# KIDS 'N SPACE:

## EXPLORATION INTO SPATIAL COGNITION OF CHILDREN LEARNING 3-D COMPUTER GRAPHICS

by

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Submitted to the Media Arts and Sciences Section,
School of Architecture and Planning in partial fulfillment
of the requirements for the degree of

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## Abstract

This thesis provides an analysis of children's interactions with a 3-D computer graphics learning environment, J3D. This study integrates a statistical analysis and a case-study approach. The analysis draws upon 1) spatial cognition; 2) learning and development in spatial cognition; 3) mental imagery; 4) representations of space and object and view transformations in 3-D computer graphics; and 5) gender differences and cognitive styles related to spatial cognition. In J3D, topological, projective, and Euclidean spatial relations are intimately linked and interrelated to form a cohesive spatial environment. Through visual, verbal, and mathematical modes, children explore transformations of objects and views of these objects by using positive and negative numbers and decimal fractions solidly bound to the Cartesian coordinate system.

After working on a pre-designed set of J3D exercises and exploring J3D on their own, an experimental group of ten fifth grade children improved in their spatial understanding and their ability to solve spatial problems both on and off of the computer as measured by a battery of spatial tests and tasks. All of these children also improved in their organization of spatial concepts as seen in the language they used to describe space, in their drawings representing 3-D space, in their understanding and use of the Cartesian coordinate system, and in their ability to coordinate object and view transformations. No effects of gender, initial spatial abilities, or cognitive style as measured by three types of spatial abilities tests were found.

Through a microanalysis, individual's diversities in spatial thought and strategies were revealed, and each of the children's style was found to be pervasive in their cognitive and spatial functioning. This analysis also suggests that spatial cognition and mental imagery are indistinguishable in the process of imagining movements or change of views, and that the reference frame is a crucial element in the construction of these spatial transformations.

The present analysis shows ways in which an integrative approach, drawing upon the fields of developmental psychology, cognitive science, education, and computer graphics can provide a better understanding of children's learning spatial concepts. This analysis also demonstrates that spatial cognition is more than an innate ability. Rather, it is a learnable set of concepts and skills that should be incorporated into the school curriculum.

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Judy E. Sachter, Massachusetts Institute of Technology
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# CHAPTER 1

# INTRODUCTION

## 1. RESEARCH STUDY INTRODUCTION

The purpose of this research is to assess the quantitative and qualitative changes in spatial concepts in children resulting from interventive experiences using three dimensional (3-D) computer graphics. The study is focused on the representations and strategies that different children use when constructing, transforming, and displacing objects through a command language based 3-D computer graphics environment. I investigated how children learn about spatial and metric concepts while creating and animating images with the system. Particular attention was paid to strategies used to solve spatial problems.

3-D computer graphics is a environment in which several aspects of spatial cognition come together to form a coherent spatial learning environment. In the particular environment employed here, children are given a fixed 3-D coordinate system, and a set of operations allowing them to locate, modify, and displace objects relative to this fixed reference frame. Children can modify their own point of view relative to these objects. This environment was designed to provide children with a microworld for learning to manipulate transformations of objects and views of these objects through the use of both positive and negative numbers and decimal fractions solidly bound to the Cartesian coordinate system through the command language.

The population was selected from fifth grade children. The children's spatial "abilities" were assessed through a large battery of spatial tests, then a subset of the group was chosen in order to look more closely at how children with varying spatial abilities function in a 3-D computer graphics environment. I was not concerned with spatial performance *per se* but with individual strategies used to solve spatially related problems in structured tasks,

assisted tasks, and free play tasks. The main questions of interest were: What do children learn from building graphic objects, transforming them, or creating animation? What is confusing to children in this process? How do children overcome obstacles? When and how do they ask for help? How do they choose their frame of reference through the specification of object transformation and view changes, and how do they relate to the computer environment?

### 1.1. Research Rational

Spatial cognition is a particularly good indicator of how we impose coherence upon an ever moving environment. How do we construct and organize objects in space so that we can describe locations and movements? What objects do we choose as fixed referents? How do we describe relationships among objects? How do we organize these relations within a larger frame of reference? From a psychological point of view, a reference frame can be defined as "that which is kept invariant" by a subject in order to give meaning to other things. Piaget views spatial development as a homogeneous progression from local towards global frames of reference. To him, coordinate systems as described by mathematicians are nothing but an elaborate version of the most stable and reliable reference system for describing objects and movements in space.

My own assumption is that stage theory is not sufficient to capture some deep individual differences in spatial understanding. If it is true that people with different styles in different contexts tend to choose different "objects" as referents, then it becomes crucial to analyze how these different individuals come to solve spatial problems. Alternate strategies and stages of development may be determined by the conscious or unconscious choice of reference frame. The reference frame is an integral part of recognition, information retrieval, and spatial thought. It also plays an essential role in performance on spatial tasks. The chosen reference frame often depends on the context, the materials, and the complexity of the problem posed. All of these variables can effect the ways people choose a reference frame and thus how they perform in spatial tasks.

Gender differences are likely to relate to style differences, both indicating very particular problem-solving processes in solving spatial tasks. Yet these differences in spatial "abilities" may in turn effect people's math and science achievements, since both math and science require spatial skills. Spatial ability is also at play in architecture, biology, chemistry, engineering, physics, mathematics, and design.

There are also some indications of gender differences in the use of computer environments. Girls seem to become more easily involved in computer activities when they use graphics software which allows them to create and manipulate pictures rather than do mathematics or programming. Research has also shown that an appropriate explication of some basic spatial principles can minimize gender differences on certain spatial tasks, can change cognitive styles and strategies, and can improve performance in tasks which indicate spatial abilities. For this reason, 3-D computer graphics may help to explicate the spatial content and reduce some of the gender differences.

**1.1.1. Research Environment.** This study was done at an inner city Boston school whose population is ethnically diverse. Two of the three 5th grade Advanced Work Classes (AWC) were participating in Project Headlight (Papert, 1986, 1987). All of the children had some experience using computers and two of the selected classes had been programming in Logo about forty-five minutes a day for over a year. The students in these three classes interacted in math groups which were divided into three levels. This study attempted to reflect the diversity of the school in the choice of participants. I selected an equal number of boys and girls. A major criteria was the children's spatial "abilities" and math competence. Children selected for this study were excused from their regular classes and invited to work in a separate room with the computers, the experimenter, and sometimes other subjects.

I worked with children who were at the age at which it is virtually possible to construct Euclidean and Projective concepts. In the domain of spatial cognition, stages are not necessarily related to age. However, fifth grade children (ages 10-11) are mostly at the stage of what Piaget calls late concrete

operations. In the development of spatial concepts, this corresponds to the period when Euclidean and Projective systems are constructed, and when mental imagery becomes dynamic and anticipatory. This will be discussed in greater depth below in the chapter on development of spatial concepts in the child.

**1.1.2. Computational and Learning Environment.** I constructed an interactive 3-D computer graphics program I call J3D (Just 3D) for the purpose of this study. J3D runs on an Apple Macintosh computer and provides an environment for upper elementary children to actively explore 3-D Cartesian coordinate space perceptually and cognitively. It enables the children to view and transform synthetic objects in this environment. Children can thus manipulate one or more basic geometric polyhedral objects by changing their size, position, and orientation in space. These transformations allow children to position objects on the screen to create a more complicated object or set up a scene and view it in perspective from an arbitrary view point.

I use a command language interface where verbal commands are entered through a keyboard. The syntax of the commands obliges the children to name the desired spatial transformation and to consciously and explicitly use the Cartesian coordinate system as the frame of reference in this simulated space. This environment, with its capability of displaying a 2-D perspective view of 3-D objects provides concrete visual feedback of this synthetic 3-D world as well as control through the abstract notation of spatial transformations and the Cartesian coordinate system. J3D sets the Cartesian coordinate system into a coherent framework with a concrete basis. The numeric values of coordinate space become less abstract as they are used as a language for dealing with space to create imagery. The use of language in the form of the syntax also helps to decompose a parallel nonverbal process and slow it down enough to allow the children and the researcher a discrete level of control over these operations.

This system offers multiple representations of spatial concepts through visual, verbal, and formal modes. The integration of these different representations is designed to enhance children's understanding of spatial concepts as

well as make this environment accessible for children of a variety of styles.

Aspects of this system were based on my own preferences as a learner and a teacher. Visual images of concepts are very useful to me and I think that they are for other people as well. My preference is to be given some structure in new or complex situations; many students seem to feel more secure in exploring novel situations under structured conditions. Explicit information gives me a higher level of control in the learning process and in creating my art in any medium. However, more open-ended problems allow me to use my creativity. As a result of some of these preferences, I chose to use a system that required a fairly low-level of control by the children. This increases the learning curve but it also then addressed the concepts underlying 3-D computer graphics for the children at a fairly basic level.

Another consideration in the development of this system was the difference between practical and explicit knowledge. An interface that allows children to just push, point, and generally interact with the system in gestural ways does not provide explicit descriptions in understanding of spatial concepts. This also taps the higher verbal abilities of girls. Another preference that was built into this system is the use of the Cartesian coordinate system rather than Polar coordinates, or a space that is more relational. I prefer fixed than relative reference systems, because it helps me to not get lost. I also find it more difficult to think of 3-D space as a navigational environment where one thinks of space in terms of relative relations rather than in terms of a more absolute and global frame of reference. My focus is on representation not navigation of spatial environments. In this environment I imagine myself as a "director" with some perspective on the whole scene and I want to view and control the whole scene. My focus is on spatial representation and not on navigation through an environment. It is set up more as an environment for mentally manipulating objects than finding ones way in a territory as emphasized in 3-D logo. The reference system that I give is external and absolute rather than egocentric and relative.

My system is psychologically relevant because it meets and represents spatial problems. It makes formal metric concepts concrete through the use of the Cartesian coordinate system. My interests as a researcher are not so

much in the mathematical operations that different children use, but in what these operations mean for the children. To what extent can children appropriate these operations as tools for exploring the environment? What are the contexts in which children find these operations meaningful? To what extent does the use of these operations bear on their understanding of measurement? The system also provides a window to observe spatial processes and to address style issues. What do children choose as operators? How do children choose and coordinate the possible multiple frames of reference and how do they coordinate the rotation and perspective problem. How do children understand and use this spatial domain? What is the stability of spatial knowledge for children on the computer and the various paper and pencil tasks.

Through my interventions in this experiment, I expect qualitative changes rather then quantitative changes. My goal is to introduce children with various strategies for organizing space and for achieving spatial transformations to multiple strategies within the operations available in the system. Offering a larger repertoire of strategies might change the strategies that are normally used. I examine the strategies used on specific tasks prior to the intervention and after the intervention. I examine if there are differences between boys and girls in their strategies and representations in this particular environment.

An important issue to touch on is what is given and not given in this microworld. It is easier to think of J3D as an environment for mentally manipulating objects rather than tracing their edges or figuring out their geometry, as is often a major aspect of 3-D Logo. I supply geometric primitives so that the children may construct imaginary objects or scenes rather than deal with the mathematical understanding of geometric forms. Think of this environment more as a stage or container in which the children can manipulate or transform object. Through these transformation commands, these primitives or "objects" are continually linked with the Cartesian coordinate system. The children must anticipate transformations, transfer their ideas into a formal notation, and explicitly request feedback through image or textual means for self evaluation and correction. This allows children the ability to correct their own behavior. As a result, it becomes an intrinsic measure, through the

intervention, predictions, keeping a history, checking errors, gaining feedback, and correcting predictions. (There are many possible referents in this environment, none of which are visually represented. The Cartesian coordinate system is always present through the parameters of the command.)

Another important issue is to investigate my own presence with the children's interactions with the microworld. Is the experimenter important because of the framework she provides? How much scaffolding should I provide? Are the children able to understand and use the structure built into the system? How much help do children ask from me and when? What influences the children and their interactions with this simulated environment. How much is the system's on on-line help, the experimenter, or a combination used to gain information? Does it vary among the children where they attain assistance and feedback? How much feedback do the children need to be functional? Is personal contact with the experimenter as important or more so than the content?

In my study I intend to try to understand the influences of these different aspects of the learning process in this particular context. Since issues of style are important to me, I shall focus my study on different ways in which different children go about learning the system, when they ask questions, to whom, about what, and how they relate to the experimenter. How much feedback do the children need to use the system? What is their sense of autonomy socially, psychologically, emotionally, or intellectually in this context? Is there a tendency or style that is pervasive in their emotional, cognitive, social, and physical interactions? We build reality from our intellect, but even this is an aspect of the physical world. Do the children deal with it as a social context? All these questions are essential indicators of learning styles.

The method of inquiry that I will use will be that of a progression from quantitative analysis of standard spatial tests between groups of children (control and experimental, boys and girls), qualitative analysis of Euclidean and Projective spatial tasks and J3D related tasks within the experimental group of children, and finally an analysis of a few individual children in depth.

## 1.2. Experiential Motivator for the Study

My own psychogenesis can serve as an introduction to my interest in spatial cognition, mental imagery, cognitive styles and gender differences in the domain of space, and 3-D computer graphics.

My main interest as an artist was in representing 3-D space on a 2-D surface and in the organizing of design elements in order to create and emphasize my message in the final composition. In the process of becoming an art teacher, I had to relearn what I knew intuitively. I had to learn how to make my knowledge of design and technique explicit so that it could be communicated to others.

In studying 3-D computer graphics I learned how to think in 3-D and manipulate objects and points of view more freely in my imagination. Computer graphics also gave me a language to think with and ability to name transformations. Math for the first time in my life had some relevance -- to create art and to represent space. Through programming I learned to slow down my thoughts, to organize and parse them.

As a teaching assistant in computer graphics and animation, again I had to verbalize what I knew and make it explicit. While teaching computer graphics to artists, I realized that computer graphics, which for me had become a medium and a tool, was also an environment where mathematics, science, and art were integrated.

Currently I am interested in understanding how individuals come to understand or misunderstand the spatial concepts involved in 3-D computer graphics and animation. In a sense what I have discovered about others spatial thinking through teaching, I now want to understand explicitly.

Through this process I have become aware of my personal preference for how I learn and see the type of environments that accommodate my cognitive and learning style. I have often said "If I can't see it, I don't understand it." When I am learning something, I spend most of my time constructing visual models in my mind. This is why 3-D computer graphics has become a favorite "microworld" of mine. 3-D computer graphics helps me in my understanding of space - the perception, conception, and mathematics of space have become

much more integrated as a result. If there are other people with my style, possibly this environment will enable them to learn the formal and informal aspects of spatial content and about the Cartesian coordinate system.

The following chapters address the rationale for this study in terms of my interests and concerns. An overview of the experimental environment and population are discussed as well as some of the reasons for the choices made in the design of the software.

This brings me to the four domains upon which I base my theoretical background: **spatial cognition** and its development in the child; **mental imagery** and its development in the child; **gender differences** and **cognitive strategies** in the domain of space; how space and transformation are represented in **3-D computer graphics**, and how the above four domains interact in the simulated world of 3-D computer graphics.

Chapter 2 explores the concepts and research that significantly contributes to the understanding of spatial development in children. I shall focus especially on the development of Projective and Euclidean concepts with more emphasis on the development of the reference frame (measurement, perspectivism, and construction of a coordinate system) and its importance in spatial cognition.

Chapter 3 is on mental imagery or rather, on mental transformations, especially mental rotation and perspective change. These concepts and their development in children are a necessary theoretical foundation for addressing both the mental transformation and the spatial concepts used in 3-D computer graphics.

Chapter 4 addresses the issues of gender differences and cognitive styles, both relevant to the study of space. Gender differences are important to address because of the magnitude of differences in spatial ability. One possible reason for this is the difference in the strategies used to solve spatial problems. Cognitive styles are an important aspect of how people learn and solve problems. The criteria of assessment for several cognitive styles are closely related to the utilization of spatial thought. The gender differences in cognitive styles may be a result of differences in strategies or spatial abilities.

Chapter 5 discusses how space and movements of objects in space are represented in the synthetic 3-D computer graphics environment. The basic concepts in 3-D Computer graphics are introduced. J3D, is discussed and the syntax or commands in the system are presented.

Chapter 6 discusses how spatial cognition, mental imagery, cognitive style, and gender differences intertwine in J3D, an environment that I built to enable children to explore spatial transformational concepts visually, linguistically, and mathematically.

Chapter 7 briefly discusses the Pilot Study and general results of the Pilot.

Chapter 8 is an overview of the design for the thesis experimental study and implementation. With an overview of the pre- and post-test batteries for the control and experimental groups and the computer related pre- and post-tasks and activities.

Chapter 9 addresses the methodology for the research and analysis. The quantitative data is presented and discussed.

Chapter 10 presents the task analysis of the Euclidean, Projective, and J3D spatial tasks developed for this study. The children's performance on these tasks are analyzed and discussed.

Chapter 11 introduces the qualitative analysis of the description of spatial relations, one Euclidean, and one Projective and Transformational task of several children and the J3D learning activities for two children.

Chapter 12 contains the conclusions and discussion.

# CHAPTER 2

# SPATIAL COGNITION

## 2. WHAT IS SPATIAL COGNITION?

Many great thinkers, like Aristotle, Newton, and Einstein to name but a few, have contemplated the nature of space and come up with different theories relating to its structure and form. Others, such as Kant, Berkeley, Cassirer, Werner, and Piaget have contemplated the origin and development of spatial knowledge. While, others have introduced distinctions between perceptual, physical, representational, and conceptual aspects of space. Mach (1906), for example, distinguished between the physiological, psychological, and physical aspects of space.

Space has many qualities and meanings perceived at different levels of experience. There is no one cohesive theory in the field of spatial cognition. The theories and empirical findings vary depending on the authors, the experimental setting, and the population under study. Other variations stem from different disciplinary perspectives, such as philosophy, geography, mathematics, architecture, urban planning, education, neuroscience, art, or the various branches of psychology.

There are many meanings of the term **spatial cognition**. Liben (1988) attributes this to the many interpretations of the concepts of space and whether space is viewed as physical or a conceptual abstraction. Theorists approaching spatial cognition from different perspectives have come up with a range of ways to define and assess spatial cognition. This multiplicity in the terms used, the behavior studied, and the various methodologies has led to many theories and conflicting reports in the research. Given the many aspects of space and the many ways of contemplating space, what then is spatial cognition? In the many psychological perspectives there are no clear-cut

answers. Let us first briefly review the psychometric and neuropsychological perspectives of spatial cognition, then the developmental and cognitive perspectives in greater depth.

### 2.1. Psychometric Perspective

The psychometric approach to intelligence studies the pattern of results of intelligence tests. A primary issue has been the structure of mental abilities. Investigators have looked at the correlations of the various sub-tests to determine if intelligence is a unitary ability or made up of various unrelated abilities. British and American psychologists differed along these lines. The British psychologists favored a general factor of cognitive abilities, while the Americans favored multiple factors.

In the early decades of this century in Britain Spearman derived his general "G" theory of intelligence. Vernon, also a Britain, expanded this theory to include a secondary bi-polar factor with V (verbal) and S (spatial) at the extremes of the G (general) factor. The V factor included numerical and educational sub-factors and the S factor include mechanical and practical sub-factors (Eliot, 1983). This model is somewhat supported by neurological findings in brain research that indicates specialization of the right hemisphere for visual-spatial and left hemisphere verbal (Harris, 1975).

American psychologists, on the other hand, analyzed correlations from test-batteries because they recognized a large number of ability factors on an equal basis. Thurstone's model (Thurstone, 1938) of Primary Mental Abilities (PMA) was made up of many factors. These were V (verbal), I (inductive reasoning), D (deductive reasoning), N (numerical), S (spatial), W (word fluency), M (rote memory), and P (perceptual speed). A version of PMA is currently used in many of our schools. Thurstone showed that spatial knowledge is separate and distinct from verbal and analytic aspects of intelligence. Others have showed that even the spatial factor is not unidimensional by coming up with multiple spatial factors (i.e. spatial, spatial relations, visual spatial, visualization, kinaesthetic) (Anderson, 1954).

Some "spatial abilities tests" have been shown to access perceptual solutions rather than cognitive solutions. Some behaviors occur in space, yet this

does not imply spatial representation. Spatial behavior is often complex and often intimately associated with nonspatial behavior. For example, a great deal of the research on spatial cognition has been related to maps and way-finding which requires a non-spatial component (recognition of landmarks and seriation) and two spatial components (spatial attention and spatial memory) (Kritchevsky, 1988).

### 2.2. Neuroscience Perspective

Kritchevsky, a neuroscientist, states that spatial cognition "refers to any aspect of an organism's behavior which involves space and is mediated by cerebral activity" (Kritchevsky, 1988). He proposes a models of spatial functions which categorized six elementary spatial functions and identified the major regions of the brain where they have been indicated. These elementary spatial functions are spatial perception (object localization, line orientation, and spatial synthesis), spatial memory (short-term and long-term spatial memory), spatial attention, spatial mental operations (mental rotation), and spatial construction. These spatial functions were based on patterns of abnormalities found in brain damaged adults, however Kritchevsky believes that they can add to a further understanding of spatial behavior in normal adults and in spatial development. These modality independent spatial functions also seem to give credence to the American Psychometric view of spatial functions being multi-dimensional.

### 2.3. Developmental Perspective

Taking a developmental approach, Hart and Moore define spatial cognition as "knowledge and internal or cognitive representation of the structure, entities, and relations of space ... the internalized reflection and reconstruction of space in thought" (Hart, 1973).

The developmental study of space is difficult because the evolution of spatial relationships proceeds at two different levels, one at the perceptual level and the other at the level of thought and imagination (Voyat, 1982). Werner and Piaget, like Kritchevsky and many others who study spatial intelligence, acknowledge a distinction between spatial perception and spatial

cognition. Werner thought of "perception as both a subsystem of cognition and a function of cognition" (Hart, 1973). Piaget and Inhelder (1967) conceptualized perceptual space and conceptual space as separate and as two major aspects of cognition. The **figurative** aspect relates to the direct or pictorial perception of successive states and the **operative** aspect refers to action, the result of which is some transformation, construction, or change of reality. Laurendeau and Pinard elucidate the interaction between figurative and operative aspect of cognition.

> Between these two structures a reciprocal influence or functional interaction must operate; at all levels of development, the information provided by perception (or mental image) serves as raw material for the intellectual action or operation, and reciprocally, these intellectual activities exert an influence (direct or indirect) on perception, enriching and increasing the flexibility of its functioning with development (Laurendeau, 1970).

Piaget and Inhelder believe that perceptual space develops from Topological to metric and Projective. This sequence is repeated at a later age in the development of representational space. They emphasized **motor activity** as an underlying factor in both perception and representation. To Piaget, spatial concepts are internalized actions and these actions are internalized in stages (Piaget, 1967).

Other researchers, while retaining aspects of Piaget's focus on the distinction between perception and representation diverge on the significance. Olson and Bialystok (1983) agree with Cassirer (1944) and Piaget on the distinction between perception and representation and with the idea that representation is the primary component of thought. They also agree on the implicitness of perceptual space and explicitness of representational space, although Olson and Bialystok attribute this difference to the explicitness of the structural description. They differ, however, with Cassirer on attributing development to culture and with Piaget attributing it to the internalization of actions. Olson and Bialystok suggest that neither is true.

> Structural descriptions reflect the basic coding processes of the human mind. From these resources, descriptions are constructed which are

> invariant across actions for example and which can be explicitly represented in the form of a symbol, but they originate in neither (Olson, 1983).
>
> Spatial development may be attributed to two concurrent processes -- the increasing elaboration of the spatial aspects of the structural descriptions of objects, and the gradual explication, or "representation" of the features of those descriptions. ... These explicit representation make possible the perception of **form** whereas structural descriptions permitted the perception of **objects**. ... The spatial features of the display must be explicitly represented to permit the solution of some spatial problems (Olson, 1983).

The role of activity in the development of spatial cognition which Piaget and Inhelder attribute so much importance is for Olson and Bialystok important, but for a different reason. They claim that activity is important because it brings the child into contact with alternative choices, rather than as interiorized knowledge. To Olson and Bialystok, the child's correct choices are encountered in the process of activity, and it is these choices that are later retrieved from memory not an action schema. Activity, for Olson and Bialystok, is only one of several such means of detecting and explicating spatial relations. Mathematics, language, visual arts, and instruction are others. Spatial development, they believe, involves both elaboration and explication of spatial relations.

### 2.4. Development of Spatial Cognition

The general principle of development is defined by Werner and Piaget as the "degree of organization and, thus, is not limited to the processes changing over time, but may be used for the conceptual ordering of contemporaneous systems" (Hart, 1973). Piaget's genetic epistemology is both empirical and theoretical. His main goal was to study children's gradual attainment of increasingly effective intellectual structure. His emphasis in studying mental activity is on what the individual **does** in interactions with the world. Knowledge of reality is not given to a passive observer, but must be discovered and constructed by the activity of the child (Ginsburg, 1969). Piaget explicitly mentions that there are two mental processes involved in

mental growth. One is "*development*, which results in genuine learning, and *learning* in the narrow sense" (Ginsburg, 1969). Development is spontaneous and vital. Learning, by contrast, is provoked and limited to certain situations. Piaget felt that learning occurs primarily as a result of development. Sometimes this distinction between development and learning is difficult to make. Hart and Moore also discuss this difference between learning and development.

> Learning implies quantitative changes in the reception and retention of information or subject matter. It refers to the situation in which information is presented to the individual who changes through reacting to it and corrects initial attempts in response to indications about her prior successes. On the other hand, development implies qualitative changes in the organization of behavior. Most often it refers to the situation where the individual changes as a function of interaction between current organization and discrepancy with the environment (Hart, 1973).

Both Werner and Piaget emphasize the notion of structure in their theories of development. Their findings are similar. Werner specified three developmental progressions; progressive self-object differentiation, progressive constructivism, and progressive perspectivism (Hart, 1973).

Piaget distinguishes between structural and functional aspects of cognitive development. The functional aspect remains invariant throughout ontogeny, whereas the structural aspect changes. The process of development is influenced by maturation, experience or contact with objects, social transmission, and equilibration. Piaget's theory of equilibration is, to my view, his most important contribution to the field of developmental psychology. It brings together structure and function. It conceives of stages or levels of development as plateaus of equilibration between the organism and its environment. It conceives of the child's behaviors as strategies used by the organism to compensate the perturbations of the outside world. The sequence of stages is hierarchical with each stage standing for a level of equilibration. The stages are differentiated by the degree of integration, consolidation, and coordination. Piaget specifies four major periods of development -- the sensorimotor, preoperational, concrete operational, and formal operational. "Each

level is composed of an organized totality of mutually dependent and reversible behavior sequences known as *schemas* (or *schemes*)" (Hart, 1973).

Developmental issue are of primary importance in working with children. The most important aspect of my research will be to understand and analyze children's understanding and use of spatial concepts involved in 3-D computer graphics. This requires an understanding of the development of spatial cognition in the child. To accomplish this, let me first review Piaget and Inhelder's findings on the stages of development of spatial cognition, second relate these stages to the types of spatial understanding, or content of spatial cognition; Topological, Projective, and Euclidean, and third introduce Piaget and Inhelder's most important experiments.

### 2.5. Piagetian Stages of Spatial Development

Spatial cognition was first discussed by Cassirer. He proposes a hierarchy that begins with organic or active space, and proceeds to perceptual, and symbolic or abstract space. For Cassirer, what divides us from the animal world is the human "ability to comprehend and represent the idea of abstract space -- the space of 'pure intuition', bereft of any necessary concrete referent" (Cassirer, 1944) Werner's theory of the development of space was similar but more comprehensive than Cassirer. It included three levels of development; sensorimotor, perceptual, and contemplative. Piaget and Inhelder's empirical studies are the most extensive attempts to understand the child's development of spatial thought. They applied Piaget's structuralist frame to the study of the development of space (Piaget, 1967), geometry (Piaget, 1960), and mental imagery (Piaget, 1971) in the child.

In the **sensorimotor stage** (0-2 years), the child evolves from reflex activity to the ability to coordinate actions and internalize thought. This stage culminates in the simultaneous development of mental imagery, object permanence or object concept, and in the ability to distinguish self from surrounding and to move freely in a limited spatial terrain. Finally and most important, actions become internalized into thought patterns which mark the beginning of representational space. At the end of the sensorimotor stage, the child can take shortcuts (combination), return to the point of origin

(reversibility), and detour around obstacles (associativity). These groupings of spatial displacements underlie the further development of spatial intelligence (Hart, 1973). The appearance of mental imagery make possible delayed imitation and results in the first attempts at drawing. "The symbolic function thus evolved facilitates the acquisition of language or a system of collective signs. Hence from being purely perceptual, space [becomes] partly representational" (Piaget, 1967).

In the **preoperational stage** stage (2-7 years) the child can represent the external world using symbols and begins to mentally operate on real or symbolic objects. Certain basic transformations can be performed, although the representation of space is still static and not coordinated into reversible structures. The child's thinking is still egocentric or tied to her own point of view (Piaget, 1967, Hart, 1973).

In the **concrete operation** stage (7-12 years) the child's conception of space gradually develops into mobile, flexible, and reversible structures. This stage begins with the appearance of concrete operations and ends with the organization of operations into logical structures. Spatial thought is still dependent on the presence of real or represented objects. The child becomes free of an egocentric approach to space and achieves a degree of abstraction through the logical coordination of space from multiple points of view (Piaget, 1967, Hart, 1973).

In the **formal operation** stage (11-15 years and beyond) spatial operations are completely removed from real actions, objects, or space. The adolescent can grapple with the whole universe of spatial possibilities through the mathematical multiplication and coordination of space (Piaget, 1967, Hart, 1973).

**2.5.1. Content of Spatial Thought.** Piaget and Inhelder characterize three major types of relations or properties of space -- Topological, Projective, and Euclidean space. **Topological** properties (proximity, separation, order, and continuity of surface or enclosure) are the qualitative relations internal to the object. **Projective** space is the coordination of both the viewpoints -- actual and virtual -- and the figures considered in relation to these viewpoints, such

as straight line, triangle, or parallel lines which remain invariant under Projective or perspective transformations. **Euclidean** or metric properties require the concepts of straight lines, distances, measurements, parallels, and angles to be fused into a single operational whole. These relations, completed by the construction of the reference frame, are established between objects and allow their location within an organized whole. The coordinates are the logical multiplications between relations of order in two and three dimensions and express structures of Euclidean space, which link together objects, their positions and displacements, and relative distances (Piaget, 1967). Piaget and Inhelder explain this in greater detail.

> [A] reference frame is not simply a network composed of relations of order between the various objects themselves. It applies equally to positions within the network as to objects occupying any of these positions enables the relations between them to be maintained invariant, independent of potential displacement of the objects. Thus the frame of reference constitutes a Euclidean space after the fashion of a *container*, relatively independent of the mobile objects *contained* within it (Piaget, 1967).

Mathematically and psychologically, Projective space and Euclidean space derive independently from topological space. Topological concepts of the straight line and elementary Projective relations with the addition of "viewpoint" forms Projective concepts. These same topological notions are used to derive the Euclidean concepts of distance and measurement. Both Projective and Euclidean space form comprehensive systems whereas topological relations remain internal to each object regarded as an isolated thing in itself (Piaget, 1967). The reference frame is a construct which underlies Projective and Euclidean space and becomes involved in performance on various spatial, developmental, and cognitive style tasks. Age 9, which is mid-concrete operations, is a decisive point in the development of spatial concepts, because the framework appropriate for the comprehension of Euclidean and Projective systems is completed. At this age, mental imagery in cooperations with these operations becomes dynamic and anticipatory.

In summary, Piaget and Inhelder's general findings on the development of spatial thought are:

> [T]hat the representation of space arises from the coordination and internalization of action; ... that the genesis of the image arises from the internalization of deferred imitation; ... that there are four levels or structures of spatial organization; ... that there are three classes of specific spatial relations which form the content of spatial cognition: topology, projective, and euclidean or metric relations; ... that the understanding of topological relations precedes the understanding of projective and Euclidean or metric relations; ... [and that projective and Euclidean relations] develop in parallel, although the final equilibrium of Euclidean relations is achieved slightly later that projective (Hart, 1973).

**2.5.2. Piagetian Experiments.** In 3-D computer graphics the concepts of Projective and Euclidean space become intertwined. Understanding Projective space through the interpretation of the image on the screen is important, as is the ability to choose a point in space from which to view a synthetic scene. Both of these tasks require an understanding and application of Projective concepts. Understanding and application of Euclidean space is essential in all of the transformations performed on synthetic objects in 3-D computer graphics, because these transformations are intimately linked with the Cartesian coordinate system. Below are summaries of a few of Piaget and Inhelder's experiments on Projective and Euclidean space which are pertinent to the spatial concepts involved in 3-D computer graphics. Experiments 1 and 2 reveal developmental issues related to Projective concepts and experiments 3 and 4 are related to Euclidean concepts. Experiment 5 requires the integration on of both Projective and Euclidean concepts as does 3-D computer graphics.

**Experiment 1: Projective Lines and Perspective.** This experiment has three parts. In the first part of the experiment the child is asked to imagine what apparent shape a needle and a disc will present when placed in a number of different positions. The purpose is to determine how the child will represent objects seen from different perspectives. The child must understand the difference in actual and apparent shape of an object, namely that the shape of a Projective straight line remains unaltered from whatever point it is seen. Only its length is changed. The drawing or imagining of Projective straight

line presuposses a Projective or Euclidean space.

In the second part of the experiment a doll is used as an "observer" and the doll is placed at right angles to the child. The child is asked how the object will appear to the doll. To find one's own viewpoint implies others therefore perspective is a reversible operation. The notion of a single viewpoint is an essential requirement for the notion of "left-right," "above-below," and "front-back."

In the third problem, the child is asked to draw railroad tracks and fence posts as they recede into space. From this experiment it is possible to see how the child represents one of the most common distortions of perspective -- the convergence of parallel lines. This experiment also allows the experimenter to investigate the childs concept of intensive and extensive quantity without the use of any metric system by noting the distance between ties or fence posts as they recede. This experiment shows the child's combined coordination and differentiation of perspective. At stage I (4-5 years) the child is unable to represent shape or understand perspective. In stage II (7-8 years) the child totally or partially fails to distinguish between different viewpoints and her representations lack perspective. By stage III (7-12 years) the child progresses from partially distinguishing between different views even in her own drawings, to attaining operational coordination of points of view, and applying systematic perspective to drawings. Perspective does not appear until the formation of a coordinate system or system of references (Piaget, 1967).

**Experiment 2: Coordination of Perspectives.** This classical Piagetian experiment is on coordination of perspectives. A 3-D model representing three mountains, each differing in color, location, size, and objects on the summit is placed on a table. The child is seated in front of the model and a doll is placed at alternate quadrants. The child is asked to either pick the correct view which the doll sees from photographs, or to represent it with cut out "mountains." "The development of perspectives requires a comprehensive, global construct, one which enables objects to be linked together in a coordinate system, and viewpoints to be linked by Projective relations corresponding

to various observers" (Piaget, 1967). At stage I (4-6 years) the child does not understand what the experimenter wants. In stage II (6-9 years) the child fails to realize that the "observer" sees different things from different views. There are attempts to represent alternate views but only in terms of one dominant feature. However, the child usually relapses to her own point of view. This egocentric reference prevents the child from reversing right-left and front-back relationships. By stage III (7-12 years) the child understands that left/right and front/back relationships change with differing point of view. By the end of this stage the child systematically coordinates all points of reference (Piaget, 1967).

**Experiment 3: System of References - Horizontal-Vertical.** The child is asked to anticipate the position of the water level in a bottle, when the bottle is presented in different orientations. Cylindrical and spherical bottles are used. This experiment tests the child's understanding of the horizontal axis. To test the child's understanding of the vertical axis, Piaget and Inhelder present a cork with a stick rising out of it and floating on the water. The child is asked to draw the stick with the jar at different orientations. In the third part of this experiment the child is asked to place poles "nice and straight" on a sand mountain. At stage I (3-6 years) the child is unable to distinguish the surface of planes for fluids or solids. The child only thinks in topological terms of something inside the bottle and shows the poles lying on the mountain. In stage II (5-8 years) the child initially shows levels parallel to the base of the bottle and the poles perpendicular to the mountainsides. The liquid is usually shown as expanding with the jar tilt. Even after seeing the jar being tilted the child does not change the representation because there is an absence of an appropriate schema. Gradually the child represents the water level in the tilted jars as no longer being parallel to the base but is connected at the corners of the bottle no longer parallel to the base. The water level in the spherical bottle is drawn correctly because there is no internal references as there is in the cylindrical bottle. The poles are still perpendicular in the drawings, but they are vertical in the sand. The spatial orientation is mainly determined by the particular configuration represented not by an external system of references, as a result, there is only partial coordination.

By stage III (7-12 years) the child discovers the vertical and the horizontal for all positions of the bottles through trial and error. By the end of the stage, the child is able to anticipate correctly the horizontal and the vertical because of a coordinated system of references (Piaget, 1967, Voyat, 1982).

**Experiment 4: Locating a Point in a Plane.** The child is given two pieces of paper, identical except that one has a red dot on it. One piece of paper is placed at the top right-hand corner of the table and the other at the bottom left-hand corner. The child is also supplied with a ruler, strips of paper, a stick, and a length of thread which can be used for measuring. The child is asked to put a dot in the same place on the other sheet. At stage I (4-7 years) the child first locates the point purely intuitively -- visually. Over the course of this stage, the child comes to use the rod to measure, but only as an aid in visual estimation. In stage II (6-8 years) the child makes a qualitative estimate by measuring with a ruler in one dimension -- either vertical or horizontal, indicating uncoordinated one-dimensional measurement. By stage III (7-10 years) the child discovers by trial and error measurement in two-dimensions. Often the child starts with a single oblique measure from the corner, but gradually becomes aware of a need to determine the angle. The child comes to understand that the two measures must be perpendicular. Finally the child coordinates measure in two dimensions and uses Cartesian coordinates. Measurement in three-dimensions is synchronous (Piaget, 1960).

**Experiment 5: Rotation and Development of Surfaces.** The child is seated at a table with pencils, scissors, paper, a paper cube, and a pyramid. A rectangular sheet of paper is folded once to make a "ridge roof." The child is asked to draw the shape of the "roof" if it was opened out flat. Then the child is shown the cube and the pyramid and asked to show how they would look if they were unfolded. This experiment is especially interesting because the solution involves the use of anticipatory imagery and coordinated notions of both Projective and Euclidean concepts. Piaget and Inhelder's analysis of the solution will be discussed later in references to 3-D computer graphics. At stage I (4-5 years) the child fails to understand the problem. The child is unable to imagine a genuine rotation. In stage II (5-7 years) the child

generally draws the unfolded (developed) figure identical to the intact figure or similar and smaller because the loss of one dimension (three to two dimensions). Sometimes there is an attempt to indicate the developed surface through the addition of one or more surfaces, although they are vague or disconnected. There are unsuccessful attempts at imagining the rotation and drawing it. In stage III (7-11 years) some children attain a partial solution by showing all the correct surfaces although they are incorrectly combined or the unfolding action itself is represented with the surface only partly opened or a stroke indication the direction of the rotation. The child can imagine and draw the rotation of a cone or cylinder but not a cube or pyramid. By stage IV (11-13 years) the child can draw the developed cube and pyramid with little or no difficulty.

### 2.6. Spatial Reference Frames

Pufall defined a **spatial reference system** as a "systematic representation of spatial relations among objects which provides a set of coordinates for expressing transformations of such relations" (Pufall, 1973). The development of the reference frame progresses from merely subjective to a more objective representation of spatial relations. A deeper understanding and discussion of the reference frame is necessary because as mentioned earlier this construct underlies Projective and Euclidean space. It also becomes involved in performance on various spatial, developmental, and cognitive style tasks.

Piaget and Inhelder (1967) and Mach (1906) point out that the first reference frame used is the "postural system" of our own body. Our sense of orientation, balance, and position is regulated by gravitational effects on the vestibular system with its three orthogonal semicircular canals. Through lying down or standing upright we become aware of our orientation in space in reference to our body. For Piaget the "concepts of horizontal and vertical are by nature physical rather than mathematical" (Piaget, 1967).

Mathematicians formalized this internal orientation in the development of a coordinate system. Classical Euclidean geometry, lacking a numerical coordinate system, addressed relationships between objects in an axiomatic way. Descartes combined algebra and geometry and created the **Cartesian**

**Coordinate System**, which consisting of three mutually perpendicular axes that intersect at the origin. This system creates a one-to-one mapping among real numbers, points in physical space, and points (ordered triplets) in a mathematical coordinate system. This innovation provided a coordinate system that allowed for both the representation of physical objects and the mathematical transformation of objects, such as generalized rotation, translation, and size scaling of geometric objects.

Physical objects are perceived and coded with respect to a cognitive coordinate system (Marr, 1978, Rock, 1973) which "has an effect on recognition, information retrieval and on spatial transformations" (Just, 1985). The mental descriptions of objects are thought to be comprised of an implicit reference to a coordinate system. This cognitive coordinate system "consists of an implicit origin and some directional axes" (Just, 1985). Often in the cognitive coordinate system, the origin is the center of gravity of the object. The cognitive coordinate system becomes evident is mental rotations tasks by the choice of the axes of rotation. Just and Carpenter believe that this choice of axes is partially determined by the task demands. In fact, they argue that the most important psychological difference between a formal mathematical coordinate system and a cognitive coordinate system is "the variation in possible coordinate systems within which an object can be embedded" (Just, 1985).

In the mental representation of objects, along with the implicit origin and axes of the objects, there is also implicit information about the viewing point from which this object is viewed. The occurrence of a viewing point also implies Projective space with all the distortions and occlusion that occur in perception. In the cognitive coordinate system there are upper limit restrictions imposed by the amount of working memory. It is not possible to keep a main cognitive coordinate system and an embedded coordinate system in an activated state simultaneously (Just, 1985).

The Cartesian coordinate system is fundamental to 3-D computer graphics as is the cognitive coordinate system with its implicit Projective system and memory limitations. The relevance or role these two systems play in children's understanding of 3-D computer graphics will be discussed later.

The reference frame is not necessarily a stable global construct. There are often more than one reference frame in a particular situation. What becomes an important issue is how, why, and when a particular reference frame is chosen. The referent for the reference frame can change and the assignment of the referent is often an automatic or unconscious process. Each object or environment has its own reference frame. For example, an object can have its own intrinsic or, as Olson and Bialystok call it, *canonical* references (front-back, top-bottom, and/or left-right) and the environment this object is in also has a reference frame. These multiple reference frames are often embedded.

Let's take an imaginary example to better understand the complexity of the reference frame. An object, for instance a toy airplane, is placed on a table, the table is in a room, the room is in a building, the building is in a town, the town is on a mountain at a particular elevation at a particular longitude and latitude in the northern hemisphere of the world. A child who is in this room is presented with a task, any spatial task. Depending on the problem being solved the choice of frame of reference can be a local referent, the airplane or the child herself, to a global referent, the room or world. An individual is not necessarily conscious of the choice. The task itself may determine the choice or inversely the choice of the correct reference for the task will determine the solution or performance on the task.

Alternate strategies and/or differences in stage of development may be determined by the conscious or unconscious choice of reference frame. As Just and Carpenter point out, the "strategy difference in spatial tasks can be explained in terms of cognitive coordinate systems that the subject adopts" (Just, 1985). For example, in the Piagetian water level experiment discussed earlier the reference frame that the child uses to solve the problem determines the solution. When a young child uses the bottom of the cylindrical bottle as a referent, this choice results in the incorrect solution. When the bottle is turned upside down or positioned diagonally, the water is still represented as being parallel to the bottom of the bottle. The child's choice of the upright bottle as the referent was the most local of the many possible referents, whereas the table or the room would have been a better indicator for the

horizontal level of the water. For this reason it is easier for a child to draw the level of water in a spherical bottle. The spherical bottle, a more neutral referent, is not used as the reference frame, therefore the child chooses a more global referent (e.g. the table). Witkin's work on cognitive styles (Witkin, 1981) is primarily based on the Rod and Frame Test (RRT) and Embedded Figures Test (EFT) which was designed to distinguish field dependence or field independence. The solution to these two tests is based on the choice of the reference frame that is used to perform the task. Witkin's work will be discussed further in the next chapter 4.

The construction of the coordinate system is for Piaget the culmination of Euclidean thinking. Rock (1973), Marr and Nishihara (1978), Metzler and Shepard (1974), Just and Carpenter (1985), Pinker (1980), and Olson and Bialystok (1983) all believe that physical objects are perceived with respect to an implicit coordinate system. This implicit coordinate system is part of the mental representation of objects and fundamental to spatial thought.

Olson and Bialystok base their theory of spatial cognition on the existence of this implicit coordinate system. To them the

> origin of spatial concepts lies in the structural descriptions constructed to represent objects and events. Spatial information pertaining to the internal form of objects, to relations between objects, and between objects and their environments constitute part of the implicit structural descriptions which underlie the perception and representation of objects and events (Olson, 1983).

This structural description, they argue, is in the form of a proposition which relates an argument to a relatum through a spatial predicate. For example, "The lamp is on the table" would be in the form of "on(lamp,table)," where "on" is the predicate, "lamp" is the argument, and "table" is the relatum. All dimensional predicates are organized in terms of axes, planes, directions, distance, and positions in three dimensions. The most important predicates are related to Euclidean space defining three axes -- front-back, top-bottom, and left-right. The concepts for these three dimensions, as expressed in language, develop in order of difficulty, with the dimension of top-bottom first, then

front-back, and finally left-right.

Olson and Bialystok have declared four general classes of relatum; **ego, observer, object**, and **environment**. These provide a reference frame for the interpretation of a spatial predicate and in turn spatial thought. Spatial predicates may easily be assigned to all four. Even though spatial predicates are "automatically assigned to ego, objects and events in the visual environment, their voluntary assignment to particular aspects of displays or particular points of view is accomplished only through explicit spatial concepts" (Olson, 1983). However, it is in the assignment that they frequently come into conflict. Consequently, choosing the appropriate reference frame or relatum for the proposition becomes one of the more difficult problems is spatial cognitive tasks.

These relatum are not merely descriptions, they are "categories of the mind" (Olson, 1983) which are hierarchical in terms of their invariant properties. The environment provides the most stable reference system because it is common to itself, the objects, and the people in it. The next most invariant properties lie in a canonical object, whose orientation is fixed. For example the front of a car is always the front of the car, but for a noncanonical object such as a ball the front may become the side or the back of the ball depending on the point of view. The ego is the least invariant source of description between objects because it changes for each person and changes when the person moves (Olson, 1983).

Canonical objects, those having intrinsic spatial parts (i.e. top, bottom) or reference frame, can be used as a reference frame for other objects. This is also true of ego, which actually can be thought of as a special object. An object or the ego can become a privileged object or a referent to reconstruct a scene or to make spatial judgments. Again the Piagetian experiment of horizontality and verticality is a good example. A child can solve the water level problem earlier with the spherical bottle than the cylindrical bottle. The cylindrical bottle is a canonical object, with intrinsic top, base, and sides, therefore it is used as the referent for the water level rather than a more global referent.

As mentioned earlier, the reference frame is an integral part of recognition, information retrieval, and spatial thought. The reference frame also plays an essential role in performance on spatial tasks. The particular reference frame used depends on the context, the materials, and the complexity of the problem posed. All of these variables can effect the strategy and the reference frame used and consequently the performance on a task. These factors also make spatial cognitive task very style sensitive. A large number of the diagnostic instruments for determining cognitive styles involve the determination of the reference frame as an underlying spatial component. Since as we know the choice of reference frame can affect the performance on a task, it is important to look differences in spatial cognition in chapter 4. Mental imagery also plays a major role in the performance on spatial tasks. In order to better understand the psychological considerations of transformation in 3-D computer graphics, I will first review the relevant concepts in mental imagery.

# CHAPTER 3

# MENTAL IMAGERY

## 3. WHAT IS MENTAL IMAGERY?

Transformational imagery (mental rotations) and the development of imagery in children are both relevant to the study of spatial cognition and 3-D computer graphics. The study of mental imagery, like spatial cognition, is characterized by many opposing views and a great diversity of findings depending on the aspect of imagery studied, the context, the task, and the disciplinary perspective.

What is mental imagery? We have all experienced imagining ourselves moving around our environment, moving an object, putting our hand out to catch a ball, designing a new object, or even imagining what a partially obscured or hidden object looks like. Imagery has many uses. Its main evolutionary purpose is probably to anticipate events in our environment. For this reason imagery has some of the constraints imposed by natural phenomenon in the real world, yet it is not so tied to nature as to prohibit inventions as in the arts.

Marks (1985), in his review of different psychological paradigms used to study mental imagery, discusses the different research interest of various fields of psychology, such as neuropsychological, experimental-cognitive, developmental, psychoanalytic, and behavioralist. Due to these different fields of interest with their different approaches, perspectives, and uses of imagery, it is no wonder that the imagery literature varies as to what is its function, how it works, how it develops, how it is stored and retrieved from memory, when it occurs, and so on. This paper will address only the neuropsychological, experimental-cognitive, and developmental issues related to mental imagery.

Kosslyn (1980) and Paivio (1983) maintain that imagery is a basic form of thought. Paivio developed a theory of a dual coding system of thought, the verbal and imaginal (Paivio, 1983). Kaufmann takes this model and asks when each coding, verbal or imaginal, is used in the process of problem solving. He found that the verbal symbol system is used for a task that is very familiar, because it is quick, stable and useful for generalized problem solving. In contrast, he found that the imagery symbol system is used with tasks that are novel and require information-integrating functions and are highly complex and require extra processing (Kaufmann, 1986).

Kosslyn (1980) breaks imagery into different functions. These functions are the long-term visual memories, the visual buffer (shared with perception), the image generation process, and the inspection process. Using Kosslyn's model of mental imagery as a base, Farah (1988) searched for the location of imagery functions in the brain. Farah found that imagery is a separate system from other forms of memory. Long-term visual memory is bilaterally represented in the brain and shared by imagery and recognition systems. She found that the left the hemisphere contains the structures for image generation. Kritchevsky (1988) and Metzler and Shepard (1974) reports that the structure for spatial processes are, however, located on the right side of the brain. In addition, studies of the human brain have indicated left hemisphere specialization for language and right hemisphere specialization for spatial cognitive processes. These underlying structures appear to be present from birth. "This specialization increases as general cognitive growth increases, with important maturational points at age 5 and puberty" (Lillo-Martin, 1988). This is interesting because images, rich with implicit spatial information, are used to solve mental rotation tasks which Kritchevsky found implicated in the right parietal. Corballis (1982) discusses research which found greater EEG activity over the left than the right hemisphere during mental rotation. One explanation has been that the more complex or sophisticated the task the more the left-hemisphere is involved because of the sequencing required. This may also be a reason why Piaget and Inhelder found mental imagery to first focus on end states and later in development on the transformation process itself. This change occurs at about the same time that order due to seriation

becomes operational.

