Essays in Organizational Economics

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Abstract
The first chapter examines the interaction of heterogeneous firms in a competitive market in which firms motivate their workers using relational incentive contracts. In the steady-state rational-expectations equilibrium, aggregate TFP is fully characterized by a weighted average of firm-specific sustainable effort levels. Relational contracts amplify exogenous productivity heterogeneity and lead to dispersion in the net marginal revenue product of labor. Improvements in formal contracting disproportionately benefits low-productivity firms, leading to a greater dispersion of the net marginal revenue product of labor in weaker contracting environments. Thus, cross-country differences in contracting institutions can partially explain the observed pattern that misallocation is more pronounced in developing countries.

The second chapter explores organizational responses to influence activities—costly activities aimed at persuading a decision maker. Rigid organizational practices that might otherwise seem inefficient can optimally arise. If more complex decisions are more susceptible to influence activities, optimal selection may partially account for the observed correlation between the quality of management practices and firm performance reported in Bloom and Van Reenen (2007). Further, the boundaries of the firm can be shaped by the potential for influence activities, providing a theory of the firm based on ex-post inefficiencies. Finally, boundaries and bureaucratic institutions interact: more concentrated decision-making and bureaucratic institutions are complements.

The third chapter (co-authored with Robert Gibbons and Richard Holden) analyzes a rational-expectations model of price formation in an intermediate-good market under uncertainty. There is a continuum of firms, each consisting of a party who can reduce production cost and a party who can discover information about demand. Both parties can make specific investments at private cost, and there is a machine that either party can control. As in incomplete-contracting models, different control structures create different incentives for the parties’ investments. As in rational-expectations models, some parties may invest in acquiring information, which is then incorporated into the market-clearing price of the intermediate good by these parties’ production decisions. The informativeness of the price mechanism affects the returns to specific investments and hence the optimal control structure for individual firms; meanwhile, the control structure choices by individual firms affect the informativeness of the price mechanism. In equilibrium the informativeness of the price mechanism can induce ex ante homogeneous firms to choose heterogeneous control structures.

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Chapter 1

Relational Incentive Contracts and Persistent Misallocation

1.1 Introduction

In the absence of perfect formal contracts, trust is an important input to the production process. Recent empirical work by Bloom, Sadun, and Van Reenen (2011) suggests that lack of trust constrains firm size by limiting the decentralization of important operating decisions. As a result, productive firms that would like to expand cannot, which potentially has important implications for the aggregate TFP of an economy. In this paper, I develop a simple model of relational incentive contracts to analyze the consequences of limited trust on the steady state distribution of firm size and aggregate TFP.

I view trust as a self-enforcing agreement of a repeated principal-agent game (Bull, 1987; MacLeod and Malcomson 1989; Levin 2003) between a firm owner (principal) and many managers (agents). The principal would like to incentivize each agent to exert effort, but she is unable to do so using a formal contract. Instead, she can promise to pay a pre-specified bonus if the

---

1 Firm owners may lock up spare parts for machines, depriving local managers of the ability to perform repairs when a machine breaks down, for if they did not, the managers might steal the parts, sell them, and replace them with low-quality parts. They may require managers to obtain approval from the owner to make capital investments greater than $500, hire non-temporary personnel, or make sales and marketing decisions.

2 Or, since the stage game of the principal-agent problem is a trust game, the principal gives the agent discretion over some resources, and the agent can convert those resources into output or use them in a socially wasteful, privately beneficial, way.

3 Discretionary payments take the form of monetary bonuses in the relational incentive contracting literature. They are typically interpreted as raises, promotions, additional freedom, and improved working conditions that can be awarded to an agent in a contingent way.
agent chooses a particular effort level. The principal lacks commitment, so in a one-shot game, after the agent’s effort has been sunk, the principal would always prefer not to pay the bonus (to renege); thus, a forward-looking agent will not choose a positive effort level. However, through repeated interaction, the principal can use some of the future surplus it generates as collateral for this promise.

I augment the canonical relational contracting model with two additional elements: multiple agents and endogenous competitive rents. In any given period, the principal hires potentially many agents. Using the same logic as Bernheim and Whinston (1990) and Levin (2002), I argue that a multilateral relational contract always dominates a sequence of bilateral relational contracts. The intuition is straightforward: to implement a given effort vector using a sequence of bilateral relational contracts, the effort vector must satisfy the aggregate reneging constraint as well as additional individual reneging constraints. Using a multilateral relational contract simply implies that the principal can reduce the number of constraints he faces, and he always performs weakly better by doing so (and in fact, if efforts are substitutes across agents, strictly better). This makes the principal’s choice of the number of agents one of deciding upon the size of the workforce and then treating the workforce as a single agent. The dynamic enforcement constraint then requires that the aggregate reneging temptation (which here will be the aggregate volume of promised bonuses) must be smaller than the expected net present value of future competitive rents. If there are diminishing returns to scale, the aggregate promised bonus (and hence principal’s reneging temptation) increases linearly in the number of agents the firm employs, whereas the marginal returns are decreasing, and thus the dynamic enforcement constraint can provide a cap on the size of a firm’s workforce.

The product of the principal-agent problem is output that is sold into a competitive product market. The second additional element is the endogeneity of competitive rents. In this model, there are many firms, each producing output and selling it on the competitive market. Each firm makes a conjecture about the stream of future output prices, which determines its ability to sustain cooperation through an optimal relational contract—and hence its output—in any given period. Equilibrium prices are determined by supply and demand, and thus the expected net present value of future competitive rents must be consistent with market clearing in all future periods. I therefore focus on rational expectations equilibrium and show that under some regularity conditions,

\[1\] I show that under constant returns to scale, there does not exist a competitive equilibrium. This is because CRS implies zero competitive rents, and relational incentives require strictly positive rents. These are incompatible.
a stationary rational expectations equilibrium always exists.

In the stationary rational expectations equilibrium, the model reduces to a Lucas span-of-control model with an additional constraint that depends on the contractual frictions. This allows me to provide a simple characterization of aggregate TFP as a weighted average of the sustainable effort levels in the firms in the economy. Since the efficiency of a firm is determined by its competitive rents, aggregate TFP is determined by variables that determine the levels of competitive rents a firm earns, including prices and firm turnover. This creates a non-standard feedback between the price level of the economy and the efficiency of firms in the economy.

I also show that the quality of the formal contracting environment a firm operates in is an important determinant of TFP. Firms with lower productivity potential earn less in equilibrium, and thus cannot sustain as much effort. Improvements in formal contracting institutions disproportionately benefit such firms, leading to a greater dispersion of total factor productivity in weaker contracting environments if one holds all else equal. Thus, cross-country differences in contracting institutions can partially explain the observed pattern that misallocation is more pronounced in developing countries. However, differences in formal contracting institutions will also lead to differences in the price level. I show that when one takes these general equilibrium effects into account, improvements in formal contracting institutions leads to compression in firm size—small firms produce more and large firms produce less.

This paper is related to the recent literature on misallocation and economic growth (Banerjee and Duflo 2005; Jeong and Townsend 2007; Restuccia and Rogerson 2008; Hsieh and Klenow 2009; Bartelsman, Haltiwanger, and Scarpetta 2008), which has argued that cross-country differences in the ability to efficiently allocate resources can explain a substantial portion of the differences in per-capita GDP. These papers argue that misallocation of productive resources is ubiquitous, but it is more pronounced in developing countries than in developed countries. Hsieh and Klenow show that improving the allocation of capital and labor in China and India to U.S. levels would result in a one-off increase in per-capita GDP by 30-50% and 40-60% respectively.

In order to design effective policy aimed at improving the allocation of resources, one first needs to understand why they were not allocated efficiently to begin with. Restuccia and Rogerson and Hsieh and Klenow remain agnostic as to the mechanism (positing firm-specific capital and labor taxes or wedges), but several recent papers in the macro tradition (Banerjee and Moll 2010; Moll 2010; Buera, Kaboski, and Shin 2010; Midrigan and Xu 2010) have focused on the role of under-
developed financial markets. Others include Peters (2011), who argues that in a monopolistic competition framework, heterogeneity in entry rates leads to heterogeneity in markups, which in turn leads to a distortion in relative output prices and thus misallocation. Collard-Wexler, Asker, and De Loecker (2011) argue that much of the misallocation is driven by adjustment costs. Guner, Ventura, and Xu (2008) and Garicano, Lelarge, and Van Reenen (2011) highlight the importance of existing size-dependent policies on whether or not firms operate at their efficient scale.

The normative implications of each of these explanations differs. For example, if misallocation is driven solely by adjustment costs, then there is little scope for policy in reducing this. If, on the other hand, heterogeneity in markups is the driving factor, then we want to understand why there is heterogeneity in entry rates and perhaps remedy this by selectively reducing entry barriers in certain industries. If underdeveloped financial markets are the problem, then top-down improvements in financial markets could reduce misallocation. My model generates persistent misallocation in a perfectly competitive environment with no adjustment costs or credit rationing, and is therefore complementary to existing views. It suggests that policy should focus on the quality of formal contracting environments.

This paper is also related to the organizational economics literature on persistent performance differences among seemingly similar enterprises. Most closely related here are Chassang (2010) and Gibbons and Henderson (2011), who argue that firm-level heterogeneity in productivity is due to differences in (ex ante identical) firms' success in developing relational contracts that put them on the production possibilities frontier. In contrast, I posit that all firms are successful in implementing optimal relational contracts and thus all operate on the production possibilities frontier. However, small differences in firm-level productivity potential translate into differences in continuation values and potentially large differences in sustainable effort levels and thus TFP. Relational incentive contracts can therefore amplify existing differences. The analysis in this paper is silent on firm dynamics, unlike Chassang (2010) and Ellison and Holden (2009). It provides a theory of steady state misallocation, not a theory of the process that leads to it.

---

5In a static model, mismatch between the quality of ideas of an entrepreneur and the funding necessary to take the idea to fruition can lead to misallocation. However, in the long run, entrepreneurs with the best ideas should be able to save their way out of capital constraints. This argument forms the basis of the title of Banerjee and Moll (2010)'s paper, "Why Does Misallocation Persist?"

6The firms in Hsieh and Klenow (2009)'s study are large manufacturing firms that are in the top 2% of the firm size distribution. Such firms are likely to be well-connected to financial markets and therefore are less likely to be subject to credit rationing of this sort.

7Also related is Ellison and Holden (2009), who show the potential for path-dependence in the efficiency of organizational rules.
Finally, there is a growing literature on the macroeconomic implications of contractual incompleteness (Caballero and Hammour 1998; Cooley, Marimon, and Quadrini 2004; Acemoglu, Antras, and Helpman 2007). My analysis is most similar to Acemoglu, Antras, and Helpman (2007), who examine the role of incomplete contracts and unresolved hold-up on technology adoption. In contrast, I explore how the success of attempts to resolve contractual incompleteness using relational contracts varies with underlying firm characteristics.

Section 2 sets up the basic model. Section 3 characterizes the solution in the complete contracts case. Section 4 analyzes optimal relational incentive contracts in the absence of formal contracts, and section 5 explores applications of the main results and extends the model to incorporate the possibility of formal contracting. Section 6 concludes.

1.2 Setup and Technology

There is a unit mass of firms, indexed by $i \in [0, 1]$. Each firm is run by a risk-neutral principal who is the residual claimant. As in standard models of production, output requires both capital and labor. As in standard principal-agent models, an agent is productive only if he exerts effort. Contracting institutions are weak and thus effort cannot be directly contracted upon. Throughout, we will assume that there is a large enough mass of risk-neutral agents (so that in equilibrium, they are indifferent between working and not). Play is infinitely repeated, and we denote by $t = 1, 2, 3, \ldots$ the period. All players share a common discount factor, which we will express in terms of a discount rate $\frac{1}{1+r}$. The product of the principal-agent problem is output, which is homogeneous across firms and sold into a competitive product market. Aggregate demand is assumed to be stationary,
where $p_t$ is the output price in period $t$, and downward-sloping $(D' < 0)$.

Each period consists of seven stages. In the first stage, the principal $i$ decides whether or not to pay the fixed cost of production, $F$. If she chooses to, in the second stage, she decides how much capital $K_{it}$ to rent at rental rate $R_t$ and the mass $L_{it}$ of agents to whom she would like to make an offer. In stage 3, the principal offers each agent $\ell \in [0, L_{it}]$ a triple $(s_{it}, e_{it}, b_{it})$, where $s_{it}$ is a payment that potentially depends on a contractible measure $\rho_{it}$ of agent $\ell$'s effort, $e_{it}$ is a proposed effort level, and $b_{it}$ is a bonus that the principal intends to pay agent if and only if he chooses the proposed effort level. In the fourth period, each agent $\ell$ decides whether or not to accept this proposed contract or reject it in favor of outside opportunity that yields utility $W > 0$. If agent $\ell$ accepts the contract, in stage 5, he chooses an effort level $\hat{e}_{it} \geq 0$ at cost $c(\hat{e}_{it}) = \hat{c}e_{it}$. This effort is commonly observed, and in stage 6, the principal decides whether or not to pay agent $\ell$ a bonus if $b_{it}$. Output for firm $i$ is then realized and sold into the market at price $p_t$ in stage 7.

Firms have heterogeneous productivity potential. Let $A_i$ denote the potential of firm $i$. Assume $A_i \sim G(A_i)$, where $G$ is a distribution function. Given capital $K_{it}$ and a mass $L_{it}$ of workers who choose efforts $\hat{e}_{it} = \{\hat{e}_{it}\}_{\ell \in L_{it}}$, firm $i$'s production in period $t$ is given by

$$y_t(\hat{e}_{it}, K_{it}, L_{it}) = A_i K^0_{it} \left( \int_0^{L_{it}} (\hat{e}_{it})^{1-\alpha-\theta} d\ell \right)^{1-\alpha-\theta}.$$ 

*Throughout, we assume that perfectly enforceable contracts can be written on $\rho_{it}$ but that no contracts can be written directly on effort.

*Some are founded on better ideas, possess more appropriate managerial skills for the environment they operate in, or are more successful in adopting good management practices.
Throughout, make the following assumption.

**Assumption 1.** $\theta < 1 - \alpha - \beta$.

Assumption 1 ensures that the first-order conditions for the unconstrained problem are sufficient, and it also implies that the efforts of the agents are substitutes. In period $t$, if principal $i$ pays all bonuses, her profits are

$$p_i y_i (\hat{e}_{it}, K_{it}, L_{it}) - R_t K_{it} - \int_0^{L_{it}} (s_{it\ell} (\rho_{it\ell}) + b_{it\ell}) d\ell - F.$$

We will analyze the principal’s optimal solution to this problem when different performance measures are available. The next section analyzes the case where $\rho_{it\ell} (e_{it\ell}) = e_{it\ell}$, so that formal contracts can be written directly on effort (obviating the need to use relational incentives), and the section that follows examines the pure relational incentives case, where $\rho_{it\ell} (e_{it\ell})$ is constant. Intermediate cases are considered in Section 5.

Throughout, I assume that the rental rate of capital is exogenously given and constant at $R$. Additionally, the product market structure is as in Lucas (1978). Alternatively, as I show in Appendix B, this model is equivalent to a monopolistic competition model, where $A_i$ is a function of the size of the market for the variety that firm $i$ produces. Finally, the mass of firms in the economy is fixed at 1. In Appendix C, I allow for endogenous firm entry. As in Hopenhayn (1992), a firm can pay a sunk cost $F^e$ to enter the market and draw a value $A_i \sim G$. The resulting mass of entrants is determined by an indifference condition.

### 1.3 Complete Contracts

As a benchmark, consider the case where $\rho_{it\ell} (e_{it\ell}) = e_{it\ell}$, so that the principal can use the contractible portion of the payment, $s_{it\ell}$, to both pin the each agent to his (IR) constraint and directly choose his effort (say, by setting $s_{it\ell} (\hat{e}_{it\ell} \neq e_{it\ell}) = -\infty$). Because in this case, there are no intertemporal linkages in the problem, each firm can solve its profit-maximization problem period-by-period. Given a price level $p_t$, principal $i$ wants to choose $L_{it}, \{e_{it\ell}\}_{\ell \in [0, L_{it}]}$, and $K_{it}$ to solve the following

---

1. I will be focusing on the steady state of this economy. Consequently, it is possible to microfound the stationary aggregate demand function and constant rental rate by specifying an underlying consumer choice model, but for simplicity, I do not do this.
problem.

\[
\max_{K_{it}, L_{it}, \{e_{it}L_{it}\}} \sum_{t=0}^{T} p_t A_t K_{it}^{\alpha} \left( \int_{0}^{L_{it}} (e_{it})^{1-\alpha-\theta} d\ell \right) - RK_{it} - \int_{0}^{L_{it}} s_{it} d\ell - F
\]

subject to each worker's individual rationality constraint, which will hold with equality

\[ s_{it} - c e_{it} = W. \]

By Assumption 1, the firm's problem is concave in \( \{e_{it}\}_{t} \), so any optimal solution must satisfy \( e_{it} = e_{it}^* \) for all \( t \). Recognizing this and substituting the (IR) constraint into (1.1), the problem becomes

\[
\max_{K_{it}, L_{it}, e_{it}} p_t A_t e_{it} K_{it}^{\alpha} L_{it}^{1-\alpha-\theta} - RK_{it} - (W + c e_{it}) L_{it} - F.
\]

There will be some shutdown value of productivity potential, \( A_S \), for which \( A_i < A_S \) implies that a firm with potential \( A_i \) should optimally not produce. The solution to this problem is captured in the following proposition.

**Proposition 1.1** Let

\[
A_S = \frac{F^\theta}{p} \left( \frac{c}{\theta^2} \right) \left( \frac{R}{\alpha} \right)^\alpha \left( \frac{W}{1 - \alpha - 2\theta} \right)^{1-\alpha-2\theta}
\]

\[
H(p_t, A_i) = (p_t A_i)^{\frac{\alpha}{R}} \left( \frac{1 - \alpha - 2\theta}{W} \right)^{\frac{1-\alpha-\theta}{\theta}}.
\]

The unconstrained solution to firm i's problem is

\[
e^{FB} = \frac{W}{1 - \alpha - 2\theta} \frac{\theta}{c}
\]

\[
I^{FB}(A_i) = \frac{1 - \alpha - 2\theta}{W} H(p_t, A_i) e^{FB}
\]

\[
K^{FB}(A_i) = \frac{\alpha}{R} H(p_t, A_i) e^{FB}
\]

if \( A_i \geq A_S \). If \( A_i < A_S \), firm i optimally does not produce.

Since the solution to the period \( t \) problem does not depend on variables from any other period, and demand is stationary, output prices will be constant, \( p_t = p \) for all \( t \). A competitive equilibrium
is then a price level $p$, and a vector of firm-level choices $\{K_i, L_i, \epsilon_i\}_{i \in [0,1]}$ such that these choices are optimal given the price level, and the price level clears the market.

Note that $e^{FB}$ does not depend on $A_i$. The optimal balance between hiring another worker and increasing the amount of effort that existing workers exert is analogous to the intensive and extensive margins of labor demand. Equilibrium total factor productivity for a firm with potential $A_i$ is given by

$$TFP_i = \frac{y_i}{K_i^\alpha L_i^{1-\alpha-\theta}} = A_i \left(e^{FB}\right)^\theta.$$

This implies the following.

**Proposition 1.2** In the complete contracts model, a firm’s equilibrium total factor productivity depends only on that firm’s productivity potential, $A_i$, and the first-best level of effort, $e^{FB}$.

This proposition will stand in contrast to the results from the following section, where effort is not directly contractible.

### 1.4 Relational Incentive Contracts

We now turn to the heart of the model and assume that $p_{itt} = 0$ for all $e_{itt}$. Effort is non-contractible, and therefore $s_{itt}$ is constant. The principal would like to incentivize her agents to exert effort, but she can only do so by making a promise that she will pay a pre-specified bonus if the agent chooses a particular effort level. The principal cannot commit to doing so, so in a one-shot game, after the agent’s effort has been sunk, the principal would always prefer not to pay the bonus, and thus, a forward-looking agent will not choose a positive effort level. However, the principal may use future competitive rents as a partial commitment device.

Her ability to do so depends on the clarity with which her failure to pay bonuses gets communicated to her current and potential future agents, and it also depends on the ease with which she can replace her current agents. Throughout, I make the following strong assumptions of perfect observability and no labor market frictions.

**Assumption 2.** A firm’s current workforce and its potential future workforce commonly observe the effort choices of individual workers and whether or not they were paid their promised bonuses.

**Assumption 3.** Workers can be rematched with a different firm at no cost.
Assumption 3 implies that quasi-rents are equal to competitive rents, and Assumption 2 ensures that the totality of a firm's future competitive rents can be used as collateral in its promises. Relaxing Assumption 2 to the case of imperfect public monitoring makes the goal of dynamic enforcement more difficult to achieve but does not qualitatively change any of the results (see Levin (2003)). Relaxing Assumption 3 allows labor market frictions in addition to competitive rents to be leveraged in dynamic enforcement. This is the basis for the Shapiro and Stiglitz (1984) observation that equilibrium unemployment can serve as a worker discipline device.

1.4.1 Dynamic Enforcement

Under what conditions can the agents "trust" the principal to pay the promised bonus? Working backwards, under what conditions will the agent choose a proposed effort level $e_{it\ell}$? Throughout, we will look for equilibria in trigger strategies, which in this game constitute an optimal penal code (see Abreu 1988). This will be made more precise below.

Suppose agent $\ell$ believes the principal will pay bonus $b_{it\ell}$ if and only if he chooses effort $\hat{e}_{it\ell} = e_{it\ell}$. Then he will choose $\hat{e}_{it\ell}$ (instead of $\hat{e}_{it\ell} = 0$, which minimizes his private costs of effort) if

$$b_{it\ell} + \frac{1}{1 + r} \left( U_{it+1,\ell} - \hat{U}_{it+1,\ell} \right) \geq c(e_{it\ell}), \quad (1.2)$$

where $U_{it+1,\ell}$ is the continuation utility agent $\ell$ receives from $t + 1$ on if the relationship is not terminated, and $\hat{U}_{it+1,\ell}$ is the continuation utility the agent he receives if separation occurs. Thus, he will choose the proposed effort level if and only if the sum of the bonus and the change in the continuation values exceeds the cost of effort provision.

If the agent chooses any effort level other than the proposed effort level, the principal has no incentive to pay the bonus and therefore will not. If the agent chooses the proposed effort level $e_{it\ell}$, the principal will pay the promised bonus $b_{it\ell}$ if

$$\frac{1}{1 + r} \left( \Pi_{it+1,\ell} - \tilde{\Pi}_{it+1,\ell} \right) \geq b_{it\ell}, \quad (1.3)$$

where $\Pi_{it+1,\ell}$ and $\tilde{\Pi}_{it+1,\ell}$ are, respectively, the profits generated by the continuing relationship between the firm and worker $\ell$ from $t + 1$ on and the profits generated if the firm and worker $\ell$ separate at $t + 1$. Thus, the change in continuation value for the firm (i.e. the surplus generated

---

11 Ghosh and Ray (1996) and Kranton (1996) show that relaxing the observability assumption with respect to future potential agents can lead to interesting relationship dynamics.
by the firm's relationship with worker \( \ell \) must exceed the size of the promised bonus.

We know from MacLeod and Malcomson (1989) and Levin (2003) that (1.2) and (1.3) can be pooled together to provide necessary and sufficient conditions for the agent to choose the proposed effort level and the principal to pay the promised bonus. That is, if we let \( S_{i,t+1,\ell} = U_{i,t+1,\ell} + \Pi_{i,t+1,\ell} \) and \( \tilde{S}_{i,t+1,\ell} = U_{i,t+1,\ell} + \Pi_{i,t+1,\ell} \), we need that

\[
\frac{1}{1 + r} \left( S_{i,t+1,\ell} - \tilde{S}_{i,t+1,\ell} \right) \geq ce_{it}\ell
\]  

(1.4)
is satisfied. \( \tilde{S}_{i,t+1,\ell} \) is not a straightforward object. After terminating a relationship with worker \( \ell \), the firm would in principle find it optimal to alter its promises to other workers, and the continuation value that results for the firm then depends on its relationships with its other workers. As in Levin (2002) and Bernheim and Whinston (1990), however, the firm always does best if it uses a multilateral relational contract, in which case the continuation strategy following either an inappropriate effort level or the failure of the principal to pay a bonus to a worker involves all current and future agents to exert zero effort, the principal to not pay promised bonuses, agents to reject the relational contract, and the principal not to pay the fixed cost of production. This allows us to focus only on the aggregate reneging temptation, which can be expressed as

\[
\frac{1}{1 + r} \left( S_{it+1} - \tilde{S}_{it+1} \right) \geq \int_0^{L_{it}} ce_{it}d\ell,
\]  

(1.5)

where \( S_{i,t+1} \) represents the total variable profits generated by the principal and the agents she hires net of their outside opportunities, and \( \tilde{S}_{i,t+1} \) is the firm's outside option. Since the stage game has an equilibrium in which agents reject any contract they are offered, they choose zero effort if they do work, the principal does not pay any bonuses, and the principal instead scraps the firm, we can use this equilibrium as the worst possible punishment. By Abreu (1988), this will form the basis of an optimal penal code, and thus \( \tilde{S}_{i,t+1} = \frac{1+r}{r} F \).

