Vector Quantization for Spatiotemporal Sub-band Coding

by

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Abstract

This thesis investigates image coding using a combination of sub-band analysis/synthesis techniques and vector quantization. The design of a vector quantizer for image sub-bands is investigated, and the interplay between multi-rate filter banks and the vector coder is examined. The goal is twofold, first, a vector quantizer that is bounded by a distortion criterion versus an apriori fixed limit is essential to more optimally allocate bits in a sub-band coding system. This is due to the dynamic nature of the energy distribution in the sub-bands. Second, parameters for the vector quantizer that are psychophysically and statistically well matched to the characteristics of image sub-bands must be determined. Chrominance information is treated independantly due to reduced acuity in humans to chroma information.

Ultimately a moving picture coding system is proposed which affords significant bit rate reductions while maintaining a high degree of flexibility with respect to the feature set required in an interactive multi-media environment. Wherever applicable, the tradeoffs between coding efficiency and flexibility are discussed. Issues involving source material and coder/decoder complexity are also examined.

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Advances in microprocessor technology have made it possible to decode compressed image data streams at reasonable cost. Consequently, developing schemes that enrich the coded data by building a flexible representation is as vital as efficiency when designing image coding systems. Schemes providing a large gamut of features and organizing image data in an easy to manipulate manner support a much broader application base. For example, many established interframe prediction coding techniques are quite limited in terms of the features they support. While highly efficient and acceptable for the purposes of point to point communications, they possess characteristics which make common intuitive operations such as random access, fast search and bidirectional play either impossible or quite difficult to implement. The reason
is the resultant data streams are comprised solely of interframe prediction error. For
digital broadcast and video teleconferencing these issues are irrelevant by virtue of the
application context. However, they are significant in the case of multi-media systems
which place an emphasis on premastered, stored data streams. This premise sets the
tone for this investigation.

Resolution scalability is a highly desirable trait for image coding systems. From
both a psychovisual modeling and feature standpoint, it is advantageous to segment
the spatiotemporal spectrum into distinct regions for coding. This prompted the se-
lection of the sub-band paradigm as the platform for this work coupled with a vector
quantizer to provide the necessary compression. Traditionally, vector quantizers have
been applied to baseband images, but recently there has been an interest in apply-
ing this technique to image sub-bands. [2] [47]. Certain modifications to traditional
schemes will be shown to be psychovisually better suited to image sub-bands. Inter-
band energy dynamics tracking will also be incorporated into the quantization engine
so as to more optimally allocate bits within the spatiotemporal spectrum. This will
compensate for the burst nature of many of the image sequence’s constituent sub-
bands. Filter characteristics will be discussed with an emphasis on the temporal axis
of the three dimensional sub-band transform.

No comprehensive metrics exist that reflect human perception, thus making it
difficult to elegantly quantify all aspects of this type of system. Rigorous analyses
of the methods employed will be made when possible, but the most interesting out-
comes in this work are direct results of the abandonment of stringent mathematical
modeling. In particular, it will be shown that minimizing maximum error instead of
mean squared error in the vector quantizer when applied to certain sub-bands will yield higher perceived picture quality at a given bit rate, even though this ultimately results in a higher overall error rate. Furthermore, using temporal filter banks that have gradual roll-offs is more perceptually optimal when a lossy coding system is inserted between the sub-band analysis and synthesis stages although using these types of filters reduces coding efficiency by decreasing energy compaction and frequency separation. Again, the overall error rate is higher, but the obtrusiveness of the coding artifacts are greatly reduced.

The remainder of this work is divided into five chapters. Chapter two delves into the details of sub-band analysis. Chapter three discusses vector quantization techniques used to code sub-bands. Chapter four deals with coding chrominance signals in accordance with well known psychophysical properties. Chapter five proposes a complete image coding system based on the building blocks described in the previous chapters and Chapter seven concludes and lays the groundwork for future investigation. Appendix A is useful for those readers who are interested in hardware implementations of systems like the one proposed in chapter six in that it computes the computational cost of an arbitrary spatiotemporal sub-band transform implemented via convolution.

Before proceeding there are two terms that must be defined. The first is a common distortion metric known as the signal to noise ratio (denoted by SNR). It has several forms but the one used in this work follows:
SNR = 20 \log \left( \frac{DR(x)}{RMS\ (noise)} \right) \quad (1.1)

where:

\[ RMS\ (noise) = \sqrt{\frac{1}{N} \sum_{i=0}^{N} (x_i - \hat{x}_i)^2} \quad (1.2) \]

and the peak to peak dynamic range is expressed as:

\[ DR(x) = MAX(x) - MIN(x) \quad (1.3) \]

The second term that must be defined is one that determines the average number of bits needed to characterize a sample of a discrete signal from an information theory standpoint. This is otherwise known as the entropy of the signal. The bit rates quoted in this work are taken from the following equation:

\[ \text{bits/sample} = \sum_{i=0}^{N} -p(x_i) \log_2 p(x_i) \quad (1.4) \]

where \( p(x_i) \) is the probability of the sample value \( x_i \). Many practical algorithms
exist that allow one to get close to, or even fall below, the theoretical limit such as Huffman coding, run length encoding and arithmetic coding [37] [38] [29]. The reason it is possible to outperform this lower bound is that some entropy coders exploit the joint statistics between samples of a signal. The entropy calculation described in Equation 1.4 only utilizes first order statistics.
Chapter 2

Sub-band Analysis

This chapter investigates techniques which allow three dimensional image spectra to be analyzed into distinct regions and later be resynthesized with little or no overall distortion. Because there is a wealth of literature on spatial sub-band analysis, the focus will be on the temporal dimension and the overall performance of perfect reconstruction multi-rate filter banks as applied to lossy coding systems. References on the subject include works by P. P. Vaidyanathan [46], Karlsson and Vetterlli [20], Woods and O’Neil [49], Adelson et. al. [1] and Crochiere and Rabiner [4]. The discussion will be limited in scope to 1D separable FIR band splitters in light of the target applications.
2.1 Modeling the Human Visual System Using Spatiotemporal Sub-bands

Waveform coding systems designed to exploit spectrally local characteristics of both a source signal and a visual model must employ a set of band pass filters to subdivide the input signal's spectrum into distinct regions. Ultimately, the coded waveform will be resynthesized using a set of reconstruction filters that are matched to their analysis counterparts. The motivation for using such schemes for image coding in conjunction with a visual model is quite strong due to the human visual system's nonuniform response across different portions of the spatiotemporal spectrum. Schreiber [40], Troxel et al. [45], Glenn [12] [13] and Kelly [23] all discuss the reduced distortion sensitivity to high frequency spatial detail that humans exhibit. Glenn also shows that this spatial masking effect is amplified by a temporal one if the retinal image of the object is moving. The caveat here relies on the viewer's propensity to track the object in question. The tracking phenomenon is reinforced by Girod [10] who also examines this in a study which demonstrates there is little or no temporal masking effects should the observer track the object in question. In yet another study, Girod [11] constructs a nonlinear model of human threshold vision to determine when and where spatiotemporal masking can be used. The results according to the nonlinear model demonstrated the maximum savings due to masking are below .5 bit/sample on average.

These results illustrate the difficulty in modeling the human visual system. The best one can hope to achieve is to select a suitable model for the application at hand.
It would be much simpler to identify what, if any, spatiotemporal detail reductions would be transparent if one could predict what a viewer would track. Eye motion complicates the problem severely because motion estimates must be made relative to the retina to be useful. For example, if a viewer is tracking an object moving across a stationary background, the background is moving and the object is stationary from the viewer's perspective. If a system was designed to blur moving detail from the camera vantage point the result would be contrary to the ultimate goal, in that detail that is more difficult to resolve would be preserved, and that which, is easily resolved would be discarded. Because all viewers may not track the same objects in any given scene, and viewing conditions may vary, the only way to guarantee discarded detail is irrelevant is to incorporate eye tracking into the codec [10] and fix the viewing environment, which is highly impractical.

Spatially, the human response is also not easily modeled overall, but there are well known components of the early visual pathway that are surprisingly well approximated using simple filter banks. Butera[2] recognized the merit of this approach and employed a sub-band decomposition within an image coding system to model these components. He cites Wilson's work [48] which observes that groups of receptors in the human visual system are organized into roughly one octave wide circularly symmetric band pass channels. Each of these receptive units therefore responds to detail at a given scale and is isotropic in nature. This suggests a spatial sub-band decomposition of an image into one octave wide regions is a suitable and practical means of modeling the human visual system. It also has been shown through experimentation that humans have lower acuity with respect to diagonal edge detail than more vertical or horizontal orientations. It is important to note that source modeling alone estab-
lishes the same fact. Most images do not have flat spectra, in fact the amplitude of the spectral components decreases rapidly in proportion to spatial frequency. Octave subdivisions, therefore, make sense in that they divide the spectra into pieces which have roughly equal amounts of information.

There is less support for octave divisions temporally. In fact, it is well known that the human visual system is inherently bimodal with respect to temporal stimuli. At sufficiently low temporal frequencies, the eye acts as a differentiator, while at sufficiently high frequencies it can be modeled as an integrator. The point at which the transition is made depends on whether the stimulus is presented at the periphery of the receptive field or in the center as well as the illuminant amplitude [22]. Because the temporal response of the human visual system is highly context dependent, it is much more advantageous to establish a source model for the temporal dimension with which to adhere. Visual system characteristics can be used to temper the design of the overall temporal analysis/synthesis system. Due to its complex, context dependent response, the possibility of truly matching a practical image coding system to every detail of the human visual system is unlikely. It is very easy, however, to take advantage of some of its basic characteristics to develop a more visually tuned coder.

### 2.2 Temporal Source Model Considerations

The first point to note when designing systems which utilize temporal processing is that the varieties of input devices available yield a large set of possible input sam-
pling rates. Furthermore, many systems apply shutters (or lack thereof) to vary the amount of temporal bandlimiting before sampling. Because there is little support for any particular division scheme along the temporal axis, the logical approach is to choose the split points that accommodate the widest variety of input sampling rates. The pretense for this is to insure sub-bands from sources with varied sampling rates represent motions over an absolute velocity range. The most prevalent frame rates are those stemming from conventional imaging systems such as movie and television cameras. The 24Hz, 30Hz and 60Hz rates are widely seen in today's imaging systems in this country, all being conveniently divisible by 6Hz. Therefore temporal subdivisions at 6Hz intervals would be quite useful in that many input materials could be mapped into such a framework without interpolating. In general, input sequences should be frame rate converted to the system specified input rate. Traditionally, motion compensation systems are used to perform the necessary interpolation. In the case of the European 50 Hz standards, this is the only way of obtaining reasonable results to date.

An important point to note is that all the conventional sampling rates are not sufficiently high to prevent aliasing of some motion. Appropriately bandlimiting the image sequence often introduces an unacceptable amount of motion blur. In fact, sharp but temporally aliased motion pictures are usually preferable to blurred ones at most frame rates in common use. Most camera systems introduce some temporal bandlimiting by the nature of the sensor systems used and the integration period during frame times to accurately represent the image. This effect differs from system to system depending on the type of sensor and shutter employed, but most people have become accustomed to some degree of motion blur. This makes a strong argu-
ment for reducing the emphasis on moving spatial detail in an image coding system designed for very low bit rate storage and transmission and, in fact, accounts for the lack of energy in these bands. While it has been shown that eye motion overrules the psychophysical justifications, some information reduction must be done to avoid artifacts induced by overloading the coder when severe bit rate limitations are imposed. Because discarding moving detail seems to be tolerable in reasonable amounts, it is a good candidate solution. Usually this step will yield better overall image quality in comparison to systems which retain all the spatiotemporal components and overload the coder.