Pinker, a cognitive psychologist, discusses imagery as one of two sub-topics of what he refers to as "visual cognition." The first sub-topic is the representation of information involving the visual world presently before a person. The second subtopic is visual imagery which "is the process of remembering or reasoning about shapes or objects that are not currently before us but must be retrieved from memory or constructed from a description" (Pinker, 1985). This view is similar to Piaget's differentiation between perceptual space and representational space. Piaget characterizes a visual image as "a figural evocation of objects, relations, and even classes, etc. [Imagery] converts them into concrete and simili-sensible form, though at the same time it possesses a high degree of schematization" (Piaget, 1967).

Through their study of the mental scanning of images, Kosslyn and Pinker (1978,1980) found that the 3-D "structure of objects are encoded in long-term memory in 3-D object-centered coordinate systems. When these objects are imagined, this information is then mapped to a 2-D 'surface display' in which the perspective properties specific to a given viewing angle can be depicted" (Pinker, 1980). This three-dimensional quality of imagery is part of the implicit spatial information available in imagery. This internal environment can be explored through the "minds eye" or used in anticipating changes in the external 3-D world. Implicit in the "minds eye" is a viewing point and perspective properties as in visual perception. Just and Carpenter (1985), Olson (1983), and Shepard (1982) also make note of this implicit viewing point as being part of the structural description or mental representation of an object. In J3D this becomes more explicit in the creation of graphical objects and in the specification of views.

### 3.1. Development of Imagery in the Child

Piaget and Inhelder conceptualized cognition in terms of two major aspects -- the **operative** and the **figurative**. The operative aspect can be thought of as dynamic. It refers to actions which result in some transformation, construction, or change of reality. "By contrast, the figurative aspect refers to action by which the child produces a 'copy' of of reality rather than

its transformation" (Ginsburg, 1969). Piaget and Inhelder introduce three further distinctions in the figurative aspect. The first is **perception** and implies that the object or event must be physically present and experienced through the senses. The second is **imitation** by which the childs actions produce a copy of the actions of people and things. The third is "**mental imagery** which refers to personal and idiosyncratic internal events which stand for or represent absent objects or events" (Ginsburg, 1969).

Piaget and Inhelder have carried out the only really thorough investigation of the development of imagery. They conceptualize the image not as a reproduction of perception, but as an "active and internalized imitation" (Piaget, 1971). They classify imagery into two groups according to the content and degree of internalization. The degree of internalization depends whether the image is immediate or deferred. The first group is **reproductive imagery** which evokes objects or events already known. The content of reproductive imagery is the **static** image, the **kinetic** image, and the **transformational** image. The static image is of objects or configurations that are devoid of motion. The kinetic image evokes movements figurally, in other words it only deals with linear displacements. The transformational image represents in a figural manner transformations already know. Piaget and Inhelder refer to transformations as changing of the form not just the position of an object.

The second group is **anticipatory imagery** which by figural imagination represents events previously *not* perceived. The content again is **kinetic** or change of position and **transformational** which is a change in the form. Piaget and Inhelder broke transformational imagery into transformations that only bear on the results or product of a transformation (end states) and transformations that bear on the process of modification itself (Piaget, 1971).

Through their research with children of different ages Piaget and Inhelder found that the development of imagery did not go through stages as did other areas they had studied. They found two decisive moments and a single break of general significance. The first of these moments is the appearance of the image, which occurs at 2 years of age about the same time as the formation of the symbolic function. The second decisive moment comes at about 7 or 8 years of age with the emergence of anticipatory images. Prior to

this change the child is unable to represent or imagine either kinetic or transformational images. After this age the child becomes capable of thinking in terms of transformations due to the operational structures brought about by the equilibration of intellectual processes. At first, children are only able to represent initial and ending states of the transformations. Later they are able to represent the process or intervening states. With the formation of the operations the child can start to link static states, can deduce and anticipate with the assistance of images, can anticipate movements of transformations in a given order due to operational seriation, and finally can anticipate some conservations.

Piaget and Inhelder point out the special nature of the image in spatial cognition. The visual image with its spatial properties acts as an auxiliary to operational functioning because both the *form* and the *content* of the image are spatial. "Geometrical intuition" is the only field where the spatial image tends towards a real isomorphism between the symbolizing form and the symbolizing content.

> An image with logico-arithmetical content entails the conversion of nonspatio-temporal transformations into a necessarily spatial form. The spatial image, on the other hand, represents spatial content in forms that are likewise spatial. And spatial operations (displacements, projections, etc.) relate to transformations actually taking place in space and presenting figurative, not exclusively operative characteristics. Thus the transformations themselves are in a sense figures in space (Piaget, 1971).

Piaget and Inhelder talk clearly about the interrelationship of mental imagery with spatial cognition, development of operations, and spatial representation in mathematics in their "essential results" of the development of imagery in the child.

> First, the anticipatory image can only be formed with the help of the operations. And second, the image proves to be indispensable for a representational reconstruction of movements and transformations already known and possibly well known to the subject. It seems, therefore, that an operational framework of a logico-mathematical kind is necessary, not only, as one would expect, for notional interpretation of perceptual data, but also

> - and this more surprisingly - for the imaginal evocation of such data. True, this logico-mathematical framework ... is a combination of elementary spatial operations and correspondence procedures. But this does not mean that they are any the less logico-mathematical and that they cannot in consequence lead to the formation of strictly deductive notions, such as conservation of length and surface, numerical equivalence, and so on. ... The fact of the matter is that conservation presupposes a system of quantitative compensations beyond the image's own capabilities, whereas the logico-mathematical framework, once constituted, rebounds on the image and makes objective anticipation possible. ... The image ensures finer analysis of "states," and even aids figural anticipation of "transformations," in spite of the irreducibly static character of such a figuration. This makes the image an indispensable auxiliary in the functioning of the very dynamism of thought - but only as long as it remains consistently subordinate to such operational dynamism, which it cannot replace, and which it can only express symbolically with degrees of distortion or fidelity varying according to circumstances (Piaget, 1971).

As Piaget and Inhelder have pointed out, there is a relationship between transformational imagery and operations, especially those operations related to reference frame and Euclidean space. They distinguish between the anticipation of the outcome of a transformation and the anticipation of the actual process of the transformation in its successive states. Piaget and Inhelder's research focused on children's representation of successive states of transformations in their investigation of anticipatory imagery.

Other researchers have studied imagery in children by duplicating the Piagetian experiment or creating new finer grained experiments in the Piagetian spirit. Some have corroborated their findings, while other have disputed them. For example, Marmor (1975,1977) found that very young children could use anticipatory imagery on a mental rotation task in which the child had to decide if two objects were equivalent. She conclude that her findings did not uphold Piaget and Inhelder's findings on the importance of operational thought in anticipatory imagery. However, Marmor's research focused on end states which children are capable of achieving at a younger age.

Other have carried out experiment in which the same children were given imagery tasks along with operational or spatial tasks. One such researcher, Dean has also done several studies on the Piagetian mental imagery tasks in conjunction with operational thought. Her findings were different from Marmor's, however because of concentrating on the representation of the *process* of transformation. In one study, Dean (1976) compared children's drawings of transformation in relation to their understanding of Euclidean spatial relations. Her research supports Piaget and Inhelder's findings that indicate an essential correspondence between children's coordination of reference frame with imagining movement and spatial relations. Dean and Scherzer (1982) have also investigated the links between drawing of 2-dimensional movements and mental imagery. They found that the problems that children have in their drawings are the same problems that children have in preparing for a mental image, therefore drawings can be used as a valid measurement of the quality of anticipatory imagery. More recently Dean and Scherzer (1986) have shown that the child's knowledge of sequence relations is necessary for the representation of successive states in a rotation movement, but not in deciding if two objects are equivalent through mental rotation.

De Lisi et al.(1976), studied children's anticipatory imagery and spatial operations. They found that children's performance on rotation tasks were related to their performance of the Piagetian coordination of perspective and water level tasks. For all of the tasks they found three types of understanding of movements in space: transposition, intermediate, and transformation. Transposition reflected an understanding of movement as a displacement. Transformation reflected a coordination of change of position and change in the object's features. Intermediate include elements of the other two, but they were not coordinated. They concluded that "rotation performance made spatial operatory levels explicit. In turn, these levels predicted success rates and types of errors on two other imagery problems" (De Lisi, 1976). In a subsequent study McGillicuddy-De Lisi and De Lisi took a closer look at the content of transformations through the use of a peg-board. The discrete nature of the medium allowed for the investigation of the strategy the child used to accomplish the transformations and the effects of anticipation on the form of

the transformations. The manner in which the children imagined geometric transformations corresponded to their understanding of "reference axes, distance relations, relations of equality and measurement" (De Lisi, 1981).

As Piaget and Inhelder emphasized, anticipatory imagery and spatial operations especially Euclidean geometric operations are intimately linked. In 3-D computer graphics these become explicitly linked by the way that children interact with the system. This will be discussed in more depth later in this paper.

### 3.2. Mental Rotation

Mental rotations up until the 1970's were studied primarily in psychometrics and cognitive development in children. In psychometrics, mental rotations are the basis of the PMA Spatial Relations test, Cube Comparison test, Block test, Hands test, Flags tests, and many other spatial tests which have been created to study intelligence and spatial ability. Piaget and Inhelder, in their developmental studies, carried out the coordination of perspectives, the development of surfaces, the appearance of toppling sticks, and other anticipatory imagery tasks requiring mental rotations as part of the process. Piaget and Inhelder found that mental rotations are not performed prior to the concrete operational stage. Others have showed that mental rotation abilities change through the life span. The rate of rotation doubles between middle childhood and adulthood (Kail, 1980,1985, Marmor, 1975) and then declines in old age (Berg, 1982).

Shepard introduced mental rotations to experimental cognitive psychology in 1971 (Shepard, 1971). He and his associates did many timed studies of subjects' response to tasks requiring mental rotation of 2-D and 3-D figures. Shepard's idea of timing the subjects' response to mental imagery tasks "has become one of the flourishing *paradigm* of modern experimental psychology" (Corballis, 1982). Mental rotation is one of the few concepts to bridge psychometrics, experimental, and developmental psychology. Corballis believes that it is a good *paradigm* because it is a "vehicle for studying much wider questions" such as the "role of imagery in human cognition, the nature of mental representation and mental process, and the appropriateness of

models based on analogy with the digital computer" (Corballis, 1982).

Shepard and Metzler created ten 3-D shapes, five were isomers or mirror forms of the other five. Each was made up of "cubes attached face-to-face to form a rigid armlike structure with exactly three right-angled elbows. ... For each of the ten objects, 18 different perspective projections [20 degree increments] were generated by digital computer" and photographed (Shepard, 1971). Two of these pictures at a time were presented to the subjects, who were timed on their response of "same" or "different." The reaction time increased monotonically with the angle difference between the two forms. The results suggested that the subjects rotate one form into congruence with the other through intermediate orientations. Subjects reported that the mental rotations were smooth and continuous and in some respects analogous to physical rotation. From this experiment Shepard proposed that subjects carry out a mental analogue of a physical rotation.

### 3.3. The Nature of Imagery

Since the time of Locke, imagery has played an important role in the debate about the nature of the representational processes (Steiger, 1983). Imagery still plays a major role in this debate. Shepard's findings on the mental rotation tasks have become a focus of this debate. Is imagery analogue or propositional? It is important to clarify that this debate entails two elements. The representation of the image itself and the process of transformations of the mental image. Some people, such as Metzler and Shepard (Metzler, 1974, Shepard, 1971), Shepard (1982), Cooper and Podgorny (1976), and Kosslyn (1980) and others argue that imagery is analogic or "quasi-pictorial" and the process of transformation is also analogic. While Pylyshyn (1981,1979), Olson and Bialystok (1983), and Just and Carpenter (1985), and others argue that spatial representations and imagery are propositional in nature even though the subjective impression of mental rotations is of a smooth, continuous motion.

One of the main premises of the analogue proponents is that the object is rotated holistically. Cooper and Podgorny (1976) showed that the complexity of random 2-D polygons had no effect on the rate of rotation, therefore

rotations are analogue. Pylyshyn tested this hypothesis by having subjects decide if one figure was a subfigure of the other. He found that the rotation rate was not due to holistic analogue rotation images, rather it was an "articulated and piecemeal process in which the analysis of the stimulus figure interacts with the comparison task" (Pylyshyn, 1979). Just and Carpenter (1985) also found that subjects rotate complex 3-D objects (Shepard-Metzler shapes) part by part or piecemeal in separate rotation episodes. Both of these experiments indicate that rotations are propositional.

Barfield (1986), in his study of 3-D mental rotations for CAD systems, came up with a "hybrid" model to account for these different arguments. He maintains that the representation is propositional and the process is analogue if the object is simple, however if the object is complex the process is propositional. This difference he argues is due to memory limitations. Corballis addresses the differences between the analogue and the propositional theories by also saying that imagery is made up of both propositional and analogous elements. "Images are stored in propositional fashion but 'displayed' in analogue format" (Corballis, 1982). In a recent study Corballis (1986) found that the process of rotation itself is automatic, however it takes attentional control to set up the mental structures for the rotation. This is similar to Just and Carpenter's findings from eye fixations data collected during rotations. They revealed that most of the time spent on the rotation task, especially for complex figures, was spent in encoding the form prior to the rotation. They also found that the process of determining the frame of reference for the rotation is automatic and not under conscious control (Just, 1985). The orientation of the object is extracted as a single proposition and the rotation is carried out in discrete steps incrementally.

The work carried out by Just and Carpenter (1985,1976) based on the Cube Comparison task and on the Shepard-Metzler rotations provides a theoretical account of individual differences in strategy for these two tasks. They attribute the difference to different cognitive coordinate systems that subjects adopt and they emphasize that these differences are not a quantitative differences, rather they are a qualitative differences. In their experiments, they used eye-fixation data coupled with timed responses of "same" or

"different" for pairs of cubes. Subjects were interviewed about their strategy. They developed a procedural computer simulations of low-spatial and high-spatial strategies to verify their findings.

Just and Carpenter (1985) found four alternate strategies for the Cube Comparison task. The first strategy is **standard trajectories** which refers to rotations around axes that are perpendicular to the faces of the cube. They found that these were most often used by low-spatial subjects. The second strategy is **alternate trajectories** which refers to axes that are not perpendicular to the faces of the cube. These are generally shorter than standard trajectories and the choice of these arbitrary trajectories are determined by the task. These were most often used by the high-spatial subjects. The third strategy is **orientation-free description.** This strategy requires an object-defined coordinate system which is invariant with the object orientation. The relationship of adjacent parts is coded. Consequently no rotation is required. This strategy requires a thorough understanding of the structure of the object. This process was not as slow as the standard trajectories or as fast at the alternate trajectories. However it does show that a rotation task can be accomplished by non-spatial means. The fourth strategy is **perspective change** where both the object and the viewer position are coded into a cognitive coordinate system with the object as the origin. In this strategy the objects' position is maintained and the viewing point is changed. The axis-finding process entails a choice of which view of the object to take. This strategy had been reported on on Shepard-Metzler rotation tasks, but was not used by their subject. It is thought to be a more difficult strategy. The reasons for this are complex and will be discussed in the section on "Perspective-taking."

Just and Carpenter obtained eye fixation data that showed that the process used in mental rotations is discrete with fairly large step sizes. The process is not ballistic, rather it is "monitored after every rotation step to determine if the new orientation is sufficiently close to the target orientation" (Just, 1985,1976). A rotation is made up of three processes; **search, transformation and comparison**, and **confirmation.** Search entails finding a pair of matching letters on the cube. Transformation and comparison is the process of rotations and checking against the target. This process is repeated until it

can be determined if the two faces are similar. Confirmation is the process of checking subsequent faces with the same transformation and takes the same amount of time for each face.

Just and Carpenter found that there was no significant difference between the high and low spatial groups on the search process. The analysis of the fixation duration indicated that the major source of difference with the rotation strategy was in the transformation and confirmation process. The confirmation process was the major source of error for both spatial groups. The low-spatial subjects had a tendency to lose track or not complete a trajectory during the confirmation process, especially when a letter was imagined moved to a hidden face. A second source of error was not confirming all the letters, which is a bookkeeping problem. They found that from trial to trial there were differences in how subjects reacted to an inconsistency. Sometimes the subject would stop and other times they would do the entire process again. The high-spatial subject rotated at twice the rate of the low-spatial subjects. In the computer model this was simulated by incrementing rotations for high-spatial subjects by 30 degrees and for low-spatial subjects by 15 degrees.

This same methodology and computer simulation was applied to the Shepard-Metzler rotations forms. With these shapes there is only one axis per trial. Thus it produces the same strategy in all of the subjects -- the standard trajectory rotation. Again, the process they found was search, transformation and comparison, and confirmation. Errors occurred more often when subjects looked between non-corresponding ends of the figures. Just and Carpenter found that the source of individual differences was the same in both tests.

> In summary, the eye-fixation results indicate that low-spatial subjects take longer to perform a mental rotation task (increasingly longer at greater angular disparities) because their rotation rates are slower and because they are less efficient at mentally keeping track of their work in more demanding problems. Their poor bookkeeping forces them to do extra work, occurring in the episodes we have called *subsequent rotation* and *subsequent confirmation* (Just, 1985).

These differences of strategy, bookkeeping, and speed of rotation are likely the source of difficulty of the Shepard-Metzler rotation task for children.

### 3.4. Perspective-taking

"Perspective ability involves the ability to imagine or to represent how objects look relative to one another from another person's point of view" (Cox, 1977). Piaget showed that children made egocentric errors in the three mountain Coordination of Perspective experiment until the age of 9 or 10. This experiment has been an important focus of discussion on egocentrism, the ability to take another's point of view, and coordination of perspectives. Many have duplicated this experiment varying the type of object or array, the number of choices, and the means of response. All of these effect the age of competence at which this problem can be solved, ranging from 2 years to 10 years of age.

Laurendeau and Pinard found that Piaget and Inhelder's experiment was difficult because it required coordination of multiple Projective dimensions and of multiple perspectives produced by several objects. The diversity of observer positions and the complexity of mental operations needed to reconstruct the perspectives corresponding to the different alternative pictures and the inadequacy or absence of reference points or topological clues also added to the difficulty (Laurendeau, 1970). Many people have shown that by controlling the complexity of the task younger children are able to successfully take another's point of view. Donaldson (1978) recognizing how abstract this problem is for young children showed that if study was put in a context young children could understand, they could solve this task at a very young age. Fishbein (1972) showed that the stimulus complexity (one object or an array of several object), the mode of response, and the use of another person rather than a doll representing the "observer" effected successful performance for children as young as 3 1/2 years old.

Researchers found that different positions of the "observer" (90, 180, 270 degrees from the subject) were more difficult that others, however *which* position was more difficult was different in the various experiments. Cox (1977) designed an experiment in which the effect of masking or obscuring of objects

from different positions was controlled. He showed that the 180 degree position for the observer is the most difficult perspective change because it entails both a front-back as well as a right-left reversal.

In summary, the factors that have been shown to effect the difficulty of this task are:

(1) An array of several objects is more difficult than a single object (Fishbein, 1972, Donaldson, 1978).

(2) A non-canonical object or array is more difficulty than a canonical object or array (Olson, 1983).

(3) Choosing from a set of 8 pictures is more difficult than from a set of 4 pictures (Donaldson, 1978).

(4) Choosing from a set of pictures is a more difficult response than turning the array to a designated position for the observer to see (Fishbein, 1972).

(5) Choosing from a set of pictures is a more difficult response than indicating where the camera was located when the picture was taken (Olson, 1983).

(6) Taking the point of view of a doll is more difficult that taking the point of view of another person (Fishbein, 1972).

(7) Staying in the same place makes it more difficult than if the child is able to move to the position of the observer (even if blindfolded).

Piaget and Inhelder's three mountain task is very difficult for all of the above reason. There were three mountains, all of which were non-canonical. Some of the mountains masked other mountain from different points of view. The child was seated at a table and asked to respond to "what does the doll see?" by choosing from a set of 8 pictures.

Olson and Bialystok analyzed the perspective problem by looking more closely at the reference frame and subsequent spatial coding on many of the variation of this experiment. They found that the most important determinate of difficulty is the presence or absence of distinctive features of the

display which may be directly related to the position of the observer by means of a spatial predicate such as front, back, adjacent, etc. They found that

> The egocentric errors in perspective tasks appear, then, to be related to the pictorial mode of response rather that to a general characteristic of preoperational thought as suggested by Piaget and Inhelder. ... Egocentrism errors are most likely caused by an interaction of the visual array and the pictorial response mode which encourages the child to focus on the unanalyzed representation rather than on the discrete formal qualities of the display. ... Children have difficulty extracting an explicit form representation and making their response on the basis of it when they are faced with an image of the whole display whether real or pictured. This may be complicated as well by the child's inability to consider two complex representations of the same display, one that he knows to be true to his vision, and one that would be true of the observers view.
>
> The non-egocentric errors displayed a different pattern. When the correct orientation requires a two-part linguistic description (e.g., "front and side" which was a term used for the front corner) kindergarten children tended to leave off one component of this description (Olson, 1983).

Besides pointing out what can be changed so that younger children can accomplish this task, we need to look at why these changes enable younger children to perform successfully. Comparing the perspective task with a rotation task offers some insights into this question.

### 3.5. Perspective and Rotation

The rotation and perspective problems are logically and algebraically similar, but they are not psychologically similar. Why is this so? Just and Carpenter enumerate some of the differences in rotation and perspective change strategies.

> First, they appear to be used selectively for different types of stimulus objects. If the object is small, mobile, and manipulative, contrast, if the object is large and immobile, ... then people are more likely to mentally keep it stable and imagine their own position changing. ... A second difference, is that mental rotation is sometimes accompanied by an

imagined manipulation of the object with ones hands. By contrast, perspective change involves an imagined transformation in body position that is sometimes accompanied by reports of proprioception of such a change. A third distinction is that children of a particular age can perform a mental rotation task but cannot perform an equivalent perspective-change task. A fourth possible distinction is that mental rotation produces intermediate representations that correspond to intermediate orientations of the rotated object that lie between the initial and final orientation. By contrast, it seems possible to take opposite perspectives without passing through intermediate stages (Just, 1985).

Huttenlocher and Presson looked at the differences between rotation and perspective-taking tasks. In their first experiment they used parallel tasks of array-rotation and viewer-rotation. The viewer-rotation was more difficult. They thought this was due to the subjects interpreting the instructions literally. The subject either imagined rotating the array or moving to a new position (Huttenlocher, 1973). In their second experiment, however, they showed that the perspective change is not always more difficult. The perspective task was easier if the question asked was about specific items rather than the appearance of the entire array. They thought this was due to the coding of the array in relation to the larger spatial context, the room, rather than the viewer in relation to the array. They argued that the reason adults were better at this was because of their greater memory capacity (Huttenlocher, 1979).

Olson and Bialystok attribute the difference in difficulty in rotation and perspective-taking tasks to an increasing ability to code complex displays by means of compound predicates and an increasing ability to isolate and activate the critical spatial predicates needed to solve the task. They argue that the results of the Huttenlocher and Presson experiments are due to the superiority of a verbal response because it isolates the critical predicates. Olson and Bialystok emphasize the importance of explication and language of spatial thought in development.

Those aspects of the structural descriptions which become explicit as form representations, become subject to conscious control in speech, drawing,

> and so on. But those aspects of structural descriptions which do not come to be explicated as form perception, are not lost or ignored. They remain as part of our implicit knowledge of objects and forms, they can be activated as images, they can be transformed somewhat as the real objects of which they are structural descriptions, and some of them can be made explicit by new developments in language, mathematics, art, and so on. These later cultural activities make aspects of structural descriptions explicit and again subject to conscious control. And, finally, it is those aspects of description which are explicit and subject to control that are assigned to the left hemisphere.
>
> Spatial development may be attributed to two concurrent processes -- the increasing elaboration of the spatial aspects of the structural descriptions of objects, and the gradual explication, or "representation" of the features of those descriptions. (Olson, 1983)

As we will see in the chapter five and six, children using 3-D computer graphics are obliged to deal with spatial concepts explicitly through the object and view transformations, their specification through both language and mathematics, and visually from the subsequent computer image. Their understanding of the similarities and differences of object and view transformations presents an interesting issue to be looked at more deeply, because 3-D computer graphics offers the children dynamic control over both. Prior to looking at transformations in 3-D computer graphics, it is important to gain a better understanding of individual differences in spatial cognition and mental imagery and what affect they have on J3D as a learning environment.

# CHAPTER 4

# DIFFERENCES IN SPATIAL COGNITION

## 4. GENDER DIFFERENCES AND COGNITIVE STYLES

Gender differences and cognitive styles are an important element to consider when studying spatial cognition. The issue of gender differences in spatial "ability" has been one of the most persistent differences found in all abilities research (McGee, 1979). Researchers converge in showing that women are less performant in the area of spatial abilities than men. The interpretation of such a fact is an important consideration. Some people attribute this difference to biological, experiential, and/or strategy differences. Of these possible sources of difference the most relevant to this paper is the differences in strategy. Strategy difference may pertain to acquiring a strategy, choosing the correct strategy for the task, and/or efficiently using a particular strategy (Linn, 1986, Globerson, 1985).

Cognitive styles deeply influence how people learn and function. They are of particular interest in the study of spatial cognition for a variety of reasons: (1) The assessment tasks of some cognitive styles have an underlying spatial component as well as exhibiting some gender differences, possibly as a consequence of the spatial aspects. (2) In the field of education, it is essential to take into consideration style and gender. Our goal is to encourage educators to provide children with flexible learning environments where children of all different styles can progress by using the tools that are more salient to their style. This is especially relevant in environments for exploring spatial concepts. (3) In addition, spatial cognition, gender, strategies, and style have implications for user interface design considerations and capabilities for a 3-D computer graphics system where children or adults construct spatial ideas, images, and products.

Let me first discuss what is usually referred to as spatial abilities and gender differences in spatial abilities. Then issues of cognitive styles that are of most interest to this study will be discussed because they tackle issues of control rather than performance. Finally, the educational aspects of spatial abilities, cognitive styles, and strategies of relevance to spatial cognition will be discussed.

### 4.1. Spatial Abilities

As with the study of spatial cognition and mental imagery, spatial abilities are studied from different perspectives. Each perspective carries with it particular objectives, methods, and projected outcomes. The sources of these differences are still under debate because it is very difficult to separate the genetic, hormonal, biological, sex-related brain differences, and psychosocial influences (Halpern, 1986).

In the field of psychology, spatial abilities have been investigated from four different perspectives: the **psychometric** perspective comparing correlations between different spatial tasks to define factors in spatial ability; the **strategic perspective** which attempts to identify the qualitative difference of strategies; the **cognitive perspective** which attempts to identify the processes and their different qualitative efficiency; and the **differential** perspective comparing different populations (Linn, 1985). The psychometric perspective is of interest because the study of spatial cognition originated in this branch of psychology. Many subsequent studies have used the psychometric spatial abilities tests for various purposes. Addressing the strategic, cognitive, and differential perspectives is more difficult because these perspective have become intermixed and somewhat blurred.

Psychometric research has focused on finding intellectual factors. The spatial factor was distinguished from verbal and numerical factors through documenting individual differences on spatial tests. There has been some debate as to whether spatial abilities is a unitary factor. The use of different terms, different tests, and different results have contributed the controversy. The two spatial factors most often referred to are **spatial visualization** and **spatial orientation.** Spatial visualization usually refers to the "ability to

mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object" (McGee, 1979). Spatial orientation usually refers to the "comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to remain unconfused by the changing orientation in which a spatial configuration may be presented" (McGee, 1979).

Traditionally factor analysis research has grouped factors too grossly. Others researchers in the field have considered the terms confusing or defined the terms in different ways. More recently, the strategies and processes approaches have attempted to analyze individual ability differences in a more fine grained manner by focusing not only on the performance, but on the nuances and processes of the differences. For example, Just and Carpenter (1985) maintain that the major problem with traditional factor analysis is that it assumes the same strategy among all the subjects. They believe that these varying

> factors may actually refer to three distinct processes engendered by the use of different coordinate systems. The visualization (rotation) factor may result from the mental manipulation within a coordinate system defined extrinsically to the object. The object in this case is represented with respect to an axis that is usually provided by the visual environment or the retinal upright. The factor described as spatial orientation seems to be two distinct processes -- using orientation-free descriptions and perspective change. The orientation-free descriptions are generated within a object-referenced coordinate system, whereas the perspective-change strategy may result from a coordinate system that includes both the object and the observer, with the object at the origin (Just, 1985).

Recent research has also tried to find relationships between factors and processes. Linn (1985,1986) carried out a meta-analysis of the research findings from the four perspectives mentioned above to examine sex differences in spatial ability. Based on factor-analytic, correlational, and process-oriented studies, she formulated three broad categories of spatial ability. These are **spatial perception** (P), **mental rotation** (R), and **spatial visualization** (V). Spatial perception refers to the determination of the horizontal and vertical in spite of distracting information and is measured by the Rod and Frame Test

(RFT) and the Piagetian Water Level task. Mental rotation requires the ability to accurately and rapidly rotate a 2-D or 3-D figure and is measured by various Shepard-Metzler mental rotation tests, the PMA Spatial Relations test, the Hands test, the Flags test, and Cards test. Spatial visualization refers to spatial tasks that require complicated, multi-step analytic processing of spatially presented information. This ability may or may not include spatial perception or rotation. It requires an analytic approach with flexible adaptation of procedures used to solve the task. This is measured by the Embedded Figures Test (EFT), Hidden Figures, Blocks Counting, Cubes test, Paper Folding, Surface Development, Paper-form Board, and DAT Spatial Relations sub-test.

### 4.2. Gender Differences and Spatial Abilities

Many researchers have shown that no significant sex differences can be found in general intelligence, yet there is a decided advantage for females in verbal abilities and for males in spatial and quantitative abilities. The magnitude of differences for verbal abilities are small, for spatial abilities large, and for quantitative abilities intermediate (Halpern, 1986). The difference between females and males in spatial ability is so well documented and persistent that "the magnitude of the sex difference, when in the favor of males, gives an indication of the spatial content of the test" (Eliot, 1983).

In studies of mental rotation of the Shepard-Metzler forms the male superiority at the task is well documented. Tapley et al. (1977), Kail et al. (1979), and Bryden et al. (1987) believe that the reason for this is that men are both more accurate and carry out metal rotations faster than women. As you recall, Just and Carpenter thought that the Shepard-Metzler rotations only had one possible strategy. Sanders et al. (1982) correlated performance on the Shepard-Metzler mental rotation and the Card rotations test. They found that males scored significantly higher on both tests. From comparing the results on the two tests, they thought that the sex differences may become exaggerated when the stimuli is more complicated and/or rotated in depth rather than the plane.

Some current studies on gender differences in spatial ability have attempted to determine the real magnitude of sex differences and others have tried to determine if these differences are a result of different strategies. From her meta-analysis, Linn concluded that "sex differences in spatial ability are large only for mental rotations, medium for spatial perception, and small for spatial visualization." (Linn, 1986) She found that

> Sex differences in spatial ability appear on tasks for which efficient solution requires rapid manipulation of symbolic information and on tasks that require recognition of the vertical and horizontal. Spatial visualization tasks, where efficient solutions depend on effective use of analytic procedures to select strategies for manipulating symbolic information, do not appear to yield sex differences.

She proposes that some of these difference may be due to strategy differences. The spatial visualization tasks where there are small sex differences, may be due to the complexity of the items which may encourage strategy shifting. She concludes that

> one mechanism governing gender differences in spatial ability may be a constraint on solution strategy.
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> Females and males do not appear to differ in ability to select the best strategy. Rather, they may differ in the repertoire of strategies available to them. Tasks that require a single specialized strategy may reveal gender differences because the most efficient strategy is less well developed in females than in males. Speed of mental rotation is an example of such a strategy. One approach for responding to the observed pattern of gender differences is to help females gain specific skills to add to their repertoire for solving problems (Linn, 1986).

Halpern also addresses this notion of gender differences in strategies. She argues that "it is possible that women use different approaches in solving spatial problems than men" (Halpern, 1986). Females have more of a tendency to rely on verbal strategies while males tend to rely on spatial strategies to solve the same problems.

Many investigators have found no indication of a male advantage in spatial ability until about the 10th grade. In a recent study, Johnson and Meade (Johnson, 1987) administered a large battery of paper spatial tests to 1,875 K-12 students. However, their results indicate a male advantage as young as 4th grade with a large increase at 10th grade. Their findings showed that the Hands (R) and Blocks (V) tests exhibited the largest male advantage, the Cubes (V), Flags(R), and PMA Spatial Relations (R) exhibited an intermediate male advantage, and the Mental Rotations (R) and Hidden Figures (V) exhibited the smallest male advantage, Their findings did not correspond with those of Linn on any of the dimensions measured.[†] They conclude that their research as well as others in the field was in need of a theory relating test performance and sex differences to basic cognitive abilities.

Current research suggests that gender differences in spatial ability may contribute to – or even underlie – gender differences in a wide range of disciplines requiring mathematics and problem solving skills.

> [There is] abundant evidence that spatial ability is correlated more highly with skills needed in fields such as engineering, physics, mathematics, architecture and design, than is verbal ability. And it is becoming clear that sex differences in the skills required in these specialist fields are in fact secondary consequences of sex differences in spatial abilities (Eliot, 1983).

Other research suggests, some of these gender differences may be due to differences in the problem-solving process. The processes of acquiring strategies, selecting a strategy, or efficiently applying the strategy could each be the source of such differences (Linn, 1986).

Gender differences have also been found in the computer environment. There has been some concern about these differences in educational circles. Hawkins (1985) differentiates between the computers as a topic and

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[†] The Johnson-Meade (JM) battery of spatial and Piagetian paper tests were used as part of the pre- and post-tests on 5th grade students(45). I believe that there are some problems with the particular mental rotation test that was part of this battery. All the rotations were in the plane(z-axis), in contrast to all depth (y-axis) rotations in the other tests with which it was compared. Some children determined the plane rotation by turning the page, while other children determined the answer without physically rotating the figure.

computers as tools. She noticed that when computers are a topic they are usually in the domain of mathematics. However, when they are a tool they are more often used for the purposes of language, art, music, information gathering, science, mathematics, and technology. She argues that sex differences emerge in relation to the context or the domain for which the computer is used. The context of the situation seems to be important for girls. Girls became much more engaged with computers when they were presented with graphics software with which they could create pictures and designs.

Gender differences, both in spatial cognition and in the computer environment are very important to keep in mind for people working in 3-D computer graphics. J3D is an environment where both these issues come into play and they had to be very carefully considered in the design of the research experiment.

### 4.3. Cognitive Style and Spatial Cognition

Cognitive style refers to "individual differences in modes of perceiving, remembering, and thinking" (Halpern, 1986). There are many theories and measures of cognitive style. Many of the assessment tests for cognitive style are based on how an individual uses, organizes and processes information. Several of these tests also have an underlying spatial element. Some researchers believe that differences in cognitive style may reflect strategy differences (Linn, 1985, Globerson, 1985).

The projective approach to personality assessment in the Rorschach Test and the TAT is based on the notion that by structuring unstructured materials people reveal some deeper, perhaps unconscious, facets of their personality. Both of these tests use information gathered from responses based on ambiguous visual stimuli. Similarly, children's art has been used as a way to interpret and understand the formulation and expression of ideas and percepts. For example, the Draw-a-Man (or woman) test has been used as a measure of intelligence in younger children Piaget often asked children to draw solution to a problem.

The range of cognitive styles have been measured by various other tests that again rely on visual stimuli. The following are some examples of the

cognitive style and the tests used as indicators: tolerance for unrealistic experience (Rorschach), conceptual differentiation (Size Constancy Test), constricted-flexible control (Stroop Color-Word Test), leveling-sharpening (Schematizing Test), scanning (Size Estimation Test), contrast reactivity (Lines Contrast Test), reflection-impulsivity (Conceptual Style Test (CST) and Matching Familiar Figures (MFF) Test).

Another well known cognitive dimension is field articulation, field-dependent and field-independent is determined by the Embedded Figures Test (EFT) and Rod and Frame Test (RFT). The EFT and the RFT are designed to distinguish between field-dependence or field-independence. In the EFT the subject is asked to find a geometric figure located in an embedded context. In the RFT the subject is required to bring a rod into a vertical position when presented with a variety of disorienting cues. These assessments for field-dependent and field-independent cognitive styles have revealed gender differences. A larger proportion of males exhibit a field-independent style of processing. However, as Halpern (1986) points out, on *non*spatial tests for field articulation there are no gender differences.

Proponents of the field theory of cognitive restructuring believe that people are self consistent in the extent of their cognitive restructuring competence across spatial-visual tasks. Some of these theorists have interpreted traditional spatial cognition tasks from a cognitive style perspective. The differentiation of articulated or global and degree of autonomous (self-object differentiation) functioning can be determined from these tests. Witkin and Goodenough (1978) believe that the Piagetian water-level problem is linked to field independence and may be a disembedding problem. Another example of a restructuring dimension is "perspectivism" or "decentration." Consequently, success on Piaget's three-mountain problem has been related to field independence. As discussed previously, these tasks are closely related to spatial thought. Again, as in the RFT and EFT, gender differences in favor of males have been found in these two tasks (Liben, 1980,1984). As discussed previously, older children perform more successfully on these two tasks than younger children. This same pattern of age differentiation has also been found in field articulation with a progression towards becoming more field

independent with age.

Witkin points to evidence that field-independent people rate higher in individual autonomy. He associates this with competence in cognitive restructuring. Especially in ambiguous situations, he believes that

> the internal frames of reference available to field-independent people enable them to structure situations on their own. Field-dependent people, on the other hand having less access to internal referents, are, in general, more likely to have recourse to external sources of information which may be helpful to them in acts of structuring.
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> The internal referents available to field-independent people provide them with a fund of mediating mechanisms for use in restructuring a field on their own, when required to do so by the task at hand. Restructuring may entail organizing a field which lacks inherent structure, imposing a different organization on the field than the one it contains, or breaking up an organized field so that its parts are rendered discrete from ground. The designation 'restructuring' seems appropriate for all these acts since they involve making changes in the field, or 'going beyond information given,' rather than following the field 'as is.' (Witkin, 1978)

He argues that subjects who exhibit autonomy of external referents in perceptual tasks also exhibit autonomy in social behavior. Therefore, there is an interrelation among autonomy, cognitive restructuring, and interpersonal competence. However, Witkin and Goodenough distinguish between style and goal attainment ability. They propose that cognitive style is a dimension of individual functioning in perceptual, intellectual, and social domains which is connected in its formation with development of the person as a whole. This implies that individual differences are in the process not the content (Witkin, 1978).

The development of cognitive restructuring skills, Witkin believes, are rooted in the basic characteristics of self-nonself segregation and individual autonomy. He also points out that "self-consistency may be moderated by unique effects of the particular sense modality (visual or auditory), medium (figural or symbolic), and processing (simultaneous or sequential) involved in

any given cognitive restructuring task" (Witkin, 1981). In some respects, this is similar to what Weir (1986) talks about as factors in learning styles: spatial, parallel, global, holistic styles as compared to the verbal, serial, detailed, analytical styles. Globerson (1987), also addresses these same issue. She believes that multiple modes and multiple representations are important in the learning environment because they both engage different styles and stimulate the construction of multiple representations. Multiple representation -- verbal and visual, formal and informal -- encourage the reflection and integration of representations.

### 4.4. Educational Issues

Weir (1986), Witkin (1978), and Globerson (1987) believe that mode of information has some effect on the style and the ease of learning. For example, a student who is primarily visual may have difficulties if all the instruction is theoretical and verbal. This may cause the student to be very field-dependent in some learning situations. For the learner, this dependency may make cognitive restructuring more difficult. The student may also require assistance in organizing or structuring the content. One way of doing this it to make the underlying structure of the content explicit.

The style and mode of learning may have a great influence on a person's interest and education. Witkin (1978) thinks that people may adapt their life choices in education and vocation along the lines of their style. In longitudinal studies done on college students, a choice or a shift of choice in major or vocation correlated with cognitive style. For example, field-independent students tend to favor mathematics, natural sciences, and the health professions in particular dentistry. Evidence suggests that these students have more interest in the theoretical, the abstract, and the artistic. Field-dependent students have a more "people" emphasis. They favor elementary or social-science teaching, business administration, welfare, helping, and humanitarian professions. These professions which field independent people tend to prefer are almost identical with the professions that have been thought to require higher levels of spatial ability.

Dentistry is considered a field to be favored by more field-independent people. Recall that I mentioned that spatial cognition plays a part in the field determination. Dentistry is also an example of how spatial cognition and style play a part in influencing peoples lives. In a conversation with my child's dentist, Dr. Tessini, I learned that when he applied to Dentistry school, prospective students were tested on their ability to build a 3-D model. When he was a student, a study was done correlating results of success in building the model (as a predictor of success at dentistry) and a battery of spatial test. There was such a high correlation that the model was no longer required for admittance, that afterwards the spatial tests were used to determine admittance. It would be interesting to look into how a battery of spatial tests (which are know to be gender sensitive) could effect admittance for females interested in dentistry. Presently, many schools are beginning to eliminate even the spatial test as a criteria for admittance.

Does this mean that interest and profession may be determined or restricted by cognitive style or spatial abilities? Witkin found evidence that visual artists and musicians tend to be high in restructuring skills. This result could stem from an education in art and music which explicitly teaches analysis of musical or artistic compositions. Witkin believed that it may be possible to foster the development of cognitive restructuring skills by perceptual training, and through this training it may become possible to help people acquire the characteristics associated with both styles (field dependent - independent). In other words, their style can become more mobile with perceptual training. Mobility, rather than fixity, means a person could have available more diverse ways of functioning and be more adaptable. Training may enhance mobility, which in turn may help the students learn in diverse situations, and may even open up new possibilities for them.

Weir points out that "acquiring expertise in a field of activity involves controlling the process of problem-solving by building an understanding of problem-solving itself. Much of this is normally not available to consciousness"(Weir, 1986). Papert made a similar point, when he mentions that the

mathematician George Polya has argued that general methods for solving

> problems should be taught. ... Because of Polya's influence, it has often been suggested that mathematics teachers pay explicit attention to heuristics or 'process' as well as content.
>
> Powerful ideas have the capacity to help us organize our own way of thinking about a particular class of problem, we don't have to reorganize ourselves in order to use them. We put our skills and heuristic strategies into a kind of tool box - and while their interaction can, in the course of time, give rise to global change, the act of learning is itself a local event (Papert, 1980).

In learning how to select the appropriate problem-solving method or the appropriate style for a particular kind of problem, the student's style becomes more mobile. The student gains the ability to choose a cognitive style that suits the problem or the particular context.

Traditionally, this has been the approach in art education, because the *process* is taught as an integral part of the solution or product. McFee (1977) discusses individual differences in learning and the strategies in teaching to different cognitive styles. For example, she mentions that global field-dependent children often need more help in learning to see in three dimensions because they are getting different information from the same experience as field-independent children.

Globerson studied cognitive styles from a strategies perspective. She found that field-independent children perform better on a wide range of academic, developmental, and intelligence tasks than field-dependent children. Globerson gives two explanations for this. The first is **ordinal**, which is "based on ranking according to cognitive abilities" (Globerson, 1985). The second is **differential**, which "is based on the assessment of people's different capabilities, without ranking." (Globerson, 1985) The differential simply acknowledges individual differences in information-processing strategies, competencies, inclinations, and motivation. She links the ordinal explanation with cognitive development, because children of a certain style perform better on intelligence and developmental tasks. This she believes, also links style differences to developmental differences. The field-dependent children were more

susceptible to perceptual cues and more often used a perceptual strategy.

Globerson used the Piagetian water level problem to look more closely at the differential explanation. She found that children used either what she called a *floor strategy* (more global referent) or a *bottle strategy* (more local referent). Her analysis indicated that different stylistic children applied different specific strategies *contentwise*, but *processwise* there did not seems to be any differences. She then conducted a study of *style-appropriate training* where she controlled the variables of perceptual salience of the stimulus, self awareness (reflective abstraction), and developing new strategies. All of these variables "capitalized on students' strengths, compensated for their weaknesses, and enhanced their awareness of relevant self- and situational-aspects" (Globerson, 1985). She found that under style-appropriate learning conditions low performers (field-dependent) could become high performers (field-independent). She concludes with a very strong point, "that cognitive style is a performance rather than a competence variable" (Globerson, 1985).

It is difficult to access the difference in cognitive style and strategy. Are they related or are they the same thing? Is strategy part of cognitive style or part of the assessment for cognitive style? Many researchers have only addressed cognitive style or strategies. Globerson looked at the differences, but which comes first? What is needed is a long term developmental study with tasks that look at affect and strategies.

Globerson's study links cognitive styles, development, and strategies. It also seem to link these with spatial cognition. One of the variables that Globerson controlled was the saliency of perceptual clues. She did this by emphasizing these "clues" and aiding the learner to find the "hidden cues." Both of these variables explicate the underlying spatial structure present in the stimulus.

How can cognitive style be determined from perceptual tests? Perception is quite automatic and people generally are not conscious of how they see and interpret visual and spatial information in the world. One reason is that they are not explicitly aware of visual syntax or the underlying spatial structure, what Olson and Bialystok refer to as the form. These unanalyzed

perceptions and subsequent representation of these visual tests makes it possible to find the underlying cognitive structuring and style. Therefore we are assessing a "default style" or as Olson would say our unanalyzed perceptions.

This default style or strategy used to solve certain kinds of perceptual problems may or may not be appropriate in different situations. Some people's style or strategy can change according to the particular situations or task. Therefore, as Witkin pointed out, cognitive style or strategies can be thought of as adaptable. If styles are adaptable, then possibly different styles can be learned. Globerson has shown that in a style-appropriate learning environment this is possible. If a default style is inappropriate for solving a particular problem or gets in the way of learning, then learning a new field of knowledge should include strategies as part of the discipline or content. This instruction, especially in the domain of space, needs to address the explication of the underlying spatial structure.

An important question is the extent to which spatial ability is effected by training. Many researcher have addressed this question. The methods for teaching or training among the many studies have varied with the aims of the researchers. Many of the studies and methods have been specifically designed to improve performance in specific domains, such as mathematics and chemistry. Other studies were designed to improve performance on particular spatial tasks, such as the EFT. Using Linn's three categories of spatial ability as a framework for discussing the literature on instruction in spatial abilities, let me briefly review some of the findings.

### 4.5. Spatial Perception

Spatial perception refers to the determination of the horizontal and vertical in spite of distracting information and is measured by the Rod and Frame Test (RFT) and the Piagetian Water Level task. The studies in this category usually address pointing out relevant information to subjects. Studies on spatial perception have shown improvement on performance.

As mentioned above, Globerson showed that through style-appropriate training low-performing children improved their performance on the Piagetian Water Level task. Liben's findings are similar to Globerson's. Liben and

Goldbeck (Liben, 1984) found that female subjects who were explicitly given information about the physical phenomenon relevant to the Piagetian Water Level task tended to perform better. They argue that this indicates "that sex-related differences in college students' success on Piagetian spatial tasks can largely be attributed to performance factors rather than to competence deficits" (Liben, 1984).

### 4.6. Spatial Visualization

Spatial visualization refers to spatial tasks that require complicated, multi-step analytic processing of spatially presented information. This ability may or may not include spatial perception or rotation. It is measured by the Embedded Figures Test (EFT), Hidden Figures, Blocks Counting, Cubes test, Paper Folding, Surface Development, Paper-form Board, and DAT Spatial Relations sub-test. Spatial visualization has often been associated with mathematical achievement by researchers. As a result many of the studies done on spatial visualization have been for the purpose of improving mathematics abilities. The greatest number of studies on training or education of spatial thought have been done on spatial visualization. All have shown improvement in spatial visualization after training.

Brinkmann (1966) designed a self-instruction program to teach the visualization of spatial relations. It was given to 8th grade students as part of their math class. The program significantly effected scores on content (geometry) and the DAT Spatial Relations test.

Smith et al. (1981,1979) used tangrams for the instruction of spatial visualization. One aspect of their research was a concern about gender differences in mathematics and spatial abilities. They pointed to a crucial age for instruction for females. They found that "the timing of instruction appears to be crucial. Both girls and boys can benefit from instruction at the fourth-grade level; but if instruction is postponed to early adolescence, only the boys benefit" (Smith, 1979). They conclude that training in spatial concepts should begin early so that both sexes may benefit. However in a recent study, Ben-Chaim et al. (1988) showed that through training, children of both sexes in grades fifth through eighth could benefit from training.

Connor et al. (1977,1978) carried out training on Children's Embedded Figures Test (CEFT). In two studies they found that subjects improved with training on the CEFT. This improvement was also observed in the performance on the Folding Blocks Test. They showed that gender differences favoring males are modifiable through training. In fact, the results of both studies showed that there was more significant improvement for girls than for boys on the CEFT. Linn, as you recall, found the gender differences small for this category.

Talley (1973) had students use physical objects to construct molecular models of chemical species and interactions. He found that the instruction with physical models as an aid improved achievement in freshman level chemistry and in the Paper Folding and Surface Development test scores.

In another study, Yates (1986) looked at the effect of mental imagery training on the Paper Folding and the Cube Comparison tests. He found that subjects performed significantly better on the Paper Folding after training, however the training had no effect of the Cube Comparison test.

### 4.7. Mental Rotation

Mental rotation requires the ability to accurately and rapidly rotate a 2-D or 3-D figure and is measured by various Shepard-Metzler mental rotation tests, the PMA Spatial Relations test, the Hands test, the Flags test, and Cards test.

Kail (1986) found that extended practice (1500 trials) can eliminate age differences in the rate of mental rotation of letters and numbers characters. He found the subjects at all ages (9-20 years) improved their rate of mental rotations. He attributed the improvement to unitization -- the size of the rotation step, which Just and Carpenter believed to be the difference in high and low spatial performers. He believed that the elimination of the age difference may have been from the increase in step size through practice. This, in turn, made the age difference in processing resources less important. These figures were rotated in the plane, which as mentioned earlier, plane rotations are easier than rotations in depth. All of these trials were carried out on 8 pairs of mirror characters which were repeated often. The effect of practice

on the rate of rotations for 3-D objects has not as yet been studied.

McClurg and Chaile (McClurg, 1987) investigated whether fifth, seventh, and ninth grade students improved on a Shepard-Metzler rotation test after using computer games that required spatial skills. The spatial components identified in the games were perception and discrimination, differentiation of opposite obliques, visualization of transformations in series, the use of referent systems, and the development and updating of cognitive maps. They found that despite initial differences between the males and females, both sexes of all three grades benefitted from the games.

As we have seen, researchers have shown that intervention can minimize gender differences on certain spatial tasks, can change cognitive styles and strategies, and can improve performance on many spatial tasks. As the research cited in the previous section has shown improvement is possible in spatial perception, spatial visualization, and mental rotation. All three categories of spatial abilities can benefit from instruction, experience, and practice at explicating the essential spatial information. Since in J3D it is necessary to explicitly address space and spatial transformation, I believe it could provide a learning environment for spatial cognition and mental imagery. In the next chapter there is an introduction to the concepts involved in 3-D computer graphics and the syntax of J3D.