The remaining object to pin down in (1.5) is \( S_{i,t+1} \), which is the expected net present value of future competitive rents. This depends on the whole future stream of prices and future promises. Given a conjecture \( \{p_{\tau}\}_{\tau=t}^\infty \) that is shared by the principal and the agents, \( S_{it+1} \) is given by

\[
\sum_{\tau=t+1}^\infty \left( \frac{1}{1 + r} \right)^{\tau-t-1} \left[ p_{\tau} A_i K_i \left( \int_0^{L_{it}} (e_{it})^{\frac{\theta}{1-\alpha}} \, d\ell \right)^{1-\alpha-\theta} - RK_i - WL_i - \int_0^{L_{it}} ce_{it}d\ell - F \right].
\]  

(1.6)
\( \{p_t\}_{t=1}^{\infty} \) are determined jointly by the production capabilities and relational contracts of all the firms in the economy as well as demand conditions.

### 1.4.2 Rational Expectations Equilibrium

Throughout, we will focus on rational expectations equilibria in which all firms conjectured the same price sequence, and this price sequence in fact clears the market in each period. Because it is not essential for the model, assume the capital market is perfectly competitive, and the interest rate sequence is pinned down by a consumer Euler equation.

**Definition 1.1** A rational-expectations equilibrium (REE) is a sequence of prices \( \{p_t\}_{t=1}^{\infty} \), a sequence of capital and labor choices \( \{L_t, K_t\} \), a sequence of relational contracts \( \{s_{it}, b_{it}, c_{it}\}_{it} \), and a sequence of effort choices \( \{e_{it}\}_{it} \) such that at each time \( t \)

1. Given promised bonus \( b_{it} \), worker \( \ell \) for firm \( i \) optimally chooses effort level \( e_{it} = e_{it} \)
2. Given the conjectured price sequence \( \{p_t\}_{t=1}^{\infty} \), firm \( i \) optimally offers relational contract \( \{s_{it}, b_{it}, c_{it}\}_{it} \)
   and chooses capital and labor levels \( \{K_{it}, L_{it}\}_{it} \)
3. \( \{p_t\}_{t=1}^{\infty} \) clears the output market for all \( t \)

Throughout, I will focus instead on stationary REEs with constant prices \( p_t = p \). The following proposition establishes existence a unique stationary REE.\(^{12}\)

**Proposition 1.3** Suppose \( D \) is smooth and satisfies \( \lim_{p \to 0} D(p) = \infty \), \( D' < 0 \), and suppose \( G \) is absolutely continuous. There exists a unique stationary REE.

**Proof.** Suppose all firms conjecture price sequence \( p_t = p \) for all \( t \). Fix a firm \( i \) and assume all other firms use a stationary relational contract \( (s_{jt}, b_{jt}, c_{jt}) = (s_j, b_j, c_j) \) and choose constant capital and labor levels \( (K_{jt}, L_{jt}) = (K_j, L_j) \). Then from firm \( i \)'s perspective, the environment is stationary. Suppose firm \( i \) chooses \( (K_{it}, L_{it}) = (K_i, L_i) \) for all \( t \). By Levin (2003), firm \( i \) can replicate any optimal relational contract with a stationary relational contract. Thus,

---

\(^{12}\)In a related model, I have shown that there may exist a nonstationary REE with price cycles. The basic intuition is the following. Suppose all firms believe that output prices will be high and then low and then high and so forth. Then from the perspective of a period in which prices are high, the future looks relatively grim, as prices will be low in the future. This constrains the level of effort firms can sustain as part of an optimal relational contract today, which leads to a restriction in quantity and hence a high price today. From tomorrow's perspective, future prices will be high, and thus the firm's competitive rents are sufficient for sustaining high levels of effort. Quantity is high and therefore prices are low. Thus, this two-point alternating price sequence is consistent with equilibrium.
which in turn makes the firm’s choice of a constant capital and labor sequence optimal. This implies a constant aggregate production sequence, so that \( p_t = p \) if such a price exists.

Aggregate supply is upward-sloping, since future competitive rents, and hence today’s output, are increasing in \( p \) for all firms. Further, it is smooth, since \( G \) is absolutely continuous. Since aggregate demand has an infinite choke price and is decreasing and smooth, existence and uniqueness of such a price \( p \) follows. ■

### 1.4.3 Optimal Relational Incentive Contracts

The remaining sections characterize optimal relational contracts in the stationary REE and examine the aggregate implications of the dynamic enforcement constraint. We know from the previous proposition that in the steady state, it is an equilibrium for firms to choose stationary relational contracts. By Assumption 1, production is concave in individual effort choices. Since workers are symmetric, any optimal relational contract will involve \( e^\ell_i = e_i \) for all \( \ell \). At the steady state, per period profits for firm \( i \) are given by

\[
\pi_i = p A_i e_i^\theta K_i^\alpha L_i^{1-\alpha-\theta} - RK_i - (W + c e_i) L_i - F.
\]

In an optimal relational contract, firms maximize their per-period profits subject to their pooled dynamic enforcement constraint. That is, each firm takes \( p \) as given and solves

\[
\max_{K_i, L_i, e_i} \pi_i \tag{1.7}
\]

subject to

\[
\frac{\pi_i}{r} \geq L_i c e_i. \tag{1.8}
\]

In the formulation of the production function, I have assumed that if all workers choose the same effort levels, production exhibits decreasing returns to scale in \( K \) and \( L \). This is a standard assumption in models in which firms of different productivities co-exist in equilibrium (e.g. Lucas (1978)). Alternatively, the entire model can be reformulated as a monopolistic competition model in which each firm’s production exhibits constant returns to scale in \( K \) and \( L \), but each firm faces a downward-sloping demand curve (see Appendix B). I show in Appendix A that if revenues exhibit constant returns in \( K \) and \( L \), there is no competitive equilibrium.
We can think of the interest rate the firm faces as an effective interest rate that combines firm turnover (i.e., an exogenous probability of firm destruction), pure time preferences, monitoring technology on the part of the firm (i.e., can the firm see whether or not a worker has chosen the correct effort level?) or on the part of the population of workers (i.e., can future workers see if the firm has paid the bonuses?).\footnote{See Appendix A for a derivation of the effective discount rate in this case.} In other words, think of $r$ as fairly large.

The next proposition characterizes the solution to the constrained problem (1.7) subject to (1.8).

**Proposition 1.4** In this model, the solution to the constrained problem satisfies

$$
\frac{e^*(A_i)}{e^{FB}} = \frac{L^*(A_i)}{L^{FB}(A_i)} = \frac{K^*(A_i)}{K^{FB}(A_i)} = \mu^*(A_i),
$$

where $0 \leq \mu^*(A_i) \leq 1$ is (weakly) increasing in $p$ and (weakly) decreasing in $R, W,$ and $r$. Further, if we define

$$
A_L \equiv (1 + r)^\theta A_S
$$

$$
A_H \equiv \begin{cases} (1 - r)^{-\theta} A_S & r < 1 \\ +\infty & r \geq 1, \end{cases}
$$

then

$$
\mu^*(A_i) = \begin{cases} \frac{1}{1 + r} \left(1 + \left(1 - (A_L/A_i)^{1/\theta}\right)^{1/2}\right) & A_i \geq A_H \\ 0 & A_L \leq A < A_H \\ A < A_L. \end{cases}
$$

This proposition is proven in the appendix. The following figure characterizes this solution as a function of $A$. In the complete contracts model, $e^*(A_i)$ equals zero if $A_i$ is not large enough for the firm to cover its fixed costs of production and $e^*(A_i) = e^{FB}$ otherwise. When formal contracts are unavailable, there are three additional regions. For $A_S \leq A_i < A_L$, the firm should produce but is unable to. For $A_L \leq A_i < A_H$, the dynamic enforcement constraint is binding, and the firm is unable to produce efficiently. For $A_i \geq A_H$, the firm is unconstrained and thus produces according to first-best.
Equilibrium total factor productivity for a firm with potential $A_i$ is then

$$TFP_i = \frac{y_i}{K_i^\alpha L_i^{1-\alpha - \theta}} = A_i \mu^*(A_i)^\theta \left(e^{FB}\right)^\theta.$$

This leads to the following proposition.

**Proposition 1.5** In the relational contracting model, a firm’s equilibrium total factor productivity depends on the firm’s productivity potential as well as its sustainable effort-level, which is increasing in $p$ and decreasing in $R, W$, and $r$.

A firm’s total factor productivity depends on the effective discount rate a firm faces and is therefore decreasing in firm turnover and increasing in the clarity with which deviations are communicated. The quality of communication technology and the strength of social connections may therefore play a role in determining a firm’s total factor productivity. In addition, total factor productivity is jointly determined with the equilibrium price—a firm’s production possibilities set is endogenous to market conditions, unlike in the standard Neoclassical growth model.

### 1.5 Aggregate Implications of this Approach

The first set of implications concerns the impact of relational solutions to contractual incompleteness on the allocational efficiency of production in an economy. I show that firms do not produce up
until the point where the net marginal revenue product of labor \( (MRP_L - W) \) is zero; further, heterogeneity in productivity potential leads to heterogeneity in net marginal revenue product of labor. Additionally, I provide a simple characterization for aggregate TFP.

I then compare two otherwise identical economies: one with noncontractible effort, the other with perfectly contractible effort. Holding prices constant, allowing for perfectly contractible effort has two effects. First, TFP rises more for firms that are more constrained, leading to a compression of the TFP distribution for existing firms. Second, lower-potential firms that were previously unable to produce now do so. Since allowing for complete contracts increases production for a given price, the equilibrium output price will fall. This leads to a contraction of output in more productive firms and increases the minimum scale necessary for production. The net effect is a compression of output. I confirm these findings numerically.

1.5.1 Aggregate TFP and Misallocation

Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) are agnostic about the mechanism behind the misallocation of capital and labor in an economy. This misallocation is modeled as resulting from labor and capital wedges in a firm’s first-order conditions—different firms have different net marginal revenue products of labor and capital in equilibrium. In this model, firms equate the net marginal revenue product of capital to zero; however, they face heterogeneous distortions on the labor margin.

To see this, note that at the constrained optimum, the ratio of labor, capital, and effort to their first-best levels are identical, and therefore their ratios are the same as the first-best ratios—firms that do not achieve first-best still nevertheless have the same capital/labor and output/labor ratios as firms that are unconstrained. However, since more constrained firms demand less effort from their workers, in equilibrium, they pay lower wages. Thus, their labor share of total revenues is lower than at the first best. Firms in this economy thus tend to be suboptimally small. This is reflected by a disparity between the marginal revenue product of labor and the effective wages a firm pays a worker, \( W^*(A_i) = W + e^* (A_i) \). To see this, note that at the optimum,

\[
MRP_L (A_i) - W^* (A_i) = (1 - \mu^* (A_i)) e^{FB}.
\]

An unconstrained firm should equalize the marginal revenue product of labor to the wage rate, which is indeed the case when \( \mu^* = 1 \). The more constrained a firm is (as measured by a lower \( \mu^* \)),
the greater is the disparity between the marginal revenue product of labor and its compensation. Such firms would like to expand, but are unable to.

What are the implications of this for aggregate total factor productivity? Let $Y^{AGG} = \int y^*_i di$, where $y^*_i$ is firm $i$’s per-period production under its optimal relational contract, and let $K^{AGG} = \int K^*_i di$ and $L^{AGG} = \int L^*_i di$, where $K^*_i$ and $L^*_i$ are, respectively, firm $i$’s capital and labor demands in the optimal relational contract. Aggregate TFP is then aggregate output divided by an appropriately weighted measure of aggregate observable inputs, or

$$TFP^{AGG} = \frac{Y^{AGG}}{(K^{AGG})^\alpha L^{AGG})^{1-\alpha}} = (e^{FB})^\theta \left( \int A^{1/\theta} \mu^* (A) dG (A) \right)^\theta$$

where $TFP^{AGG}_{FB}$ is the aggregate TFP that would arise if effort were perfectly contractible, and $\omega (A) = \frac{A^{1/\theta}}{\int A^{1/\theta} dG (A)}$.

Aggregate inefficiencies are a weighted average of firm-level inefficiencies, and thus factors that determine the efficiency of production in individual firms also determine the efficiency of aggregate production. These include endogenous objects such as output prices, and exogenous objects such as the importance of effort in production ($\theta$), and the firm’s effective interest rate, which is composed of a time-preference parameter, the rate of turnover, and measures of the underlying monitoring structure.

In what sense is this economy inefficient? Obviously, a social planner that need not respect dynamic enforcement constraints (say by directly choosing effort) can achieve more output for the same level of input. Suppose instead that a social planner can reallocate competitive rents, keeping their sum equal to the aggregate competitive rents, through fixed cost subsidies and taxes. Then a reduction in competitive rents of an unconstrained firm (by marginally increasing its fixed costs) leads to no reduction in its output; if these rents are reallocated to a constrained firm (by marginally reducing its fixed costs), they increase its output in proportion to the shadow cost of its dynamic enforcement constraint. When contracts are incomplete, and there is limited commitment to promised contingent compensation, the price mechanism need not allocate competitive rents efficiently.
1.5.2 Differences in Formal Contracting Institutions

Suppose there are two countries, comp and inc. In country comp, firms have no access to useful verifiable performance measures (so that $\rho_{it} = \emptyset$), while in country inc, firms have access to perfect verifiable performance measures (so that $\rho_{it} = e_{it}$). The two countries are otherwise identical, in the sense that firms in the two countries face the same aggregate demand curve $D(\cdot)$, the same interest and rental rates ($r$ and $R$, respectively), workers have the same outside options ($W$), and productivity potential is drawn from the same distribution $G$. Assume throughout that the two countries do not trade with each other.

In country inc, low-potential (low $A$) firms are more constrained. To see this, recall that for a firm with potential $A$, if this firm operated in country inc, its TFP would be

$$TFP^{\text{inc}}(A) = A\mu^*(A)^\theta (e^{FB})^\theta,$$

and if it operated in country comp, its TFP would be

$$TFP^{\text{comp}}(A) = A(e^{FB})^\theta.$$

For such a firm, the ratio of these two values is

$$\frac{TFP^{\text{inc}}(A)}{TFP^{\text{comp}}(A)} = \frac{A\mu^*(A)}{A} \leq 1,$$

which is increasing in $A$. Superior contracting institutions disproportionately improves effort provision in such firms and may lead to a compression in productivity by thinning out the left tail of the distribution of productivity. This is consistent with the idea that countries with better contracting institutions (i.e. the US) tend to have relatively fewer firms operating in the left tail of the productivity distribution than countries with worse contracting institutions (i.e. China or India). I simulate the $n_s = 1,000,000$ firms in a model with the $\alpha = 0.45, \theta = 0.25, W = 0.5, R = 0.45, r = 0.8, c = 1, \log A \sim N(1, \frac{1}{4}), F = 2.7, D(p) = p^{-1/5}$. The productivity distribution for firms in the two economies is shown in figure 3.

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14 A richer model would allow for a more subtle comparison. For example, countries could perhaps be indexed by $\omega \in [0, 1]$, where firms in a country with index $\omega$ have access to a performance measure $\rho_{it} = 1\{e_{it} \geq \omega e^{FB}\}$. In this case, high-quality (but imperfect) formal contracts may undermine relational incentives, as in Baker, Gibbons, and Murphy (1994). This is a potentially interesting direction for future research.
Total factor productivity is more compressed in \textit{comp} than in \textit{inc}: \( \frac{\text{std}(\log TFP^{inc})}{\text{std}(\log TFP^{comp})} \approx 1.3 \). Differences in contracting institutions across countries can partially explain differences in the dispersion of total factor productivity, as documented by Hsieh and Klenow (2009) in Table 1.

<table>
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<tr>
<td>( \sigma_{\log TFP} )</td>
<td>1.06</td>
<td>1.23</td>
</tr>
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</table>

However, since each firm in country \textit{comp} will produce more than its counterpart in country \textit{inc}, aggregate supply will be greater for a given price level. Since in both countries, aggregate demand is the same, this implies lower output prices in country \textit{comp}. Figure 3 shows that the equilibrium output price is \( p^{\text{inc}} = 0.7525 \) in country \textit{inc} and \( p^{\text{comp}} = 0.6758 \) in country \textit{comp}. Since \( r < 1 \), \( A_H < \infty \), there will be some firms in country \textit{inc} that have sufficiently high potential that they are not constrained in equilibrium. Because of this difference in prices, such high-potential firms in country \textit{comp} produce less than their counterparts in \textit{inc}. 

Figure 3: Productivity Dispersion
In particular, there will be some level of potential, \( \tilde{A} \), such that for \( A < \tilde{A} \), output for a firm in \( \text{comp} \) will be greater than its counterpart in \( \text{inc} \), and for \( A > \tilde{A} \), output will be greater in \( \text{inc} \) than in \( \text{comp} \). This is shown in Figure 5. For these parameter values, we will have \( A_s^{\text{inc}} = 3.8 \), \( A_s^{\text{comp}} = 4.3 \), \( A_l^{\text{inc}} = 4.4 \), and \( A_H^{\text{inc}} = 5.7 \).
The thin lines represent the differences between production in \textit{inc} and \textit{comp} holding output prices constant at $p^{inc}$. That is, the thin line represents the differences in production that result solely from a difference in the contracting environment. For $A < A^{inc}_S$, firms do not earn enough to cover their fixed costs of production and therefore should not produce in either country. Firms with $A^{inc}_S < A < A^{inc}_L$ are unable to produce using only relational contracts but if perfect contracts are available, they will earn enough to cover their fixed costs. Firms with $A^{inc}_L < A < A^{inc}_H$ are constrained and thus produce more in \textit{comp} than in \textit{inc}, though this difference is smaller the greater is $A$. Finally, firms with $A > A^{inc}_H$ are unconstrained in either economy and therefore produce the same amount.

The thick lines compare equilibria in the two countries by accounting for differences in the price level. Firms with $A^{inc}_S < A < A^{comp}_S$ are able to cover their fixed costs under the higher price level, $p^{inc}$, but not under the lower price level in \textit{comp}. Only those firms with $A^{comp}_S < A < A^{inc}_S$ are present in \textit{comp} but not in \textit{inc}. For firms with $A > A^{inc}_L$, there are two effects—perfect formal contracts allows for more effort provision, but lower prices leads to less desired production. The net effect of these two effects gives rise to a cutoff $\hat{A}$ such that $A^{inc}_L < A < \hat{A}$ firms produce more and $\hat{A} < A$ firms produce less in \textit{comp} than in \textit{inc}.

Taking output to be a measure of firm size, we can see that there will be greater dispersion in the size of firms in \textit{inc} than in \textit{comp}. This is shown in Figure 6.

![Figure 6: Output Dispersion](image-url)
1.6 Conclusion

Expanding an organization requires making promises. Using the logic of relational contracting models, I show that a firm's potential competitive rents can be used as collateral in these promises. However, the returns to expansion are decreasing, while the aggregate promises a firm must make in order to sustain expansion grow linearly in the size of the firm. This implies that in the steady state, some firms may wish to expand but are unable to do so. Lack of trust in organizations thus leads to a persistent cap on firm size.

This has implications for the allocation of productive resources in an economy. Relative to an economy with perfect formal contracts, an economy in which firms must rely solely on relational contracts will be characterized by a thick left tail of fairly unproductive firms. Output prices will be higher, and this will lead to a greater dispersion in the size of firms. Some firms will be "too large" and others will be "too small."

There are several potential avenues for future research. Since the model focuses on the steady state, it is silent about the growth path of individual firms. If we view firm growth as being made possible only by non-contractible investments from employees of a firm, then the rate at which a firm grows may be limited by the potential productivity of a firm. In particular, it may be that there is a critical level of potential productivity \( \hat{A} \) such that firms with productivity \( A < \hat{A} \) face extremely sluggish growth (or perhaps none at all) and firms with productivity \( A > \hat{A} \) are able to grow significantly faster. Such a model may be able to generate results consistent with the new Hsieh and Klenow (2011) facts on firm growth.

This model could also be embedded into a Melitz (2003) style international trade model to explore the effects of trade liberalization on individual firm TFP. There would be a market expansion effect for exporting firms, and the increased competitive rents this would imply could lead to increased effort provision within the firm. On the other hand, the decreased price level that would result could negatively impact the productivity of firms that do not export. The net welfare consequences of these two effects is ambiguous. These effects would complement the reallocation effects of Melitz.

1.7 Appendix A: Proofs and Derivations

Proposition 1.6 If production exhibits constant returns to scale in labor and capital, there does not exist an REE.
Proof. Suppose production is \( y_{it}(e_{it}, K_{it}, L_{it}) = A_i e_{it}^\theta K_{it}^\alpha L_{it}^{1-\alpha} \). Then, in period \( t \), the firm with the highest value of \( A_i \) will continue to produce as long as \( p_t y_{it} - R_t K_{it} - (W + c_{it}) L_{it} \geq 0 \). Market clearing with finite demand thus implies that \( p_t y_{it} - R_t K_{it} - (W + c_{it}) L_{it} = 0 \) for all \( t \). This in turn implies that the left-hand side of the dynamic enforcement constraint is zero, which implies that no production can be sustained. ■

Proposition 1.7 Suppose with probability \( q_p \), deviations by the principal go unnoticed by future potential agents, and with probability \( q_A \), deviations by an agent go unnoticed by the principal. Then the effective interest rate in (1.8) is \( \tilde{r} = \frac{r}{q_A q_p} \).

Proof. If we rewrite (1.2) and (1.3) recognizing that (a) the principal will choose \( s_t \) to pin each agent to his \((IR)\) constraint and that (b) the optimal relational contract will be stationary, and we introduce \( q_A, q_B > 0 \), these become

\[
\begin{align*}
\pi_{i t}^F &\geq \frac{1}{q_A} (e_i^F) \\
q_p &\pi_{i t}^F \geq b_i^F.
\end{align*}
\]

If we pool these across agents, this becomes

\[
q_p \pi_i \geq \int b_i^F \geq \frac{1}{q_A} Lc e_i
\]

or \( \pi_i \geq \frac{r}{q_A q_p} L_i c e_i \equiv \tilde{r} L_i c e_i \), which is the desired result. ■

Proposition 1.8 In this model, the solution to the constrained problem satisfies

\[
\frac{e^* (A)}{e^{FB}} = \frac{L^* (A)}{L^{FB} (A)} = \frac{K^* (A)}{K^{FB} (A)} = \mu^* (A),
\]

where \( 0 \leq \mu^* (A) \leq 1 \) is (weakly) increasing in \( p \) and (weakly) decreasing in \( R, W, \) and \( r \). Further,

\[
\mu^* (A) = \begin{cases} 
1 & A \geq A_H \\
\frac{1}{1+r} \left( 1 + \left( 1 - (A_L / A)^{1/\theta} \right)^{1/2} \right) & A_L \leq A < A_H \\
0 & A < A_L, \end{cases}
\]
where

\[
A_L = \frac{F^\theta}{p} \left( \frac{1 + r}{\theta^2} \right) \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - 2\theta} \right)^{1 - \alpha - 2\theta},
\]

\[
A_H = \frac{F^\theta}{p} \left( \frac{1}{1 - r \theta^2} \right) \left( \frac{R}{\alpha} \right)^{\alpha} \left( \frac{W}{1 - \alpha - 2\theta} \right)^{1 - \alpha - 2\theta}.
\]

**Proof.** Throughout this proof, I drop the \( i \) subscript for the firm. Proposition 3 allows us to focus on the stationary problem. Worker symmetry and decreasing returns to effort imply that \( e_\ell = e \) for all \( \ell \in [0, L] \). The firm’s problem is then

\[
\max_{K, L, e} p A e^\theta K^\alpha L^{1 - \alpha - \theta} - R K - (W + ce) L - F
\]

subject to

\[
p A e^\theta K^\alpha L^{1 - \alpha - \theta} - R K - (W + ce) L - F \geq r L c e.
\]

Since an increase in \( K \) increases the objective function as well as the left-hand side of the constraint, capital will be chosen efficiently, given \( L \) and \( e \). Define

\[
\pi (K^* (L, e), L, e) = py (K^* (L, e), L, e) - R K^* (L, e) - (W + ce) L - F.
\]

The firm’s problem is then to \( \max_{K, L, e} \pi (K^* (L, e), L, e) \) subject to \( \pi (K^* (L, e), L, e) \geq r L c e \). Suppose the firm is constrained at the optimum. Define \( L(e) \) such that the constraint holds with equality. The unconstrained problem is then

\[
\max_e r L(e) c e.
\]

Taking first-order conditions, the firm chooses \( e \) such that \( \frac{L'(e)}{L(e)} e = -1 \). Implicitly differentiating the constraint with respect to \( e \) and substituting this into the first-order condition yields

\[
\frac{py (K^*, L^*, e^*) - R K^*}{L^*} = \frac{1 - \alpha}{1 - \alpha - 2\theta} W
\]

and we know from the constraint that

\[
\frac{py (K^*, L^*, e^*) - R K^*}{L^*} = (W + (1 + r) c e^*) + \frac{F}{L^*}.
\]

(1.9)
implies

\[ L^*(e^*) = \left( \frac{1 - \alpha - 2\theta}{W} \right)^{\frac{1}{\theta}} (pA)^{\frac{1}{\theta}} (\frac{\alpha}{R})^{\frac{\theta}{2}} e^*, \]

and substituting this into (1.10), we have that \( e^* \) solves a quadratic equation. The linearity of \( L^*(e^*) \) results from the assumptions that production is constant returns to scale in \((K, L, e)\) and effort costs are linear in \( e \). Without this assumption, \( e^* \) would then be the solution to a nonlinear equation. If we define \( A_L \) as in the statement of the proposition, the solution to this quadratic equation is

\[ e^* = \frac{1}{1 + r} \left( 1 + \left( 1 - \frac{A_L}{A} \right)^{1/2} \right). \]

Optimal labor and capital are linear in \( e^* \). It is then easy to show that the constraint is binding for \( A < A_H \). For \( A \geq A_H \), the solution to the constrained problem is the same as the solution to the unconstrained problem.

