
<th>AUTHORS</th>
<th>FOCUS</th>
<th>METHODOLOGY</th>
<th>SIGNIFICANT RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark yield cost of default</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Commercial Whole Loans-</td>
<td></td>
<td>Negative relationship with default rates &amp; 5yr. cumulative property value change.</td>
</tr>
<tr>
<td></td>
<td>Yield cost of default</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ciochetti &amp; Riddiough (1994)</td>
<td>Commercial Whole Loans - Post foreclosure performance</td>
<td>Analyze complete disaggregate cash flow histories for sample of defaulted loans</td>
<td>Foreclosure period avg slightly over 1yr. Recoveries at 75-80%. State law has signif. impact.</td>
</tr>
<tr>
<td>Titman &amp; Torous (1989)</td>
<td>Commercial Whole Loans- Investigate contingent claims model</td>
<td>Two state variable (prop. value &amp; interest rate) contingent-claims rational option model</td>
<td>Model provides accurate estimates for rates. Prop. Val. volatility &amp; payout rate have major rate effects</td>
</tr>
</tbody>
</table>
Table 2-A (cond.)

<table>
<thead>
<tr>
<th>AUTHORS</th>
<th>FOCUS</th>
<th>METHODOLOGY</th>
<th>SIGNIFICANT RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vandell (1992)</td>
<td>Commercial Whole Loans-</td>
<td>Option based default</td>
<td>High transaction costs of default result in underexercise of option.</td>
</tr>
<tr>
<td></td>
<td>Estimate parameters associated</td>
<td>factoring in transaction costs of default</td>
<td>Much variation explained by LTV.</td>
</tr>
<tr>
<td></td>
<td>with default</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Riddiough &amp;</td>
<td>Commercial Whole Loans-</td>
<td>Monte-carlo.</td>
<td>Signif effects from amortization, LTV, payout, prop. volatility.</td>
</tr>
<tr>
<td>Thompson (1993)</td>
<td>Recognize default transaction cost</td>
<td>with probabilities accounting for option value</td>
<td>Rate &amp; term structure less signif.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corcoran &amp; Kao</td>
<td>CMBS-</td>
<td>Link ACLI delinquencies with NCREIF returns to infer CMBS credit by assessing whole loan def.</td>
<td>Modest rise in real estate prices, and amortization each have significant impact on CMBS</td>
</tr>
<tr>
<td>(1994)</td>
<td>Real estate market assessment of</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CMBS credit</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Childs, Ott &amp;</td>
<td>CMBS-</td>
<td>Combined Backward”</td>
<td>Diversification may decrease value of junior tranche. Apparent differences due to term structure and security structure</td>
</tr>
</tbody>
</table>
2.2 Whole Loan Research

We have seen, that both the size and investment impact of the whole loan market is substantial. Due to the unavailability of data however, there has been little formal research in this area. A large proportion of this market is in the private hands of banks and insurance companies, and they are under no obligation to disclose disaggregated contractual and/or performance information. Many companies view such information as proprietary and will not disperse data considered to supply a competitive advantage. None the less, this area of research still supplies the greatest amount of applicable insight into the valuation of CMBS.

Traditional default assessment in this area has focused on cross-sectional mortgage evaluation. That is, defaults were measured as a percentage of a loan portfolio at any given point in time. A more appropriate and telling measure however, would track loans throughout their lifetime. Snyderman (1991) utilized such a "longitudinal" approach to default study and refuted the conventional wisdom surrounding the borrower’s decision. The study’s default projection of 15.4% tripled the previous belief of frequencies centered in the 5% range.

This important study also indicated that frequencies varied with loan cohort. The fact that certain years of origination produced significantly different levels of default frequency indicated that there appeared to be more advantageous periods in which to make loans. Snyderman’s (1994) second study confirmed that default rates did in fact have a negative relationship with the previous five years cumulative change in property value. We would then expect endogenization of property value to be an important consideration for modeling whole loans and the resultant CMBS.

In order to properly price the cost of default, it is important to understand the actual loss that results once a foreclosure occurs. The Ciochetti & Riddiough (1994) analysis of post
foreclosure performance indicated loss recoveries in the range of 75% -80%, and these results provide a valid benchmark for modeling borrower behavior in a CMBS structure.

Recent modeling approaches have borrowed from the contingent claims approach of representing the liability as an option on the total value of the assets. This approach is popular for corporate bond pricing, and an option model was tested for commercial mortgage debt by Titman & Torous (1989). A significant portion of observed default premia was accounted for by their model, which suggests option pricing may provide accurate estimates of commercial mortgage rates.

The model used by Titman & Torous (1989) assumed “ruthless default” scenarios. That is, default is considered to occur as soon as the mortgage value exceeds building value. By providing for the existence of borrower transaction costs of default, Vandell (1992) on the other hand illustrated that such costs may result in underexercise of the default decision. This improved design provides for greater explanatory power by more accurately reflecting borrower behavior, a factor that should also contribute to more precise modeling and valuation of CMBS.

Riddiough & Thompson (1993) represented the importance of mortgage contract terms and conditions when determining whole loan pricing. Their approach determined that significant effects result from the underwriting requirements of loan to value (LTV) and amortization, factors that may differ for each loan in a CMBS pool. Furthermore, Riddiough & Thompson (1993) illustrated that property characteristics and mortgage contract design are more important than interest rate environment for assessing pricing impact, an important consideration in the development of the model of this thesis.

In order to incorporate the numerous state variables required for such an assessment, Riddiough & Thompson (1993) implemented a monte-carlo pricing model for their study.
This illustrates an important benefit of the monte-carlo technique. As opposed to rational option models, monte-carlo modeling more easily incorporates complex cash flow patterns and numerous state variables. Modeling CMBS tranche structuring and consideration of a greater number of endogenous variables is therefore more straightforward under this approach.

A more important benefit of the approach stems from the fact that the previously aforementioned and important borrower transaction costs (Vandell (1992)), are not accurately observable to the lender. As a consequence, utilizing probabilities to simulate more non-discrete, “fuzzy”, default boundaries may be more appropriate for modeling borrower default decisions.

One last key point regarding the Riddiough & Thompson (1993) study relates to their clever consideration of a significant criticism of monte-carlo models. Typically, these models fail to incorporate the value of the embedded option, and it is therefore argued that pricing is inaccurate. Recognizing this concern, the authors set a default probability function that varied with time to maturity. Although this function was somewhat ad hoc, their method of addressing the criticism was none the less a valid attempt at improving the model.

In summary, the body of this literature makes progressive strides toward the understanding of the borrower’s default decision, and its resultant effect upon whole loan pricing. This thesis will attempt to directly relate these methodologies regarding mortgagor behavior to the pooled loan framework of commercial mortgage backed securities. In the same manner as Riddiough & Thompson (1993), the thesis model will also utilize a monte-carlo approach that incorporates option value. Changes in property characteristics and mortgage contract design will then be tested for their effect upon the distributed cash flows of various CMBS tranches.
2.3 Commercial Mortgage-Backed Securities Research

Unfortunately, there has been extremely little formal research conducted directly on CMBS pricing. The few studies in this area utilize the same general framework as previously outlined and do provide some insight into the understanding of these complex securities.

As learned from studies valuing whole loans, underlying real estate risk is a significant factor in assessing loan value, and it is therefore important to endogenize property price. Corcoran & Kao (1994) demonstrated that this risk extends directly to commercial mortgage backed securities. By linking delinquencies from American Council on Life Insurance data with returns outlined by the Russell NCREIF index, they developed a framework for assessing the credit risk of CMBS. Their results indicate that the effect of a modest rise in property prices has substantial impact on CMBS. In addition, as amortization has similar effects upon LTV, they determined that it too has subsequent consequences on the overall security.

By factoring in the uncertainty of property price, Childs, Ott, & Riddiough (1995) quantified required yield spreads for several typical CMBS tranche structures. Utilizing a combined approach of rational contingent claims decision making, and monte-carlo analysis, their model determined that tranche structure is an important pricing factor. In addition, their results outlined diversification effects unique to each tranche. Of particular interest was the indication that diversification may actually increase the required yield spreads for a first loss junior tranche.

Understanding the importance of endogenized property price upon CMBS valuation, the analysis undertaken by this thesis will attempt to confirm these structure and diversification effects. Furthermore, by building upon prior modeling of decision behavior, the thesis will also strive to assess the effects of loan extension. The following chapter details the approach utilized for this evaluation.
3. The Pricing Model

3.1 Introduction

Accurate assessment of CMBS investment requires a pricing model that properly incorporates the uncertainties inherent in the underlying mortgage collateral. In the context of unsure property values, mortgage value is affected by the irregular behavior surrounding the borrower’s default decision. Further uncertainty results from the servicer’s decision regarding borrower extension requests. Therefore, in order to obtain a meaningful value for any security comprised of mortgage assets it is imperative to properly address these pricing considerations.

As outlined by the previous chapter, the modeling techniques utilized by recent researchers have generally fallen into the categories of rational option pricing, and monte-carlo simulation. Although each technique is a valid attempt at explaining borrower behavior, monte-carlo simulation has several benefits for modeling complex borrower behavior, and is the approach used for this study. In an effort to outline these benefits, this section will further compare both techniques.