2.3 Sub-band Transforms and Image Coding

Visual system considerations aside, sub-band transforms have several characteristics which make them highly desirable for use in image coding applications. Because of the natural concentration of energy in the lower spatiotemporal sub-bands that image spectra exhibit, transforms of this type provide a means of separating out the energy rich portions of the spectrum. Because energy distributions will fluctuate over time, band adaptive coding scheme should be applied to optimally code the waveform. An added benefit of systems such as this is they inherently rearrange the data into a hierarchy of spatiotemporal resolutions.

While many current systems employ spatial decompositions only [47] [49], there is strong motivation to employ three dimensional spatiotemporal analysis techniques
because of the natural coherence groups of consecutive frames in an image sequence exhibit. Given a temporal transform with reasonable energy compaction characteristics, significant increases in coding efficiency can be achieved. However, there are substantial drawbacks involved in temporal sub-band analysis. The tighter the constraints on frame delays and/or decoder memory limits the more difficult it is to take advantage of these characteristics. Also, the atomicity of the coded data stream begins to decrease because adding temporal subdivisions directly impacts the amount of data necessary to decode a single frame. This is a serious issue when random access is critical. A loose analogy can be made here with predictive coding systems. As the temporal transform window size grows, the access problems approach those of purely predictive coding schemes with the added drawback of increased output frame delay.

An important point to note when designing transforms for lossy coding systems is that the distortion cancellation properties of perfect reconstruction QMF filter banks (described in Section 2.3.1) rely on the complete preservation of information between the analysis and synthesis stages. Different transforms exhibit different immunity to errors that are introduced between the analysis and synthesis banks. Unlike frequency domain scrambling systems or picture analysis and progressive transmission schemes, coding for low bit rate transmission and storage almost always involves substantial data loss between the analysis and synthesis phases. For these systems, minimizing the overall transfer function distortion is not enough. A good convention to follow is to select transforms whose individual basis functions serve as adequate decimation/interpolation filters. Transforms that rely too heavily on cooperative distortion cancellation will not perform as well when used in lossy coding systems. This is especially apparent temporally. Burst errors introduced by the coder will persist for
longer time periods after signal reconstruction given a synthesis filter bank with a non-trivial number of taps. The persistence time is directly proportional to the number of taps in the impulse response of the synthesis bank and the shape of the impulse response envelope. (See Section 2.3.3 for more details).

2.3.1 Perfect Reconstruction Multi-rate Filter Banks

The term sub-band analysis/synthesis system implies that the spectrum of an input signal is divided into distinct pieces. To accomplish this task it is necessary to associate a band pass filter(s) with each sub-band. This filter would be applied to an input in order to extract only those sinusoidal components that are contained within the associated sub-band. The ensemble of filters corresponding to the distinct frequency sub-bands can be collectively termed a filter bank. Because the output of each of the constituents of the filter bank will by definition have fewer components than the original signal it is possible to decimate the output of each filter to its new Nyquist rate. Systems of this type are termed “maximally decimating” and have the property of conserving the total number of samples after the analysis stage. This also lends the term multi-rate to the name of this class of transforms due to the non-constant sampling rate through the system. The synthesis stage is equally straightforward. It is comprised of a set of upsamplers (one for each sub-band) and corresponding interpolation filters. The output of which are summed to ideally yield the original signal. A block diagram for an arbitrary M band system is depicted in Figure 2.1. The diagram implies that the M bands have equal extents, hence, the constant decimation
rate across sub-bands. This is not a necessary condition.

The perfect reconstruction characteristic initially seems to require a filter bank composed of ideal band pass filters. Because these filters are unrealizable, ones with imperfect characteristics must be used, but this implies their responses will either be separated to avoid overlap resulting in an overall notching of the spectrum or they will be allowed to overlap resulting in aliasing (see Figure 2.2). Either choice does not satisfy the perfect reconstruction criterion.

The first point to note is that any notching in the analysis transfer function...
Non-overlapping analysis filters

Ideal analysis filters

Overlapping analysis filters

Figure 2.2: **Top:** Notched 2way band splitter. **Middle:** Overlapped 2way band splitter. **Bottom:** Ideal band splitter

that completely blocks a sinusoidal component from passing through the system, introduces an unrecoverable error. It is therefore necessary to allow the component filters to overlap resulting in aliasing. To begin to understand how this might be canceled in the overall system let us begin by writing the expression for an M fold decimator. Note that this has the effect of stretching the spectrum of the decimated signal by the decimation rate denoted by $M$.

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} X(e^{j(\omega-2\pi k)/M}) \quad (2.1)$$

If the signal has not been prefiltered by an appropriate amount or the prefilter
used has non ideal stop and transition bands, the stretched spectra may overlap after
decimating (see Figure 2.3 for an illustration of this for $M = 2$).

Figure 2.3: **Aliasing as a result of decimating the pictured spectrum (top) by a factor of 2**

The counterpart to the decimator is the M fold upsampler which is also a time
varying system. Analogous to the decimator, the upsampler corresponds to a com-
pression of the signal spectra. The expression for this in the frequency domain follows
(Note that the cascade of a decimator and an interpolator would be time invariant):

$$Y(e^{j\omega}) = X(e^{j\omega M})$$  \hfill (2.2)

Now that the basic building blocks have been described, the overall expression for
an arbitrary M band multi-rate filter bank can be written. Let M analysis filters be denoted by $H_M(e^{j\omega})$ and M synthesis filters by $F_M(e^{j\omega})$. Given equations 2.1 and 2.2 the overall expression for the system can be written as:

$$X'(e^{j\omega}) = \frac{1}{M} \sum_{n=0}^{M-1} X(e^{j(\omega - \frac{2\pi n}{M})} \sum_{k=0}^{M-1} H_k(e^{j(\omega - \frac{2\pi k}{M})}) F_k(e^{j\omega})$$  \hspace{1cm} (2.3)

The intricacies of the design of multiband filter banks is beyond the scope of this work. Let it suffice to say that it is possible to design filter banks such that the aliasing/imaging effects cancel leaving an overall transfer function of:

$$T(z) = \frac{1}{M} \sum_{n=0}^{M-1} H_k(e^{j\omega}) F_k(e^{j\omega})$$  \hspace{1cm} (2.4)

See Simoncelli [42] or Popat [35] for a elaborate discussion of design techniques. If $T(z)$ is an alias free system with no amplitude or phase distortion it then possess the perfect reconstruction property. The two band case of this general M band system is elegant and quite useful. It is the familiar quadrature mirror filter bank (QMF) introduced by Esteban, Croisier and Galand [5] [8]. Limiting the analysis to two bands yields a much cleaner expression for the overall input/output relationship which is shown below. The first term is the decimator/upsampler independent spectrum while the second represents the aliased overlap copies (see Equation 2.6).
Furthermore, choosing the synthesis filters to be

\[ F_0 (e^{i\omega}) = H_1 (-e^{i\omega}) \]  \hspace{1cm} (2.6)

\[ F_1 (e^{i\omega}) = -H_0 (-e^{i\omega}) \]  \hspace{1cm} (2.7)

yields the overall LTI transfer function

\[ T (e^{i\omega}) = \frac{1}{2} \left[ H_0 (e^{i\omega}) H_1 (-e^{i\omega}) - H_1 (e^{i\omega}) H_0 (-e^{i\omega}) \right] \]  \hspace{1cm} (2.8)

Adding the further constraint of

\[ H_1 (e^{i\omega}) = e^{-j\omega(N-1)} H_0 (-e^{-j\omega}) \]  \hspace{1cm} (2.9)
insures there is no aliasing, amplitude or phase distortion. The filter bank is then said to poses the perfect reconstruction property. It is possible to design an $M$ band (quasi) perfect reconstruction bank for $M > 2$, but it is beyond the scope of this work to explain the design methods in detail (See Popat [35]). Note that the term QMF does not truly extend to the multiband case because it is impossible for more than two unique filters to be mirrors of one another. The name is still applied to $M$ band systems though for historical reasons.

### 2.3.2 Pyramid Coding

The perfect reconstruction multi-rate filter bank provides many degrees of freedom to the designer of image coding systems. One of the most crucial involves picking the optimal subdivision of the spatiotemporal spectrum. There have been many attempts to design image coding algorithms that employ these techniques but most settle on one of two approaches. The first is to divide the spectrum into equal sized regions and the second is to use octave subdivisions i.e. each successive split cuts away half of the spectrum along a given axis. There have been attempts to divide the spectrum into regions using split planes that are not orthogonal to any of the spectral axes [42] [21] using nonseparable filters but for purposes of this work they will not be considered.

This work employs a technique commonly known as a QMF pyramid transform to spatially decompose images for later coding. The early work in pyramid transforms [1] grew out of vision research as a multiresolution image representation tool that
would facilitate pattern recognition, image manipulation and compression. Initially, an image was simply convolved with Gaussian lowpass filters of different widths and subsampled to its new Nyquist frequency. Each of these sub-images represented the original at a different scale. Laplacian pyramids were also investigated in a similar fashion. They can be computed by subtracting the bands resulting from two Gaussians of different variances.

For applications where sample conservation is critical, QMF pyramids are more desirable. The basic principle here is to apply a set of QMF band splitting filters to an image and subsample the output of each to its new Nyquist frequency [20] [42]. Unlike Gaussian or Laplacian pyramids, the total number of samples in the subimages equals that of the source. The term "pyramid" here refers to the recursive application of the filter bank to the lowpass band resulting from the previous application. The bands resulting from each application constitute a "level" of the overall pyramid. Note that because the same filter bank is applied recursively, the subdivisions are self similar at each level. Because the QMF banks can be made to be perfect reconstruction, the pyramid can be resynthesized with no distortion, barring digital precision shortcomings. The 9 tap QMF analysis/synthesis bank employed in this work (designed by Simoncelli [42]) to perform the spatial band splitting is pictured in Figure 2.4.

There is a simple mapping between the actual sub-bands and a more intuitive meaning which stems from the orientation selectivity of the filter banks employed. At each level of the spatial pyramid, each sub-band can be thought of as the output of two filter-decimate components (when using separable filters) of the lowpass component of the previous level. One filter-decimate is applied vertically and the other horizontally.
Figure 2.4: 9 tap QMF analysis/synthesis bank [42]. Top Left: Impulse response of the low pass filter. Top Right: Impulse response of the high pass filter. Bottom: Magnitude response of the two filters pictured above displayed simultaneously.
Each sub-band therefore, represents the output of one of four possible combinations of the high pass and low pass QMF filters selected. A table of the sub-band symbols and their origins follow with more graphical interpretations presented in Figures 2.5 and 2.6.