1.8 Appendix B: Monopolistic Competition Version

In this appendix, I show that this model can be equivalently formulated as a monopolistic competition model in which production exhibits constant returns to scale in capital and labor. Under a restriction on the effort cost function, total factor revenue product, \((TFPR = p - A)\), is equated across firms, as in Hsieh and Klenow. Higher productivity firms produce more and therefore have lower output prices. Distortions in the allocation of productive resources across firms can then be inferred from heterogeneity in \( TFPR \). Suppose each firm faces a demand curve of the following form

\[ q = \hat{M} p^{-\frac{1}{1-\sigma}}, \]

where \( \sigma \in [0, 1] \) is a measure of the elasticity of demand. The inverse demand is then \( p = M^{1-\sigma} q^{-(1-\sigma)} \) and thus revenues are given by

\[ R = pq = \hat{M}^{1-\sigma} q^\sigma \]

where \( \hat{M} \) is a function of market size and the price aggregate. Per-period profits are then

\[ \pi = \hat{M}^{1-\sigma} q^\sigma - RK - (W + c(e)) L \]
where \( q = \hat{A}e^{\tilde{\theta}}K^{\tilde{\alpha}}L^{\tilde{\beta}} \). Define \( \theta = \tilde{\theta}\sigma, \alpha = \tilde{\alpha}\sigma, \beta = \tilde{\beta}\sigma, \) and normalize effort so that \( \theta + \alpha + \beta = 1 = \sigma (\tilde{\alpha} + \tilde{\theta} + \tilde{\beta}) \). Then, assume \( c(e) = ce \), which is not without loss of generality. Per-period profits become

\[
\pi = \tilde{M}^{1-\sigma} \left( \hat{A}e^{\tilde{\theta}}K^{\tilde{\alpha}}L^{\tilde{\beta}} \right)^{\sigma} - RK - (W + ce) L
\]

\[
= Me^{\theta}K^{\alpha}L^{1-\alpha-\theta} - RK - (W + ce) L
\]

where \( M = \tilde{M}^{1-\sigma}\hat{A}\). We know from the previous analysis that the unconstrained solution to this problem is

\[
e^{FB} = \frac{W}{1 - \alpha - 2\theta c}
\]

\[
L^{FB}(M) = M^{\frac{1}{\sigma}} \left( \frac{\alpha}{R} \right)^{\frac{2}{\sigma}} \left( \frac{1 - \alpha - 2\theta}{W} \right)^{\frac{1-\alpha}{\sigma}} c^{FB}
\]

\[
K^{FB}(M) = M^{\frac{1}{\sigma}} \left( \frac{\alpha}{R} \right)^{\frac{2+\delta}{\sigma}} \left( \frac{1 - \alpha - 2\theta}{W} \right)^{\frac{1-\alpha-\delta}{\sigma}} e^{FB}
\]

TFPR is then

\[
TFPR = p\hat{A} = \hat{A}\tilde{M}^{1-\sigma}q^{\sigma} = K\tilde{M}^{1-\sigma}\tilde{M}^{(1-\sigma)(1-\frac{1}{\sigma})}\tilde{A}^{\frac{\tilde{\alpha}+\tilde{\beta}}{\sigma}} A^{\frac{1-(\tilde{\alpha}+\tilde{\beta})}{\sigma}}
\]

TFPR is thus independent of \( \hat{A} \) if and only if \( \tilde{\alpha} + \tilde{\beta} = 1 \), that is, if production exhibits constant returns to scale in capital and labor, then TFPR is independent of \( \hat{A} \). This restriction amounts to

\[
\frac{1}{\sigma} = (\tilde{\alpha} + \tilde{\beta} + \tilde{\theta}) = \left( 1 + \tilde{\theta} \right)
\]

or \( \tilde{\theta} = \frac{1-\sigma}{\sigma} > 0 \).

### 1.9 Appendix C: Free Entry

In the main text of the paper, I have assumed that there is a mass \( N = 1 \) of firms. This appendix follows Hopenhayn (1992) in endogenizing the mass of firms in the economy. Suppose there is a period 0 at which a firm can pay a cost \( F^e \) to enter the economy and take a productivity potential draw \( A \sim G(A) \). Throughout, I will focus on the stationary REE. Suppose a mass \( N \) of firms has
entered. Aggregate supply in each period as a function of the steady state price level \( p \) is given by

\[
S(p; N) = N p^{1-\theta} \left( 1 - \frac{\alpha - 2\theta}{W} \right)^{1-\theta} \left( \frac{\alpha}{R} \right)^{\theta} e^{FB} \int_{A_L(p)} A^{1/\theta} \mu^*(A; p) dG(A),
\]

where I have made the dependence of \( \mu \) on \( p \) explicit. Note that \( A_L(p) \) is decreasing in \( p \) and \( \mu \) is increasing in \( p \), so \( S(p; N) \) is increasing in \( p \). Demand is stationary and given by \( D(p) \), which is smooth, downward-sloping, and satisfies \( \lim_{p \to 0} D(p) = \infty \). The unique equilibrium price \( p^*(N) \) thus solves

\[
S(p^*(N); N) = D(p^*(N)).
\]

Define first-best gross profits for a firm with productivity potential \( A \) as

\[
\pi_{\text{gross}}(A; p) = \theta (pA)^{1/\theta} e^{FB} \left( \frac{\alpha}{R} \right)^{\theta} \left( 1 - \frac{\alpha - 2\theta}{W} \right)^{1-\theta}.
\]

Net profits can be shown to be

\[
\pi(A; p) = \max \{ \mu^*(A; p) (2 - \mu^*(A; p)) \pi_{\text{gross}}(A; p) - F, 0 \}.
\]

Expected gross profits are given by

\[
\bar{\pi}(p) = \int \pi(A; p) dG(A)
\]

The free entry condition is then given by

\[
F_e = \frac{\bar{\pi}(p)}{r}
\]

Since \( D \) is downward-sloping and \( S \) is increasing in \( p \) and \( N \), \( p^*(N) \) is decreasing in \( N \). The free entry condition pins down \( N \) by \( \bar{\pi}(p^*(N)) - rF_e = 0 \). Since \( \bar{\pi} \) is increasing in \( p \), this is monotonically decreasing in \( N \). Since the choke price is infinity, we will have that \( \bar{\pi}(p^*(0)) > rF_e \). As long as \( \bar{\pi}(p^*(\hat{N})) < rF_e \) for some \( \hat{N} \) large, there will be a unique value \( N^* \) that satisfies the free entry condition.
Chapter 2

Influence-Cost Models of Firm Boundaries and Structures

2.1 Introduction

This chapter explores the organizational implications of influence activities—costly activities aimed at persuading a decision maker—both within and between firms. Such activities are commonplace in business relationships. Employees may devote a significant fraction of their otherwise-productive time building their credentials and seeking outside opportunities to convince management that they are ideal for promotion to a key position (Milgrom and Roberts 1988). Division managers may lobby corporate headquarters for larger budgets to pursue pet projects (Wulf 2009). Buyers of intermediate goods may try to persuade sellers to provide favorable delivery time slots, to give them first pick of the highest quality batches of goods, or to assign specific personnel to their case. Such activities are often privately costly and can lower the quality of decision making, and thus part of the organizational design problem is aimed at mitigating them.

Organizations often adopt rigid practices that seem inefficient from a neoclassical perspective but can make sense from the viewpoint of reducing influence activities. A seniority-based promotion rule can sometimes promote a less talented worker or one who is not a good fit for the new position, but it effectively reduces the incentives for workers to waste time "buttering up the boss" (Milgrom 1988; Milgrom and Roberts 1988). Low-powered managerial incentives can stifle motivation but can help reduce an own-division bias in lobbying for corporate resources. Closed-door organizational practices that hamper communication can make it difficult to implement continuous
quality improvement initiatives, but a more open policy may invite lobbying. Moreover, as Milgrom and Roberts (1990) point out, "even the very boundaries of the firm can become design variables." That is, divesting a business unit can create barriers to influence (Meyer, Milgrom, and Roberts 1992). Influence activities are not absent between firms, however—many business relationships are on-going and involve significant relationship specificity, and hence a firm does care about (and thus may hope to influence) what its business partners do.¹

This paper seeks to provide a unified theory of the costs and benefits of integration that is based on the logic of influence-activity mitigation. In order to do so, I embed a tractable model of influence activities it into an organizational design problem. As in Grossman and Hart (1986) and Hart and Moore (1990), this is carried out under a common economic environment (i.e. preferences, information structure, contracting possibilities, and decision sets do not exogenously vary with the control structure). In a model with two decision rights, integration is unified control and non-integration is divided control. The tractability of this approach allows me to explore the impacts of alternative mechanisms for influence-activity mitigation, such as those described above, and their interaction with control structures.

To ground ideas, suppose two parties are in a working relationship. Contracts are incomplete—the parties are unable to meet ex ante and specify a complete state-contingent decision rule—and, in the course of their relationship, decisions must be taken. The rights to make these decisions are contractible ex ante, but neither the rights to make decisions nor the actual decisions to be made are contractible ex post.² That is, when a particular contingency arises, the interested parties cannot costlessly meet and efficiently bargain over the decision that is to be taken. Control is thus exercised—the party with the control right unilaterally chooses his ideal decision given his information. Additionally, there are decision externalities—each party cares directly about the decisions the other party makes.

Finally, information regarding the ideal decision is most easily discernible by the parties who care about the decision to be taken. As such, a decision maker must often rely on reports and messages that originate from parties who have a direct interest in altering the decision maker's beliefs and the ability to do so. The party may seek out additional information that favors his view,

¹As Williamson (1971) and Klein, Crawford, and Alchian (1978) emphasize, even in a perfectly competitive environment for homogeneous intermediate goods, the fundamental transformation ensures that firm boundaries do not eliminate all externalities between parties involved in a transaction.

he may neglect to mention certain points that do not, or he may attempt to tell a story consistent with the facts but heavily biased in its conclusion. In any case, crafting such an argument takes time that would be better spent on more productive tasks—the direct cost of influence activities is the opportunity cost of the influencer's time. As such, these costs are convex—engaging in influence activities crowds out less productive tasks before more productive tasks. Of course, the decision maker recognizes that the influencer has the incentive to manipulate information in this way and will take this into account when making a decision. Nevertheless, following the logic of Holmstrom (1999), equilibrium may involve non-zero levels of influence activities, for if the decision maker anticipated none, the influencer would have the incentive to carry out some.

The model shows that for a fixed control structure, the equilibrium level of influence activities a party engages in is greater the greater is (a) his concern for the decisions being made, (b) the degree of ex post disagreement, (c) the effectiveness with which beliefs can be manipulated, and (d) the number of decisions not under his control. The last point implies that, all else equal, dividing control reduces the costs of influence activities: divided control leads both parties to crowd out mundane activities, whereas concentrated control leads one party to essentially specialize in influence activities, crowding out potentially important tasks. Parties operating in volatile environments in which beliefs are highly sensitive to manipulable information should thus perhaps become non-integrated.

On the other hand, there may be benefits to concentrating control: coordinating the two decisions could be important, or one party might simply have more to lose from not having his ideal decision implemented. The parties may thus opt for integration and choose to reduce influence activities using alternative instruments, such as closed-door policies or restrictions on the discretion of the decision maker. Influence-cost theory can thus help shed light on why certain puzzling management practices persist (as documented by Bloom and Van Reenen (2007)) and can provide a selection-based, rather than causal, explanation for their finding of a positive correlation between the quality of management practices and plant-level performance. Further, the theory predicts interactions between boundaries and organizational practices: rigid organizational practices and integration are complementary. Non-integrated relationships should be governed by less restrictive rules than relationships within integrated firms.

While the analysis above pertains to the boundaries of the firm, similar insights apply to control structures within firms. In the coordination versus local adaptation framework of the recent papers by Alonso, Dessein, and Matouschek (2008) and Rantakari (2008), I derive a similar trade-off.
Centralizing control with a third-party headquarters facilitates coordination, but it does so at the expense of high levels of influence activities. Decentralization hampers coordination and reduces influence activities. As in the baseline model, centralization and rigid organizational practices are complementary. This is consistent with the (Bloom, Sadun, and Van Reenen 2011) findings of positive correlations between decentralization and the quality of management practices and between decentralization and firm performance.

My analysis assumes that information-gathering activities are separable from influence activities, which I define to be (weakly) information degrading. All else equal, an organization may like to incentivize information acquisition (see Zermeno (2011) for recent work on motivating information acquisition by an unbiased expert) and discourage influence activities. But the two need not be separable, and thus I am ruling out potentially beneficial effects of influence activities—since an individual must be credible to be persuasive, he must gather useful information in order to influence a decision maker (see Laux (2008) for recent work along these lines).

This paper is related to the literature on influence activities in organizations (Milgrom 1988; Milgrom and Roberts 1988, 1990, 1992; Schaefer 1998; Scharfstein and Stein 2000; Laux 2008; Wulf 2009; Friebel and Raith 2010; Lachowski (2011)) but is closest in spirit to Meyer, Milgrom, and Roberts (1992) who explore the idea that the boundaries of the firm can serve as design variables to mitigate influence activities. The key difference is that in their model, divestiture of a division is equivalent to assuming that decision rules cannot depend on the information the division possesses, whereas in my model, divestiture of a division amounts to divided control. I view informational restrictions on decision rules as an additional instrument (as in Milgrom and Roberts (1988)) and analyze the interaction between the two. It is closest in analysis to Gibbons (2005) who explores the role of the allocation of a single decision right on equilibrium influence activities. This paper goes farther in that it analyzes the simultaneous choice of boundaries and organizational practices. In doing so, it provides a theory of the firm based on ex post inefficiencies (Matouschek 2004; Hart and Holmstrom 2010) that is related to Williamson's classic "haggling" versus "fiat" argument. My treatment of rigid organizational practices is also related to the literature on endogenous bureaucracy (e.g. Prendergast (2003)).

Section 2 develops a simple model of the allocation of control in the presence of influence activities. Section 3 defines and characterizes the equilibrium of this influence activity model for a fixed governance structure, and sections 4-6 endogenize the governance structure. Section 4 analyzes the optimal allocation of control (control structure) for a fixed set of organizational
practices, section 5 analyzes organizational practices for a fixed control structure, and section 6 examines the optimal choice of both. Section 7 shows that similar logic can also form the basis for a theory of the internal structure of decision making rather than boundaries, and section 8 concludes.

2.2 The Model

There are two managers, denoted by $L$ and $R$ and two decisions that must be made, $d_1$ and $d_2$. The payoffs to the managers for a particular decision depend on an underlying state of the world, denoted by $s \in S$. The state of the world is unobserved; however, the two managers can commonly observe an informative signal, $\sigma$. But, as Milgrom and Roberts (1988) point out, information regarding the ideal decision typically originates with the parties who care about the ultimate decisions taken and have the means to misrepresent the information. As a result, the signal can be manipulated by both managers in a way that will be made precise shortly.

For example, the two managers may make use of a common asset such as the reputation of the final product that emerges from their production process. The upstream manager may prefer that the reputation be geared toward showcasing the durability of the inputs. The downstream manager may prefer that it emphasize novelty. Decisions must be made regarding the direction to emphasize. Both managers want the final product to succeed, and success largely depends on consumers' preferences, which are uncertain. Depending on who is making these decisions, one or both managers may have the incentive to try to change the other's beliefs by, say, alter the phrasing of certain questions that are asked in consumer focus groups.

Formally, assume that each manager can choose a level of "influence activities," denoted by $\lambda^i \in \Lambda$ at private cost $k(\lambda^i)$. The private cost represents the opportunity cost of time wasted manipulating the signal. As such, the costs of influence activities are increasing and convex. Throughout, I assume that the influence activities are chosen prior to the observation of the public signal and without any private knowledge of the state of the world, and they affect the conditional distribution of $\sigma$ given $s$. Further, I assume that this effect is linear. After the signal has been observed, 3

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3Throughout, I assume the two parties are "locked in" with each other, regardless of the allocation of control, and thus each directly care about both decisions. A richer model might allow for endogenous dependence between the two players. How this interacts with firm boundaries is an interesting question for future research.

4I show in Appendix B that the qualitative results of this model can also be generated as a separating equilibrium in a noisy signaling game. However, the multiplicity of equilibria in signaling games makes such an approach relatively unappealing.

5The assumption that $\lambda^A$ and $\lambda^B$ do not affect the conditional variance of $\sigma|s$ rules out the Milgrom and Roberts
the party(ies) with control of the decision right must immediately choose a decision. There is neither time nor opportunity for the two parties to get together and bargain over the decision to be made. Further, I assume that the parties cannot bargain over a signal-contingent decision rule ex ante.

![Figure 1: Timing](image)

The timing of the model (shown in figure 1 above) is as follows: (1) \(L\) and \(R\) bargain over a control structure \(g\) from a set \(G\), which I will describe soon; (2) \(L\) and \(R\) simultaneously choose (unobservable) influence activities \(\lambda^L, \lambda^R \in \Lambda \subset \mathbb{R}\) at cost \(k(\lambda^i)\), where \(k\) is convex and symmetric around zero, with \(k'(0) = k(0) = 0\); (3) \(i\) and \(j\) publicly observe the signal \(\sigma = s + \lambda^i + \lambda^j + \varepsilon\); (4) The manager with control chooses decision \(d \in \mathbb{R}\); (5) Payoffs are realized.

Throughout, assume that all random variables are normally distributed \((s \sim N(0, h^{-1}), \varepsilon \sim N(0, h_c^{-1}))\) and independent, and managers have quadratic costs of influence, \(k(\lambda^i) = \frac{1}{2}(\lambda^i)^2\) and gross payoffs of the following form

\[
U^i(s, d) = \sum_{\ell=1}^{2} \left[ -\frac{\alpha^i}{2} (d_{\ell} - s - \beta^i)^2 \right], \alpha^i > 0, \beta^i \in \mathbb{R}.
\]

Manager \(i\) prefers that \(d_1 = d_2 = s + \beta^i\), and hence the two managers disagree on their ideal decision conditional on the state of the world. The problem is not interesting if \(\#L = \#R\), so without loss of generality, assume \(\beta^L - \beta^R = \Delta > 0\). Two aspects of symmetry have been assumed here. First, the amount by which manager \(i\) cares about how close the decision is to his ideal decision is assumed to be the same across decisions. That is, the \(\alpha^i\) coefficient on the loss functions for both decisions

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(1988) idea that "when... underlying information is so complex that unscrambling is impossible, decision makers will have to rely on information they know is incomplete or inaccurate." I explore this idea in more detail in Appendix C.

\(5\) Ex post noncontractibility is a central feature of many recent papers in organizational economics. See footnote 1 for a list of several such papers.

\(7\) Appendix C explores related issues of optimal rule design.

\(8\) The prospect of influence-activity mitigation provides a force toward divided control. What is important in generating the trade-off in this model is that the same party who cares more about one decision also cares more
is the same. Secondly, the amount by which the two managers disagree about the ideal decision is equal across decisions. Throughout, assume that \( \alpha_L \geq \alpha_R \).

There are four possible allocations of control. Control can be unified and held by either manager \( L \) or manager \( R \) or it can be divided. If manager \( L \) has control of decision 1, and manager \( R \) has control of decision 2, control is said to be non-integrated. Otherwise, control is reverse non-integrated. Consistent with many theories of the firm (i.e. the "IO" view, Transaction Cost Economics, and Property Rights Theory), divided control will be referred to as non-integration (and will be denote by \( g = NI \)) and unified control as integration (\( g = I \)).

2.3 Equilibrium

Suppose manager \( i \) has control of a decision. Manager \( j \) cares about the decision to be taken and recognizes that this decision depends on \( i \)'s beliefs. Thus, manager \( j \) has a direct interest in what manager \( i \) believes and will do whatever is in his power to change \( i \)'s beliefs. But, as Cyert and March (1963) argue, "We cannot reasonably introduce the concept of communication bias without introducing its obvious corollary - 'interpretive adjustment.'" That is, manager \( i \) recognizes that manager \( j \) has the incentive to influence the signal, and he will correct for this in his beliefs. As in career-concerns/signal-jamming games, this "interpretive adjustment" does not eliminate the incentives to carry out influence activities, for if the decision maker expected no influence activities, then the influencer would have a strong incentive to engage in them. Thus, conditional on a control structure, \( g \), the solution concept is perfect Bayesian equilibrium, as in career-concerns/signal-jamming games. Denote manager \( i \)'s beliefs about the vector of influence activities by \( \hat{\lambda}(i) \).

**Definition 2.1** Given a control structure, \( g \), a **Perfect Bayesian Equilibrium** of the resulting game consists of choices of influence activities, \( \lambda^L \) and \( \lambda^R \), and a decision function \( d^g(\sigma; \hat{\lambda}) \), such that: (1) each component of \( d^g(\sigma; \hat{\lambda}) \) is chosen optimally by the manager who controls that decision under \( g \), given his beliefs about the state of the world, which depend on conjectures about

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\(^9\) Relaxing this does not qualitatively change any results. Allowing for different \( \Delta \)'s across decisions simply adjusts the weights that are placed on each decision in the governance structure choice period.

\(^10\) At this point, I do not consider the possibility of allocating control to a third party. Section 7 considers this possibility.

\(^11\) Though there are four potential allocations of control, only two will ever be optimal: unifying control with manager \( L \) or dividing control by giving decision 1 to \( L \) and decision 2 to \( R \). This allows us to avoid introducing additional notation for the remaining control structures: \( R \)-control and reverse non-integration.
the level of influence activities, \( s|\sigma, \hat{\lambda}(i) \); (2) influence activities are chosen optimally given the allocation of the decision right; and (3) beliefs are correct: \( \hat{\lambda}(i) = \lambda^* \).

Let us begin by solving for an equilibrium for an arbitrary control structure \( g \). Suppose manager \( i \) has control of decision \( \ell \) under governance structure \( g \). Let \( \lambda^* \) denote the equilibrium level of influence activities. Manager \( i \) will choose \( d_i^* \) to minimize his expected loss given his beliefs. Since he faces a quadratic loss function, his decision will be equal to his conditional expectation of the state of the world, given the signal and his equilibrium conjecture about influence activities, plus his bias term, \( \beta_i \). That is,

\[
d_i^g(\sigma; \lambda^*) = E_s[s|\sigma, \lambda^*] + \beta_i.
\]

The decision manager \( i \) chooses differs from the decision manager \( j \neq i \) would choose if he had the decision right for two reasons. First, \( \beta_i \neq \beta_j \), so for a given set of beliefs, manager \( i \) prefers a different level of \( d_\ell \) than manager \( j \) does. Secondly, it may be that, out of equilibrium, beliefs are incorrect. That is, manager \( i \) knows \( \lambda^i \) but only has a conjecture about \( \lambda^j \). The normal updating rule implies that the conditional expectation of the state of the world from the perspective of individual \( i \) is a convex combination of two estimators of the state of the world. The first is the prior mean, 0, and the second is a modified signal, \( \hat{s}(i) = \sigma - \hat{\lambda}^L(i) - \hat{\lambda}^R(i) \), which must of course satisfy \( \hat{\lambda}^i(i) = \lambda^i \). The weight that \( i \)'s preferred decision rule attaches to the signal is given by the signal-to-noise ratio, \( \varphi = \frac{h_s}{h_s + h_o} \). That is,

\[
E_s[s|\sigma, \hat{\lambda}(i)] = (1 - \varphi) \cdot 0 + \varphi \cdot \hat{s}(i).
\]

Given decision rules \( d_{\ell}^g(\sigma; \lambda^*) \) for \( \ell = 1, 2 \), we can now compute the equilibrium level of influence activities that each manager will engage in. Influence activities for manager \( j \) are more privately beneficial (out of equilibrium) the greater is the difference between the equilibrium decision rule and manager \( j \)'s decision rule, the more manager \( j \) cares about his loss from having a suboptimal decision rule, and the more weight the decision maker places on the manipulable signal. Manager \( j \)'s level of influence activities will solve

\[
k'(\lambda^j) = E_{s,e} \left[ \sum_{\ell=1}^2 -\alpha^j \left( d_{\ell}^g(\sigma; \lambda^*) - s - \beta_j \right) \frac{\partial d_{\ell}^g}{\partial \sigma} \frac{\partial \sigma}{\partial \lambda^j} \right] = N^{-j} \alpha^j \Delta \varphi, \tag{1}
\]
where $N^{-j}$ is the number of decisions that manager $j$ does not control under governance structure $g$. Further, since given any beliefs about $\lambda^i$, the unique optimal decision rule of manager $i$ is a pure strategy, and given that manager $i$ chooses a pure strategy decision rule, there is a unique value of $\lambda^i*$ satisfying (1). Thus, the pure strategy equilibrium characterized in this section is the unique equilibrium of this game.$^{12}$ These results are captured in the following proposition.