Rational option based models consider the implicit value of the borrower’s default put option. Due to the uncertainty in property prices, and the irreversibility of the default decision, there is value for the borrower in maintaining this right to default. Although exercising the option limits the downside exposure for the borrower, keeping the option alive retains possible upside benefits. In other words, market volatility may return the borrower to a position of positive equity, provided the option has not already been exercised. Rather than basing this decision on a simple loan to value comparison then, a borrower will instead compare property value against a mortgage value that also accounts for the value of the put option.
Unfortunately, the reverse time path approach of a rational model requires ex-ante determination of borrower default transaction costs, and it is difficult to accurately ascertain such costs at the time of loan origination. These costs may be significant, and they vary widely with diversity in contract arrangements, state and local foreclosure regulations, and individual borrower behavior. In addition, indirect costs, such as those associated with securing future credit, further exacerbate the problem by being essentially unobservable to the lender.

Another criticism of the rational option model is that it assumes the borrower has no short term cash flow problems. In other words, the approach postulates that every debtor has the ability to finance a cash deficiency in order to keep his default put option alive. Periodic payments must be made to continue the option rights, and not all borrower’s will necessarily hold this ability. Loss of a major tenant for instance may very well affect the timing of default.

In a rational model, all borrowers are considered to make the same judicious decision based upon encounter of set criteria. The monte-carlo method on the other hand, accounts for a greater level of uncertainty in borrower behavior by utilizing less distinct boundaries of decision criteria. Here, the effects of unobservable borrower transaction costs are endogenized by placing probabilities on the decisions of the debtor, and the result is a more realistic reflection of the range of complex behavior exhibited in the marketplace.

In addition, the monte-carlo method is more flexible. A probability based, forward path analysis makes it easier to consider variations in the borrower’s behavior, and multiple state variables are more easily incorporated into the analysis. Perhaps even more importantly however, the forward path approach is a much better fit for distributing cash flows that are dependent upon previous allocations, a factor critical to modeling CMBS.
As opposed to the rational option model, a major criticism of the monte-carlo approach is that it does not consider the value of the borrower’s default put option. The approach taken by this thesis addresses this problem by utilizing a default probability function developed by Riddiough & Thompson (1993), and this function is detailed in the following section.

3.2 Whole Loan Model

The CMBS monte-carlo pricing model developed for this research is rooted by a model that prices whole loans. As outlined in Chapter 1, the most important risk affecting commercial mortgages is that of default. The emphasis of the whole loan model is therefore centered on the examination the borrower’s default decision. Due to the fact that most commercial mortgages contain lockout or yield maintenance clauses, prepayment risk will not be considered.

The model prices a commercial mortgage with a fixed rate of interest for a fixed term, and requires fixed payments at discrete time intervals. A balloon payment will vary with the rate of amortization, and the loan is assumed to be non-recourse to the borrower. In the event of default, foreclosure is assumed to occur immediately with the lender having fixed transaction costs of foreclosure. The model allows for variation of all loan terms, and transaction costs, and although only immediate foreclosure is considered, any time lag may be easily incorporated into the model.

Figure 3-1 illustrates the flow of the model. Basically, the monte-carlo analysis simulates numerous possible cash flow outcomes for the life of the loan. Property price is updated at the start of each period, and default is then considered based upon equity level. If a default does not occur, the lender receives the scheduled mortgage payment. The default decision is then viewed for the following period based upon a new updated property price, and the process
continues until maturity or default. In the alternative, if a default does occur, the lender obtains the property value, less the lender’s foreclosure costs, and the loan is terminated.

Extension risk is also priced into the loan via decision simulation. If the loan has reached the end of its contract term, an extension is considered. If no extension occurs, the loan is immediately paid in full. On the other hand, if extension does occur, no payoff is made and the loan continues under the same terms and conditions as before for an additional two year term. Default continues to be considered at each period during the extended term, and if the loan survives the two year extension without a default, no further extension is considered, and the loan is assumed to pay off in full.

Finally, the loan value is calculated as the present value of these varying payment streams. Monte-Carlo analysis simulates numerous, discreet scenarios of such possible varying cash flow streams and averages the results to obtain a specific mortgage value. The greater the number of iterations, the more the mortgage value approaches its true limit.
This loop iterates many times and an average of loan values is obtained. The greater number of iterations, the closer the approximation to the "true" value.

Figure 3-1

Logic Flowchart- Whole Loan Model
3.2.1 The Decision Frequency Functions

3.2.1.1 Default Frequency

The default decisions are modeled by placing probabilities upon the likelihood of borrower default. A borrower’s default decision is based upon his economic interest in the property, and the probabilities therefore vary continuously according to the borrower’s equity level for the current period. The greater the equity, the lower the probability of default.

As previously outlined, part of the borrower’s economic interest is embedded in the default option. In order to account for the inherent value of the option, the default frequency function varies with time remaining in the contract term of the mortgage. This is the same approach utilized by Riddiough & Thompson (1993), and the probabilities used in this analysis are identical to theirs. Although more empirical work is needed in this area, this approach produced default frequency results consistent with those observed in the marketplace.

Riddiough & Thompson (1993) observed that default is less likely early in the loan term, as borrowers are less willing to lose the default option that may be of benefit latter in the loan term. The value of this option becomes less valuable as the term draws near, and as a result, defaults occur at less severe levels of negative equity as the end of the loan term approaches.

The point probabilities used for this analysis follow this logic and are listed in Table 3-A. The term $E_t$ is a normalized measure of net equity level. That is: Property Price at time $t$, divided by the Mortgage Value at time $t$, $(P_t/M_t)$. The default probability values at the commencement of the loan term are represented by $f(E_0,0)$, while $f(E_T,T)$ is the probability that is assumed at the end of the loan term. It is important to note the inverse relationship between the probability of default and net equity level, as well as the reduced default probabilities at loan commencement.
### Table 3-A

Default Probabilities

<table>
<thead>
<tr>
<th>$E_i$</th>
<th>1.2</th>
<th>1.1</th>
<th>1.0</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(E_{0},0)$</td>
<td>0.0001</td>
<td>0.001</td>
<td>0.01</td>
<td>0.03</td>
<td>0.07</td>
<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>$f(E_{T},T)$</td>
<td>0</td>
<td>0.05</td>
<td>0.1</td>
<td>0.25</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

To properly account for the borrower's option value, the following quadratic weighting system was used to interpolate probabilities at intermediate loan periods:

\[
f(E_t, t) = [f(E_0,0) * (1 - (t/T)^2)] + [f(E_T, T) * (t/T)^2]
\]

**Eq 3.1**

where: $t = \text{intermediate period}$

and, $T = \text{maturity period}$

This weighting system produces a probability function that initially moves away from the lower bound ($f(E_0,0)$) at a gradual rate and then rapidly approaches the upper bound ($f(E_T, T)$) as the end of the term draws near. Figure 3-2 graphically illustrates this relationship by comparing probabilities at: i) origination ($E_0$), ii) halfway through the loan term ($E_{0.5}$), iii) eighty percent through the loan timeframe ($E_{0.8}$), and iv) loan maturity ($E_T$).
To keep the inverse relationship smooth across the range of net equity, a continuous default function was implemented by using linear interpolation to determine the probabilities associated with intermediate levels of net equity. Note, that even under a negative equity situation, default is not certain, as is the case for ruthless default models (see Titman & Torous, 1989). In addition, there is a possibility of default even at positive levels of equity, a possibility consistent with the findings of Ciochetti & Riddiough (1994).
Finally, the renewed default option value associated with the extension term of an extended loan is accounted for by adjusting the default probability function (Equation 3.1). This adjustment resets the maturity \( T \) to: \( T + 24 \), while the current period \( t \) is not altered. In this manner, the probabilities for default decrease immediately following the grant of an extension. Note that this decrease would not be as severe as those that would occur under a new origination, and that the default probabilities will again increase as the new maturity draws nearer. Overall, these are likely to be reasonable assumptions regarding borrower behavior.

### 3.2.1.2 Extension Frequency

Due to the lack of any empirical evidence regarding extension frequencies, an ad hoc extension probability function was postulated based upon principles similar to those governing the default probability function. First of all, the possibility of extension is assumed to hold an inverse relationship with net equity level. In addition, extension may occur even at positive levels of net equity and, it is certain to occur at net equity levels low enough to warrant certain default at maturity. Table 3-B displays the increasing probabilities that accompany borrower net equity decreases.

As a result of the assumption that extension may only occur at the end of the loan term, this probability function was not assumed to vary with time remaining to maturity. This is consistent with the fact that this option does not belong to the borrower, but rather ultimately relies with the lender, and no option value should be implied.

Due to the ad hoc nature of the extension probabilities, two separate sets of point probability estimates were utilized to assess the effects of extension. The alternate scenarios outlined by the independent estimates are intended to provide a reasonable range for the likelihood of
extension. The extension probability is not a function of the combination of these two arrays, as is the case for the default frequency function. Instead, each set of estimates is independently incorporated and separately tested in the analysis. Both arrays are outlined in Table 3-B, where it can be seen that the second set of estimates provides a more conservative conjecture of extension probability.

Table 3-B

Extension Probabilities

<table>
<thead>
<tr>
<th>$E_q$</th>
<th>1.1</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(E_q)$ # 1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.75</td>
<td>1.0</td>
</tr>
<tr>
<td>$f(E_q)$ # 2</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In order to incorporate a continuous probability function, the probabilities associated with intermediate values of net equity were determined through simple linear interpolation.