# CHAPTER 5

# OVERVIEW OF 3-D COMPUTER GRAPHICS

## 5. RESEARCH ENVIRONMENT: A DESCRIPTION OF J3D

3-D computer graphics is a simulated environment which has borrowed ideas from may disciplines -- computer science, physics, mathematics, engineering, perception, art, and animation. There can be a great deal of overhead in learning and controlling this new medium. In 3-D computer graphics people often work more directly with numerical representations of objects, time, and space, as well as with the workings of the computer, software, and peripherals that in 2-D paint programs.

The ability to perceive, visualize, and organize form and space in both two and three dimensions is necessary for the utilization of 3-D computer graphics. When using the computer, the children must learn to plan and previsualize (anticipate) space, both 2-D and 3-D. An understanding of perspective is important in being able to previsualize the results of the perspective transformation of the models. Temporal-spatial skills and experience are essential for achieving good timing in 3-D animation.

For my research I developed J3D a C-language based 3-D computer graphics program which runs on HP-Bobcat and Macintosh computers. As part of the development, various tools were built into this system to gather records of the commands the children typed. J3D provides a flexible viewing and transforming environment. Children can manipulate one or more basic geometric polyhedral objects by changing their size, position, and/or orientation in space. The objects can be thought of as malleable building blocks to be used in the construction of images. The transformations allow children to position objects, to create more complex objects, and/or set up a scene. Children may then view the scene in perspective from an arbitrary eyepoint. This

allows them to choose a point of view in space and see these objects represented through perspective or orthographic projection from the chosen point in space.

In J3D the children interact with the computer through a keyboard entered command language. (See Appendix A for a complete manual and for a short list of commands available on-line through the help command on the system). A command language was chosen as a result of the importance of language in spatial cognition. Language, J3D syntax, becomes an important aspect for this microworld as a learning environment. Other types of interfaces which allow more natural interaction through knobs, dials, slides, and/or mouse etc., also allow the children to interact in a way that actions or gestures are used to communicate with this microworld rather than language. Language is an important factor in spatial thinking as is an explicit spatial referent. This software constrains the specification of all transformation commands to the use of the Cartesian coordinate system. The syntax of the command language obliges the children to both name the spatial transformation desired and consciously and explicitly use the coordinate system as the frame of reference. J3D's capability of displaying 2-D perspective views of 3-D objects gives children concrete visual feedback of this synthetic 3-D world and control through the abstract notation of spatial transformations and the Cartesian coordinate system. This sets the Cartesian coordinate system into a coherent framework with a concrete basis. The numeric values of coordinate space become less abstract as they are used as a language for dealing with space to create images.

It is necessary in 3-D computer graphics to create a symbolic model of the world in the computer memory. Objects, once created, can be manipulated by changing their size, position, and orientation in space. In J3D the children can choose both a window from which to view this world and the direction of view. The building of models, the organization of space, and view point, are where children exercises control in this process. The direction comes from the child, but the actual changes are done mathematically by the software and displayed on the screen at the child's command. Children only need to understand the effects of the mathematical representation in the

software. However, a short discussion of the representation of models, transformations, and view parameters adds to a better understanding of this particular environment.

## 5.1. Modeling The World

In a digital computer, the world must be modeled mathematically. Each 3-D model must have a complete definition in 3-space. The frame of reference for this space is the **Cartesian coordinate system**, which provides us with a standard mathematical reference system.

## 5.2. The Cartesian Coordinate System

The 3-D object is defined numerically by plotting coordinate points within the Cartesian coordinate system. The center of the coordinate system is called the **origin**. The X axis runs horizontally with positive X to the right and negative X to the left of the origin. Y is the vertical axis with Y going up and -Y going down from the origin. The X and Y axes create the XY plane. Extending the Z axis with Z advancing towards the observer and -Z recede way from the observer from the origin, creates depth or the third dimension. This creates two more planes, the XZ which extends horizontally and the YZ which extends vertically to create the Cartesian coordinate system. Each coordinate point in this space can now be defined as an "X, Y, Z" triplet. Points are always represented as an ordered triplet.

**5.2.1. Symbolic Models.** Realistic models are often complex, therefore we are concerned with how to simplify a form and still represent the desired object. In J3D a model is represented by a collection of points, edges, and faces. For example, a cube is made up of eight points which represent the **vertices**. **Edges** are lines connecting points. A sequence of edges or vertices are **faces**. It is fairly simple to represent a cube in 3-space, but objects with curved lines and curved volumes become more difficult. One way to think of this is similar to the connect the dot drawings found in children's books. A straight line becomes a line between two points. To approximate a curved line several points can be plotted to form a curve and straight lines are drawn to connect the points. For example, a simple circle could be made up of six

points and would look like a hexagon. A detailed circle could be made up of 360 points. The points are so close together that the length of the edges are so short that the circle appears to be a continuous curve. Therefore, the "simple" circle would appear to be made up of straight edges, the "detailed" circle would appear to be a continuously curved line.

Until this point only models that represent objects as sets of points have been discussed. Connecting these points with lines or edges provides a line drawing of the 3-D object, called a **wireframe** view of the object. To define a surface, not just the edges, a surface can be approximated by modeling it as a set of **polygons**, i.e., as a **polyhedron**. A cube is a simple polyhedron with each side composed of a polygon made up of four connected points. Polyhedral or polygonal models are completely covered with flat, straight edged tiles.

Having discussed the surface of an object, we can refer to its "inside" and "outside." This is done by using a convention of ordering the vertices of the polygon. A face whose vertices are ordered clockwise as seen from the observer is determined to be **frontfacing**. This order is used when describing the polygon by listing every point or vertex around the periphery of the surface. The reverse order as seen from the eyepoint would indicate that the polygon is facing away or **backfacing**.

Keeping track of frontfaces and backfaces is important for speeding up the display algorithm. If the face is backfacing then it can be discarded or **culled** and not sent on to the display program. At present J3D displays line drawn objects with a backface cull which can be turned on or off for each object. When the backface is off only the visible surfaces, the outside of the object, are displayed. If the backface is on, the object will appear transparent because backfacing polygons are also drawn.

An object is a collection of adjoining polygons. It is important to store an object in memory in a compact form. Since neighboring polygons share vertices along common edges, data can be easily and compactly defined by listing each vertex once. The data set for a cube, listed below, is made up of 8 vertices and 6 polygons. The file format for this data is the data line with the first number representing the number of vertices in the data and the second

number representing the number of polygons in the data. This means that the first 8 lines after the data line are coordinate points, each an XYZ triplet. These are followed by the polygon descriptions, the first number on each line, represents the number of vertices in the polygon the rest of the numbers are the vertex numbers. These refer to the list of coordinate points. For example the first polygon is made up of 4 vertices, (points 1 2 3 4). Once the coordinate points and polygons for the cube are defined, it can be displayed as an object on the computer screen. Below is the J3D format for a cube with a title line after the number of points and polygons.

| | |
|---|---|
| Data | 8 6 |
| | title - cube |
| Vertices (1) | -1 1 1 |
| (2) | 1 1 1 |
| (3) | 1 -1 1 |
| (4) | -1 -1 1 |
| (5) | -1 1 -1 |
| (6) | 1 1 -1 |
| (7) | 1 -1 -1 |
| (8) | -1 -1 -1 |
| Polygons (1) | 4 1 2 3 4 |
| (2) | 4 2 6 7 3 |
| (3) | 4 6 5 8 7 |
| (4) | 4 8 5 1 4 |
| (5) | 4 1 5 6 2 |
| (6) | 4 7 8 4 3 |

This object is defined with its center or origin at the origin of the coordinate system. It extends from 1 to -1 in X, Y, and Z, therefore it is two units in all dimensions. All manipulations of an object are performed on a copy or **instance** of the original object. As a result several instances of the same original object can be transformed individually. Each copy can be given a name to assist in keeping track of the many objects in a scene. The command to bring

an object into the environment is

**call <model-name> <object-name>**
**ie. call cube box**

### 5.3. Describing A Changing World: Object Transformation

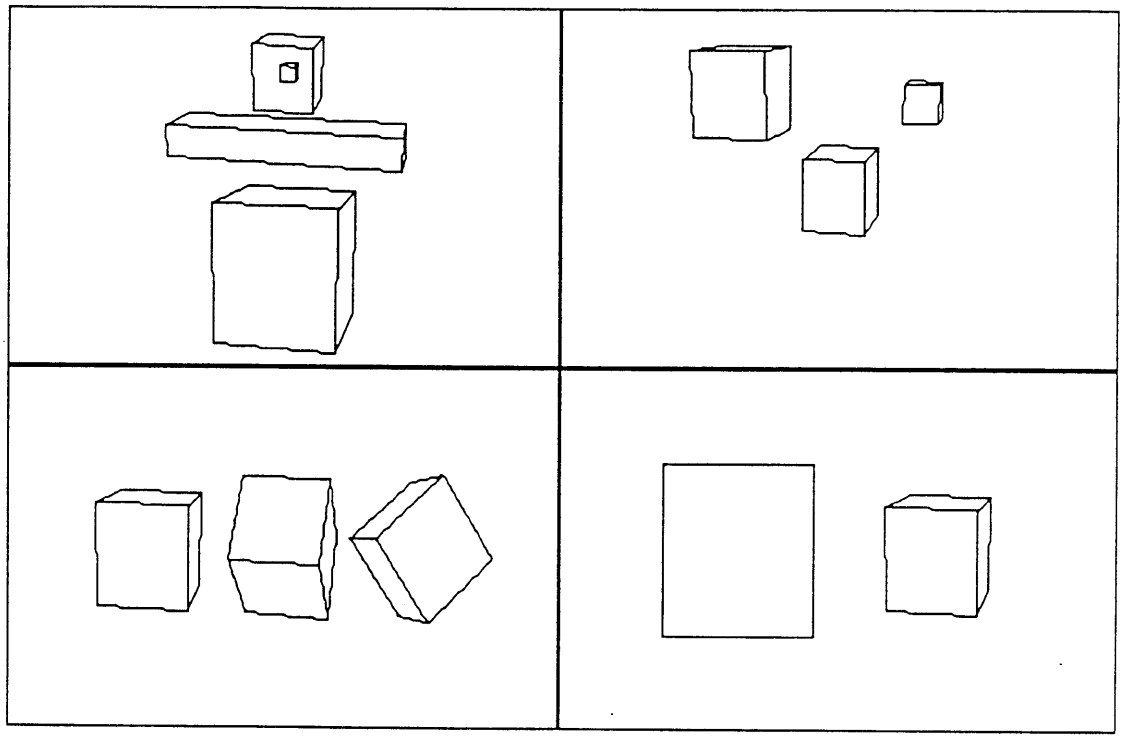
Once various objects are defined and stored in the computer as a file there are several things that can be done with each object to create a scene or a complex object. Objects can be **scaled** or made larger or smaller. They can be **translated** or moved around in space. They can be **rotated** or turned in space. These manipulations of the object are called **transformations**. Standard mathematical techniques of coordinate geometry, trigonometry, and matrix methods are used. They are expressed mathematically in matrix algebra by a single entity called the **transformation matrix**. Several transformations of an object can be combined or **concatenated** into one matrix. The software creates and manipulates the matrices, however the scale, translation, and rotation matrix representations are not seen by the child.

**5.3.1. Scaling Objects.** In scaling, large numbers expand the object and small numbers contract it. One is free to scale independently in each axis and thus change the shape or proportions of an object. Objects can also be scaled equally in each axis to obtain a larger or smaller version of the same object. In this way it is very easy to turn a cube into a rectangular parallelopiped by scaling it more in X than in Y. Children best understood scaling as multiplication of components of the object vertices by the scale factor. The scale factor in J3D is either an X, Y, Z value or a single value which is used for uniform scaling. The command syntax in J3D is:

**scale <object-name> <x y z>**
**ie. scale box 4 .5 1**
**ie. scale box 2**

This cube was designed with its origin at the origin of the coordinate system, therefore when it is scaled the scaling will be symmetrical. The scale factor is always multiplied by the original model. For an example, when scaling the cube by 4 in X, .5 in Y, 1 in Z, each X component of a vertex would be

Figure 1: J3D tranformations.



multiplied by 4, each Y component would be multiplied by .5, and each Z component multiplied by 1 would remain the same. The default scale in J3D is the value needed to get the original cube through multiplication, which is 1 1 1. Therefore to change the outer limit or **bounding box** of the original cube the new scale factors are multiplied by the original values as you can see in the bounding box on the right.

| Original bounding box | New bounding box |
|---|---|
| ( -1 X to 1 X ) | ( -4 X to 4 X ) |
| ( -1 Y to 1 Y ) | ( -.5 Y to .5 Y ) |
| ( -1 Z to 1 Z ) | ( -1 Z to 1 Z ) |

When scaling of an object whose origin is not coincident with the origin of the coordinate system the scaling will not always be symmetrical. If the model for a cube is designed with its origin in the lower left hand corner. Any points with a coordinate value of zero will, of course, remain zero when multiplied by the scale factor. The two bounding boxes below illustrate this concept. The result of this scaling by 4 in X will create elongated cube stretched in the direction of the positive X axis.

| Original bounding box | New bounding box |
|---|---|
| ( 0 X to 1 X ) | ( 0 X to 4 X ) |
| ( 0 Y to 1 Y ) | ( 0 Y to 1 Y ) |
| ( 0 Z to 1 Z ) | ( 0 Z to 1 Z ) |

This same principle holds true when the cube is scaled by 4 in X, Y, and Z. If there are no points on the origin of an axis, then the object will appear to move when it is scaled. The bounding box below is for a cube designed with its lower left hand corner at 1 1 0.

| Original bounding box | New bounding box |
|---|---|
| ( 1 X to 2 X ) | ( 4 X to 8 X ) |
| ( 1 Y to 2 Y ) | ( 4 Y to 8 Y ) |

( 0 Z to 2 Z )  ( 0 Z to 8 Z )

This type of object can be difficult to control because the scaling can appear to move the object. Although, in effect, it is really the coordinate system being stretched. Like a sheet of rubber, when stretched an object in the middle will appear to expand, but an object near the outer edge will appear to move.

In the upper left quadrant in Figure 1[†] we can see the original cube at the top. In the middle you can see the cube scaled by 4 .5 1. It is larger in X, smaller in Y and the same in Z. At the bottom the cube is scaled uniformly by 2 in all axes and is twice as large. Scaling objects smaller than their original size is where scaling becomes more difficult for children, because they must use decimal numbers as fractional scalars. For example to scale the cube 1/4 its original size it has to be scaled by .25. We can see the small cube inside the original cube at the top to get an idea of this size difference.

Scaling is an operation whose artifacts in computer graphics can be fascinating and allow for "accidental effects." Scaling an object by a small number can cause the object to disappear or by a large number will cause it to cover the entire screen. In a line drawing program such as J3D, these two extremes will visually appear the same. One of the most interesting design possibilities of scaling is the ability to turn an object inside out by scaling by a negative value. This reverses the frontfacing and backfacing polygon descriptions. Therefore, an object can be designed with another object inside which can provide an exciting use of this "accidental effect" when scaling.

**5.3.2. Translating Objects.** In translation, a value is either added to or subtracted from each coordinate component of the original model to move it around in space. As with scaling this can be done independently along each axis or in more than one axis to place the object where desired in space. A positive number moves the object in the direction of the positive end of the

[†] All images in Figure 1 are created with the eye at 3 3 15 to show greater depth information, except the lower right quadrant.

axis. A negative number moves it towards the negative end of the axis. In the example of the cube listed above, the object has its origin at 0 0 0. This is also the default value for place in J3D, so that an object when it first comes into the system comes in where it was built in relation to the coordinate system. In order to move the object a translation is given, for example 0 0 -20 to move it back. The syntax in J3D is:

**place <object-name> <x y z>**
**ie. place box 0 0 -20**
**ie. place box -2 3 3**

In the second example we can think of 2 being subtracted (or -2 added) to all of the X values in the original data and 3 being added to each Y and Z value of all the points. If the translation is -2 3 3 these values will be added or subtracted respectively from each coordinate of the object. It is easier to think of translation as moving the center of the object to -2 3 3, for example, rather than keeping track of the addition and subtraction of these values from each coordinate point in the object. When placing objects adjacent to others objects, the child has to take the current bounding box into consideration. If we look at the upper right quadrant of figure 1 we can see where the original cube comes in right in the center of the picture. The cube above and to the left in the picture was moved -2 in X (to the left 2 unit), 3 in Y (up 3 units), and 3 in Z (forward 3 units). The other cube appears smaller because it has been moved back -20 in Z.

Translation is used to organize and position objects in space. Instancing and placing copies of the same object at various intervals can be used to build more complex images, for example a picket fence, bars for a crib, or stairs.

**5.3.3. Rotating Objects.** Rotations are mathematically more complex. They are done through matrix multiplication using sines and cosines, which are hidden from the child. Rotations in J3D are only around the Cartesian axes and they are always in world space. Rotations are specified by the axis of rotation, and angle of rotation in degrees, either positive or negative for the rotation direction. The direction of a rotation is dependent on the particular space and whether the value is positive or negative. The space used in J3D is

**right-handed** space, therefore to visualize the direction for a rotation, imagine grasping the axis with the right hand. When the thumb is pointing towards the positive end of the axis, the fingers will curl in the direction of the positive rotation.

An **on-origin** rotation is when an object is rotated around its own origin. In J3D, the models available for the children to use were all designed with the origin of the object coincident with the origin of the coordinate system. As a result of this, a rotation on the X axis is a rotation on the X axis of the coordinate system. **Off-origin** rotations are different from on-origin rotations. This can be thought of in terms of the solar system. An on-origin rotation is like the earth spinning on its own axis, and an off-origin rotation corresponds to the earth orbiting around the sun. An object rotating around a point, not on its center, will rotate around the point as if in a circular path or orbit. In J3D off-origin rotations can be achieved by attaching an object to another "pivot" object and then the pivot object is rotated. We can think of this in terms of the reference frame for the rotation. If the object is rotated about itself it is a local coordinate system, however if it is attached to another object and that other object is rotated it is rotated about the other objects coordinate system.

A **simple rotation** is a rotation around a single axis in the coordinate system. A **concatenated rotation** is a rotation made up of more than one rotation occurring simultaneously. A concatenated rotation consists of at least two rotations occurring simultaneously, i.e. in X and Y at the same time. The order of the rotations is important when rotations are concatenated. Keeping track of rotations can become very complex and requires skill and a good sense of spatial orientation. The order of rotation is important because matrix multiplication is, in general, not commutative. That is, (A * B) may not be equal to (B * A) when A and B are matrices. If one object is rotated first in X then in Y, and another object is rotate first in Y then in X, even when the angle and axes of rotation are the same, the final orientation in space may be different due to the order of the rotations. If you want an example, use a book. Y is the spine and X is the bottom edge. Notice how the axes rotate as the book rotates. The command for rotation in J3D is:

**rotate <obj> <num-axes> <axis> <angle> [<ax> <ang> <ax> <ang>]**
**ie. rotate box 1 x 45**
**ie. rotate box 3 x 45 y 50 z -30**

Here you see that to rotate an object in J3D the user must specify the number of rotation there will be, which axis and what angle, etc for as many axis of rotations specified. In the lower left quadrant of Figure 1 we can see the final orientation from these commands. On the left is the original cube with no rotations. In the middle is the rotation 45 in X with the cube just tilting forward 45 degrees. The cube on the right was tilted forward 45 (side to side axis), turned 50 in Y (up and down axis), and then turned again -30 in Z (the in and out axis). As you can see by this last rotation it is important to realize how complex rotations can become and that several rotations both on and off-origin can be going on at once increasing the complexity. Some systems even allow for rotations about an arbitrary axis. In J3D the number of rotations and the order of the concatenations is set up by the child and carried out by J3D.

**5.3.4. Object or Scene Description.** The basic operations of scaling, rotating, and translating can be used to create a scene as well as an object. A more complex object or scene can be constructed out of geometric primitives, eg. cone, cube, sphere, and cylinder. To create the desired size and proportions scaling is used. Translation is used to position the parts in the desired relationship to other parts. To correctly position the part, it is necessary to take into consideration the original object, its current size, and the displacement of its boundaries. The rotation can be figured by knowing the orientation of the original object and the new desired orientation, and specifying the axes and degree of separation from the original. One method introduced to the children to aid them in organizing and planning a scene or an object was to create a projection of it. A projection makes it possible to only deal with two axis at a time, or is a 2-D way of representing 3-D.

Some computer graphics and animation systems allow objects to be attached to each other in **hierarchical** relationships. This option can be very powerful for setting up relationships among objects and maintaining these

relationships. For example, to move a train built out of primitives, it is necessary to figure out the position and orientation for each part of the train. This could be very time consuming, and of course leaves room for errors. If all of the parts of the train are specified and attached to only one part, for example the engine, we then only need to move the engine to the new position. All the parts would maintain the same relationship to the engine and move as well. Attaching objects is not like permanently gluing them together. As the train is moved to a new position, we want the wheels to move with the rest of the train as well as rotate appropriately. By attaching the wheels to the boxcar and then rotating the wheels this effect can be achieved. Generally the order of attachments is very important and can also become extremely complex. J3D allows two types of hierarchical capabilities. The attach command allows you to set up the hierarchy of objects. Whatever transformations are applied to an object are applied also to any objects attached to it.

**attach <object-name> <to-object>**
**ie. attach wheel boxcar**
**ie. attach caboose boxcar**
**ie. attach boxcar engine**

The group command allows the child to group several objects together and refer to them with only one name. This command is very similar to attach, however the grouped objects are not hierarchically attached.

**group <name> <number-of-objects> object-name> <object-name> etc.**
**ie. group train 3 engine boxcar caboose**
**ie. group rocket 2 cone cylinder**

Objects may be detached by using the detach command. There is an option to either leave the parent transformation or remove it when the object is detached. The default leaves the object with the parent transform so that it does not suddenly change when it is detached.

**detach <obj> <from obj> [<0|1>]**
**detach wheel boxcar**

## 5.4. Viewing The Model: The Illusion of Depth

The basic problem addressed by visualization techniques in 3-D computer graphics is depth cuing. In Western civilization artists working in 2-D media, such as drawing or painting, have explored various techniques to create the illusion of 3-D space on a 2-D surface. Traditionally scale, position, occlusion, sharp verses fuzzy details, converging parallels, and linear perspective have been used as visual cues for space organization. For example, increase in scale can be interpreted as nearness, and conversely, decrease in scale is interpreted as spatial distance.

Linear perspective is a geometric system which uses the spatial indication of size, position and converging parallels and converts size and distance into a unified spatial order as seen from one viewpoint. This visual logic of linear perspective is programmed into the J3D.

**5.4.1. View Transformations.** There are three components necessary for understanding the view transformation; where the observer is viewing the scene from, what the observer is looking at, and what is the field of view. Consequently, once all the objects are arranged as desired in a scene, a point in space, an **eyepoint**, must be selected to view the scene. The eyepoint is placed by locating it in the coordinate system through an X Y Z triplet. The point in space at which we are looking must also be selected. This point is sometimes called the **center of interest**. It is placed by specifying its X Y Z position. These two points create a line of sight. The **view angle**, is the angle of view on either side of this line. The view angle actually defines a view pyramid. The eyepoint is at the apex of this pyramid. The view angle may be specified in J3D but the default is 45 degrees. The line of sight runs down the center of the view pyramid. A 45 degrees angle indicates a 22.5 degree view on either side of this line of sight. It is important to know the angle for planning a scene. The center of interest or **coi** will always appear exact at the center of the display monitor.

Setting the view angle is similar to changing the focal length of a camera lens, in that the size of the angle controls how much of a scene will be seen in the display. A small angle is similar to a long focal length or a telephoto lens

and yields a narrow field of view. Similarly, a large angle is like a short focal length or wide-angle lens, and it provides a very wide field of view. Anything outside the viewing field is not seen through the lens. In computer graphics any object or part of an object outside the view angle will be **clipped** and not displayed.

Transformations used in modeling and viewing are handled in slightly different ways. When referring to objects and the transformations of scale, translation, and rotation we are working in **world space**. In this space we manipulate an object by changing its coordinates. View transformations are used to move a coordinate system that measures the position of objects. View transformations can be thought of as global transformations, because they effect all of the objects in the scene. These two coordinate systems are embedded. When the *eyepoint* and *coi* are at their default positions, world space, eye space, and screen space are coincident. However, these can be changed. Consequently, understanding and coordinating the many reference frames can be a complex task. When the child transforms objects, and changes the viewing parameters, the anticipation of the resultant image can become very complex.

Once the view parameters are defined, all the coordinates of the objects in the scene are transformed through matrix multiplication to line up with the *eyepoint* fixed at the origin. This new coordinate system is called **eye space**. These new X Y Z coordinates of the scene are then projected onto a 2-D "picture plane" which is called **screen space** or **image space**.

There are various techniques used to project eye space points to screen space. J3D has the capability of perspective and orthographic projection. The child can specify which one they want in the display command, however the perspective projection is the default. The perspective projection uses linear perspective and foreshortening to represent depth. The orthographic is a form of projection without foreshortening effects of perspective. The perspective projection is the most familiar, because it is remarkably similar to the photographic image and is frequently used in the visual arts. This projection is done by a line from the *eyepoint* through each visible vertex of an object. The intersection of this line and the 2-D plane is the new X and Y coordinate

in screen space. Thus the 3-D eye space is transformed to a 2-D screen space. After the view transformations are complete the 2-D image is displayed.

The commands for the view parameters in J3D are as follows.

**eye <x y z>**
**ie. eye 3 3 15**

**coi <x y z>**
**ie. coi 0 0 0**

**view <angle>**
**ie. view 35**

In the lower right quadrant of Figure 1 we can see two different objects. They are both the same cube viewed from different eye points. They have been offset to the side to make it easier to see the differences. On the left, is a cube which looks like a square. This is because we are looking at it directly straight on from the default eye point of 0 0 10. Which is directly straight on and back 10 unit in Z. The other cube is seen from 3 3 15. This particular eye point I use often, because it provides a better more depth information.

J3D also has a **blueprint** command which displays four views. It display an oblique perspective projection of the object in the upper right quadrant, with an orthographic projection down each axis for the other three quadrants, in effect producing top, side, and front views. This in effect allows the child to see both types of information at once. The perspective projection is good for seeing how something looks in depth and the orthographic projection is useful when constructing new complex objects because it does not distort the geometry like a perspective projection. The default perspective eye point is 3 3 10. The blueprint command also allows the child to request a particular eyepoint for the perspective image as an option. Multiple views are useful for seeing what a scene looks like from several views at once. The children may display four views at once through the blueprint command or they may display one chosen view to the entire screen or up to four different views in different quadrants.

**display [<quadrant> <projection>]**
**ie. display**
**ie. display 1 1**

**blue [<x y z>]**
**ie. blue**
**ie. blue 3 5 12**

### 5.5. Information Utilities

J3D offers several commands to obtain information on the status of objects and the view parameters. One of these is the **show** command. This command prints out information on the terminal screen. Using just the *show* command will list the objects that are current in the environment. The *show object-name* command will show all the parameters of the named object or *show all* will do so for all the objects. The *show view* command will show all the view parameters. Another command, **reset** changes the parameters back to default. Therefore *reset view* changes the *eye* and the *coi* parameters back into coincidence with the coordinate system - their default positions. *Reset object-name* or *reset all* puts one or all of the objects back at their default parameters. These commands allow finding out about objects and the eyepoint.

### 5.6. Computer Animation

One advantage of the computer is its ability to deal with complex relationships precisely and rapidly. The computer is able to repeat mathematical operations over and over again either identically or with small incremental changes. Besides specifying the parameters of the objects and the view of a scene, we can focus on the changes of these parameters over time for animation. Any quantity that can be controlled can be changed for an animation sequence. Thus, we are able to manipulate and change the objects and the "camera."

The objects can be changed through the use of transformations -- scale, translate, and rotate. Principles of motion and timing can be applied to these

transformations. Scaling can be used to make an object grow larger, or shrink to nothing, or even turn inside out. Rhythmically scaling an object larger then smaller could be used to simulate breathing. This is done by incrementally changing the scale value over a particular number of frames. There are 24 frames per second. For example, scaling an object larger in increments of .02 over twelve frames then decrementing the same object by .02 over over twelve frames. This would cause an object to change from its original size to about 1/4 its size and back to its original size over one second of animation. By repeated this over and over again could appear like the object was breathing.

Translations can be used to organize and place objects in 3-D space. Incrementally placing instances of an object at various intervals can be used to build more complex objects, or to move one object to another position in space. By varying the distance between moves for each frame we can either speed up or slow down the motion. For example, to move an object 3 unit in X can be done over 24 frames by moving the object incrementally by .25 each frame. If the object moved the same distance in 6 frames or .5 each frame, it would appear to move faster in the animation because the step size is larger.

Rotation, like scaling and translating, can be used effectively in animation. By incrementally rotating an object it can appear to spin, tumble, or roll. The speed of this motion again depends on the size of the increments. For example, if an object is rotated by .5 degrees per frame, it will turn 12 degrees a second, or revolve completely around twice in one minute of animation.

All of the above operations can also apply to objects that are attached hierarchically. We can build a bicycle out of separate parts. The wheels, the handle bars, pedals, seat, and rider can be attached to the frame of the bicycle. As we translate the bicycle frame down the street all of the parts come along. We can rotate the wheels and the pedals at the appropriate speed for the translation, and at the same time turn the handle bars. This motion of rigid articulated objects can become extremely complex.

The ability to arbitrarily choose any view is one of the most salient features of 3-D computer graphics. Cinematic conventions, (e.g., pan, truck, crane shot, and dolly), can be simulated, by moving the *eyepoint* and *coi*. The view angle can be adjusted to simulate the focal length or view angle of a camera lens.

J3D supplies several facilities for saving the work done on the computer. Children may **save** or **append** a script or file to create a scene. They may start a **history** which is a record of everything they type into the computer. There is a very simple editor that children may call up from within J3D through an **edit** command. This editor allows the children to enter, yank, and put commands into a file. Any of the files created or saved by the system (edit, save, append, or history) may be brought back into the workspace of the computer to be executed as if they were typed in. The children must type in an **@** followed by the name of the file and all the commands in the named file will be executed. In this way, the children can create animation in this system by setting up a list of commands and displaying the resultant images. When this process is repeated over and over with small changes in the sets of commands parameters between displays, for simple scenes it appears to animate. In J3D children must explicitly deal with unit iteration for animation sequences, because there is no assistance for interpolation between frames or sets of parameters.

Let me review some of the qualities of J3D. J3D is an interactive three-dimensional(3-D) computer graphics environment which enables upper elementary children to actively explore 3-D Cartesian coordinate space, perceptually and cognitively. The children communicate with the computer through a command language. This language provides commands for object transformations or operations of scale, translation, and rotation, as well as the placement of the *eyepoint* or synthetic camera from which to view the objects. Children may then view a perspective projection of these 3-D objects from the chosen view point. The software constrains the specification of all transformation commands to the use of the Cartesian coordinate system. The use of a command language obliges the children to both name the spatial transformation desired, as well as the advantage of obliging conscious use of the

coordinate system as the frame of reference for the simulated space. This environment, with its capability of displaying a 2D perspective view of 3-D objects gives children concrete visual feedback of this synthetic 3-D world and control through the abstract notation of space. This sets the Cartesian coordinate system into a coherent framework with a concrete basis. The numeric values of coordinate space become less abstract as they are used as a language for dealing with space to create imagery. The concepts involved in 3-D computer graphics are discussed in greater detail in previous documents (Sachter, 1983,1984).

As a result my own experiences in learning 3-D computer graphics, my understanding of space -- perceptually, conceptually, and mathematically have become more integrated. I am more capable to imagining transformations in space as well as different views of objects in space. It is my hope that 3-D computer graphics may enable others to learn the formal and informal aspects of spatial content. The next chapter addresses how spatial cognition, mental imagery, and 3-D computer graphics psychologically form a cohesive environment in which to study spatial cognition.

# CHAPTER 6

# ENVIRONMENT FOR SPATIAL EXPLORATION

## 6. SPATIAL COGNITION, MENTAL IMAGERY, AND J3D

In the previous chapter, 3-D computer graphics and some of the commands available in J3D were introduced. Now it is important to look more closely at the psychological aspects of J3D through the integration of spatial cognition, mental imagery, and 3-D computer graphics. At the same time it is important to keep in mind the affect of gender differences and cognitive styles.

3-D space can be model with 3-D computer graphics. In 3-D computer graphics Topological, Projective, and Euclidean spatial relations are intimately linked and interrelate to form a cohesive spatial environment. J3D makes many of these spatial concepts explicit. This microworld requires the use of the Cartesian coordinate system, positive and negative numbers, decimal fractions, and spatial concepts and transformations in an explicit and interactive way. These aspects of computer graphics make it possible for the researcher to look more closely at individual differences in spatial understanding.

Mental imagery has a bearing on this computer graphics environment because the children need to be able to anticipate transformations in J3D. In choosing a command they had to anticipate the final desired state of objects, choose the appropriate transformation by converting the image into language and naming the transformation, and finally relate the imagined transformation to the Cartesian coordinate system through the command parameters. In anticipating the appearance of objects on the screen, the children had to consider the original state of the object and the current state of the objects, as well as the effects resulting from current point of view or views. In animation the children must also anticipate the *form* of the transformation, not just its end state. Therefore, the operations of the spatial transformations can be

thought of as both concrete operations on synthetic mathematical objects, mental operations on mental images of these objects, and formal operations on visual objects.

The multiple representations integrated in J3D -- its visual, verbal and mathematical modes -- are important for spatial exploration. These multiple representations of spatial concepts are both formal and informal. As mentioned above, children use imagery to think about what images and transformations they want. They use language to name the transformations. They use mathematics and the coordinate system to specify parameters of transformations. They see the image on the screen and adjust or correct the result. These multiple representations are integrated in J3D. The content of space is dealt with explicitly and children had the opportunity to actively explore these concepts and received both visual and/or verbal feedback on request. The system also compelled the children to translate back and forth between these different representations. The complexity of the system encouraged "mindful abstraction" (Globerson, 1987) and active control by the learner. All these characteristics merge to create an environment accessible for children with different cognitive styles. The environment was designed to allow girls to become engaged. It has, in effect, been shown that girls prefer to work with computers when graphics are involved (Hawkins, 1985). Around fifth-eighth grade, children also seem to have a fascination with perspective and "realistic looking" images that show depth. For many of the children this complex and difficult process may become worth it to create pictures with perspective. The children have an opportunity to be creative and express personal ideas in the process of exploring space. Those with artistic interests can use imagery to explore mathematics and those more interested in mathematical notions can explore imagery through mathematics. In other words, there are enough reasons for children of different interests and styles to become engaged in exploring the spatial content of this environment.

J3D was designed to provide the researcher with the opportunity to track how children used this system through saving a record of all the commands that the children entered. This record can aid in the study of the children's processes. It enables the researcher to recreate the children's

images and animation for display or analysis.

The complexity of spatial cognition, mental imagery, and J3D makes it difficult to discuss the content of this environment and all the psychological implications. These interrelate in a way that becomes very circular, but points to the resonance of spatial concepts and mental images in J3D. In the following sections, I present the different aspects involved in understanding the system which are organized into spatial components, mathematical components, and artistic and animation components.

### 6.1. Spatial Components

Piaget talks about the reference frame as a network of relations of order between object and the positions of objects as constituting Euclidean space, like a *container* with the mobile objects *contained* within it. Reference frames are an integral part of our visual systems and spatial thought. Consequently, they play a major role in spatial tasks. Reference frames may be embedded and the choice of the most adequate one may define the outcome. As previously mentioned, Olson and Bialystok (1983) distinguish four relatum for spatial predicates essential to spatial thought: ego, observer, object, environment. The most stable referent is the environment. It is common to itself and to everything it contains.

In J3D the Cartesian coordinate system is the dominant reference frame. All of the objects that are available to the children were built centered on the origin of the coordinate system. Prior to moving the objects, the *eye*, or the *coi*, all the reference frames coincide. Furthermore, they are coincident with ego for the children sitting in front of the computer screen. However, when the *eye* is moved the children must coordinate eye-space and world-space, and reconcile what they are seeing. Therefore, when the children were first introduced to the system, the reference frame for the object (contained) and the coordinate system (container) are identical. In J3D all referents are explicit except for ego. The eye (observer) and the objects are specified by the children in relationship to the environment -- the Cartesian coordinate system. Terms like front-back and right-left of the object change when the *eye* changes, but X, Y, and Z do not. This can also cause some confusion if the

children are using ego to relate to this world. The *eye* at default is at (0 0 10), so the children are back 10 units looking along the Z axis at the origin. Even when the *eye* changes, the transformations on objects are still in relation to the coordinate system. For example, if the *eye* is moved so that you are looking along the X axis, but you are still thinking of the coordinate system as if you were still looking along the Z, transformation of objects can appear incorrect. A cube scaled larger in X will not appear to have changed when looking at the resulting image from down the X axis. This can sometimes cause confusion because children know that they changed something, but they see nothing different. Another example, is if an object is rotated 90 degrees in X and then the eye is moved from the Z axis to the X axis, again there will appear to be no change.

These examples point out the complexity of the system and the amount of mental overhead that children must address. Keeping track and possibly coordinating both the object parameters and view parameters, while at the same time, interpreting and debugging their images is very difficult even for adults. Besides the sheer bookkeeping needed to keep all the variables apart, some psychological readjustments or reorientations also seems necessary. When children start to do transformations on an object they may confuse the axis in which the change should occur. They have to keep in mind both the referent for the object and the point of observation, and make those mental adjustments when looking at the image. Keeping it all together is no easy task.

Various commands (show, help, ?, reset) were built into the system for information and to allow users to immediately reorient and start exploration again if they became "lost" or disoriented by resetting the parameters. This feeling of being "lost" or a need to reorient may also take place when the *eye* is moved. I personally have experienced this need for reorientation and have seen children become confused after moving the eye to a new position. When they start to do transformations on an object the children may confuse the axis in which they think the change should occur. It is as if you had to constantly keep in mind the referent for the objects and the point from which you are looking at the object and making mental adjustments.

Yet another readjustment can be described as an accommodation to the scale of the reference frames or an expansion from the synthetic world into the present physical environment. In the process of choosing a new eyepoint the space can be thought of as expanding out of J3D and in a sense, out of the computer. Once the eyepoint is chosen, the reference and the space contracts back into the world of the computer. At this point, children can often use either the computer or themselves as the referent in relation to the object on the screen and to the particular view they want of that object. Some children even asked me where a particular coordinate would be, as if it were floating in the air near us. I have also seen children hold their hand out in relation to the computer to figure out where to place the *eye*, when they want to change the view.

This "telescoping effect" -- in and out -- may be one of the reasons why changing the eyepoint can be disorienting for those who are not as conversant with spatial concepts because it is these shifts, as mentioned above, which can make the various reference frames difficult to coordinate.

There can be several reference frames besides the coordinate system at play in this environment, and as mentioned previously, they may be embedded. For example, it could be the child (ego) sitting in front of the computer, the computer and its screen, or the image on the screen could be a reference frame. Moving the *coi* to some place besides the origin of the coordinate system may also complicate the reference frame, because you are no longer looking at the origin of the reference system. Often, objects in the scene are used as landmarks to place other objects. Since everything in this system can change, the choice of a reference frame is often complex, even though the coordinate system remains stable. What appears to be in flux is how the children relate to what they see, how they thinks about it, and what is chosen as the current reference frame.

**6.1.1. Object Transformations.** Object transformations are always in relation to the original mathematical model of the object. Therefore, it is important to have a basic understanding of the structure of the geometric primitives. The operations of scaling, translating, and rotating are mathematical

transformation applied to the model. The children had to understand that these transformations were relative to the original model, and not to the current state or presentation. This requires some mental bookkeeping. Children had to keep in mind the original model and the current transformations when looking at the image of the object on the screen. They also had to keep in mind the position of the *eye* and *coi*, especially after they had been moved. For example, scaling an object larger or smaller requires an understanding of the original model, both its dimensions and its extent. An intuition of "twice as large" when scaling by 2 aids in this, as does "half as big" when scaling by .5. The intuition of "a little smaller" than .5 (e.g. relative to the current rather than the original dimensions. This same strategy is also used in translation and rotation.

The operation of translation makes clear Piaget's idea of the coordinate system as the container and the object moving within this space as contained. Translation requires an understanding of direction, unitization, and displacement – in essence measurement. Translation can be thought of in two ways. It can be the addition or subtraction of numbers from the original model. Or it can be a displacement of the object's origin in relation to the origin of the coordinate system.

Children often think of "up" for Y and "down" for -Y, "right" for X and "left" for -X. This strategy works quite well, until the *eye* is moved. Up and down remain invariant, however right and left may not. Movement in the Z axis does not seem linguistically as intuitive. "Front" and "back" or even "in front" and "behind" refer to the object or ego. "In" and "out" are terms for depth, however the ego relationship with the default *eye* on the Z axis might cause confusion. In and out from where? Is the child, the origin, or the screen the referent? The idea of positive and negative numbers is another important concept for understanding translation. Children often think of the operation of subtraction when they "see" a minus sign and of addition when they "say" positive. In translation this strategy is useful although it entails that the object becomes the referent. Under this condition, viewing the minus sign as a direction of displacement in relation to the origin of the coordinate system seems a more robust strategy.

Rotation involves extracting both the axis and angle of rotation from a mental image and/or a model. The rotations are more complex and not as "intuitive" to the children. The children must decide on the axis or axes of rotation, the order of rotations (if there is more than one), and finally, the direction and angle of rotation in order to attain the desired orientation in space. Rotations in mental imagery are very close to automatic (Corballis, 1986), but in interactive 3-D computer graphics, the specification of all the necessary information requires conscious control of the process. There were some children who did very well on mental rotations (pre-tests), but reported that they did not know how they came to their answers. However, in the post-test, they were able to report a rotation strategy. Possibly this is because in J3D the specification of all the necessary information required conscious control of this process. Extracting the axis was sometimes difficult for the children. Often their choice of axis was obtained by trial and error. I observed children become confused about what was the relationship of the axis to the object during rotation. This tendency to think of an object rotated in X as rotating *into* the X axis (a Z rotation) is an example of the interference between the referent and the operation. Here the language or name, an X rotations, is confused with the X axis and the relationship of the object to that axis. A useful image proposed to the children was to imagine a skewer piercing the object and the skewer was the axis that is turned. The problem seemed to lie in the relationship of the rotation to the coordinate axes. Another misconception was the use of positive and negative numbers to control direction (as in translation). Here the physical analog of grasping the axis and looking at the curl of finger seems the only strategy used besides trial and error.

**6.1.2. View Transformation -- Projective Space.** There are several ways of choosing an eye point. This process is not as difficult if the *coi* is still at the origin, because, no matter which coordinates are chosen, they are still "aimed at" the origin of the coordinate system. Thus, the numbers are relative to the origin. Children can then choose a place to look at things (if centered on the origin also) in relation to the scene. Another method used by the children was trial and error. To choose an eyepoint, they would just pick three numbers

and display the image to find out what could be seen from there. This strategy can have a disorienting effect if children do not think about where those numbers would put the eye. Most often the choice of eyepoints was orthogonal to the original eye point. For example, if they want to see something from behind they would use "eye 0 0 -10" instead of "eye 0 0 10." If this was not exactly what they wanted, they fine tuned along one axis at a time.

Coordinating perspectives is a very difficult task for children. This can be seen in their difficulty with perspective-change tasks. In J3D it is possible to see 4 different views at once. These different views can either be specified by the children and displayed in different quadrants one at a time, or the children can use the *blueprint* command which displays multiple fixed views. The use of this command, which children seemed to enjoy, may imply the coordination of the four different views. This command gave a great deal of visual (spatial) information about the form of an object or the spatial relationships of more than one object. The display from this command also gave the children a way of understanding 2-D representations of 3-D space through viewing both perspective and orthographic projections of objects and scenes simultaneously.

A distinction needs to be made between rotation and perspective change as discussed previously. In computer graphics however, the distinction becomes a matter of choice in strategies. In the physical world, we generally rotate a small object and walk around a larger one. In our imagination we do the same. In 3-D computer graphics the reason for using a particular strategy seems to change. For example if children want to see the back side of an object, they would have to turn it with the axis and angle of rotation. The same result can be obtained with a change of view. In this case, children would have to come up with a point in space from which to see that side of the object. The rotations may seem easier. However, if the children wanted to see the top right side of the object, they would have to do multiple rotations or come up with an adequate point in space from which to see that spot. It depends on the child which is easier or more difficult. In some cases (such as a scene with multiple objects) it may be easier to change the eye to see the back side, rather than to rotate all the objects while maintaining their

relationships to each other. To rotate the scene, all the objects would have to be either *attach*ed or *group*ed. What children had to keep in mind using J3D was the relationship between the point of view, the moving of the object, and the center of interest. Everything could be changing.

### 6.2. Mathematical Component

The mathematical concepts involved will just be briefly mentioned here. Children playing with J3D, used all three axes of the number line, explored the idea of positive and negative numbers and fractions, and used fractions for addition (translation), subtraction (translation), multiplication (scaling), and division (to compute the interpolation between key transformation). They used mathematical concepts as tools for planning transformations on graph paper and as parameters to commands in J3D. They began to explore relative or proportional numbers when working on graph paper (which does not have a reference to scale) and when anticipating or estimating values for transformation. These values were judged by what was seen on the screen, yet they were in proportional relationship to the screen determined by the distance of the *eye* from the objects.

In 3-D computer graphics, the mathematics is closely linked with Topological, Euclidean, and Projective relationships. Just as children were studying geometry in their math groups, they used concepts of point, line, angle, polygon, vertex, and their transformations in J3D. Children saw the effect of all of these operations on geometric shapes and forms.

### 6.3. Art and Animation Component

Artistic choice and engagement in the creation of images are two motivating factors that make it possible for children to master the complexity of J3D. J3D required a large overhead of details to be kept in mind. Children at about 10-12 years old have a fascination for either drawing or looking at representations in "perspective." This made J3D an exciting medium to explore design issues in creating models, constructing complex objects or scenes, and most of all creating animation. Artistic choice is involved when children spend a great deal of time fine-tuning a scene and choosing a point in

space from which to best show their creation. Artistic choice and problem solving are both at play in working within the constraints of the system. Important design decisions had to be made when addressing the issues of simplifying complex forms while trying to maintain a desired representation.

### 6.4. Summary of J3D as a Learning Environment

J3D could be a cogent learning environment for children to explore spatial concepts. It offers multiple representations of spatial concepts through visual, verbal, and formal modalities. The integration of these modalities encourages and enhances children's understanding of spatial concepts and makes this environment accessible to children of different cognitive styles.

In J3D, the coherence of spatial concepts provides children with the opportunity to investigate and learn about Projective and Euclidean space while creating images and animation. Children actively use these concepts making them explicit through perceptual and cognitive explorations in spatial problem solving.

Children are obliged to deal with spatial concepts explicitly in J3D through the specification of object and view transformations with both command language and mathematics. All operations in the system are constrained to the Cartesian coordinate system which becomes the dominant reference frame. This in turn can aid in the coordination of reference frames and with object transformations. This I believe to be important for two reasons. First, an understanding of the reference frame along with the ability to decipher underlying spatial information plays an essential role on performance in spatial problem solving. Second, the reference frame and metric relations underlie the operations required for anticipatory imagery, which in turn can be brought to bear on spatial problem solving.

Children may also gain mastery of spatial concepts by coordinating both rotation and perspective. Since, J3D offers both a choice and dynamic control over these operations, some children may come to understand the relationship between mental rotation and perspective change through exploring the differences and similarities in object and view transformation.

I believe that through manipulating synthetic objects and synthetic views, the children have an opportunity to learn to build a repertoire of strategies. The questions I hope to see answered by this research are if the children come to understand that they can achieve the same effect through different strategies? Do they understand the relation between these different strategies well enough that they could group them as a whole operation? For example, either rotating an object or changing the *eye* can result in the same effect, in the same way scaling an object smaller, moving it back in space, or moving the *eye* back can all result in the same image. Do children move the *eye* or the object? The decision becomes one of moving an object, the relationship between objects, or determining the place from which these objects should be viewed. One strategy difference may be in understanding this distinction between global and local changes? Through exploration these transformations could become a grouping, because children could achieve the same visual effect through different strategies. The understanding of this grouping would be an optimum coordination of these operations.

As we have seen in the literature, the appropriate intervention and explication of essential spatial information can minimize gender differences on certain spatial tasks, can change cognitive styles and strategies, and can improve performance in all three categories of spatial abilities: spatial perception, spatial visualization, and mental rotation. It is my hope that experiences using J3D will also help the children in the study to improve their performance in these spatial abilities.

# CHAPTER 7

# PILOT STUDY

## 7. PILOT STUDY INTRODUCTION

The prerequisite skills that the 5th graders need to take advantage of this environment are some familiarity with decimal fractions, and with the operations of addition, subtraction, multiplication, and division, as well as the ability to understand a 3-D object represented by a 2-D image through both "reading" the image and attempting to draw with perspective. In using the system, children also have to anticipate the outcome of a command, break it down to component parameters for X, Y, and Z.

The questions I wanted to address through the Pilot Study were: (1) Are 5th grade children (about 10-11 years old) able to function in this environment? Or do they become frustrated? (2) Do boys and girls enjoy spending the time and concentration necessary to learn about the system? Do they learn how to use it? (3) What prior knowledge do children need to bring to this environment? (4) What do the children learn?

### 7.1. Pilot Pre-tests

All the students in two AWC 5th Grade classes were given three paper and pencil spatial tests. The Primary Mental Abilities (PMA) Spatial Relations Abilities test (SRA), for 4-6th grades, the Card Rotation Test(2-D), and the Vandenberg Mental Rotation Test (3-D). I chose ten students, six boys and four girls, with a range of spatial abilities determined by the ratings on

the PMA SRA, † because it is considered a reliable indicator of spatial ability. Five of these children were in the control group and five in the experimental group. The experimental group included three boys and two girls, with SRA ratings of "very high," a "little above average," "average," a "little below average." The control group included three boys and two girls, with the same spread of ratings. All ten student were then given several tests and questions developed for this Pilot. ‡ Children were asked to draw what another "person" (a toy) had seen and perform several measurement and math problems.

### 7.2. Pilot Experimental Sessions

The children in the study were given an hour lecture on 3-D computer graphics using slides I designed at The Ohio State University (Sachter, 1984) to illustrate the concepts involved. These slides contain an explanation of the Cartesian coordinate system and how object and view transformations are performed and controlled. 3-D computer generated animations were shown as part of the instruction.