**Proposition 2.1** For a given control structure, there exists a unique Perfect Bayesian Equilibrium of the game that follows. Further, in that Perfect Bayesian Equilibrium, the levels of influence activities are given by a simple formula

$$|\lambda^j*| = |\Delta| N^{-j} \alpha^j \varphi,$$

where $N^{-j}$ is the number of decisions player $j$ does not control and $\varphi = \frac{h_e}{h_e + h_s}$ is the signal-to-noise ratio.

All else equal, manager $j$ will choose a higher level of influence activities the more disagreement ($\Delta$) there is, the more he cares about the decision ($\alpha^j$), and the more informative the signal is ($\varphi$). This last comparative static can be decomposed further. $\varphi$ is high the larger is $h_e$ (i.e. when the signal is more precise) and the smaller is $h$ (i.e. when there is more ex ante uncertainty). The rest of this paper will concern itself with alternative methods of mitigating these influence activities.

### 2.4 The Coasian Program

Property Rights Theory (Grossman and Hart 1986, Hart and Moore 1990, and Hart 1995, hereafter PRT) is often lauded for its methodological insistence on specifying a contractual environment that is common across prospective allocations of residual rights of control. In doing so, it is able to provide a unified description of the costs and benefits of integration without resorting to alternative explanations for the costs of one governance structure and the benefits of another. However, PRT assumes ex post efficiency (via Coasian bargaining), instead focusing on how the allocation of control affects managers’ bargaining positions and hence the sensitivity of their expected split of the surplus to their ex ante investments. While the approach has proven fruitful in a variety of

$^{12}$If higher levels of influence activities increases the precision of the signal, there could be multiple pure strategy equilibria. In this case, if the decision-maker believes that high (low) levels of influence activities have been chosen, he will place much (little) weight on the signal. This in turn will induce the other manager to choose high (low) levels of influence activities. This is analogous to the multiplicity argument in Dewatripont, Jewitt, and Tirole (1999).
fields, it has had difficulty confronting the idea that ex post inefficiencies are equally (and perhaps more) important determinants of firm boundaries, and thus as Hart (2008) points out, "in order to make progress on the Coasian agenda, we must move away from Coase (1960) and back in the direction of Coase (1937). We need to bring back haggling costs!" But a satisfactory formalization of Williamson (1971)'s appealing argument that non-integration may produce "haggling," so that decision-making by "fiat" under integration may be more efficient has been difficult. This section will lay the framework for analyzing a version of the "haggling" versus "fiat" trade-off, but a more complete analysis is deferred until section 6.

From the perspective of period 1, before $\lambda^L$ and $\lambda^R$ are chosen, the two managers bargain over a control structure, $g^*$, correctly anticipating its effects on equilibrium influence activities (which are unique, conditional on $g$) as well as on the equilibrium decision rules. I assume that the managers can freely make transfers at this stage, so that the governance structure $g^*$ will be the solution to the following Coasian program

$$
\max_{g \in G} \{ W(g) \} = \max_{g \in G} \left\{ E_{s,e} \left[ \sum_{i \in \{L,R\}} U^i(s, d^g(\sigma^i; \lambda^*)) \right] - \sum_{i \in \{L,R\}} k(\lambda^{*i}) \right\},
$$

The quadratic loss structure of the problem leads to a mean-variance decomposition of the first term and thus a simple characterization of $W^g$ as follows

$$
W^g = - (ADAP + ALIGN(g) + INFL(g)).
$$

That is, ex ante expected welfare can be decomposed into the sum of three costs: (1) an adaptation cost that arises from basing decisions on a noisy signal rather than directly on the state of the world; (2) an alignment cost that is due to the fact that for each decision, one manager will not be able to implement his ideal decision rule; and (3) an influence-cost component, which can be interpreted as "haggling costs." The exact expressions for these terms are derived in proposition 7 in the appendix.

The $ADAP$ term does not depend on the control structure, so $g$ is chosen to minimize the sum of $ALIGN(g)$ and $INFL(g)$. Two polar cases help identify the relevant trade-off for this

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13 Masten (1986) develops a model based on Tullock (1980) to highlight the costs of divided control but it is silent on the costs of unified control. Recent work by Hart and Holmstrom (2010) adapts several behavioral elements from Hart and Moore (2008) and Hart (2009) to argue that different control structures create different feelings of entitlement and hence different risks that parties will feel "aggrieved" and thus "shade" by supplying only perfunctory effort on noncontractible tasks.
model. First, let us look at a "pure adaptation" model in which \( k(\lambda) = \infty \) for all \( \lambda \neq 0 \), so that influence activities are impossible by assumption. To minimize alignment costs, the managers want to allocate control of both decisions to the manager who has more to lose from not having his ideal decision rule implemented. Since \( \alpha^L \geq \alpha^R \), the optimal control structure involves unifying control with manager \( L \) (\( g^* = I \)).

Next, let us look at a "pure influence" model in which \( k(\lambda) = \frac{1}{2} \lambda^2 \) and \( \alpha^L = \alpha^R \). Under any control structure, each decision will be \( \Delta \) away from one of the manager's ideal decisions. Since \( \alpha^L = \alpha^R \), both managers care equally about the resulting loss. That is, \( ALIGN(g) \) does not depend on \( g \) and thus the control structure will be chosen to minimize influence costs. Here, the managers will optimally choose to divide control. To see why, notice that by proposition 1, the total amount of time wasted on influence activities (\( \sum_j x_j^2 \)) is independent of \( g \). Since influence costs are convex, \( INF\!L(g) \) is minimized under divided control.\(^{14}\) That is, \( g^* = NI \) is optimal.

In the richer model in which \( \Lambda = \mathbb{R} \) and \( \alpha^L > \alpha^R \), these opposing forces lead to a non-trivial trade off, provided \( \alpha^L \) is not too large relative to \( \alpha^R \). There is a critical value of the signal-to-noise ratio \( \varphi^* \) such that if \( \varphi < \varphi^* \), control will optimally be concentrated and if \( \varphi > \varphi^* \), control will optimally be divided. This leads to the following proposition.

**Proposition 2.2** Assume \( \alpha^R < \alpha^L < \sqrt{3} \alpha^R \). Divided control is optimal if and only if

\[
\varphi^2 \geq \frac{\alpha^L - \alpha^R}{3 (\alpha^R)^2 - (\alpha^L)^2}.
\]

The condition that the manager \( L \) cares more about the decision than manager \( R \) but not too much more (i.e. \( \alpha^L < \sqrt{3} \alpha^R \)) is best understood by considering the case in which manager \( R \) is essentially indifferent about both decisions but manager \( L \) is not. Then it is clear that control should be concentrated with manager \( L \). Also, note that the level of disagreement, \( \Delta \), does not matter for the optimal control structure. The reason for this is that with quadratic preferences and quadratic influence costs, both \( ALIGN \) and \( INF\!L \) are proportional to \( \Delta^2 \) and thus differences in \( \Delta^2 \) do not affect the relative trade-off between minimizing alignment costs and influence costs.\(^{15}\)

When are influence costs large relative to alignment costs? Condition (2) implies that whenever the signal-to-noise ratio is large, the costs of integration exceed the costs of non-integration. Further

\(^{14}\)More generally, with an arbitrary increasing and convex cost function \( k \), we need that \( k''/k' > k'''/k'' \). This is satisfied for \( k(\lambda) = c\lambda^k \) for all \( \xi \).

\(^{15}\)More generally, an increase in \( \Delta \) makes integration relatively less appealing if \( k'' > 0 \) and makes integration relatively more appealing if \( k'' < 0 \).
unpacking $\varphi$ (which is equal to $\frac{h_{x}}{h_{x} + h_{e}}$), non-integration is preferred whenever the level of ex ante uncertainty is high (i.e. $h$ small) or the signal is very informative (i.e. $h_{e}$ large) and thus will be relied heavily upon. Influence-activity mitigation therefore provides a basis for a theory of the optimal control structure.

This section began by arguing that this model would provide a framework for thinking about the "haggling" versus "fiat" trade-off. In what sense is this the case? Interpreting the opportunity costs of influence activities as the costs of "haggling," this model generates the prediction that such costs should be greater under integration than under non-integration. Put differently, the model in this section suggests that the cost of "fiat" (interpreted here as unified control) is increased "haggling," and thus the current model does not deliver the Williamson (1971) trade-off. This will be resolved in section 6, which allows for integration to be coupled with organizational practices aimed at reducing "haggling."

### 2.5 Rigid Organizational Practices

Recall that under a control structure in which party $i$ controls $N^{-j}$ decision rights, manager $j$'s equilibrium influence activities are $|\lambda| = |\Delta| N^{-j} \alpha^j \varphi$ (Proposition 2.1). The previous section emphasized the scope for using $N^{-j}$ as an instrument for mitigating influence activities (Proposition 2.2). However, as Milgrom and Roberts (1988, 1992) highlight, there are many other methods available for mitigating influence activities. These include rigid decision-making rules, flat incentive schemes, defensive information acquisition, closed-door policies, etc. While the adoption of many of these organizational practices would otherwise seem inefficient, they begin to make sense when one considers the effect they may have on the incentives for influence activities.

In the context of this model, any institution that reduces $\alpha$, $\Delta$, or $\varphi$ will reduce equilibrium influence activities. For example, adopting closed-door policies in which decision makers are effectively insulated from relevant information could correspond to a decrease in $h_{e}$ (and hence in $\varphi$). Such a policy will reduce the private return to influence activities and will thus discourage them. This would not be costless, since it would also effectively reduce the amount of information the decision maker has available to make a decision. Similarly, putting into place incentive schemes that effectively make a manager indifferent about the decision being taken (which could correspond

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16 Because influence activities do not affect the conditional variance of $\sigma$ given $s$, they have no equilibrium effect on the quality of ex post decisions. The model thus focuses only on the opportunity costs of influence activities. A richer model could allow for both opportunity costs and degradation in decision quality.
to a decrease in that manager's $\alpha$) would have such an effect as well. The costs of low-powered incentive schemes, of course, is diminished motivation for putting in (here unmodeled) effort. The decision makers could hire outside consultants to acquire information about the state of the world. This "defensive information acquisition" would lead to an increase in $h$, a reduction in $\varphi$, and thus a reduced incentive to influence. However, hiring an outsider who, by definition, is not an insider and thus not privy to the relevant information, is costly.

For the purposes of the present section, I analyze a fairly blunt instrument with costs that are endogenous to the model. Assume party $L$ has both decision rights. The model is as above, except that in the first period, instead of bargaining over the control structure, $L$ and $R$ bargain over whether or not to carry out their relationship under an open or closed door policy. They may trim out personnel whose job it is to gather relevant information, they may purposefully load up their schedules and keep themselves too busy to pay attention to everything that crosses their desks, or they may limit the frequency and length of meetings with each other. Let $\theta \in G = \{0,1\}$ denote this choice. Under an open door policy (denoted by $\theta = 0$), the rest of the game proceeds as usual. Under a closed door policy (denoted by $\theta = 1$), no public signal is realized in period $3.17$ Let $W(\theta)$ denote the expected ex ante equilibrium welfare under organizational practice $\theta$. The Coasian program is

$$\max_{\theta \in G} \{W(\theta)\}.$$ 

If no public signal is realized, neither manager will have the incentive to exert any influence over it, and thus $\lambda^L = \lambda^R = 0$. This is potentially worthwhile if manager $R$ would otherwise have a strong incentive to influence the signal (i.e. if $\varphi$ is large). Since there is no additional information on which to base his decisions, $L$ will set both decisions equal to the prior mean. If the prior is very imprecise (i.e. $h$ is small), this is potentially very costly, but if there is already a wealth of information (i.e. $h$ is large) about the decision to be made, then it might not be very costly to have a closed door policy. This is captured in the following proposition.

**Proposition 2.3** In this model, when control is unified, a closed door policy ($\theta = 1$) is preferred to an open door policy ($\theta = 0$) whenever $\varphi h > \Phi(\Delta^2, \alpha^L, \alpha^R)$, where $\Phi$ is increasing in $\alpha^L$ and decreasing in $\alpha^R$ and $\Delta^2$.

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17I can instead allow for a more convex set of policies. For example, if $\varphi$ denotes the signal-to-noise ratio, the players could bargain over a level of "noise" they could put into the signal, which reduces $h_\varphi$ up to the point where the effective signal-to-noise ratio is given by $(1 - \theta) \varphi$. This can be interpreted as shutting off certain lines of communication. The rest of the analysis would proceed similarly. Proposition 2.7 in Appendix A analyzes this case.
The logic of influence-activity mitigation can help shed light on why certain rigid organizational practices persist. A recent series of papers starting with Bloom and Van Reenen (2007) documents substantial dispersion in management practices across firms, and in particular, highlights the prevalence of firms with puzzling ("bad") management practices. They conduct a survey inquiring about eighteen specific management practices of individual manufacturing plants (e.g. about whether or not the firm adopts continuous improvement initiatives, the criteria the firm uses for promotions). Each response is scored on a 1 – 5 scale, with 1 being considered a "bad" management practice and a 5 being considered "good". They construct a firm's management score by taking a normalized average of the scores for each individual practice and find that firms with higher management scores perform better (have higher sales, higher profitability, are less likely to exit, and have greater sales growth) than firms with lower management scores.

The negative correlation between "bad" management practices and firm performance is consistent with selection, as the following figure illustrates.

![Figure 2: Endogenous Practice Selection](image)

A firm operating in an environment with greater levels of disagreement (i.e. with a higher $\Delta$) will, all else equal, perform worse than a firm with a lower $\Delta$. Further, such a firm will be plagued by greater influence activities (since $\mathcal{N}$ is increasing in $\Delta$) and thus will find that adopting a closed-door policy is relatively more appealing. There will be some cutoff value $\Delta^*$ such that
firms with $\Delta < \Delta^*$ will have open door policies and better performance and firms with $\Delta > \Delta^*$ will have closed door policies and worse performance. Thus, a simple selection story along these lines could account for a negative correlation between closed-door policies ("bad" management practices) and firm performance. Further, since firms choose their management practices optimally, any outside intervention resulting in a change in management practices would lead to a decrease in firm efficiency. In particular, an intervention aimed at altering management practices for poorly performing firms would lead to a decrease in the performance of such firms.\(^{18}\)

### 2.6 Practices and Control

In analyzing the effects of a change in the allocation of control, further differences between control structures (beyond the identity of the party(ies) with the control) should be derived, not assumed. Such an approach is substantive, not merely aesthetic. For example, if integration is viewed as a concentration of control bundled with inefficient bureaucracy, this naturally begs the question of why can we not concentrate control without the concomitant inefficient bureaucracy, perhaps through contractual allocation of control rights? In this section, I will show that rigid organizational practices may arise optimally in response to concentrated control without regards to the methods used to concentrate control (i.e. in my model, formal integration and "contractual integration" are the same). For continued simplicity, I will focus on the stark instrument of closed/open door policies. The model is similar to the previous section, except now $L$ and $R$ bargain over the control structure in addition to the (closed/open door policy). That is, in the first period, $L$ and $R$ bargain over $(g, \theta) \in G = \{I, NI\} \times \{0, 1\}$. The rest of the analysis proceeds as above.

In order to draw a parallel to the "haggling" versus "fiat" argument of Williamson (1971), I first introduce some terminology. A choice of $g$ is referred to as a control structure, and a choice of $\theta$ is referred to as an organizational practice. A governance structure is the joint choice of a pair $(g, \theta)$, as it forms a complete description of how the transaction is to be governed. Only three governance structures will shown to be chosen in equilibrium: $(I, 0)$, $(I, 1)$, and $(NI, 0)$. These are referred, respectively, as "directed transaction," "hierarchy," and "market." Markets are characterized by divided control and flexible organizational practices. The defining feature of hierarchy is that

\(^{18}\)This, of course, assumes that management practices are chosen optimally. To the extent that certain practices are not adopted due to managerial unawareness or mistakes, such interventions could potentially improve the performance of firms. Bloom et al. (2011) conducts such an intervention and shows that improving management practices leads to improved performance in Indian manufacturing firms.
decision making is carried out by fiat, in the following two senses: (1) all relevant decisions are made by a single decision maker (control is unified) and (2) the decisions made are somewhat arbitrary, in that they are made without regard to changes in the environment (rigid organizational practices are adopted).

Under either control structure, setting \( \theta = 1 \) eliminates the incentive for (and hence the presence of) influence activities. Given that the costs of influence activities is zero when \( \theta = 1 \) for both \( g = I \) and \( g = NI \), it is clear that \( g = I \) will be preferred whenever \( \theta = 1 \). Closed-door policies are thus inconsistent with non-integration. Fixing \( \theta = 0 \), Proposition 2 implies that there will be some \( \hat{\varphi} \) such that non-integration is preferred if and only if \( \varphi > \hat{\varphi} \). Let \( W(g, \theta) \) denote the expected equilibrium welfare under decision right allocation \( g \) and organizational practice \( \theta \). It can be shown that

\[
W(g, \theta) = -(ADAP(\theta) + ALIGN(g) + INFL(g, \theta)),
\]

where the exact expressions for these three components are given in Appendix A. The Coasian program is therefore

\[
\max_{(g, \theta) \in G} \{W(g, \theta)\}.
\]

It is worth noting that the only term that depends both on the control structure and the organizational practices is \( INFL(g, \theta) \). Define \( I > NI \). The intuition described above suggests that \( INFL(g, \theta) \) exhibits decreasing differences in \( g \) and \( \theta \), and hence \( W(g, \theta) \) exhibits increasing differences in \( g \) and \( \theta \). Let \( \chi \) denote a vector of parameters of the model. The complementarity between \( g \) and \( \theta \) gives us the following proposition.

**Proposition 2.4** Let \( \alpha^R < \alpha^L < \sqrt{3} \alpha^R \). Then \( W(g, \theta) \) exhibits increasing differences in \( g \) and \( \theta \). Further, \( \min_\chi \theta^*(I, \chi) \geq \max_\chi \theta^*(NI, \chi) \).

This implies the empirical proposition that transactions within firms are more rule-driven and rigid than transactions carried out in the market, which has been discussed by Williamson, "Interorganizational conflict can be settled by fiat only rarely, if at all... intraorganizational settlements by fiat are common... ." (1971, emphasis in the original) The following figure results from a full

\[
^{19} \text{In fact, this holds even if } \theta \text{ is a continuous variable between } 0 \text{ and } 1, \text{ where a choice of } \theta \text{ affects the noise of the signal such that the signal-to-noise ratio becomes } (1 - \theta) \frac{b \chi}{\kappa + \kappa_e}.
\]
There are three boundaries of note in this figure. I refer to the vertical boundary between "Directed Transaction" and "Market" as the "Meyer, Milgrom, and Roberts boundary": a firm rife with politics should perhaps disintegrate.²⁰ The diagonal boundary between "Directed Transaction" and "Hierarchy" is the "Milgrom and Roberts boundary": rigid decision-making rules should sometimes be adopted within firms. The presence of these two boundaries highlights the idea that non-integration and bureaucratization are substitute mechanisms: sometimes a firm will prefer to control influence activities with the former and sometimes with the latter.

The third boundary is the "markets versus hierarchies boundary" (Williamson 1975). Sometimes, the market mechanism, with its high-powered incentives and open lines of communication invite such high levels of influence activities ("haggling") that it should be superseded by a hierarchy (unified control) coupled with rigid organizational practices. This becomes increasingly true the greater is the level of ex post disagreement between the parties (Δ) and the greater is the level of ex ante uncertainty (as measured by a small value of h or a large value of ϕ). The latter is consistent with many of the classical empirical papers in support of Transaction Cost Economics (Masten

²⁰This is consistent with Forbes and Lederman (2009)'s argument that the principal obstacle to integration between major airlines and regional carriers is that integration invites the regional carrier's work force (which is comparatively less well-compensated than the major's) to lobby for higher pay.

Figure 3: Optimal Governance Structures
(1984); Masten, Meehan, and Snyder (1991); Lieberman (1991); Hanson (1995)), where measures of the uncertainty or complexity of the environment a firm operates in serves as the empirical proxy for the level of contractual incompleteness, which is the actual object of interest in TCE.

Figure 4 below depicts the relationship between the level of uncertainty surrounding a transaction and the potential "haggling" costs under each of the three potential governance structures. The bolded segments depict the actual "haggling" costs under the optimal governance structure. In section 4, I argued that the cost of unified control was an increase in "haggling." Holding organizational practices fixed, this is indeed the case, as shown by the difference between the $Infl(NI, \theta = 0)$ and $Infl(I, \theta = 0)$ lines. However, changing organizational practices in addition to the control structure completely eliminates "haggling," as shown by the difference between the $Infl(NI, \theta = 0)$ and $Infl(I, \theta = 1)$ lines. A firm with $\varphi > \varphi^{**}$ that decides to integrate will adopt rigid organizational practices, opting for unresponsive decision making by "fiat" rather than responsive decision making and "haggling."

Whereas Williamson views "bureaucratic costs of hierarchy... [as] a deterrent to integration," this model views the bureaucratic costs of hierarchy as the lesser of two evils, the alternative to which is high levels of influence activities. This view cautions against the popular advice that one should "bring the market inside the firm" as a way of strengthening incentives - doing so will often create more problems than it solves. Rather, this model underscores the importance of aligning
these instruments with each other and with the environment the organization operates in.

2.7 Internal Structure of a Multidivisional Firm

I interpret the model in the previous sections as a model of the boundaries of the firm. However, the same logic that determines firm boundaries and organizational practices in the previous sections can also be used to explain the internal structure of firms. Should control within an organization reside with headquarters or with division managers? Should the organization adopt rigid or flexible practices? The model in this section is related to a pair of recent papers (Alonso, Dessein, and Matouschek (2008), hereafter ADM, and Rantakari (2008)) exploring the performance implications of various control structures on a firm's ability to adapt to local circumstances and to coordinate decisions across divisions. These papers emphasize the role of strategic communication in transmitting soft information within the organization, whereas this section will instead emphasize the role of influence activities in affecting the transmission of hard information. These two complementary approaches differ substantially in their implications.

There are two division managers and a headquarters, denoted by \( L, R, \) and \( HQ \) respectively, and one decision to be made for each division: \( d_L \) and \( d_R \). Each decision is ideally tailored to the local state of the division, \( s_i \sim N(E[s_i], h^{-1}), i \in \{L, R\} \), where \( E[s_L] - E[s_R] = \Delta > 0 \), but information directly relevant to the state of the world is unobserved. Instead, an informative (but manipulable) signal relevant to each division is commonly observed. This signal is linear in the division’s local state, \( s_i \), the level of influence activities the manager of that division engages in, \( \lambda_i \), and a noise term, \( \epsilon_i \sim N(0, h_{\epsilon}^{-1}) \). That is, \( \sigma_i = s_i + \lambda_i + \epsilon_i \) for \( i \in \{L, R\} \). However, coordination across divisions is also important. There are two possible control structures: control can either be unified and held by the headquarters \( (g = cent) \) or divided and held by the respective division managers \( (g = dec) \). Additionally, as in the previous section, the organization can adopt a closed door policy \( (\theta = 1) \) or an open door policy \( (\theta = 0) \). Under the closed door policy, no signals are realized.