- **LL** - Low pass filter applied both vertically and horizontally
- **LH** - Low pass filter applied vertically and high pass filter applied horizontally
- **HL** - High pass filter applied vertically and low pass filter applied horizontally
- **HH** - High pass filter applied both vertically and horizontally

### 2.3.3 Temporal Band Splitting

As mentioned earlier, there is no evidence that octave subdivisions temporally (or any subdivision scheme for that matter) have any psychovisual support. It is therefore worthwhile to choose subdivisions which accommodate as many frame rates as possible. **Therefore, for this work**, a temporal QMF bank will be employed but it will not be used in the pyramid style described above. The band splitting will be done using equal sized subdivisions with the extent of the bands in the frequency domain set at 6Hz. Different filter banks will be employed for different frame rate source material so as to maintain the 6Hz bandwidth of each resulting temporal sub-band. For example, 24 fps source material would require a 4 way band splitter while 30 fps
Figure 2.5: Frequency domain subdivisions in a typical 3 level QMF pyramid. LL denotes lowpass filtering vertically and horizontally. LH denotes lowpass filtering vertically and highpass filtering horizontally. HL denotes highpass filtering vertically and lowpass filtering horizontally. HH denotes highpass filtering both horizontally and vertically.
Figure 2.6: Top: Original image from the sequence “Alley”. Bottom: Pyramid decomposition of the above image corresponding to the frequency domain subdivisions shown in Figure 2.5. Level 1 Gain = 16. Level 2 Gain = 8. Level 3 Gain = 4.
material would require 5. The impulse and frequency responses of the filter banks employed in this work are pictured in Figures 2.7, 2.8, 2.9 and 2.10. Notice that the 4 way band splitter is built from the cascaded application of a 2 way band splitter, i.e. the spectrum is split in two followed by a recursive application of the same bank to each of the pieces yielding the four desired sub-bands. To investigate the effects of non-orthogonal transforms, the 4 way system is also of this nature. This implies that the analysis and synthesis banks are distinct. In fact, the pictured response is one designed by Simoncelli [42] which uses a simple 3 tap filter for synthesis. Because he has constrained the tap values to be integer multiples of 2 it is possible to implement the synthesis stage using only shifts and adds, making a hardware gout implementation more tractable. Only 24 fps and 30 fps source material is investigated but the model easily extends to other 6Hz multiples. Ultimately, the system frame rate should be fixed and arbitrary inputs converted using a motion compensation system, as mentioned earlier, so as to fix the temporal analysis filter bank but this was not explored in this work.

One of the more interesting results this work has to offer is in the area of defining good properties for temporal transforms. A comparison of two 5 way band splitting filter banks will be used below to illustrate the key characteristics that impact picture quality in a lossy coding system. The basic rule is that it is possible to trade between “ringing” artifacts resulting from the Gibbs phenomenon and increased coding efficiency through better band separation. On the surface, this shouldn’t matter because this discussion is limited in scope to perfect reconstruction filter banks which supposedly cancel any filter artifacts upon reconstruction. Upon closer inspection, though, one finds that this is not the case if a lossy coding system is inserted between
Figure 2.7: 2 way band splitting filter bank (15 tap) used recursively to achieve the 4 way split - Left Column: Analysis transform basis functions. Right Column: Synthesis transform basis functions
Figure 2.8: **Magnitude response of the 2 way band splitting filter bank** - Top: Magnitude response of the analysis bank (15 tap). **Bottom**: Magnitude response of the synthesis bank (3 tap).
Figure 2.9: 5 way band splitting filter bank (7 tap) - Left Column: Analysis transform basis functions. Right Column: Synthesis transform basis functions
Figure 2.10: 5 way band splitting filter bank (7 tap) - Top: Magnitude response of the analysis bank. Bottom: Magnitude response of the synthesis bank.
the analysis and synthesis stages. The reason is that the coding artifacts will exhibit filter bank dependent manifestations.

As in most image interpolation systems, filters with sharp cutoffs result in large over/under shoots and the familiar ringing or Gibbs phenomenon. This effect is analogous to spatial ringing constrained to moving edges. The same edge replication structure is exhibited whose extent is proportional to the frequency response of the filter bank and the velocity of the moving edge in question. For systems which utilize block coders, this added structure can decrease the spatial correlation of the data. Since this artifact is associated with moving edges in the input sequence, the overall manifestation of the problem is a halo of noise surrounding moving objects.

Designing filters with more gradual transition bands minimizes the amount of ringing in the analyzed sequence. This results in significant basis function overlap in the frequency domain when these characteristics are incorporated into the design methods of perfect reconstruction multi-rate filter banks. Many stop band spectral components, therefore, "leak" significantly into each sub-band. This energy leakage sacrifices coding efficiency because the resultant sub-bands have more common information than their maximally separating counterparts which forces the post coding fidelity to drop at a given bit rate.

The conjecture now is that it is better to sacrifice efficiency and incur more coder error because the resultant artifacts will be less perceptually bothersome. Put in other words, a higher overall noise level (within reason) is better than structured, spatially localized halo artifacts. An experiment was conducted to demonstrate this
using two filter banks that exhibit different degrees of frequency separation. Two banks were designed using the methods outlined by Popat [35]. Differing amounts of frequency separation was achieved by varying the impulse response size allowed. A shorter impulse response results in more frequency domain overlap using Popat’s design technique.

The two filter banks, one 7 taps in size and one thirteen, are pictured in Figures 2.9 and 2.10 and Figures 2.11 and 2.12 respectively. Note their very different frequency responses, especially in terms of band separation characteristics. A 30 frame excerpt from the ISO MPEG\(^1\) standard test sequence “Table Tennis” was selected for this experiment. To compute the two transforms, each of the basis functions for each was convolved with the image sequence temporally with the subsequent output decimated by a factor of 5. Examples of the 5 resultant temporal sub-bands are pictured in Figure 2.14 for the 7 tap bank and Figure 2.15 for the 13 tap one. The associated region of support for these bands is pictured in Figure 2.13. Compare bands 2-4 in both figures for the spatial extent of the “ring” surrounding moving edges and note that The 13 tap filter bank produces significantly more of this effect. Bands 2 - 4 from both filter banks were then coded using a vector quantizer. Tables 2.1 and 2.2 list the individual bands and their associated coding parameters as well as their post entropy coding bit rate. The bands emanating from the 13 tap bank were coded first. The overall distortion was clamped using an error limit vector quantizer (see Section 3.3 allowing the code book size to freely arrive at the sizes listed in Table 2.1. The bands resulting from the 7 tap bank were then coded with the same vector coder but this time the code book sizes were dictated to be the same as those in the 13 tap

\(^1\)International Standards Organization / Moving Pictures Experts Group
experiment. The difference between post entropy coding bit rates for both the codes and codebooks were insignificant enough to allow for a fair comparison of the filter bank performance.

<table>
<thead>
<tr>
<th>Band</th>
<th>Vector Dims.</th>
<th>SNR (Fixed)</th>
<th># Vectors</th>
<th>bits/code</th>
<th>bits/vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4x4</td>
<td>36 db</td>
<td>4119</td>
<td>9.86</td>
<td>91.80</td>
</tr>
<tr>
<td>3</td>
<td>4x4</td>
<td>34 db</td>
<td>2534</td>
<td>8.34</td>
<td>80.58</td>
</tr>
<tr>
<td>4</td>
<td>4x4</td>
<td>32 db</td>
<td>1211</td>
<td>7.23</td>
<td>66.27</td>
</tr>
</tbody>
</table>

Table 2.1: Bit rate and vector quantizer parameters for bands 2-4 resulting from the 13 tap analysis/synthesis filter bank. Note: Maximum error clamped to specific SNR's.

<table>
<thead>
<tr>
<th>Band</th>
<th>Vector Dims.</th>
<th>SNR</th>
<th># Vectors (Fixed)</th>
<th>bits/code</th>
<th>bits/vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4x4</td>
<td>34.13 db</td>
<td>4119</td>
<td>10.05</td>
<td>91.68</td>
</tr>
<tr>
<td>3</td>
<td>4x4</td>
<td>30.89 db</td>
<td>2534</td>
<td>8.66</td>
<td>80.57</td>
</tr>
<tr>
<td>4</td>
<td>4x4</td>
<td>28.14 db</td>
<td>1211</td>
<td>7.64</td>
<td>70.13</td>
</tr>
</tbody>
</table>

Table 2.2: Bit rate and vector quantizer parameters for bands 2-4 resulting from the 7 tap analysis/synthesis filter bank. Note: Code book sizes fixed to match those in Figure 2.1.

As to be expected, both coded versions had visible errors whose character differed significantly (see Figure 2.16). The results support the conjecture made earlier in that it is possible to trade global noise for correlated, spatially localized artifacts at a given bit rate by altering the transition band characteristics of the analysis/synthesis filter bank used. The manifestation of this can be seen in Table 2.2 where the post coding SNR's for each vector quantized band resulting from the 7 tap bank are lower than those fixed in the 13 tap case. Because the vector coder employed was designed to minimize maximum error, overall sharpness was well preserved in the 7 tap case, as can be seen in Figure 2.16 confining the noise to be correlated with the low amplitude
detail (this assumes that there is no quantizer overload). The longer filter bank produced sub-bands with a higher overall SNR at a given bit rate at the expense of significantly more halo error surrounding moving objects. Because this type of error is more correlated than those introduced by less efficient filter banks, it is easily detectable.

In summary, this experiment demonstrates that maximally separating filter banks result in more correlated errors with larger spatial extents than their more gradual transition band counterparts. The reduction in coding efficiency that shallower transition bands introduce results in a lower overall SNR in the sub-bands after coding at a given bit rate. The characteristics of these errors, though, make them more tolerable than those which result from coding sub-bands emanating from more statistically efficient filter banks even though such banks afford a higher overall SNR in a given sub-band at a given bit rate.

From a computational standpoint, these results have some interesting implications. Given that the 7 tap bank employed herein is a reasonable filter bank to use in an image coding application, one can make the conjecture that relaxed transition band characteristics allow the use of shorter kernel filter banks. Furthermore, the similarity between the 7 tap basis functions and those of a 5 point discrete cosine transform (which has an efficient $n \log n$ implementation) warrants the investigation of more computationally efficient filter banks. The DCT or LOT (lapped orthogonal transform) are excellent candidates and may prove to be fruitful.
Figure 2.11: 5 way band splitting filter bank (13 tap) - Left Column: Analysis transform basis functions. Right Column: Synthesis transform basis functions
Figure 2.12: 5 way band splitting filter bank (13 tap) - Top: Magnitude response of the analysis bank. Bottom: Magnitude response of the synthesis bank.
Figure 2.13: 13 frame excerpt from the sequence "Table Tennis"
Figure 2.14: 5 sub-bands resulting from applying the 7 tap 5 band transform pictured in Figures 2.9 and 2.10 to the middle 7 frames pictured in Figure 2.13. **Top Left:** Band 0. **Top Right:** Band 1 (gain = 2). **Middle Left:** Band 2 (gain = 3). **Middle Right:** Band 3 (gain = 5). **Bottom:** Band 5 (gain = 6).
Figure 2.15: 5 sub-bands resulting from applying the 13 tap 5 band transform pictured in Figures 2.9 and 2.10 to the frames pictured in Figure 2.13. Top Left: Band 0. Top Right: Band 1 (gain = 2). Middle Left: Band 2 (gain = 3). Middle Right: Band 3 (gain = 5). Bottom: Band 5 (gain = 6).
Figure 2.16: Top: Original. Middle Left: 13 tap analysis/synthesis bank w/bands 2-4 vector coded. Middle Right: 7 tap analysis/synthesis bank w/bands 2-4 vector coded. Bottom Left: Error signal resulting from 13 tap analysis/synthesis system w/vector coding (gain = 8). Bottom Right: Error signal resulting from 7 tap analysis/synthesis w/vector coding (gain = 8).
Chapter 3

Vector Quantization

This chapter explores a well known statistical coding method known as vector quantization. The technique was first developed to code speech waveforms but its applicability to images was quickly recognized. Recently this technique's application to image coding has been extended to include image bands [2] and has been shown to perform well. A discussion of the statistical grounds for using a vector quantizer to code sub-bands is presented. The burst nature of the information distribution across spatiotemporal sub-bands is discussed and a paradigm termed "error limit coding" is proposed to compensate.