3.2.2 The Property Price State Variable

The state variable used by the model to capture the effects of mortgage value is Property Price. The borrower uses property price, plus default transaction costs, as a reference for comparison against outstanding mortgage value. As the previous sections have illustrated, borrower decisions, and therefore pricing, are contingent upon this dynamic variable as a result of its effect upon current equity position.
Although commercial property prices also rely upon stochastic interest rates, term structure volatility is not endogenized in this model. Childs, Ott, & Riddiough (1994) demonstrated that a constant interest rate assumption produces relatively little effect when pricing call protected mortgages. Upon comparing the two approaches, they discovered small differences between required risk premiums over ranges of both property value volatility and term to maturity. Their conclusion emphasizes that accurate representation of the current term structure is more important for pricing non-callable mortgages.

In assessing the stochastic movements of property price, the model uses a log-normal diffusion process. Constant mean and variance parameters have been assumed, and the change in Property Price is given by the following equation:

$$dP = (\alpha - b)P dt + \sigma_p P dZ$$  \hspace{1cm} \text{Eq. 3.2}$$

where:
- $\alpha$ = instantaneous total expected return on property
- $b$ = continuous rate of property payout
- $\sigma_p$ = instantaneous standard deviation of property prices
- $Z$ = standardized Wiener process

Following standard contingent claim requirements, it can be assumed that investors price derivative securities independent of risk preferences. The following risk adjusted price process may then be used:

$$dP = (r_f - b)P dt + \sigma_p P dZ$$  \hspace{1cm} \text{Eq. 3.3}$$

where:
- $r_f$ = riskless spot rate of interest
This then allows for discounting at the risk free rate of interest. To implement the process, an annual payout rate of eight and one half percent (8.5%) was utilized, while time increments were always monthly. The volatility and risk free rate parameters were varied and these assumptions are presented in more detail by Chapter 4.

3.2.3 Input/Output Specifications

Any set of fixed loan contract terms may be analyzed. This includes any desired LTV, interest rate, amortization, loan term, and number of payments per year. The state variable parameters of risk free rate, payout rate, and property price volatility are also separately input and may take on the full range of typical values.

The monte-carlo technique of generating numerous pricing solution paths gives us approximations for exact solutions. The number of state variable path iterations is only limited by computer run-time, where run-time increases linearly with the length of the loan term. Five thousand (5000) iterations generally produces reasonable convergence to the “true” mortgage value while requiring only short run time.

3.3 CMBS Model

The CMBS model is an extension of the whole loan model. The model derives cash flows from an embedded whole loan model in the same manner that commercial mortgage backed securities derive their cash flows from their underlying mortgages. Basically, numerous loans are individually simulated every period. Rather than price each loan separately however, the
periodic cash flows from this pool of loans are instead agglomerated and then distributed on a priority basis to the various tranches that make up the security.

Just as in the whole loan model, property price for each loan included in the pool is updated at the beginning of each period. The default decision is then separately considered for each loan, and individual loan cash flow is dependent upon the outcome of that decision. This process continues for each loan, for each period until the end of the loan term. Once maturity is reached for a particular loan, that loan is then considered for extension, and the resultant individual loan cash flow is once again set by the outcome of that decision.

Each period, total interest and total principal received from all loans in the pool are distributed to the various tranches according to the particular priorities associated with each tranche. Each tranche’s resultant cash flow is appropriately discounted, and the process continues until all loans have reached their original maturity or, if applicable, the end of their extended term. Tranche value is simply calculated as the sum of the discounted cash flows. The entire process is performed over and over to obtain convergence to a true tranche value. Finally, this tranche value is set as the initial cash outlay for a stream of non-risky cash flows (scheduled, no-default payments) and the yield to maturity is calculated through an iterative process.

3.3.1 Security Structure

A tranche structure typical of many commercial mortgage backed securities is utilized. This structure prioritizes distribution in a top down manner. That is, the higher the tranche, the higher its priority for return of principal.

Each period every tranche receives periodic interest earned on its then outstanding tranche balance. In the case in which tranche contract rates of interest are set below the underlying
loan contract rates, excess interest available from the loans is distributed to an “Interest Only” tranche.

All principal from the entire pool is then distributed to the highest priority tranche. Once the uppermost tranche has received all of its outstanding balance, the periodic principal payments are granted to the next highest priority tranche until it too has been retired. Principal continues to be distributed by tranche priority.

A tranche may also be retired as a result of allocation of default losses. Any loss resulting from the shortfall between loss recovery (property value less lender foreclosure costs) and the outstanding loan balance, is allocated in a bottom up manner. That is, the lowest priority tranche still alive bears this loss. If a tranche has lost its entire balance due to default, the next higher priority tranche then bears these losses. Once again, the periodic distribution process continues based on the new balances of each tranche.

3.3.2 Input/Output Specifications

Each loan may have its own individual set of contract terms, and the number of loans allowed in the pool is limited only by computer memory. In addition, loans may have property prices that are either fully correlated, or completely non-correlated with other loans in the pool. Although this results in a very large number of possible designs, computer run time increases with the number of loans included in the pool.

The model also allows for any number of tranches, and any tranche contract rates or pool proportions (percentage of the entire pool balance allocated to a particular tranche) may be individually assigned. To keep consistent with the whole loan discounting structure, this study assumes all loans had contract rates equal to the risk free rate and that all tranches also shared
this rate. Under the same risk neutral assumption outlined in the state variable section of this chapter (see Eq. 3.3), discounting again occurred at the risk free rate of interest.

Monte-carlo runs of five thousand iterations were run and the following output was produced for all of the various scenarios:

1) **Spread**- The difference between the risk free rate and the risky yield to maturity

2) **% Default**- Average Default Frequency as a percentage of the total number of loans

3) **% Extension Frequency**- Average Extension Frequency as a percentage of the total number of loans

4) **% Average Loss Recovery**- Average Loss Recovery as a percentage of the weighted average loan amount

The next chapter outlines and analyzes the overall results.
4. Results

4.1 Test Parameter Specifications

Variation of the state variable parameters will alter the periodic property prices and ultimately affect pricing of the underlying mortgages. A change in property price alters the borrower’s net equity level and influences the default decision. This then has a direct impact upon pricing, as a change in losses affects cash flows. In addition to this impact, differing loan contract terms will also modify the equity position and directly affect pricing in the same manner.

To determine the relative effects of each of these variables, comparative static sensitivity analysis is performed through systematic variance of each parameter. The reference for the comparative analysis is a “base case” scenario from which parameters were varied one at a time. Table 4-A details these alternate values alongside the comparative base case scenario.

Table 4-A
Parameter Variation

<table>
<thead>
<tr>
<th>State Variables</th>
<th>BASE CASE</th>
<th>ALTERNATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Free Rate ($r_f$)</td>
<td>8.00% annual</td>
<td>10.00% annual</td>
</tr>
<tr>
<td>Property Value Volatility ($\sigma_p$)</td>
<td>17.50% annual</td>
<td>22.50% annual</td>
</tr>
<tr>
<td>Contract Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan to Value Ratio</td>
<td>70 %</td>
<td>80 %</td>
</tr>
<tr>
<td>Amortization</td>
<td>Interest Only</td>
<td>25 year</td>
</tr>
<tr>
<td>Term to Maturity</td>
<td>3 years</td>
<td>7 years</td>
</tr>
</tbody>
</table>
The base case Property Price parameters are taken from market observations, and those estimated in previous literature, while the variant Contract Terms attempt to replicate several originations typical in the market.

In order to determine the effects of pool size (measured by the number of loans comprising the pool), a minimum of four scenarios were typically run. The four sizes included securities with 1, 2, 5, and 10 loans. Utilization of this format provides insight into diversification effects, due to the fact that the model provides for individual property price paths for each loan ("Independent Draw"). That is, in some scenarios, each loan is considered to be totally uncorrelated with the other loans in the pool. In such a case, a greater number of loans in the pool produces greater diversification of the security.

In order to extract additional understanding of the effects from diversification, the model allows reversal of the zero correlation assumption, and instead can provide identical property price paths for each loan ("Dependent Draw"). For cases with identical state variable parameters, the loans are 100% correlated. Pricing differences here lie only in the variance of individual borrower decisions.

To distinguish the effects of tranche structure, results from several alternate distributions were viewed. The structures utilized are outlined by Table 4-B.

**Table 4-B**

**Tranche Structures**

<table>
<thead>
<tr>
<th>Tranche Priority</th>
<th>Structure 1</th>
<th>Structure 2</th>
<th>Structure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>70 %</td>
<td>70 %</td>
<td>70 %</td>
</tr>
<tr>
<td>Mezzanine</td>
<td>25 %</td>
<td>20 %</td>
<td>10 %</td>
</tr>
<tr>
<td>Junior</td>
<td>5 %</td>
<td>10 %</td>
<td>20 %</td>
</tr>
</tbody>
</table>
Lastly, in order to better understand the specific consequences of exposure to extension risk, the model incorporates a choice for inclusion of such risk. In other words, the model also prices the tranches under a scenario in which extension is not allowed. The dual ability to price securities either by endogenizing this risk, or by omitting it, grants the capacity to single out the significance of such exposure.

The number of monte-carlo iterations was held constant at five thousand (5000) for all cases. This number of simulations should produce results sufficient enough to perform meaningful analysis, and the level of variance surrounding the results appears to be reasonably small at this level of iteration.

### 4.2 Discussion of Numerical Results

#### 4.2.1 Base Case

Spreads for the base case scenario are presented in Table 4-C. All three tranche structures are presented in an order that is ascending by size of the junior tranche, and therefore descending by mezzanine tranche size. The number of loans in the pool increases moving left to right.