All experimental children worked for five hours using the system. Two Macintosh computers were set up next to each other, which allowed two children to work at a time. During the first session, I showed the children the geometric polyhedral primitives available on the system; cone, cube, cylinder, soccer ball, and hut (a simple house form). Then they were free to explore the scale, place, and rotate commands and operations. The children actively explored the different operations and learned how changing different parameter values would change the appearance of the object. After the first session children worked on individual projects of their own choice. They either created a complex object or built a scene. One child created a stereo, another a rocket, and another a box with a ribbon.

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† I was unable at the time to find group tests that addressed 3-D spatial abilities in children. The majority of 3-D spatial tests were for high school students or adults developed as aptitude tests for vocational placement. As a result the timing on the the card rotation and Vandenberg tests were too short for 5th grade children.

‡ I want to thank Prof. Edith Ackermann for her assistance in the development of these questions.

I worked closely with the children and intervened when they requested it, or if they appeared stuck, frustrated, or misunderstood a concept. I also intervened to point out alternative strategies that could be used to achieve the same effect. I worked with the children to develop ideas for images fitting their interest when they lacked an idea. The greatest amount of my time, however, was spent explaining and clarifying concepts related to the number line, positive and negative direction along an axis, and especially decimal fractions.

In the second to the last session, I showed the children how they could use a history file to create a short animation. I introduced the children to the idea of key-frames or extremes, and explained how the parameters between key frames could be incrementally increasing or decreasing a parameter until the desired next extreme was achieved. There were no utilities built into the system to assist the children in the interpolation of parameter between key-frames. I believe that this might encourage them to practice the use of decimal numbers. The children used simple objects, such as a square, so that the system could display the changes more quickly and the animation would appear to be in real time. After the children were satisfied with the extremes, they used Macwrite to edit their file and add the in between frame commands to their animation scripts. Below is an example of a sequence where an object gets larger or is scaled up in the X axis. This sequence can then be called in as a script and displays fast enough to appear animated. Below is an example of a script, and the changes that a child added.

ORIGINAL FILE

```
dis
scale sq 2 1 1
dis
```

EDITED FILE

```
call sq sq
scale sq 1 1 1
dis
scale sq 1.2 1 1
```

```
dis
scale sq 1.4 1 1
dis
scale sq 1.6 1 1
dis
scale sq 1.8 1 1
dis
scale sq 2 1 1
dis
```

After five hours on the system, I gave the children final tasks on scale, translation and rotation. I wanted to gain a better idea of each child's understanding and mastery of these transformations. I showed the subjects a picture of an original object and an object that had been transformed, and I asked them to tell me, in their own words, the difference between the two pictures. "In your own words tell me what has changed? Can you write down the commands that would cause this change?" If they could not write the command and the parameters, I asked them to use the computer to figure it out. A record of the commands and of the children's questions and comments were kept from this session. As a final task, I gave the children a printout image of a complex object created on the computer. It was made of primitives the children were familiar with. I asked them to duplicate the picture as exactly as they could. I timed how long it took them to complete this activity. I also gave all ten experimental students three written post-test; the card rotation test (part 2), the two mental rotation tests (one timed and one not timed), and a post-test of the tasks which I had developed.

I interviewed the children who had worked on the computer. I asked them what they thought they had learned, what was the easiest and what was the hardest thing about the system, and last of all what they would like to see changed in the software to make it easier to use the system.

### 7.3. General Pilot Results

From the Pilot Study I was able to answer the questions I asked earlier. (1) 5th grade children ages 10-11 years old are able to function in this

environment. They did not become too frustrated or feel overwhelmed by the complexity of the system. (2) Both boys and girls who worked on the system said that they had enjoyed it. Often children asked if they could come in and work during recess or lunch. All the children were willing to spend the time and concentration necessary to learn about the system and became engaged enough to try to learn how to use it. (3) The prior knowledge that the children most needed to bring to this environment was an understanding of decimal numbers fractions. (4) What the children learned I will address below.

The children learned their way around the Cartesian coordinate system fairly quickly. They learned that the X axis ran horizontally, the Y axis ran vertically, and that the Z axis was in and out. By the second day, they could tell me that "to the right" was towards the positive end of the "X" axis, and "to the left" was towards the negative end of the "X" axis. Up was positive "Y" and down was negative "Y." The children were able to use the screen with the eye in the default position as their frame of reference, and relate it to the Cartesian coordinate system. However, when they started to work on the system they often got confused about the direction along an axis (if it was positive or negative), and about which axis the desired transformation was in. This confusion seemed to get more intense as children began using numbers with decimals to scale objects smaller than the original object or to place objects, at values that contained decimals. They seemed to become confused by the "mystical" notation of either a decimal point or a minus sign before the number. It was as if they forgot what these symbols represented. They saw the meaning of a minus sign as an operation, rather than a direction and what the decimal signified. As the children began working on their own projects, placing an object exactly where they wanted it, they often required both a negative and decimal, such as -.5. Several times they would not know if they should use a decimal or a minus sign, for example to scale an object smaller they would sometimes use a minus sign and a whole number, thereby making it larger and turning it inside out. However, when asked, all the children thought that they had learned something about "X, Y, Z" or the "coordinate system."

Another problem that came up, was, I believe, due to the fact that the children had previously worked in Logo. In Logo, when you say RIGHT 45, the turtle turns right 45 in relation to its present state. In J3D, all the commands are relative to the original object, and not its present state. Therefore, if an object was rotated 30 degrees and the children wanted to rotate it 10 more degrees from its present state, they would have to rotate it 40 degrees in J3D. This is also true for translation and scale. The transformation commands are not relative to the last position but are an operation applied to the original object. Another thing the children often did, which I believe is related, was to use the reset command to get an object back to its original parameters, as if they had to get the original back before they could change it. This phenomenon seems very similar to the tendency, in Logo, to use the HOME command as a way of knowing exactly where the turtle is - a way of gaining or regaining orientation. I believe that many of the children requested a command to reset all of the objects at once, which I have since added to the system.

The biggest ccmplaint that the children had about the system was the limit of ten objects. They thought this was not enough to create the complexity they wanted. I find this complaint of the children exciting, because it shows a willingness to deal with a great deal of complexity in a changeable environment. The children wanted a lot of different building blocks to work with.

As part of the final interview, I asked the children what they would change in the software to make working on the system easier. I actually used their feedback to revise various aspects of the software. I have now included "reset" for the view and for all the objects. I have introduced an on/off command that can be used globally for all objects. I also increased the maximum number of objects from 10 to 15, without substantially slowing down the program or making it too large to run on a regular model Macintosh. I built an editor that the children could use from within the program, rather than having to quit the program, start the editor, and return to J3D. For research purposes I decided to use another version of J3D called H3D. The H is for history. As soon as the software starts up, the children are asked to type in

their name. The system then starts up a history file of every one of the children's commands, and saves them to a file with the children's name and date.

In the card rotations post-test, several of the experimental children's scores went down drastically. I found this result interesting since 2-D rotations are almost always used to test spatial abilities in children. One explanation might be that children do not consciously distinguish between 2-D and 3-D rotations. Another is that after working in 3-D graphics, the children were possibly thinking more 3-dimensionally. In other words, instead of rotating the cards in the plane (Z rotation) they might just as easily mentally flip the card in depth (X or Y rotation) and therefore conclude that the shapes were the same rather than mirror images. This later explanation seems more plausible for two reason. First, the 3-D mental rotation scores went up a little in these same children. Second, I re-administered the test, stressing the 2-D rotation, insisting that the card could not be flipped, and the scores went back up. In another post-test I presented two images to the experimental children and asked "what had changed from picture A to picture B." The children of "higher" spatial ability were able to talk about all the possible transformations that could have occurred to change the image. While the other children were able to talk about one or more of the possible transformations. All of the children did improve, however, in relation to the pre-test, in their ability to understand the possible transformations.

I did not see any significant change in ability in the mental rotation tasks. Part of the problem may be that the computer tasks were not directed enough to insure that all children had a similar experience in exploring rotations in some depth. This, I hoped would be alleviated by having specific and varied task for the children dealing with different levels of understanding and control of the various object transformations.

# CHAPTER 8

# DISSERTATION STUDY: EXPERIMENTAL DESIGN

## 8. EXPERIMENTAL DESIGN

The purpose of my research was to study children's understanding of spatial concepts, and transformations. I wanted to know how they use this knowledge in different spatial and metric tasks. J3D was used as a tool to provide the experimental children with explicit experiences in using the Cartesian Coordinate System and spatial transformations in constructing and animating images.

### 8.1. Overview of the Experimental Design

The study began in March, 1988, and was completed the first week of June. To give the reader a general idea of the duration of the study below is a brief schedule of the experiment.

The research started with administering a large battery of spatial tests to three 5th grade classes at the Hennigan School in Boston (N=45). Twenty children with a range of spatial abilities from average to very high were selected based on these tests. Half of these children were selected for the experimental group to work with J3D as described below. These children were then interviewed and completed computer tasks in order to discover what they had learned from the J3D intervention. All 45 children were then given a subset of the pre-test as a post-test. The duration for the entire study was three months. There was about 20-25 hours of computer related tasks and activities with the ten experimental children, and six hours of testing with the ten control children.

| | |
|---|---|
| March 7-11 | Large battery of spatial tests given to 45 fifth graders<br>Time: 4 * 1 hour tests (see Appendix B) |
| March 11-19 | Tests analyzed and according spatial abilities and gender<br>20 children selected (10 J3D, 10 control children) |
| March 22 | J3D group introduction to J3D and pre-J3D tasks.<br>Time: 1 hour |
| March 23 | 10 J3D children start J3D activities on the computer<br>Time: 15 * 45-60 minute sessions (11.5-15 hours). |
| May 16-20 | 10 J3D children post-interviews and review projects.<br>Time: 1 hour |
| May 23-27 | 10 J3D children post-J3D tasks on the computer.<br>Time: 1 hour |
| June 1-3 | Post tests are given to 45 children in two sessions.<br>Time: 1 hour |

### 8.2. Instruments

I administered a large diverse battery of tests to the children in order to gather data on as many relevant spatial concepts as possible. As previously discussed, J3D involves spatial concepts related to both Euclidean and Projective space, as well as, the transformations involved in mental imagery. One of the problems addressed in the design of the study was to attempt to sample the diversity and complexity involved in spatial tasks related to J3D. As Piaget pointed out, the image has a special function in spatial cognition, because the spatial properties of the image aid operational functioning and the spatial operations, the transformations, are figures in space. Through J3D, the concepts of mental imagery and spatial cognition are also related to mathematics in representing space and spatial transformations. As a result, multiple aspects of mental imagery, Euclidean, and Projective space were taken into account to develop the battery.

The tests were administered in four parts to all of the fifth graders over a period of one week. In the first three sections, children underwent were a replication of the Johnson and Meade (JM) battery (Johnson, 1987). It was a paper and pencil version of standard spatial tests. Children also underwent some piagetian spatial tasks, and some directions tasks. The only changes made on their tests were those necessary to make the directions tasks relevant to the local area. These tests are described in greater depth below. The last section of the battery was designed to look at concepts that were more closely related to fractions, measurement, positive and negative numbers, reference frame, Cartesian coordinate system, taking another's point of view or a perspective change, and the use of language to describe space. A complete copy of all the pre-tests, including the JM tests and their instructions for administration can be found in Appendix B, and all the post-tests are presented in Appendix C. Those pre-tests that are identical to the post-tests will only have the instruction sheet included as a place holder for that test. The following sections provide details on each test.

**8.2.1. Standard Spatial Tests.** The JM battery of standard spatial tests was used for a number of reasons. First, it provided a baseline indicator of children's spatial "abilities" for selecting participants for the control and experimental groups. Second, it was made of group tests that had already been administered to many children of the age to participate in this study. Third, the administration process and directions had been well thought out. It made it easy to replicate here. Finally, the replication of the Johnson-Meade study made it possible to compare the children in my study with other fifth grade children (in case this type of comparison felt necessary). The tests which were used for the analysis in this study are listed below. The information in parenthesis () is what is involved or indication by these tasks. The page number of the test is in brackets, (e.g. [318] or for pre- and post- tests [318,345]):

**(1). Paper and Pencil Standard Spatial Tests: Pre-Test Only:**

A. [PMA]SRA (2-D rotations - used as main standard, 4 min.) [296]

B. Hidden Figures (field dependent/independent, 3 min.)[301]

C. Block Counting (imagery, proportions, bookkeeping, 4 min.)[307]

Figure 2: Flags, Rotations, Cubes (FRC) Battery.

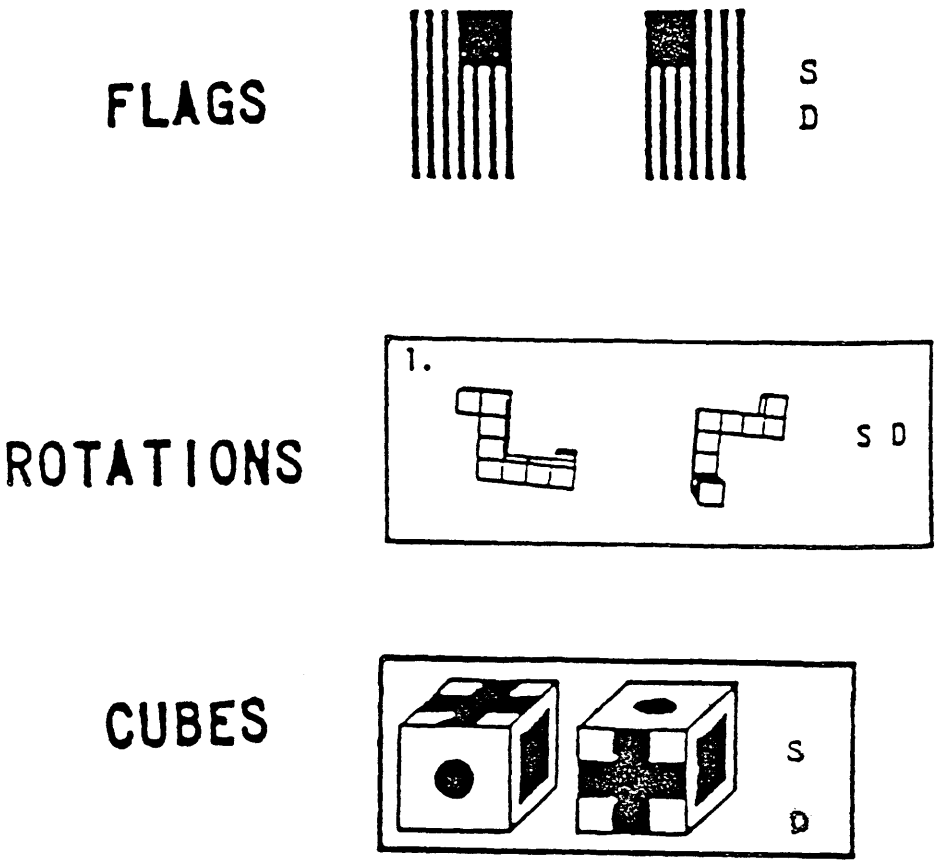
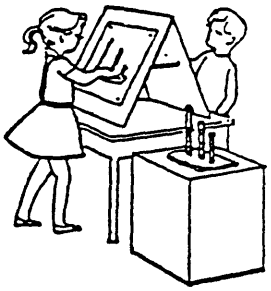
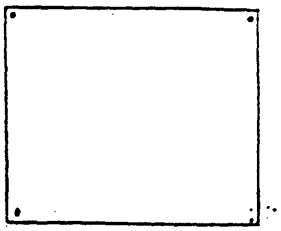


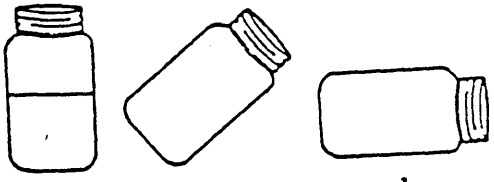
Figure 3: Water Level, Fishing Lines, Perspective (Piagetian).

These children are drawing three sticks of different lengths just as they see them. Pretend you are the boy. Make his drawing on this paper.



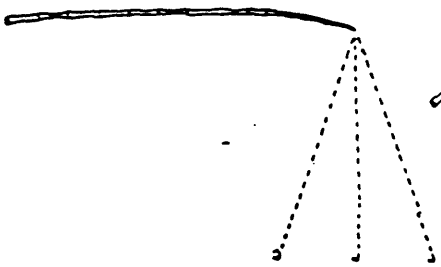


Draw the water line as it would be if the jars were half full of water. The first one has been done for you. (The cap is on tightly—it will not leak.)

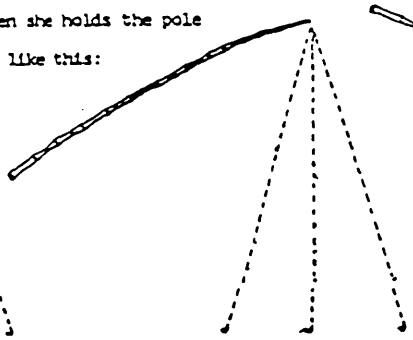



This girl is pretending she is fishing. She stands on her front porch and drops her line so that the hook almost touches the ground. Trace how the line will look when she holds it in these 3 ways:

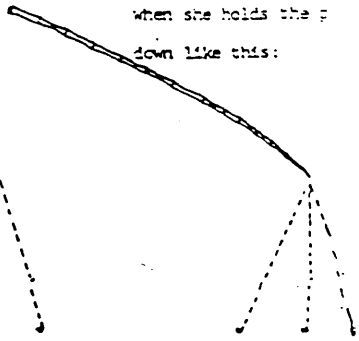
when she holds the pole straight like this:



when she holds the pole up like this:



when she holds the p down like this:



(Trace down one line for each of the 3 poles)

D. Hands test (rotation, 2 min.)[293]

**(2). Paper and Pencil Standard Spatial Tests: Pre- and Post-Tests:**

A. Flags (2-D rotations, 2 min.)[318, 345]

B. Rotation (3-D rotations Shepard-Metzler forms, 4 min.)[322, 349]

C. Cubes comparison test (mental rotations, 3 min.)[312, 347]

D. mouse and the maze (perspective) [311,348]

**(3). Paper and Pencil Piagetian Tasks: Pre- and Post-Tests:**(Figure

A. draw water level (system of references - horizontal)[292, 348]

B. mark fishing line (system of references - vertical)[292,346]

C. draw what another sees (perspective) [292, 346]

In replicating the Johnson-Meade study, the technique for carefully explaining and giving practice problems on each task was also replicated. This insured that all the children understood what was being asked of them. All the tests in paragraphs (1) and (2) above were timed. When the time was up, the children could switch to a red-leaded pencil and continue to work until they were done with that test. As a result, the children had two scores for each test which will be called the *timed* and the *total* score (on both the pre-tests and post-tests). The mean score of the untimed results of the 7 tests in paragraphs (1) and (2) above were used in the selection process. Only three of tests in paragraph (2), the Flags, Rotations, and Cubes (FRC, see Figure 2), were re-administered as part of the post-tests and used for the pre- and post-test comparisons. The limited amount of time was the main reason for the reduction in the number of tests administered. The size of the battery was reduced to what could be done in only two sessions.

**8.2.2. Euclidean and Projective Spatial Tasks.** In the first three parts of the battery mentioned above, the children had to mark the correct answers to the posed problems. These standard spatial tests provided a *quantitative first approximation* of the children's abilities to solve spatial problems. However, they did not provide much in the way of insight into the *qualitative strategies* underlying the children's thinking. After the standard spatial tests in paragraphs (1) and (2) were completed, we included several questions asking the children to "explain to younger children how they performed or how they

solved a particular type of problem." This was done in an attempt to elicit the children's strategies for solving spatial problems. These questions were asked on both the pre- and post-tests. A few questions at the end of each of the four sections also asked the children which problems were easy or difficult for them. We hoped that these qualitative reflective responses would give an indication of how the children perceived their performance, rather than just their scores.

The fourth part of the battery was originally developed for the Pilot Study and refined for this study. Below is a list of the tasks dealing with metric systems (that will be discussed in the analysis). Several of these tasks or problems, were borrowed from a battery of tests on mathematical problem solving by Lesh, Landau, and Hamilton (1983) and used by Harel (1988) in her research. These tests can be seen in Appendix B for the pre-test and in Appendix C for the post-test. I put the page number in [brackets] after the tasks in the following list.

**(1). Euclidean Spatial Tasks:**

A. recognize a point in 1-D (unit, subdivision) [337, 361]

B. snake length (length, unit, measure) [337, 360-361]

C. temperature (negative numbers, measurement) [337, 360]

D. locating a point in a plane (coordination of measurement 2-D)

D1. with a ruler [338]

D2. using grid paper [362]

**(2). Projective Spatial Tasks:**

A. mouse & maze (imagine left/right turns) [311,348]

B. choose set of orthographic views (multiple views, perspective)

B1. for object [333, 359]

B2. for multiple objects - scene [334, 355]

C. Generate Representation of Space (Projective):

C1. draw so looks 3-D (memory, perspective) [329, 351]

C2. draw a cube & square (verbal/visual representation) [330,352]

C3. write description of spatial layout (image to verbal)[332,354]

Table 1: Selection scores.

| MATCHING CONTROL & EXPERIMENTAL ON SPATIAL MEAN SCORE, STANDARD DEVIATION, AND MATH GROUP | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ALL SPACE | | | | MENTAL ROTATION | | | | SPAT. VISUAL | | | |
| BOYS | mg | smn | std | | hnd | sra | flg | rot | hfg | blk | cub | |
| Eb1. | David | 1 | .84 | .10 | .82 | .87 | 1.0 | .69 | .71 | .86 | .91 | |
| Cb1. | Johnb | 1 | .85 | .08 | .69 | .95 | .90 | .93 | .82 | .84 | .81 | |
| Eb2. | Mikek | 1 | .75 | .13 | .94 | .87 | .88 | .57 | .69 | .72 | .59 | |
| Cb2. | Erict | 1 | .76 | .11 | .76 | .89 | .92 | .62 | .67 | .80 | .69 | |
| Eb3. | Melvi | 1 | .73 | .12 | .78 | .89 | .88 | .71 | .71 | .67 | .50 | |
| Cb3. | Ethan | 2 | .70 | .16 | .61 | .87 | .90 | .71 | .55 | .80 | .44 | |
| Eb4. | Pauld | 2 | .67 | .13 | .55 | .79 | .85 | .57 | .70 | .78 | .47 | |
| Cb4. | Joshu | 2 | .69 | .14 | .65 | .87 | .90 | .55 | .67 | .67 | .50 | |
| Eb5. | Nigel | 3 | .65 | .15 | .47 | .73 | .94 | .50 | .63 | .75 | .56 | |
| Cb5. | James | 3 | .66 | .12 | .47 | .71 | .79 | .81 | .56 | .72 | .53 | |
| GIRLS | mg | smn | std | | hnd | sra | flg | rot | hfg | blk | cub | |
| Eg1. | Bonni | 1 | .83 | .11 | .94 | .92 | .94 | .62 | .85 | .77 | .81 | |
| Cb1. | Caden | 2 | .81 | .10 | .65 | .89 | .96 | .79 | .76 | .72 | .91 | |
| Eg2. | Rhoda | 1 | .76 | .15 | .92 | .84 | .92 | .69 | .65 | .80 | .47 | |
| Cb2. | Stati | 1 | .78 | .07 | .80 | .87 | .88 | .71 | .79 | .77 | .66 | |
| Eg3. | Roxan | 2 | .72 | .12 | .82 | .92 | .77 | .60 | .70 | .67 | .53 | |
| Cb3. | Eboni | 1 | .71 | .12 | .53 | .84 | .79 | .76 | .80 | .75 | .53 | |
| Eg4. | Tiffa | 2 | .63 | .07 | .59 | .65 | .65 | .76 | .62 | .62 | .50 | |
| Cb4. | Carin | 3 | .64 | .04 | .63 | .71 | .62 | .62 | .64 | .67 | .56 | |
| Eg5. | Jenni | 2 | .59 | .17 | .49 | .87 | .44 | .64 | .65 | .70 | .31 | |
| Cb5. | Tammy | 1 | .59 | .15 | .45 | .84 | .71 | .55 | .63 | .62 | .34 | |

E/C = Experimental / Control
b/g = Boy / Girl subject
mg = Math group
smn = Spatial Mean (7 Tests)
std = Standard Deviation
hnd = Hands Test
sra = PMA Satial Relations (SRA 4-6th grade)
flg = Flags Test
rot = Rotation Test
hfg = Hidden Figures Test
blk = Blocks Test
cub = Cubes Test

C4. draw what another would see (perspective) [292,346]

C5. draw what object looks like from 3 different views [331,329]

The above tasks were proposed to the children to gather clues into their current understanding of measurement (in relation to the Cartesian coordinate system), of representation of 3-D space on a 2-D surface, and of interactions of transformations of object and view changes. The measurement tasks were designed to progress from measurement in 1-D to the construction of the coordinate system and the plotting of a few coordinate points in 2-D. Several tasks were designed to gain insight into the children's understanding of incremental transformations and into their knowledge about J3D.

### 8.3. Choosing a Population

As mentioned earlier, fifth grade children (ages 10-11) are at an age when formalization of space is virtually possible. However children of this age seem to have a high degree of variability in their performance on spatial tests as seen in the high standard deviations. This reflects the kind of fluctuation often apparent when there is a shift in the way something is thought about. This marks a potential for change, not necessarily the stability appearing after a change. It may even reflect the influence of the situation or context on how this tenuous understanding is applied. Spatial thought, even in many of Piaget's experiments seems to fluctuate. Piaget (Laurendeau, 1970) explains this as decalages, Ackermann (1990) believes that performance on spatial task is affected by style, modality of description, and context, and therefore not always consistent or stable.

Twenty children (ages 10;2-12;0), ten in the control and ten in the experimental group, were selected on a number of parameters. First, the groups included the same number of boys and girls. They were matched as close as possible by the mean scores on seven standard spatial tests (Flags, Hands, SRA spatial relation, Hidden figures, Cubes, Blocks, and Rotations). These children's individual scores are presented in Table 1 and the group scores in Table 2. They are marked with a code (E)xperimental, (C)ontrol, (b)oy, and (g)irl (e.g. Eb1 stands for a child who is in the (E)xperimental group, a (b)oy and the highest scoring child in this category). The matching

was between the control and the experimental children, between experimental girls and experimental boys, between control boys and experimental boys, and between control girls and experimental girls. Second, once children were closely matched by mean scores they were also matched by math groups, because the mathematics was an integral part of J3D. Third, the children's attempt to include perspective in their drawings was seen as an important prerequisite. The children were thus asked to produce drawings of geometric forms immediately after briefly seeing their models. Fourth, the teachers were asked to consult about the children who fit all these parameters. I wanted to be sure that these children were also generally consistent about attending school, were good at sticking with projects that were challenging, and that they did not have a major part in the up coming school play due to time constraints. In summary, the means scores on seven tests, math group, gender, drawings, and standard deviations were matched as carefully as possible for twenty children.[†] Ten children for the control group and ten for the experimental group with mean scores equal (mean .72 and s.d .08). There were five control girls (mean .71 and s.d. .08) carefully matched with five experimental girls (mean .70 and s.d. .09). The same was true for the ten boys (equal means .73 and s.d. .07). There was also an attempt to match the boys and girls as carefully as possible. The mean scores between the control and the experimental groups were equal (mean .72 and s.d. .08), with the boys means score only slightly greater than the girls (boys girls .71 and experimental girls .70).

My experimental group included one boy and one girl of very high spatial ability ranging to one boy and one girl with average spatial abilities.[‡] The children chosen for the study were ranked by spatial scores from 3rd to 41st

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[†] It was not possible to match the girl with the highest spatial ability with the boy of the highest spatial ability, because the top four scoring children were boys. Therefore, the boy closest to the top girl was chosen as the highest spatial boy for the study. It was also difficult to match boys with lower spatial ability to girls with lower spatial ability because the bottom ten scoring children were girls.

[‡] The girl scoring lowest on the battery was still ranked as average by the SRA. I believe that because these children had been pre-selected for the AWC classes from the higher academic children from several elementary schools. I chose children from this group because of the higher math skill.

from the whole range of scores (N=45). I believe that J3D was difficult for children of lower spatial ability and the children that were chosen from the lower end of the spectrum were chosen because of their known persistent work habits. The ranking of high to average was obtained from untimed scores on the (PMA)SRA.

### 8.4. Experimental J3D Related Activities

The tests and the activities for the children were refined after the Pilot Study so that they were more closely related to the J3D tasks and concepts. The software was redesigned and fine tuned to allow more consistent collection of children's work and to provide the children with several of the

| | |
|---|---|
| March 22 | J3D group introduced to J3D and administer pre-J3D tasks (Lecture, slides, animation) |
| March 23-31 | Researcher with child to learn J3D system, 1-3 sessions (Object and view transformation exercises) |
| April 4-15 | Researcher with two children for Project 1, 2-4 sessions (Use object and view transformations to create scene) |
| April 13-27 | Researcher with two children for Project 2, 1-2 sessions (Create object data for the animation) |
| April 15-May 5 | Researcher with two children for Project 3, 3-7 sessions (Create animation and camera moves (eye)) |
| May 5-13 | Researcher with two children for Project 4, 1-3 sessions (Child's choice) |
| May 16-20 | Researcher with child post-J3D activity, 1 session (post-interview and projects review) |
| May 23-27 | Researcher with child post-J3D tasks, 1 session (paper and pencil, and computer tasks) |

commands requested by the Pilot Study children.

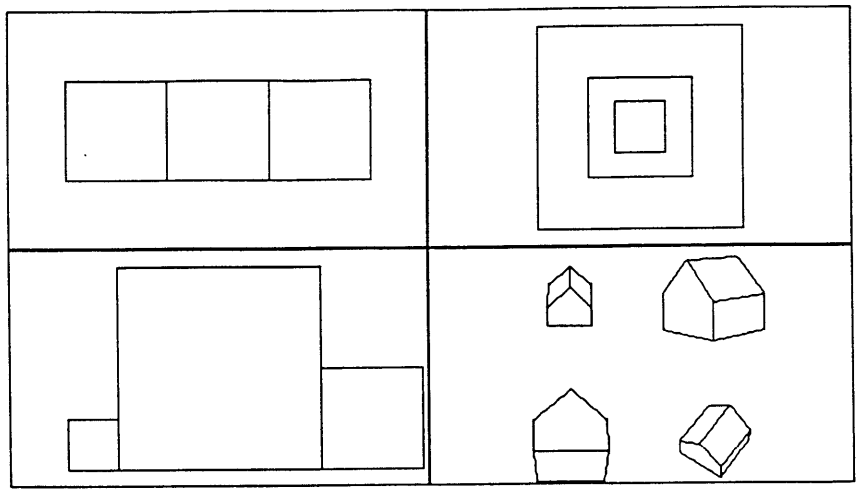
The primary goal of this study was to introduce new concepts and materials, specifically the concepts and syntax of J3D. This was done as a means to assist the children in building a conceptual domain. I proposed the vocabulary of the system, talking about what they could do with it, while pointing out alternative strategies for constructing pictures on the screen.

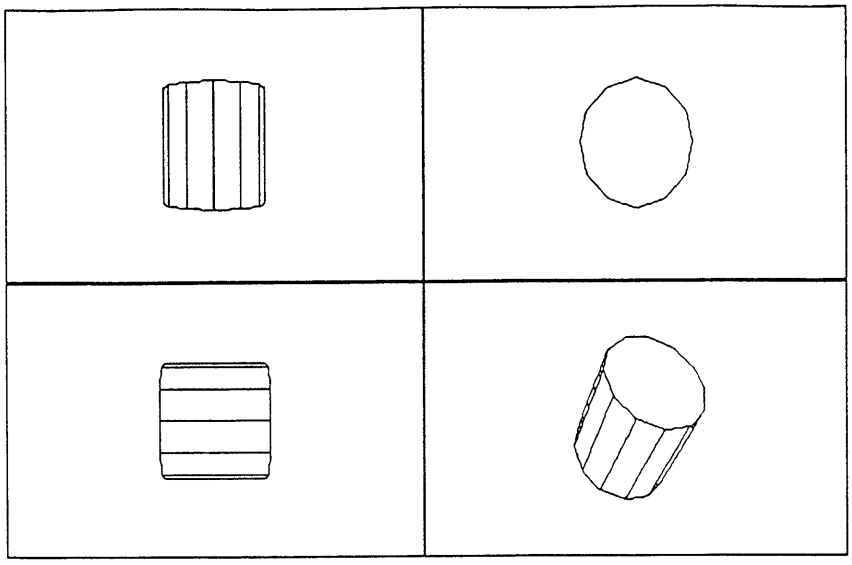
I conducted the learning sessions by moving from very uniform and structured tasks to open-ended projects. The latter often involving the integration of many transformations. The first few exercises focused on only one specific operation, while the later projects were left open to the children's decision on how many operations and the complexity of the project. All the tasks were ordered by the complexity and integration of transformations required. My interventions shifted from the child being more-, to less-assisted and structured. In the beginning session, I worked very closely, one-to-one with the children. These first sessions were a thinking "aloud" step-by-step process. After the introduction to the system, two children worked at the same time each on a separate computer. Later, I was assisting children at their request, or when they seemed stuck or confused. In the final tasks the children were asked to work on their own as much as possible. If assistance was needed, clues were offered to aid the child in focusing on what was important for the solution of a task. This was in the form of a strategy reminder rather than an introduction to new material or concepts.

Each of the children went through the same sequence of experiences, however they went through them at their own pace -- faster or slower -- depending on the children's difficulties and styles. The complexity of the projects were also open to the individual. Though often for an overly complex design, which was possibly too difficult in this medium, simplification of the design was discussed with the child. Below is a list of the experimental activities that the children were asked to do related to J3D.

Figure 4: Computer Exercies.

```
;TASK 1
call sq sq
call sq sq1
call sq sq 2
pla sq1 2 0 0
pla sq2 -2 0 0
dis 1

;TASK 2
call sq sq
call sq sq1
call sq sq 2
sca sq1 2 2 2
sca sq2 .5 .5 .5
dis 2

;TASK 3
call sq sq
call sq sq1
call sq sq 2
sca sq1 2 2 2
sca sq2 .5 .5 .5
pla sq2 -2.5 -1.5 0
pla sq 3 -1 0
dis 3

;EYES TASK
cal hut d
eye 0 12 20
dis 4
eye -10 2 -10
dis 4
eye 0 -7 -13
dis 4
eye 5 18 7
dis 4

;TASK ROTATION
call cylnd cy
dis 1
rot cy 1 x 90
dis 2
rot cy 1 z 90
dis 3
rot cy 2 x 45 z-30
dis 4
```





**8.4.1. Introduction to 3-D Computer Graphics.** The children were first introduced to 3-D computer graphics through slides, lecture, and 3-D computer generated animation. This entire session lasted about one hour and only the ten children in the experimental group participated in this activity. At the end of the lecture, several tasks -- involving scale, translation, rotations, and change of view -- were administered. The children were asked "What had changed" between two pictures. They were asked to give the best answer they could, and if they thought of more than one answer to write them all down. Below is a list of the tasks that were given to the children during this first session. (see Appendix B for these pre-J3D tasks).

**Pre-J3D Transformation Tasks for ten experimental children:**

A. Transformational tasks:
   1. drawing the rotation of a stick [mental imagery - motion]
   2. exercises on transformations available on the computer
      a. translation (-Y)
      b. scale larger (X) and scale smaller (X, Y, & Z)
      c. rotation (X)
   3. compare two scenes (change of view - rotation)
   4. timed computer-rotation test (Shepard-Metzler forms)

B. Euclidean Space tasks:
   1. plot a point on graph paper

**8.4.2. Learning to use J3D.** Each child participated in one to three individual learning sessions on using the J3D. These sessions included a series of exercises which were structured to teach J3D commands one at a time and to show the purpose of the utilities, or ways of getting information. Each exercise explored one type of object transformation and view transformation. The children were shown the existing objects on the system and different ways of displaying them. They were asked to keep any ideas, notes, or comments in a notebook that was supplied to them. Any notes or drawings made in explaining any concepts to a particular child in the learning sessions were put into their notebooks. The following exercises involve the transformations that were discussed and analyzed previously in chapter 5 in the section on *Object*

*Transformations.* Here the particular tasks will be discussed, but the reader is reminded that their is a previous explanation of J3D in chapter 5 and Appendix A. The images that the children were shown as a model to recreate in the learning sessions can be seen in Figure 4. Figure 4 also included the commands used to create these images. In the top squares starting in the upper left quadrant is the translation exercise. The upper right quadrant contains the scale exercise and the lower left contains the combination of the scale and translations exercise. Finally, the lower right quadrant is an example of the children's explorations into trying new eye points. The bottom squares contain the rotation exercises.

(1) The **translation** exercise had the child call in three squares and place them so that there were three squares in a lateral row. This exercise was used to teach how to call in objects, and to become familiar with translation in the X axis, both positive and negative. The instruction involved pointing out or explaining to the children about the size of an object when it originally is called in. They needed to consider both its extent and position in the translation. They were made aware that an object with the extent from 1 to -1 in X was actually 2 units. The child had to consider the whole object and its displacement.

(2) The **scale** exercise used the three squares called in for previous exercise. The children were taught how to reset all the square and then scale them. The children had to figure out how to solve the problem of the 3 embedded squares by scaling two of the three objects. In scaling as in translation the children needed to consider the size of the original object. In this exercise one object was not scaled, one was twice as large, and one was half as large. The most difficult part was scaling smaller than the original, because this procedure included decimal fractions. This was complicated for the children because of the concept of "multiplying by a fraction to make something smaller" was new to them. Almost every child's strategy was to start with the smallest square, which happened to be more difficult because of the mathematics concepts involved. This exercise took the longest time due to the amount of new concepts that had to be introduced.

Figure 5: Computer Project 1.

```
;dav.eyes
;Mon. 4/11/1988, 10:31:11
call pyr.asc  b
scale b 1.5 3 1.5
call soccer.asc  h
rotate h 1 z 28
call cylnd.asc  ll
scale ll 0.25 1 0.25
place ll -0.5 -1 0
call ncube.asc  la
scale la 1 0.5 0.5
place la -1.55 2.1 0
rotate la 1 z 155
call cylnd.asc  rl
scale rl 0.25 1 0.25
place rl 0.5 -1 0
call ncube.asc  ra
scale ra 1 0.5 0.5
place ra 1.55 2.1 0
rotate ra 1 z -155
call circle.asc  eye
scale eye 0.5 0.5 0.5
place eye -0.35 0 1
rotate eye 1 x 90
call circle.asc  eye1
scale eye1 0.5 0.5 0.
place eye1 0.35 0 1
rotate eye1  1 x 90
call sq.asc  mo
scale mo 0.15 0.15 0
place mo -0.05 -0.5 1
group head 4  h  eye  eye1  mo
place head 0 4 0
rotate head 1 x 0
group body 5  b  ra  rl  la  ll
group person 2  head  body
eye  0 0 30
dis 1
eye  20 20 20
dis 2
eye  -20 -20 -20
dis 3
eye  10 20 15
dis 4
```

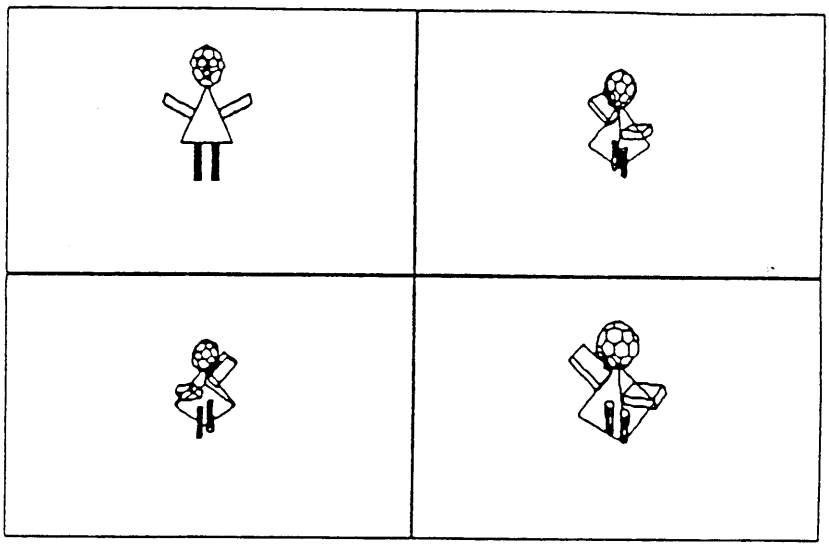
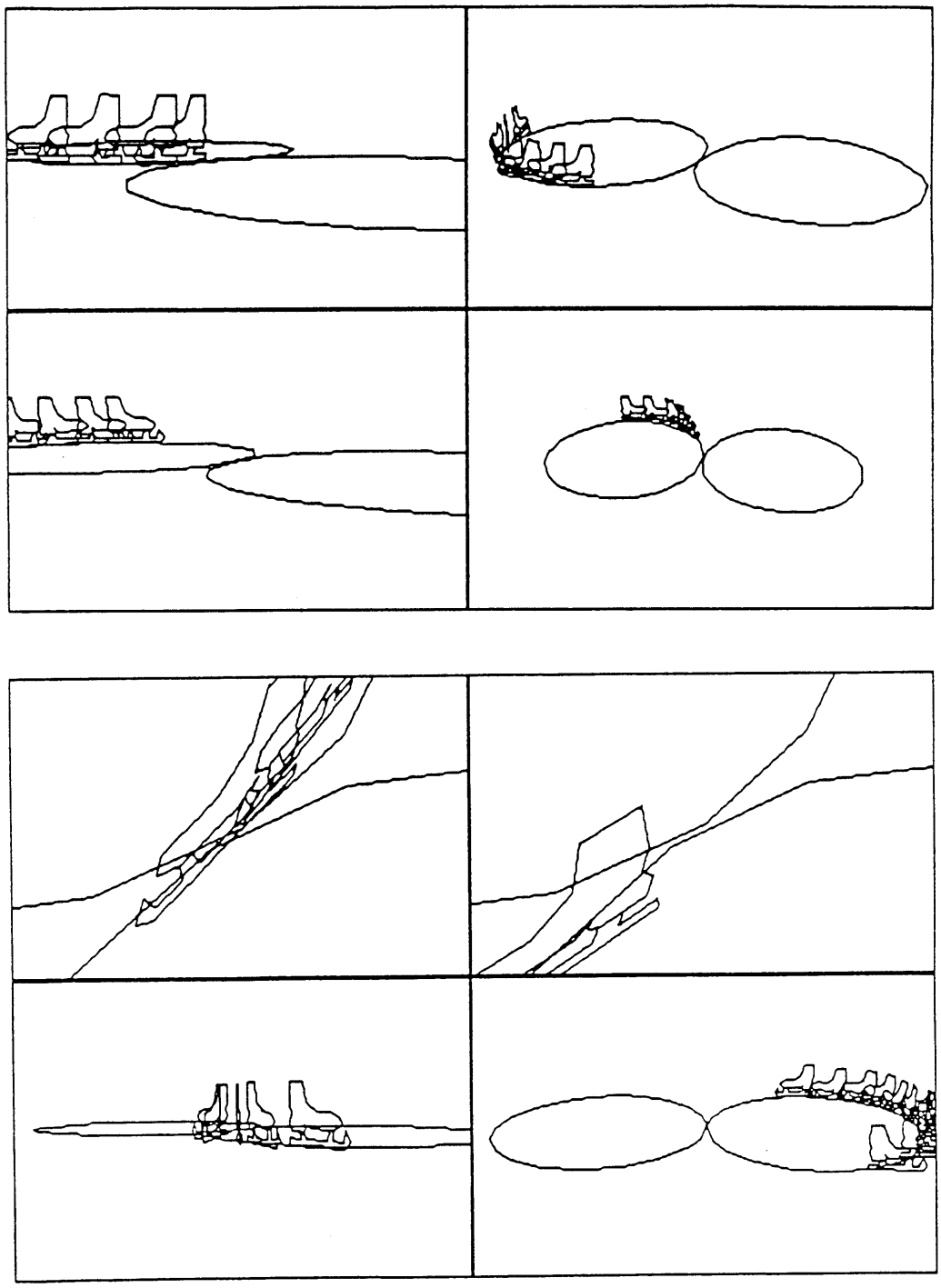


Figure 6: Computer Project 2.



(3) The **scale and translation** exercise combined the previous exercise (scale) and translation. This exercise was designed to provide the children some experience with placing objects that had been scaled. This required keeping the current scaled dimensions of an object in mind while translating, which included all concepts from the above two exercises and their coordination.

(4) The **rotation** exercises introduced rotation and the axes of rotation. This exercise included 3 exercises. There were exercises for rotation in X, Z, and a concatenated rotation in X and -Y. This exercise was designed to provide the children experiences in distinguishing the axis of a rotation and the resulting orientation. In Figure 4 the bottom set of squares contain the rotations exercise. In the upper left quadrant is the original object as it comes into the system. The upper right quadrant shows a rotation in X, the lower left is in Z and the lower right is X and -Z.

(5) The **view parameters, objects, and blueprint** exercises introduced the view parameters -- the eyepoint and center of interest. The children were asked to place the eye so that the view of the object was from different points of view, ie. "place the eye so that I can see this from above and behind it." The blueprint command was introduced and used to look at all the available objects so the children could plan their first project. It was also used to get the children used to seeing and understanding multiple views of one object or scene, as well as thinking about where an object was being viewed from.

**8.4.3. Projects for Using J3D.** All of the projects were designed to allow the children to express what they wanted with J3D. They were thought of as a guide to exploration of the processes of J3D transformations. Each project had several components built on the complexity of the previous project.

(1) In **Project 1** the children were asked to plan and build a complex object using the geometric primitives available in the system. They were introduced to the concept of malleable building blocks to construct objects or scenes. The children were given graph paper to

sketch their ideas. This project required that they use the different transformations as a means to change an objects' size, orientation, proportions, and placement in constructing their picture. After the object was completed the children were asked to choose three eye points which "best showed of their work to another as if it were a sculpture." This exercise required the children to change eye point positions. Support was provided in designing, planning, and executing the project if the children asked for it. Two children worked at the same time on separate computers. An example of the children's work on this project is presented in Figure 5, which show Eb1 David's four images and the commands used to create these on the computer.

(2) In **Project 2** the children were asked to create a simple object by making data or plotting the coordinate points and defining the polygons. The children were asked to draw the object that they wanted to create and support for the design was available if requested. The children were offered one-to-one help in marking the coordinate points that would make up their data and in extracting one or two points to get them started in recording the numbers needed for their data file. To make the construction of the objects easier for the children, they were asked to define 3-D objects with no depth. This procedure simplifies the polygon description by only requiring the definition of two polygons (front and back) for an object. This exercise provided practice in both understanding how to design, document, and extract the coordinate points needed to create digital data for J3D. Figure 6 shows a skate, which is Tiffa's object, here shown in her animation.

(3) For **Project 3** the children created a simple animation sequence. Most of the children used the object that they had created in their animation. They were supported with planning and designing if they requested assistance. There was an introduction to the editor (Hedit) built for the system. The concept of key-frame and in-betweens were introduced and demonstrated. The children were encouraged to work on their own, using the calculator on the system to compute the parameters for in between frames when necessary. When the children

were satisfied with their animation, they were asked to add dynamic camera changes. A further introduction to view parameters was provided along with a discussion of how a change of the eye point and center of interest could be used to achieve different "camera moves" or film conventions such as cut, pan, dolly, etc. Again, support was provided when ever requested. Figure 6 contains all the frames of Eg4 Tiffa's animation on a skate doing an eight formation. It is separated into overlays of all the frames in sequence from the different eye positions she chose. She both designed the skate and created the animation. Figure 7 presents Melvi's baseball (data) and animation.

(4) **Project 4** was just one to two sessions which provided the children with time to do whatever they wanted to do with J3D using what they had learned. This project was of their own choice with no constraints and very little assistance. Figure 8 is Roxan's final project.

**8.4.4. Final Post-J3D Computer Tasks.** The children were asked to do some final exercises both on paper and with J3D as a post-tasks. The children were given a similar paper and pencil test with pairs of pictures as in the pre-J3D tasks. However, in the post-J3D tasks they were also asked to write down what command they would enter into the computer. When the children had answered all the questions for one set of pictures, they were asked to create the second picture in J3D. These exercises, were proposed in a one-to-one session which was conducted as a working dialogue. The only support offered was through intervening to point out or remind the children of previous strategies they had used or previous experience that they could apply.

**Post-J3D Transformation Tasks for 10 experimental children:**

A. Transformational tasks:

1. rotation of stick [mental imagery - motion]
2. exercises on transformations available on the computer
   a. translation (-X, -Y) and two objects offset (-X, Z)
   b. scale larger (X, Y) and scale smaller (X, Y, & Z)
   c. rotation (X, Y, Z, & (X & Z))
3. comparing two scenes (change of view - rotation)

Figure 7: Computer project 3.

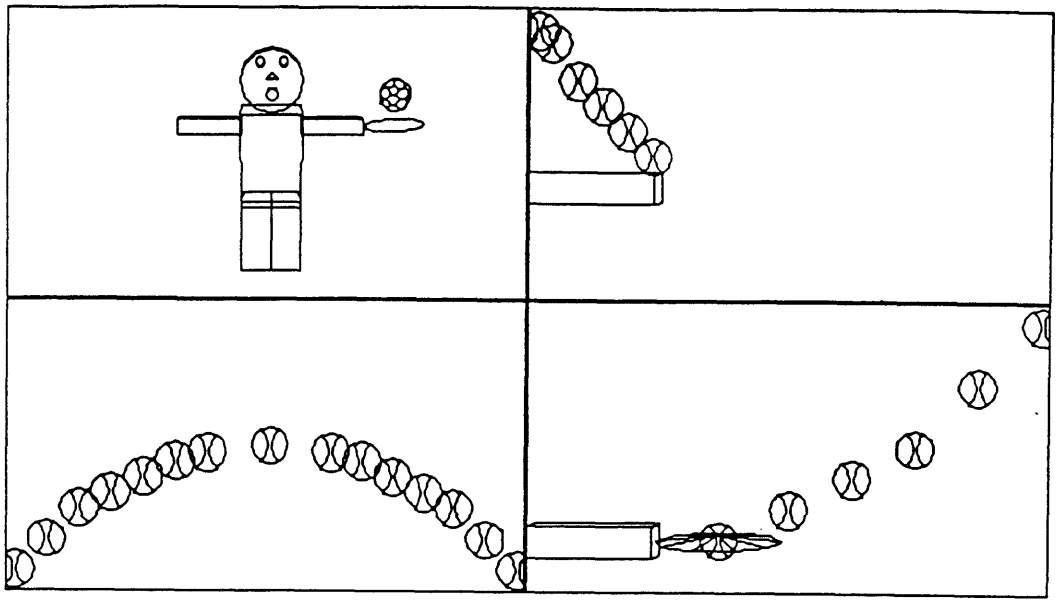
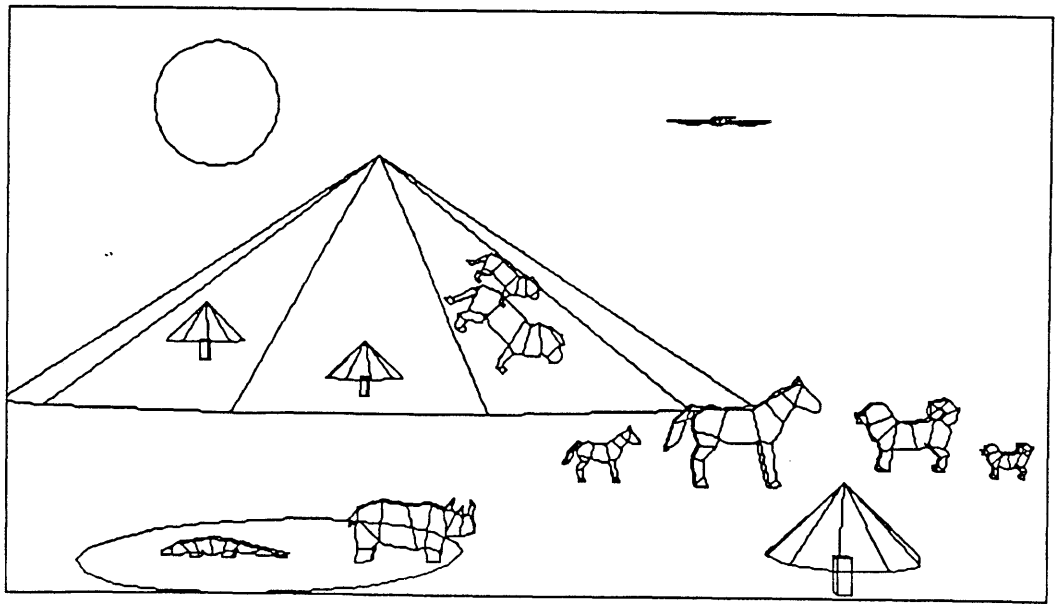


Figure 8: Computer project 4.