Some familiarity with the topic is assumed. For a thorough review of vector coding
3.1 Background

A quantizer is defined to be a system that maps a domain to a range in a many to one fashion. Thought of in terms of coordinate systems, a quantizer divides the domain space into regions and maps all points residing in each to a single representative value. In most cases, the term quantizer refers to the type of operation described above applied in one dimension (also known as a scalar quantizer). A vector quantizer therefore, partitions a multidimensional space into cells and maps all values in the domain that are localized to a cell to a single representative value. Each individual vector can therefore be thought of as a coordinate in a multidimensional space. Coders that operate on vectors are preferable from a rate-distortion theory standpoint because they can take advantage of correlations that exist between multiple dimensions.

In the case of images, the intuitive space is that of all possible gray values as dictated by the dynamic range specifications of the input signal. For single pels, this space is limited to one axis where the number of cells dictates the maximum number of intensity levels that can exist in one image after quantization. This notion can be extended to an arbitrary number of dimensions by grouping samples together to form a multidimensional coordinate or “vector”. If the image were then divided up into groups of samples where all the groups were of like dimensions, a multidimensional
space would be defined where each vector would correspond to a single point. In most image coding applications, the vectorization process usually divides the input image into rectangular blocks of pels. Vector quantizing an image therefore corresponds to fragmenting it into N dimensional blocks and calculating a new, smaller set to represent them. This new set of blocks is known as the code book. The image is then encoded by selecting a code book entry to represent each original vector thereby only requiring the transmission of one index to retrieve multiple image samples. Of course, bandwidth must be reserved for table transmission so the block size (which is directly related to table size) must be adjusted so as to provide compression without necessitating a table that is too large else the savings created by the vector coding will be offset by the table bandwidth. Note that in some cases, vectors are formed from samples occurring in several distinct data channels such as corresponding blocks of samples in the red, green and blue channels of an image. This will be known from here on as jointly coding channels.

Many schemes have been devised to partition a given vector space into distinct cells but most exhibit $O(n^2)$ order statistics. These are the class of iterative decent techniques used to converge to the minimum distortion code book of a given size. To alleviate some of the computational burden, the input signal is randomly sampled (to form the "training set") to reduce the number of constituent vectors used in calculating the code book. This has been shown to only minimally impact image quality.

The first and most common of these convergence based schemes was developed by Linde, Buzo and Gray [26] (termed LBG). It operates in the following manner.
An initial guess for a representative population (code book) is made. The Euclidean distance between each input vector and each representative is calculated and the input vector is assigned to the representative that is nearest to it. After all input vectors are assigned to representatives, the aggregate error introduced by coding each input vector with the current code book is calculated. Each vector in the code book is then replaced by the centroid of the cluster of vectors that map to it and the procedure is repeated. When the distortion reaches a minimum, the calculation is complete. There are obvious problems with this though. First, it is impossible to bound the convergence time or to guarantee that the algorithm will converge at all and second, if the algorithm does converge on a minimum there is no guarantee that that minimum is global.

Because the LBG method is computationally intense for all but trivial population sizes for both code book generation and the subsequent image encoding with the derived code book, it is useful to employ other algorithms which have more reasonable order statistics. Representations for multidimensional spaces is a classic problem in computational geometry to which much attention has been paid. If some constraints are made as to the location of split hyperplanes, one can devise tree based algorithms that have efficient \( n \log n \) implementations for both code book generation and image encoding.
3.2 Tree Based Algorithms for Vector Quantization

3.2.1 History

Tree-like data structures are appealing for both the search and subdivision of data due to the efficient $n \log n$ computational complexity they exhibit. Kd-Trees (K dimensional trees), introduced by Bentley [9], make it possible to characterize multidimensional data as a binary search tree at the expense of some constraints placed on the subdivision process. They were first applied to the problem of color palate selection (which is inherently vector quantization) by Heckbert [15] and later formally discussed in terms of general purpose vector quantization by Equitz [7]. Kd-Trees are general binary trees in that only a single binary decision may be stored at any node. When used for data organization, these decisions usually involve characterizing an object as to the "left" or numerically less than or equal to some split point or to the "right" or numerically greater than some split point. The extension that makes Kd-Trees unique is that the dimension which is subdivided at each node may be different. This mechanism provides a convenient way of organizing vector data using only locally scalar partitions with all calculations limited to only that node's constituency.

Building a vector quantizer in this paradigm is very simple. Each node represents a single partition of the entire space and contains pointers to its children, split
parameters such as the dimension to be split and split point along that axis, and a single vector to represent the cell population. Splitting the space is also simple and self-similar at each node. First one populates the root of the tree with the entire multidimensional space. This initial hyper-rectangle is then iteratively subdivided by continually splitting the leaf with the highest overall distortion along a single dimension until some bound condition is met. The split hyperplanes are constrained to be orthogonal to the axis being split (this is a direct result of the one-dimensional nature of each node). Figure 3.1 depicts an arbitrary Kd-tree with $K = 2$. Methods of selecting the split dimension and split point represent some of the degrees of freedom that are useful for tuning the quantizer to specific applications and will be discussed later. The distortion criterion used for leaf selection though, has the most impact on the granularity of the step sizes at various locations throughout vector space.

Unlike the LBG method, this style of vector quantizing is deterministic in that the number of operations needed to split a given space into a predetermined number of regions is known a priori. Again, it is important to note the distinction between the Kd-tree algorithm and the more general LBG style methods. The canonical Kd-tree foregoes arbitrary split planes and multidimensional distortion measures for a reduction in computational complexity and determinism. Sproull [43] devised a means of incorporating arbitrary split planes into the Kd-tree with the added expense of more computation at each decision point. This extra cost still does not approach the expense of a full LBG calculation because of the exponentially decreasing amount of data that need be analyzed with each split in a Kd-Tree.
Figure 3.1: **Kd-Tree**: A typical Kd-Tree ($K = 2$) viewed as both a binary tree and a subdivided 2 dimensional space.
3.2.2 Implementation Specifics

This section will describe, in detail, the basic implementation of the Kd-tree based vector quantizer employed in this work. Some interesting extensions will be discussed in the next section but the presentation here is limited to canonical form.

First we must begin by defining the three basic control parameters that dictate the outcome of the vector coder.

- **Node/Dimension Selection:** This parameter dictates how leaves are prioritized for subdivision. Candidates for this include selecting the leaf (and dimension within that leaf) based on the variance, cell extent, mean squared error (with respect to the representative), maximum error (with respect to the representative) or maximum number of constituents. A distortion function is derived based upon one of the aforementioned metrics to calculate each leaf's "selection value". This function is constrained to return a monotonically increasing result in proportion to the amount of distortion incurred by representing a given constituency with its representative. This function must also return a value of 0 if the node in question should undergo no further subdivisions. With each iteration the leaf with the highest selection value becomes the candidate for subdivision. Note that this value is calculated for each dimension in the cell and the highest one is retained as the nodes actual selection value. The dimension that resulted in this maximum score is set to be the split axis should the leaf be selected for subdivision.
• **Split Point Selection:** This parameter determines how the split point along the chosen split axis is calculated. Candidates for this are the mean, median, mode or midpoint of the split dimension.

• **Representative Calculation:** This parameter describes the method used in calculating the representative vector for a given cell. Candidates for this include selecting the centroid of the cell's population, the midpoint of the cell or the constituent vector with the most occurrences in the original signal that resides in that cell.

• **Bounding Condition:** This parameter sets the criterion which must be met before the subdivision process terminates. Candidates for this are the codebook size, total distortion incurred by quantizing and Peak distortion incurred by quantizing. Note that the term “distortion” here can be replaced with any metric desired.

The split iteration is simple. The tree initially begins as a single node whose constituency is the entire vector space. The selection value, split point and representative vector are also calculated. The following loop is then executed until some ending condition is met:

1. Find the leaf with the maximum selection value.

2. Make the leaf a node and create two new leaves. Make one the left child and one the right child. Assign all vectors in the node to either the left or right child depending on whether the vector is less than or equal to or greater than the split point respectively.
3. Calculate the selection value/dimension, split point and representative vector for both new children.

4. If bound condition is not met, Goto step one.

### 3.3 Error Limit Coding

The bit allocation problem for vector quantizers is difficult. Unlike scalar quantizers whose statistics are limited to a computationally tractable one dimension, vector quantizers exploit the joint statistics of the individual elements of a vector. It would not be feasible to truly calculate these statistics for any non-trivial sized source image. To circumvent this problem, a new paradigm for bounding vector quantizers is proposed known as error limit coding. As its name implies, an error limit quantizer's code book growth is bounded by a distortion criterion instead of the usual code book size limitation. Systems, such as this, which have nonuniform output rates must be coupled with a buffering system for transmission and reception to compensate for the inherent bit rate fluctuation.

Given that the amount of information in the source signal varies substantially over time, it is convenient to take advantage of periods of reduced data requirements by sending some portion of the data for later, higher bandwidth segments. If the source signal is broken into parallel components, error limit clamping becomes a convenient way of distributing the available bandwidth. Spatiotemporal sub-band
decompositions exhibit exactly these characteristics in that the proportion of the signal energy that is devoted to each band varies substantially over time. This has been observed by Vetterli [20] and is well characterized in Figure 3.2 which represents the variance of the individual sub-bands over time.

The selection of an appropriate distortion criterion is the next relevant topic that must be addressed. Because certain sub-bands are extremely sparse (few of their samples have significant amplitudes), it is not wise to use global error metrics. As one can infer from the histograms of certain spatiotemporal sub-bands pictured in Figure 3.3, it is possible to achieve low mean squared errors or high signal to noise ratios by simply zeroing the band. This will usually have detrimental effects on the overall picture quality of the reconstructed images because of the small amounts of significant spatially local information which is wrongly discarded. To avoid this problem, a spatially local metric should be adopted. The one found to produce the best results was that of clamping the maximum Euclidean distance between a constituent vector and its representative. For computational reasons the squared form of this measure (mean squared error) was adopted to avoid the added complexity of having to evaluate a square root for each constituent vector in a cell. Another advantage of the mean squared error is that it is a linear distortion metric i.e. the distortion of the whole signal equals the sum of the distortion of its parts. This is a convenient property should it become necessary to keep a running estimate of global distortion while quantizing because it is possible to additively update this metric upon splitting a leaf without recalculating the distortion over the entire signal.