Due to the increased risks associated with lower priority tranches, one would expect spreads to increase as tranche priority decreases, and the model’s output supports this conclusion for all structures, in all pool sizes. The senior tranche holds steady across structure because it always contains the same level of subordinate credit enhancement. That is, the combined size of the junior and mezzanine tranches is always thirty percent (30%) of the security in this analysis.
Table 4-C
Base Case Spread Requirements

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>3</td>
<td>8.00%</td>
<td>I.O.</td>
<td>17.5%</td>
</tr>
</tbody>
</table>

On the other hand, the level of protection afforded to the mezzanine tranche by the junior tranche varies. As increased junior tranche size affords greater loss protection for the mezzanine piece, one would expect decreasing mezzanine spreads for increasing junior tranche size across all pool sizes. Again, the model supports this relationship.

There is also an inverse relationship between junior tranche size, and junior tranche spread requirements. This generally holds true for all pool sizes, and again this result is intuitive. The larger the tranche size of this first loss piece, the more likely a portion of its original balance will remain after absorption of default losses.

All of these structure relationships are evident in Figure 4-1, which compares individual tranche spreads within and across all tranche structures. It will become apparent that these relationships hold across all tested scenarios.
Base Case Spread Requirements Across Tranche Structure and Pool Size

Figure 4-1

Base Case Spread Requirements

It is also very apparent that pool size has a drastic effect upon all tranches no matter what the structure. You will note a general decrease in the required spreads for the two most senior tranches as you move from left to right across Table 4-C. This is an indication that significant benefits are available to these two tranches as a result of diversification, because diversification benefits result from reduced cash flow volatility, and a greater number of non-correlated loans will produce more stable cash flow.

Countering this intuition however, are the results from the junior tranche. Figure 4-1 clearly illustrates that all structures produce junior piece spreads that increase with pool size (front to
back along Figure 4-1). This indicates that diversification is detrimental to this tranche, and lends further support to results obtained by Childs, Ott & Riddiough (1995).

The reasoning behind this outcome has to do with the fact that this tranche is the first to receive losses. As such, it is consistently out of the money with respect to return of principal, and the cost of increased downside cash flow volatility is therefore small. Benefits from volatility however are reflected by the greater chance of low losses, a factor that reduces required yields.

Further examination of the results in Table 4-C lends supplementary credence to this explanation. Note that the effects of this diversification effect decrease as the tranche size increases. The increases in required spreads from a pool of one loan to a pool of ten loans is much greater for a five percent junior piece (Structure 1) than for a junior piece comprised of twenty percent (Structure 3). In addition, note that the smaller the size of the junior tranches, the less the accrual of diversification benefits to the mezzanine tranche.

The results from Tranche Structure 3 indicate a limit to the detrimental effects of diversification for the junior tranche. A larger pool run (50 loans) for each tranche shows that there does in fact appear to be a limit across all structure levels, and these results are outlined in Table 4-D.
Table 4-D

Base Case Spreads Up to 50 Loans

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>3</td>
<td>8.00%</td>
<td>I.O.</td>
<td>17.5%</td>
</tr>
</tbody>
</table>

**SPREAD (basis points)**

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>Number of Loans in Pool</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70%</td>
<td>19</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>50</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>1443</td>
<td>1871</td>
</tr>
<tr>
<td>2</td>
<td>70%</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>169</td>
<td>155</td>
<td>27</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>221</td>
<td>445</td>
<td>708</td>
<td>815</td>
<td>810</td>
</tr>
<tr>
<td>3</td>
<td>70%</td>
<td>18</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>143</td>
<td>51</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>216</td>
<td>314</td>
<td>379</td>
<td>377</td>
<td>376</td>
</tr>
</tbody>
</table>

Note that the larger the size of the first loss tranche, the fewer loans it takes to reach the diversification limit. This makes sense as larger size means that a greater proportion of the tranche would behave as a mezzanine piece. That is, proportionally less and less of the tranche would be in a first loss position, and it would trend away from the reverse behavior of a first loss piece.

In a similar manner, the size of the junior tranche also affects the diversification limit of the mezzanine tranche. As the level of credit protection provided to a mezzanine tranche is increased, less and less of the mezzanine piece would behave as a first loss tranche. As a result, the larger the junior tranche, the fewer number of loans required for the mezzanine piece to reach the maximum benefit from diversification.
4.2.2 Correlated Output

Further evidence in support of this interesting diversification behavior comes from a series of results collected under the assumption of fully correlated loans (Dependent Draw). Table 4-E outlines the base case scenario spreads that result under this framework.

Table 4-E

Base Case- Fully Correlated Loans

<table>
<thead>
<tr>
<th>LTV</th>
<th>Dependent</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>3</td>
<td>8.00%</td>
<td>1.0</td>
<td>17.5%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>Spread (basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
</tr>
</tbody>
</table>

A comparison of these results with the non-correlated base case generally indicates greater spreads for the higher priority tranches. The required spreads need to increase in order to compensate for the reduced diversification benefits. In addition, these increases are particularly more prevalent in the larger pool situations, where one would expect diversification differences to be more pronounced.

The decrease in spreads as pool size increases illustrates a diversification benefit due to the addition of variant borrower behavior. One loan imparts no diversification benefits as a result
of variant borrower behavior, for there is only one borrower for a single loan pool. The benefit is obtained in the cases with more than one borrower, but the overall benefit is reduced due to the fact that we are not gaining from diversified property prices.

Again, we encounter a detrimental effect from diversification for the junior tranche. As expected, the effects are not as severe due to the non-correlated aspect of the loans. The effect of junior tranche size on this behavior has the same trend in this scenario. The smaller the first loss piece, the greater the negative effects.

4.2.3 Alteration of State Variable Parameters

As previously discussed, one would expect altered property prices, and therefore altered tranche pricing, as a result of changes effected to state variable parameters. Increased property value volatility should increase the likelihood of default (Riddiough & Thompson, 1993) and therefore increase required tranche spreads. On the other hand, a term structure change of an increased risk free rate should lower spreads due to the fact that default risk is being discounted at a higher rate of interest.
4.2.3.1 Property Price Volatility

Table 4-F
Property Volatility Comparison- Spreads

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>With Extension Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>3</td>
<td>8.00%</td>
<td>I.O.</td>
<td>Volatility</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Base (17.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>vs. 22.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPREAD (basis points)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>22.5%</th>
<th>22.5%</th>
<th>22.5%</th>
<th>22.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
<td>BASE</td>
</tr>
<tr>
<td>70%</td>
<td>19</td>
<td>60</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25%</td>
<td>186</td>
<td>383</td>
<td>211</td>
<td>449</td>
<td>109</td>
<td>328</td>
<td>50</td>
<td>220</td>
</tr>
<tr>
<td>5%</td>
<td>235</td>
<td>441</td>
<td>473</td>
<td>637</td>
<td>1036</td>
<td>1961</td>
<td>1443</td>
<td>2949</td>
</tr>
<tr>
<td>20%</td>
<td>169</td>
<td>365</td>
<td>155</td>
<td>387</td>
<td>27</td>
<td>145</td>
<td>7</td>
<td>73</td>
</tr>
<tr>
<td>10%</td>
<td>221</td>
<td>438</td>
<td>445</td>
<td>630</td>
<td>708</td>
<td>1465</td>
<td>815</td>
<td>1857</td>
</tr>
<tr>
<td>70%</td>
<td>18</td>
<td>63</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10%</td>
<td>143</td>
<td>335</td>
<td>51</td>
<td>203</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>20%</td>
<td>216</td>
<td>418</td>
<td>314</td>
<td>701</td>
<td>379</td>
<td>819</td>
<td>377</td>
<td>872</td>
</tr>
</tbody>
</table>

Table 4-F outlines the results for a high property value volatility (22.5%), and the data match our expectations. A comparison to the base case indicates that the spreads are higher for all tranche structures at all pool levels. This result is graphically illustrated by Figure 4-1 where a side by side volatility comparison is made across all tranche structures and all pool sizes.
The reasoning behind this increase is indeed a result of increased default loss. Table 4-G compares default frequencies and default severity (loss recovery) of the base case against those obtained utilizing a higher rate of property value volatility. In all cases, the increase in rate of default and the decrease in loss recovery of the high volatility scenarios are significant. The greater movement in property prices that results from increased volatility produces a greater likelihood of the loan encountering a situation of negative equity. As default probability increases with reduced equity, there is an accompanying overall increase in default frequency. This greater movement in property price also increases the likelihood of lower property values at foreclosure and more severe levels of default tend to occur.
Table 4-G

Property Volatility Comparison - Default & Extension Incidence

<table>
<thead>
<tr>
<th>DRAW INDEPENDENT</th>
<th>With Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV 70% Term (yr)</td>
<td>Rate 8.00%</td>
</tr>
<tr>
<td>Loans Extended</td>
<td>Amortization 1Q</td>
</tr>
<tr>
<td>Loans Defaulted</td>
<td></td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>Number of Loans in Pool</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>70, 25, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>BASE 22.5%</td>
<td>10.68%</td>
<td>14.58%</td>
<td>10.82%</td>
<td>14.58%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td></td>
<td>8.28%</td>
<td>14.64%</td>
<td>8.65%</td>
<td>14.01%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td></td>
<td>69.97%</td>
<td>63.49%</td>
<td>69.64%</td>
<td>63.15%</td>
</tr>
<tr>
<td>70, 20, 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>BASE 22.5%</td>
<td>10.38%</td>
<td>13.66%</td>
<td>11.19%</td>
<td>15.00%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td></td>
<td>8.24%</td>
<td>14.52%</td>
<td>8.66%</td>
<td>14.66%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td></td>
<td>71.09%</td>
<td>63.94%</td>
<td>70.27%</td>
<td>63.34%</td>
</tr>
<tr>
<td>70, 10, 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>BASE 22.5%</td>
<td>10.74%</td>
<td>15.20%</td>
<td>10.76%</td>
<td>15.06%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td></td>
<td>8.46%</td>
<td>14.44%</td>
<td>7.87%</td>
<td>14.68%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td></td>
<td>71.25%</td>
<td>62.73%</td>
<td>70.51%</td>
<td>64.06%</td>
</tr>
</tbody>
</table>

In addition, it is important to note that the increase in property volatility also has a strong positive impact upon the rate of extension. This of course is to be expected, as extension probability is also a function of borrower equity, and the same reasoning as that underlying increased default then applies. The effects of extension will be more thoroughly discussed in a latter section.