4. timed computer rotation test (Shepard-Metzler forms)

B. Euclidean Spatial tasks:

1. plot a point and construction of references
2. extracting the value of three points from labeled graph paper

C. J3D task:

1. create a copy of a complex object with J3D (using several primitives and transformations)

**8.4.5. Final Post-J3D Interview.** In addition, a final interview was designed to investigate: how they understood the system; how they would teach it to another child; what they found important; what language they used; what explanations they had created for themselves; and what explanations they had internalized from the experimenter. This was also intended as a way for the children to present their work, and to talk about plans, problems, and feelings about their work.

**General Post-J3D Interview Questions for 10 experimental children:**

1. Do you think you can explain to one of the students in your class, one who has not worked on the system, how the system works and what you can do with it?
2. How would you explain it to them? What else would you tell them?
3. Do you think they could begin to work from what you said? What would you have to add?
4. In the first class what would you begin to teach them? In the second?
5. What else would they need to know to use the system?
6. Where do you think other children could have difficulties -- where you should explain better?
7. What do you think you learned from this project?
8. What do you think you learned about 3-D computer graphics?
9. What did you like the most about the system?
10. What did you like the least? Can you think of anything I should change?
11. Was there anything you learned from using this system that you think is related to your regular class work?
12. Should others learn this system?
13. Can you tell me the difference in Logo and this system?

**Questions on Each Project:**

1. What projects did you do?
2. What did you learn from this project?
3. How did you get your idea for this project?
4. Describe how or the steps of what you did to get this final picture.
5. What was the hardest thing?
6. What was the easiest thing?
7. What do you like about your picture?
8. What would you change to make it better if you had the time?
9. Project 1: Where are we seeing this scene from?
   Project 2: What did you have to do to make this animation? Plans, editor? What is changing?
10. What commands did you use? Any new ones?

A great deal of data was gathered on the children in this study. The next chapter will present and discuss the analysis of the data. First the results of the tests scores from the ten experimental children and 10 carefully matched control children are discussed in the comparative pre-post analysis. Then the ten experimental children are discussed further on tasks related to J3D.

# CHAPTER 9

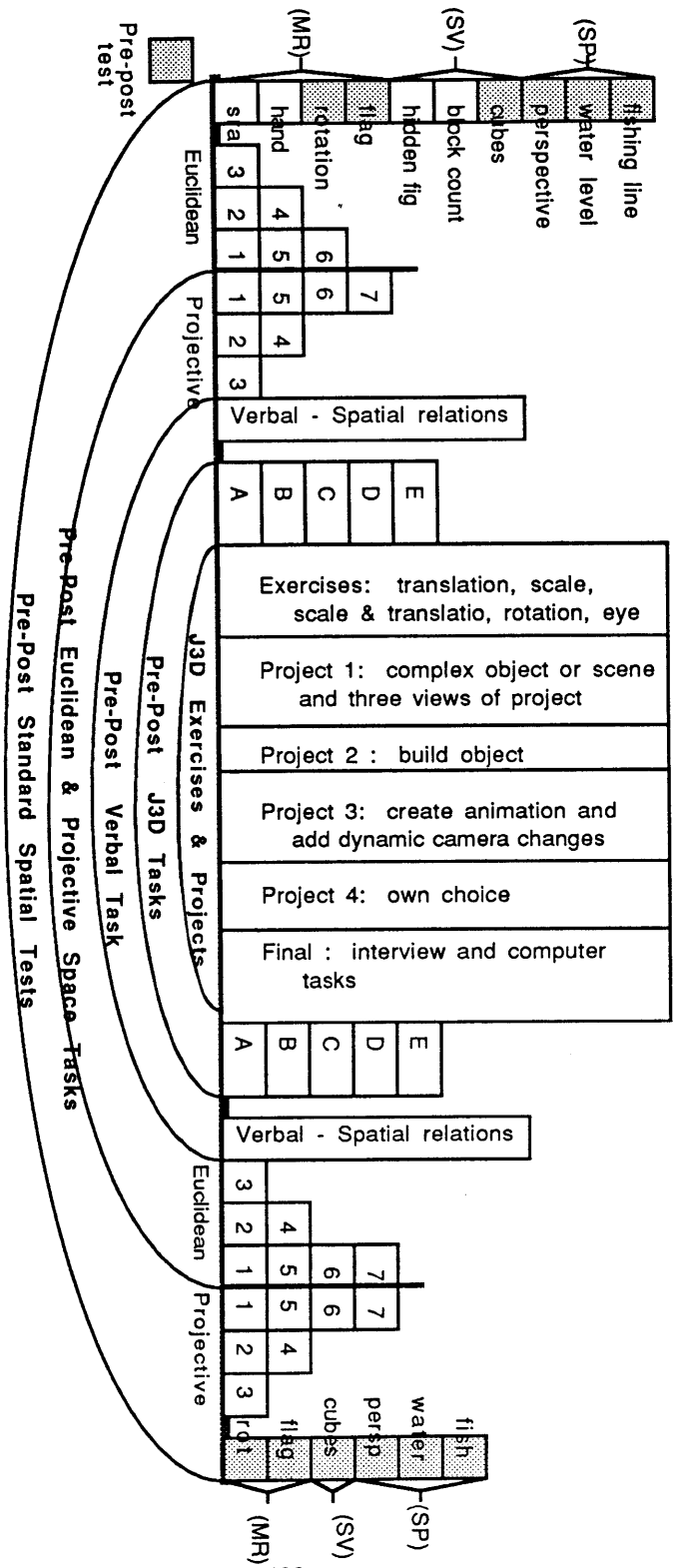
# COMPARATIVE ANALYSIS: STANDARD SPATIAL TESTS

## 9. DATA ANALYSIS: BETWEEN GROUPS OF CHILDREN

The original design of this study was to determine if or how J3D could contribute to spatial cognition, and what components of spatial thinking are developed and used in the J3D environment. As a result, a great quantity and variety of tests and tasks were administered to the children in order to sample a wide field of related concepts in spatial thought and mental imagery. I hoped that the broad sampling in this study would point to the thinking and learning elements that were related and interrelated.

The analysis of the children's performance on all the tests and tasks, as well as the records of their work with J3D was very intricate. Some of the complexity lies in the data being of a visual and spatial nature. The process of this analysis was to first come up with a method for data analysis and assessment of patterns of change, resulting in a comparative analysis of quantitative and qualitative data using two methods. The first method was a statistical analysis of the quantitative data that can be used to compare the differences *between* the control and the experimental children, which will be the focus of this chapter. The second method uses the qualitative data for comparing the differences *within* the group of experimental children which will be the focus of chapter 10 and 11. The results for the first method of analysis are presented in this chapter, while the results from the second method are in chapter 10. This methodology was based on the work by Harel (1988) in which she used a similar approach to looking at the quantitative changes in the learning of fractions and Logo programming as well as the qualitative learning and development of individual children.

Figure 9: Study and Analysis Diagram.



## 9.1. Method of Analysis: An Overview

A first level of analysis was a quantitative analysis *between* Groups by comparing the performances of the control and experimental children on various standard spatial tests. A second level of analysis was a qualitative analysis of performance *within* the group of experimental children. Figure 9 is a diagram representing both a history of the intervention and the data to be analyzed. It is organized in such a way as to look at it historically in a linear fashion from the beginning (at the left) towards the end of the study (to the right). It also provides an overview of the analysis when looked at simultaneously from the outside and working towards the center. In this way, the study can be looked at as sets of embedded parentheses or concentric analysis. The outer-most ring of analysis is the most global and encompasses general trends. As we move from the outside to the inside the data becomes individual in that it is both more personal for the children as well as more situated. In the diagram these sets of parenthesis are connected by the curved lines and go from standard pre- and post-tests, Euclidean and Projective pre- and post-tasks, Verbal pre- and post-task, pre-J3D and post-J3D tasks and finally the intervention itself with J3D projects and activities.

Data on the outside provides us with the first approximation of spatial "abilities" from the standard spatial tests of the JM battery. From this data we are able to carry out a quantitative comparative evaluation between the different groups of children. The next level inward addresses Euclidean and Projective spatial tasks which provide a more qualitative profile or spatial framework. Moving further inward, the verbal test on spatial relations allows us a glimpse of every-day language used by children to organize space and communicate this to someone else. Finally, the inner two levels are the intervention using J3D, with pre- and post-J3D tasks on the outside and the individual or child-centered learning exercises and projects in the center.

The quantitative analysis of the standard spatial tests supplies some important empirical evidence about the intervention's impact on those spatial "abilities" that are being measured by these tests. However these results do not provide any information about the reasoning underlying the performance on these tests. The children's responses to questions regarding strategies used

in the tests were also gathered to gain some indication of a stability or shift in strategy on these particular tests after the intervention. Therefore, after looking at the general trends revealed by the quantitative data, I will go one level deeper and propose a more textured qualitative analysis of the children. Their performances on the tasks on Euclidean and Projective space was looked at more closely to ascertain the various processes underlying spatial notions, and to see whether they are affected by the experience of working with J3D. These tasks were also analyzed as component parts, or building blocks, of more complex and integrated spatial thought.

Established standard spatial tests were used for the comparative study *between* the control and experimental groups. However, for the rest of the study, where new ideas are being combined and explored, it was imperative to develop a performance model or "task analysis" of all the tasks, both on and off the computer. This task analysis once achieved, helps me proceed to the qualitative analysis *within* the experimental group. This analysis was carried out along various parameters: gender, spatial ability, as well as learning and use of J3D. As Cole and Means point out,

> "comparative cognitive psychology must be based on the union of a theory of the task with a theory of the subject. It is impossible to understand the differences between people that lead to different performances without first understanding the component processes of the task eliciting those performances" (Cole, 1981).

Since this study is an *exploration* into the inter-connection of spatial thought, mental imagery, and 3-D computer graphics a greater emphasis has been placed on the task analysis. Due to the sheer quantity of data, the qualitative analysis was a sampling of all the possible methods of looking at this data. It can be thought of as different lenses which can magnify differences, or point out patterns of similarities in this new interdisciplinary area.

### 9.2. Results: Control and Experimental Groups

Three types of spatial problems were analyzed quantitatively for each child. They are categorized into what Linn (1986) called spatial perception data, spatial visualization data, and mental rotation data. The differences

between these spatial "abilities" are briefly reviewed below.

**Spatial perception** (SP) is the spatial ability relevant to the determination of the horizontal and vertical system of references in spite of distracting information. SP tasks reveal the children's understanding of horizontality and verticality which are fundamental to the development and understanding of the coordinate system. Spatial perception is also relevant to tasks used by Witkins to determine field-dependence and field-independence. These type of tasks are also important because the performance on them is dependent on the children's choice of reference frame. The Piagetian **water level** and the **fishing lines** tasks, used in the pre- and post-test, are examples of this type of spatial problem solving.[†]

**Spatial visualization** (SV) tasks require complicated, multiple-step analytic processing of spatially presented information. This ability may or may not include spatial perception and/or rotation, and requires an adjustment of procedures to solve the task. The Hidden Figures and Block Counting in the pre-tests, and the **Cubes Comparison** test in both the pre- and post-tests are spatial visualization tasks. As discussed earlier in chapter two, Just and Hart used the Cubes Comparison test in their research. They found that subjects used three different strategies - rotation, perspective change, or orientation-free description.

**Mental rotation** (MR) problems require the ability to accurately and rapidly imagine rotating a 2-D or 3-D figure. The children must compute the rotation and axis of rotation, either consciously or unconsciously. In the pre-tests the Hands and the PMA(SRA) tests are of this category, and in the pre- and post-tests the **Flags** and **Rotations** tests. Johnson and Meade found the results of the Flags test to be highly reliable, and the result of the Rotations test unreliable for children of this age.[‡]

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[†] In the statistical results, the score on perspective change is also included in this category, because it was considered part of the Piagetian tasks that relate to reference frame.

[‡] The paper and pencil version of the 3-D rotations test of Shepard-Metzler forms in the JM battery are rotations in the plane (Y axis) and not in depth (the Z axis). This particular test was created like a 2-d rotation test of 3-D rotations. In other words, it seems to have been developed by using the pictures of the rotations in the Y axis and cutting them out and turning them in the Z axis to create the test questions. As a result, these

I will compare the results of the tests of the experimental children with those of the control children in three ways: (1) pre-pre differences, (2) post-post differences, and (3) pre-post differences. As a result of the intervention, one could hope to see improvement on the spatial tests for the experimental children, especially among the girls. This study was also designed to investigate what spatial concepts were relevant to, or affected by J3D, and to see if there were any patterns of change that would indicate this.

The Johnson and Mead (1985) method of scoring was used to transform all scores (proportions of correct responses) to a common scale so that scores near .5 indicate chance and a score of 1.0 was a perfect scores. This was done for both the *timed* and *total* scores. Their method also made it possible to compare mean scores of the different tests.

As mentioned in the previous chapter, the children were chosen for this study by their mean scores and standard deviations of seven standard spatial tests (Table 1). The control and the experimental groups were matched as close as possible keeping with all relevant requirements. Of these seven tests only three were used for the pre-post test comparisons. Although there had been no significant differences in the scores between matched pairs for the seven test battery total score, according to Sandler's A test on the FRC (three test) comparison the scores show significant improvement (Tables 2 and 5). These significant differences were found in the *total* scores on the pre-pre test comparisons between the control and experimental children ($A=.228$, $p<.025$), the boys and girls ($A=.205$, $p<.01$), the experimental boys and girls ($A=.346$, $p<.05$), and the control and experimental girls ($A=.378$, $p<.05$). No significant differences were found for *timed* scores for any of the pre-pre or post-post comparisons for any of the children.

If we look at these findings more closely, the significant differences appeared among the experimental girls. The girls in the experimental group had significantly lower mean scores on the three spatial pre-tests.

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problems could be solved by turning the paper and some children even report doing this.

Table 2: Pre-pre and Post-post comparison scores.

PRE-PRE & POST-POST TEST MEAN SCORES
7 TEST MEANS = Sra, Flg, Rot, Cubes, Hfig, Blks
3 TEST MEANS = (FRC) Flg, Rot, Cubes:

| | 7 TOTAL MEAN | 7 TOTAL SD | 7 TIMED MEAN | 7 TIMED SD | 3 TOTAL MEAN | 3 TOTAL SD | 3 TIMED MEAN | 3 TIMED SD | 3 TOT SIG |
|---|---|---|---|---|---|---|---|---|---|
| Pre Control | .72 | .08 | .47 | .08 | .71 | .10 | .44 | .14 | .025 |
| Pre Experimental | .72 | .08 | .46 | .07 | .68 | .10 | .40 | .07 | |
| Post Control | | | | | .76 | .08 | .49 | .07 | ns |
| Post Experimental | | | | | .74 | .09 | .49 | .11 | |
| Pre Boys | .73 | .07 | .45 | .08 | .72 | .08 | .40 | .10 | .01 |
| Pre Girls | .71 | .09 | .48 | .07 | .67 | .12 | .44 | .12 | |
| Post Boys | | | | | .76 | .07 | .48 | .10 | ns |
| Post Girls | | | | | .74 | .10 | .50 | .09 | |
| Pre Control Boys | .73 | .07 | .44 | .08 | .73 | .08 | .41 | .13 | ns |
| Pre Control Girls | .71 | .08 | .50 | .08 | .69 | .12 | .48 | .13 | |
| Pre Experimental Boys | .73 | .07 | .46 | .08 | .71 | .08 | .39 | .06 | .05 |
| Pre Experimental Girls | .70 | .09 | .46 | .05 | .64 | .11 | .40 | .08 | |
| Pre Control Girls | .71 | .08 | .50 | .08 | .69 | .12 | .48 | .13 | .05 |
| Pre Experimental Girls | .70 | .09 | .46 | .05 | .64 | .11 | .40 | .08 | |
| Post Control Girls | | | | | .75 | .08 | .47 | .07 | ns |
| Post Experimental Girls | | | | | .73 | .11 | .52 | .10 | |
| Pre Control Boys | .73 | .07 | .44 | .08 | .73 | .08 | .41 | .13 | ns |
| Pre Experimental Boys | .73 | .07 | .46 | .08 | .71 | .08 | .39 | .06 | |
| Post Control Boys | | | | | .77 | .07 | .51 | .07 | ns |
| Post Experimental Boys | | | | | .76 | .06 | .45 | .11 | |

N = 20 Control Group N = 10 [ Boys = 5 Girls = 5 ]
Experimental Group N = 10 [ Boys = 5 Girls = 5 ]

There was no siginificant difference in scores on the timed tests between pre/pre or post/post.

Table 3: Pre-post changes in speed, accuracy, and Piagetian Tasks.

| PRE-TEST TO POST-TEST CHANGE IN % CORRECT SCORE | | | | | |
|---|---|---|---|---|---|
| BOYS | FRC SPEED (TIMED) | FRC ACCURACY (TOTAL) | PIAGET H | V | P |
| Eb1. David | +++ | | | + | |
| Eb2. Mikek | +++ | + | | o | |
| Eb3. Melvi | + | + | + | + | |
| Eb4. Pauld | - | ++ | | | |
| Eb5. Nigel | + | ++ | + | | + |
| Cb1. Johnb | | + | | | |
| Cb2. Erict | ++++ | -- | | | |
| Cb3. Ethan | +++++ | ++ | | | |
| Cb4. Joshu | | ++ | | | |
| Cb5. James | +++ | ++ | | | |
| GIRLS | | | | | |
| Eg1. Bonni | +++ | +++ | | + | |
| Eg2. Rhoda | +++++ | ++ | | | |
| Eg3. Roxan | ++++++ | +++ | | | |
| Eg4. Tiffa | ---- | --- | | | |
| Eg5. Jenni | +++ | ++++++ | | | + |
| Cg1. Caden | -- | + | | | |
| Cg2. Stati | ++ | + | o | | |
| Cg3. Eboni | ----- | ++ | + | | |
| Cg4. Carin | +++ | ++ | | - | |
| Cg5. Tammy | ++ | + | + | | + |

Pre-Post score change (mean Flag, Rot, & Cube)
+ increase - decrease
= 0 ++++ = 16-20
+ = 0-5 +++++ = 21-25
++ = 6-10 ++++++ = 26-30
+++ = 11-15 +++++++ = 31-35

Piaget: Perspective(P)
Horizontal(H)
Vertical(V)
o=some improvement
-=errors previous correct
+=corrected prior errors

Table 4: Overview of all pre-post significance values.

PRE-POST SCORE SIGNIFICANCE by SANDLER'S A TEST (blanks=NS)

| Test -> | BATTERY SCORES | | | | SEPARATE TEST SCORES | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Category-> | FRC (MR & SV) | | FRCP (ALL) | Piag (SP) | Flags (MR) | | Rotation (MR) | | Cubes (SV) | |
| Type -> | TOT | TIME | TOT | TOT | TOT | TIME | TOT | TIM | TOT | TIME |
| Experiment | .005 | .025 | .0005 | .01 | .05 | .025 | | | .025 | .005 |
| Control | .01 | | .01 | .05 | .005 | | | | .01 | |
| ExGirls | .025 | | | | .05 | .05 | | | | .05 |
| ExBoys | .025 | .05 | .01 | .025 | | | .025 | | | .05 |
| CoGirls | .025 | | .05 | | .05 | | | | | |
| CoBoys | | .05 | | | .005 | .05 | | .025 | .05 | |
| Eg2Rhoda | | .05 | .025 | | | | | | | |
| Eg5Jenni | .05 | | .005 | | | | | | | |
| Cg1Caden | | .05- | | | | | | | | |
| Cg5Tammy | | .05 | | | | | | | | |
| Cb2Erict | | .025 | | | | | | | | |
| Cb4Joshu | | | .025 | | | | | | | |

N = 20 Control Group N = 10 [ Boys = 5 Girls = 5 ]
Experimental Group N = 10 [ Boys = 5 Girls = 5 ]

(SP) = Spatial Perception
(SV) = Spatial Visualization
(MR) = Mental Rotation
(ALL) = SP, SV, MR
P/Piag= Piagetian horizontal, vertical, perspective tasks
FRC = Flags, Rotation, and Cubes tests
FRCP = Flags, Rotation, Cubes, and Piagetian tests

Table 5: Pre-Post FRC and Piagtian scores for groupings.

N = 20 Control Group N = 10 [ Boys = 5 Girls = 5 ]
Experimental Group N = 10 [ Boys = 5 Girls = 5 ]

SIGNIFICANCE OF PRE/PRE COMPARISON PARAMETERS
SANDLER A-TEST on FRC (Flg, Rot, Cubes):

| | TOTAL RESPONSES | | TIMED RESPONSES | |
|---|---|---|---|---|
| PRE-CONTROL / PRE-EXPER | A | P | A | P |
| All children | .228 | .025 | .918 | ns |
| Boys-Boys | .504 | ns | 6.980 | ns |
| Girls-Girls | .378 | .05 | .991 | ns |
| Boys-Girls | .205 | .01 | 1.193 | ns |
| POST-CONTROL / POST-EXPER | A | P | A | P |
| All children | .950 | ns | 71.777 | ns |
| Boys | 3.000 | ns | 1.004 | ns |
| Girls | 1.272 | ns | 2.590 | ns |
| Boys-Girls | 1.782 | ns | 13.98 | ns |

SIGNIFICANCE OF PRE/POST COMPARISON PARAMETERS
SANDLER A-TEST on FRC (Flg, Rot, Cubes):

| PRE / POST | A | P | A | P |
|---|---|---|---|---|
| All Control | .206 | .01 | .733 | ns |
| Control Boys | .654 | ns | .345 | .05 |
| Control Girls | .296 | .025 | 49.5 | ns |
| ALL Experimental | .172 | .005 | .267 | .025 |
| Experimental Boys | .309 | .025 | .387 | .05 |
| Experimental Girls | .278 | .025 | .516 | ns |

SIGNIFICANCE OF PRE/POST COMPARISON PARAMETERS
SANDLER A-TEST on FRCP (Flg, Rot, Cubes, Piagetian):

| | | | | |
|---|---|---|---|---|
| All Control | .211 | .01 | .333 | .05 |
| Control Boys | .441 | ns | .424 | ns |
| Control Girls | .351 | .05 | .635 | ns |
| ALL Experimental | .080 | .0005 | .197 | .01 |
| Experimental Boys | .250 | .01 | .306 | .025 |
| Experimental Girls | .432 | ns | .436 | ns |

Table 6: Pre-post FRC, F, R, C scores for groups.

SIGNIFICANCE OF PRE/POST COMPARISON PARAMETERS
SANDLER A-TEST on FRC (Flags, Rotations, & Cubes:

| Pre Post | TOTAL RESPONSES A | TOTAL RESPONSES P | TIMED RESPONSES A | TIMED RESPONSES P |
|---|---|---|---|---|
| All Control | .206 | .01 | .733 | ns |
| Control Boys | .654 | ns | .345 | .05 |
| Control Girls | .296 | .025 | 49.5 | ns |
| ALL Experimental | .172 | .005 | .267 | .025 |
| Experimental Boys | .309 | .025 | .387 | .05 |
| Experimental Girls | .278 | .025 | .516 | ns |
| PRE/POST FLAGS TEST | | | | |
| All Control | .167 | .005 | 2.112 | ns |
| Control Boys | .231 | .005 | .322 | .05 |
| Control Girls | .324 | .05 | 10.867 | ns |
| ALL Experimental | .319 | .05 | .279 | .025 |
| Experimental Boys | 3.093 | ns | 2.050 | ns |
| Experimental Girls | .354 | .05 | .322 | .05 |
| PRE/POST ROTATIONS TEST | | | | |
| All Control | .769 | ns | .605 | ns |
| Control Boys | .469 | ns | .296 | .025 |
| Control Girls | 113.500 | ns | .717 | ns |
| ALL Experimental | 1.220 | ns | 4.260 | ns |
| Experimental Boys | .293 | .025 | 2.600 | ns |
| Experimental Girls | 6.042 | ns | 1.109 | ns |
| PRE/POST CUBES TEST | | | | |
| All Control | .190 | .01 | .371 | ns |
| Control Boys | .340 | .05 | .456 | ns |
| Control Girls | .429 | ns | 1.387 | ns |
| ALL Experimental | .238 | .025 | .187 | .005 |
| Experimental Boys | .445 | ns | .375 | .05 |
| Experimental Girls | .493 | ns | .345 | .05 |

Table 7: Pre-post significance values for individuals.

SIGNIFICANCE OF PRE/POST COMPARISON PARAMETERS
SANDLER A-TEST on FRC (Flg, Rot, Cubes):

| RESPONSES TOTAL Pre-Post | FRC A | FRC P | FRCP A | FRCP P | TIMED FRC A | TIMED FRC P |
|---|---|---|---|---|---|---|
| Control All | .206 | .01 | .211 | .01 | .733 | ns |
| Control Boys | .654 | ns | .441 | ns | .345 | .05 |
| Cb1 Johnb | 2.324 | ns | .506 | ns | 2.324 | ns |
| Cb2 Erict | .690 | ns | 3.300 | ns | .350 | .025 |
| Cb3 Ethan | 1.415 | ns | 1.415 | ns | .429 | ns |
| Cb4 Joshu | .438 | ns | .324 | .025 | 1.0 | ns |
| Cb5 James | .570 | ns | .451 | ns | .505 | ns |
| Control Girls | .296 | .025 | .351 | .05 | 49.500 | ns |
| Cg1 Caden | 2.125 | ns | 2.125 | ns | .410 | -.05 |
| Cg2 Stati | .680 | ns | .479 | ns | 1.022 | ns |
| Cg3 Eboni | 2.805 | ns | .753 | ns | .492 | ns |
| Cg4 Carin | .879 | ns | 8.500 | ns | .699 | ns |
| Cg5 Tammy | .524 | ns | .543 | ns | .389 | .05 |
| Experimental All | .172 | .005 | .080 | .0005 | .267 | .025 |
| Experimental Boys | .309 | .025 | .250 | .01 | .387 | .05 |
| Eb1 David | 8.500 | ns | .907 | ns | .421 | ns |
| Eb2 Mikek | 1.969 | ns | .713 | ns | .709 | ns |
| Eb3 Melvi | 1.280 | ns | .805 | ns | 8.500 | ns |
| Eb4 Pauld | .566 | ns | .566 | ns | 204.500 | ns |
| Eb5 Nigel | .476 | ns | .418 | ns | .782 | ns |
| Experimental Girl | .278 | .025 | .432 | ns | .516 | ns |
| Eg1 Bonni | 1.899 | ns | .666 | ns | 1.289 | ns |
| Eg2 Rhoda | .460 | ns | .315 | .025 | .339 | .05 |
| Eg3 Roxan | .428 | ns | .428 | ns | .603 | ns |
| Eg4 Tiffa | .726 | ns | .726 | ns | 1.130 | ns |
| Eg5 Jenni | .349 | .05 | .276 | .005 | 1.107 | ns |

N = 20 Control Group N = 10 [ Boys = 5 Girls = 5 ]
Experimental Group N = 10 [ Boys = 5 Girls = 5 ]

**9.2.1. Speed Versus Accuracy on Spatial Tests.** Two scores were attained from each test, *timed* and *total.* It was thus possible to look at changes in both speed and accuracy in performance. Often there is a trade off between speed and accuracy in spatial problem solving. Looking at this trade-off can give clues as to the type of strategy used or the ease of performing the task. It takes longer to solve a spatial problem using a linguistic rather than a spatial strategy, and we know that girls often use their linguistic strength to solve spatial problems (Halpern, 1986). It has also been shown that gender differences favoring males are more often evident when a test requires a rapid response. However, speed of a task may not be as important an indication of spatial ability as has previously been thought. Carpenter and Just (1986) have suggested that it is the accuracy scores on psychometric tests that appear to measure what is called spatial ability. They even point out that performing rapidly on simple spatial tests is not related to being able to solve harder spatial problems, because performance on difficult tasks involves many subprocesses. They maintain that "accuracy reflects success in selecting, sequencing, and coordinating component processes, and encoding, maintaining, and transferring information between processes. All this executive processing may differentiate individuals as much as the speed of any one spatial transformation" (Carpenter, 1986).

Table 3 is a graphic representation of the control and experimental children's changes on both the *timed* and *total* mean scores for the Flags, Rotation, and Cubes Battery (FRC). We can see that all the children changed in either speed or accuracy. None of the children's scores remained fixed. If we only look at changes greater than 10 percent ( more than ++ or --), we can see that one child sped up and maintained accuracy (Eb1 David), one child slowed down but increased in accuracy (Cg3 Eboni), one child increased in speed but made more errors (Cb2 Erict), and one child both slowed down and made more errors (Eg4 Tiffa).[†] However, these just appear

[†] To make conscious process that are normally unconscious may slow down the process and affect the performance because of the increase in meta-functions. This often takes the form of a U-shaped learning curve, with performance dropping before it starts to improve.

to be random individual changes. The only pattern of change that becomes evident is the increase in both speed and accuracy for four of the girls in the experimental group (Eg1 Bonni, Eg2 Rhoda, Eg3 Roxan, and Eg5 Jenni).

From the data presented in Table 3, we can see an improvement even in the control children, but the largest changes in the pre- and post-tests scores were among the experimental girls in both speed and accuracy. This finding is especially exciting to me because the hopes of seeing this kind of change in the girls was my original reason for wanting to do this study.

From Table 3 we can also see patterns of change in the Piagetian horizontal, vertical, and perspective tasks. Note that only the experimental children improved on the fishing lines (vertical) task, but children in both groups improved on the water level (horizontal) and the perspective change tasks. The greatest number of changes on the Piagetian tasks was found among the experimental boys, while in extreme contrast there were no changes on any of the Piagetian tasks for the control boys. I am not sure why. My best guess is that the spatial processes related to choosing a stable reference frame did not change over the duration of this experiment for the boys.

**9.2.2. Significant Results of Pre-Pre and Post-Post Analysis.** Table 3 allowed us to visualize the quantity of change from the pre- to the post-tests. However, we still need to know if these changes are statistically significant, because of the effect of very high standard deviations. Table 4 contains a summary of all the significant findings between the pre- and post tests seen in Tables 5-7. All the result were calculated using Sandler's A test. Table 4 is organized with the left side containing significance scores for the batteries, *timed* FRC, *total* FRC, and FRC and Piagetians (FRCP). The right side contains the scores on the separate tests which made up the batteries. Starting at the top, the scores are organized from the most inclusive groups (experimental and control) and becomes more narrow in scope towards the bottom where we see the results for individual children.

At first glance we see a great number of occurrences of significant improvement for all children, even the control children. Before going on with the analysis it is important to mention some of the possible reasons for this.

A first reason is that the subjects in this study are children. It is thus important to question if the changes we see could be developmental changes. The entire duration of this study from beginning to end was barely three months, which makes me doubt that we could see deep developmental changes. Moreover, the control group is introduced precisely as an attempt to disentangle this issue. Second, there were six hours of pre- and post-tests for this study. Six hours are roughly one-third of the entire intervention for the experimental children. I believe that the detailed instructions, the tests, and the tasks involved experiences that were very intensive. These activities could be considered learning experiences by themselves. These tasks, like the intervention itself, made the children conscious of spatial problem solving. They spent four days in one week taking tests on spatial problems. Finally, the children in all three math groups coincidentally started units on geometry at the same time as the pre-tests for this study began. The children studied geometry in math during the entire study. Instruction in geometry has often been used in interventions (Brinkermann, 1966, Smith, 1979, 1981) to improve spatial cognition. The children were taught about angles, polygons, perimeter, circles, circumferences, area, finding distances on maps, solid figures, reading graphs and charts, slides, flips and turns. Many of these concepts were spatial and metric in nature and they were being explicitly addressed in the math groups. Therefore, my control and experimental groups were participating in other spatial learn experiences simultaneously. Many of the concepts were similar to what the experimental children were working with in J3D. However, in their math classes the transformations (slide, flip, and turn) were more 2-D rather than 3-D. Psychologist who rely too blindly on control-experimental and pre-post tend to forget that outside of their intervention children get many other chances to learn.

**9.2.3. Significant Results of Pre-Post Analysis.** In spite of the control children's improvement, the experimental children still out performed the control children on the post-tests in all of the battery results. In Table 4, we can see that the control group showed significant improvement on the *total* FRC ($A=.206$, $p<.01$) and the FRCP ($A=.197$, $p<.01$). The experimental group showed improvement in all batteries with ($A=.172$, $p<.005$) on *total* FRC,

(A=.267, p<.025) on *timed* FRC, and (A=.08, p <.0005) on the FRCP. The differences in the control and the experimental groups after the intervention becomes even more pronounced when the Piagetian scores are combined with the other scores. This FRCP battery included all three categories of spatial ability and this is where we see significant results in the experimental children. It is an important result, because performance on the Piagetian tasks involved the determination of the reference frame. The main premise of my intervention was that the explication of the Cartesian coordinate system would help children improve their understanding and use of the reference frame. It also indicates that as a group, the experimental children showed significant improvement after the intervention for both *total* and *timed* scores in all categories of spatial ability -- spatial visualization, spatial perception, and mental rotation.

While the pre-pre and post-post comparisons indicated more significant changes in the experimental girls than any of the other groupings of children, this is not reflected in Table 4 (statistical results of the pre-post comparisons). Yet what we do see is the same significance of improvement for all girls in the study on the FRC, both the experimental (A=.278, p<.025) and the control (A=.296, p<.025). The control girls also improved on the FRCP (A=.351, p<.05).

The experimental boys had the same significance score (A=.309, p<.025) on the *total* FRC as the girls, however they also showed improvement on the *timed* FRC (A=.387, p<.05) and especially the FRCP (A=.250, p<.01). Whereas the control boys only showed improvement on the *timed* FRC (A=.345, p<.05). The scores of the experimental boys seems to be reflected in the scores of the experimental group more so than the girls scores, however we know that this is not statistically possible. If we examine the boys and girls results on the FRC battery, we notice that only the boys, both control and experimental, showed significant improvement on the *timed* FRC.

Why don't the changes in speed for the experimental girls (which appeared in Table 3) appear here? I believe it is an effect of the nature of the

variable of their performance on these tests.[†] Actually, major "erratic" increases in scores seemed to cause the scores to "drop out" of the statistically significant range. For example, if we look at Roxan in Table 3 we can see that her score on the *timed* FRC increased by 27%! Her mean score on the pre-test was .35 and .62 on the post-test, with a standard deviations of .07 and .18 respectively. However great the magnitude was for this change in speed, it does not appear in Table 4. We do see, however, that statistically the experimental boys show improvement on all of the batteries, even though if you look at Table 3 there does not appear to be any great major changes as indicated by percent of change in the mean scores.

As previous research has shown, gender differences in favor of boys are more prevalent when spatial tests are timed. Therefore it is thought that boys do have an advantage on timed tests. However, as we have seen in Table 3, the experimental girls have also showed a great deal of improvement in speed, but this did not show up for them as a group statistically. Again this is probably due to the high variability[‡] on these batteries, especially for the girls.

Looking at the results of individuals for the batteries, we don't see any major patterns of change for the twenty individuals, except for two experimental girls, Rhoda and Jenni. We see the most significant change in Jenni because she scored the lowest on the seven test pre-test battery (Table 1). We can see her improvement in both Table 3 and Table 4. Her score on the FRC increased from 46% to 74%, and on the FRCP went from 56% to 75% correct. This transfers into a statistically significant improvement on the FRC ($A=.349$, $p<.05$) and the FRCP ($A=.276$, $p<.005$).

Let me now move on to look at the right side of Table 4 where the separate test scores are documented. These scores give us a more detailed picture of what pieces or separate tests contributed to the results we saw on the

[†] One indication of this is the high standard deviation. This variability also shows up in the computations of what is significant.

[‡] High variability seen in high standard deviations.

test battery results. The experimental children showed improvement on the Piagetian tasks, on the Flags (total and timed) and on the Cubes (total and timed), while the control children showed improvement on the Piagetian, Flags (total) and Cubes (Total). Only on the Piagetian tasks do the experimental children show greater improvement than the control children. It appears that the control children out performed the experimental children on the *total* of the Flags and Cubes test, but I believe this is another instance of a time-accuracy trade off. However if we just look at changes for spatial perception, mental rotation (grouping the Flags and Rotations), and spatial visualization, we see that both the control and the experimental children improved in all three categories. If we look at these three categories by gender, we see that all of the children improved in the mental rotation category, with only the experimental boys significantly improving in accuracy on Rotations and the control boys improving in speed. Only the experimental boys improved significantly in spatial perception. In the spatial visualization category all the experimental children improved in speed and only the control boys improved in accuracy.

According the Just and Carpenter's views on accuracy, true improvement in complex spatial thought is more likely to be revealed through the *total* scores rather than the *timed* scores. However, we can see a pattern of improvement in Table 4 in both the *total* and *timed* scores for all the experimental children, especially the boys. This pattern also appears for the experimental girls in Table 3. I would expect the experimental children to have more experience working with complicated spatial problem solving, which if it had any effect on the children's thinking would show up in the improvement in the *total* scores. So, why are we also seeing improvement in the *timed* scores? Not only did the experimental children demonstrate more significant improvement in their *total* scores compared to the control group, they also had significant improvement in the speed of performing these tests. This may indicate that they improved on complex problem solving (Total) as well as gained mastery or facility (Timed) in performing these tasks. Looking at the children's personal evaluations on which tests were the hardest and which were the easiest, I see that nine out of the ten children thought that the

Rotations test was the hardest and the Flags test the easiest. In looking at Table 4, we can see that their comfort with these tasks is also revealed in their scores. On the Flags test (which they thought was easier), the experimental children improved in both accuracy and speed, but in the Rotations test (which they report more difficult), there are no statistically significant changes. Another indication of this "comfort" or facility with spatial problem solving may be reflected in the Cubes scores. This test is in the spatial visualization category, which is the spatial ability that involves complicated analytic processing on spatial information. Here we see that the experimental children improved again in both their *total* and *timed* scores.

As mentioned previously the girls in the experimental group had significantly lower mean scores on the FRC pre-tests. Since, there were no significant differences on any of the comparisons between post-post test scores, this finding suggests that there was significant improvement for the girls who participated in the intervention. They had significantly lower pre-test scores than the other children in the study prior to the intervention and had no significant differences after the intervention.

Now, let me summarize the results of the main effects of the study. The design of the study is a three factors mixed design, with repeated measures on one factor. The three factors are Experimental vs. Control factor, Boys vs. Girls factor, and Pre vs. Post factor; the Pre vs. Post factor is counted as the repeated measures factor, for the scores were measured at two different times for the same participants. An Analysis Of Variance (ANOVA) was performed on the data, and the principal results of the ANOVA are presented in Table 8.

The table shows that there is no significant main effect of the Experimental vs. Control condition, no significant effect of the Boys vs. Girls condition, and no significant interaction effects between these two conditions. However, there is a significant main effect of the Pre vs. Post condition ($p < .0001$); no significant interaction effects were found between this condition and the other two, nor between the three conditions.

Table 8. Summary Table of Main Results of the ANOVA

| Source | SS | df | ms | F | p |
|---|---|---|---|---|---|
| Total | 5733.77 | 39 | -- | -- | -- |
| Between subjects | 4208.27 | 19 | -- | -- | -- |
| Experimental vs. Control | 99.22 | 1 | 99.22 | <1 | -- |
| Boys vs. Girls | 42.01 | 1 | 42.02 | <1 | -- |
| Exp-Con X Boys-Girls | 7.23 | 1 | 7.23 | <1 | -- |
| Error$b$ | 4059.80 | 16 | 169.20 | -- | -- |
| Within Subjects | 1525.50 | 20 | -- | -- | -- |
| Pre-Post | 801.02 | 1 | 267.00 | 28.93 | <.0001 |
| Pre-Post X Exp-Con | 11.03 | 1 | 3.68 | <1 | -- |
| Pre-Post X Boys-Girls | 3.03 | 1 | 1.01 | <1 | -- |
| Pre-Post X Exp-Con X Boys-Girls | 45.95 | 1 | 15.32 | 1.66 | -- |
| Error$w$ | 664.20 | 16 | 9.23 | -- | -- |

The overall results indicate that all the children improved their performance on the FRCP battery, which represents all three types of spatial thinking. The control children also benefited even from the time spent on tests and tasks addressing spatial thought. I believe that if this little time spent explicitly addressing spatial cognition can have an effect on the control children, that this is all the more reason to include spatial tasks as part of the regular school curriculum. No main effect for Boys vs. Girls was found. This is also an important finding, because we can see that girls as well as boys can benefit from explicitly addressing spatial thinking.

### 9.3. Discussion on the Rotation Test Results

The most unexpected results were the 3-D rotation scores. I originally predicted that performance would improve from working with 3-D space and transformations. Moreover, I predicted that both speed and accuracy on the Rotation test would improve. However, looking at Table 4, shows that of all the tests, that the Rotation test is the only one in which the children as a

group did not improve in either *total* or *timed* scores. There are significant changes for boys in *total* for the experimental and on the *timed* for the control. Since, Rotations tests using the Shepard-metzler forms often show a greater gender difference than 2-D rotations in favor of males, this is not truly surprising. Nevertheless, there could be several reasons for why no other significant changes occurred.

The first reason could be an effect of the high standard deviation on this test. Johnson and Mead did not think that this test was reliable for children of this age. To them, the test should not even be administered to children younger than fourth grade. It may well be that this task is just very difficult for children. In fact, nine of the ten children in the control group reported that they thought this was the hardest test. I none the less believe that some of the possible problems may be due to this particular version of this test. As mentioned earlier, this particular test was constructed through cutting, turning (Z), and pasting on paper the cut images of the 3-D rotations. A number of things caused me to wonder about this test.[†] First of all, Johnson and Meade did not find major differences between boys and girls on this test; nor did I. However, when I used the Vandenberg Rotation test in my Pilot Study, I found major differences in performance between boys and girls. This confirms what other researchers have found in using the Vandenberg test. The difference of between the two tests may affect the performance of the subjects, and in turn play a major role in the differences observed.

I also administered a timed rotation test on the computer to all the experimental children. The tests used the same Shepard-Metzler forms and the same-different responses. In the computer test, the forms were rotated either in the Y axis (like the Vandenberg test) for one part of the test, and the Z axes (like the JM test) for the other part of the test. Having these separate parts allowed me to ask the children if they noticed anything different between part one and part two (without mentioning the rotations).

[†] In figuring out the answers for the master tests, I found this test much harder for me than other rotations tests I had used.

Most of the children reported that to them the rotations in the plane (or Z axis) were more difficult. It turns out that the children made more errors on the Z axis rotations and were slower in answering them as well. These tests do not reflect a large sampling since they only had ten problems in each part.

I also found that the children performed better on the computer test than the paper test. Several of the children reported that it was easier to see the 3-D effect on the computer. However, the rotation tests, both the paper and pencil and the computer version, did not show any major changes from the pre- to the post-test. Finally, all the children reported that they thought "rotations were easier on the computer," and they scored higher on the computer than the paper test.

Another reason why we may not see any significant changes in the rotation tests is that the children did not have enough experiences with rotations. All the research showing improvement in either timed or total results on the 3-D Rotations test actually used a drill and practice approach. In other words, there was a great deal of repetition. This was not the case in this study. The children used rotations according to the needs of their projects. With some children, this meant they did not work with rotations very much. Although the experimental children worked with 3-D transformations, they seldom did so with complex abstract objects like the shepard-metzler shapes. They built and rotated complex 3-D objects, something recognizable like a dog or a robot which was easier to code, remember, and rotate. Maybe, it would have had an effect on the post-test if the children were familiar with the shepard-metzler shapes.[†]

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[†] I found that in building the mathematical models for these objects and in working with them to develop the tests for the children, I became much more familiar with them. I also understood better what the isomorphic "mirrored" properties were for those forms. However, for this study I did not make these forms available for the children to play with. I wanted to know if just exploring transformation, especially rotation could affect the results on this test without the familiarity with the form.

## 9.4. Strategy Responses to the Standard Spatial Tests

As mentioned earlier, the quantitative data analysis is a first level of analysis for revealing any changes in spatial cognition *between* groups of children. I was much more interested in a deeper level of searching for patterns of performance *within* the group of children who participated in the J3D experiment. However, before going on to the qualitative analysis of the data, I will briefly look at some examples of the children's responses to strategy questions, for example on the Rotations tests.

In this study all of the children reported using a rotations strategy on the Cubes Comparison task. As you recall this task is usually considered a spatial visualization task and requires a multi-step process or choice of strategy. This uniformity of strategy found among the children's strategy reports may possibly be a result of the explicit JM instructions (Appendix B) which the children were given. The JM directions and model demonstration introduced the solution as a rotations. The JM method of models, detailed instructions, and several practice problems may have ensured that all the children understood the problem, however this also could well have predisposed all the children to use the same strategy. A difference in strategy could be a possible reasons for gender differences. Models were used to demonstrate clearly how to achieve a task, for example: two like and two unlike cubes were shown to the children, and then turned, to explain how to check for same or different cubes. This explicit demonstration of a particular strategy may have flattened out some of the gender differences. Since gender differences are thought to be a difference in the choice made or choices available as a repertoire of strategies, the JM directions could have had an effect on the results. This may also be why their results on the magnitude of gender differences were not as large as other studies. This may be why the Johnson-Mead study and mine do not show major gender differences even on tests that have shown gender differences in the past, however there is no way from my study to ascertain this for sure.

I generally found that the children's responses did not change in any major way from the pre-test to the post-test. In fact, there seemed to be no major change of strategy for any single child or any difference of strategy

among the children for any of the tests. What seems to have changed is the children's ability to *explicitly discuss* the *process* and to become more *consciously aware* of the process itself. An extreme example of this is David, the boy with the highest spatial ability among the experimental subjects. David could only respond with a "?" to the Rotations test. In the post test, I asked him why he answered this questions with a "?." He said he *did not know how* he did it. He was unable to express how he achieved the task, even though he was quite good at it. Another child, Bonni, the highest spatial girl, changed in how she thought about these forms. Bonni, like several other children, focused on the representation of the forms, or "how it looked," rather than on the transformation used to solve the problem. In the pre-test she said "Pretend you can take the blocks off the page." In the post-test, her representation was even more specific to the structure of the object rather than the representation. She said "If 3 arms are the same and one (is) diffrent,[†] figurs diffrent." She never explicitly said that these forms are imagined as rotating, but I did gain a sense of how she thought about the forms themselves, and a possible insight into how she represented them. Finally, Nigel is a child who's strategy did not change much. In the pre-test he said, "I just turned them around in my mind"; and in the post-test said, "I turned it in my Head." Even though Nigel's strategy did not change, his speed and accuracy on this task improved 19%. Where David had no access to verbalize the process, Bonni could talk about the visual and structural representation of the object itself, and Nigel could clearly talk about the movement of the transformation. These are three examples that illustrate the importance of the qualitative data and we can start to see differences in how different children thought about one standard test, the Rotations test.

From the statistical method we have been able to see the significant changes made by the children on standard spatial tests. In the next chapter the analysis will entail two processes. The first part will be a task analysis of

[†] When reports are taken from the children's writing, I have left their spelling errors and duplicated their response as accurately as possible.

all the Euclidean, Projective, and J3D tasks and related activities that the children were asked to do. Once these tasks and their relationships are better understood, they can then be used to compare the experimental children's performance on the Euclidean and Projective spatial tasks, the concepts underlying these tasks, and their possible relationship with performance on J3D transformational tasks. By first gaining this general view of patterns of performance on these spatial tasks it will then be possible to begin a finer-grain analysis of some of the tasks by using the results as a base from which to look at how individual children solve different problems.

# CHAPTER 10

# ANALYSIS OF SPATIAL AND J3D TASKS

## 10. TASK AND DATA ANALYSIS: ALL EXPERIMENTAL CHILDREN

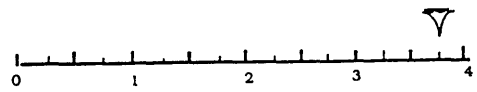
The focus of the analysis of the qualitative data focused on searching for patterns of change -- similarities and differences in spatial cognition **within** the group of experimental children. While the quantitative analysis gave us a way to compare changes between groups of children and within individual children, the qualitative analysis gives us a sense of the representation and spatial understanding of individual children. The qualitative data provided us with many details about the experimental children's understanding of spatial operations and strategies, and of their use of natural language to talk about space, and use of J3D.

Understanding the component processes of the tasks themselves must precede understanding how these processes affect children's performance on various tasks. Therefore, the task analyses becomes integral to the qualitative analysis of the data gathered from these tasks. One purpose of both the task and data analyses was to come up with a method of both analyzing the processes involved, the data, and looking for patterns of change in the data. The task analysis in section 10.1 provides a reference to compare the children's pre- and post-J3D task performances within the experimental group and their individual differences and spatial understanding.

This chapter will look more carefully at the children's performance on Euclidean and Projective spatial tasks in order to ascertain the various component processes that make up this spatial content area, and to assess whether they are affected by the experience of working with J3D. These tasks will also be analyzed as component parts (or building blocks) of more complex and integrated spatial thinking.

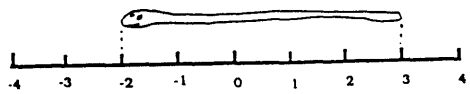
Figure 10: Euclidean space tasks for qualitative analysis.