Another important property of error metrics for sparse, bursty signals is that they
Figure 3.2: Variance "Cardiogram": Variance of selected sub-bands plotted as a function of time for the test sequence "Alley".
Figure 3.3: **Histograms of selected sub-bands.** Top Left: Histogram of the LH channel of pyramid level 1. Top Right: Histogram of the LH channel of pyramid level 2. Bottom: Histogram of the LH channel of pyramid level 3. Note that all of the above originate from a selected pyramid assembled from the test sequence "Table Tennis"
be dynamic range independent. Metrics such as the signal to noise ratio were initially investigated and then abandoned for sparse signals because of their spurious, signal dependent performance. Given that the available input dynamic range is known and fixed, it is advantageous to determine a psychovisually well suited absolute error limit. This yields much more consistent results across diverse source material.

Modifying the Kd-Tree algorithm to be bounded by some distortion criterion is trivial. For the simple maximum error bound, all that need be stored at each node is the maximum mean squared error between any constituent vector and its representative. Upon each split iteration, the list of leaves is scanned for maximum error and it is compared to the bound. If the error limit is met, the quantization is halted. A global error error metric can be tallied trivially should this type of bounding be deemed suitable. Because mean squared error is an additive metric, it is possible to update it upon each split by subtracting the contribution of the leaf to be split from the total and adding the error contributed by each new child (see Equation 3.1. The only constraint here is that the metric be derivable from the mean squared error such as the root mean square or signal to noise ratio.

\[
E'_{\text{total}} = E_{\text{total}} - E_{\text{split leaf}} + E_{\text{left child}} + E_{\text{right child}}
\]  

(3.1)

A parameter that goes hand in hand with the error bound is selection method used to rank cells for subdivision. The choice for this which natural complements the peak error bound and yields the best results is that of selecting the cell with the single largest distance between any of its constituents and their representative
along a single dimension. While this may yield a lower overall signal to noise ratio, the resultant picture quality of the reconstructed image will be better because of the natural favoring of sharp edges. One problem with this approach is that it results in many vectors that are used infrequently because of the concentration of subdivisions in areas of vector space that are sparse. This increases the information “richness” of the coded signal causing decreased entropy coder performance.

3.4 Jointly Coding Spatial Detail in QMF Pyramids

The 2D QMF pyramid transform (discussed in Section 2.3.1) decomposes a source signal into spatially oriented band pass components at various scales. These bands are also in most cases not statistically independent. This implies that there are efficiency gains that are exploitable by jointly encoding the horizontal, vertical and diagonal bands. The justification for this is a direct result of the statistics natural images. As illustrated in Figure 3.4, few edges that occur in images that contain man made structures are obliquely oriented. Furthermore, for small spatially localized regions of an image, horizontal and vertical detail will not coincide. Hence the benefit of combining small corresponding regions of pels from the horizontal, vertical and diagonal channels to form a single vector for coding.
Figure 3.4: **Edge Orientation**: Probability distribution of the angle (with respect to horizontal) of edges for an outdoor scene containing man-made structures (from F. Kretz).
Chapter 4

Chrominance Coding

This chapter discusses color perception and its implications with respect to picture coding. A method of coding chroma information using vector quantization techniques is proposed that affords significant data reduction while maintaining near transparent quality.
4.1 Color Vision

It is well known that color perception is a three degree of freedom system and therefore characterizable by three linearly independant primaries. Grassman’s laws imply that these primaries obey the laws of linearity with respect to the human visual system without loss of generality. It is therefore convenient to consider these primaries as basis functions that span a three dimensional color space. Because of the convenient linearity constraint, it is possible to derive linear transformations to map a set of samples acquired with respect to one primary space to another. For the purposes of coding images for bandwidth reduction, it would be advantageous to select a coordinate system whose axes were not of equal importance with respect to the response of the human visual system. This would partition the input data into three parallel channels of skewed visual importance. It then would be possible to code each of the three channels differently and in such a way that the algorithm selected and the error introduced was well matched to the specific human response to that channel.

Many experiments have been performed to determine how people respond to incident light and conclude that humans have different responses to chromatic and achromatic information embodied in the stimuli. One of the many contrast sensitivity tests that examine this topic was performed by Mullen in 1984 [30]. She demonstrated that humans have higher acuity for achromatic gratings than for constant luminance chromatic gratings. Her results are summarized in the graphs of contrast sensitivity versus spatial frequency shown in Figures 4.1 and 4.2.
Figure 4.1: Contrast sensitivity as a function of spatial frequency for the red-green chromatic grating and a green monochromatic grating [30]

Figure 4.2: Contrast sensitivity as a function of spatial frequency for the blue-yellow chromatic grating and a yellow monochromatic grating [30]
These and other similar results suggest that a transform which maps source primaries into an orthogonal luminance/chrominance space is psychovisually well matched. The NTSC color television system operates in such a space, commonly known by the symbols YIQ (Y = luminance, IQ = chrominance). The I axis spans the orange-cyan color hues which play a major role in flesh tone rendition with the Q axis encompassing the green-magenta information [39]. This color space was chosen for this work. The matrices which characterize the linear transformation from the commonly used RGB primaries to YIQ space and back are given by equations 4.1 and 4.2 respectively [39].

\[
\begin{align*}
    Y &= \begin{bmatrix} 0.299 & 0.587 & 0.114 \end{bmatrix} R \\
    I &= \begin{bmatrix} 0.596 & -0.274 & -0.322 \end{bmatrix} G \\
    Q &= \begin{bmatrix} 0.211 & -0.522 & 0.311 \end{bmatrix} B \\
\end{align*}
\] (4.1)

\[
\begin{align*}
    R &= \begin{bmatrix} 1 & 0.956 & 0.623 \end{bmatrix} Y \\
    G &= \begin{bmatrix} 1 & -0.272 & -0.648 \end{bmatrix} I \\
    B &= \begin{bmatrix} 1 & -1.105 & 0.705 \end{bmatrix} Q \\
\end{align*}
\] (4.2)

Temporally, color perception differs substantially from that of luminance. Studies by Glenn [12] and Kelly [23] [24] show the critical fusion frequency for chroma signals is substantially below that of their luminance counterpart. Butera [2] conducted a study which confirms this observation. He constructed several image sequences such that the chrominance information was updated at exactly half the rate of the luminance (15 Hz). The results varied from imperceptible to marginally perceptible
demonstrating the viability of such a scheme in an image coding environment. The caveat here is that the temporal granularity of the decoded frames with respect to chrominance differs from that of luminance. If still picture extraction is required, one must conform to the lowest granularity of all picture components thereby restricting still availability.

4.2 Source Model Considerations

Many psychophysical experiments were carried out by researchers developing the initial color television systems. These were analogous to the ones conducted by Mullen [30]. Because of the experimental results indicating reduced chrominance acuity in humans, color television system developers decided to bandlimit the chrominance signals before transmission to conserve bandwidth. Consequently, the chrominance information available in imagery originating from color television cameras is limited to 1.5 Mhz for the I channel and .5 Mhz for the Q channel in the horizontal direction. Although many commercially available television components only demodulate .5 Mhz of I. Given the luminance information is bandlimited to approximately 4Mhz, the ratio of luminance to chrominance resolution is 8:1. While this ratio only applies horizontally in television systems due to the raster scan scheme employed, Butera applied it vertically as well for the purposes of image coding by assuming an isotropism constraint [2].

The isotropism constraint must be qualified here though. While it is true the
human visual system's frequency response is isotropic, it does not imply bandlimiting in the vertical direction to match the horizontal limit will be undetectable. While it is true humans have reduced chroma acuity, it is not true that $\frac{1}{8}$ of the luminance resolution represents the upper limit of what is visible. Furthermore, all resolution issues are functions of viewing conditions whether they involve chromatic or achromatic signals. The true objective here involves using general psychophysical observations to reduce the data rate for the purposes of image coding. Because it has been shown that humans are relatively less sensitive to chromatic bandlimitations, this represents a good avenue to exploit but retaining a luminance/chrominance resolution ratio of higher than 8:1 will, in general, yield superior image quality.

4.3 Using Vector Quantization to Code Chrominance Information

The above deductions prompted an investigation into the application of vector quantization techniques to code chrominance signals. The goal was to maintain a lower luminance/chrominance ratio than in Butera's scheme while minimizing the portion of the available data needed to represent the chrominance information. A 4:1 luminance/chrominance ratio was therefore selected as a target.

The use of Gaussian pre and post filtering systems for image decimation and interpolation is thoroughly investigated by Pian [34] and has been shown to work well
especially when combined with a sharpening filter before interpolation to compensate for the slow roll-off. A 23 tap Gaussian filter was therefore employed to bandlimit the chroma for coding and to interpolate it after decoding (see Figure 4.3). Gaussian filters have smooth envelopes in both the spatial and frequency domains thereby minimizing the effects of the Gibb’s phenomenon. The sharpener was omitted in this proposed system due to its propensity to aggravate coding artifacts. Because humans have been shown to tolerate highly bandlimited chrominance signals, these roll-off effects do not pose a severe problem.

The grounds for selecting vector quantization for the task of chroma coding resides with the high degree of spatial chromatic correlation found in natural imagery. Most perceived surface variations are the result of luminance shading not chrominance variation. Figure 4.4 shows several frames decomposed into one luminance and two chrominance channels which illustrates this observation. The application of vector quantization to color coding is, in fact, well established. Heckbert [15] successfully demonstrates its usefulness with respect to color palette selection by quantizing vectors formed by grouping the red, green and blue components of color pictures. DuBois [6] carried out a similar experiment with the addition of a transformation to YIQ space before vector coding. His supporting argument is in accord with the observation made earlier in that such transforms concentrate information in one of the three color axes.

Vector quantization is highly suited to coding such signals. The details of the vector coder used are described in chapter 3. The mode of operation chosen was that of error limit coding with parameters set to minimize maximum error with clamped SNR.
Figure 4.3: Gaussian decimation/interpolation filter applied to the chrominance channels. Top: Impulse response. Bottom: Magnitude response.
Figure 4.4: Top Row: Luminance. Middle Row: I. Bottom Row: Q.
Error limits between 28 and 32 db were found to yield near transparent quality. These limits were arrived at empirically and test results did exhibit some signal dependence in the previously mentioned range. This warrants a more thorough investigation into perceptually matched distortion criterion for vector quantization of chrominance signals. Uniform perceptual spaces, which may be well suited to address this problem are discussed by Bender [18].

Table 4.1 summarizes the results of the coding experiments performed. Five seconds of the sequence ”Alley” were coded with 1 code book devoted to a 1 second block of frames. The block size selected was 4x4 pels in dimension with the I and Q channels coded jointly yielding an ultimate block size of 4x4x2. Because the ”Alley” sequence originates from 24 fps film, the chroma update rate was set to 12Hz. Note the gains over Butera’s method of simply transmitting a more severely bandlimit signal without coding (see Figure 4.2).