As with the base case, spreads increase with decreasing tranche priority, and the senior piece remains steady across structure. In addition, we continue to find that mezzanine spreads decrease with increased junior tranche size, and the required spreads of the junior position are negatively affected by an increase in its own size.

Once again we encounter the same diversification benefits for the top priority tranches, as well as the same negative impact from diversification to the junior piece. The smaller this
piece the larger this detrimental impact. The same indications of overall limits for both diversification effects also apply. The larger the junior tranche, the fewer loans required to near this limit.

4.2.3.2 Term Structure

Table 4-H outlines a comparison of results from differing interest rates. As expected, spreads decrease with an increase in the riskfree rate. This is an important factor, for it indicates that the model is able to accurately factor changes in the current term structure. As previously mentioned, under certain assumptions, this consideration is more important than a stochastic rate assumption (Childs, Ott, & Riddiough, 1995), and therefore has strong ramifications for our pricing considerations.

Table 4-H

Interest Rate Comparison- Spreads

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate Base (8.00%) vs. 10.00%</th>
<th>Amortization I.O.</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>3</td>
<td></td>
<td></td>
<td>0.175</td>
</tr>
</tbody>
</table>

SPREAD (basis points)

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>Number of Loans in Pool</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BASE 10.0%</td>
<td>BASE 10.0%</td>
<td>BASE 10.0%</td>
<td>BASE 10.0%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>19</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>186</td>
<td>211</td>
<td>109</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>235</td>
<td>473</td>
<td>1036</td>
<td>676</td>
<td></td>
</tr>
</tbody>
</table>
Review of these results will uncover the identical overall trends and behavior as outlined under the base case. That is, spreads increase with priority, junior piece tranche size directly affects spreads of the mezzanine and junior tranches, and diversification is a benefit to the two top tranches but a detriment to the bottom tranche. Once again, the smaller the size of the junior piece, the greater the effects of this detriment.

4.2.4 Alteration of Loan Contract Terms

The loan to value ratio and rate of amortization both play a direct role in the calculation of net equity. As such, we would expect a negative effect on pricing from a larger loan to value ratio (as equity is decreased), and a positive pricing effect from loan amortization (as equity is increased).

Our intuition is supported by prior research. Riddiough & Thompson (1993) demonstrated these effects for whole loan pricing, and Childs, Ott, & Riddiough (1995) illustrated the benefits of partial amortization on CMBS tranche pricing. In addition, Riddiough & Thompson (1993) find a negative impact on whole loan pricing as a result of a longer term to maturity. One would therefore expect the same loan term impact on the required spreads of a commercial backed security.

4.2.4.1 Loan to Value Ratio

Table 4-I displays the pricing results obtained from a security comprised of underlying loans originated at an eighty percent (80%) loan to value ratio. The expected results are apparent in
the significantly higher spreads overall as compared to the base case. At smaller pool sizes, the negative effects even reach the senior tranche.

Only at the upper pool levels is the senior tranche unaffected. This is probably due to the dominance of the same diversification effects previously outlined. In fact, review of Table 4-I puts forth the identical effects and trends from diversification as those encountered under the base case scenario. Furthermore, it can be seen that the overall interactions between spreads and tranche priority, as well as those between spreads and tranche size are the same as those found at the base case level.

Table 4-I

Loan to Value Comparison- Spreads

<table>
<thead>
<tr>
<th>DRAW-</th>
<th>INDEPENDENT</th>
<th>With Extension Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV</td>
<td>Base (70%)</td>
<td>Rate 8.00%</td>
</tr>
<tr>
<td>Base (70%) vs. 80%</td>
<td>Term (yrs)</td>
<td>Rate</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.00%</td>
</tr>
<tr>
<td>SPREAD (basis points)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tranche Structure</td>
<td>Number of Loans in Pool</td>
<td>1</td>
</tr>
<tr>
<td>BASE</td>
<td>BASE</td>
<td>80.0%</td>
</tr>
<tr>
<td>1</td>
<td>70%</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>25%</td>
<td>166</td>
</tr>
<tr>
<td>3</td>
<td>5%</td>
<td>235</td>
</tr>
<tr>
<td>2</td>
<td>70%</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
<td>169</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>221</td>
</tr>
<tr>
<td>3</td>
<td>70%</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>143</td>
</tr>
<tr>
<td>5</td>
<td>20%</td>
<td>216</td>
</tr>
</tbody>
</table>

The much greater overall level of spreads is also explained through a significant increase in default. Table 4-J compares the severity and frequency of default for these two scenarios.

Although there appears to be only slight increases in default severity (reduced loss recovery), overall there are enormous differences in the number of defaults. These losses obviously translate directly to greater required spreads.
Table 4-J
Loan to Value Comparison- Default & Extension Incidence

<table>
<thead>
<tr>
<th>LTV Base (70%) vs. 80%</th>
<th>DRAW INDEPENDENT</th>
<th>With Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term (yr)</td>
<td>Rate 8.00%</td>
<td>Amortization I.O.</td>
</tr>
<tr>
<td>Base</td>
<td>3</td>
<td>0.175</td>
</tr>
<tr>
<td>70, 25, 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.68%</td>
<td>10.82%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.28%</td>
<td>8.65%</td>
</tr>
<tr>
<td>Recovery (% of Avg Loan)</td>
<td>69.97%</td>
<td>69.64%</td>
</tr>
<tr>
<td>70, 20, 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.38%</td>
<td>10.73%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.24%</td>
<td>8.33%</td>
</tr>
<tr>
<td>Recovery (% of Avg Loan)</td>
<td>71.09%</td>
<td>70.54%</td>
</tr>
<tr>
<td>70, 10, 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.74%</td>
<td>10.74%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.46%</td>
<td>8.50%</td>
</tr>
<tr>
<td>Recovery (% of Avg Loan)</td>
<td>71.25%</td>
<td>70.43%</td>
</tr>
</tbody>
</table>

A security comprised of 80% LTV loans also manifests an increased level of extension. The increased level of default results from a greater probability of reduced equity. This reduced equity similarly affects extension, as it too is a function of net equity.

4.2.4.2 Amortization

The model produced the expected results for a security comprised of partially amortizing loans. Base case comparison as outlined in Table 4-K indicates that even partial amortization of the underlying loans is beneficial for the security. As expected, the already well protected senior tranche receives less benefit from the periodic pay down than does the exposed first loss junior tranche.
Once again, the primary trends surrounding tranche size and priority are evident in this case. In addition, the effects from diversification of the security follow the same guidelines as those discovered for the base case.

As expected, the default rates for securities comprised of partially amortizing loans are lower than those corresponding to the non-amortizing base case. Apparently, even a small amount of periodic principle produces sufficient enough impact upon net equity levels to make a difference in borrower decisions. Further inspection of Table 4-L suggests that the defaults that do occur under this scenario are actually a little more severe than their base case counterparts. This is due to the fact that in this study, loss recovery is calculated as a percentage of original loan value rather than as a percentage of outstanding loan balance.
Table 4-L
Amortization Comparison - Default & Extension Incidence

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yr)</th>
<th>Rate</th>
<th>Amortization Base (I.O.) vs. 25 yr.</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70%</td>
<td></td>
<td></td>
<td>17.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DRAW-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>INDEPENDENT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Extension frequency is also reduced as a result of amortization. This result is intuitively consistent considering this function’s reliance upon net equity value.

4.2.4.3 Term to Maturity

Overall, significantly greater spreads are required for a security comprised of loans that have longer terms to maturity. Examination of Table 4-M compares the three year term loan backed securities of the base case against those backed by loans due in seven years. This result is simply explained by longer exposure to the possibilities of default loss. Property price is stochastic, and the greater the period of time this variable is under scrutiny, the greater the likelihood for development of a problem encounter with negative equity.
Table 4-M

Term to Maturity Comparison- Spreads

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>Base (3) vs. 7</td>
<td>8.00%</td>
<td>I.O.</td>
<td>17.5%</td>
</tr>
</tbody>
</table>

SPREAD (basis points)

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BASE</td>
<td>7 yrs</td>
<td>BASE</td>
<td>7 yrs</td>
</tr>
<tr>
<td>1</td>
<td>70%</td>
<td>19</td>
<td>41</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td>186</td>
<td>269</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>235</td>
<td>314</td>
<td>473</td>
</tr>
<tr>
<td>2</td>
<td>70%</td>
<td>18</td>
<td>43</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>169</td>
<td>279</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>221</td>
<td>338</td>
<td>445</td>
</tr>
<tr>
<td>3</td>
<td>70%</td>
<td>18</td>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>143</td>
<td>241</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>216</td>
<td>310</td>
<td>314</td>
</tr>
</tbody>
</table>

The next table (4-N) clearly shows the drastic increase in default frequency and severity that explains the greater spread requirements for this type of security. Basically, problem loans have more opportunity to become even worse, and the result is more losses producing greater spread requirements.
Table 4-N

Term to Maturity Comparison- Default & Extension Incidence

<table>
<thead>
<tr>
<th>LTV</th>
<th>Term (yrs)</th>
<th>Rate</th>
<th>Amortization</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>Base (3) vs. 7</td>
<td>8.00%</td>
<td>I.O.</td>
<td>17.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Number of Loans in Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>70, 25, 5</td>
<td>BASE 1</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.68%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.28%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>69.97%</td>
</tr>
<tr>
<td>70, 20, 10</td>
<td>BASE 1</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.38%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.24%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>71.09%</td>
</tr>
<tr>
<td>70, 10, 20</td>
<td>BASE 1</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.74%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.46%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>71.25%</td>
</tr>
</tbody>
</table>

The pervasive trends involving the relationships between priority, tranche size and required spreads generally hold true for this overall series of scenarios, although there is slightly more variation in the lower sized pools than previously encountered. In addition, the unique diversification effects affecting the various tranches are also encountered for securities of this maturity class.