Which number goes with the point?




a. $1\frac{4}{4}$ b. $3\frac{4}{5}$ c. $1\frac{5}{5}$ d. $3\frac{3}{4}$ e. not given

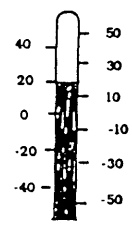
How long is the snake?



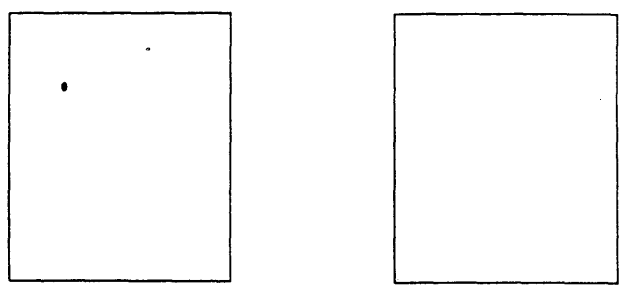

a. 3 b. 4 c. 5 d. -2 e. not given

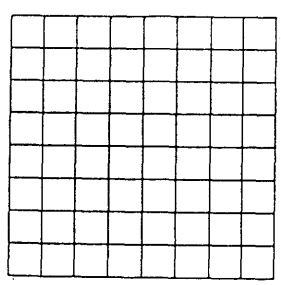
If the temperature drops 30 degrees, what will the temperature be?

a. -30 b. 10 c. 0 d. -10 e. not given




Please put the dot that is on the left rectangle in exactly the the same position on the rectangle on the right. You can use a ruler if you want.





Please put in the reference numbers in the grid and plot the following 2 points:

-1 2 0
3 -.5 0

## 10.1. Task Analysis of Euclidean, Projective, and J3D Tasks.

This task analysis was developed in order to better understand the *underlying spatial components* of the tasks in this study, and to determine which of these spatial components changed through working with J3D. The task analysis also becomes a *strategy-reference* that children used in solving these task. As discussed earlier, different strategies may or may not allow the children to achieve equivalent performance on these various tasks. There could be strategy differences both among the different children and within a single child in respect to different tasks, or situations within one task. One important issue, that can not be ignored, is the influence that I, the researcher, had on the strategies that children used. Similar to the way the detailed instructions on the JM battery affected the strategy for the problem solving on the standard spatial tests, the style of my intervention may well have influenced strategies on tasks related to J3D. Another important point, that a task analysis by definition makes explicit the underlying processes needed to solve the task. In order to know what these processes are, the researcher uses what they have learned about the subjects responses. The process thus becomes somewhat circular. The definition of the task consists of ways in which people solved the task and understanding how people solve the task is part of the analysis. Wherever possible, I have attempted to separate the analysis of the task and the results.

Below, I present an analysis of tasks, that I have organized into Euclidean space, Projective space, and J3D specific tasks. Figure 10 contains the following tasks.

**10.1.1. Analysis of Euclidean-Space Tasks.** Euclidean or metric concepts involve straight lines, distances, measurements, and angles fused into a single operational whole. The process of measurement is itself made of several concepts: conservation of length, subdivision, change of position, transfer, and of course a system of references. Unit iteration is the series of unit differences converted to metric quantity. More interesting, iteration of metric units and series of whole numbers are isomorphic, as are fractions of units and fractions of numbers. *In J3D these units are not only isomorphic, they become*

*identical.* The completion of many of the tasks described below are indications of the children's construction of the reference frame. This system is made explicit and formal through the use and understanding of the Cartesian coordinate system. J3D is built in such a way that in order for children to address the spatial content, they must use tools involving numbers; both positive and negative, and decimals. We must remember that measurement is the spatial use of numbers.

The tasks discussed below were proposed to better understand the components that go into the construction of the coordinate system, and how each of the children used these concepts. They are also used to gain an understanding of the concepts necessary for using J3D, and of the stability of these concepts after having used J3D.

(1) **Recognize a Point in 1-D:** This was a multiple choice problem in which the children chose the right number to represent the point. The problem involves a process of subdivision and the representation of the fraction in mixed numbers. This problem was used to indicate children's understanding of mixed fractions and the concept of unit and unit subdivision -- which is a foundation of measurement. The application of these principles to J3D was in determining the unit and subdivision of the unit. Most errors occurred in choosing a correct numerical representation. It was difficult, from this task, to determine the cause of an error. It could have been the result of many things. For example, it could be caused by not understanding mixed fractions, not knowing what part of the unit the arrow was pointing, what was the unit, and how to translate that to a numerical form.†

(2) **Temperature - Measurement in 1-D:** This was a multiple choice problem. Children had to determine a temperature after the temperature had dropped by so many degrees. The task requires the use of

† There were many other tests and tasks related to an understanding of fractions and different representation of fractions that will not be discussed in this thesis. These mathematical issues need to be looked at more carefully, therefore they will be addressed in future work.

negative numbers. Children had to determine the unit of measure (degrees), to decide the series of unit distances or unit iteration, and to keep in mind that the referent went from positive to negative numbers. This required linking several references, both the unit of measure and the point of reference for unit iteration. The children had to think about the transfer of the unit as well. The tasks also requires conservation of length, the *contained* being movable in relation to a fixed *container*. The problem could be solved mathematically through subtraction if the children understood that the result could be a negative number. If they understood this, it was an indication that they could understand positive and negative numbers (at least in a context where positive and negative numbers are familiar such as temperature). This problem is crucial in understanding Cartesian coordinate system, and specifically, in translating along a positive and negative axis in J3D.

(3) **Snake Length:** This multiple choice problem required that the children determine the length of a snake stretched out in relation to a number line. There were two snake problems on both the pre- and post-tasks. One problem placed the snake along line marked in 1/2 units and labeled every other mark with positive whole numbers. This problem gave an indication of both the conservation of length and the ability to use unit iteration and convert the units appropriately to report the length, (e.g. 5 units = 2 1/2). The second problem placed the snake along a range of positive to negative whole numbers. This problem was similar to the temperature problem. However the temperature was a not familiar context for dealing with negative numbers for the children, whereas the idea of linear measurement with positive and negative numbers was new most of them. Again the solution deals with recognizing the correct referent and unit. The ends of the snake also seemed distracting for some children. They chose an answer based on the location of the head or the tail of the snake, rather than on the length of the snake. The concepts involved in these tasks are extremely relevant to those needed for J3D. Conservation of length is

crucial in understanding the extent of the dimensions of an object, an essential concept in translation. This was especially relevant when the extent ranges from a positive to a negative number. All the objects on the system were designed to extend from minus one to plus one in all axes, which means the child has to keep in mind in translation that the object is two units in extent which goes from -1 to +1.

(4) **Water Level and Fishing Lines - Reference Frame:** The water level task and the fishing lines task are both important non-metric indicators of the children's understanding of horizontal and vertical referents. These two tasks are also included in the standard spatial test quantitative analysis, but I believe they are even more important for the purpose of a qualitative analysis. Understanding the reference frame is definitely an important component of understanding the coordinate system and underling influences on transformations.

(5) **Coordination of measurement in 2-D:** The children were asked to relocate a point from one rectangle to the exact same place on a displaced identical rectangle. This task entails locating a point in a plane or plotting a point in 2-D, what Piaget refers to as the coordination of measurement in 2-D. In the pre-task, the children were given two rectangles, one with a mark on it, and a ruler to use if they desired. They were asked to mark a dot in the exact same spot on the other rectangle. The children had to *generate* their own strategy to solve this problem.[†] In the post-task the children were given two rectangular grids set at 45 degree angles from each other. The left grid had a dot on it. The children were asked to place a dot on the right grid in the exact same spot as on the left one. Although this was a post-task I believe that it was easier than the pre-task because the grid was an explicit referent. In other words, the correct referents were supplied

[†] I believe there may have been some problem with the way I set up these two rectangles. They were parallel on the piece of paper. This made possible the strategy of using the ruler just as a guide for the second point rather than a referent for measurement. It may have slightly encouraged the use of a line of sight strategy.

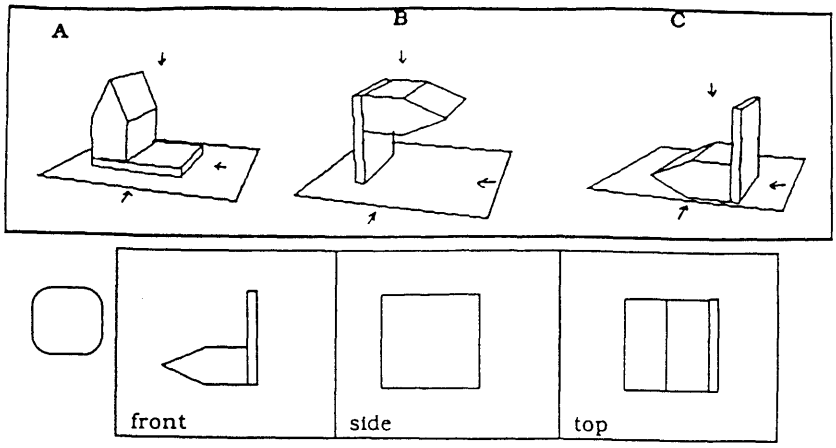
and were dominant. All the children had to do was correctly count along both referents for the correct intersection. The children were asked to report on how they had accomplished this task on both the pre- and post-task. Success on these tasks indicate coordination of measurement in 2-D and the completion of the construction of the coordinate system. Measurement in 2-D (3-D is synchronous) and of course the Cartesian coordinate system are used in translation, scale, and for placing the eye and coi in J3D.

(6) **Plot a Point in 2-D:** After showing the children instructional slides on the Cartesian Coordinate system, they were asked to put a point at -7 in X, and 10 in Y on a piece of graph paper that had the axes and reference numbers printed on it. In the post-task, the children were asked to extract three coordinates from the three points of a triangle drawn on a grid with the axes and numbers marked. I believe that it is easier to *extract* the two coordinates (X Y) than to *generate* them, because the two elements (X and Y) don't necessarily have to be coordinated. The children can look at the axis in each direction to determine the number in each axis. To generate the coordinate requires that the children themselves coordinate the two axes. To extract the correct coordinate leaves one free to focus on one axis at a time while determining the elements of the coordinate point. Many children were able to extract the correct point before they could generate it. This task is related to the Cartesian coordinate system that is used in almost every aspect of J3D.

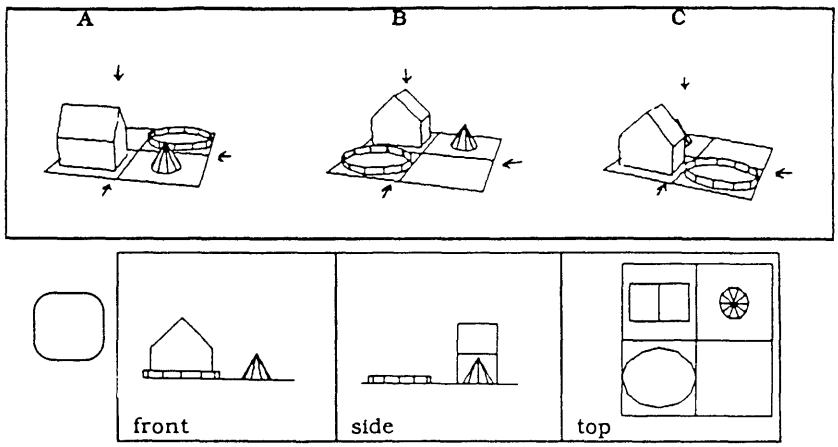
(7) **Construct Referents and Plot Points in 2-D:** This task requires that the children plot two points (-1 2 0, 3 -.5 0) on a grid, and more importantly, that they explicate, or label, the axis and numerical referents. This task was only administered as a post-task with the help of the researcher. The children, worked by themselves, and the researcher aided the children through questions, or reminder of previously introduced concepts or strategies. The children had to decide what was important as a referent and then to plot the two points. As mentioned previously, this type of task relates to the coordinate system, and thus

Figure 11: Projective space tasks for qualitative analysis.

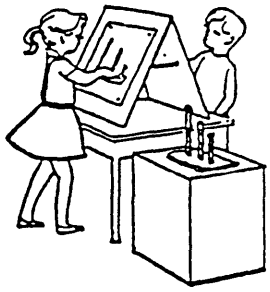
5. The little space cadet took a set of three photographs (top, front, side) of the object below when it was in three different positions. He can't remember which set of photographs went with which position. Can you help him ? Place the letter of the correct position in the box to the left of each set of photographs.

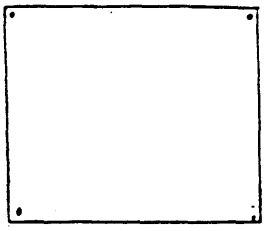



6. The little space cadet took a set of three photographs (top, front, side) of the three different scenes below. He can't remember which set of photographs went with whichscene. Can you help him ? Place the letter of the correct scene in the box to the left of each set of photographs.




These children are drawing three sticks of different lengths just as they see them. Pretend you are the boy. Make his drawing on this paper.





is very important for J3D. It essentially allowed the children to demonstrate what they knew about the coordinate system by having to generate it.

**10.1.2. Analysis of Projective-Space Tasks.** The following tasks (Figure 11) are related to Projective spatial concepts. As mentioned earlier in chapter two, Projective space is the coordination of both viewpoints -- actual and virtual -- and the figures considered relative to these viewpoints. Therefore tasks which require drawing, describing, or choosing objects from a particular point of view, are relevant to Projective spatial thought.

(1) **Draw and Write the Difference between Square and Cube:** In the pre- and post-task I asked the children to both draw and write about the difference between a square and a cube. This exercise was given to see how the children represented a cube and also how they talked about these differences. As mentioned before when talking about the children's rotation strategies, the children actually had two very different ways of thinking about this task. One was focused on the actual parts of the drawing (2-D representation) on the paper, (i.e. the number of lines or edges to make the shape or the idea that there are three "squares in the cube"). The other was to focus on the properties of the object itself, such as a square is 2-D and a cube is 3-D.

(2) **Mouse in the Maze:** For the Mouse-in-the-maze task the children had to determine if a mouse should "turn" right or left at an intersection, as it attempts to liberate itself from the maze. The children had to imagine if the turn was right or left for the mouse in reference to it's current heading. Therefore the children had to consider to the current heading of the mouse and give a left or right direction to continue through the maze. I believe that doing this task relates to taking another person's point of view. One problem the children had with this task was in the *choice of the referent.* If children choose the whole map on the page as the reference this was too global a reference to solve the task. They had to think about the mouse in reference to the map, which is a more local reference than the whole page. This

required a shift in point of view. The children must keep oriented with the mouse and choose the correct referent from that point of view. This task is relevant to J3D, because the children often have to imagine what something looked like from somewhere else and to maintain a link with a particular reference while working through a problem.

(3) **Choose Orthographic Views of Object:** In both the pre- and post-tasks one complex object created on the computer was placed on a reference platform in different orientations. There were three of these problems on the pre-task, four on the post-task. The way the problem was posed to the children was that a "little guy" (toy person) with a camera had taken three pictures of each object from the front, side, and top, but he had forgotten which set of pictures belonged to which object. The children were asked to figure out which three pictures belonged with which object. The three pictures (blueprint) of the object were created using parallel projections to avoid depth clues. The children could have determined the correct set by just matching the front views, which were easily visible. This task was introduced to see if the children could imagine how the object would look from different points in space, and also if they could coordinate these different views to rebuild the object. Both of these skill are often used in J3D.

(4) **Choose Orthographic Views of Array of Objects:** The same exercise as above was also done with an array of objects. This could be solved in two ways. The children could pick a salient object and test their transformation against that object, or they could check with two or more of the objects relations to each other. Again, an easier strategy could have been to check only the front views.

(5) **Draw Orthographic Views of Object:** In the previous three exercises the children only needed to recognize the correct answer. In the next few exercises the children were asked to *generate the correct representation.* In both the pre- and post-tasks I held up a rawhide mallet and a toy person with a camera. In both tasks I asked the children to draw what the photographs would look like if taken from that point of view. There were three different points of view (the front, the side,

and the top). This exercise allowed me to see how well the children could imagine something from a particular point of view, as well as how they represent perspective information in their drawings.

(6) **Draw Objects to Look 3-D:** In both the pre- and post-tasks I asked the children to draw a picture of a pyramid, hexagonal form (a six sided "cylinder"), a smoothly curved cylinder, and a cone so that they looked 3-D. The children were shown the object for just a few seconds, and then it was hidden so that they would have to draw it from memory. To analyze and rate the children's drawings, I used Mitchelmore's (1978) plane schematic, solid schematic, pre-realistic, and realistic categories. I believe this exercise and the one above gave me indications of the children's understanding of *representing 3-D objects in a 2-D medium.* If the children were able to generate this type of representation, I could be fairly sure that they were able to correctly "read" a perspective display of an object.

(7) **Draw What the Boy Sees:** In the pre- and post-tasks there is a picture of a little boy facing a set of three sticks of different length. The children were asked to draw what the boy sees. This task required imagining that either you are in the boys position and drawing the sticks from the "other side"; or imagining rotating the base (that the sticks were on) and drawing the result. Doing this drawing correctly implies a coordination of perspectives. This task is relevant to J3D, because it is important to be able to imagine what something will look like from another point of view. In many respects this task is similar to the two tasks above. This task, along with the water level and fishing lines task made up the Piagetian component on the previous battery.

**10.1.3. Analysis of J3D Related Pre-Post Tasks.** When I first learned 3-D computer graphics I wanted to rotate the "eye" in one of my animations in order to slowly move around and view a scene from the other side. In many computer systems it is not possible to rotate the "eye". I told my teacher I wanted to rotate the "eye" "over there". He looked at me like I was crazy and said "just rotate the whole scene. It's the same thing." This had never

occurred to me! I had thought of the scene as a stable world that I had built. I knew that I could move to another place to view the scene, but I didn't think of the possibility of moving the scene around. Then all of a sudden I realized that both ways create the same effect, and that in 3-D computer graphics both were equally possible. Part of my strategy had previously been based on experiences in the real physical world not the virtual world of J3D. As mentioned earlier, the strategy used when we want to imagine the other side of something depends on the size and nature of what we are imagining. If it is an object we often think of rotating the object, however if it is a large building we think of it from another point of view. From this experience with my teacher, I realized that the same visual effect could be achieved through different means. Understanding this "grouping" (i.e. it is logically the same to rotate an object and to change the viewpoint) made me think about space differently than I had before. This kind of grouping of transformation becomes very powerful in spatial problem-solving, and I believe, an indication of spatial ability in general.

In the simulated world of 3-D computer graphic, as in the mind, there are many occurrences like this where groupings of transformation are possible. J3D provides an environment for exploring and discovering these groupings, such as: rotation vs. changing the "eye", scaling vs. moving the "eye" or moving the object, and translation vs. moving the "eye" and/or the "coi" (center of interest). So this coordination would be the optimum understanding of transformations that I hoped to see in the children after working in J3D. In the Pilot Study, I did see this kind of understanding in the children of higher spatial ability.

The following tasks were designed to ascertain if the children had come to understand some of these groupings as they used J3D. In designing the computer related tasks I purposefully chose a single object with no other referent except for its prior position and the relationship with the screen or "picture frame". The use of one object to study transformations is useful because it can be ambiguous. The only clues are the object, it's size, orientation, and relation to the picture frame. This ambiguity is useful to the researcher. Often there can be more than one way to attain the same final

image from the previous image. Also giving the children two scenes, made it easier for them to talk about the changes.

The children were presented with two pictures side by side. These pictures were done on the computer and printed from the computer screen. The children were asked "What has changed between picture A and picture B"? The pre-task was given to the children as a group paper and pencil task (right after the lecture introducing 3-D computer graphics). At that time, I verbally asked the children to give as many answers as they could think of. In the post-task, I asked for as many possible answers in writing. The post-task was also different in that it was a one-to-one interaction. The child and I sat at a desk where s/he first wrote the answers for one question, then we would discuss the answers. I also asked the children to write down the commands they would use to create their choice of transformations on the computer. In working with the children in the post-task I would ask "Can you think of any other way you could get this picture? Which do you think is the easiest? Which one do you think I used to create this picture? The questions were used to help the children focus on which transformation seemed the easiest to complete the task in J3D. Another important aspect of these tasks is that they allowed me to get a sense of the vocabulary that the children used to talk about objects and scenes. The post-task helped me capture information on: children's understanding of transformations; the language that they used; their ability to choose the different operation; and how they complete tasks in J3D using the correct syntax and appropriate X, Y, & Z values. An analysis of the concepts of 3-D computer graphics has been discussed previously in chapter 5 (system analysis), 6 (psychological analysis), and 7 (Pilot results). The following subsections describe the different J3D-related tasks and the tasks can be seen in Figure 12.

**TASK A. Translation of a Cube - down in Y:**

The first and most essential part of this task, and all the following tasks, involves being able to look at two images, compare them, and decide what are the differences between them. The children had to figure out all the possible ways the second image could be achieved using J3D. One of the issues is to see what the children use as the stable referent. While this

particular problem was created using translation, there were also some partial solutions. For example, some children mentioned that this translation task could be solved through rotation or an eye change because they only looked at the object. They did not notice that the relationship of the object to the picture frame had also changed. One child (Eb1) only noticed the relationship with the frame and put down coi. This would have moved the cube in relation to the frame, but left the angle of viewing the cube identical to the original. Some of the children had not consciously thought about the fact that if the object moves in relation to the eye that it would look different. I was looking for a "mastery" of J3D conceptually, syntactically, and numerically. On the post-task, I thus wanted to find out what computer command the children generated. Did they achieve changes that looked very much like the one they were trying to match? (The exact answer was place cube 0 -2 0.)[†]

### TASK B. Scaling of Cylinder - Larger in X:

Many of the same processes used in translation are also used in scaling. This task allowed me to see if children were able to scale in the correct axis. When they figured out the right axis for scaling, they also had to choose number to scale by. Therefore this also allowed me to see what kind of number they chose, and whether they thought about keeping the cylinder round. I also made note of how many attempts it took to achieve a solution which satisfied them. There was only one solution for this task. Since all the children did it correctly, it is not included in the diagram in Figures 15-18. (The exact answer was: scale cylnd 2 1 1).

### TASK C. Scaling of Cylinder - Scale smaller in X, Y, & Z:

This particular task had the most possible transformation. It could have been achieved by scaling, translation, or changing the eye. This, more than any other task, indicated if the children understood the groupings of the object and view transformations. Here again, whatever the choice for

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[†] The children were told that all the picture were created with the eye at 3 3 15. This position was a little to the right, up a little and back 15 units. This is oblique enough to show the depth of the space.

Figure 12: J3D tasks for qualitative analysis.

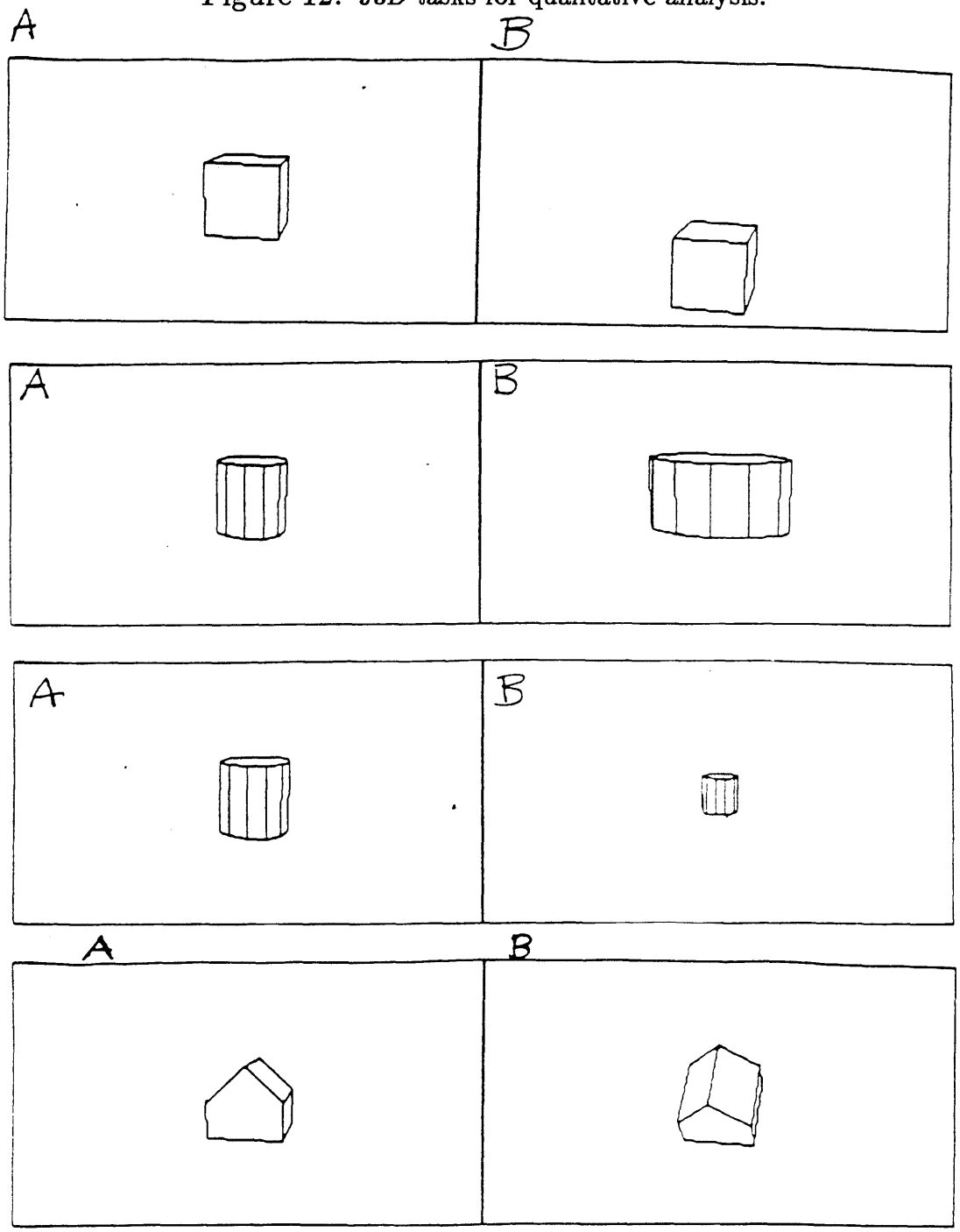
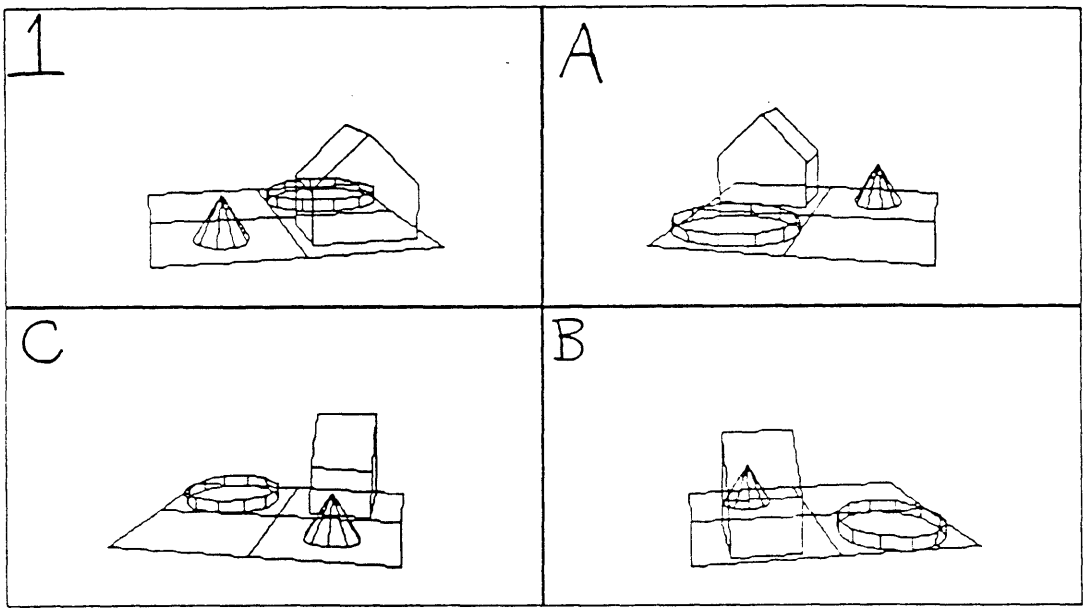
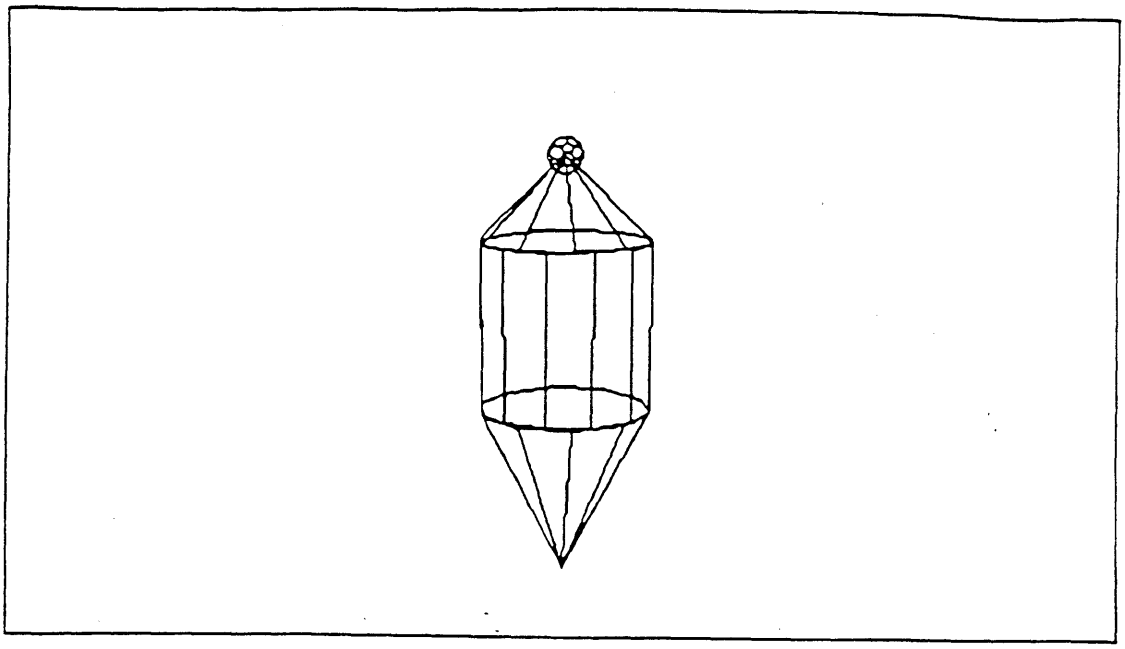


Figure 12a: J3D tasks for qualitative analysis.



the solution on the computer was, it was more important to see how much trial and error was involved in the answer. (The exact answer was: scale cylnd .5 .5 .5. and the other possible solutions were place cylnd -2 -2 -10 (compensation for perspective), or eye 7 7 25).

### TASK D. Rotation of Hut in X:

This task could be achieved either by rotating the object or changing the eye. The rotation could be considered as a local change and the eye as a global change. However, in this particular task no other referent indicates which way it was created. Here again, it was important to look at how easy it was for the children to achieve a satisfactory solution within a given strategy. For example, how many rotation commands in a row were needed to achieve the desired orientation? Was the problem in choosing the right axis or the needed angle? If they chose to change the eye, did they understand right away where they were placing the eye, and subsequently, where they would then be viewing the scene from? (The exact answer was rotate hut X 45, or eye 1 11 10).

### TASK E. Change of View of Scene:

This task, like the scaling task, was an indicator of coordination of rotation and change of perspective. It allowed me to see which children were able to understand and/or coordinate these transformation. The task used a grid divided into four squares on which several objects were placed to create a scene. A hut, a cylinder, and a cone were each placed on different squares. This task can satisfactorily be achieved through either changing the eye or a global rotation. It can also be done through place or translation, however, this was only a partial solution because the hut would also have to be rotated. This task was originally created by displaying the same scene as viewed from the front (original, +Z), side (+X), back(-Z), and the other side (-X). The easiest way to get this is through changing the eye, because it is the most global change and it requires the fewest commands in J3D. To achieve this through rotation all the objects would have to be attached or grouped and this would required many more commands to J3D. To use rotation the children had to understand what rotates, what is the referent for rotation, and the nature of local and global rotations. Since just turning the hut does not produce the desired effect, it

is considered a partial solution. The children were not asked to recreate this task on the computer. However, they were asked to predict how they would implement their solution on the computer. This sometimes may have had the affect of causing them to realize they had not thought through a possible solution far enough, or had incorrectly anticipated an outcome. Sometimes through thinking about how to implement their solution in J3D, they would switch their solution or strategy.

To summarize, the task analysis process was somewhat complicated because many of the sub-components of these tasks are interdependent. At times it was difficult to really know where the children's difficulty lay. For example, imagine a child who have a great deal of difficulty with negative numbers. This child is asked to do a translation problem, in which s/he must place an object at -2.5 .5 0. It would be difficult to determine if the child's problem was in attaining the correct translation; understanding the transformation itself; knowing the coordinate system; or mixing up the axes. It could also be a problem with positive and negative numbers, decimal numbers; or the negative numbers related to the coordinate system (direction); or a combination because of a negative decimal fraction. As mentioned previously, complex spatial tasks not only depend on the correct components, but also on an *executive function* which mobilizes and coordinates the necessary components. In other words, some children may have the necessary component processes available to perform a certain task, but they may lack "sufficient resources to do it *and* to do all the other components of the task as well" (Cole, 1981).

### 10.2. Results: Experimental Children's Spatial and J3D Tasks

Due to the enormous amount of data I created summary sheets (examples in Appendices D and E), which helped me analyze how each of the children did on the different tests and tasks. These summary sheets do not show at a glance eventual changes in the quantitative data, or any patterns of change that could provide insights into consistency across the various spatial tasks to J3D. Looking at each task for each of the children (as in the above task analysis) provides rich information on the thinking of the individual children, yet it does not provide an easily graspable profile of each of the children. It is in patterns of change that I am currently most interested in.

As a result, I decided to present the qualitative spatial tasks in a more visual and accessible way, using my hypothesis of how spatial components get organized. As most research in Euclidean spatial concepts show, the mastery of measurement in 1-D precedes mastery of measurement in 2-D. Also measurement itself is very complex depending on the task and the context. Each 1-D measurement tasks, provided me with insights into a particular sub process of measurement. For example, the temperature task provided insight into the children's mastery of the unit of measure, transfer, and negative numbers. While the snake task provided insight into conservation of length, negative numbers, fractions, and transfer. These tasks provide a more fine-grained understanding of the children's knowledge of measurement in 1-D.

In general, my hypothesis was that if children were able to perform Euclidean and Projective spatial tasks, they would also be able to comprehend the reciprocal transformations on the computer tasks. I also predicted that those children who were able to achieve most of the Euclidean and Projective tasks would more likely be able to coordinate the two types of spatial concepts. I thought that if the coordinated Euclidean tasks (#5-7) and coordinated Projective task (#6 and 7) were coordinated, then the children would also be able to coordinate the object and view transformation. This idea becomes clearer through the layout of the data and the patterns that they revealed.

**10.2.1. Spatial Framework - Euclidean and Projective Tasks.** Figures 13 and 14 are diagrams of the spatial "building blocks", or components necessary to construct spatial understanding. Each child's "spatial framework" is represented as a pyramid of such spatial components, or building blocks. The left side of each pyramid represents the Euclidean spatial components and the right side represents the Projective spatial components. The pyramids on the left show which spatial components were used prior to the intervention, and on the right we find which were used after working with J3D.

The numbers in each building block correspond with the tasks (analyzed above) on Euclidean and Projective space). The blocks are organized hierarchically. The lower levels of the pyramids are component processes that

need to be combined to allow more complex spatial tasks, represented higher up in the pyramid. For example, on the Euclidean side of the pyramid, the lowest level (blocks 1-3) represent 1-D measurement tasks, the second level (blocks 4-5) represent measurement in 2-D, and the upper level (blocks 6 and 7) represents tasks that require a coordination and generation of measurement in 2-D. In other words, these pyramids are organized from simpler tasks at the bottom to more complex tasks on the top.

The shaded areas represent that a task has been accomplished, and thus understood. Notice that there are more shaded components near the base of the pyramids and less near the top. When the shading only covers a portion of the block, it means that children completed only part of the components of that task. From looking at the shaded blocks it is possible to see what spatial components the children have understood in each task. Thinking about it another way, one could say that the diagram exposes what I call "holes" in the children's spatial thinking. These building blocks allow us to grasp each childs spatial framework, and to understand what type of spatial problems they could or could not solve. This framework provides a reference for understanding what children brought to the learning experience and how that experience modified their spatial reasoning. It could also provide a way to predict where children might have more difficulties with J3D.

The diagrams are organized by gender, with Figure 13 for girls and Figure 14 for boys. On each page, the children are ordered from top to bottom in their ranking by the pre-test scores (seven tests used to select them). Under the children's names are their pre- and post-test scores on the FRCP battery. The children with highest spatial scores are higher up on the page and those with lower scores are near the bottom. The FRCP is used here as a reference for the children's scores on the standard spatial tests because it includes all three types of spatial abilities in the one composite score. There are three main patterns that become apparent from the children's "spatial framework".

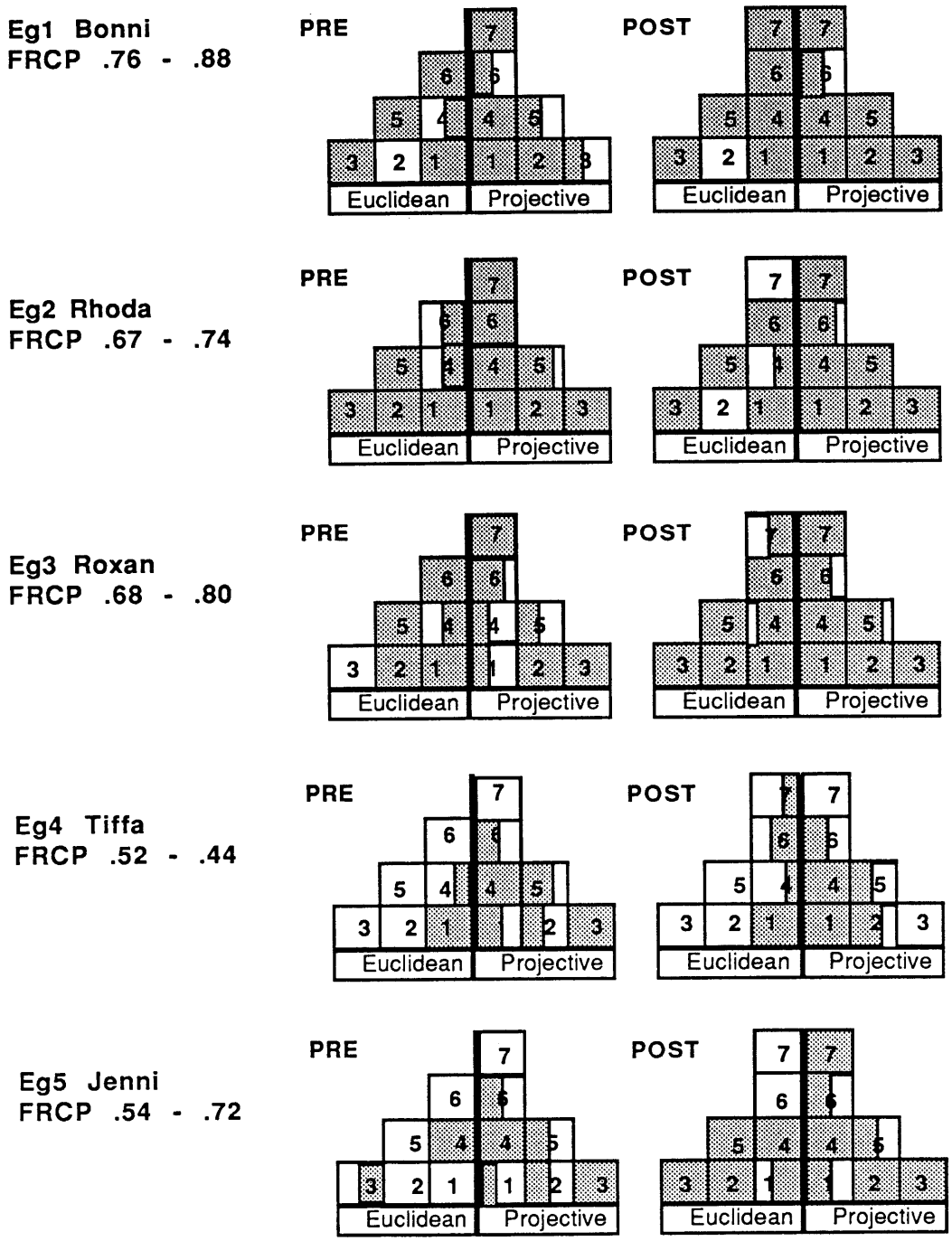
A first visible patterns has to do with quantity of area shaded in the pyramids. The amount of shaded areas coincides with the children's ranking on the standard spatial tests. There are more shaded areas in the pyramids near the top of the page and more white space for those near the bottom of

the page. This means that Euclidean and Projective spatial concepts are in some way related to performance on the FRCP battery which included the abilities of mental rotation, spatial visualization and spatial perception. This is not so surprising because, as mentioned in chapter 3, De Lisi (1976, 1981) found that children's performance on coordination of perspective and the water level task was related to the children's performance on rotation tasks. Taking this one step further, the authors found that children's understanding of transformations correspond to their understanding of the references axes, distance relations, and measurement. Therefore, the children's performance on these standard spatial tests should match their understanding of both Euclidean (measurement and reference frame tasks) and Projective (perspective tasks) components. This is exactly what is visible by the ranking and percent of the shaded areas in these pyramids.

A second pattern is the aggregate of shaded areas. Notice that we find fewer "holes" in the post-tasks. Again, as in the quantitative analysis, we can see an improvement in every child in the post-tasks. As a way of comparing the information from this spatial framework with the results on the FRCP, this representation was converted to a percentage score. Each square was considered one unit and the amount shaded was divided by the total number of tasks. The resulting scores and the scores from the other tests and batteries are in Table 9. By just examining the combined Euclidean and Projective (EP) task scores and the FRCP scores, it is possible to see a correspondence between the two batteries. In two cases, the scores between the two batteries were very close -- and even identical for two of highest spatial children, Eg1.Bonnie and Eb1.David. The scores for all but one child[†] are the same or higher on the post score for the EP battery than the FRCP. However they are all within a close range of each other. There was a significant improvement for all the children from the pre to the post on the Euclidean tasks ($A=.196$, $p<.005$), on the Projective tasks ($A=.225$, $p<.025$), and on the EP or combined Euclidean and Projective tasks ($A=.136$, $p<.005$). If we look

[†] Again Tiffa's scores went down, however this time it was only 1 percent.

Figure 13: Spatial framework of the girls.



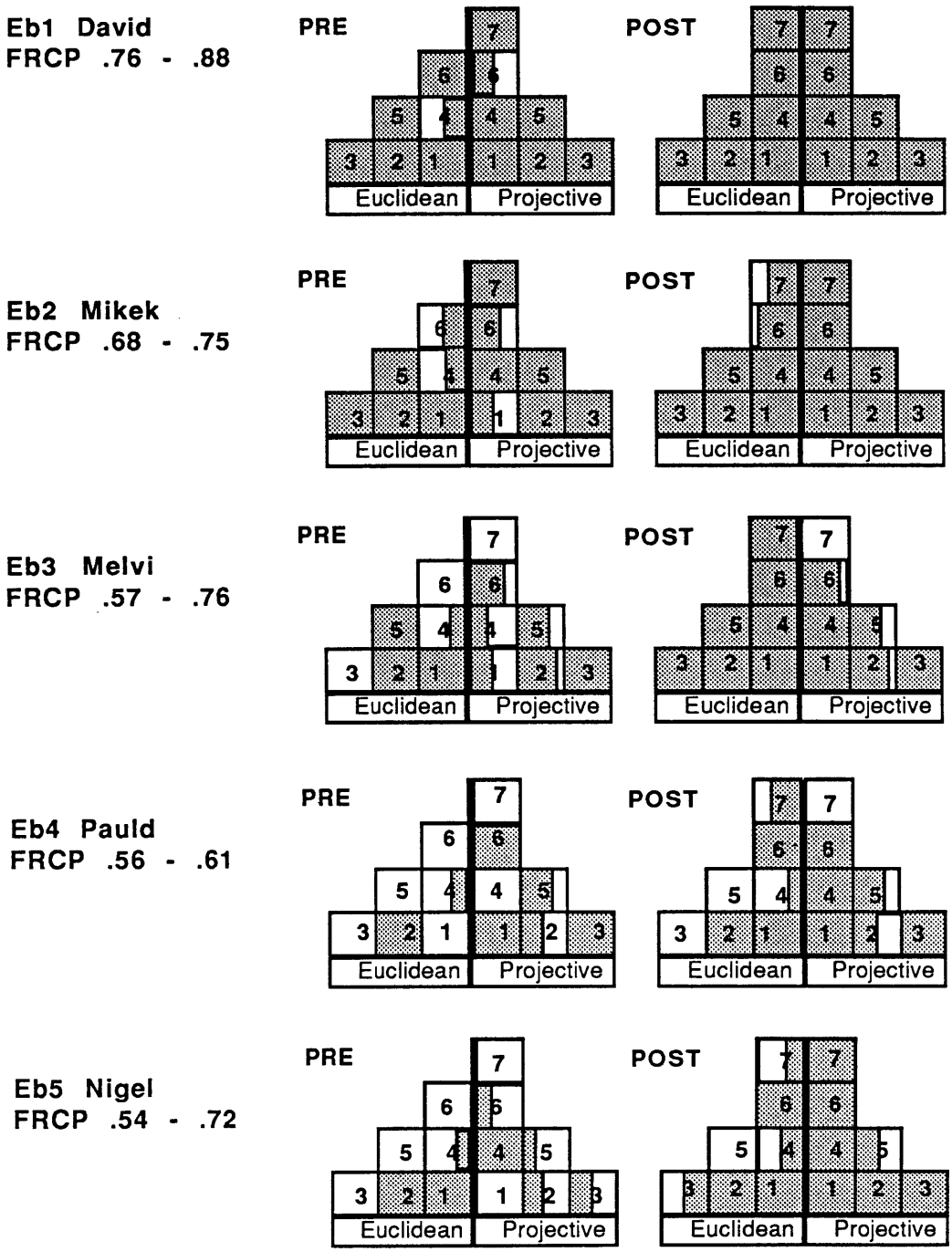

**EUCLIDEAN TASKS**

1. Recog point 1-D
2. Temperature 1-D
3. Snake 1-D
4. Horizontal & Vertical 2-D
5. Point in plane 2-D
6. Plot point in 2-D
7. Construct coord. system 2-D

**PROJECTIVE TASKS**

1. Draw/write square & cube
2. Mouse maze
3. Orthographic (object)
4. Orthographic (scene)
5. Draw 3-D objects
6. Draw orthographic views
7. Perspective task

Figure 14: Spatial framework of the boys.




**EUCLIDEAN TASKS**

1. Recog point 1-D
2. Temperature 1-D
3. Snake 1-D
4. Horizontal & Vertical 2-D
5. Point in plane 2-D
6. Plot point in 2-D
7. Construct coord. system 2-D

**PROJECTIVE TASKS**

1. Draw/write square & cube
2. Mouse maze
3. Orthographic (object)
4. Orthographic (scene)
5. Draw 3-D objects
6. Draw orthographic views
7. Perspective task

just at the boys, we see a significant improvement on the Euclidean tasks ($A=.264$, $p<.025$), on the Projective tasks ($A=.335$, $p<.05$), and on the EP ($A=.241$, $p<.01$). The girls only show significant improvement on the EP ($A=.276$, $p<.025$).

A third pattern is visible in the symmetry between the Euclidean side and Projective side of the pyramid. Piaget has shown that Euclidean and Projective space both form comprehensive systems. More important is that using a stable and external reference frame underlies both systems. Since my intervention focused on the reference frame, it is not surprising that we see an improvement in both systems, and that they have stayed loosely balanced from right to left. As Piaget believed, and as this graph shows, both system develop simultaneously. As mentioned earlier, J3D provided an environment where Euclidean and Projective space formed a cohesive whole and where children explored them both.