The timing of the model is as follows: (1) \( L \) and \( R \) bargain over a governance structure \( (g, \theta) \in \)
\{\text{cent}, \text{dec}\} \times \{0, 1\};^2^3 (2) \text{ L and R simultaneously choose (unobservable) influence activities } \lambda^L, \lambda^R \in \Lambda \subset \mathbb{R} \text{ at cost } k (\lambda^i) = \frac{1}{2} (\lambda^i)^2; (3) \text{ L, R, and HQ commonly observe two signals } \sigma_i = s_i + \lambda^i + \epsilon_i, i = L, R; (4) \text{ the party(ies) with control choose decisions; (5) payoffs are realized. The key differences between the present model and the model in the previous section are (a) the presence of division-specific signals, (b) the prospect of allocating control to a third party, and (c) the payoff structure, to which I will now turn. Division manager } \text{i has gross payoffs of the following form}

\[ U^i (s_i, s_j, d_i, d_j) = \frac{1}{2} (d_i - s_i)^2 - \frac{r}{2} (d_i - d_j)^2, \]

and the headquarters has a gross payoff of \( U^{HQ} = U^L + U^R. \)

Both managers desire to match their decisions to their local states and the decisions of the other division. The key source of friction is that, on average, manager L wants a higher decision than manager R does, and he only partially internalizes the coordination losses this imposes. The headquarters fully internalizes the coordination externalities, and absent influence activities, it would always be optimal to centralize control with the headquarters.\(^2^4\)

As before, given a governance structure, the solution concept will be pure strategy perfect Bayesian equilibrium. Under any governance structure, the equilibrium decision rule \( d^\theta_i \) is a convex combination of estimators \( s_i \) and \( s_j \). When \( \theta = 0 \), the equilibrium decision rules are given by

\[
d^\theta_i \big| \theta = 0 = C^\theta (r) \left( (1 - \varphi) E[s_i] + \varphi \left( \sigma_i - \hat{\lambda}^i (g) \right) \right) \\
+ (1 - C^\theta (r)) \left( (1 - \varphi) E[s_j] + \varphi \left( \sigma_j - \hat{\lambda}^j (g) \right) \right),
\]

and when \( \theta = 1 \), no public signals are available, and thus the equilibrium decision rules are given by

\[
d^\theta_i \big| \theta = 1 = C^\theta (r) E[s_i] + (1 - C^\theta (r)) E[s_j].
\]

The control structure affects the equilibrium decision rules in two ways. First, it can be shown that \( 1 > C^{dec} (r) > C^{cent} (r) > \frac{1}{2} \), and both are decreasing in \( r \). That is, under decentralized decision making, decision rules exhibit more of an own-division bias. This is because under decentralization, the division managers do not fully internalize the coordination externalities they

\(^2^3\)Alternatively, if one permits the headquarters to take into account the anticipated private costs of influence activities by the managers (say because the headquarters has to attract them to work for the firm), one can think of the headquarters as unilaterally choosing a governance structure.

\(^2^4\)The coordination motive for unified control was absent in the previous model, since the ideal decisions were independent from each other.
impose on the other division. Secondly, the control structure determines who makes decisions and hence determines whose information about influence activities is used to make decisions. Under centralization, the headquarters makes decisions based on conjectures about influence activities. Under decentralization, division managers know their own influence activities but not the influence activities of the other division manager.

Given a governance structure \((g, \theta)\) and a decision rule \(d^{g, \theta}\), in period 2, player \(i\) chooses his level of influence activities to solve

\[
k'(\lambda^i) = E_s \left[ (s_i - d_i^{g, \theta}) \frac{\partial d_i^{g, \theta}}{\partial \sigma_i} \right] + \tau E_s \left[ (d_L^{g} - d_R^{g}) \frac{\partial (d_R^{g} - d_L^{g})}{\partial \sigma_i} \right].
\]

Under decentralization, manager \(L\) influences the signal upward in an attempt to convince manager \(R\) that he will take a higher decision than he actually will. Out of equilibrium, this will induce \(R\) to take a higher decision in order to coordinate with him, which is preferable on average for \(L\). Manager \(R\) will influence the signal downward for the same reason. I refer to this as the "influence for coordination" motive. Because of the additive signal structure, these attempts at manipulating each others’ decisions will have no effect on the equilibrium decision, however. Under centralization, the motivation for influence activities is different. Since the headquarters always chooses to coordinate decisions more than is privately optimal for each manager, each manager will attempt to influence the signal in order to bias the decision in their division’s direction. I refer to this as the "influence for adaptation" motive. When \(\theta = 1\), neither manager has any incentive to influence their division’s signal. Equilibrium influence activities under each governance structure are then

\[
\begin{align*}
\lambda^{L*} & = \frac{1}{2} \frac{r}{1 + 2r} \Delta \varphi (1 - \theta) - \frac{r}{1 + 4r} \Delta \varphi (1 - \theta) \\
\lambda^{R*} & = \frac{1}{2} \frac{r}{1 + 2r} \Delta \varphi (1 - \theta) - \frac{r}{1 + 4r} \Delta \varphi (1 - \theta)
\end{align*}
\]

Under an open door policy, the level of influence activities is greater under centralization than under decentralization, leading to a non-trivial trade-off. Centralization leads to more coordinated decisions, but it does so at the cost of greater influence activities. This is described in the following proposition (which is proven in the appendix).

**Proposition 2.5** Fix \(\theta = 0\). Then decentralization is preferred to centralization whenever \((\varphi q (r) - \frac{1}{\varphi}) h \Delta^2 > 2\), where \(q (r), q' (r) > 0\). As \(r, h, \Delta^2\) increase, decentralization becomes relatively more ap-
Decentralization reduces the costs of influence activities. As \( h_e, \Delta^2, \) and \( r \) increase, influence activities become relatively more appealing and hence so does a control structure that mitigates them. The last of these is perhaps surprising, given the widespread intuition that as coordination becomes more important, an organization should become more centralized. This is a feature of ADM and Rantakari. In the present model, an increase in \( r \) increases the "influence for adaptation" motive significantly, since under centralization, the increase in \( r \) moves each division’s decision farther away from its manager’s ideal. In ADM and Rantakari, however, an increase in \( r \) improves the quality of communication of soft information, since it actually increases the alignment of preferences within the firm.

Closed door policies eliminate the incentives for influence activities. Absent influence activities, centralization is always preferred. Thus, as in the model of firm boundaries, closed door policies are inconsistent with decentralization. Conditional on centralization, when should an organization opt for closed door policies?

**Proposition 2.6** Fix \( g = \text{cent} \). If \( \varphi > \tilde{\varphi}(r) \), then \( \theta = 1 \) is preferred to \( \theta = 0 \) whenever \( H(r, \Delta^2, \varphi, h) > \hat{H}(r) \), where \( H \) is increasing in \( \Delta^2, \varphi, \) and \( h \).

I derive the exact expressions of \( \tilde{\varphi}, H, \) and \( \hat{H} \) in the appendix. The intuition for this proposition is relatively similar. An increase in \( \Delta^2 \) and \( \varphi \) increases influence activities and thus increases the benefits of adopting organizational practices aimed at mitigating them.

Finally, as in the previous section, organizational practices and control structures interact: the returns to rigid organizational practices are greater when control is concentrated (and thus influence activities would otherwise be high). If we refer to \((\text{cent}, 0), (\text{cent}, 1), \) and \((\text{dec}, 0)\) as "Centralization," "Centralized Bureaucracy," and "Decentralization," respectively, this can be seen
Thus, rigid organizational practices are expected to positively covary with centralized decision making. Further, for $\Delta$ and $\phi$ sufficiently large (so that Centralization is not optimal), a further increase in $\Delta$ simultaneously increases the relative attractiveness of Centralized Bureaucracy and decreases the performance of the firm across all governance structures. This leads to the possibility of a selection-based, rather than causal, story for a positive correlation between decentralization and firm performance.

Bloom, Sadun, and Van Reenen (2011) conduct a survey to collect data on measures of decentralization of key decisions within firms (asking questions such as "Who has the authority to hire new full-time workers?" and "How much capital investment can a plant manager make without authorization from headquarters?"). In addition, for each of the surveyed firms, they also collect data on management practices (as in Bloom and Van Reenen (2007)). Among their findings are a positive correlation between their measure of decentralization and their measure of management practices (table 4, column 2) and a positive correlation between decentralization and firm performance (table 5), both of which are consistent with the model in this section.
2.8 Conclusion

This paper develops a unified theory of the costs and benefits of integration based on the logic of influence-cost mitigation. Managers waste time persuading decision makers, and firm boundaries and organizational practices are determined on account of this. It provides an interpretation of haggling costs—the opportunity cost of time spent attempting to persuade decision makers—and unpacks what it means to make decisions by fiat into (1) unified control and (2) decision making that is detached from relevant information. In doing so, it shows that decision making by fiat in the first sense exacerbates haggling costs: if one of two parties specializes in making decisions, the other will specialize in trying to convince him that they should be made in one way rather than another. On the other hand, fiat in the second sense implies that influence activities fall on deaf ears, thus eliminating haggling altogether.

When does the cost of decision making by fiat involve only an increase in haggling (so that there is no trade-off per se between haggling and fiat), and when does it involve unresponsive decisions (so that one must choose between haggling and unresponsive decision making)? I show that when transactions are sufficiently complicated (i.e. there is much ex ante uncertainty, disagreement, or parties have a large stake in the decisions), optimal decision making by fiat always involves the latter. Complementarities between unified control and rigid organizational practices thus lead to a notion of "Williamsonian integration" in which there is a trade-off between rigid decision making by fiat and haggling costs. The extent of the costs of each of these components is related to the level of uncertainty in the environment and hence this explanation is consistent with the Williamson (1973) idea that "substantially the same factors that are ultimately responsible for market failures also explain failures of internal organization."

Influence-activity mitigation also provides the foundations for a theory of the internal organization of a multidivisional firm. Centralized control structures aid in coordinating decisions but can lead division managers to lobby the headquarters to bias decisions in favor of their division. In order to reduce influence activities, a firm can either decentralize decision making or adopt rigid organizational practices ("bureaucratize"). This approach justifies a wide range of policies present in firms, and its empirical implications are consistent with correlations found in recent papers by Bloom and Van Reenen (2007) and Bloom, Sadun, and Van Reenen (2011). These include the positive correlations between good management practices and firm performance, good management practices and decentralization, and decentralization and performance.
2.9 Appendix A: Omitted Proofs and Computations

Proposition 2.7 In the full model of section 6, ex ante expected equilibrium welfare as a function of the allocation of decision rights \( g \in \{I, NI\} \), the organizational practices \( \theta \in [0,1] \), and a vector \( \chi \) of parameters, is given by

\[
W(g, \theta, \chi) = -(ADAP(\theta, \chi) + ALIGN(g, \chi) + INFL(g, \theta, \chi)).
\]

Further, these three components can be expressed as

\[
ADAP(\theta, \chi) = \frac{\alpha^L + \alpha^R}{h + h_\epsilon} + \theta \varphi \frac{\alpha^L + \alpha^R}{h}
\]

\[
ALIGN(g, \chi) = \begin{cases} 
\frac{\alpha^R \Delta^2}{2} & g = I \\
\frac{\alpha^L + \alpha^R}{2} \Delta^2 & g = NI
\end{cases}
\]

\[
INFL(g, \theta, \chi) = \begin{cases} 
(1 - \theta)^2 \left( \frac{\alpha^R}{2} \right)^2 \varphi^2 & g = I \\
(1 - \theta)^2 \frac{1}{2} \left( (\alpha^R)^2 + (\alpha^L)^2 \right) \Delta^2 \varphi^2 & g = NI
\end{cases}
\]

Proof. Suppose the managers have agreed upon a control structure \( g \) and a level of organizational practices \( \theta \in [0,1] \). The variance of the signal is then given by \( \tilde{h}_\epsilon = (1 - \theta) \frac{h + h_\epsilon}{h + \theta h_\epsilon} \), which reduces the signal-to-noise ratio in the updating formula to \((1 - \theta) \varphi \). Condition (1) then implies that

\[
\lambda^* = (1 - \theta) N^{-\varphi} \alpha^L \Delta \varphi,
\]

so that \( INFL(g, \theta, \chi) = \sum_{j \in \{L,R\}} \frac{1}{2} \left( \lambda^* \right)^2 \), which is equal to the expression given in the statement of the proposition. We know from section 3 that

\[
d^*_{ \ell } (\sigma; \lambda^*) = E_s[s|\sigma, \lambda^*] + \beta^i = (1 - \theta) \varphi(s + \tilde{\varepsilon}) + \beta^i,
\]

where \( \tilde{\varepsilon} \sim N\left(0, \tilde{h}_\epsilon \right) \). Substituting this into the definition of \( W(g, \theta, \chi) \) gives us

\[
W(g, \theta, \chi) = - \sum_{i \in \{L,R\}} \alpha^i \frac{1}{2} E_{s,\ell} \left[ (d^*_{ \ell } (\sigma; \lambda^*) - s - \beta^i)^2 \right] - INFL(g, \theta, \chi).
\]

The bracketed term can be decomposed into sum of the a variance and a bias term. Since the for decision \( \ell \) is 0 if \( i \) controls \( \ell \) under \( g \), the bias term is equal to \( ALIGN(g, \chi) \) given above. The
variance term is given by

\[
ADAP(\theta, \chi) = \sum_{l=1}^{2} \sum_{i \in \{L, R\}} \frac{\alpha^i}{2} Var\left(d_{i}^{*g}(\sigma; \lambda^*) - s\right)
\]

\[
= (\alpha^L + \alpha^R) Var\left(d_{i}^{*g}(\sigma; \lambda^*) - s\right) = \frac{\alpha^L + \alpha^R}{h + \epsilon} \left(1 + \theta \frac{\epsilon}{h}\right),
\]

which is the desired result. \(\blacksquare\)

**Proposition 2.8** In this model, when control is unified, a closed door policy \((\theta = 1)\) is preferred to an open door policy \((\theta = 0)\) whenever \(\varphi h > \Phi(\Delta^2, \alpha^L, \alpha^R)\), where \(\Phi(\Delta^2, \alpha^L, \alpha^R)\) is increasing in \(\alpha^L\) and decreasing in \(\alpha^R\) and \(\Delta^2\).

**Proof.** Applying proposition 7, \(W(I, 1, \chi) > W(I, 0, \chi)\) whenever

\[
\varphi h > \frac{1}{2} \frac{(\alpha^L + \alpha^R)}{(\alpha^R)^2 \Delta^2} \equiv \Phi(\Delta^2, \alpha^L, \alpha^R),
\]

and \(\Phi\) clearly satisfies the described comparative statics. \(\blacksquare\)

**Proposition 2.9** For a general increasing convex cost function \(k\), in the pure influence model in which \(\alpha^L = \alpha^R = \alpha\), divided control is optimal if \(k''/k' > k'''/k''\). This condition is satisfied for \(k(\alpha) = c\alpha^\xi\) for all \(\alpha > 0\).

**Proof.** Under non-integration, \(|k'(\alpha^j)| = |\Delta| \alpha \phi\) and under integration, \(\lambda^L = 0\) and \(k'(\lambda^R) = 2|\Delta| \alpha \phi\). Total influence costs are \(2k(k'^{-1}(|\Delta| \alpha \phi))\) under non-integration and \(k(k'^{-1}(2 |\Delta| \alpha \phi))\) under integration. A sufficient condition for the latter to be larger is that the function \(k(k'^{-1}(x))\) is convex in \(x\). Let \(h(x) = k'^{-1}(x)\). Then

\[
\frac{d^2}{dx^2} k(h(x)) = k''(h(x)) \left(\frac{k''(h(x))}{k''(h(x))} + k'(h(x))\right)
\]

\[
= \left(\frac{k'(h(x))}{k''(h(x))}\right)^2 \left(\frac{k''(h(x))}{k'(h(x))} - \frac{k'''(h(x))}{k''(h(x))}\right).
\]

This is positive for all \(\lambda\) if the parenthetical term is positive for all \(\lambda\). Finally, note that if \(k(\lambda) = c\lambda^\xi\), then the parenthetical term is \(\frac{1}{\lambda} > 0\), so this is satisfied. \(\blacksquare\)

**Proposition 2.10** Given a governance structure \((g, \theta)\), there exists a pure-strategy PBE of the
multidivisional firm model with

\[ d_i^{\varphi, \theta} = C^g(r) \left( (1 - (1 - \theta) \varphi) E[s_i] + (1 - \theta) \varphi \left( \sigma_i - \lambda^i(g) \right) \right) \\
+ (1 - C^g(r)) \left( (1 - (1 - \theta) \varphi) E[s_j] + (1 - \theta) \varphi \left( \sigma_j - \lambda^j(g) \right) \right), \]

and influence activities given by

<table>
<thead>
<tr>
<th>[ \lambda^L^* ]</th>
<th>[ \lambda^R^* ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{dec} \quad \left( \frac{r}{1 + 2r} \right)^2 \Delta \varphi (1 - \theta) &amp; - \left( \frac{r}{1 + 2r} \right)^2 \Delta \varphi (1 - \theta)</td>
<td></td>
</tr>
<tr>
<td>\text{cent} \quad \frac{r}{1 + 4r} \Delta \varphi (1 - \theta) &amp; - \frac{r}{1 + 4r} \Delta \varphi (1 - \theta)</td>
<td></td>
</tr>
</tbody>
</table>

**Proof.** To see this, plug in the decision rules to verify that they indeed form an equilibrium given beliefs about influence activities. Influence activities then solve (3). ■

**Corollary 2.1** Under the multidivisional firm model, welfare can be decomposed as follows

\[ W(g, \theta) = - (Adap(g, \theta) + Coord(g, \theta) + InfI(g, \theta)), \]

where

<table>
<thead>
<tr>
<th>[ Adap(g, \theta) ]</th>
<th>[ Coord(g, \theta) ]</th>
<th>[ InfI(g, \theta) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{dec}, \theta = 0 \quad r^2 \left( \frac{1}{1 + 2r} \right)^2 (2 \frac{E}{h} + \Delta^2) + \frac{r}{h} \varphi^2 \Delta^2 \left( \frac{r}{1 + 2r} \right)^2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{cent}, \theta = 0 \quad 4r^2 \left( \frac{1}{1 + 4r} \right)^2 (2 \frac{E}{h} + \Delta^2) + \frac{r}{h} \varphi^2 \Delta^2 \left( \frac{r}{1 + 4r} \right)^2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\text{cent}, \theta = 1 \quad r^2 \left( \frac{1}{1 + 2r} \right)^2 \Delta^2 + \frac{1}{h} r \left( \frac{1}{1 + 2r} \right)^2 \Delta^2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Proposition 2.11** The following are true. \( W(\text{dec}, 0) > W(\text{cent}, 0) \) if \( \left( \varphi q(r) - \frac{1}{\varphi} \right) h \Delta^2 > 2 \) where \( q(r) = \frac{(1 + 5r + 8r^2)(1 + 3r)}{(1 + 2r)^2(1 + 4r)^2} \) \( > 1 \) and \( q'(r) > 0 \). Thus, \( (\text{dec}, 0) \) becomes more appealing relative to \( (\text{cent}, 0) \) when \( \varphi, r, h, \) and \( \Delta^2 \) increase. \( W(\text{cent}, 1) > W(\text{dec}, 0) \) if \( \tilde{q}(r) \varphi h \Delta^2 > 1 \), where \( \tilde{q}(r) = \frac{r^4}{(1 + 2r)^2(1 + 2r + 2r^2)} < 1 \) and \( \tilde{q}'(r) > 0 \). Thus, \( (\text{cent}, 1) \) becomes more appealing relative to \( (\text{dec}, 0) \) when \( \varphi, r, h, \) and \( \Delta^2 \) increase. \( W(\text{cent}, 0) > W(\text{cent}, 1) \) if \( \varphi^2 > \frac{1 + 4r}{(1 + 2r)^2} \) and \( r^2 \Delta^2 < \frac{r^2(1 + 2r)^3(1 + 4r)}{\varphi^2(1 + 2r)^2(1 + 4r)} \) and \( \varphi^2 > \frac{1 + 4r}{(1 + 2r)^2} \). The rhs of the first inequality is decreasing in \( \varphi \) and \( h \), and thus \( (\text{cent}, 0) \) becomes more appealing relative to \( (\text{cent}, 1) \) when \( \varphi, r, h, \) and \( \Delta^2 \) decrease.
**Proof.** These are relatively straightforward by noting that

\[
W(\text{dec, 0}) - W(\text{cent, 0}) = \frac{r^2}{(2r + 1)^2 (4r + 1)} \left[ \left( \varphi^2 \left( \frac{5r + 8r^2 + 1}{(2r + 1)^2 (4r + 1)} \right)^2 \right) - 1 \right] \Delta^2 - 2\frac{\varphi}{h}
\]

\[
\propto \left( \varphi (r) - \frac{1}{\varphi} \right) \Delta^2 - 2
\]

\[
W(\text{dec, 0}) - W(\text{cent, 1}) = \varphi \left( \frac{1 + 2r + 2r^2}{(1 + 2r)^2} \right) \frac{1}{h} \left( 1 - \frac{r^4}{(1 + 2r)^2 (1 + 2r + 2r^2)} \right) \varphi h \Delta^2
\]

\[
\propto 1 - \tilde{q}(r) \varphi h \Delta^2
\]

\[
W(\text{cent, 0}) - W(\text{cent, 1}) = \frac{1}{(1 + 4r)^2} \left[ \varphi^2 - \frac{(1 + 4r)^2}{(1 + 2r)^2} \right] \left( \frac{(1 + 4r)(1 + 2r)^3 \varphi}{\varphi^2 (1 + 2r)^2 - (1 + 4r) - r^2 \Delta^2} \right)
\]

The comparative statics for the relative welfare computations are straightforward. □

### 2.10 Appendix B: Interim Signaling Version

Suppose there are two decision rights. Consider the game with the following timing: (1) \(L\) and \(R\) bargain over a control structure \(g \in G\); (2) \(s^L \in S\) is drawn and observed by \(L\) (but not \(R\)) and \(s^R \in S\) is drawn and observed by \(R\) (but not \(L\)); (3) \(L\) and \(R\) simultaneously choose influence activities \(\lambda^L, \lambda^R\) at costs \(\frac{1}{2} \lambda^2\). Public signals \(\sigma^i = s^i + \lambda^i\) are publicly observed; (4) whoever has control chooses decisions \(d\); (5) parties receive gross payoffs (letting \(s = s^L + s^R\))

\[
U^i(s, d) = -\sum_{t=1}^{2} \frac{\alpha^i}{2} (d_t - s - \beta^i)^2.
\]

Suppose \(L\) has control of \(N - R\) decisions, and suppose \(L\) conjectures the equilibrium strategy \(\lambda^{R*}(s^R)\) of \(R\). He chooses each decision \(d\) to solve

\[
\max_d E_s \left[ -\frac{\alpha^L}{2} (d - s - \beta^L)^2 \right] s^L, \sigma^R
\]

or

\[
d^* (s^L, \sigma^R) = E [s|s^L, \sigma^R] + \beta^L = s^L + (\sigma^R - E [\lambda^{R*}(s^R)|\sigma^R]) + \beta^L.
\]

Given this decision rule, \(R\) chooses \(\lambda^{R*}(s^R)\) to solve

\[
\max_{\lambda^R} N^{-R} E_s \left[ -\frac{\alpha^R}{2} (d^* (s^L, \sigma^R) - s - \beta^R)^2 \right] s^R - \frac{1}{2} (\lambda^R)^2
\]

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Taking first-order conditions (and imposing the equilibrium restriction that $\lambda^L(s^L) = 0$)

$$
\lambda^R(s^R) = N^{-R} \alpha^R (\Delta + E[\lambda^R(s^R)|\sigma^R] - \lambda^R(s^R)) .
$$

Taking expectations of both sides, $E[\lambda^R(s^R)|\sigma^R] = N^{-R} \alpha^R \Delta$, and therefore $\lambda^R(s^R) = N^{-R} \alpha^R \Delta$.

The incentives to influence the signal are thus the same in this model as in the baseline model with $\varphi = 1$.

2.11 Appendix C: Organizational Rules

This appendix outlines a simple model of endogenous organizational rules. If managers can commit ex ante to a decision rule (as a function of an informative, but manipulable signal), when will they prefer a responsive decision rule that takes the signal into account, and when will they prefer to make decisions "in the dark"? There is a natural trade-off that parallels the haggling versus fiat argument of section 6: a responsive decision rule makes use of potentially valuable information, but it also invites influence activities.

In order to make progress on this question, I depart from the model in section 2 and analyze a binary-state, binary-signal model. Since influence activities by definition affect the conditional distribution of the signal (given the state of the world), in this binary case, they necessarily also affect the conditional variance of the signal and thus its information content. This leads to the additional (Milgrom and Roberts 1988) effect, absent in the model in section 2, that "when... underlying information is so complex that unscrambling is impossible, decision makers will have to rely on information they know is incomplete or inaccurate." Optimal decision rules may thus also be unresponsive on account of this.