Finally, extension frequency again increases for this contract term change. The results have illustrated that the greater length of time for property drift to occur, the greater the occurrence of negative equity. This, once again, not only has an effect upon default, but also upon the incidence of extension.
4.2.5 Extension Risk

Overall, the various sets of results produced from the diverse parameter shocks have indicated an increased incidence of extension. It is important to understand the ramifications of this in terms of how it affects risk. This section will illustrate the significance of this exposure and aid our understanding of this un-tested area.

Our first intuition regarding extension revolves around the timing of cash flows. First, recognize that this model does not endogenize alternate term structure shapes, therefore issues surrounding duration are not considered. However, the model will address the issue of how extension risk affects default losses. The opportunity for far greater yield impact is embedded in this effect, and it should therefore become a concern of higher priority.

No intuition may be gained from previous literature directly focused in this area. We are able however to incorporate our understanding of other factors affecting mortgage and tranche value and apply them to extension risk. The most direct application is fostered in the preceding section.

Longer times to maturity have produced significantly larger spreads due to their consequent effect upon default loss. As discussed, by allowing for longer drift time, property prices have more opportunity to encounter situations of negative equity. One would then similarly expect to encounter some upward effect upon required spreads as a result of an extension decision. After all, an extended loan is simply a longer term to maturity loan. This upward trend is not predicted to be large however, for only some proportion of those loans should extend.

A factor of greater concern is that extended loans are typically in the midst of financial hardship. The increased spread requirements of increased LTV loans has already been demonstrated, and extended loans tend to be at extremely high levels of LTV. In most cases,
the collateral is not sufficient enough to cover the outstanding balance. Basically then, the situation is one of financing very troubled loans, and the result is a pool left with problem mortgages.

4.2.5.1 Extension Array #1

The differences between base case results with and without the factor of extension risk are put forth in Table 4-O. Although the direction of these differences are consistent with our expectations, the magnitude of the spread differences was surprising. These results indicate a very significant effect from extension exposure. Except for the zero spreads of the senior tranche at high pool levels, this exposure is reflected by all tranches, across all structures and pool sizes.

Table 4-O
Extension Comparison - Spreads

<table>
<thead>
<tr>
<th>DRAW-INDEPENDENT</th>
<th>WITH &amp; WITHOUT Extension Risk #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV 70% Term (yrs) 3 Rate 8.00%</td>
<td>Amortization I.O. Volatility 17.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPREAD (basis points)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 70% 5 with ext/out ex</td>
<td>19</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25% 94</td>
<td>211</td>
<td>92</td>
<td>109</td>
<td>33</td>
</tr>
<tr>
<td>5% 124</td>
<td>473</td>
<td>273</td>
<td>1036</td>
<td>523</td>
</tr>
<tr>
<td>2 70% 6 with ext/out ex</td>
<td>18</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20% 78</td>
<td>155</td>
<td>56</td>
<td>27</td>
<td>4</td>
</tr>
<tr>
<td>10% 117</td>
<td>445</td>
<td>225</td>
<td>708</td>
<td>327</td>
</tr>
<tr>
<td>3 70% 6 with ext/out ex</td>
<td>18</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10% 63</td>
<td>51</td>
<td>10</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>20% 112</td>
<td>314</td>
<td>151</td>
<td>379</td>
<td>170</td>
</tr>
</tbody>
</table>
The senior tranche is affected the most on a percentage wise basis, but this is due to the fact that this well insulated tranche has very small spreads to begin with. The junior tranche is affected slightly more as pool size increases, although this may simply be the result of less noise with increasing pool size.

An explanation of the sizable overall differences is again well explained by default analysis. Table 4-P depicts the frequency and severity of default under both scenarios, and the results clarify the underlying issues. Across the board, drastic increases in default frequency are compounded by significant decreases in loss recovery.

Table 4-P

<table>
<thead>
<tr>
<th>Extension Comparison- Default Incidence</th>
<th>DRAW- INDEPENDENT</th>
<th>WITH &amp; WITHOUT Extension Risk #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV 70%</td>
<td>Term (yrs) 3</td>
<td>Rate 8.00% Amortization I.O.</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>10.68% 0.00%</td>
<td>10.82% 0.00%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>69.97% 75.97%</td>
<td>69.84% 76.20%</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.38% 0.00%</td>
<td>11.19% 0.00%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.24% 4.36%</td>
<td>8.66% 4.58%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>71.09% 76.27%</td>
<td>70.27% 76.42%</td>
</tr>
<tr>
<td>% Loans Extended</td>
<td>10.74% 0.00%</td>
<td>10.76% 0.00%</td>
</tr>
<tr>
<td>% Loans Defaulted</td>
<td>8.46% 4.50%</td>
<td>7.87% 4.31%</td>
</tr>
<tr>
<td>Recovery(% of Avg Loan)</td>
<td>71.25% 76.07%</td>
<td>70.51% 76.58%</td>
</tr>
</tbody>
</table>
This table illustrates that for all cases, this rate of increased extension nearly doubles the rate of default. Furthermore, loss recovery is decreased by approximately eight percent. In other words, the overall outlook for a troubled loan is not good, and once such a loan is extended, the situation tends to worsen. A very large proportion of these troubled loans consequently default, and the effect is compounded by the fact that loss recovery decreases for the loans hence foreclosed. This effect is strong enough to create a significant drop in the overall average of pool default frequency and default severity. In short, these loans reduce the financial strength of the overall security and a large increase in spreads is required.

4.2.5.2 Extension Array #2

Due to the ad hoc nature of the extension probability assumptions, an alternate set of assumptions was also analyzed. This second set of extension probability estimates are more conservative than those of the initial array, and our expectations are therefore for reduced extension effects under this second set of assumptions.

The magnitude of pricing differences encountered under the first extension set highlighted the concern that the initial extension assumptions may have been unrealistic. A strong sensitivity to these assumptions might be producing misleading pricing results, and it is possible that a conservative set of assumptions might effect only minor pricing increases.

This is not the case. The pricing results obtained under the second set of extension probability assumptions are also significantly different than those encountered under the case assuming no extension risk. Table 4-Q compares the results from all three cases: i) No extension risk, ii) Extension risk set #1, and iii) Extension risk set #2.
Table 4-Q

Extension Risk # 1 & # 2 Comparison- Spreads

<table>
<thead>
<tr>
<th>DRAW- INDEPENDENT</th>
<th>WITH &amp; WITHOUT Extension Risk #1 &amp; Risk #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV 70%</td>
<td>Term (yrs) 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPREAD (basis points)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Tranche Structure</th>
<th>Number of Loans in Pool</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ext</td>
<td>ext</td>
<td>no ext</td>
<td>ext</td>
<td>ext</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>70%</td>
<td>19</td>
<td>16</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>25%</td>
<td>186</td>
<td>158</td>
<td>94</td>
<td>211</td>
</tr>
<tr>
<td>5%</td>
<td>235</td>
<td>196</td>
<td>124</td>
<td>473</td>
</tr>
<tr>
<td>70%</td>
<td>18</td>
<td>15</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>10%</td>
<td>169</td>
<td>145</td>
<td>78</td>
<td>155</td>
</tr>
<tr>
<td>10%</td>
<td>221</td>
<td>193</td>
<td>117</td>
<td>445</td>
</tr>
<tr>
<td>70%</td>
<td>18</td>
<td>15</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>10%</td>
<td>143</td>
<td>119</td>
<td>63</td>
<td>51</td>
</tr>
<tr>
<td>20%</td>
<td>216</td>
<td>174</td>
<td>112</td>
<td>314</td>
</tr>
</tbody>
</table>

Both sets of assumptions affect the required spreads. This generally holds true for all tranches, across all structures and across all pool sizes, with the only exception being the already low spreads (zero) of senior tranches for large sized pools. The second set of extension assumptions do indeed produce lower results than those encountered under the first, but this conservative second set of assumptions still indicates strong effects from exposure to extension risk. Although the probability estimates do certainly play a factor in pricing, the overall results would indicate that the pricing effects are not overly sensitive to the probability estimate assumptions.

Finally, Table 4-R explains the spread differences in a manner similar to Table 4-P. Even though this set of assumptions produces smaller levels of extension, we still encounter increases in default frequency on the order of fifty percent. The loss recoveries are also
reduced in magnitude, and are only slightly smaller than those encountered under the initial set of extension assumptions.