<table>
<thead>
<tr>
<th>Second</th>
<th>Vector Dims.</th>
<th>SNR (Fixed)</th>
<th>Code Book Size</th>
<th>bits/code</th>
<th>bits/codebook</th>
<th>total bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4x4x2</td>
<td>28 db</td>
<td>106</td>
<td>6.32</td>
<td>14183.14</td>
<td>90998.78</td>
</tr>
<tr>
<td>2</td>
<td>4x4x2</td>
<td>28 db</td>
<td>178</td>
<td>6.66</td>
<td>24465.21</td>
<td>95870.41</td>
</tr>
<tr>
<td>3</td>
<td>4x4x2</td>
<td>28 db</td>
<td>354</td>
<td>7.52</td>
<td>48111.10</td>
<td>108305.73</td>
</tr>
<tr>
<td>4</td>
<td>4x4x2</td>
<td>28 db</td>
<td>379</td>
<td>7.40</td>
<td>50014.23</td>
<td>106571.45</td>
</tr>
<tr>
<td>5</td>
<td>4x4x2</td>
<td>28 db</td>
<td>301</td>
<td>7.29</td>
<td>41634.36</td>
<td>105029.09</td>
</tr>
</tbody>
</table>

Table 4.1: Experimental results from error limit vector coding 5 consecutive 1 second sequences. The input was temporally subsampled to 12 fps and spatially bandlimited by a factor of 4 both horizontally and vertically to a resolution of 160x120. Note: bit rates quoted are post entropy coding

These results demonstrate that vector quantization affords significant compression when applied to chroma coding even at near transparent quality. As an aside, an
Table 4.2: Experimental results using Butera's algorithm to code 5 consecutive 1 second sequences. The input was temporally subsampled to 12 fps and spatially bandlimited by a factor of 8 both horizontally and vertically to a resolution of 64x48. Note: bit rates quoted are post entropy coding experiment was conducted where chroma signals from adjacent frames were jointly vector quantized by combining corresponding spatial blocks into one vector, obviating the need for temporal subsampling. This required only slight increases in bit rate to achieve acceptable quality and guaranteed that chrominance and luminance channels are available at the same temporal granularity. A more thorough investigation is necessary though before conclusive results can be reported.
Chapter 5

A Spatiotemporal Sub-band Coding System

This chapter combines the previously described elements into a flexible image coding system. A digital intermediate format is proposed so as to minimize the need for system tuning for different sources. This format is inherently hierarchical in both space and time, affording multiple output frame rates and spatial resolutions. This flexibility is made possible by a spatiotemporal sub-band decomposition as discussed in Chapter 2. Figures 5.2 and 5.3 serve as a pictorial summary of the overall scheme. It is important to note that this chapter proposes a system so as to provide a more concrete presentation of the material investigated in this thesis and to demonstrate
its effectiveness. The specific architecture is meant to serve as a guideline and can be reworked to fit the application at hand. Although the proposed system is very viable, the overall design philosophy behind it is what the reader should devote attention to.

5.1 System Overview

There are many factors influencing the development of image coding systems. These include possible viewing conditions, source material specifications and display formats that must be supported. All of these free variables affect distortion visibility. Unfortunately, it is difficult to architect a system which accommodates arbitrary parameters, especially if human intervention is to be kept at a minimum. In order to affect the greatest control over the ultimate subjective picture quality, this work adopts a fixed digital intermediate format that supports many desirable input and output spatiotemporal resolutions. The coding parameters are tuned to this model in the hopes of being widely applicable to many input sequences for bit rates between 1 and 2 Mbits/second. All material is converted to and from this format before and after coding.

The codec can therefore be thought of as consisting of five major sequentially applied components. The first and last in the chain are obviously the pre and post processing modules responsible for the conversion to and from the fixed input specification. The middle three functions involve the spatiotemporal transform from the input specification to the digital intermediate format, a coding stage, and an inverse
transform. This chain of events is summarized in Figure 5.1.

5.2 Preprocessing

The pre-processing unit is source signal dependent. Therefore, it is only practical to discuss the output requirements it must conform to. The following list enumerates the specifics:

- **Color Space**: YIQ is the chosen space for this work. Equation 4.1 characterizes the linear transformation between the commonly used RGB space and the specified YIQ space. Transforms from many other color spaces to YIQ can be found in Pratt [36].

- **Frame rate**: The predetermined input frame rate is set to be 30 frames per second for luminance and 15 frames per second for chrominance, both progressively scanned. There are many candidate frame rate conversion schemes to select from [25]. The most effective of these are those employing motion compensation techniques but this task could be accomplished in many ways. Some others include bandlimited interpolation and frame replication. Note in almost all cases, frame rate conversion will introduce some visible artifacts. Also note the chrominance frame rate could be arrived at by simply temporally subsampling as discussed in Section 4.1. This mechanism is a carry over from Butera’s [2] work and while being a useful idea it is inconsistent with respect to
Figure 5.1: High Level Codec Block Diagram.
Figure 5.2: Encoder.
Figure 5.3: Decoder.
the, later described, luminance partitioning. An extension of Butera’s scheme is proposed in the next item that preserves luma/chroma consistency but it was not implemented in this thesis.

- Spatial Resolution: The representation assumes a 640x480 luminance raster and a 160x120 chrominance raster.

One of the test sequences processed (“Table Tennis”) originated from an interlaced NTSC source. It was deinterlaced so as to conform to the above specification using a vertical/temporal interpolation filter to calculate the missing fields of the sequence. The filter was a simple three tap bilinear interpolator in the temporal direction and a half band Gaussian vertically.

5.3 Luminance Sub-band Analysis

This section will concentrate on the specifics of the sub-band decomposition employed. See Chapter 2 for a theoretical discussion of the topic. The decomposition is performed with respect to both space and time using 1D FIR filters. Spatially, 1D bandsplitters are recursively applied both horizontally and vertically to perform a 4 level pyramid decomposition (See Section 2.3.2 for details and Figure 2.4 for plots of both the impulse and frequency responses of the QMF filter bank employed. Temporally, five even subdivisions are made in the frequency domain using the five way band splitting filter bank depicted in Figures 2.9 and 2.10.
The effect of this hybrid pyramid/uniform subdivision scheme is pictured in Figure 5.4. Note the octave subdivisions spatially coupled with uniform partitions temporally. An initial data reduction step is applied here. Progressively more spatial detail is discarded commensurate with temporal frequency at the rate of one pyramid level per temporal division beginning with spatial detail greater than 12Hz (See Figure 5.5. This style of data reduction has been proposed by Schreiber [41] as a component of High Definition Television systems. Note that top octave diagonal detail may also be discarded, as mentioned in Chapter 2. The basic assumption here is that blurring moving objects is a reasonable way of reducing the amount of information to be coded as discussed in Section 2.2. Making the sub-band selection scheme adaptive, would be an improvement but was not addressed in this work.

5.4 Encoding

The next step is to subsequently code the retained sub-bands using a vector quantizer. The specific parameters are summarized in Table 5.4. Sub-bands are grouped for coding on the basis of their spatial scale. Each of these sub-groups is assigned a block size, appropriate error bound and vector space subdivision criterion. After vector coding all the retained sub-bands, final bit rates are calculated. Should the resultant rate be too high, certain sub-bands are thresholded and requantized. The threshold selection scheme is based on the same assumption used in discarding the initial set of sub-bands in that higher spatiotemporal bands are thresholded first and more severely than lower ones. The need for iteratively settling to the desired bit rate is undesirable.
Figure 5.4: Partitioned Luminance Spectrum.
Figure 5.5: Retained Luminance Sub-bands.
and can be circumvented by incorporating entropy constraints in the vector quantizer [3].

One code book is formulated from all sub-bands at a given “level” of the spatial pyramid (see Figure 2.5 for a pictorial description of the spatial decomposition) i.e. sub-groupings of the set of retained sub-bands are formed by selecting all of those at a given spatial scale regardless of which temporal subdivision they originate from. Within these sub-groupings, individual vectors are formed by dividing each sub-band into NxN spatial blocks. Because humans can tolerate more noise in detail at higher spatial frequencies, the quantizer block size is doubled at each level of the spatial pyramid starting from the origin and working outward. Levels three and four use the trivial spatial block size of 1x1. Level two moves to 2x2 spatial blocks, etc. All the corresponding spatial blocks from each level are concatenated to form one vector as described in Section 3.4. The vector coder is used in error limit mode for pyramid levels 1 and 2. Because of the sparse nature of bands at this level, maximum error minimization was the chosen metric as described in Section 3.3. Level 3 was experimentally shown to substantially more populated with significant information so a maximum signal to noise ratio was used to bound code book growth here to insure global fidelity.

Butera [2] and Stempleman [44] describe a technique known as multiscale code book generation. This involves several levels of indirection to formulate a code book. The mechanism here is to assemble larger vectors for higher frequency spatial bands by grouping smaller vectors from code books derived for lower spatial frequency sub-bands. The actual vectors contained in the resultant code book are therefore made up
of indices into the code book for lower frequency pyramid levels. Each indirect vector element is therefore expanded into yet another vector, with each of these arranged in a tile like fashion to ultimately form the actual vector. Obviously the resultant vector's dimensions are constrained to be multiples of their constituents. This technique was not employed in the experiments conducted for this work but it is a useful way of reducing the code book bandwidth and should be investigated.

Level 4 of the spatial pyramid represents the most critical information. The lowest temporal band from this set is statistically very different than the rest of the ensemble. It is derived by applying only lowpass spatial and temporal filters, thereby containing the DC component for the sequence. Because it is well known that perturbations of this component on a block basis results in visible seams, known as block artifacts, this band must be dealt with individually and treated with extreme care. Butera also recognized this fact thereby deciding to transmit this component without quantizing. It is possible, however, to code this band without introducing perceptible artifacts. By using a scalar quantizer or a vector quantizer in its degenerated 1x1 state to code this image and clamping the maximum SNR to 50 db using the error limit mode, it is possible to achieve transparent coding quality while taking advantage of the dynamic range characteristics of the band to provide some compression. Other coding methods such as DPCM or ADPCM would probably be superior for these components but this has not been investigated.

To take advantage of temporal coherence, code books are assembled for one second intervals of the input sequence. Care must be taken to insure there are no scene boundaries within the one second blocks of images that form the region of support
for the quantizer. Chrominance coding is consistent with the experiment described in Section 4.3. Table 4.1 summarizes the parameters for the vector coder employed. To briefly recap, both the I and Q signals are jointly coded using a vector quantizer. As in the case of luminance, codebooks are constructed for one second blocks of data with the spatial vector dimensions set at 4x4.

All codes and code books are assumed to be entropy coded before transmission using one of the many algorithms available such as Huffman coding, run length coding and arithmetic coding. The experimental results in this work quote post entropy coded bit rates when evaluating the cost of transmitting a signal as described by equation 1.4. It is possible to achieve results that are close to or even fall below this theoretical lower bound depending on the number of dimensions the scheme uses to exploit correlations.