As Just and Carpenter have pointed out, it is difficult to keep more than one cognitive coordinate system active at a time. It is also difficult, especially for children, to keep more than one task component active at one time. The complexity of some tasks have caused an even greater mental "load" for the children because of the complexity of the coordinate system, the complexity of reference frames, and the complexity of the number of components. The mental "load" put on the children by the task components may affect the task performance. As more of the components become easier to handle and more automatic, less mental resources are needed to handle that component. Recall that Eb1 David scored highest on the Rotations test, but reported that he didn't know how he did them. David could do these tasks without consciously thinking about what he was doing. On the other hand, this may be why in complex learning situations the learning curve can take such a "u-shape" as seen with Eg4 Tiffa. When many of the components are not yet coordinated there could be a regression in performance. One of the reason could be that the concepts are still tenuous, and when they get "stirred up" it provokes a mental load in juggling the concepts. As Cole mentioned some children could possess the necessary processes but not have sufficient resources to do the task and all the other components parts of the task at the

Table 9: Mean and significant improvement scores for tasks.

| MEAN SCORES | FRC | FRCP | EP | TR | Eucl | Proj | Piag | Flg | Rot | Cube |
|---|---|---|---|---|---|---|---|---|---|---|
| PRE Eb1.David | .87 | .82 | .93 | .63 | .92 | .93 | .67 | 1.00 | .69 | .91 |
| POST Eb1.David | .87 | .91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .74 | .88 |
| PRE Eb2.Mikek | .68 | .68 | .85 | .50 | .83 | .86 | .67 | .88 | .57 | .59 |
| POST Eb2.Mikek | .73 | .75 | .97 | 1.00 | .93 | 1.00 | .83 | .79 | .64 | .75 |
| PRE Eb3.Melvi | .70 | .57 | .57 | .38 | .56 | .57 | .17 | .88 | .71 | .50 |
| POST Eb3.Melvi | .72 | .76 | .88 | 1.00 | 1.00 | .76 | .85 | .96 | .74 | .47 |
| PRE Eb4.Pauld | .63 | .56 | .42 | .50 | .22 | .61 | .33 | .85 | .57 | .47 |
| POST Eb4.Pauld | .70 | .61 | .66 | .81 | .57 | .75 | .33 | .92 | .57 | .62 |
| PRE Eb5.Nigel | .67 | .54 | .36 | .44 | .36 | .36 | .17 | .94 | .50 | .56 |
| POST Eb5.Nigel | .77 | .72 | .79 | .75 | .62 | .95 | .58 | .96 | .60 | .75 |
| PRE Eg1.Bonni | .79 | .76 | .77 | .63 | .78 | .76 | .67 | .94 | .62 | .81 |
| POST Eg1.Bonni | .83 | .88 | .89 | .88 | .86 | .93 | 1.00 | .98 | .55 | .97 |
| PRE Eg2.Rhoda | .69 | .67 | .90 | .50 | .83 | .96 | .58 | .92 | .69 | .47 |
| POST Eg2.Rhoda | .76 | .74 | .80 | .75 | .69 | .96 | .67 | .94 | .81 | .53 |
| PRE Eg3.Roxan | .63 | .68 | .79 | .50 | .83 | .75 | .83 | .77 | .60 | .53 |
| POST Eg3.Roxan | .79 | .80 | .92 | .63 | .91 | .93 | .83 | 1.00 | .64 | .72 |
| PRE Eg4.Tiffa | .64 | .52 | .45 | .38 | .22 | .68 | .17 | .65 | .76 | .50 |
| POST Eg4.Tiffa | .52 | .44 | .44 | .63 | .33 | .55 | .17 | .69 | .50 | .38 |
| PRE Eg5.Jenni | .46 | .56 | .56 | .38 | .25 | .60 | .86 | .44 | .64 | .31 |
| POST Eg5.Jenni | .74 | .75 | .75 | .88 | .68 | .85 | 1.00 | .69 | 1.00 | .53 |

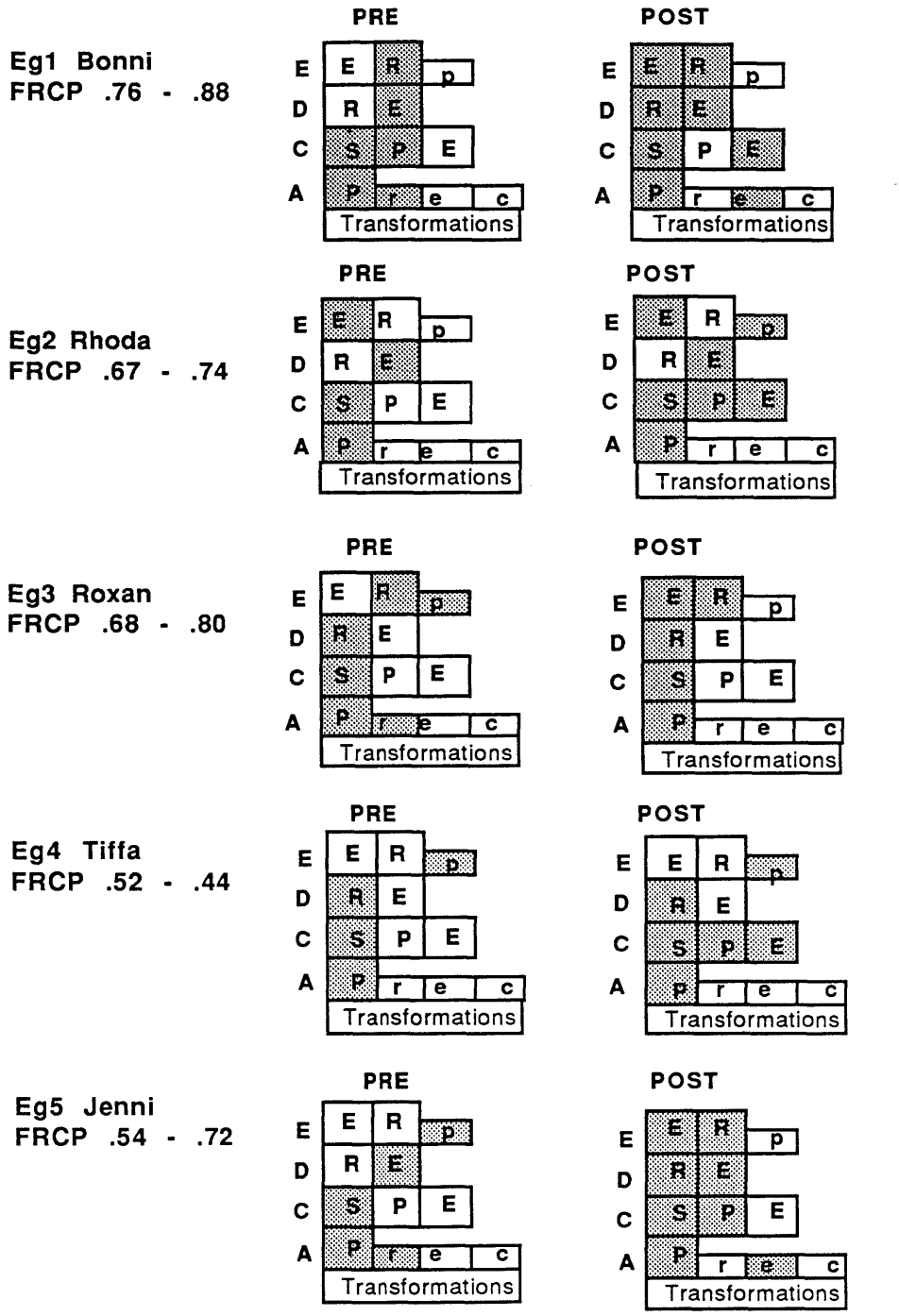
PRE-POST SCORE SIGNIFICANCE by SANDLER'S A TEST (blanks=NS)

| Test -> | FRC | FRCP | EP | TR | Eucl | Proj | Piag | Flag | Rot | Cube |
|---|---|---|---|---|---|---|---|---|---|---|
| Experiment | .005 | .0005 | .005 | .0005 | .005 | .025 | .01 | .05 | | .025 |
| ExGirls | .025 | | .025 | .01 | | | | .05 | | |
| ExBoys | .025 | .01 | .01 | .005 | .025 | .05 | .025 | | .025 | |

Experimental Group N = 10 [ Boys = 5 Girls = 5 ]

FRC = Flags, Rotation, and Cubes tests
FRCP = Flags, Rotation, Cubes, and Piagetian tests
Piag = Piagetian horizontal, vertical, perspective tasks
EP = Euclidean and Projective spatial tasks
Eucl = Euclidean spatial tasks
Proj = Projective spatial tasks
TR = J3D Transformation Tasks

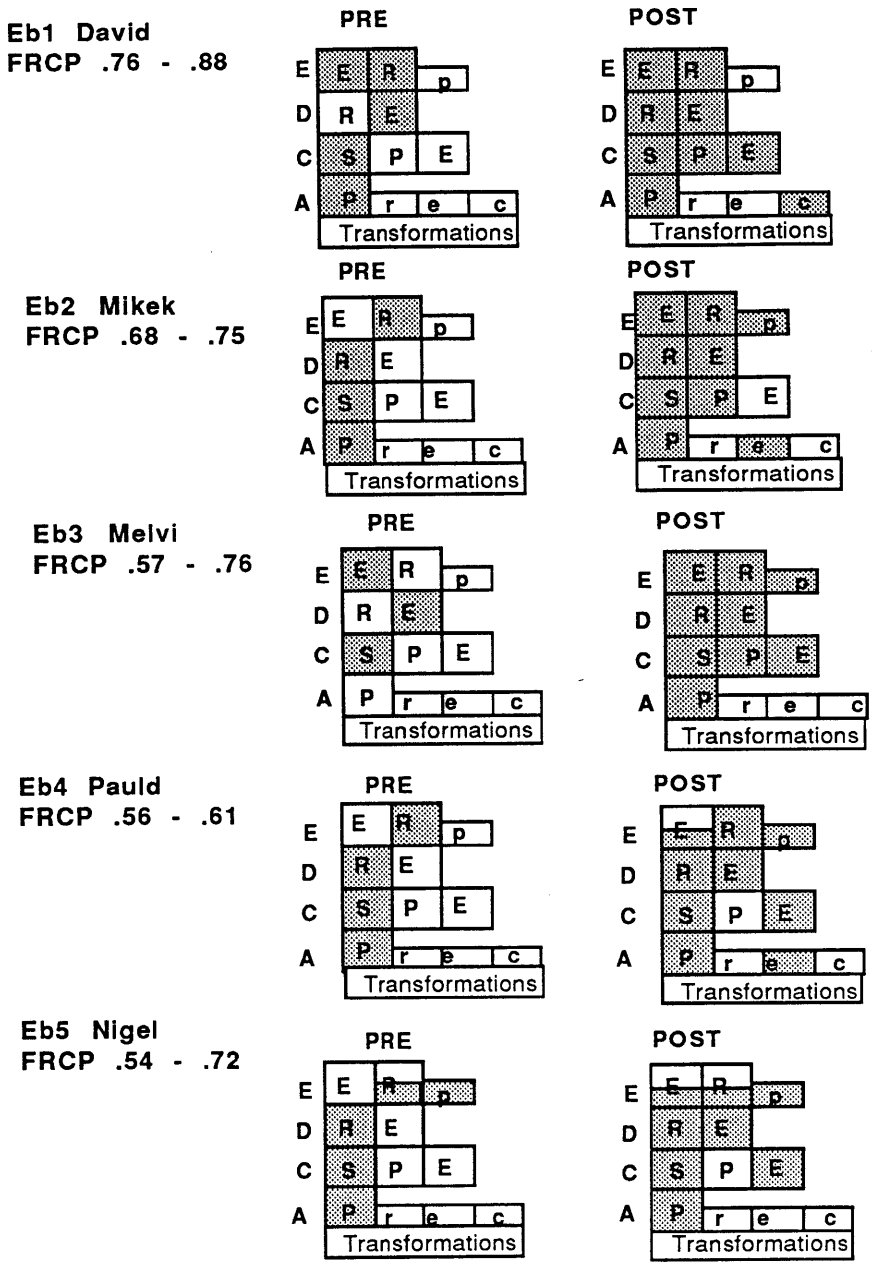
Figure 15: Transformational framework of the girls.




A. Place CUbe -Y
C. Scale cylinder smaller
D. Rotate hut in X
E. Change eye for scene
(B. Scale cylinder large X
not in graph, all correct

P = Place (Translation)
S = Scale
R = Rotation *
E = Eye *
C = Coi *
* = Partial solution
(p, r, e, & c)

Figure 16: Transformational framework of the boys.



same time. Eg4 Tiffa's scores, for example (Table 3) indicate a regression on both accuracy and speed of the spatial tests. This same tendency appeared in her spatial framework, in many of her scores (Table 9), as well as in her comfort and skills in using J3D.

I was surprise to find systematic improvement in both high and low spatial children: The intervention seems to actually have had a clear impact on all the children who took part. All the children were able to complete the computer tasks and projects. Some with more help than others, all could explore the spatial concepts that are so much a part of every aspect of working in J3D, and all improved in related off-computer spatial tasks, after using the system.

**10.2.2. Transformation Framework - J3D Related Tasks.** The "spatial framework" diagram gives us a sense of what spatial processes children used to solve various tasks. Using the "spatial framework" as a reference we can now look for patterns in children's ability to recognize and create the different possible transformation on the computer related tasks (A-E). Using the idea of the pyramid structure, I organized the pre- and post-J3D tasks in a hierarchy of what I believed to be the complexity of the task. This new "transformational framework" can be seen in Figures 15 (boys) and 16 (girls). The tasks were ordered from (A) **P**lace, (B) Scale larger in 1 axis, (C) Scale in all axes, (D) **R**otation, and (E) **E**ye change. The graph shows the tasks ordered from A (**P**lace) at the bottom to E (**E**ye) on the top. B was eliminated from the graph because there was only one possible solution and all the children chose the correct transformation. The column of blocks on the left contain the transformations (P,S,R,E) used to create and print these tasks. The square blocks extending to the right of the left column are alternate solutions. These alternate solutions were discussed in the previous task analysis and are labeled in the graph. The half blocks represent the partial solutions possible for each task. These partial solution were not included in the scoring process. The

shaded blocks represent an understanding[†] of the transformation, not necessarily ease in doing the exercises in J3D.

The children's performance improved from the pre- to the post-J3D tasks. After the intervention the children were able to imagine more transformations than before working in J3D. The same method as in the previous section was used to calculate a numerical value for this structure. These scores are listed in Table 9 under TR for TRansformations. As in the Euclidean and Projective tasks there was significant improvement for all the children ($A=.116$, $p<.0005$), the boys ($A=.216$, $p<.005$), and the girls ($A=.239$, $p<.01$).

Only one child (Eb1 David) coordinated the view and rotation problem (E) prior to the intervention. However, if you look at the "spatial framework" of his pre-task, this is not surprising. He was the only child who mastered Euclidean and Projective processes prior to the study. After working with J3D, eight out of the ten children could solve task E either through change of view or rotation. Six of these children understood completely how to do these two transformations in J3D.

It is important to mention that all children were able to use J3D and with varying degrees of prompting to solve all post-J3D tasks (A-E). Figures 17 and 18 show that it is possible to use J3D (and the coordinate system embedded in it) without being able to coordinate measurement in 2-D. We can actually see this in the graphs of several children, such as Rhoda, Tiffa, Jenni, Pauld, and Nigel.

One of the reasons might be that children can extract correct coordinate points without necessarily being able to plot them. For example, several children, solved Euclidean Tasks 4, 5, or 6 before being able to solve Euclidean task 7. In order to place a point it is important to coordinate the two axes, whereas to extract a point, thinking about one axis at a time is enough. This same principle holds true for J3D. Since the children could think about one parameter in one axis at a time they were able to use the system. Children

[†] Answers were from both written and transcripts of oral responses.

like Tiffa, Pauld, Nigel, and Jenni, changed one parameter at a time, while David, Melvi, Bonni, and Mikek often changed more than one parameter at a time. Below is a portion of a history file of the commands Jenni used to solve a task with two cubes off-set by their width in X and depth and in Z (with their edges just touching). She had to place one cube off set from the other in two dimensions. Notice that she started by changing one axis at a time. She first changed the X axis. When she located it where she wanted, she started changing the Z axis. Then she went on to the Y axis.

| | |
|---|---|
| call cube cc | *Bring in two objects* |
| cal cube c | |
| dis | |
| pla c 3 0 0 | *Changes X axis* |
| dis | |
| pla c -1.5 0 0 | *Satisfied with X value* |
| dis | |
| pla c -1.5 0 2 | *Starts on the Z value* |
| dis | |
| pla c -1.5 0 4 | |
| dis | |
| pla c -1.5 0 3 | |
| dis | |
| pla c -1.5 0 3.5 | *Satisfied with Z value* |
| dis | |
| pla c -1.5 -1 3.5 | *Starts on the Y value* |
| dis | |
| pla c -1.5 .25 3.5 | *Satisfied with Y value* |
| dis | |

David in the same task, immediately changes of two parameters at a time. He also keeps the dimensions of the cube in mind when dealing with the off set.

| | |
|---|---|
| call cube c | *Bring in two objects* |
| call cube c1 | |
| place c1 -1 0 1 | *Moved cube in X and Z simultaneously* |
| dis 4 | |

place c1 -2 0 2 *Adjusted cube in X and Z simultaneously*
dis 4

These examples illustrate the difference in the nature of the trial and error in different children. Some children try one axis at a time to determine the correct axis. Jenni is "lowest" spatially. She adjusts by axis in order to achieve the picture. Although she had not yet coordinated measurement in 2-D, she could use J3D by just keeping one axis in mind at a time. For each command she merely had to focus on one dimension at a time and remember what each of the parameter changes (X, Y, Z) meant for the place, scale, or eye commands.

Along with Piaget and Inhelder (1971), Dean (1976), De Lisi (1976), I believe that there is a link between anticipatory imagery and spatial operations especially in Euclidean-geometry. I also think that if children are able to coordinate Euclidean and Projective space, they could also coordinate the reciprocal transformations. For example, coordinating object transformations (rotation) and eye transformations (perspective) would show a more sophisticated understanding of these spatial processes. The relations between Euclidean and Projective space and transformations (imagery) in J3D become clearer through Figures 17 and 18. In these figures, the "spatial framework" and the "transformational framework" are presented side-by-side to allow us to look for these links.

As it turns out, knowledge and mastery of Euclidean tasks helped the children in working with the commands in J3D, especially in coming up with the correct values for the X, Y, Z parameters. Coordination of Euclidean concepts alone, though, did not appear to directly influence the degree of coordination in the transformations. As a matter of fact, in Figures 17 and 18 we can see indications that the coordination of Projective space (Projective tasks # 6 & #7) may correspond more with the coordination of transformations (J3D task E) than it does with coordination of Euclidean space (Euclidean tasks #6 & #7). Moreover, the EP score (combined Euclidean and Projective task scores) rank in a very similar order to the ranking of the FRCP scores (Table 9). This finding leads me to believe that Euclidean concepts and the reference frame underlie spatial thinking in general, while Projective concepts

Figure 17: Spatial & transformational framework of the girls.

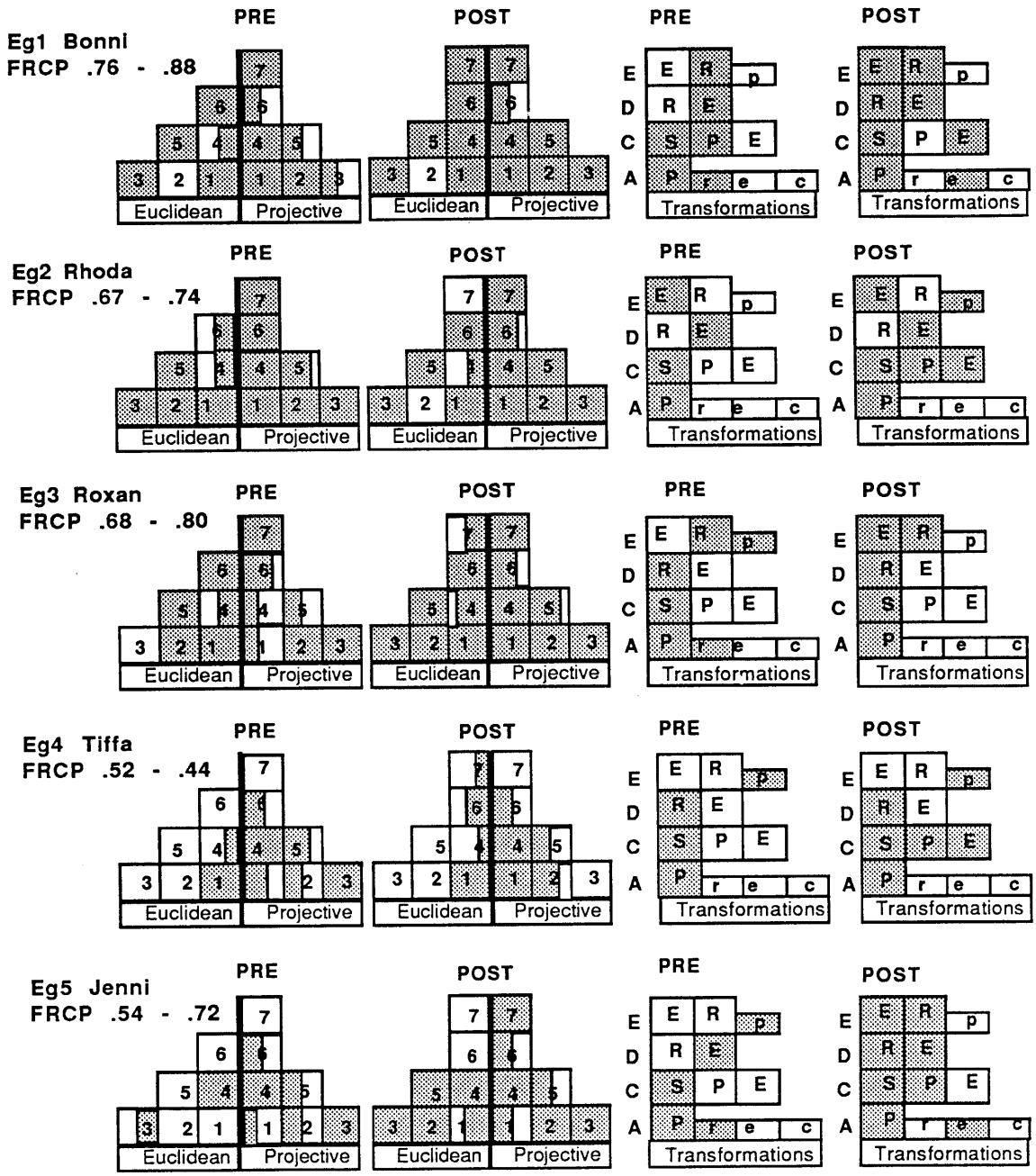
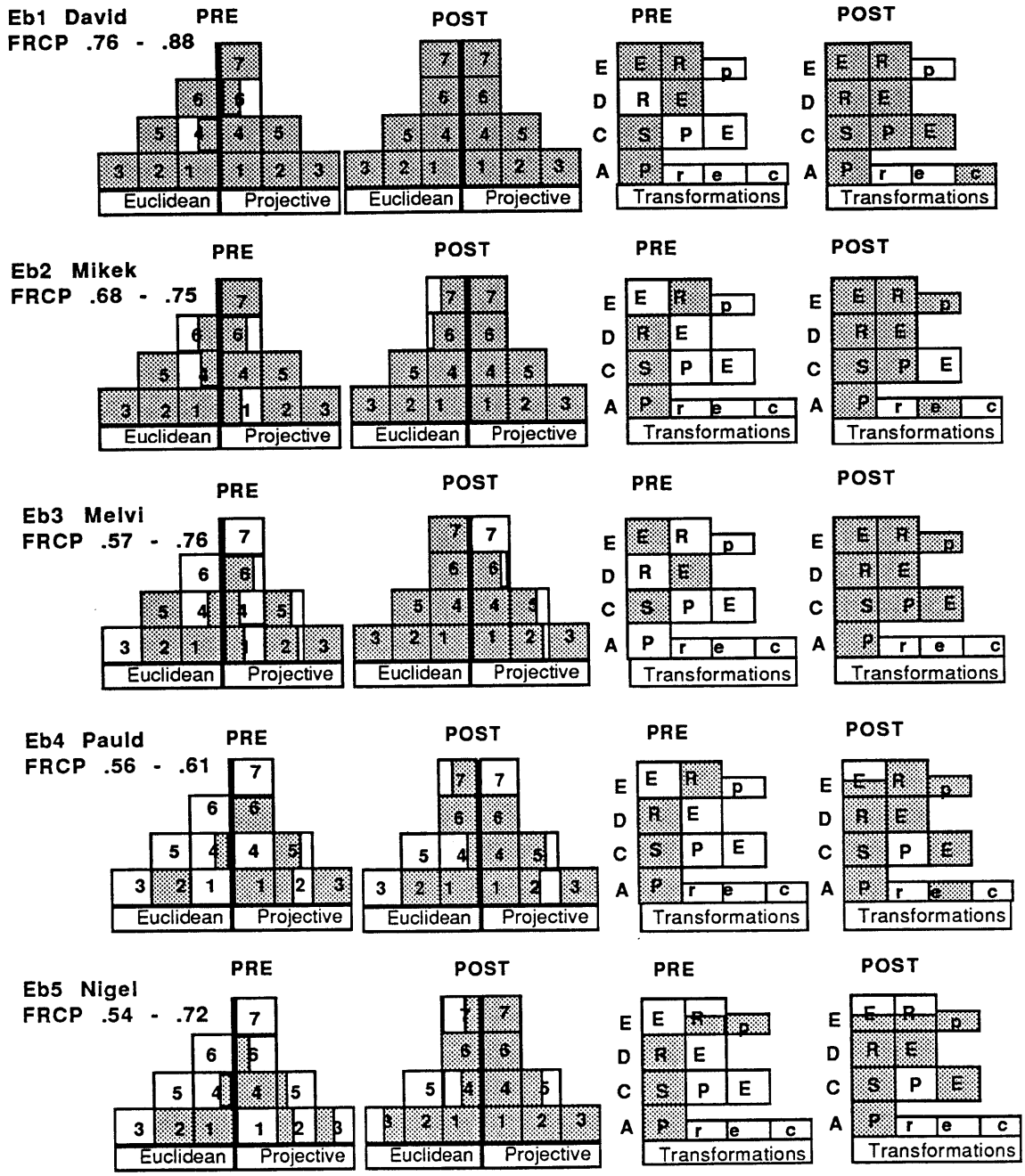


Figure 18: Spatial & transformational framework of the boys.



are more closely related to anticipatory imagery or transformations. It is the combination and integration of these concepts, that I believe we will actually indicate the spatial thinking level of the children.

One thing looked at (since it relates to the Projective tasks), is how the children think about the objects that they are viewing on the computer screen. I expected they would gain understanding of 2-D representation of 3-D objects from working in J3D. I also expected that from working in J3D the children could more easily imagine that these 2-D images were just a "window" into a virtual 3-D world. The children did improve in thinking about these 2-D images. There is a subtle change that can be seen in their reports on the strategies they used in the standard spatial tests, and in their ability to draw 3-D objects and to draw different views of an object. This ability to think about a 3-D object (and imagine it moving, rotating, and stretching in space) goes together with the underlying ability to think of these objects in 3-Dimensions.

At the beginning of the study children tended to think more about representing 3-D objects in 2-D than about the object itself in 3-D. This seemed to change through the study. In the pre-tests, the children seemed to think of these 3-D representations (on paper or on the computer) in terms of their position in 2-D space or in terms of how they would draw (represent) them in 2-D rather than as virtual 3-D objects. We saw an example of this in Bonni's strategy report on the Rotations test. She talked about the shape as being able to come off the page. This was a way to talk about a 2-D representation being expanded into 3-D with full movement in space. Psychologically it was no longer attached to the page.

Thinking about these virtual objects as actually 3-D objects becomes one of the problems for the lower spatial children in the J3D group. These children didn't consistently think of the virtual objects as 3-D. Often, when looking at the screen and seeing the object represented in 2-D, these children would start trying to adjusting the image by choosing the 2-D screen or picture as their reference not what it was the representation of in 3-D. Two interesting incidents illustrate this point. In the example of Jenni's and David's commands above, we can see how they differ in their thinking about

space. There was a very different way of thinking about space. For example, when David changed his command from (-1 0 1) to (-2 0 2), I believe he was thinking about the cube as a 3-D object. His adjustment shows that he considered the object in 3-D and changed both the X and Z (or width and depth) of the object at one time. Jenni, on the other hand, was making her adjustments and fine tuning in the picture so that it *looked* like it was correct. She didn't think of the object as 3-D, but as 2-D. The difference in what it represented (3-D) and how it looked (2-d) was something she often confused when working at the computer. Jenni thought about it in this way even though the changes she made where in 3 axes. Notice that she was able to understand and imagine all the possible transformation for creating a particular image, but when she started working in J3D her focus shifted to the 2-D screen, not the virtual 3-D world that the screen was a "window" into.

Another interesting incident occurred with Roxan that relates to this same issue of 2-D versus 3-D. In her final project Roxan was creating a scene using many of the animals (alligator, horse, lion, dog, and rhino) that I created. She was building a picture with a mountain, a tree, and two of each animal, one larger for the parent and one smaller for a baby. Some animals appeared close to the viewer and some farther away. Everything looked really great and there was a lot of depth in the picture. I was asking her a question about something in the scene and asked her to move the eye to the side (15 0 0). When she displayed the scene from that view everything was lined up or in the same depth in Z. She had used a combination of scale to make the animals smaller as if they were further away and placed objects only in X or Y, never Z. She had even compensated for the perspective in placing things higher in Y as they moved farther away. It was like using 2-D painting conventions or the principle of perspective to represent depth. J3D would have done all this for her if she had moved things in Z either closer or farther away from the viewer. I asked her about this and she said that she did not think about doing that. This incident had a strong affect on her thinking about space. Later in an interview she mentioned that she had learned "that if you moved the shapes back they got smaller and I did the whole project just doing the scale and place." Later in the interview when asked what else she had

learned from this project she said "to use objects as other things instead of just for what they are." When asked to explain what she meant, she said "This line here is a circle. Like you flipped it over and it looks like a line. So it is now just a surface." This shows a new freedom for her in being able to think about things from different views and different orientations and as a result use them to represent all together different things. This shows she was thinking about these as virtual objects in 3-D.

This increased facility in representing objects also goes along with the increase facility in imagining what they look like from different views and being able to transform them. I had hoped that after the children explored and used the system by transforming objects and changing the view that they would gain a deeper understanding of all the possible changes in the object and in the view. I believed that after this experience they would be able to make better sense of building invariants of both representations and reciprocal transformations. Again the example I use is the rotation of an object verses seeing it from a different point of view. What is so important to remember is that if you change your point of view, it is not the same, psychologically, as if you maintain the same position and change the object. These two things have to be coordinated and considered the same at the end stage -- coordination of both the rotation problem and the perspective problem. We can see patterns that point to these links in Figures 17 and 18 and in the incidents that I have related. J3D provided the children the opportunity to coordinate these aspects.

The next chapter we will look more at individual children through a few selected tasks and learning experience.

# CHAPTER 11

# CHILDREN LEARNING WITH J3D

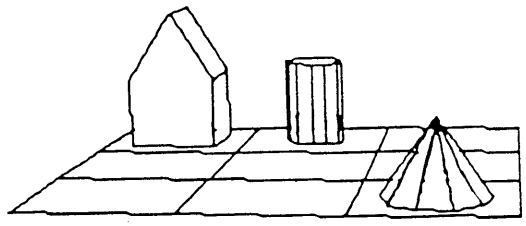
## 11. Task and Data Analysis: Individual Experimental Children

Chapter 9 presented the quantitative analysis comparing the performance of the control and experimental children on various standard spatial tests. The analysis in chapter 10 provided us with a more qualitative profile of each of the experimental children's spatial framework. This framework also allowed us to compare the performance of the experimental children on Euclidean, Projective, and Transformational tasks. We have slowly moved from looking for patterns of similarities or changes among all the children towards looking at individual children. If you refer back to Figure 9, you can see that we have slowly moved towards the center of the diagram and have progressed to more *qualitative concerns about the individual child*: what each child brought to the experiment, how they learned about J3D, how they used it, and how this affected their spatial thinking.

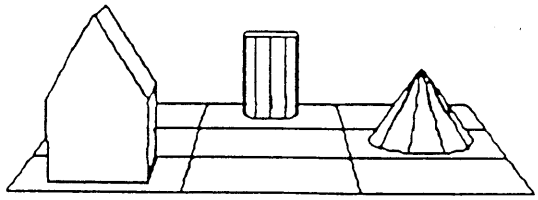
The first section of chapter 11 (11.1), will briefly look at children's descriptions of spatial relations. This task informs us about the every-day language used by children to organize space. It also allows us to determine eventual changes in either language or spatial organization as a result of working with J3D and with me, the researcher. In 11.2 we will look at three children's performances on two tasks: the construction of the coordinate system and the coordination of transformations. Section 11.3 will follow two children through part of the process of learning J3D and look at how they learned in different ways.

Figure 19: Pre-post description of scene task.

4. Your mission is to help you friend duplicate the scene below. In a short letter to your friend, tell them about the little scene well enough for them to recreate it when they get your letter. You can only help your friend by using words and not drawings.



4. Your mission is to help you friend duplicate the scene below. In a short letter to your friend, tell them about the little scene well enough for them to recreate it when they get your letter. You can only help your friend by using words and not drawings.



## 11.1. Verbal Description of Spatial Layout Task

This task was designed to provide some insights into the more natural non-metric language used to talk about spatial relations. In both the pre- and post-tasks I asked the children to write a description of a spatial layout for other children, in such a way that these other children could recreate the same image or scene from this description. This task is good to analyze how the children described spatial relations, and what referents they chose to describe these relations. The rationale for developing this task was based on Olson's (1983) views of the role of language in spatial cognition. In other words, I hoped that the children's description of space will reveal changes in their spatial thinking, or in the way they organize space.

This particular task was especially difficult because horizontal relations of non-canonical objects are the most difficult to articulate. A scene was designed using non-canonical objects (cone and cylinder) for horizontal and depth relations. The only exception was a hut, who's shape was different in depth than width. This object was more salient because it was "taller" than the other objects, and for this reason, it was often used as a referent in the solution of the task. The other non-canonical object was the grid on which everything was sitting. The grid was symmetrical horizontally and in depth. It is a neutral object, and can be used as a referent, with a numerical or a coordinate system. Figure 19 presents the question and the image that were presented to the children in the pre- and post-tasks.

In their descriptions in general, children make some things explicit and leave others implicit. Therefore it becomes important to look at the reasons why something might not be made explicit or why it may be left out. For example, it may be because it is not understood or else, because it is understood so well that there is no need to make it explicit.

**11.1.1. Task Analysis of Verbal Description of Spatial Layout.** Below are what I believe to be the four major parts of the task, and the possible strategies needed to solve each part. As is often the case in spatial cognition, it is difficult to separate out the parts of this task because they often overlap.

(1) **Description of Objects:** Addressing the objects or shapes either through their geometric *name* (e.g. cone, hut, cylinder, grid) or through *describing* a likeness (e.g. "tin can type of thing") are the two main approaches to specifying the objects.

(2) **Setting Description:** The setting is either implicitly or explicitly described. Descriptions of the setting seems to fall into three types of arrangements; sequential or relational construction (procedural), formal description, or systematic numbering.

(3) **Description of Relations Between Objects or Location of Objects:** Various systems could be used to represent relations between objects. We wanted to see which relations the children specified between objects and how they organized them. One method of organizing the parts was to come up with a system using a sequence, such as rows and columns of the base grid (4 squares in the pre-test, 9 squares in the post-test). Another method combined the use of sequence and system of referents. It is not easy to describe relations between objects set up in 3-D using only one referent. The spatial information from one reference in this context will rely on an anchor point to set up relations. One problem is that the children often leave the anchor point out of the description. To avoid ambiguity, it is almost necessary to use two referents to locate an object, even in two dimensions. The most global referent is often the most stable. Yet in this particular task finding the global referent have requires that the children specify the referents in a more formal way. Some children used two or more referents in describing relations between objects. They described relation between two objects, relations between the objects and the paper they were on, relations between the observer and objects, relations to the grid or base, relations derived from coordinate location, and relations derived from systematic numbering. It is possible to employ more than one type of relation. Since the children were presented with a 2-D drawing of a 3-D space on pre- and post-tasks it would be interesting to see if children talk about the scene as if it were a 2-D drawing or a 3-D spatial representation as Bonni did on her Rotations test.

(4) **Method of Description:** I analyzed the consistency of children's descriptions of relations and choice of referents. Could they be reconstructed by someone else from the description? Are there any obvious changes from the pre to the post-test description? These questions become important in analyzing qualitative differences in how the children organize space.

**11.1.2. Analysis of Verbal Descriptions Data.** We found many changes in the children's description of the spatial relations in this problem. Examples of the kinds of changes the children went through for each part of this task will be discussed: object description, setting description, description of relations, and method of description. A final example will show some of the more subtle changes in spatial thought, due to the language used to specify the referent.

**Change in Object Description:** **Roxan** went through a striking change in her ability to name the objects in the picture. In the pre-task, she only *describes* the objects, but in the post-task she was able to *name* the objects. For example, calling one of the objects a "cone" and describing it as "a triangle with a round bottom" is quite a change.

> **Roxan Pre:** The first one is like a house turning sideways, the next one is like a long octagon and the next one is like a triangle with a round bottom.
>
> **Roxan Post:** The base is checkered 3 on each side a hut is in the front left square the cone is to the right and in the middle square on the right. The cylinder is all the way in the back in the middle square.

In the pre-task no setting or relations between objects were even mentioned by Roxan. Yet in the post-task, she was able to explicitly describe the setting and use it as a 3-D reference for placing the objects. The difference in her answers is striking. She went from not defining, or explicitly thinking of any relations between objects, to making explicit Euclidean relations such as front/back, left/right, and top/bottom.

**Change in Setting Description:** **Tiffa** did not provide enough information in her description (pre-task) to allow us to recreate the scene without the image. It is as if she almost assumed that we were there with her, seeing what she saw.

> **Tiffa Pre**: You make a big square, then draw one horizontaly, then draw one horizontaly, then draw another one horizontaly. Get a small house and put on the first square in the corner on the left. Then put a cylinder on the next square next to it. Then put the cone on the right hand side at the bottom corner.
>
> **Tiffa Post**: Draw a rectangle. Make two lines horizontally and two lines vertically. Make a hut and put it on the left hand side in the 1st box in the first row. Put the cylinder in the second row, and the third box. Put the pyramid in the third row in the second box.

She told us three times to "draw one horizontaly." By looking at the picture we can figure out what she meant, but without the picture we can't recreate the setting from her description. In the post-task the description of the base was clearer and more explicit, but she still did not describe what the base looked like, and thus provide enough information to recreate the scene. In her letter, she wrote that there were "two lines horizontally and two lines vertically." Where were these lines, what was the spacing between them, and what image did they create? Another child had described the setting like Tiffa, yet she added that it looked like a "tic-tac-toe thing." This additional information allows us to recreate the setting. Tiffa was more focussed on telling us how to draw the setting, than on what it looked like or what it represented. As a result, her description, was like a 2-D drawing and not like a 2-D representation of a 3-D scene. In the post-task, her conception remains 2-D, yet the description slightly shifts in the explicitness of the references. For example, in the pre-task she wrote "Get a small house and put on the first square in the corner on the left." This description is clear, even if it leaves out the referent for "first." The second object was anchored to the house, which we know is in the corner on the left in the first square. The referents are chained, however the anchor point is never clearly defined. The relation Tiffa establishes between the house and the cylinder is also

ambiguous, because "next" is only a relation of proximity and not useful for reconstruction. In the post-task, Tiffa actually used three referents. For example, "put it on the left hand side (referent: base, page, or viewer) in the 1st box (referent) in the first row (same dimension as "left" referent)." She is still missing the anchor in this second description, because we don't know exactly where the first box or row is. Her descriptions in both the pre- and post-task were consistent within each task and between the tasks.

**Change in Description of Relations:** **Mikek** clearly described the base even in the pre-test (as presented below). The scene was a rectangle divided into 9 equal parts.

> **Mikek Pre**: Make a rectangle. Divide it into nine equal parts. In the top left space, draw a small house. On the space next to it draw a cilinder. Then on the bottom right hand space, draw a cone.
>
> **Mikek Post**: Use a box, divide it into 9 equal rectangles. In the rectangle to the right, the middle one you place a cone. On the rectangle furthest to the left and closes to you is a hut. The middle rectangle farthest away from you, you place a cylinder.

This could be referring to the top of the base, but I believe the top and bottom could also come from referring to the sheet of paper that they're on. In either case, Mikek was thinking about the base in 2-D, because he used the preposition top and bottom to describe the base. In the post-task, he changed the description of the base from a rectangle (divided up "into 9 equal parts") to a box (divided "into 9 equal rectangles)." (A box is more often thought of as 3-D and a rectangle or square as 2-D.) Also, the prepositions he uses in the post-task to describe the spatial relations are no longer "top" and "bottom," but terms like "near" and "far," which indicate depth or 3-D (in relation to the implicit viewer). In the post-task he used two references; "farthest to the left" implies a viewer, and "the middle one" refers to the base. When placing the hut, his use of the viewer, "you," as a referent becomes explicit. The most interesting change though, is his shift in thinking about the scene as a 2-D drawing (in the pre-test) to a 2-D representation of 3-D space (in the post). Terms like close, farthest are used to refer to the 3rd dimension, depth. In

the pre-task, his ambiguous placement of the "cilinder" could not be completely recreated. In the post-task, it is possible to re-construct the scene from his description.

**Change in Method of Description:** **Jenni**'s description in the pre-task has little to do with what was asked from her. She may not have understood the task.† We can see from her answer, however, that she could name some of the objects. We also get a hint of her procedural approach to this task.

> **Jenni Pre**: First you have to take a cone and put it on top of the cylinder and that is a tree. Second you have to put a triangle in top of a square. Then, make a drawing of the sky and colored it and that is it. By! [Bye!]
>
> **Jenni Post**:
>
> Dear Friend,
>
> I know that you got a little scene of a hut, cone, and a large cylinder. You got three lines. The lines have three rectangles. Well, put the hut were it says 3. First glue the bottom of the hut. Then put the hut on the rectangle number three. Put glue on the bottom of the cylinder. Then put the cylinder on the number 4. In the 2 row. Then, put glue on the bottom of the cone. Then, put the cone on the rectangle # 8. In the 3 row. There you finished.
>
> Your best friend
> Jenni

In the post task, the base was not well defined. She wrote "you got three lines ... the lines have three rectangles." I think that she thought of the base as three sets or rows of 3 rectangles. In her numbering system she even counted the "rectangles" as three sets of three. She wrote numbers in the squares starting in the back left corner with 1 and counting forward to 3. Then she went to back square in the middle and she started with number 4 and counted forward to 6. She then went to the back right corner and

† Jenni was from the Spanish-English Bilingual class. She spoke and understood English well, but sometimes I had some trouble understanding her accents. It is important to note though, that her regular Mathematics and Computer classes were given in English.

starting counting forward with 7 and ended with 9 in the front right corner. She then numbered the rows 1, 2, 3 from left to right. This system of numbers works very nicely for reconstructing the scene. Note that the children where told to use only words, and not send drawings to their friend. Without her numbering system there is no way for someone else to reconstruct the scene just from the verbal instructions. It is clear that Jenni was thinking about the objects in this scene as 3-D. In fact for her, they are concrete objects. In her letter, she gave her friend very careful step by step instructions for construction. She wrote "put glue on the bottom of the cone" and then, "put the cone on the rectangle #8."

**Change in Choice of Language:** **David** was able to accomplish this task very well in the pre-task. He also changed over the course of the study. In the pre-task, he was very clear and explicit when describing objects, setting, relations, and method. He simply stated that you have a "3 X 3" grid, and then he located the objects using the grid and the implicit position of the viewer as his two referents. Everything was placed in terms of depth (far left, far middle, near right) in relation to the viewer or ego as the dominant referent. This tells us that he thought of the scene as 3-D, seen from a particular view, his, in space. In the post-task David used the same method. However, instead of using the viewer as the dominate referent, he used the base (which is so obvious to him that he forgot to even mention it). Here we see prepositions like "front-left," "rear-center," and "middle-right." All of these terms have been hyphenated which further indicates his referring to positions in 2-D in relation to the base which is though of as 3-D (the top and bottom or Y axis here are implicit). This appears like a very subtle shift, but it is a shift from topological relations to Euclidean relations, from a more relational to a more stable and global invariant reference.

**David Pre**:

Dear Tommy,

First get a grid of 3"X3." Put a house on the far left corner. Then put a cylinder on the far middle space. Finally, put a cone on the near right corner.

Your Pal,

David

**David Post:**

Dear Tom,

Put the hut in the front-left square. Then put the cylinder in the rear-center square. Finally put the cone in the middle-right square.

Your pal,
David

## 11.2. A Look at Three Children Through Two Tasks

The two most difficult task and marked the coordinations of concepts in Euclidean, Projective, and J3D tasks. These were Euclidean task (#7) and Transformational task (E). The Euclidean task requires that children label referents and plot two points on a grid. Solving the Transformational task (E) requires both a coordination of Projective concepts as well as a coordination of Transformational concepts. Analyzing these two tasks, helps us understand how individual children solved these spatial problems. Three children will be discussed: David, Jenni, and Tiffa. These three children represent extremes in spatial ability among the experimental children. David is the child with the highest spatial pre-test scores, and Tiffa and Jenni had the lowest spatial pre-test scores. Jenni and Tiffa are different in that Tiffa's post-test scores went down and Jenni's post-test scores significantly improved.

Let me briefly introduce David, Jenni, and Tiffa. David was a stocky, white boy, who was fairly serious, especially about school and learning. He was often involved in his own thoughts and projects. Jenni was a very petite, vivacious, and out going Hispanic girl, who was always interested and distracted by what was going on around her. Tiffa was a very tall, quiet and soft spoken, mature black girl. She was known for being a hard worker and sticking with problems that were very difficult for her. David and Jenni will be the two children investigated most deeply in the section 11.3 on learning J3D.

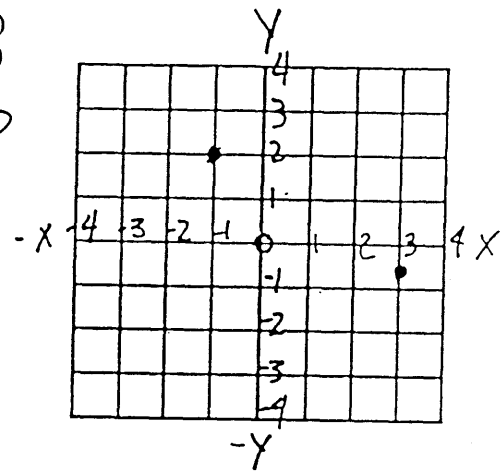
**11.2.1. Construction and Coordination of the Coordinate System.** This post-J3d task was described in chapter 10 section 10.1.1, problem #7 and was only given as a post-task. Only three of the ten children were able to

Figure 20: David, Tiffa, and Jenni construction of coordinates.

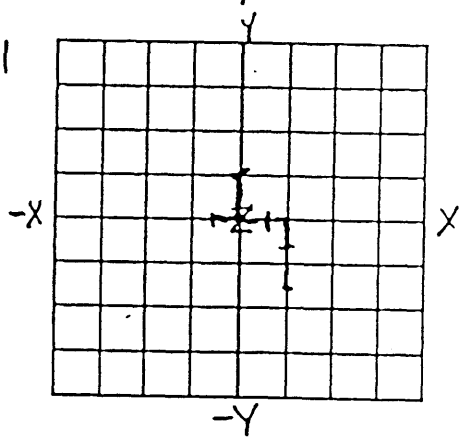
Please put in the reference numbers in the grid and plot the following 2 points:

-1 2 0
3 -.5 0

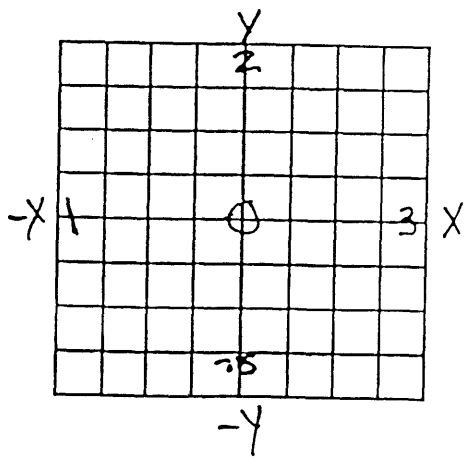
DAVID




JENNI




TIFFA



complete this task correctly, four other children were able to do parts of the task (i.e. label the axes, or label the numbers, or plot the points). This task was difficult for the children. Even those who could complete all parts, had to be prompted. David, Jenni, and Tiffa's answers to be discussed below, are shown in Figure 20. David, the highest spatial subject, demonstrated a level of spontaneous response not seen in the other children. The children were asked to plot two points; (-1 2 0) and (3 -.5 0).

**David.** This task was not very difficult for David. He had coordinated measurement in 2-D prior to the study. In the pre-task, when asked how he had measured in 2-D (Euclidean #5), he wrote,

> I measured how far from the side & top in the first box & put the dot in the same place in the second box.

Of the three children, David is the one who was able to plot a point in the pre-test (Euclidean Tasks #6). During this task, David immediately marked the X and Y axis and plotted the two points. When asked if there was any other information he thought would be helpful, he added the positive numbers to the graph. When asked again if he thought this was enough information for someone to understand, he added the negative numbers and said "It is now."

**Jenni.** This was a more difficult task for Jenni. She had not coordinated measurement in 2-D prior to the study, nor was she able after the study. In the pre-task, she placed two points, one next to the correct number in each axis separately. The two axes were not coordinated into one single point. She was able to label each axis correctly, both positive and negative in the post-task. Jenni even indicated the Z axis by putting a Z at the origin. During the interview, I asked her three times if she could think of any other information that would be needed on the graph in order to plot the two points. This question was asked several time to see if she could think of any other information needed on the graph (such as numbers along the axes), and to determine how confident she was in her answer. Jenni could use the coordinate system in J3D and knew the axes, but she had not yet coordinated them. Below is a

short portion of the transcript of this session, when Jenni was placing a point (-1 2 0) on her graph,

> *Judy: What does the first number mean?*
>
> *Jenni: Minus, it's minus right here, this one I got is minus one, then right here?* [She drew a line from zero to the left halfway along the line to the first intersecting line and marked it with a short line.]
>
> *Judy: All right.*
>
> *Jenni: -1 and two. 2 and zero. Over here.* [She then drew another line from zero to one full unit up in Y.]

Jenni drew lines from the origin and placed a point in -X, and drew then another line from the origin to a point in Y (see Figure 20). If each square of the graph was equivalent to 2 units then the end points were placed at the correct distances from the origin. She labeled the axes, but had not labeled the reference numbers along the axes. The axes helped to orient her, but the absence of reference numbers prevented her from using the grid as a consistent global reference frame. As a result, the first point (a set of two lines) seemed to be in one scale, and the second point (two line from the origin to two points) in another scale. While Jenni was able to measure in 1-D and extract the coordinates for a point by the end of the study, she was unable to generate and plot a single point in 2-D. We can see that it is possible to use the Cartesian coordinate system and J3D without being able to coordinate measurement in 2-D. My assumption is that if she could have spent more time with J3D she could have coordinated measurement in 2-D.

**Tiffa.** Like Jenni, Tiffa had not acquired coordinated measurement in 2-D before the study nor afterwards. She was able to find her way about the coordinate system, one axis at a time. In the pre-task, Tiffa put a point above 7 on the X axis and to the left of 10 in Y axis, instead of placing a single point at (-7 1 0). As we can see in Figure 20, she used the numbers to label a point rather than as a reference from which to mark a point. In the post-task, she wrote "1" for the point (-1 2 0), almost at the left edge of the grid (-X axis), "2" almost at the top edge of the grid (Y axis), "3" almost at the right edge (X axis) and -.5 at only 1 square less than the lower edge (-Y axis). The 1, 2,

3, and -.5 are all very close to being the same distance from the origin that she marked. She did not put in reference numbers, but she labeled the axes. Below is a portion of the session, which reveals Tiffa's understanding of the axes and directions along the axes. Her difficulty is to make what she knew explicit, and making use of what she knew to solve the task at hand. Tiffa needed prompting to even get started on this task.

*Judy: What information would you need to be able to read this grid? Remember when you made your data? What information did we have to put on the graph paper?*

*Tiffa: The points, the picture, and and that's about it.*

*Judy: Would we need anything else? If we wanted to plot a point, would there be any other information we would need to know about those three numbers?*

*Tiffa:* [silence]

*Judy: What do those three numbers mean?*

*Tiffa: X, Y and Z?*

*Judy: OK, so what would we need on this the* [grid] *to let us know what those number really are?*

*Tiffa: X, Y and Z.*

*Judy: Great. Could you do that for me?*

*Tiffa: Uh, just put this here?*

*Judy: What do you think?*

*Tiffa:* [She writes the axes on her paper.]