The effect of extension risk on default frequency is graphically illustrated by Figure 4-3. The distinct increase in the percentage of loans that default as a result of either set of extension assumptions may be clearly seen across all structures and pool sizes.
The end result is once again a continually deteriorating set of loans that effects the value of the overall security. It is important to note that these results can be directly related to the aforementioned market pricing discrepancies. The spread requirements for extension risk may account for a sizable portion of the excess spreads witnessed for rated CMBS, and the exhibited spread requirements will assist further quantification of this issue.
5. Summary & Conclusion

The monte-carlo CMBS valuation model developed for this research appears to properly account for numerous intricacies involved in pricing these securities. In addition, the analysis has provided both supportive and original insight into the pricing uncertainties of CMBS by appropriately factoring alternative marketplace parameters.

The pricing consequences from security design were examined under numerous scenarios, and overall, predicted results fit characteristic behavior observed in the marketplace. The most straightforward of these predictions calls for an increase in required spreads as tranche priority decreases. This result is consistent with the expected effects from the reduction in credit enhancement as one moves down the tranche priority ladder. Protection from default losses is reduced with priority, and the model quantifies the pricing effects of such losses.

An extension of this same rationale dictates that spreads will decrease along with increased size of subordinate tranches. The output does indeed reflect this increased level of protection at all levels of the analysis. Furthermore, the spreads for the junior tranche also decrease as the size of the junior tranche increases.

Further comparative static pricing effects were considered for varying security pool sizes (as measured by the number of loans in the pool). These results consistently outlined inverse relationships between spreads and pool size for both the senior and mezzanine tranches. This result is consistent with the expected benefits from property diversification.

On the other hand, diversification was consistently shown to be detrimental to first loss tranches. Increased cash flow volatility due to lack of diversification is beneficial to the junior tranche since investors generally expect to be out of the money with respect to full return of
principal. Increased cash flow volatility increases the probability of no losses which lowers the required yield spreads. Accordingly, the smaller the junior tranche, the greater the detriment as a result of diversification.

Additionally, the model has indicated the possibility of an asymptotic limit for each of these differing effects. That is, the inverse relationship between pool size and diversification benefit, (or detriment in the case of the junior piece), holds only up to a certain pool size. These limits are individually dependent upon tranche structure. For example, the smaller the junior tranche size, the greater number of underlying loans required to reach its limit of detrimental diversification, while the greater the credit enhancement for a superior tranche, the fewer number of loans required to reach its limits of diversification benefit.

These divergent diversification effects were both confirmed by results obtained from pools of non-correlated assets. A positive next step in the development of this model would provide for the incorporation of a loan correlation matrix, and thereby provide the ability to quantify any series of securitized loans, at any level of property price correlation.

Further testing allowed by the model included singling out the effects from changes in state variable parameters and loan contract origination terms. Results here were entirely consistent with prior whole loan research. In other words, for all remaining variable alterations the results illustrated positive effects resulting from: i) decreased property price volatility, ii) reduced initial loan to value ratios, iii) shorter terms to maturity, and iv) positive loan amortization.

Finally, perhaps the most important results follow from the model’s ability to isolate extension risk. Under two distinct sets of assumptions regarding the likelihood of extension, the model illustrated strong effects as a result of exposure to this risk. Furthermore, these results occurred under an assumption of a flat term structure. In a scenario of an increasing
yield curve, the effects would have been even more dramatic. Results such as these might indeed explain a portion of the excess spreads appearing in the market, and are important in assessing the true risks of commercial mortgage backed securities.

Although the two sets of extension risk pricing results do not indicate extreme sensitivity to the differing sets of assumptions, both sets are ad hoc in nature and further study is therefore required in this area. Empirical research on extension frequencies would provide more reliable parameter value estimates and therefore contribute valuable pricing insight.

Furthermore, although the model is able to effectively adjust for changes in the current term structure of interest rates, it does not endogenize volatility in the term structure. Such an improvement would further the understanding of extension and its effects, as timing issues under changing spot rates would be incorporated.

An added benefit is that this approach would also allow the review of callable loans. There are numerous contracts in which pre-payment of commercial loans is allowed, and the pricing of securities comprised of such loans are likely to be effected.
Bibliography


APPENDIX

"C" Source Code- CMBS Pricing Model

// PRICES ****************** CMBS ******************

#include <conio.h>
#include <iostream.h>
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include <time.h>

int main()
{

    clrscr();

    // ***** VARIABLES *****

    char Extension;

    int count, period, zone, n, loan, tranche, TotTranche;
    int iteration, TotLoans;
float riskfreerate, DefFreq=0, Principal, TotPrincipal = 0, Extend = 0;
float NetEquity, DefInterpolation, RandomDef, Interest, TotInterest = 0;
float SumLossRec=0, AvgLossRec, time, upperprob, lowerprob, ExtInterpolation;
float DefPerc, deltat, Normal, norm, sumnorm=0, LoanPercAvgRec, RandomExt;
float discrate=0, TotOLB = 0, InMaxMaturity =0, DefaultLoss = 0;
float Diff, EquilibRate, TempVal = 0, MaxMaturity;

double discfact;

//+++++Default Arrays+++++
float Boundry[] = {1.2, 1.1, 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4};
float E0[] = {0.0001, 0.001, 0.01, 0.03, 0.07, 0.2, 0.4, 0.7, 1.0};
float ET[] = {0, 0.05, 0.1, 0.25, 0.5, 0.7, 1.0, 1.0, 1.0};

//+++++Extension Array+++++
float ExtBoundry[] = {1.1, 1.0, 0.8, 0.6};
float ExtProb[] = {0.1, 0.5, 0.75, 1.0};

//+++++Loan Arrays+++++
int PMTperPer[100], Ext[100] = {0};
float OLB[100], LoanAmt[100], PMT[100], inmaturity[100], maturity[100], loanrate[100];
float payoutrate[100], propvol[100], LenderCost[100], InBldgVal[100];

double BldgVal[100];
//+++++Tranche Arrays+++++

float TranchePerc[10], IntDist[10], PrincDist[10], TrancheOLB[10];
float TrancheRate[10], TrancheVal[10] = {0}, SumTrancheVal[10] = {0};
float Spread[10];

float NonDefCashFlow[144][10] = {{0},{0}};

randomize();

// ***** INPUTS *****

cout << "\nHow many loans are there ? ";
cin >> TotLoans; //Loan parameter input
for (loan = 0; loan < TotLoans; loan++)
{
cout << "\nWhat is the Loan Amount for Loan # " << (loan+1) << " ? ";
cin >> LoanAmt[loan];
cout << "\nWhat is the periodic payment for Loan # " << (loan+1) << " ? ";
cin >> PMT[loan];
cout << "\nWhat is the term to maturity (years) for Loan # " << (loan+1) << " ? ";
cin >> inmaturity[loan];
cout << "\nHow many payments per year are there for Loan # " << (loan+1) << " ? ";
cin >> PMTperPer[loan];
cout << "\nWhat is the initial Building Value for Loan # " << (loan+1) << " ? ";
cin >> InBldgVal[loan];
cout << "\nWhat is the annual loan rate (input as decimal i.e. 0.06) for Loan# " << (loan+1) << " ?";
cin << loanrate[loan];
cout << \\nWhat is the building payout rate (decimal) for Loan # " << (loan+1) << " ?"; 
cin >> payoutrate[loan];
cout << \\nWhat is the annual property volatility ? (decimal) for Loan # " << (loan+1) << " ?"; 
cin >> propvol[loan];
cout << \\nWhat are the Lender's costs in event of foreclosure for Loan # " << (loan+1) << " ?"; 
cin >> LenderCost[loan];

inmaturity[loan] *= PMTperPer[loan];
if (inmaturity[loan] > InMaxMaturity)
    { InMaxMaturity = inmaturity[loan]; }
TotOLB += LoanAmt[loan];

} //end of loan parameter input

cout << \\nInclude extension risk ? (CAPITAL Y or N)"; 
cin >> Extension;
if (Extension == 'Y')
    { cout << \\nExtension Risk is included";
    }
else {cout << \\nExtension Risk is NOT included";
    }

cout << \\nHow many tranches are there excluding I/O tranche ?"; 
cin >> TotTranche;

cout << \\n****** NOTE: Tranches are numbered highest to lowest. i.e. Tranche 1 \n"; 
cout << " is the highest priority tranche and IO is the last tranche\n";
// Tranche parameter input
for (tranche = 0; tranche < TotTranche; tranche++)
{  cout << "What percentage of the Total security consists of Tranche # " << (tranche+1) << " ? ";
   cin >> TranchePerc[tranche];
   cout << "What is the contract rate of interest for Tranche # " << (tranche+1) << " ? ";
   cin >> TrancheRate[tranche];
}

cout << "The IO tranche will have no contract rate of interest"

cout << " and will be 0% of the total security";

// end of Tranche parameter input

cout << "What is the risk free rate? ";
cin >> riskfreerate;
cout << " How many monte carlo simulations do you require? ";
cin >> iteration;

// ******* ITERATION LOOP *******

for (count = 0; count <= iteration; count++)
{  for (tranche = 0; tranche < TotTranche; tranche++)
   {  TrancheVal[tranche] = 0;
      TrancheOLB[tranche] = TranchePerc[tranche] * TotOLB;
   }

   TrancheVal[tranche] = 0;  //to handle IO tranche
   TrancheOLB[tranche] = 0;
MaxMaturity = InMaxMaturity;

for (loan = 0; loan < TotLoans; loan++)
{ OLB[loan] = LoanAmt[loan];
  BldgVal[loan] = InBldgVal[loan];
  maturity[loan] = inmaturity[loan];
  Ext[loan] = 0;
}