The parameters used in experiments for this thesis are summarized in the table below:

<table>
<thead>
<tr>
<th>Sub-band</th>
<th>Vector Dimensions</th>
<th>Vector Updates</th>
<th>Error Codes</th>
<th>Error Book</th>
<th>Error Limit 6Hz</th>
<th>Error Limit 12Hz</th>
<th>Error Limit 18Hz</th>
<th>Error Limit 24Hz</th>
<th>Error Limit 30Hz</th>
<th>Threshold Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>320x240x(LH,HL,IL)</td>
<td>4x4x2</td>
<td>4800</td>
<td>6Hz</td>
<td>1Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
</tr>
<tr>
<td>L2</td>
<td>160x120x(LH,HL,HH)</td>
<td>2x2x3</td>
<td>4800</td>
<td>6Hz</td>
<td>1Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
</tr>
<tr>
<td>L3</td>
<td>80x60x(LH,HL,HH)</td>
<td>1x1x3</td>
<td>4800</td>
<td>6Hz</td>
<td>1Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
</tr>
<tr>
<td>L4</td>
<td>80x60</td>
<td>1x1x1</td>
<td>4800</td>
<td>6Hz</td>
<td>1Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
</tr>
<tr>
<td>IQ</td>
<td>160x120x(I,Q)</td>
<td>4x4x2</td>
<td>1200</td>
<td>15Hz</td>
<td>1Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
<td>2Hz</td>
</tr>
</tbody>
</table>

Table 5.1: Vector quantizer parameters. †Note: Diagonal detail is discarded at this level.
5.5 Decoding

Decoding vector quantized signals is very simple. The data stream simply consists of codes which are indices into a table of vectors known as a code book. After the appropriate entropy decoding is complete for both the tables and the codes, the sub-bands are reconstructed by selecting the vector (group of samples) from the corresponding table for that band as indicated by the code. Because the spatial location of the code is preserved through storage and transmission, decoding a sub-band is a simple matter of tiling the output band with the vectors addressed by their corresponding codes. Because horizontal, vertical and diagonal detail are jointly coded, the three bands are reconstructed simultaneously.

5.6 Sub-band Synthesis

The synthesis of the coded sub-bands involves interpolating each using the appropriate spatial and temporal filters followed by summing. Temporally, each band is interpolated by 5 using the synthesis filter from the matched analysis/synthesis pair. Spatially, the detail from each level of the pyramid is interpolated using the appropriate filters and summed to form the low frequency component for the next level (higher spatial frequency). This recursive process is continued until all components have been synthesized. The power of this format now becomes apparent. It is possible to only synthesize a subset of the available sub-bands to achieve output sequences
at various spatiotemporal resolutions i.e. 640x480, 320x240, 160x120 and 80x60 are all readily available spatial resolutions attainable with no additional computation. Temporally, an analogous situation exists where multiples of a 6Hz frame rate are directly available.

5.7 Post Processing

This is display system dependent and therefore not boundable. In many cases there would be no need to do any post given one of the available spatiotemporal resolutions is supported by the display. In others, only minimal processing need be done such as interpolating the chrominance signals to match the luminance resolution and converting to the specified output color space (usually RGB).

5.8 Experimental Results

The following tables demonstrate the results of applying the above coding system to two test sequences. The first of the two sequences originates from 24fps film and the second one from 30 fps interlaced video. Note the fluctuation in bit rates devoted to individual sub-bands over time. This demonstrates the efficiency gains of error limit vector quantization. The overall picture quality of the resultant images is good, in
fact, the error limits for the top octave detail (pyramid level 1) were too low. There
was no detectable noise introduced at those spatial frequencies and consequently too
high a bit rate was required to represent those channels. This error limit should be
increased. When analyzing the tables below, pay close attention to the incremental
costs of transmitting each pyramid level or temporal division.

Another point to note is that the “Alley” sequence was analyzed and synthesized
with a 4 way band splitting filter bank (pictured in Figures 2.7 and 2.8). This
breaks the very rules laid out in this chapter but it was necessary to avoid frame rate
conversion issues in this work. The different filter bank maintains the 6Hz division
temporally because the “Alley” sequence originated from 24 fps film.

Selected frames from the test sequences are displayed in Figures 5.6 and 5.7.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th># Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>LI 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>10.78</td>
<td>8.78</td>
<td>X</td>
<td>X</td>
<td>5729</td>
<td>563388.00</td>
<td>401085.20</td>
<td></td>
</tr>
<tr>
<td>L2 160x120x2</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>10.72</td>
<td>1.23</td>
<td>0.10</td>
<td>X</td>
<td>2642</td>
<td>345616.00</td>
<td>154897.45</td>
<td></td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>7.15</td>
<td>2.23</td>
<td>0.42</td>
<td>1.43</td>
<td>X</td>
<td>1671</td>
<td>334656.00</td>
<td>26716.60</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>—</td>
<td>2.29</td>
<td>1.30</td>
<td>1.60</td>
<td>X</td>
<td>49</td>
<td>149472.00</td>
<td>273.12</td>
</tr>
<tr>
<td>IQ 160x120x2</td>
<td>SNR = 28db</td>
<td>4x4x2</td>
<td>6.52</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>X</td>
<td>111</td>
<td>174528.00</td>
<td>754.18</td>
</tr>
</tbody>
</table>

Subtotal Bits: 1660528.78 | 597856.49
Total Bits: 2258447.27

Table 5.2: Bit Rates for Second 1 of 5 of Test Sequence: “Alley”

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th># Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>LI 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>10.76</td>
<td>9.88</td>
<td>X</td>
<td>X</td>
<td>6395</td>
<td>563388.00</td>
<td>401085.20</td>
<td></td>
</tr>
<tr>
<td>L2 160x120x2</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>10.90</td>
<td>1.64</td>
<td>0.15</td>
<td>X</td>
<td>4226</td>
<td>365624.00</td>
<td>150786.81</td>
<td></td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>7.38</td>
<td>2.64</td>
<td>1.23</td>
<td>1.66</td>
<td>X</td>
<td>1823</td>
<td>365760.00</td>
<td>29039.02</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>—</td>
<td>2.40</td>
<td>1.45</td>
<td>1.77</td>
<td>X</td>
<td>58</td>
<td>101656.00</td>
<td>337.76</td>
</tr>
<tr>
<td>IQ 160x120x2</td>
<td>SNR = 28db</td>
<td>4x4x2</td>
<td>5.80</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>X</td>
<td>55</td>
<td>167040.00</td>
<td>424.14</td>
</tr>
</tbody>
</table>

Subtotal Bits: 1751582.41 | 666634.49
Total Bits: 2418217.10

Table 5.3: Bit Rates for Second 2 of 5 of Test Sequence: “Alley”
### Second 3 of 5 for Test Sequence: Alley

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>9.33</td>
<td>8.79</td>
<td>8.23 x</td>
<td>x</td>
<td>926</td>
<td>587808.00</td>
<td>234120.84</td>
<td></td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>10.48</td>
<td>9.12</td>
<td>8.83</td>
<td>X</td>
<td>4021</td>
<td>479318.00</td>
<td>147655.57</td>
<td></td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>7.02</td>
<td>6.79</td>
<td>5.07</td>
<td>4.56</td>
<td>X</td>
<td>1523</td>
<td>588384.00</td>
<td>24436.67</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>-</td>
<td>3.74</td>
<td>1.88</td>
<td>1.21</td>
<td>X</td>
<td>96</td>
<td>237024.00</td>
<td>632.16</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>5.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>102</td>
<td>166920.00</td>
<td>678.59</td>
</tr>
</tbody>
</table>

Table 5.4: Bit Rates for Second 3 of 5 of Test Sequence: “Alley”

### Second 4 of 5 for Test Sequence: Alley

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>8.67</td>
<td>7.39</td>
<td>6.67</td>
<td>5.51</td>
<td>X</td>
<td>782</td>
<td>409428.00</td>
<td>34810.12</td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>8.27</td>
<td>7.45</td>
<td>6.36</td>
<td>5.23</td>
<td>X</td>
<td>2074</td>
<td>399744.00</td>
<td>40654.44</td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>7.06</td>
<td>6.89</td>
<td>5.75</td>
<td>4.86</td>
<td>X</td>
<td>3273</td>
<td>886176.00</td>
<td>26945.82</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>-</td>
<td>5.03</td>
<td>3.76</td>
<td>2.87</td>
<td>X</td>
<td>109</td>
<td>389376.00</td>
<td>35498.81</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>5.97</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>96</td>
<td>389376.00</td>
<td>35498.81</td>
</tr>
</tbody>
</table>

Table 5.5: Bit Rates for Second 4 of 5 of Test Sequence: “Alley”

### Second 5 of 5 for Test Sequence: Alley

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>7.26</td>
<td>6.74</td>
<td>5.92</td>
<td>5.01</td>
<td>X</td>
<td>967</td>
<td>614720.00</td>
<td>45240.71</td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>11.63</td>
<td>10.88</td>
<td>9.33</td>
<td>4.07</td>
<td>X</td>
<td>1366</td>
<td>395744.00</td>
<td>40554.44</td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>5.04</td>
<td>4.59</td>
<td>3.69</td>
<td>2.70</td>
<td>1.80</td>
<td>X</td>
<td>1818</td>
<td>500944.00</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>-</td>
<td>2.82</td>
<td>1.34</td>
<td>0.42</td>
<td>0.97</td>
<td>X</td>
<td>97</td>
<td>266400.00</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>3.51</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>91</td>
<td>266400.00</td>
<td>28312.00</td>
</tr>
</tbody>
</table>

Table 5.6: Bit Rates for Second 5 of 5 of Test Sequence: “Alley”

### Second 1 of 3 for Test Sequence: Table Tennis

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>6Hz</th>
<th>12Hz</th>
<th>18Hz</th>
<th>24Hz</th>
<th>30Hz</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>13.65</td>
<td>11.55</td>
<td>10.24</td>
<td>8.97</td>
<td>X</td>
<td>2687</td>
<td>725700.00</td>
<td>232270.80</td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>11.63</td>
<td>10.88</td>
<td>9.33</td>
<td>4.07</td>
<td>X</td>
<td>1366</td>
<td>395744.00</td>
<td>40554.44</td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x3</td>
<td>4.47</td>
<td>3.03</td>
<td>1.45</td>
<td>0.97</td>
<td>X</td>
<td>135</td>
<td>283120.00</td>
<td>2014.04</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>-</td>
<td>2.22</td>
<td>1.32</td>
<td>0.91</td>
<td>0.67</td>
<td>X</td>
<td>96</td>
<td>174240.00</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>5.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>96</td>
<td>174240.00</td>
<td>1804.63</td>
</tr>
</tbody>
</table>

Table 5.7: Bit Rates for Second 1 of 3 of Test Sequence: “Table Tennis”
### Second 2 of 3 for Test Sequence: Table Tennis

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>bits/code</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>10.43 10.29 X X</td>
<td>8456</td>
<td>596736.00</td>
<td>560233.44</td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>3x2x3</td>
<td>7.42 2.18 0.94 X</td>
<td>981</td>
<td>202976.00</td>
<td>29207.12</td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x1</td>
<td>6.25 4.07 3.91 3.50 X</td>
<td>324</td>
<td>452736.00</td>
<td>4724.15</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>3.70 3.04 2.53 1.76 X</td>
<td>70</td>
<td>317664.00</td>
<td>427.05</td>
</tr>
<tr>
<td>IQ 160x120x2</td>
<td>SNR = 28db</td>
<td>4x4x2</td>
<td>5.41 5.32</td>
<td>246</td>
<td>107007.07</td>
<td>46654.25</td>
</tr>
</tbody>
</table>