There are two managers, $L$ and $R$, a single decision to be made $d \in \{L, R\}$, and two potential states of the world $s \in \{L, R\}$. A signal $\sigma \in \{L, R\}$ is commonly observed, and the parties can specify a decision rule ex ante that depends on it. Absent influence activities, the signal is informative. Denote $q^0_k = \Pr[\sigma = s = k], k \in \{L, R\}$. Then $q^0_k > \frac{1}{2}$. The prior is given by a scalar $p_0 = \Pr[s = R]$.

When the state of the world is perfectly known, both managers agree on the optimal decision. However, the managers disagree on what decision should be taken when there is uncertainty about
the state of the world. That is, preferences are given by

<table>
<thead>
<tr>
<th>s \ d</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1, β</td>
<td>0, 0</td>
</tr>
<tr>
<td>R</td>
<td>0, 0</td>
<td>β, 1</td>
</tr>
</tbody>
</table>

When the state of the world is L, manager L receives 1 and manager R receives β < 1 if d = L. When the state of the world is L, both managers receive 0 if d = R. Without any additional information about the state of the world, manager L prefers d = L iff \( p_0 < \frac{1}{1 + β} \) and R prefers d = L iff \( p_0 < \frac{β}{1 + β} \).

![Preferred Decisions](image)

**Figure 6: Preferred Decisions**

As in the previous models, the timing is as follows: (1) L and R bargain over a decision rule \( d(σ) \); (2) L and R simultaneously decide whether or not to influence the signal at cost K; (3) the signal is commonly observed; (4) \( d(σ) \) is taken; (5) payoffs are realized. The key difference here are that the "governance structure" the managers choose is an autonomous decision rule that depends directly on the signal. For the purposes of this model, think of K small but positive to break indifference.

Before describing the mechanics of influence activities in this model, let us first characterize the first-best decision rule (i.e. the joint surplus maximizing decision rule in a world in which there is no scope for influence activities). When the decision is a "slam dunk," (i.e. \( p_0 \) is close to either 0 or 1), the first-best decision rule is not responsive. Only when there is substantial uncertainty about the state of the world should the decision rule be responsive to the signal. That is, there
exists cutoffs \( p_0 \) and \( \tilde{p}_0 \) such that

\[
d^{FB}(\sigma) = \begin{cases} 
L & 0 \leq p_0 \leq p_0 \\
\sigma & p_0 < p_0 \leq \tilde{p}_0 \\
R & \tilde{p}_0 < p_0 \leq 1
\end{cases}
\]

Influence takes a simple form. The signal is drawn from division \( L \) with probability \( \frac{1}{2} \) (i.e. \( \sigma = \sigma^L \)) and drawn from division \( R \) with probability \( \frac{1}{2} \) (i.e. \( \sigma = \sigma^R \)). At cost \( K \), manager \( i \) can ensure that \( \sigma_i = i \) with probability \( 1 \) (without regard to the true state of the world). When influence takes this form, when does manager \( L \) want to influence the signal? This, of course, depends on the decision rule the players have agreed to ex ante. If they agree upon an unresponsive decision rule, then neither player has any incentive to influence the signal.

If, however, they agree upon a responsive decision rule (i.e. \( d(\sigma) = \sigma \)), then if manager \( L \) does not influence the signal, he receives \( (1 - p_0) q^0_L + p_0 q^0_R \beta \). If he does influence the signal, he receives \( 1 - p_0 \). He thus wants to influence the signal whenever \( p_0 < p^L_0 (\beta) = \frac{1 - q^0_L}{1 - q^0_L + q^0_R} \). Similarly, \( R \) will prefer to influence the signal whenever \( p_0 > p^R_0 (\beta) = \frac{q^0_R \beta}{1 - q^0_L + q^0_R} \). It can be shown that, for \( \beta \) sufficiently close to 1, \( p^L_0 (\beta) < p_0 \) and \( p^R_0 (\beta) > \tilde{p}_0 \), so that managers only want to manipulate the signal when the decision rule in fact should not have depended on the signal to begin with. For \( \beta \) sufficiently small, \( p^L_0 (\beta) > \tilde{p}_0 \) and \( p^R_0 (\beta) < p_0 \), so that for any \( p_0 \) for which the first-best decision rule should be responsive, equilibrium influence activities ensure that the signal is completely uninformative. In this case, since \( K > 0 \), the optimal decision rule should in fact be unresponsive.

For the intermediate case, \( \tilde{p}_0 > p^L_0 (\beta) > \frac{1}{2} \) and \( \frac{1}{2} > p^R_0 (\beta) > p_0 \). Here, for any value of \( p_0 \in \left[ p_0, p^R_0 (\beta) \right] \cup \left[ p^L_0 (\beta), \tilde{p}_0 \right] \), the first-best decision rule is responsive and only one manager will influence the signal. In this case, the signal will be informative with probability \( \frac{1}{2} \) (and uninformative with probability \( \frac{1}{2} \)), and thus it may be optimal to choose a responsive decision rule (provided that \( K \) is sufficiently small). For \( p_0 \in \left[ p^R_0 (\beta), p^L_0 (\beta) \right] \), the optimal decision rule is unresponsive, because the signal will be completely uninformative. The optimal decision rule thus
unravels from the middle as $\beta$ decreases. These are captured in the following diagram.

![Diagram](image)

Figure 7: Optimal Decision Rules

Thus, equilibrium influence activities can lead to overly rigid decision rules if there is sufficient disagreement between the two managers.

Throughout, I have assumed that parties can commit ex ante to a decision rule. What if the managers are unable to commit: after all, someone must actually make the decision. In this case, there is a time-inconsistency problem. The managers might prefer an unresponsive rule in order to eliminate the incentives for influence activities, but if neither manager has manipulated the signal, it contains useful information for decision making, and thus optimally should be taken into account by the decision maker. If he cannot commit to ignoring the signal, then in equilibrium someone will manipulate it.

This opens up the possibility of using relational contracts (e.g. Baker, Gibbons, and Murphy (1994), Levin (2003)) to partially commit a decision maker to a rigid decision rule. Such a rule must explicitly be based on something (otherwise, how will other members of an organization know it was followed?) The relational contracting view would generate predictions on the types of rigid decision rules organizations would use. For instance, a rigid promotion rule should be based explicitly on public information like seniority rather than something like a randomization device (unless the actual randomization is carried out publicly). Basing such rules on variables that are not commonly observable can potentially undermine relational enforcement and thus, by unravelling, lead to influence activities. Relational commitment to rigid decision rules is a potentially interesting direction for future research.
Chapter 3

Organization and Information: Firms’ Governance Choices in Rational-Expectations Equilibrium

3.1 Introduction

Scholars and consultants in strategic management have long espoused two approaches to strategy and organization: developing innovative new products through R&D and market research, on the one hand, and producing existing products efficiently through process control and continuous improvement, on the other. But many observers quickly emphasize the difficulty of simultaneously pursuing these “exploration” and “exploitation” (March 1991) approaches. For example, “Cost leadership usually implies tight control systems, overhead minimization, pursuit of scale economies, and dedication to the learning curve; these could be counterproductive for a firm attempting to differentiate itself through a constant stream of creative new products” (Porter, 1985: 23). Furthermore, as Chandler (1962) famously argued, a firm’s strategy and organizational structure are inextricably linked. In short, “Exploration and exploitation are quite different tasks, calling on different organizational capabilities and typically requiring different organizational designs to effect them” (Roberts, 2004: 255).

In quite a different tradition, economists have long celebrated the market’s price mechanism for its ability to aggregate and transmit information (Hayek, 1945; Grossman, 1976). The informative-

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1This chapter is coauthored with Robert Gibbons and Richard Holden.
ness of the price mechanism thus raises the possibility that the market can (wholly or partially) substitute for certain information-gathering and communication activities within the firm, thereby affecting the firm's optimal strategy and organizational structure. But as Grossman and Stiglitz (1976, 1980) pointed out, market equilibrium must be internally consistent. For example, when information is costly to acquire, market prices cannot be fully informative, otherwise no party would have an incentive to acquire information in the first place.

In this paper we view firms and the market as institutions that shape each other: in industry equilibrium, each firm takes the informativeness of the price mechanism as an important parameter in its choice of organizational design, but these design decisions in turn affect the firm's participation in the market and hence the informativeness of the price mechanism. We thus complement the large and growing literature on how organizational structures and processes affect incentives to acquire and communicate information. In particular, our analysis shows how one firm's optimal organizational design depends not only on the uncertainty it faces but also on the designs other firms choose. For example, if the market price is very informative, then many firms will choose organizational designs to improve incentives for other activities (say, cost reduction), effectively free-riding on the informativeness of the price mechanism. But the Grossman-Stiglitz insight implies that not all firms can free-ride, lest the price mechanism contain no information.

To explore how the informativeness of the price mechanism and firms' organizational designs choices interact, we analyze an economic environment that includes uncertainty. Formally, the uncertainty concerns consumers' valuation of final goods, but we discuss other interpretations below. As in other rational-expectations models, the price mechanism both clears the market and conveys some information from informed to uninformed parties. The fact that the price is not perfectly informative provides the requisite incentive for some parties to pay the cost of acquiring further information.

As one example, consider firms like Apple (an explorer that excels at developing innovative products) and Dell (an exploiter that achieves low costs through rigorous supply-chain management). Although these kinds of firms may not be direct competitors in the product market, they do participate in some of the same input markets, and broad industry trends do affect demand for both kinds of firms. In principle, Dell could organize itself to conduct consumer research and R&D

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2See Milgrom and Roberts (1988), Holmstrom and Tirole (1991), and Aghion and Tirole (1997) for early work and Alonso, Dessein, and Matouschek (2008) and Rantakari (2008) for a sample of recent work; see Bolton and Dewatripont (2011) and Gibbons, Matouschek, and Roberts (2011) for surveys.)
(as Apple does), but Dell does not do this. Instead, Dell’s organizational structure and managerial attention focus on supply-chain management. Dell can, however, infer something about broad industry trends by observing prices in Apple’s input markets. This example parallels our model, in that it is the market-clearing price of an intermediate good that provides information about demand for a final good.

Demand for Apple’s products is clearly relevant to demand Dell’s products. Dell could, in principle, devote a large amount of resources to consumer research, focus groups, and new product R&D (as Apple does). Dell does not do this, however. Dell’s organizational structure is designed to focus resources on supply chain management. They can, however, learn about demand for Apple’s products from prices markets for components for Apple’s products (such as power-efficient mobile microprocessors). This example parallels our model, in that it is the market-clearing price of an intermediate good which provides information about demand for a final good.

Many other applications of our approach arise if we consider alternative sources of uncertainty, other than the value of downstream goods. For example, the uncertainty might concern whether tariff barriers will change or whether a new technology will fulfill its promise. Interestingly, however, not all sources of uncertainty will do: our rational-expectations model requires some element of common-value uncertainty (possibly partially correlated rather than perfectly common values) rather than pure private-value uncertainty. As Grossman (1981: 555) puts it, in non-stochastic economies (and certain economies with pure private-value uncertainty), “No one tries to learn anything from prices [because] there is nothing for any individual to learn.” Often, however, there is something to learn from prices, such as when there is an element of common-value uncertainty.

To pursue these issues, we develop a rational-expectations model similar to Grossman and Stiglitz (1976, 1980) but applied to a market for an intermediate good (e.g., prices and net supply are non-negative and the players are risk-neutral). In Gibbons, Holden, and Powell (2009; hereafter GHP), we developed such a model but for the Grossman-Stiglitz case of individual investors. Relative to that paper (and other rational-expectations models), the innovation here is the enrichment from individual investors to firms, where each firm chooses one of two alternative organizational designs (one of which inspires a party within the firm to collect costly information, as in Grossman-Stiglitz).

To model these firms, we develop a simplified version of the classic incomplete-contracting approach initiated by Grossman and Hart (1986), but applied to the choice of governance structure within an organization (akin to Aghion and Tirole (1997)). To keep things simple, our
incomplete-contracts model involves only a single control right (namely, who controls a machine that is necessary for production) and hence two feasible organizational designs. Regardless of who controls the machine, each party can make a specific investment, but the incentives to make these investments depend on who controls the machine. Following the incomplete-contracts approach (i.e., analyzing one firm in isolation) reveals that the optimal organizational design is determined by the marginal returns to these investments. In our model all firms are homogeneous \textit{ex ante}, so an incomplete-contracts analysis of a single firm would prescribe that all firms choose the same organizational design. Relative to the incomplete-contracts approach, the novel component of our model is the informativeness of the price mechanism, which endogenizes the returns to the parties’ specific investments and hence creates an industry-level determinant of an individual firm’s choice of organizational design.

In summary, our model integrates two familiar approaches: rational expectations (where an imperfectly informative price mechanism both permits rational inferences by some parties and induces costly information acquisition by others) and incomplete contracts (where equilibrium investments depend on the parties’ allocation of control, and control rights are allocated to induce second-best investments). Our main results are that: (1) under mild regularity conditions an equilibrium exists; (2) \textit{ex ante} identical firms may choose heterogeneous organizational designs; and (3) firms’ choices of organizational design and the informativeness of the price mechanism interact. In fact, in our model, certain organizational designs may be sustained in market equilibrium \textit{only} because the price system allows some firms to benefit from the information-acquisition investments of others. We also provide comparative statics on the proportion of firms that chose one organizational design or the other.

Grossman and Helpman (2002), Legros and Newman (2008) and Legros and Newman (2009) analyze other interactions between firms’ governance structures and the market. These papers differ from ours in two respects. First, in modeling firms’ choice of governance structures, they focus on the boundary of the firm (i.e., the integration decision) whereas we focus on the organizational design (specifically, the allocation of control within the organization). Second, and more importantly, in modeling the market, they focus on the market-clearing rather than the informativeness aspect of the price mechanism. That is, in these models, supply and demand determine prices, which in turn determine the returns to the parties’ actions and hence the parties’ optimal governance structures; meanwhile, the parties’ actions in turn determine supply and demand, so governance and pricing interact. As Grossman (1981: 555) notes, such Walrasian equilibria are not useful “as
a tool for thinking about how goods are allocated... when...information about the future...affects current prices.” In contrast to the aforementioned papers, our model focuses on the informative role of prices – transferring information from informed to (otherwise) uninformed parties. We see our approach as complementary to these others: in economies with uncertainty the price mechanism clears the market and communicates information; without uncertainty, however, governance and pricing can still interact, for the reasons explained in these papers.

The remainder of the paper proceeds as follows. In Section 2 we specify and discuss the model. Section 3 analyzes the organizational-design choice of a single firm in isolation, and Section 4 analyzes the informativeness of the price mechanism, taking firms’ organizational-design choices as given. Section 5 then combines the incomplete-contracts and rational-expectations aspects of the previous two sections, analyzing the equilibrium choices of organizational designs for all the firms in the industry and hence deriving our main results. Section 6 offers an enrichment of our model in terms of firms’ choices about their boundaries and discusses how our approach relates to existing theories of firm boundaries. Section 7 concludes.

3.2 The Model

3.2.1 Overview of the Model

We begin with an informal description of our model. There is a continuum of firms, each consisting of an “engineer” and a “marketer” who both participate in a production process that can transform one intermediate good (a “widget”) into one final good. Any firm may purchase a widget in the intermediate-good market. Each firm has a machine that can transform one widget into one final good at a cost. The engineer in a given firm has human capital that allows her to make investments that reduce the cost of operating that firm’s machine. Likewise, the marketer in a given firm has human capital that allows him to make investments that deliver information about the value of a final good.

As is standard in incomplete-contracting models, the parties’ incentives to make investments depend on the allocation of control. There are two possible organizational designs (i.e. governance structures inside the firm): marketing control and engineering control. In particular, in our model, only the party that controls the machine will have an incentive to invest. Thus, in firms where the marketer controls the machine, the marketer invests in information about the value of the final good, whereas in firms where the engineer controls the machine, the engineer invests instead in cost
reduction and relies solely on the price mechanism for information about the value of the final good. Naturally, if the price mechanism is more informative, the returns to investing in information are lower so firms have a greater incentive to choose engineer control and invest instead in cost reduction.

As in rational-expectations models, however, when fewer firms invest in gathering information, the price mechanism becomes less informative, thereby making marketer control more attractive. An industry equilibrium must balance these two forces. We show that, given a rational-expectations equilibrium, a unique equilibrium exists and is often interior (even though firms are identical \textit{ex ante}). In this sense, the price mechanism induces heterogenous behavior among homogeneous firms.\footnote{We label our parties "engineer" and "marketer" because their investments produce cost reductions and demand forecasts, respectively. We have formulated a parallel model where market research is replaced by product development. In this model, uninformed firms again invest in cost reduction (in producing the current generation of a product), but informed firms now invest in trying to create the next-generation product. In the spirit of Christensen (1997), the new product created by informed firms may be more valuable than the current product produced by uninformed firms.}

3.2.2 Statement of the Problem

There is a unit mass of risk-neutral firms. Each firm \(i \in [0, 1]\) consists of two parties, denoted \(E_i\) and \(M_i\), and a machine that is capable of developing one intermediate good (a "widget") into one final good at cost \(c_i \sim U[c, \bar{c}]\). The machine can be controlled by either party, but it is firm-specific (i.e., the machine is useless outside the firm) and its use is non-contractible (i.e., only the party who controls the machine can decide whether to operate it). If party \(E_i\) controls the machine, we say that the governance structure in firm \(i\) is \(g_i = E\), whereas if party \(M_i\) controls the machine, we say that \(g_i = M\).

Final goods have an uncertain value. Party \(M_i\) can invest at cost \(K_M\) to learn the value of a final good in the market, \(v \sim U[v, \bar{v}]\). If \(M_i\) incurs this cost, \(E_i\) observes that \(M_i\) is informed but does not herself observe \(v\). Party \(E_i\) can invest at cost \(K_E\) in reducing the cost of operating the firm’s machine. If \(E_i\) incurs this cost, \(M_i\) observes that \(E_i\) invested, so it is common knowledge that \(c_i\) is reduced to \(c_i - \Delta\), where \(\Delta \leq c\). Both of these investments are non-contractible (e.g., for \(E_i\), neither the act of investing nor the resulting cost is contractible).

We embed these firms in a cousin of our rational-expectations model of price formation in intermediate good markets (Gibbons, Holden and Powell (2010)). Firms may purchase widget(s) in the intermediate-good market. The supply of widgets, \(x\), is random and inelastic. Assume \(x \sim U[x, \bar{x}]\).
Equilibrium in the market for widgets occurs at the price $p$ that equates supply and demand (from informed and uninformed firms). In making decisions about purchasing a widget, firms that are not directly informed about $v$ (from investments by their marketers) make rational inferences about $v$ from the market price for widgets. Firms choose their governance structures (i.e., machine control) taking into account the information they will infer from the market price and hence the relative returns to their two parties' investments.

### 3.2.3 Timing and Assumptions

We now state the timing and assumptions of the model more precisely. We comment on these assumptions in Section 2.4. There are six periods.

![Figure 1: Timeline](image)

In the first period, industry-level uncertainty is resolved: the value of a final good $v$ is drawn from $U[v, \bar{v}]$ and the widget supply $x$ is drawn from $U[x, \bar{x}]$, but neither of these variables is observed by any party.

In the second period, the parties in each firm negotiate a governance structure $g_i \in \{E, M\}$: under $g_i = E$, party $E_i$ controls the machine that can develop one widget into one final good; under $g_i = M$, party $M_i$ controls this machine. This negotiation of governance structure occurs via Nash bargaining.

In the third period, parties $E_i$ and $M_i$ simultaneously choose whether to make non-contractible investments (or not) at costs $K_E$ and $K_M$, respectively. The acts of making these investments are observable but not verifiable, but the outcome of the marketer's investment (namely, learning $v$) is observable only to $M_i$, not $E_i$.

In the fourth period, production planning takes place, in two steps. In period 4a, the parties $E_i$ and $M_i$ commonly observe $c_i \sim U[g, \bar{c}]$, the raw cost of running their machine, as well as $\delta_i \in \{0, \Delta\}$,
the amount of cost reduction achieved by $E_i$’s specific investment. Also, $M_i$ (but not $E_i$) observes $\varphi_i \in \{0, v\}$, a signal about the value $v$ of the final good, where $\varphi_i = \emptyset$ is the uninformative signal received if party $M_i$ has not invested $K_M$ in period 3, and $\varphi_i = v$ is the perfectly informative signal received if $K_M$ has been invested. We use the following notation for the parties’ information sets: $s_i^M = (c_i, \delta_i, \varphi_i)$, $s_i^E = (c_i, \delta_i, \emptyset)$, and $s_i = (s_i^M, s_i^E)$. In period 4b, the market for widgets clears at price $p$. In particular, any firm may buy a widget (but will not demand more than one widget because the machine can produce only one final good from one widget).

In the fifth period, production occurs: if the party in control of the machine in firm $i$ has a widget, then he or she can run the machine to develop the widget into a final good at cost $c_i - \delta_i$. We denote the decision to produce a final good by $q_i = 1$ and the decision not to do so by $q_i = 0$. In principle, off the equilibrium path, one party might control the machine and the other have a widget, in which case the parties bargain over the widget and then the machine controller makes the production decision. We assume that cashflow rights and control rights are inextricable, so that whichever party controls the machine owns the final good (if one is produced) and receives the proceeds.

Finally, in the sixth period, final goods sell for $v$ and payoffs are realized. The expected payoffs (before $v$ is realized) are

$$\pi_{E_i}^{q_i} = 1_{\{q_i = E\}} 1_{\{w_i = 1\}} \left[ 1_{\{q_i = 1\}} \left( E[v|s_i^E, p(\cdot, \cdot) = p] - c_i + \delta_i - p \right) \right]$$

and

$$\pi_{M_i}^{q_i} = 1_{\{q_i = M\}} 1_{\{w_i = 1\}} \left[ 1_{\{q_i = 1\}} \left( E[v|s_i^M, p(\cdot, \cdot) = p] - c_i + \delta_i - p \right) \right].$$

### 3.2.4 Discussion of the Model

Before proceeding with the analysis, we pause to comment on some of the modeling choices we have made.

First, we assume that the machine is firm-specific. This assumption allows us to focus on the market for widgets by eliminating the market for machines. By allowing both markets to operate, one could analyze whether the informativeness of one affects the other.

Second, we have only one control right (over the machine) and hence only two candidate governance structures. Our choice here is driven purely by parsimony; extending the model to allow more assets (and hence more governance structures) could allow more interesting activities within organizations than our simple model delivers.
Third, we make the strong assumption that control of the machine and receipt of cashflow from selling a final good are inextricably linked. We expect that richer models based on weaker assumptions would yield similar results (if they can be solved).

Fourth, we have binary investments in cost reduction and information acquisition (at costs $K_E$ and $K_M$, respectively), rather than continuous investment opportunities. It seems straightforward to allow the probability of success (in cost reduction or information acquisition) to be an increasing function of the investment level, which in turn has convex cost.

Fifth, we assume inelastic widget supply $x$. This uncertain supply plays the role of noise traders, making the market price for widgets only partially informative about $v$, so that parties may benefit from costly acquisition of information about $v$.

Sixth, as in GHP, our assumptions that all the random variables are uniform allow us to compute a closed-form (indeed, piece-wise linear) solution for the equilibrium price function for the intermediate good. This tractability is useful in computing the returns to alternative governance structures, at the firm level, and hence the fraction of firms choosing each governance structure, at the industry level.

Seventh, as in Grossman-Stiglitz and the ensuing rational-expectations literature, our model of price formation is a reduced-form model of price-taking behavior, rather than an extensive-form model of strategic decision-making (which might allow information transmission during the price-formation process, either by the parties as described in our model or by one party who separates from his engineer and becomes something like a marketer). See GHP for an extended discussion.

### 3.3 Individual Firm Behavior

As a building block for our ultimate analysis, we first analyze the behavior of a single firm taking the market price $p$ as given. Optimal behavior involves purchasing a widget only if one is going to produce. Define the gross surplus to the parties in a firm as $GS_i = \pi_{M_i} + \pi_{E_i}$, i.e.

$$GS_i (g_i, s_i) = 1_{\{g_i=1\}} [E[v|s_i^{g_i}, p(\cdot, \cdot) = p] - p - (c_i - \delta_i)].$$

The efficient production decision is $q_i^* = 1$ if $E_{x,v} [v|s_i^{g_i}, p] \geq p + c_i - \delta_i$, and the maximized expected gross surplus in period 4 is then

$$GS_i^* (g_i, s_i) = E_{x,v} [ (v - c_i + \delta_i - p) q_i^* (g_i, s_i, p)|s_i^{g_i}, p)].$$
Recall that the controller of the machine both controls the production decisions and receives the cashflows. Consequently, the non-owner receives zero. These payoffs determine the parties’ investment incentives in period 3, as follows.