// ********** PERIOD LOOP **********

for (period = 1; period <= MaxMaturity; period ++)
{ for (loan = 0; loan < TotLoans; loan++)
  { if (OLB[loan] <= 0)
    {continue;
     }
  }
  if (count > 0)
  { sumnorm =0;
    deltat = 1.00000/12.00000;
    for (n = 0; n < 12; n ++)
    { norm = random(10001);
      norm /= 10000;
      sumnorm += norm;
    }
    Normal = sumnorm-6;

    BldgVal[loan] = BldgVal[loan] + BldgVal[loan] * (riskfreerate - payoutrate[loan])
    * deltat + BldgVal[loan] * propvol[loan] * Normal * sqrt(deltat);
    NetEquity = BldgVal[loan]/OLB[loan];
}
// ********** DEFAULT TEST **********

if (NetEquity <= Boundry[0])
{ for (zone = 1; zone < 9; zone++)
  { if (NetEquity < Boundry[zone])
    { if (zone < 8)
      { continue;
      }
    }
  }
  time = period/maturity[loan];
  upperprob = E0[zone-1]/PMTperPer[loan] * (1-(time*time))
  + ET[zone-1]/PMTperPer[loan] * time * time;
  lowerprob = E0[zone]/PMTperPer[loan] * (1-(time* time))
  + ET[zone]/PMTperPer[loan] * time * time;
  DefInterpolation = (NetEquity - Boundry[zone])
  * (upperprob - lowerprob)
  / (Boundry[zone-1] - Boundry[zone]) + lowerprob;
  break; //break zone loop once have boundry
}

RandomDef = random(10001);
RandomDef /=10000;
if (RandomDef < DefInterpolation) //DEFAULT
{ DefFreq += 1;
  Interest = OLB[loan] * loanrate[loan]/PMTperPer[loan];
  Principal = BldgVal[loan] - LenderCost[loan] - Interest;
  DefaultLoss += (OLB[loan] - Principal); // To distribute loss after making tranche interest payment
  OLB[loan] = 0; //Non-Recourse
TotInterest += Interest;
TotPrincipal += Principal;
SumLossRec += Principal;
continue;
}
} //end of Default Test: if (N.E. < B[0])
} //end of iteration 0 if

Interest = OLB[loan] * loanrate[loan]/PMTperPer[loan];
Principal = PMT[loan] - Interest;
OLB[loan] -= Principal;
TotInterest += Interest;
TotPrincipal += Principal;

//***** MATURITY & EXTENSION TEST *****

if (period == maturity[loan])
{ if (count > 0)
  { if (Extension == 'Y')
    { if (Ext[loan] == 0)
      { if (NetEquity <= ExtBoundry[0]) //Extension Test
        { if (NetEquity < ExtBoundry[3])
          { ExtInterpolation = 1.0;
          }
        }
      }
    }
  }
}
upperprob = ExtProb[zone-1];
lowerprob = ExtProb[zone];
ExtInterpolation = (NetEquity - ExtBoundry[zone])
    * (upperprob - lowerprob)
    / (ExtBoundry[zone-1] - ExtBoundry[zone]) + lowerprob;
break;  //break zone loop once have boundry

RandomExt = random(10001);
RandomExt /=10000;
if (RandomExt < ExtInterpolation)  //EXTEND
    { Extend += 1;
        Ext[loan] = 1;
        maturity[loan] += 24;
        if (maturity[loan] > MaxMaturity)
            { MaxMaturity = maturity[loan];
            }
        continue;
    }

// End of Extend Test

TotPrincipal += OLB[loan];
OLB[loan] = 0;

}  // end of maturity check

}  //end of Loan Loop
// ********** TRANCHE LOOP **********

discrate = 1 + riskfreerate/12.0000;
discfact = pow(discrate, period);

for (tranche = 0; tranche < TotTranche; tranche++) {
    IntDist[tranche] = TrancheOLB[tranche] * TrancheRate[tranche]/12.0000;
    TotInterest -= IntDist[tranche];
    if (TrancheOLB[tranche] > TotPrincipal)
        PrincDist[tranche] = TotPrincipal;
    else
        PrincDist[tranche] = TrancheOLB[tranche];
    TotPrincipal -= PrincDist[tranche];
    TrancheOLB[tranche] -= PrincDist[tranche];
    TrancheVal[tranche] += (PrincDist[tranche] + IntDist[tranche])/discfact);
    if (count == 0)
        NonDefCashFlow[period][tranche] = PrincDist[tranche] + IntDist[tranche];
}
//end of tranche distribution

if (count == 0)
    {TotInterest = 0;
     TotPrincipal = 0;
    }

// ++++ IO Tranche ++++
IntDist[tranche] = TotInterest;
TotInterest -= IntDist[tranche];
TrancheVal[tranche] += IntDist[tranche]/discfact;

// +++++ Distribute Default Losses +++++

for (tranche = TotTranche-1; tranche >=0; tranche--)
    { if (TrancheOLB[tranche] > DefaultLoss)
        { TrancheOLB[tranche] -= DefaultLoss;
            break;
        }
        DefaultLoss -= TrancheOLB[tranche];
        TrancheOLB[tranche] = 0;
    }
    DefaultLoss = 0;

} //end of period loop

if (count > 0)
    { for (tranche = 0; tranche <= TotTranche; tranche++)
        { SumTrancheVal[tranche] += TrancheVal[tranche];
        }
    }
} //end of iteration loop
for (tranche = 0; tranche < TotTranche; tranche++)
{
    TrancheVal[tranche] = SumTrancheVal[tranche]/iteration;
}

fprintf(stdin, "LTV= %f, LoanAmt= %f, Rate= %f, periods= %f", 100.00 * LoanAmt[0]/InBldgVal[0], LoanAmt[0], loanrate[0], maturity[0]);

for (tranche = 0; tranche < TotTranche; tranche++)
{
    EquilibRate = TrancheRate[tranche]/12.0000;
    for (period = 1; period <= InMaxMaturity; period++)
    {
        disefact = pow((1 + EquilibRate), period);
        TempVal += NonDefCashFlow[period][tranche]/disefact;
    }
    Diff = TempVal - TrancheVal[tranche];
    if (fabs(Diff) < .005)
    {
        Spread[tranche] = 100.00 * (12.0000 * 100.00 * EquilibRate - 100.00 * TrancheRate[tranche]);
        printf(stdin, "TRANCHE # %d (%f) ", tranche+1, TranchePerc[tranche] * 100.00);
        printf(stdin, "Value : $ %f", TrancheVal[tranche]);
        printf(stdin, "Perc of orig Val: %f percent", TrancheVal[tranche] * 100.00 / (TranchePerc[tranche] * 100.00));
        printf(stdin, "Equilibrium Rate: %f percent", EquilibRate * 12.0000 * 100.00);
        printf(stdin, "Spread : %f basis points", Spread[tranche]);
    }
}
cout << "\nTRANCHE # " << (tranche+1) << ": (" << TranchePerc[tranche] * 100.00 << " % of Pool)";
cout << "\n Value : "$ << TrancheVal[tranche];
cout << "\n % of Orig. Value: " << TrancheVal[tranche] * 100.00 / (TranchePerc[tranche] * TotOLB) << " %";
cout << "\n Equilibrium Rate: " << EquilibRate * 12.00 * 100.00 << " %";
cout << "\n Spread : " << Spread[tranche] << " basis points";

TempVal = 0;
continue;
}
if (Diff < 0)
{
    EquilibRate -= .000001;
}
else { EquilibRate += .000001;
}
TempVal = 0;
goto PeriodLoop;
} // End of Spread Calculation

TrancheVal[tranche] = SumTrancheVal[tranche]/iteration;
cout << "\nIO TRANCHE value is: "$ << TrancheVal[tranche];
fprintf(stdprn, "\n IO Tranche Value is $ %f", TrancheVal[tranche]);

cout << "\nThere are " << TotLoans << " loans in this pool, and " << iteration << " simulations were run.\n";
fprintf(stdprn, "\n There are %d loans in this pool, and %d simulations were run.\n", TotLoans, iteration);

cout << Extend << " loans extended, representing " << 100.00 * (Extend/(TotLoans * iteration)) << " % of the simulations.";
fprintf(stdprn, " \n %f loans extended, representing %f percent of the simulations.", Extend, 100.00 * (Extend/(TotLoans * iteration)));
if (Extension != 'Y')
{
    cout << "\n(extension risk was not included)";
    fprintf(stdprn, "\n    (extension risk was not included)" );
}

if (DefFreq > 0)
{
    AvgLossRec = SumLossRec/DefFreq;
    DefPerc = 100.00 * (DefFreq/(iteration * TotLoans));
    LoanPercAvgRec = 100.00 * (AvgLossRec/(TotOLB/TotLoans));

    cout << "\nThe number of loans that defaulted was: " << DefFreq;
    fprintf(stdprn, "\n    The number of loans that defaulted was: %f", DefFreq);

    cout << "\nThe percentage of loans that defaulted was: " << DefPerc << "%;"
    fprintf(stdprn, "\n    The percentage of loans that defaulted was: %f", DefPerc);

    cout << "\nThe Average loss recovery as a percentage of the average loan is: " << LoanPercAvgRec << " %;"
    fprintf(stdprn, "\n    The Average Loss Recovery as a percentage of the Average Loan was: %f", LoanPercAvgRec);
}

else
{
    cout << "\nNO Defaults Occurred";
    fprintf(stdprn, "\n    NO Defaults Occurred" );
}

return 0;