Total Bits: 1930366.07

### Second 3 of 3 for Test Sequence: Table Tennis

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Error Limit</th>
<th>Block Size</th>
<th>bits/code</th>
<th>Vectors</th>
<th>total bits/code</th>
<th>bits per code book</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 320x240x2</td>
<td>Max. MSE = 5</td>
<td>4x4x2</td>
<td>9.56 9.32 X X</td>
<td>7252</td>
<td>543744.00</td>
<td>273425.35</td>
</tr>
<tr>
<td>L2 160x120x3</td>
<td>Max. MSE = 18</td>
<td>2x2x3</td>
<td>8.55 3.23 1.85 X</td>
<td>3444</td>
<td>396136.00</td>
<td>126340.33</td>
</tr>
<tr>
<td>L3 80x60x3</td>
<td>SNR = 42db</td>
<td>1x1x1</td>
<td>4.61 4.53 3.20 3.83 X</td>
<td>749</td>
<td>465696.00</td>
<td>11689.30</td>
</tr>
<tr>
<td>L4 80x60x1</td>
<td>SNR = 50db</td>
<td>1x1x1</td>
<td>4.07 3.33 2.80 1.98</td>
<td>60</td>
<td>350784.00</td>
<td>5527.75</td>
</tr>
<tr>
<td>IQ 160x120x2</td>
<td>SNR = 28db</td>
<td>4x4x2</td>
<td>5.83</td>
<td>98</td>
<td>161568.00</td>
<td>648.24</td>
</tr>
</tbody>
</table>

Total Bits: 2021848.76

### Table 5.8: Bit Rates for Second 2 of 3 of Test Sequence: “Table Tennis”

### Table 5.9: Bit Rates for Second 3 of 3 of Test Sequence: “Table Tennis”
Figure 5.6: Excerpts From the Test Sequence “Alley”
Figure 5.7: Excerpts From the Test Sequence “Table Tennis”
Chapter 6

Conclusions and Future Work

6.1 Concluding Remarks

When designing image coding systems, one should consider the flexibility that the underlying signal transformations and signal coders afford. The sub-band transform proposed herein, is appealing not only for its energy redistribution properties but for its hierarchical organization of spatiotemporal resolution. This is in concert with what is psychovisually known about how humans see and advantageous from an applications standpoint because it is possible to reconstruct the coded sequence at a variety of
spatiotemporal resolutions.

The proposed sub-band decomposition is composed of a four level spatial QMF pyramid transform coupled with a five way uniform temporal band splitter. Retaining less spatial detail in proportion to temporal frequency is discussed as a viable method of initially reducing the data rate of a source signal. The retained sub-bands are then vector coded using distortion bounded technique proposed in this work which simultaneously provides the necessary bit allocation throughout the spatiotemporal spectrum.

From a statistical standpoint, image sub-bands have very different statistics than their signal of origin. Attempting to minimize any global distortion metric has unfavorable psychovisual implications because the significant elements of the signal are sparsely distributed throughout the band. Using global metrics such as mean squared error to bound the vector coder only result in their being underrepresented. The solution proposed, in this work, to that problem is to employ local distortion metrics such as minimizing the maximum Euclidean distance between any given source vector and its post coding counterpart.

Because this thesis addresses such a broad topic, there are many issues that warrant further investigation. Hopefully, though, the design philosophy behind this work has been grasped and future work in this area will continue to utilize methods that enhance the feature set available to users of multi-media systems and provide structured representations for incorporating movies as a viable data type in computing environments.
6.2 Future Work

Section 2.3.3 demonstrated the effects of different frequency response shapes on picture quality in a lossy coding system. Obviously, these observations must be directly incorporated into a filter bank design technique which could provide an optimum tradeoff between minimizing ringing and coding efficiency.

The error limit paradigm for vector coding sub-bands also needs further development. Currently, there is no means of clamping the total bit rate of the system without iteratively recoding with altered parameters until the desired bit rate limit is met. Entropy constraining techniques should be incorporated into the vector quantizer to allow for more control so as to avoid the need for iterative recoding.

The task of minimizing the maximum distance between a given set of objects and their constituency is traditionally known as the fire house problem in computer science. The name comes from the analogy used to explain the problem. Namely, how can the locations of a given set of fire houses be positioned so as to minimize the maximum distance between any home and a fire house. If the code book is considered to be the collection of "fire houses" and the training vectors the "homes" then it should be possible to employ one of the heuristics used to solve this problem to calculate a code book.
Appendix A

Computational Requirements of Spatiotemporal Sub-band Transforms

A.1 Spatial QMF Pyramid Transforms

This appendix describes the computational requirements, in terms of the number of multiplies and adds, to perform the sub-band analysis and synthesis as described in...
This document. It is assumed that the method of performing the transform is by convolution with the basis functions and that the band splitters are 1D FIR filters. See Chapter 2 for more details.

It is well known that convolution requires \( N_s \) multiply operations and \( N_s - 1 \) additions per output point calculated given that \( N_s \) is the length of the filter kernel. Because the QMF transforms employed in this work are sample conserving, it is trivial to calculate the number of calculations required per pyramid level spatially. Given an input image with a horizontal dimension of \( X \) and a vertical dimension of \( Y \) the number of multiplies per level is \( X \times Y \times N_s \) and the number of additions is \( X \times Y \times (N_s - 1) \). Because the QMF pyramid transform recursively subdivides the spatial low frequencies, the total number of operations for the entire spatial pyramid for one input frame is a function of the number of pyramid levels desired. Two expressions for this follow where the number of desired levels is denoted by \( (L) \):

\[
MULTS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times N_s \\
\]  

\[
ADDS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times (N_s - 1) \\
\]  

Some optimizations can be made regarding the number of multiplies, if the transform basis functions are constrained to be symmetric. Specifically, the number of multiplies can be reduced by approximately one half. The reasoning for this follows.
Symmetric kernels have their tap values mirrored about the origin. Therefore, the contribution from each identical tap pair can be described by:

\[
\text{contribution} = at + bt
\]  

(A.3)

where \(a\) and \(b\) are the sample values corresponding to the identical taps. Making a simple factorization of equation A.3:

\[
\text{contribution} = t(a + b)
\]  

(A.4)

we see that a multiply can be omitted from the calculation thereby reducing the number of calculations required to perform the convolution to:

\[
MULTS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times \frac{N_s}{2}
\]  

(A.5)

\[
ADDS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times (N_s - 1)
\]  

(A.6)

for even \(N\) and:

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\[ MULTS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times \frac{N_s + 1}{2} \]  \hspace{1cm} (A.7)

\[ ADDS = \sum_{i=0}^{L-2} \left( \frac{1}{4} \right)^i \times X \times Y \times (N_s - 1) \]  \hspace{1cm} (A.8)

for odd \( N \).

### A.2 Temporal QMF Sub-band Transforms

Limiting the discussion to sample conserving temporal transform, the number of calculations required is simply:

\[ MULTS = X \times Y \times T \times N_t \]  \hspace{1cm} (A.9)

\[ ADDS = X \times Y \times T \times (N_t - 1) \]  \hspace{1cm} (A.10)

where \( N_t \) denotes the basis function length and \( T \) denotes the number of frames to
be transformed. If the transform kernel is symmetric, the same multiply savings can be accomplished as in the spatial case.

A.3 Summary

This brief discussion serves as an example of how to calculate the computational requirements of a given transform implemented via convolution. The discussion assumes that the transforms are orthogonal i.e. the analysis and synthesis stages are computationally equivalent given that the synthesis is accomplished using polyphase interpolation. Assuming a 30 frame sequence at a spatial resolution of $640 \times 480$ pels, and the 4 level spatial, 5 way temporal transform employed in work, the computational requirements to perform the sub-band analysis (synthesis being identical) are given by Equations A.11 and A.12. The spatial filters used are 9 taps in length (symmetric) and the temporal kernels are 7 taps in length (asymmetric).

$$MULTS = \left[ \sum_{i=0}^{2} \left( \frac{1}{4} \right)^i 640 \times 480 \times \frac{10}{2} \right] + 640 \times 480 \times 7 \times 30 = 297,792,000$$

(A.11)
\[ ADDS = \left[ \left( \sum_{i=0}^{2} \left( \frac{1}{4} \right)^i \right) 640 \times 480 \times 8 \right] + 640 \times 480 \times 6 \times 30 = 428,544,000 \quad (A.12) \]
Appendix B

Acknowledgments

The Media Laboratory has been an extremely rewarding place for me during my two years at MIT. It truly was an exciting place to do graduate work and it will be missed sincerely. The hardest part about leaving any place that has been such a large part of your life for two years though, is leaving the friends that you’ve made along the way. Hopefully we’ll all stay in touch.

First and foremost I have to thank my Advisor and friend, Andy Lippman for all he has done. I admire and appreciate his insightfullness and look forward to working with him in the future. This man has the ability to teach like no one else I know. In a phrase: Stimulating is an understatement. Thanks for everything.
Q: What can you say about a guy like Walter Bender? A: Not enough. A well oiled machine to say the least. Thanks for all the advice. I could always count on you to come through.

Mike Bove (I mean Dr./Prof. V. Michael Bove). Thanks for everything and good luck in the future. I couldn’t have had a better “brain damage” filter.

Bernd Girod, I thank you. I admire and respect your precision and thoroughness and am sure you will continue to do great work. Thanks for all the input.

To Ted Adelson and Eero Simoncelli, I extend my gratitude. Thanks for the advice.

Thanks Ashok for all the temporal filter discussions.

I am deeply indebted to Bill Butera for all his help and for laying the groundwork for this thesis. Thanks for the time spent.

Gillian (or should I say St. Gillian), thank you for your support. Keep Andy in line.

To Pascal, I send out my sincerest thanks. As Butera so well put it, ”He keeps the entropy up”.

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To John Watlington, one of the best engineers I know, goes my respect and gratitude.

Running a close second for the title of entropy maximizer is Joe “Foof” Stampleman. There’s a genius somewhere in that eclectic personality.

Henry, thanks and good luck.

An officemate can make or break your work environment. I couldn’t have had a better two than Janet Cahn and Paul Lindhart. Never a dull moment.

To my good friend and the “worlds fastest programmer” Bob Mollitor I express my thanks.

To my current and future colleague, Mark Polomski, thanks. I look forward to building an exciting company with you and the gang at Fluent Machines.

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To the rest of my hack brothers and sisters, namely Joel, Lin, Alex, Blaze and Dave Shane, thanks. You’ll all be missed.

To Alan Lasky, our resident film aficionado whose name is sure to grace the credits of many a top ranked feature film in the future, thanks for the laughter.
To my parents, grandparents and brother goes my deepest love and gratitude for all their support. I know I can always count on you. Thanks for everything.

Now for something that is long overdue:

Over the past two years there has been one person who has earned this degree with me although she gets no diploma. Ann, your ability to tolerate the hours spent apart and my constant preoccupation with classes and research is deeply appreciated. This one's for you!. Love ya.
Bibliography