Let the subscript pair \((I, 0)\) denote the situation in which \(M_i\) invested and hence is informed about \(v\) but \(E_i\) did not invest in cost reduction. Likewise \((U, \Delta)\), denotes the situation in which \(M_i\) did not invest but \(E_i\) did, hence reducing production costs by \(\Delta\), and \((U, 0)\) denotes the situation in which neither invested. Now define the following:

\[
\pi_{I,0} = E_{c_i} \left[ G_{s_i}^* (M, s_i) \right] \text{ if } \varphi_i = v, \delta_i = 0, \\
\pi_{U,\Delta} = E_{c_i} \left[ G_{s_i}^* (E, s_i) \right] \text{ if } \varphi_i = \emptyset, \delta_i = \Delta, \text{ and} \\
\pi_{U,0} = E_{c_i} \left[ G_{s_i}^* (g_i, s_i) \right] \text{ if } \varphi_i = \emptyset, \delta_i = 0.
\]

Formally, these expectations are triple integrals over \((c, x, v)\) space:

\[
\pi_{I,0} = \int_{\bar{v}}^{\bar{v}} \int_{\bar{x}}^{\bar{x}} \int_{\Lambda}^{\Lambda} \left( v - p(x, v) - c_i \right) dF(c_i, x, v), \\
\pi_{U,\Delta} = \int_{\bar{v}}^{\bar{v}} \int_{\bar{x}}^{\bar{x}} \int_{\Lambda}^{\Lambda} \left( v - p(x, v) + \Delta - c_i \right) dF(c_i, x, v), \text{ and} \\
\pi_{U,0} = \int_{\bar{v}}^{\bar{v}} \int_{\bar{x}}^{\bar{x}} \int_{\Lambda}^{\Lambda} \left( v - p(x, v) - c_i \right) dF(c_i, x, v),
\]

where \(F\) is the joint distribution function.

Since one party’s expected payoff in period 4 is independent of its investment, at most one party will invest in period 3. If \(E_i\) controls the machine \((g_i = E)\), she will invest if \(\pi_{U,\Delta} - K_E \geq \pi_{E,0}\). Similarly, if \(M_i\) controls the machine \((g_i = M)\), he will invest if \(\pi_{I,0} - K_M \geq \pi_{M,0}\). We assume that \(K_E\) and \(K_M\) are small relative to the benefits of investment, so the party that controls the machine will invest.\(^4\)

To proceed, we need to compute the price function \(p(x, v)\). This involves analyzing the behavior of other firms, as follows.

---

\(^4\)This condition can be stated in terms of primitives of the model, but since this is the economic assumption we are making, we state it in this fashion.
3.4 Rational Expectations in the Market for Intermediate Goods

Recall that there is a unit mass of firms indexed by \( i \in [0, 1] \). Who buys a widget? Let \( c_M(v, p) = v - p \) be the highest cost at which a marketer who has invested in information (and hence knows \( v \)) would be prepared to produce a final good, and similarly let \( c_E(p) = E[v|p] - p + \Delta \) be the highest cost at which an engineer who has invested in cost reduction (but not information) would be prepared to produce. Suppose (as we will endogenize below) that a fraction \( \lambda \) of firms have \( M \) control (and hence know \( v \)), whereas fraction \( 1 - \lambda \) have \( E \) control (and hence costs reduced by \( \Delta \)). Demand for widgets is therefore

\[
\lambda \frac{v - p - \xi}{\xi} + (1 - \lambda) \frac{E[v|p(x, v) = p] + \Delta - p - \xi}{\xi}.
\]

The market-clearing price equates this demand with the supply, which recall is \( x \), so

\[
p = (1 - \lambda) E[v|p(x, v) = p] + \lambda v - (\xi - \xi) x + (1 - \lambda) \Delta - \xi.
\]

The conditional expectation of \( v \) given \( p \) therefore must satisfy

\[
E[v|p(\cdot, \cdot) = p] \equiv \frac{p + (\xi - \xi) x + \xi - (1 - \lambda) \Delta - \lambda v}{1 - \lambda},
\]  
where the equivalence relation indicates that (3.1) must hold as an identity in \( x \) and \( v \).

**Definition 3.1** Assume fractions \( \mu_{UL}, \mu_{U0}, \mu_{UL}, \mu_{U0} \) of firms are, respectively, informed and have cost reduction, informed and do not have cost reduction, uninformed and have cost reduction, and uninformed and do not have cost reduction. A **rational expectations equilibrium** ("REE") is a price function \( p(x, v) \) and a production allocation \( \{q_i\}_{i \in [0, 1]} \) such that

1. \( q_i = q_i^* (g_i, s_i, p) \) for all \( i \), and
2. The market for widgets clears for each \( (x, v) \in [\underline{x}, \overline{x}] \times [\underline{v}, \overline{v}] \).

The fact that the non-controller receives none of the cashflow implies that this party will not invest, so \( \mu_{UL} = 0 \). Furthermore, \( K_E \) and \( K_M \) small implies \( \mu_{U0} = 0 \). Therefore \( \lambda = \mu_{U0} \) and \( 1 - \lambda = \mu_{UL} \). The problem of finding a rational-expectations price function in this model thus becomes one of finding a fixed point of (3.1). In GHP, we solve for this fixed point in a related model, finding it to be piecewise-linear over three regions of \( (x, v) \) space: a low-price region, a moderate-price region, and a high-price region.
Proposition 3.1 Given \( \lambda \), there exists an REE characterized by a price function

\[
p_\lambda (x, v) = 1_{\{ (x,v) \in R^1 \}} p^1 (x, v) + 1_{\{ (x,v) \in R^2 \}} p^2 (x, v) + 1_{\{ (x,v) \in R^3 \}} p^3 (x, v),
\]

where \( p_j^\lambda (x, v) = \beta_j^0 + \beta_j^1 x + \beta_j^2 v \) for \( j = 1, 2, 3 \).

We prove this proposition and derive the price function in appendix A, but to build some intuition for this result, consider the figure below, which shows the three regions of \((x, v)\) space, \(R^j_\lambda\) for \( j = 1, 2, 3 \). The low-price region \( R^1_\lambda \) begins from the lowest feasible price, \( p_L \) at \((\bar{x}, \bar{v})\), and extends up to the price \( \bar{p} \) at \((\bar{x}, \bar{v})\). The moderate-price region \( R^2_\lambda \) then extends from price \( \bar{p} \) up to the price \( p \) at \((\bar{x}, \bar{v})\), where the under- and over-lined notation for prices is chosen to match the \((x, v)\) coordinates. Finally, the high-price region \( R^3_\lambda \) extends from \( p \) up to the highest feasible price, \( p_H \) at \((\bar{x}, \bar{v})\).

![Figure 2: Regions of Piecewise-Linear Pricing Function](image)

Within each region, the iso-price loci are linear. In particular, solving \( p^j (x, v) = p \) for \( v \) yields

\[
v = -\frac{\beta_j^1}{\beta_j^2} x + \frac{p - \beta_j^0}{\beta_j^2}
\]

as an iso-price line in \((x, v)\) space. Because \( x \) and \( v \) are independent and uniform, every \((x, v)\) point on this line is equally likely. Thus, after observing \( p \), an informed party projects this iso-price line onto the \( v \)-axis and concludes that the conditional distribution of \( v \) given \( p \) is uniform, with support depending on which region \( p \) is in. For example, if \( p < \bar{p} \) then the lower bound on \( v \) is \( \bar{v} \) and the upper bound is some \( \bar{v}(p) < \bar{v} \). Alternatively, if \( \bar{p} < p < p \) then the lower and upper bounds on \( v \) are \( \bar{v} \) and \( \bar{v} \), so \( p \) is uninformative. Finally, if \( p > p \) then the lower bound is some
Given this uniform conditional distribution of \( v \) given \( p \), the conditional expectation on the left-hand side of (3.1) is then the average of these upper and lower bounds on \( v \). The coefficients \( \beta_0, \beta_1, \) and \( \beta_2 \) can then be computed by substituting \( p^i(x, v) \) for \( p \) on both sides of (3.1) and equating coefficients on like terms so that (3.1) holds as an identity. The slope of an iso-price line, \(-\beta_1/\beta_2\), is decreasing in \( \lambda \), meaning that in regions 1 and 3 uninformed parties can make tighter estimates of \( v \) from \( p \) when more parties are informed.

### 3.5 Industry Equilibrium

To recapitulate, Section 3 analyzed the production decision, taking \( p(\cdot, \cdot) \) as exogenous, and Section 4 endogenized prices. In this section, therefore, we endogenize the governance-structure choices of each firm and define an industry equilibrium, as follows.

**Definition 3.2** An industry equilibrium is a set of firms of mass \( \lambda^* \), a price function \( p(x, v) \), and a production allocation \( \{q_i\}_{i \in [0, 1]} \) such that

1. Each firm optimally chooses \( g_i \), with a fraction \( \lambda^* \) choosing \( g_i = M \);
2. Each party optimally chooses whether or not to invest;
3. \( q_i = q_i^*(g_i, s_i, p) \) and \( w_i = w_i^*(g_i, s_i, p) \); and
4. The market for widgets clears for each \((x, v) \in [x, \bar{x}] \times [\underline{v}, \bar{v}] \).

The choice in period 2 is between the two possible governance structures: \( g_i = E \) or \( g_i = M \). Given \( \lambda \), the ex ante expected net surpluses from choosing the two governance structures are

\[
NS^E(\lambda) = \pi_{UE}(\lambda) - K_E, \quad \text{and} \quad NS^M(\lambda) = \pi_{IE}(\lambda) - K_M.
\]

In an interior equilibrium, firms must be indifferent between the two governance structures. Thus our goal is to find \( \lambda^* \) such that \( NS^E(\lambda^*) = NS^M(\lambda^*) \) and to characterize how \( \lambda^* \) varies as we

\footnote{Note that in this model, as in GHP but not Grossman-Stiglitz, extreme prices are very informative and intermediate prices are less informative. In fact, with the slopes of the price functions as drawn in the above figure, intermediate price are completely uninformative.}
change the parameters of the model. For simplicity we assume that \( K_E = K_M = K \). (The case where \( K_E \neq K_M \) is discussed at the end of this section.) We therefore seek \( \lambda^* \) such that

\[
\pi_{I0} (\lambda^*) = \pi_{U\Delta} (\lambda^*),
\]
or equivalently,

\[
\pi_{I,0} (\lambda^*) - \pi_{U,0} (\lambda^*) = \pi_{U,\Delta} (\lambda^*) - \pi_{U,0} (\lambda^*).
\] (3.2)

To keep notation compact, let \( \sigma_v = \frac{1}{\sqrt{12}} (\bar{v} - v) \) and \( \sigma_x = \frac{1}{\sqrt{12}} (\bar{x} - x) \). We will use the following fact (which is derived in the appendix).

**Fact 3.1** Assume \( \lambda \leq (\bar{c} - \xi) \frac{\sigma_x}{\sigma_v} \). Then

\[
\pi_{I,0} (\lambda) - \pi_{U,0} (\lambda) = \frac{1}{2} \left( 1 - \frac{1}{2c} \right) \left( 1 - \frac{\lambda}{\frac{\sigma_v}{\sigma_x}} \right) \] and

\[
\pi_{U,\Delta} (\lambda) - \pi_{U,0} (\lambda) = \frac{\Delta^2}{\bar{c} - \xi} - \frac{\lambda}{2} + \mu_x \Delta.
\]

Observe that the first expression is decreasing in \( \lambda \) and the second is increasing in \( \lambda \). This leads to the following characterization of industry equilibrium.

**Proposition 3.2** Assume \((\bar{c} - \xi) \frac{\sigma_x}{\sigma_v} \geq 1\). For all \( \bar{c}, \xi, \sigma_x, \sigma_v, \Delta > 0 \) with \( \xi \geq \Delta \), there exists an industry equilibrium. Further,

\[
\lambda^* = \frac{\sigma_v^2 + \Delta^2 - 2 (\bar{c} - \xi) \mu_x \Delta}{\frac{\sigma_v^2}{2} \left( \frac{\sigma_x}{\bar{c} - \xi} \right)^2 + 2 \Delta^2}
\] (3.3)

if the right-hand side of (3.3) is in \([0, 1]\). If the right-hand side of (3.3) is less than 0, then \( \lambda^* = 0 \); if it is greater than 1, then \( \lambda^* = 1 \).

**Proof.** If \( \sigma_v^2 \leq 2 (\bar{c} - \xi) \mu_x \Delta - \Delta^2 \), then \( \pi_{U,0} (0) \leq \pi_{U,\Delta} (0) \) and thus, since the left-hand side of (2) is decreasing in \( \lambda \), it follows that \( \lambda^* = 0 \). Similarly, if \( \sigma_v^2 \left( 1 - \frac{1}{2} \frac{1}{\bar{c} - \xi} \right) \geq 2 (\bar{c} - \xi) \mu_x \Delta + \Delta^2 \), then \( \pi_{U,0} (1) \geq \pi_{U,\Delta} (1) \), and since the right-hand side of (2) is increasing in \( \lambda \), we must have that \( \lambda^* = 1 \). Otherwise, we want to find \( \lambda^* \) such that

\[
0 = \pi_{I0} (\lambda^*) - \pi_{U\Delta} (\lambda^*)
= \frac{\sigma_v^2 + \Delta^2 - 2 (\bar{c} - \xi) \mu_x \Delta}{2 (\bar{c} - \xi)} - \frac{\lambda^*}{2 (\bar{c} - \xi)} \left( \frac{\sigma_v / \sigma_x \sigma_v^2}{\bar{c} - \xi} + 2 \Delta^2 \right).
\]
which yields expression (3.3).

Proposition 3.2 is our main result, establishing that, given our rational expectations equilibrium, there exists a unique industry equilibrium and providing an explicit expression for the proportion of firms that choose each of the governance structures. As the proposition makes clear, this proportion may well be interior.\(^6\) Recall, however, that our firms are homogeneous ex ante, so an incomplete-contract style analysis (taking each firm in isolation) would prescribe that they all choose the same governance structure. In this sense, the informativeness of the price mechanism can induce heterogeneous behaviors from homogenous firms. To put this point differently, in this model, the price mechanism can be seen as endogenizing the parameters of the incomplete-contract model so that firms are indifferent between governance structures. In a richer model, with heterogeneous investment costs, almost every firm would have strict preferences between governance structures, with only the marginal firm being indifferent.

We are also able to perform some comparative statics. First, when the ex ante level of fundamental uncertainty increases (i.e., \(\sigma_v\) is higher), the return to investing in acquiring information increases, so \(\lambda\) increases. An increase in noise (i.e., \(\sigma_x\) is higher) has an identical effect. An increase in \(\mu_x\) increases the probability of production, which disproportionally benefits E-control firms, decreasing \(\lambda\). Finally, an increase in \(\Delta\) has two effects. The first is the partial-equilibrium channel through which an increase in the benefits of choosing engineer ownership (and hence investing in cost reduction) makes engineer control relatively more appealing, reducing \(\lambda\). In an industry equilibrium, however, there is also a price effect. For a fixed fraction \(1 - \lambda\) of parties that invest in cost reduction, an increase in \(\Delta\) makes widgets more valuable, which in turn increases demand and hence average prices. Since firms with engineer control purchase widgets over a larger region of the \(c_i\) space than do firms with marketing control, the former face this increase in average price level relatively more than do firms with marketer control, so the price effect militates towards an increase in \(\lambda\). Which of these two effects dominates depends on the parameters of the model. We give formal statements of these results in the following proposition.

**Proposition 3.3** Assume \((\bar{c} - \bar{c}) \frac{\partial x}{\partial x} \geq 1\). For all \(\bar{c}, \xi, \sigma_x, \sigma_v, \Delta > 0\) with \(\xi \geq \Delta\) and \(\lambda^* \in (0, 1)\), we have that: (i) \(\lambda^*\) is increasing in \(\sigma_v\), (ii) \(\lambda^*\) is increasing in \(\sigma_x\), (iii) \(\lambda^*\) is decreasing in

\(^6\)Models of industry equilibrium from IO (Ordover, Saloner, and Salop (1990)) and trade (McLaren (2000), Grossman and Helpman (2002), Antras (2003)) typically feature strategic complementarities in governance structure, and hence generically produce equilibria in which ex ante identical firms organize identically. One exception to this is Avenel (2008), who shows that investments in cost reduction (and hence governance structures that promote cost reduction) are strategic substitutes when firms compete Bertrand.
\[ \mu_x, \text{ and (iv) if } \Delta < (\bar{c} - \xi)\mu_x, \text{ then } \lambda^* \text{ is decreasing in } \Delta, \text{ otherwise there exists a } \sigma_v \text{ satisfying } \]
\[ 0 \leq \sigma_v \leq \frac{2\Delta(\bar{c} - \xi)\mu_x}{3\Delta + (\bar{c} - \xi)\mu_x} \text{ such that } \lambda^* \text{ is decreasing in } \Delta \text{ whenever } \sigma_v > \sigma_v \text{ and increasing in } \Delta \text{ whenever } \sigma_v < \sigma_v. \]

Proof. See appendix. ■

3.5.1 REE meets incomplete contracts

A further observation is that our incomplete-contracts approach sheds new light on the functioning of the price mechanism. In particular, most partially-revealing REE models compare the benefits of acquiring information to the exogenously specified costs of acquiring information. As our model shows, however, what matters is not only these exogenous costs, \( K_M \), but also the opportunity cost of choosing a governance structure that provides incentives to invest in information (namely, the foregone opportunity for cost reduction). To analyze these opportunity costs, consider the expression for \( \lambda^* \) when \( K_E \neq K_M \):

\[ \lambda^* = \frac{\sigma_v^2 + \Delta^2 - 2(\bar{c} - \xi)(\mu_x\Delta + K_M - K_E)}{\frac{\sigma_v^2}{2} \sigma_v/\xi - \xi} + 2\Delta^2. \]

Note the presence of production parameters, such as \( \Delta \) and \( K_E \), which have nothing per se to do with market clearing or price formation. More importantly, note that comparative statics regarding the informativeness of the price mechanism, such as \( \partial \lambda^*/\partial K_M \), can depend on production parameters such as \( \Delta \).

In addition to comparative statics that illustrate the potential effects of production parameters on rational-expectations equilibrium, we can also say something about how the production environment affects markets. For example, in GHP we showed that (as in Grossman and Stiglitz, 1980) market thickness depends on \( \lambda^* \), with concomitant implications for economic efficiency and welfare. In this paper's setting, therefore, market thickness depends on production parameters such as \( \Delta \) and \( K_E \).

3.6 Markets and Hierarchies Revisited

Coase (1937: 359) argued that “it is surely important to enquire why co-ordination is the work of the price mechanism in one case and of the entrepreneur in the other” (emphasis added). Similarly, Williamson's (1975) title famously emphasized “Markets” as the alternative to hierarchy. However,
over the next 35 years the market disappeared from the literature on firms’ boundaries. Instead, the literature focused on the choice of firm boundaries at the transaction level.

While our main focus is on the interaction between the choice of organizational designs by individual firms and the informativeness of the market’s price mechanism, a straightforward extension of our model also sheds light on the interaction between the choice of individual firms’ boundaries and the informativeness of the price mechanism. Like our analysis of organizational designs, this section shows that omitting the price mechanism from the analysis of firms’ boundaries can be problematic. In particular, we find that incentives to make specific investments (which now drive firms’ boundary decisions) affect the informativeness of the price mechanism and vice versa.

To extend and reinterpret our model, consider a vertical production process with three stages \((1, 2, \text{ and } 3)\) and a different asset used at each stage \((A_1, A_2, \text{ and } A_3)\). There are again two parties, now denoted upstream (formerly \(E\)) and downstream (formerly \(M\)). The conditions of production are such that it is optimal for the upstream party \((U)\) to own \(A_1\) and for the downstream party \((D)\) to own \(A_3\), so there are only two governance structures of interest (namely, \(U\) owns \(A_2\) or \(D\) owns it). Thus, the asset \(A_2\) is analogous to the machine from our original model, but we now focus on asset ownership as determining the boundary of the firm, rather than machine control as determining organizational designs. Because upstream necessarily owns \(A_1\) and downstream \(A_3\), we interpret \(U\) ownership of \(A_2\) as forward vertical integration and \(D\) ownership as backward. Beyond this reinterpretation of governance structures in terms of firms’ boundaries, all the formal aspects of the model are unchanged.

Under this reinterpretation, analogs of Propositions 3.1 through 3.3 continue to hold. In particular, our characterizations of the rational-expectations equilibrium and the industry equilibrium continue to hold, as do the comparative-statics results. Given this reinterpretation, the next two sub-sections explore the implications of the informativeness of the price mechanism for two leading theories of firms’ boundaries: the property-rights theory (PRT) of Grossman and Hart (1986) and Hart and Moore (1990), and the transaction-cost economics (TCE) theory of Williamson (1971, 1975, 1979).

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\(^{7}\) For formal statements and proofs, see an earlier working-paper version: Gibbons, Holden and Powell (2009), available at www.nber.org.
3.6.1 PRT Meets REE

Property-rights theory emphasizes the importance of specific investments for the choice of governance structure. To mimic the PRT, we eliminate the price mechanism in our model by supposing that a dyad (i.e., parties U and D) believes \( p(x, v) \equiv p \) for all \( \lambda, x, \) and \( v \) and hence does not recognize that prices are informative.

**Fact 3.2** If \( p(x, v) \equiv p \) for all \( \lambda, x, \) and \( v, \) then the benefits from choosing \( g_i = U \) are given by

\[
\pi_{U,\Delta} - \pi_{U,0} = \frac{1}{2} \frac{\sigma_v^2}{\bar{c} - c},
\]

and the benefits from choosing \( g_i = D \) are

\[
\pi_{I,0} - \pi_{U,0} = \frac{1}{2} \frac{\Delta^2 + 2(\mu_x - p - c) \Delta}{\bar{c} - c}.
\]

The dyad therefore chooses upstream ownership if \( \sigma_v^2 > \Delta^2 + 2(\mu_x - p - c) \Delta \), chooses downstream ownership if this inequality is reversed, and is indifferent if the inequality is replaced with an equality. Generically, one of these two inequalities must hold, so the PRT prescription will be either that all dyads are forward integrated or that all dyads are backward-integrated (because the dyads are identical *ex ante*).

In our model, however, the informativeness of the price mechanism endogenizes the returns to specific investments. In particular, dyads that would have chosen to invest in information acquisition (by choosing downstream ownership of asset \( A_2 \)) under the assumptions of Fact 3.2 may now free-ride on the information contained in the market price and choose instead to invest in cost reduction (by choosing to have upstream ownership of asset \( A_2 \)). In fact, in our model, certain governance structures may be sustained in equilibrium *only* because the price system allows some firms to benefit from the information-acquisition investments of others. More specifically, as we began to explain after Proposition 3.2, the equilibrium fraction of firms choosing downstream ownership in our model, \( \lambda^* \) in (3.3), is often interior, rather than zero or one, as is generically true in a PRT analysis of *ex ante* identical dyads.

Figure 3 illustrates the difference between our analysis and PRT by plotting \( \lambda_{PRT} \) versus our \( \lambda^* \) from Proposition 2. To plot this figure, we fix \( \Delta = 1/4, \bar{c} - c = 1, \) and \( \mu_x = 0.8, \) so that a PRT analysis predicts that all firms will choose downstream ownership (i.e., \( \lambda^*_{PRT} = 1 \)) if \( \sigma_v^2 > 0.3375, \) all firms will choose engineer ownership (\( \lambda^*_{PRT} = 0 \)) when \( \sigma_v^2 < 0.3375, \) and firms will be indifferent
(\lambda_{PRT}^* \in [0, 1]) when \sigma_y^2 = 0.3375. The figure also shows our model’s equilibrium \lambda^* as a function of \sigma_x^2 for three different values of \sigma_x (namely, 1/10, 1, and 10), with \lambda^* falling with \sigma_x for a fixed \sigma_y^2. Our equilibrium converges to \lambda^* = 1 more slowly (and especially slowly for lower values of \sigma_x).

![Figure 3: Comparison with PRT](image)

As drawn in the figure, the PRT analysis ignores the informativeness of the price mechanism. As a result, \lambda_{PRT}^* > \lambda^* for all values of \sigma_y^2. Alternatively, if the price mechanism was recognized as being partially but exogenously informative, then this would shift the vertical PRT line to the right, and it could be possible that \lambda_{PRT}^* < \lambda^* for all values of \sigma_y^2. The key idea here is that the PRT takes the environment in which firms operate (here, the informativeness of the price mechanism) as exogenous, whereas we highlight the two-way interaction between firms and their environment. As a result, empirical tests of PRT that utilize the importance of specific investments (as in Fact 3.2) may be misleading, by failing to consider the role that the price mechanism plays in endogenizing the returns to specific investments.

### 3.6.2 TCE Meets REE

Turning from PRT to TCE, recall that Williamson explicitly comments on Hayek’s (1945) discussion of the price mechanism, arguing that “prices often do not qualify as sufficient statistics and that a
substitution of internal organization (hierarchy) for market-mediated exchange often occurs on this account” (1975: 5). Our model allows us to assess this observation, if we can be precise about two things: (i) what it means for prices not to “qualify as sufficient statistics” and (ii) what is meant by “market-mediated exchange.”

A natural way to assess the extent to which prices are sufficient statistics is the following.

**Definition 3.3** The equilibrium informativeness of the price system is the expected reduction in variance $E_{x,v} \left[ \sigma_v^2 - \sigma_{v|p}^2 \right]$ that is obtained by conditioning on prices.

In our model, the equilibrium informativeness of the price system is given by

$$E \left[ \sigma_v^2 - \sigma_{v|p}^2 \right] = \frac{\sigma_v^2 \sigma_x}{\alpha^2 (\tilde{c} - \xi) \lambda^*}.$$  

Naturally, this informativeness is increasing in the fraction of firms that become informed, $\lambda$. And in our model “market-mediated exchange” also has a natural interpretation: it means relying on information about $v$ from the price mechanism, rather than acquiring it directly (i.e., upstream ownership rather than downstream). In these terms, Williamson’s claim can be stated as: when $E \left[ \sigma_v^2 - \sigma_{v|p}^2 \right]$ falls, $\lambda^*$ increases.

In our model, $\lambda$ is endogenous, so it matters what causes $E \left[ \sigma_v^2 - \sigma_{v|p}^2 \right]$ to decrease and what other effects that underlying change has on $\lambda$. For example, if $\sigma_x$ increases then it can be shown that informativeness decreases and $\lambda^*$ increases, as Williamson conjectured. On the other hand, many other changes in exogenous variables can lead simultaneously to a decrease in informativeness and a decrease in $\lambda^*$. For example, it is straightforward to see that an increase in $\mu_x$ decreases both informativeness and $\lambda^*$. And an increase in $\tilde{c} - \xi$ can do likewise, as reported in the following result.

**Proposition 3.4** Assume $(\tilde{c} - \xi) \frac{\sigma_v}{\sigma_x} \geq 1$ and $\lambda^* \in (0, 1)$. Define $\omega = \frac{1}{\tilde{c} - \xi}$. If

$$\frac{1}{2} \frac{\sigma_v^2 \sigma_x}{\sigma_x^2} \frac{\sigma_x}{\sigma_x} \omega + \frac{\sigma_v^2 + \Delta^2}{\Delta^2} < \mu_x < \frac{\sigma_v^2 + \Delta^2}{2 (\tilde{c} - \xi) \Delta^*},$$

then $\frac{\partial E_{x,v} \left[ \sigma_v^2 - \sigma_{v|p}^2 \right]}{\partial \omega} > 0$ and $\frac{\partial \lambda^*}{\partial \omega} > 0$.

**Proof.** See appendix. ■
3.7 Conclusion

We view firms and the market not only as alternative ways of organizing economic activity, but also as institutions that interact and shape each other. In particular, by combining features of the incomplete-contract theory of firms' organizational designs and boundaries, together with the rational-expectations theory of the price mechanism, we have developed a model that incorporates two, reciprocal considerations. First, firms operate in the context of the market (specifically, the informativeness of the price mechanism affects parties' optimal governance structures). And second, the buyers in the market for an intermediate good are firms (specifically, parties' governance structures affect how they behave in this market and hence the informativeness of the price mechanism).

In the primary interpretation of our model in terms of organizational design we provide a formal explanation for why similar (possibly ex ante identical) firms choose different structures and strategies (specifically, exploration or exploitation). Our analysis also demonstrates that viewing an individual firm, or transaction, as the unit of analysis can be misleading. Because of the interaction between firm-level governance choices and the industry-wide informativeness of the price mechanism, equilibrium governance choices are shaped by industry-wide factors.

We also showed that our model can be reinterpreted to address firms' boundaries. Again, considering the endogenous informativeness of prices implies that both property-rights theory and transaction-cost economics abstract from potentially important issues by focusing on the transaction as the unit of analysis.

To develop and analyze our model, we imposed several strong assumptions that might be relaxed in future work. For example, to eliminate a market for machines, we assumed that machines are dyad-specific. Also, as in our paper on price formation (where we analyze individual investors instead of firms), we ignore the possibility of strategic information transmission before or during the price-formation process. We hope to explore these and other possibilities in future work.

3.8 Appendix A: Computation of Price Function

This appendix outlines the approach for constructing the price function that is used throughout the paper. In doing so, we establish the existence of a partially revealing rational expectations equilibrium and prove proposition 3.1.
Proposition 3.5 Given \( \lambda \), there exists an REE characterized by a price function

\[
p_\lambda (x, v) = \sum_{j=1}^{3} 1\{ (x, v) \in R^j_\lambda \} P^j_\lambda (x, v),
\]

where \( P^j_\lambda (x, v) = \beta^j_0 + \beta^j_1 x + \beta^j_2 v \) for \( j = 1, 2, 3 \).

As in standard Walrasian general equilibrium theory, the markets must clear for each realization of \( p_\lambda (x, v) \), but as in Grossman-Stiglitz, demand is partially determined by the function \( p_\lambda (\cdot, \cdot) \) as well as its particular realization. A REE price function must therefore be a fixed point of the following identity (which is a rearrangement of the market-clearing condition).

\[
E [v | p_\lambda (\cdot, \cdot) = p_\lambda (x, v)] = \frac{p_\lambda (x, v) + (\bar{c} - c) x + c - (1 - \lambda) \Delta - \lambda v}{1 - \lambda},
\]

where the conditional expectation is determined by Bayesian updating given a price realization and assuming the equilibrium price function.

An iso-price locus is a set of \((x, v)\) pairs over which \( p(x, v) \) is constant. We assume that \( p(\cdot, \cdot) \) is increasing in \( v \), decreasing in \( x \), and that its iso-price curves are linear with constant slope for all \((x, v)\) (conditions that will of course need to be verified).

Define \( p_L = p_\lambda (\bar{x}, \bar{v}) \) and \( p_H = p_\lambda (\bar{x}, \bar{v}) \) to be, respectively, the lowest and highest possible prices, and define \( \bar{p} = p_\lambda (\bar{x}, \bar{v}) \) and \( p = p_\lambda (\bar{x}, \bar{v}) \). There are two possible cases. Case I (with \( \bar{p} \leq p \)) and case II (with \( \bar{p} > p \)) are depicted in the following diagrams.
Further, define $R_1, R_2,$ and $R_3$ to be, respectively, the low-, mid-, and high-price regions of the $(x,v)$. That is,

$$
R_1 = \{(x,v) : p_\lambda (x,v) \leq \min \{\bar{p}, \bar{\bar{p}}\}\},
R_2 = \{(x,v) : \min \{\bar{p}, \bar{\bar{p}}\} < p_\lambda (x,v) \leq \max \{\bar{p}, \bar{\bar{p}}\}\},
R_3 = \{(x,v) : p_\lambda (x,v) > \max \{\bar{p}, \bar{\bar{p}}\}\}.
$$

Assume we are in case I. The derivation proceeds similarly for case II, and we will describe how to determine which case applies below.

Suppose $(x,v) \in R_1$. Then because $x$ and $v$ are independent and uniform, the conditional distribution $v|p_\lambda (\cdot, \cdot) = p_\lambda (x,v) \sim U[v^1 (p_\lambda (x,v)), \bar{v}^1 (p_\lambda (x,v))],$ where $v^1 (p)$ and $\bar{v}^1 (p)$ are the lowest and highest values of $v$ consistent with the realized price $p$. As illustrated in the following diagram, since $(x,v) \in R_1$, it is clear that $v^1 (p) = v$. $\bar{v}^1 (p)$ on the other hand, solves $p_\lambda ^1 (\bar{v}^1 (p), \bar{x}) = p_\lambda ^1 (x,v)$.

Since we have conjectured that $p_\lambda ^1 (x,v) = \beta_0 + \beta_1 x + \beta_2 v$, we have

$$
\bar{v}^1 (p_\lambda ^1 (x,v)) = v - \frac{\beta_1}{\beta_2} (\bar{x} - x).
$$

The conditional expectation of $v$ given the realization of the price is therefore

$$
E[v|p_\lambda (\cdot, \cdot) = p_\lambda (x,v)] = \frac{v^1 (p_\lambda (x,v)) + \bar{v}^1 (p_\lambda (x,v))}{2} = \frac{v - \frac{\beta_1}{\beta_2} (\bar{x} - x) + v}{2}.
$$  \hspace{1cm} (2)
(1) must hold as an identity, so we can substitute (2), rearrange, and use equality of coefficients to give us

\[ \beta_0^1 = (1 - \lambda) \frac{v + ((\bar{e} - \xi) / \lambda) \bar{\xi} + (1 - \lambda) \Delta - \xi}{2} \]

\[ \beta_1^1 = -\frac{1 + \lambda \bar{e} - \xi}{\lambda} \]

\[ \beta_2^1 = \frac{1 + \lambda}{2} \]

Proceeding similarly for \((x, v) \in R^2_\lambda\) (where \(v^2(p) = v\) and \(\bar{v}^2(p) = \bar{v}\)) and \((x, v) \in R^3_\lambda\) (where \(\bar{v}^3(p) = \bar{v}\)), we have

\[ \beta_0^2 = (1 - \lambda) \frac{v + \bar{v}}{2} + (1 - \lambda) \Delta - \xi \]

\[ \beta_1^2 = -(\bar{e} - \xi) \]

\[ \beta_2^2 = \lambda \]

and

\[ \beta_0^3 = (1 - \lambda) \frac{((\bar{e} - \xi) / \lambda) \bar{\xi} + \bar{v}}{(1 - \lambda) \Delta - \xi} \]

\[ \beta_1^3 = -\frac{1 + \lambda \bar{e} - \xi}{\lambda} \]

\[ \beta_2^3 = \frac{1 + \lambda}{2} \]

Recall that we made the following assumptions in order to derive this: \(p_\lambda(x, v)\) is (1) decreasing in \(x\) and (2) increasing in \(v\), (3) \(-\frac{\partial p_\lambda}{\partial x} / \frac{\partial p_\lambda}{\partial v}\) is constant for all \((x, v)\), and (4) case I applies. (1) and (2) are satisfied, since \(\beta_1^j < 0 < \beta_2^j\) for \(j = 1, 2, 3\). (3) is satisfied, because \(-\beta_1^j / \beta_2^j = \frac{\xi - \bar{v}}{\bar{v}}\) for \(j = 1, 2, 3\). Finally, we must verify that indeed case I applies. In case I, the iso-price locus (which has slope \(\frac{\bar{e} - \xi}{\lambda}\)) is steeper than the diagonal (which has slope \(\frac{\bar{v} - v}{\bar{v}}\)). Thus, we are indeed in case I if \(\frac{\xi - \bar{v}}{\lambda} \geq \frac{\bar{v} - v}{\bar{v}}\) or \(\lambda \leq (\bar{e} - \xi) \frac{\partial \pi}{\partial v}\). We assume that \((\bar{e} - \xi) \frac{\partial \pi}{\partial v} \geq 1\), so that this condition is satisfied for all \(\lambda\). This allows us to use the same price function throughout. All of the main results of the paper go through if we drop this assumption, but we are no longer able to obtain a closed-form solution for the equilibrium industry structure. Computing the price function when \(\lambda > (\bar{e} - \xi) \frac{\partial \pi}{\partial v}\) is similar to the above analysis.
3.9 Appendix B: Omitted Proofs

3.9.1 Derivation of Fact 3.1

\[ E_{x,v,c} [\pi_{U,0}(\lambda)] - E_{x,v,c} [\pi_{U,0}(\lambda)] = \frac{1}{2} \frac{1}{\bar{c} - \xi} \frac{1}{\bar{v} - \xi} \int_0^\xi \int_0^\xi (v^2 - \mu_{v|p}^2) \, dx \, dv \]

\[ = \frac{1}{2} E_{x,v} \left[ \sigma_{v|p}^2 \right] = \frac{1}{2} \frac{\sigma_v^2}{\bar{c} - \xi} \left( 1 - \frac{\lambda \sigma_v / \sigma_x}{2 \frac{\bar{c} - \xi}{\bar{c} - \xi}} \right), \]

which is continuous and strictly decreasing in \( \lambda \) and similarly,

\[ E_{x,v,c} [\pi_{U,\Delta}(\lambda)] - E_{x,v,c} [\pi_{U,0}(\lambda)] = \frac{\Delta^2}{2(\bar{c} - \xi)} + \Delta \frac{E_{x,v} \left[ \mu_{v|p}(x, v) \right] - \xi - E_{x,v} [p_\lambda (x, v)]}{(\bar{c} - \xi)} \]

\[ = \frac{\Delta^2}{2(\bar{c} - \xi)} \lambda - \frac{\Delta^2}{2(\bar{c} - \xi)} + \mu_v \Delta, \]

which is continuous and strictly increasing in \( \lambda \). For the last equalities in these two expressions, we use the following three facts:

\[ E_{x,v} \left[ \mu_{v|p} \right] = \mu_v, \]

\[ E_{x,v} \left[ \sigma_{v|p}^2 \right] = \sigma_v^2 \left( 1 - \frac{\lambda \sigma_v / \sigma_x}{2 \frac{\bar{c} - \xi}{\bar{c} - \xi}} \right), \quad \text{and} \]

\[ E_{x,v} [p_\lambda (x, v)] = \mu_v + (1 - \lambda) \Delta - \mu_x (\bar{c} - \xi) - \xi, \]

which we now prove. First note that when \( \lambda \leq (\bar{c} - \xi)^2 \), \( p_\lambda (x, v) = \sum_{j=1}^3 1_{\{(x,v) \in \mathcal{R}_j^\lambda\}} \lambda^j (x, v) \),

where

\[ p^1_\lambda (x, v) = (1 - \lambda) \frac{v + (\bar{c} - \xi) / \lambda}{2} + (1 - \lambda) \Delta - \xi + 1 + \lambda - \frac{\bar{c} - \xi}{2}, \]

\[ p^2_\lambda (x, v) = (1 - \lambda) \frac{v + \bar{v}}{2} + (1 - \lambda) \Delta - \xi + \lambda v - (\bar{c} - \xi) x, \]

\[ p^3_\lambda (x, v) = (1 - \lambda) \frac{(\bar{c} - \xi) / \lambda + \bar{v}}{2} + (1 - \lambda) \Delta - \xi + 1 + \lambda - \frac{\bar{c} - \xi}{2}, \]

\[ \lambda v - \frac{1 + \lambda - \frac{\bar{c} - \xi}{2}}{\lambda x}, \]

91.
and

\[ R_1^1 = \{(x, v) : p_1^1(x, v) \leq p_1^1(\bar{x}, \bar{v})\} \]
\[ R_1^2 = \{(x, v) : p_1^2(x, \bar{v}) < p_1^2(x, v) \leq p_1^2(\bar{x}, \bar{v})\} \]
\[ R_1^3 = \{(x, v) : p_1^3(\bar{x}, \bar{v}) < p_1^3(x, v)\} . \]

We can rewrite the prices as

\[ p_1^1(x, v) = p_1^2(x, v) - \frac{1 - \lambda}{2} \left(\bar{v} - v\right) - \frac{\bar{c} - \xi}{\lambda} \left(\bar{x} - x\right) \]
\[ p_1^2(x, v) = (1 - \lambda) \frac{\bar{v} + \bar{v}}{2} + (1 - \lambda) \Delta - \xi + \lambda v - (\bar{c} - \xi) x \]
\[ p_1^3(x, v) = p_1^2(x, v) + \frac{1 - \lambda}{2} \left(v - \bar{v}\right) - \frac{\bar{c} - \xi}{\lambda} \left(x - \bar{x}\right) . \]

For simplicity of notation, define \( R_1^i(v) = \{x : (x, v) \in R_1^i\} \). That is

\[ R_1^1(v) = \left[\bar{x} - \frac{\lambda}{\bar{c} - \xi} (\bar{v} - v), \bar{x}\right] \]
\[ R_1^2(v) = \left[\bar{x} + \frac{\lambda}{\bar{c} - \xi} (v - \bar{v}), \bar{x} - \frac{\lambda}{\bar{c} - \xi} (\bar{v} - v)\right] \]
\[ R_1^3(v) = \left[\bar{x}, \bar{x} + \frac{\lambda}{\bar{c} - \xi} (v - \bar{v})\right] . \]

Finally, note that

\[ \mu_{\nu/vp}^1(x, v) = \mu_v - \frac{1}{2} \left(\bar{v} - v\right) - \frac{\bar{c} - \xi}{\lambda} \left(\bar{x} - x\right) \]
\[ \mu_{\nu/vp}^2(x, v) = \mu_v \]
\[ \mu_{\nu/vp}^3(x, v) = \mu_v + \frac{1}{2} \left(v - \bar{v}\right) - \frac{\bar{c} - \xi}{\lambda} \left(x - \bar{x}\right) . \]

Claim 3.1 \( E_{x,v} \left[\mu_{\nu/vp}\right] = \mu_v \)

Proof. Follows directly from the Law of Iterated Expectations. \n
Claim 3.2 \( E_{x,v} \left[\sigma_{\nu/vp}^2\right] = \sigma_v^2 \left(1 - \frac{\lambda}{2} \frac{\sigma_v}{\bar{c} - \xi}\right) \)
Proof. Here, we want to compute

\[
E_{x,v} \left[ \sigma_{v|p}^2 \right] = \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} \left( v^2 - \left( \mu_{v|p} \right)^2 \right) dx dv + \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} \left( v^2 - \left( \mu_{v} \right)^2 \right) dx dv + \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} \left( v^2 - \left( \mu_{v|p} \right)^2 \right) dx dv
\]

If we substitute and rearrange, this becomes

\[
E_{x,v} \left[ \sigma_{v|p}^2 \right] = \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} \left( v^2 - \left( \mu_{v} \right)^2 \right) dx dv + \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} \left( v^2 - \left( \mu_{v} \right)^2 \right) dx dv
\]

Integrating, we get

\[
E_{x,v} \left[ \sigma_{v|p}^2 \right] = \sigma_v^2 + \frac{\sigma_v^2}{\sigma_{v|p}^2} \left( \mu_{v} \right)^2 - \frac{1}{4} \sigma_\nu^2 = \sigma_v^2 \left( 1 - \frac{\lambda \sigma_v^2}{\sigma_x^2 \xi} \right),
\]

which was the original claim. 

Claim 3.3 \( E_{x,v} [p\lambda(x,v)] = \mu_v + (1 - \lambda) \Delta - \mu_x (\bar{c} - \xi) - \xi \)

Proof. Similarly as above,

\[
E_{x,v} [p\lambda(x,v)] = \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} p_{\lambda}^1(x,v) dx dv + \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} p_{\lambda}^2(x,v) dx dv + \frac{1}{\bar{u} - u} \frac{1}{1-x} \int_{\mathbb{R}} \int_{\mathbb{R}} p_{\lambda}^3(x,v) dx dv.
\]
If we substitute and rearrange, we get

\[ E_{x,v}[p_\lambda(x,v)] = \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} p_\lambda^2(x,v) \, dx \, dv - \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \frac{1 - \lambda}{2} \left[ (\bar{v} - v) - \frac{c - \bar{c}}{\bar{x} - x} \right] \, dx \, dv + \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \frac{1 - \lambda}{2} \left[ (v - v) - \frac{c - \bar{c}}{x - \bar{x}} \right] \, dx \, dv \]

or since the last two expressions are equal but with opposite signs,

\[ E_{x,v}[p_\lambda(x,v)] = \mu_v + (1 - \lambda) \Delta - (\bar{c} - \bar{c}) \mu_x - \xi, \]

which is the desired expression.

\[ 3.9.2 \text{ Derivation of Fact 3.2} \]

Explicit computation yields the following benefit for choosing \( g = U \)

\[ E[\pi_{U,\lambda}] - E[\pi_{U,0}] = \frac{1}{\bar{c} - c} \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} (v - p - c_i) \, dc_i \, dx \, dv - \frac{1}{\bar{c} - c} \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} (v - p - c_i) \, dc_i \, dx \, dv \]

\[ = \frac{1}{2} \frac{1}{\bar{c} - c} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} (v - \mu_v)^2 \, dxdv \]

and similarly the benefits for choosing \( g = D \) are

\[ E[\pi_{U,\lambda}] - E[\pi_{U,0}] = \frac{1}{\bar{c} - c} \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} (v - p + \Delta - c_i) \, dc_i \, dx \, dv - \frac{1}{\bar{c} - c} \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} (v - p - c_i) \, dc_i \, dx \, dv \]

\[ = \frac{1}{\bar{c} - c} \frac{1}{\bar{v} - v} \frac{1}{\bar{x} - x} \int_\mathbb{U} \int_\mathbb{I} \int_\mathbb{I} \int_\mathbb{I} \left( (v - p) \Delta - \xi \Delta + \frac{\Delta^2}{2} \right) \, dxdv \]

\[ = \frac{1}{2} \frac{\Delta^2 + 2(\mu_v - p - \xi) \Delta}{\bar{c} - \xi}. \]
3.9.3 Omitted Proofs

Proof of Proposition 3. To establish that $\lambda^*$ is increasing in $\sigma_v$, note that at $\lambda = 0$, the gains from choosing integration (and hence becoming informed) instead of non-integration (and hence enjoying a cost reduction) are given by

$$
(TS^U - TS^D)(\lambda = 0) = \frac{\sigma_v^2 + \Delta^2 - 2(\bar{c} - \bar{c})\mu_x \Delta}{2(\bar{c} - \bar{c})}
$$

and at $\lambda = 1$, the gains from choosing integration over non-integration are

$$
(TS^U - TS^D)(\lambda = 1) = \frac{\sigma_v^2}{2(\bar{c} - \bar{c})} - \frac{1}{2} \frac{\sigma_v}{\bar{c} - \bar{c}} \frac{\Delta^2 + 2(\bar{c} - \bar{c})\mu_x \Delta}{2(\bar{c} - \bar{c})}.
$$

Since we are at an interior solution, $(TS^U - TS^D)(\lambda = 0) > 0$ and $(TS^U - TS^D)(\lambda = 1) < 0$. Next, note that $(TS^U - TS^D)(\lambda = 0)$ is increasing in $\sigma_v$ and $(TS^U - TS^D)(\lambda = 1)$ is increasing in $\sigma_v$ if $(\bar{c} - \bar{c}) \frac{\sigma_v}{\sigma_x} > \frac{3}{4}$, which is true since $(\bar{c} - \bar{c}) \frac{\sigma_v}{\sigma_x} > 1$. Since $(TS^U - TS^D)(\lambda)$ is linear in $\lambda$, this then implies that $\lambda^*$ is increasing in $\sigma_v$.

The comparative statics with respect to $\mu_x$ and $\sigma_x$ are straightforward. Finally, note that

$$
\frac{\partial \lambda^*}{\partial \Delta} = 2 \Delta - (\bar{c} - \bar{c}) \mu_x - 2\lambda^* \Delta.
$$

When $\Delta < (\bar{c} - \bar{c}) \mu_x$, this is clearly negative. Otherwise, if $\Delta > (\bar{c} - \bar{c}) \mu_x$, the expression is positive. For $\sigma_v > \frac{2\Delta(\bar{c} - \bar{c})\mu_x}{3\Delta + (\bar{c} - \bar{c})\mu_x}$, the expression is negative. Since $\lambda^*$ is increasing in $\sigma_v$, this implies that there is a cutoff value $0 \leq \sigma_v \leq \frac{2\Delta(\bar{c} - \bar{c})\mu_x}{3\Delta + (\bar{c} - \bar{c})\mu_x}$, a function of the other parameters of the model, for which $\sigma_v < \sigma_v^*$ implies that $\sigma_v^* > 0$ and $\sigma_v > \sigma_v^*$ implies that $\frac{\partial \lambda^*}{\partial \Delta} < 0$.

Proof of Proposition 3.4. Note that

$$
\frac{\partial \lambda^*}{\partial \omega} = \frac{2\omega^{-2} \mu_x \Delta - \frac{\sigma_v^2}{2} \frac{\sigma_x}{\sigma_x} \lambda^*}{\frac{\sigma_v^2}{2} \frac{\sigma_x}{\sigma_x} \omega + 2\Delta^2} > 0
$$

whenever

$$
\frac{1}{2} \frac{\sigma_v^2}{\sigma_x^2} \omega + \Delta^2 \frac{\sigma_v^2}{2\omega - 1} \mu_x < \frac{\sigma_v^2}{\sigma_x} + \Delta^2 \frac{\sigma_v^2}{2\omega - 1}.
$$
and
\[ \frac{\partial E_{x,v}}{\partial \omega} \left[ \sigma_v^2 - \sigma_{wp}^2 \right] \geq \frac{\alpha_2}{\sigma_x} \left( \frac{2\Delta^2}{\sigma_x^2 \sigma_{wp} \omega + 2\Delta^2} \chi^* + \frac{2\omega^{-1} \mu_x \Delta}{\sigma_x^2 \sigma_{wp} \omega + 2\Delta^2} \right) \geq 0, \]

so that equilibrium informativeness is always increasing in \( \omega \).
Bibliography