*Judy: Good. OK, now if you could plot those two points for me.*

*Tiffa:* [silence]

*Judy: Do you think you have enough information to figure out where to put those two points?*

*Tiffa:* [She labels the X and Y axes on the grid.]

*OK, then add whatever information you need so that you can plot those two points.*

*Tiffa: I don't need anything else. Let's see, let's see. Is that what you*

*want?*

Here we can see that Tiffa became unsure of what she was doing when I asked if she needed other information. She often looked at me for the cues, answers, or even for approval of her answer when we worked together. This tendency of looking at me for confirmation of her work became more pronounced near the end of the study as she dealt with more complexity.

*Judy: What have you just done? Explain to me what you just did.*

*Tiffa: Set the -1 on the negative side of this X.*

*Judy: Uh huh*

*Tiffa: This side is over at 3. Yeh, and two in Y, it's not negative so I put it at the top.*

*Judy: Do you think that's enough for someone else to understand what those numbers mean if they looked at it?*

*Tiffa:* [She shook her head yes.]

Tiffa could not yet perform all of the 1-D measurement tasks, extract a coordinate point, nor coordinate measurement in 2-D in the post-tasks. She solved this problem by using the numbers themselves as indicators of the points rather than putting the numbers on the grid to be used as referents. For example she put a -2 and a point at the left side of the grid along what she had marked as the X axis. This use of labeling indicates that, as in her other 1-D and 2-D tasks, she had not yet coordinated all the elements into a unified system of references.

**Summary.** Among these three children we see different levels of understanding of 3-D space and the Cartesian coordinate system. All three children were able to use the coordinate system within J3D, even though their ability to coordinate the system varied. David knew the system quite clearly and could make his understanding explicit for himself and for others. Tiffa and Jenni could *use* the system and *understood* the axes, but they were not yet able to *coordinate* the axes into a *whole system of references.* Jenni was able to start systematically by defining her own references (the numbers along the axes) and use these implicit referents (the numbers were not written on the

graph). Tiffa was not yet able to define these referents systematically. She used the numbers themselves (local referents) next to the points as a label. More time for Jenni and Tiffa was needed to attain a coordinated system of references. According to Jenni's starting level, she made a great deal of progress from using J3D. As mention previously, I believe that Tiffa's lower scores reflected a U-shaped learning curve because the system and the concept were very complex for her. What we see with Tiffa's scores on spatial test is only a small section of this curve, the descent. I believe if the study had continued on longer we would have also seen an ascent. Some children need more time than others. I think that Tiffa was a child who was juggling the multiple concepts and was just beginning to sort them out when the study ended.

**11.2.2. Coordination of the Eye and Rotation Transformations.** In this task, the children were asked to write what could have caused the difference between one picture and three others (Figure 12), and described in chapter 11, section 10.1.3. The data from this exercise shows the same overall pattern for David, Jenni, and Tiffa that we saw on the previous task. Jenni and Tiffa were able to solve this task, however in a way that was not as coordinated a solution as David's. The children's solutions to this task seem related to De Lisi's (1976) hierarchy of three types of movements in space; transposition, intermediate, and transformation. Transposition reflects an understanding of movements as displacement, and transformation reflects a coordination of change of position and change in objects character. De Lisi's hierarchy of transformations is similar to what I found in the children's solution to this task. Most of the lower spatial children in the study thought that translation (transposition) was the solution in the pre-task and all but one of the children, Tiffa, progressed to see the solution as either a rotation and/or change of view in the post-task (transformation). The rotation and change of view take into consideration not just the displacement in space as does transposition, but also involve the transformation of the objects and the relations between those objects. David was able to do this task prior to the study and Jenni and Tiffa were not. Jenni and Tiffa saw the changes as a translation or change of position prior to the study, in fact neither of them reported the possibility of a rotation or change of view in the pre-task. After the study, Jenni was able to

see the solution as either a rotation or change of view, but Tiffa still saw it only as a change of position. Let's examine each child's thinking on this task more closely.

**David.** In the pre-task, David did not yet understand the reciprocal relations between rotation and a change of view in the pre-task. From his answers (below) we can see that he saw each picture as a separate problem with possibly a different solutions. He reported only one type of transformation for each picture, having not yet coordinated the concepts involved. David very quickly understood what J3D offered and he tried to incorporate what he had learned from the introduction to J3D. In his pre-tasks, he used the language proposed in the lecture, and guessed the axis of rotation. He wrote,

**David Pre:**

A: Rotated on X? axis.

B: Changed point of view

C: Rotated on X? axis

**David Post:**

Eyepoints; rotation of bottom with other things grouped.

As we can see from his answers to the post-task, he understood that all of the pictures could be achieved through *either* eyepoint or rotation. Notice that he spontaneously qualified how it would be possible to succeed through a rotation. For David, the transformations of rotation and change of view became reciprocal by the end of the study. Below is a portion of the transcript where the rotation solution was discussed.

*Judy: Now, you wrote down rotation. Rotate what?*

*David: Rotation in Y.*

*Judy: Anything else I would need to know to rotate that?*

*David: Are all the things attached?* ["Attach" is a command in J3D that is used to attach one object to another object].

*Judy: So, would they have to be attached?*

*David: Or group* ["Group" is a another command in J3D that allows several objects to be called by one name and transformed the same through one set of commands.]

David knew the axis of rotation for the scene. He saw that the relations between the objects was fixed and had to be maintained. He understood how to do this with J3D. In order to use a rotation strategy, the relations between objects had to stay fixed. The two methods for fixing the relations between objects in J3D was the attach or the group commands.

When discussing the change of view strategy, I asked David if he could tell me where the eye points were for the different pictures. He immediately began figuring out the coordinate point of the eye for each view. I stopped him, and asked him to describe it in more general terms. He said "backwards" for the view that was the opposite of the anchor view (front). After he generated this type of answer, I set up a anchor for him by stating that the reference picture #1 was seen from the front.

*Judy: So, here in #1, I'm seeing it from the front. Where am I seeing it from in A?*

*David: In back.*

*Judy: And in B?*

*David: The side.*

*Judy: And in C?*

*David: The other side.*

All the children who wrote down a change of view strategy were able to respond and immediately give the correct direction from which the scene was viewed once the reference of a given view was set up for them. It was difficult, however for them to specify their own anchor. Jenni was able to set up her own anchor.

**Jenni.** In the pre-task, Jenni saw the difference in these pictures only as a change of position. She wrote:

**Jenni Pre**:

A: The hut moved dyagelly at number # 1.

B: You moved it to the side.

C: The hut moved to the side.

Notice that the relations between the hut and the other objects were not even mentioned. She refers only to the hut for A, B, and C. This object was more salient because it was taller than the other objects and it was a recognizable canonical object. Those children who said that the difference in the pictures was a change of position often mentioned only the hut. I believe that they chose the change-of-position solution because, like Jenni, they were only looking at the hut in relation to the base or to the viewer. Their focus was localized on the objects, rather than on the more global relations between objects.

In the pre-task Jenni saw the solution as a translation or change of position of the objects. However on the post-task she only thought that the pictures could be made through rotation and change of the view. She wrote,

**Jenni Post**: You rotated them A-B-C; rotation, eye points.

In the post-task Jenni wrote down that the change was the result of a rotation. When asked if she could think of any other way to explain the changes, she asked back "Eye point?" When asked again if she could think of any other explanation, she went back to a rotation strategy. However, whenever pressed, she went back to the eye point as the strategy to explain the pictures. Jenni thought that I had created the pictures by changing the eye. She was the only child who provided herself with an explicit anchor. Under the four pictures Jenni wrote: 1. base, A. back, B. side, and C. back side. Even though she had written "rotation" as her first solution, the eye strategy seem dominant for her. Below is a portion of this session discussing that she had written.

*Judy: OK. How would we have to do it if we were going to rotate it?*

*Jenni:* [she shrugs her shoulders]

*Judy: What happens if I just rotate the house? Will I get the right picture?*

*Jenni: No.*

*Judy: So tell me what is rotating.*

*Jenni: You're rotating the house and the whole picture.*

*Judy: So how could I do this on the computer, then?* [Trying to see if she remembered the attach or group commands.]

*Jenni: Eye points, with the eye points.* [When she thought about it, she probably thought that it was either too complicated or impossible to rotate all these objects together.]

*Judy: With the eye points. But if I was going to do it with rotation, how would I do it?*

*Jenni: I don't know.* [She may not have remembered the attach or group commands or she was unable to apply their operations to this problem.]

*Judy: Well, would I want to just rotate each object, or all of them?*

*Jenni: All of them. So, eye point.* [Jenni understood that if you were going to rotate everything, then the obvious transformation in J3D would be a view change.]

*Judy: Do you think I rotated them, or I changed the eye point?*

*Jenni: Changed the eye point.*

It is difficult to say whether Jenni's switching back and forth between the eye and the rotation strategies was because she understood the difference of complexity in J3D, or because she understood that the two operations were equally possible. Another possibility is that if she *looked* at the objects, they appeared rotated. However, if she *thought* about how to achieve the image in J3D, she thought about it as a change of view. What is not clear here, is whether Jenni was able to disentangle the two transformations. It seems that focusing on one strategy would almost cause her to shift to the other strategy, just as focusing on the image on the screen would cause her to shift from what she knew to what she saw.

**Tiffa.** As mentioned above, Tiffa was the only child who saw the answer to this problem in both the pre- and post-tasks as a change of position. Several other children mentioned that this was a possible solution (see Figures 15-16, E), but they understood that it had to be used along with rotation (for the

hut). However, Tiffa, the child who's spatial ability scores actually dropped from the study, maintained that change of position or transposition was the only possible solution throughout the interview in the post-task. Let's take a closer look at how she thought about this task and what actually changed in her thinking between the pre- and post-task. In the pre-task, Tiffa describes the hut's movements in 2-D simultaneously.

**Tiffa Pre**:

A: The hut moved diagonally from 1.;

B: The hut moved diagonally again from A.

C: The hut moved diagonally again from B.

Notice that she did not explicitly mention the other objects or the base, although the base can be inferred because of the use of the word diagonally. This word refers to movement in 2-D in relation to the base.

Tiffa's thinking never changed from translation. She had changed the eye and used rotation a great deal in her animations. She was able to talk about rotations and came up with eye points. It is not that she was not familiar with these transformations. I believe that she just did not see them as part of the solution for this particular problem. She looked at this problem, more locally. She focused on the position of the objects. In the post-test she started to consider the relations of several objects when she talked about moving more than just the hut. She was aware that changing the position of the hut was not enough to achieve the picture. She mentions a rotation for the hut, which indicates that her understanding it needed to be in both the right position and in the right orientation. In the interview I tried to prompt Tiffa to see other possibilities, but her thinking did not shift from translation. She started to think about rotation for just the hut, but she settled back to a change of position as a way to solve the problem. From then on, she stayed with this strategy, as we can see below.

> *Tiffa: The picture in A, yeh, the picture A, the house has jumped over to this side and you could have turned it around.* [Silence for a moment which she thought about it then she restated what she thought the answer should be.] *The home has jumped diagonally to the other side.*

*Judy: What's the difference in one and B? What's changed?*

*Tiffa: The cone jumped to the opposite side of it.*

*Judy: OK.*

*Tiffa: And what is this?*

*Judy: It's just a fence or cylinder. Just call it a cylinder, that's fine.*

*Tiffa: So, it's moved to the other side.*

*Judy: OK. C?*

*Tiffa: Well, the hut jumped diagonally to the other side, the cone has jumped diagonally to the other side a little bit, and cylinder jumped to the other side.*

*Judy: OK. That's a lot of changes. So, basically, to do that on the computer, we'd have to place everything, because everything has moved?*

*Tiffa:* [Nods her head yes.]

*Judy: Now, can you think of any other way to explain the changes?*

*Tiffa: No.*

*Judy: OK. Look at one and look at A.*

*Tiffa: Yeh.*

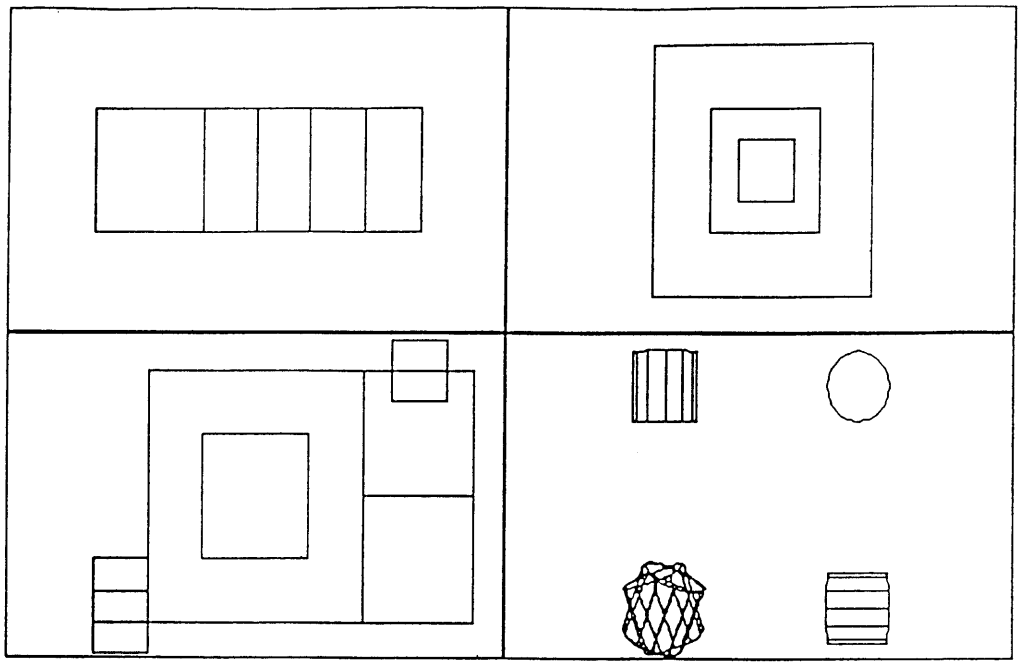
*Judy: OK. So there's no other way to change that, to get A from one, except for moving the objects around.*

*Tiffa: Right.*

Tiffa could see no other solution to this problem. However she had begun to consider more than one object's position as indicated by the position changes.

**Summary.** David, Jenni, and Tiffa were each able to solve this task, in varying degrees of coordination. Only David was able to coordinate these transformations and understand their interactions after the study. Jenni and Tiffa saw this task as a change of position rather than a rotation or change of view in the pre-tasks. In the post-task Tiffa maintained this solution, but Jenni saw it as a change of view or rotation transformation.

Figure 21: David's first day of exercises.



```
;david.3_24
;Thur. 3:24:1988, 11:19:30
call sq
call sq db
call sq db1
call sq db2
dis
pla db1 1 0 0
dis
pla db1 2 0 0
dis
pla db2 -2 0 0
dis
;save david.ex1  11:28:44
;his david.h
pla db1 0 0 0
dis
res all
dis
sho
sho db
sca db1 2 2 1
dis
sca db2 .5 .5 1
dis
;save david.ex2  11:34:04
sca db2 1
show db2
dis
del
;@david.ex2
call sq.asc  db
call sq.asc  db1
scale db1 2 2 1
call sq.asc  db2
scale db2 0.5 0.5 1
;done
dis
pla db2 3 2 0
dis
pla db2 -2.5 -2 0
dis
pla db2 -2.5 -1.5 0
dis
pla db 3 1 0
dis
pla db 3 -1 0
dis
;save david.ex3  11:41:47
del 1
dis
call cylnd d
dis 1
rot d 1 x 90
dis
dis 2
res all
dis 1
rot d 1 z 90
dis
dis 4
rot d 2 z 20 x 10
dis
rot d 2 z -20 x 15
dis
rot d 2 d -25 x 20
dis
;save david.ex4  11:52:06
del 1
off all
dis
cal bcube f
dis
blu
sho view
dis
eye 0 30 10
dis
eye -5 20 7
dis
rot f 2 x 10 z 5
dis
blu
sho view
cal cut a
;Thur. 3/24/1988, 12:02:21
```

## 11.3. Cases of Two Children's Learning with J3D

Lets investigate the situation of two children learning J3D from the researcher. The next section will introduce two children, David and Jenni. David was the child with the highest spatial ability both before and after this study. Jenni was the child with the lowest spatial ability before the study, and who showed significant improvement in her standard spatial tests (Table 4). She also improvement in her understanding of Euclidean and Projective Spatial tasks. Both children learned how to use J3D, but each child came to it with different spatial knowledge available to them. I would like to introduce these two children through contrasting their learning session on the translation, scale, combined scale and translation, and rotation learning exercises. This will be done in order to provide the reader with a better sense of the differences between the two.

David, on his pre-J3D tasks, showed a great mastery in many of the concepts necessary for J3D, such as the coordination of measurement in 2-D and coordination of perspectives. Since he was quite capable in both the mathematics and spatial concepts necessary, he presents a different process of learning J3D than those of Jenni. Jenni had not mastered measurement in 1-D and she was only able to partially incorporate perspective in her drawings. Compared with children of her age nation wide, Jenni scored average in her spatial abilities on the pre-tests, though on the standard spatial tests she ranked 41st out of 45 AWC fifth graders.[†] She was also doing quite well in relation to the school's middle math children.

**11.3.1. David Learning J3D.** David was considered one of the brightest boys in the fifth grade. He was in the top math group and had just won a computer award for one of his Logo projects in a Boston City Schools competition. It was not a surprise then, that David immediately appropriated the language of J3D that were used in the lecture introducing 3-D computer graphics.

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[†] Remember these AWC children were already a pre-selected group of higher achieving students.

The first time I worked with David at the computer, I made a note in my journal that he was "very fast." He completed all of the learning exercise in the *first* session! In fact, David surprised me. I had taught these same concepts to college students and I found that David learned as fast as most of them. He seemed attracted to J3D and got very excited about what he could do with it. He was the only child who went through all learning exercises in one session. The second time we met, he came with a design of a project that he wanted to do, without it ever being discussed or even asked of him. As soon as he learned a command he seemed to understand how it worked with the other commands and which command achieved which operation. He understood right away that moving the object or moving the eye would appear the same on the screen. In his first project with J3D (Figure 5), he had just spent a few minutes putting eyes and a mouth (all circles) on a head (soccer ball). He then wanted to start adding the arms and legs, and realized that the head filled his whole screen. He said "I am going to have to move the guy" (the person he was creating) and then immediately said "No! I will move the eye back, 'cause [if I don't] it [the guy] will be really big." This shows that David immediately understood perspective and the way perspective affects the apparent size of objects. Since the head was already filling the screen, he knew that the scale of his person was too large and he had to make some kind of adjust it. He understood right away that the scale or apparent size of objects would change if the eye moved closer or farther away from the object. This was actually a much easier solution than scaling all the objects.

David is a sociable boy who gets along very well with the other children. He had confidence in himself and his own learning. I once asked him if he thought his current command was correct, his response was simply, "I made it." David's personal focus in all the sessions was on the system, it's technical aspects, and what he was making with it. He rarely asked questions, and if he did, they were technical. He really did not want assistance and would cut me off if I tried to explain something or present an alternate strategy to that which he already understood or used. If there was a way to find out information about the system he would check *all* the ways he knew on the system before he would ask a question. He seemed to use me more as "an audience"

for his ideas or as "an expert" on the system when he needed more an expert's knowledge.

As mentioned before, David's spatial framework was already very strong prior to the study. If we look at Table 8 we can see that through the course of the study he did not change his overall score on the FRC, but solved the problems faster. He improved most on the Piagetian and J3D Transformation tasks. Besides having high spatial ability, he was also one of the best students in the highest math group.

As a result of this, David provided a special kind of measure in this study. He provided a baseline for some of the tasks and helped me to clarify what some of the problems in learning J3D could be. David was very helpful, not only because of his mathematics and spatial skills, but he was quite able to articulate his ideas. David served as a "pilot" for all my new tasks and exercises. He made it easier to pinpoint where there could be a problem and the nature of the problem was in J3D, the task, the concept underlying the task, or the mathematics and spatial skills needed to solve the task. He also provided feedback on how to present concepts, where the conceptual pitfalls might be, and if a task actually touched on what I was after. Below are some portion of the transcripts of his first day's learning session with J3D. Here it is possible to see how quickly and independently he thought and worked and how he learned about J3D. Figure 21 represents a tracing of all the changes made to create the following exercises shown in Figure 4. The place exercise is in the upper left, the scale is in the upper right, the scale-&-place is in the lower left and the rotations tasks are in the lower right. David went through all of the exercises and was introduced to the view transformations in a first session that lasted forty minutes.

**Translation Exercise.** David had just called in the three squares for the translation exercise (see Figure 4). I had just showed David a picture of the image of the three squares side by side.

*Judy: The command for this is place. For every command you need to say which object you are talking to -- Which one?*

*David: Db1.*

*Judy: Db1, put a space, and there are three things that you have to tell it.*

*David: The X the Y and the Z* [David immediately jumps in and gives me the information I was going to give him].

*Judy: Right. So, take a look at this one, it will give you some idea of scale, the default size or size it is when it comes in it goes from -1 to 1 in X, -1 to 1 in Y. This is what you are looking at right here.*

*David: Ok, I get it.*

*Judy: Before we get started show me where the X axis is?* [David was able to point correctly for each axis as well as know the positive and negative direction for each axis].

*David: So I want to move it, over here. So what part are you moving.* [This is a good question, because how you think about what is moved makes a difference. It is like deciding what is your reference frame for solving the problem].

*Judy: You are moving the whole thing. You can think of it this way. You are adding a number to it.*

*David: Ok, I get it. ... plus 1, in the X plus 1.* [Again I was cut off, when trying to provide a few alternative strategies.]

*Judy: Let's display it. Do you think you are right?*

*David: I made it.* [He displayed it.] *Oh, you need to move it 2 over because it is 2 between the middle and to each side.* [He immediately understood what the extent of the object was and revised his answer].

*Judy: Great, so let's do the third one.*

*David: Negative 2*

David immediately understood the reversibility of the space, if moving the square to the right was +2 in X then the other square was obviously a -2 in X. This again shows his clarity of understanding about the extent of the object and the constancy of length. He understood right away that he needed to consider both sides of the object from it's center (zero). He was able to keep the extent of the object in mind at the same time he thought about moving that object. This is not a surprise because we can see in his spatial framework that he was able to apply these same concepts in both positive and

negative numbers in the pre-tasks on the snake and temperature tasks.

As the reader probably noticed, I could barely get through an explanation before David would cut me off or interrupt me with an "I get it." Often if I started to point out other possible strategies or explanations a strategy, he would say "Ok, go ahead." He was like this throughout the study. He was very capable of exploring on his own and coming up with his own strategies. He made it clear to me that this was the way he wanted to work. David would listen as long as the information was new, but as soon as he understood he let me know.

**Scale Exercise.** We used the same three squares from the previous exercise and reset them to their original position then we started the exercise on scale. I showed David the picture (Figure 4) of what I wanted him to do and we began.

> *Judy: Here is the syntax for scaling, S C A, name and X, Y, and Z. The scale factor is multiplied by the numbers in each coordinate. Should we go through that for a second?* [He shook his head no. Notice that I attempted to quit offering strategies and started asking if he wanted a more detailed explanation].
>
> *Judy: Which one is the original object?*
>
> *David: The second one*
>
> *Judy: Ok, and how much larger is this outer one?*
>
> *David: 2*
>
> *Judy: Let's say it is 2. If I were going to take -1 to 1 and I multiply it by 2 -- What is 1 times 2?*
>
> *David: 2*
>
> *Judy: What is -1 times 2? Just guess.*
>
> *David: 1?*

This same pattern of guessing is evident in almost every child. Their first intuition when multiplying a negative number was to multiply normally and then subtract the value of the negative number from the answer. For one of the questions on the pre-test, I asked the children what (3 times -2 = ).

Most of them answered 4, because 3 times 2 = 6 and 6 -2 = 4. This provided an insight into where many of the children had a problem understanding negative numbers. They seemed unable to let go of the idea of the minus sign as an operation for subtraction.

> *Judy: No, you can think of multiplication as adding. You are really adding two negative numbers, (-1) plus (-1).*
>
> *David: OHH! I get it. Twice as large.*
>
> *Judy: Ok, then it is the same for Z. And display. How are we going to get the smaller one.*
>
> *David: -2*
>
> *Judy: Well, -2*
>
> *David: -1* [This response was another intuition held by most of the children. The notion that if you want to make something smaller you take something away from it].
>
> *Judy: No, maybe this is something you have not yet learned. If I multiply 1 times 1, I get 1. To make something smaller than one what do we have to do? Let's do it this way.* [I drew a number line]. *Here is zero and here is one.*
>
> *David: Uh huh, point 5!* [He immediately understood that the number had to be one-half, but more important he used a decimal fractions right away.

Once all the objects were scaled correctly, we went on to the next exercise which used the scaled objects and placed them adjacent to each other. This exercise required that the children consider the current extent of all the objects.

> *Judy: Let me remind you that you now have three object in here. One goes from -.5 to .5, one -2 to 2, and the original one. This is what you have.*
>
> *David: Uh huh, ok.*
>
> *Judy: When you move it you have to remember how far it is from zero to that side and be sure to move that over to here.*
>
> *David: Ok.*

*Judy: So think about what that offset is.*

*David: Ok,* [mumbles] *X 2, 3, because that way it will be, cause the .5 is 1 across to be on the very end. And also 1, no that's a 2 on the Y, down here and zero in Z.*

Above, David says 2, then 3. He pointed to the edge of the large square that went from -2 to 2. He then added 2 and the extent of the smallest square, -.5 to .5. He used his calculation to determine where to put the square. The command he entered was (3 2 0), and what he actually needed was (-2.5 -1.5 0). His first response was only off by half of the extent of the small square or from it's origin to it's edge. The fact that he calculated the position by considering the current extent for the objects shows his level of understanding of the principles involved. You can trace his movements for this small square. In it's first position, we can see it in the upper right portion in the lower left quadrant of Figure 21. Next it is placed at the lower left, where we see two small squares overlapped because of an adjustment. He changed the directions by placing it at (-2.5 -2 0), and finally placed it correctly at (-2.5 -1.5 0). Below we follow his progress through these adjustments.

*Judy: What did you do?* [He had just entered 3 2 0 and displayed it].

*David: I forgot the minus.*

*Judy: Right. And were the numbers correct?*

*David: Oh yeh. It's a a a 1.5?*

*Judy: You are going to go from 3 to 1.5?*

*David: 3 then, no, 2 point, 2 point 5, minus 2, and zero.* [David went from 3 2 0 to -2.5 -2 0. I told him he could change more than one thing at a time in J3D prior to displaying. Many of the other children when told they could change more than one value at a time, stayed with only changing one axis at time and usually only one change within that axis, (e.g. either sign or value).]

*Judy: Let's see it. Very close.*

*David: 2*

*Judy: Which number was wrong?*

*David: Umm, 1, so -2.5, -1.5, and 0.*

*Judy: Display.*

*David: Now I'll do the other one. It's 2 plus 1* [the extent] *from the small one. 3 to right here and 1, and display. um, -1.*

*Judy: Got it.*

*David: 3, -1, 0.* [He fixed the sign on the 1 in the Y axis.]

As you can see above, David used his understanding of space and measurement to compute the answer. He did not guess, he computed the changes from what he knew.

**Rotation Exercises.** The rotation exercises can be seen in Figure 12 (chapter 10). I had just explained that there could be a rotation in more than one axes and in up to three axes. I kept a model of a house available for the children to pick up and turn when they were trying to figure out rotations. David's exercises in rotation can be seen in the lower right quadrant of Figure 21. He did very few adjustments on the rotations.

*Judy: Which way do you think the Z will turn it?*

*David: Like that?*

*Judy: What do you think the rotation is in Logo?*

*David: Right and left.*

*Judy: Right and left, yes, and how do you see it change? That is a Z rotation. This is the front of this little house. This is looking straight down the Z axis from positive end. Let's put a little handle here.*

*David: Ohh! oh.*

*Judy: Here is the Z axis. If I grab that handle, and again remember...*

*David: Oh, oh, I get it.*

*Judy: the direction*

*David: I get it, I get it. So you grab it from the top, it would be a, uh, Y axis.*

*Judy: Yes, so where is the X axis?*

*David: It would pointing over that way.*

*Judy: Right, great. One other rule, with rotations, the direction of your*

*thumb is toward the positive end of the axis. The direction you curl your fingers is a positive rotation. So if you want it to turn that way, that's positive, what rotation?*

*David: I got it, Z.*

*Judy: Let's display the original object in quadrant 1. How would you get this for me in quadrant 2? The command is --*

*David: Um, well, you can rotate it, is it R O T?*

*Judy: It's R O T and there is a trick with rotations*

*David: In the Z axis, no that would be the X axis.*

*Judy: X axis and --*

*David: It would be 90 degrees.*

*Judy: Ok, you're right. It is 90 degrees. The syntax for it, is rotate object, what did we call it "d"?*

*David: Uh huh.*

*Judy: The reason you need a number here is because you have three axes. You have to tell it how many rotations you are going to give it.*

*David: X.*

*Judy: How many axis are you going to rotate in this time? Only one, so we put a one there, space, and tell me which axis it is?*

*David: X.*

*Judy: And what degree?*

*David: 90.*

*Judy: Great and display. Now let's go to this one. This lower right hand one.*

*David: Ok, so that's um, Z! axis.*

*Judy: Yes.* [I then showed him the picture and asked him to do the exercise which required two rotations.]

*David: Ok, one axis.* [He started typing in the command.]

*Judy: And who are you rotating?*

*David: Yes, "d," one axis, Z and 10.*

*Judy: Can you get this by one rotations, only one axis?*

*David: Oh, I don't think so.*

*Judy: How many rotations, do you think? Let's look at it and try to figure out.*

*David: You need to turn it on it's Z, I don't get it.*

*Judy: Do you need to turn it in Z?*

*David: Yes, it's tilted, it's Z and X.*

*Judy: Ok, Z and X. Let's try it.*

*David: Rot, 2, Z.*

*Judy: How many degrees?*

*David: Umm, 20.*

*Judy: Now space and X and how many degrees for the X?*

*David: Oh, 10?*

*Judy: What happened?*

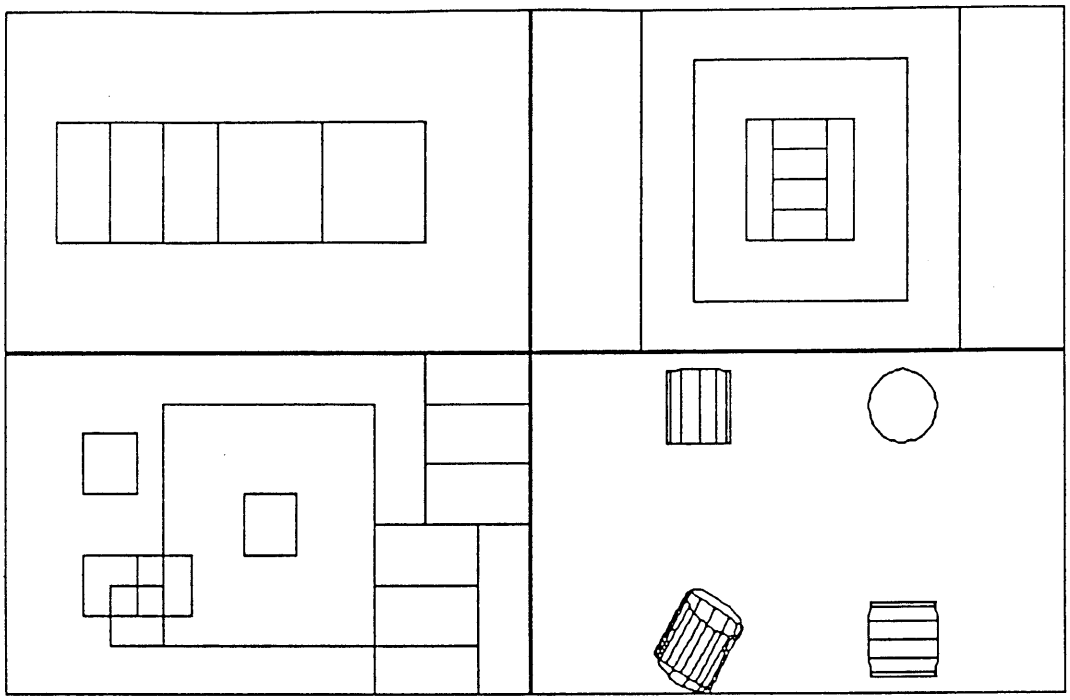
*David: Oh, I turned it the wrong way. Yeh, I was turning like that. So "d" 2 -20* [mumble] *a little bit more,*

*Judy: In which one?*

*David: In Z and X.*

David went from z 20 x 10, to z -20 x 15, and finally finished with z -25 x -20. Through David's work on these four exercises, we can see in Figure 21 there was not a great deal of trial and error. David was quick at making connections between the different concepts and understand how to use them in J3D. It is as if he only had to learn the syntax. He could quickly correct his errors on his own and generally adjusted to the system quickly. These learning exercises were easy for him and by the second session he was working on his own. In his first project, shown in Figure 5 (chapter 8), he applied concepts that he learned in these exercises. Appendix D contains David's design notes, comments, drawings, and scripts for his J3D projects.

Figure 22: Jenni's first day of exercises.



```
;jen.3_25
;Fri. 3/25/1988, 11:49:41
cal sq rut
dis
cal sq emi
cal sq aury
dis
pla rut -3 0 0
dis
pla rut -2 0 0
dis
pla aury 2 0 0
dis
;save jen.ex1
;jen.3_25
;Fri. 3/25/1988, 11:59:44
sho
sho rut
res all
dis
sca emi .5 0 0
dis
sca emi .5 1 1
dis
sca emi .5
sho emi
dis
sca aury 3
dis
sca aury 2
dis
;save jen.ex2
pla rut 4 2 0
dis
sho rut
pla rut 4 1 0
dis
pla rut 3 2
dis
pla rut 3 -2 0
dis
pla rut 3 -1 0
dis
pla emi -3 1 0
dis
pla emi -3 -1 0
dis
pla emi -2 -1 0
dis
pla emi -2.5 -1.5 0
dis
;save jen.ex3
;jen.3_25
;Fri. 3/25/1988, 12:46:12
del 1
dis
cal cylnd n
blue
dis
dis 1
dis 2
rot n 1 x 90
dis 2
;save jen.ex4
;Fri. 3/25/1988, 12:56:46
```

**11.3.2. Jenni Learning J3D.** She is a very petite, vivacious, energetic, warm, and friendly child. She was a real delight to work with. Like David she has confidence in herself and she is liked by other children and teachers. She is a very outgoing girl. Because she spoke extremely fast, and because of her accent, I sometimes had difficultly understanding her. She also spoke a great deal with hand gestures and body language. Like David, she took learning J3D very seriously.

Jenni's main focus through the projects was on making picture of things that were *part of her life.* All her objects were named after things or people that meant something to her. For her animation she used the bow that she had made for her girl on the first project (see Appendix E, which contains her projects). Her reason for doing the girl was because "it symbolized *US* females." She created a snake for her final project, because her friend and neighbors had a pet snake that fascinated her. As we go through some of her work you will notice that all her objects had names of people, (e.g. rut, auri, and emi were siblings or cousins). Learning about the J3D was important for Jenni, but so was building a relationship with others through her work. For example, on the first day that we worked together at the computer she wrote in her notebook "I learn a lot of things. How to rotate cylinderse and etc. I like having Judy for my 2 (second) computer teacher. She is fun." In slight contrast to this David on his first day wrote "I think this is fun! I want to make animation."

As much as Jenni's interest was in the social situation, when she started on her projects she was worked independently.

Jenni went through the place, scale and translation, and started the rotation exercises in the first session of forty-five minutes. At the beginning of that session I told her we were going to review the coordinate system. She said "coordinate system? Rotate, translate, and I forgot the other one." As soon as I said we had "an X axis," she finished with a "and the Y and Z." She remembered many things from the lecture. She did not have a great deal of difficulty understanding the spatial concepts of the system. Below we will follow Jenni through the same exercises that we just saw David complete.

**Translation Exercise.** We will start with the translation problem where Jenni was being introduced to J3D. She had decided that she had to move one of the boxes "left 3." She decided that this means X 3. When I asked her how she could move it to the left, she said "X, X minus"

*Judy: Let's see if your -3 was the right number. Display, D I S.*

*Jenni: WOW!*

*Judy: Close, but you moved it a little too much.*

*Jenni: Yeh, try 2.*

*Judy: Ok,*

*Jenni: Let's do aury.* [She had three squares, rut, aury, and emi.]

*Judy: Let's put this one in the right spot first.*

*Jenni: P L A rut -2 0 0, return.*

*Judy: Are you sure that is in the right spot before you display it?*

*Jenni: Yeh.*

*Judy: I think you are right, too.*

*Jenni: Yeh!*

*Judy: So what do we have to do to put the other one over here?*

*Jenni: You have to put X, I mean 2 plus, plus 2 X, pla aury 2 0 0.*

*Judy: Do you think you are right?*

*Jenni: Yeh.*

We can see the confidence Jenni had in working with the system. When she was dealing with whole numbers, both positive and negative she was able to perform quite well. It took her the same number of trials to do this exercise as for David. It took both of them ten minutes to learn about translation and to do this exercise. In Figure 22 (upper left quadrant) we can see a tracing of the commands she entered. If you examine Jenni's spatial framework in Figure 13, you will notice that Jenni had many holes in 1-D and 2-D Euclidean tasks. In the next exercise we will see the difference in Jenni and David's performance of the scale exercise. Often Jenni had problems with decimal numbers. In fact, one effect that working with J3D had on Jenni was

an improvement in fractions, especially in decimal fractions (pre-test .32 and post-test .83) and on measurement (Euclidean tasks, as we saw in the previous chapter).

**Scale Exercise.** I had just introduced scaling as multiplication, and the notion that scaling by 0 would make something disappear, (e.g. 1 X 1 = 1, and 0 X 1 = 0). At the beginning of the scale exercise, below Jenni decides to re-create the smallest of the three squares. Most of the children, when given the choice, chose to do the smaller square first, because they thought that it would be easier to do something small before doing something big. As it turns out, it is more difficult in J3D to scale something smaller than the original object, than to scale something larger than the original.

*Judy: How would I get a number between 0 and 1? Can you think of a number smaller than 1? It can't be zero remember or the square will disappear. Can you think of anything?*

*Jenni: Minus 1 times 1?*

*Judy: That will give you a -1. That is a tough one, huh?*

*Jenni: Yeh.*

*Judy: So let's look at this problem in another way. We have zero here and one here. What is part way between zero and 1.* [I drew a number line on a piece of graph paper for her.]

*Jenni: One half.* [She wrote 1/2 on the paper.]

*Judy: Can you think of another way to write 1/2? J3D doesn't understand 1/2 written like that. What's another way to write it?*

*Jenni: H A L F.*

*Judy: Good, but it won't understand that either. It wants a number. Let's do it this way. Here is zero (you are broke) and here is one (you have a dollar).*

*Jenni:* [Jenni laughs] *50.*

*Judy: How do you write 50 cents.*

*Jenni: 50 F I*

*Judy: Can't you just put a little dot here?*

*Jenni: Oh yeh!*

*Judy: Decimal point 5 0. How would I write a quarter of a dollar?*

*Jenni: Point 25.*

*Judy: How would I write 3/4 of a dollar?*

*Jenni: Point 75.*

*Judy: So if I want to multiply 1* [the original object] *and get 1/2* [an object 1/2 the size], *how do you think I would do it?*

While we were talking, I was drawing two different size squares on graph paper so that she could see what we were discussing. I also wrote down:

```
 1.0
X .5
-----
```

She immediately knew the answer when written in this form and said "point 50."

This session reflects the way this concept of multiplying to make something smaller was introduction to most of the children. As with Jenni, once the children saw the number line and thought about one as one dollar they didn't have much trouble coming up with a decimal fractions. Jenni's familiarity with these concepts in the monetary system made the task easier for her.

*Judy: You have three objects in there. Do you remember the names of the three objects?*

*Jenni: Rut, emi, and aury.*

*Judy: Which one do you want to scale smaller?*

*Jenni: Um, emi.*

*Judy: Type in emi for me, then a space.*

*Jenni: Point 5,* [Jenni types in "sca emi .5 0 0"].

*Judy: Now look at this picture. What happens if you multiply zero times your Y axis. What happens?*

*Jenni: What happens?*

*Judy: Yes, what is 0 times 0?*

*Jenni: Zero times zero is zero.*

*Judy: Right. So the Y would become zero. You will get a line just like this. Do you want to try it? You already have it typed in. Let's take a look.*

*Jenni: Ohh! I have to change the 0 0.* [She mumbles as she types "sca emi .5 1 1"].

*Judy: Ok, display it. What happened?*

*Jenni: It gets bigger!* [The 1 in Y would make it twice as large in Y as in X.]

*Judy: Ok?*

*Jenni: It has to be point 5.*

*Judy: Ok, let's display it and see if you got it right.*

*Jenni: Ooh!*

*Judy: Ok, now let's do the big one.*

*Jenni: Oh that's a tough one.*

*Judy: Think so?*

*Jenni: Yeh, I think it's 8.* [8 would have been off the screen for where the eye was placed, but she would not have known this.]

*Judy: Right here we have 0, .5, and 1. Now, show me on the screen where you think 8 would be.*

After pointing out to her where "1" was on the screen, she thought that the size might be 4, and she put her finger where she thought the edge of a box scaled by 4 would be on the screen. However, when she entered the command she typed in 3, using a single number for uniform scaling. She displayed it and it was too big. She looked at me, and I asked her "how much"? She immediately said "2" and entered the corrected command.

Jenni's adjustment to the magnitude of the numbers used in J3D went fairly quickly. Many of the children were used to the fact that fifty units in Logo are equivalent to about two inches on the screen. In J3D a square scaled by 2 was about two inches at the default eye (0 0 10). But she adjusted quickly to this new environment and the spatial units.

**Scale and Translation Exercise.** In this exercise, Jenni was asked to place the three squares that had been scaled (see Figure 4 for the original image shown to Jenni). This task was the most difficult for Jenni. Her trials can be seen in the lower left quadrant of Figure 22, which shows a trace of the her work. In this exercise, the three scaled squares were placed adjacent to each other in two dimensions. Both the size and the extent had to be kept in mind for this task. Jenni moved "rut" 4 units to the right, which put that square at the right edge of the screen with the right half of the object clipped. She then moved it down one unit in Y. These two moves form the three squares in the upper right corner of the lower left quadrant. She then moved it 3 units to the right and up 2 unit in Y. She had just placed it at 4 1 0, then she tried 3 2 0.

*Judy: So what is the number? 3 was too far and 2 was too near.*

*Jenni: Yeh.*

*Judy: So it is?*

*Jenni: It is 1 or .5 Point 5?*

*Judy: You need to move .5 more that direction.*

*Jenni: Oh, so it's minus 1.*

*Judy: No, let's do it this way. -2 is not far enough, right?* [I draw a number line and mark 0, -1 ,-2, and -3.]

*Jenni: Yeh.*

*Judy: -3 is too far.*

*Jenni: Yeh.*

*Judy: So since we are going this direction from the origin, it is -.5. What is a number between 1 and 2?*

*Jenni: Between 1 and 2 is 1 1/2.*

*Judy: Very good, so what is a number that is between 2 and 3.*

*Jenni: 1, 2 1/2*

*Judy: So you need?*

*Jenni: 2.5!*

When Jenni was focusing on how much further an object had to be moved in relation to where it currently was, she chose the wrong referent for figuring out the command. However, as soon as she saw the picture she could adjust her commands. Jenni, like many of the other children, had some difficulty with negative numbers because she thought of the negative sign as an operation. Jenni has also shown us that the notion of subtraction was related to making something smaller. She was quite able to come up with fraction to make something smaller if she did not think about it as an operation of multiplication. For example, if she could think of it as a fraction, like half, or one fourth in relation to the original object it was easier for her to scale objects smaller or larger. In her rotations exercise, as we can see in Figure 22 lower right quadrant, she had no trouble understanding rotations. It was merely a matter of learning which axis turned the object in which direction.

### 11.4. Style of Teaching and Learning.

I believe that issues of style are very central to spatial cognition. This is why I reviewed the theoretical background concerning style issues relating to gender and space earlier. I will discuss some of the styles I observed using what we have seen of David, Jenni, and Tiffa.

From analyzing the transcripts, and from my interaction with the children I came to realize that I had a relational style of working with the children. I was in fact using different techniques, hints, questions, methods, in helping the children toward their own proximal development. As we saw above, my interactions with David, Jenni, and Tiffa were different. I was able to work with them in different ways, sensitive to their styles of work and their spatial abilities. The pre-tests and pre-tasks helped me acquire knowledge about the children. This knowledge was later implemented in the way I decided to help each of them toward their zone of proximal development. I used a different language and different methods with each child, and I was reflective about their personal development and knowledge in my questioning and hints. I did not do the thinking for the children, but I found ways to interact with them, so that they could do the thinking. Some of them were able to appropriate or internalize what I had to say. The type and quality of

feedback and scaffolding reveals many style issues.

By looking at the patterns of speech, the duration, and amount of what was said to each child and how each child responded in the learning sessions, I was able to gain insight into our dynamic of discourse. Much in the style of Lampert (1988) on mathematical discourse, I tried to establish a spatial discourse between the children and I that enabled them in developing their concepts and skills. For example, the pattern between David and I shows that David often cut me off and also responded with very brief answers. He was a very independent learner. He was an "explorer" of the system, and did not want any maps or other hints. On the other hand, the pattern of speech between Jenni and I is more like a symmetrical alternation between the two of us: the duration of each part is often close to equivalent. It was more of a negotiated "dialogue," almost in the sense of what Buber (1966) calls a dialogical relation. The pattern between Tiffa and I shows long explanations by me and single word responses on Tiffa's part. From this we can already see different patterns of styles and cognitive processes in the interaction.

I also found that this pattern revealed both the quality and style of our interactions. By looking more closely at the content, it is possible to see the quality of the scaffolding that each child required. Children required different types of scaffolding; some psychological, some emotional, some social, and some cognitive. Thus I proceeded differently for each child. The data is so rich that it will have to be analyzed in greater depth in the future. However, let me give a few examples from children learning about the system and the social context in which they learned. For example, Melvi required an advocate. He was a very capable child and had almost no difficulty with the system in the first learning session. At the beginning of the second session, I found out that his father was an architect and used CAD system in his work. I told Melvi that he was using a similar type of software as his father. From this moment on, Melvi would always look at me for approval prior to displaying his pictures, or ask me if his commands were correct, even though he could actually display them and get feedback on his own. I had to continually encourage him to display the picture on his own. His fear of making a mistake and his belief that there was only one correct answer which he may not

have, somewhat impeded his progress. Another example, is Pauld, a very lonely child who needed social scaffolding. In his regular classroom he had "acted out" so much that his teacher kept him at a table by himself (he was the only child in the class alone like this). He often played the "helpless," which of course required a longer duration of interaction. He also wanted to come in during lunch hour to work, but would just sit and talk. Pauld also needs my physical proximity to be able to work. If I started to move away from him he often became distracted. He was a relational learner (Hale-Benson, 1986). Jenni also learned in a relational way, but the actual scaffolding she needed was mostly in the mathematics involved in J3D. What is interesting about this is that I thought of her as a very independent child and yet her notebook is filled with "I need more help from Judy" types of statements (see Appendix E). My final example, is Tiffa. She was able to work in J3D, however she also needed scaffolding in a way that is different from Jenni, Melvi, or Pauld. Tiffa created, what I believe to be one of the most complex and difficult animation projects (see Figure 6). She could and did work on her own, however only after we had planned both the strategies and commands together. She needed a great deal of scaffolding in planning and strategy. However, once she understood what she needed to do, she could work very hard on her own. Tiffa was usually a quite independent learner, but because of the complexity of the system, she gradually shifted to be a more dependent learner. Recall that Tiffa was a child who regressed in her spatial scores after the study. I believe that this may have been because interacting with J3D has made cognitive restructuring more difficult to her. This finding is related to Witkin's view (1981) of field independence and cognitive restructuring.

One other issue is related to the scaffolding. What a child needs for learning is how to construct their knowledge of space and how to appropriate the language and concepts from J3D and from my explanations of J3D. What each child appropriated or "made their own" appears in their explanation of how they would teach another child (Harel, 1988, 1990). For example, David tells us that he would teach others in the exact order that he had worked and solved problems on the system. He was able to say what was essential in using the system:

*Judy: How would you explain the system to other children in your class?*

*David: Well, I'd tell them that it's a system where you can have, you know, different shapes and put them together and turn them and see different places with the computer. And then I'd show them how to place, rotate, and how to scale. Then I'd show them how to group and attach.*

*Judy: Anything else you think you would need to add to help them understand how the system works?*

*David: Yeh. Tell them about x, y and z axises, the axes. And, you know, tell them that objects are like one in 3 directions.* [The extent of the objects.]

*Judy: That's right. So what would you teach them? Let's say you were going to give them a class, what would you teach them in your first class?*

*David: Um, about the axes, x, y and z, and how to rotate, place, and scale. And how to call in objects.*

*Judy: What would you teach them in the second class?*

*David: Um, how to group things. I'd tell them how to group things, how to attach things, and how to get them to put them, to know how, to show how to get it so, like in the right place so that it's right next to them so you can group them.*

David appropriated the language and the concepts of the transformations available in J3D, and his answer showed that he would teach these concepts in the order he learned them. On the other hand, Jenni appropriated the language and concepts of J3D, but she also *internalized* my explanations and language as her way of knowing the system. She also was more interested in communicating *how* she would teach to another, rather than *what* she would teach.

*Judy: Ok, if you were going to teach a class to others in your class. What would you teach in the very first class so that they could use this system?*

*Jenni: In the very first class I will teach them all the commands that kimba use and that stuff. And how to get them and the history of it and how to go to the files, and all that stuff.* [Kimba was the name of the hard disk for the computer.]

*Jenni: In the second class I will teach them how to use the keyboard and if they had a big problem I could help them and do some like projects they have to do and all that stuff. And, and two I'll teach them the data they have. And if like they need help, I will teach them how, to help them and how to go to the files, and how to go to J3D. And what to do when you get into J3D, that you have to type your name. And then the second question they ask you if you need any help.* [This was a message to the user on the system.]

*Jenni: I think, if I was a teacher, I would have to explain a lot and how to use the rotations, place, and scale because they haven't used it. They have only used Logowriter and they go like LT 90, that means LT left and RT right. And to place it goes FORWARD and BACKward. But in kimba, with the program that we are using right now, is rotate or scale, place, but I think I would teach, I would really have to explain that a whole bunch, a whole bunch of times.*

*Judy: How would you explain it?*

*Jenni: I'd explain that what they have to do is R O T for rotation and they have to put the X rotation, or the Z or whatever, or the Y. If they want to put it, to place, it will be difficult for them because they have not used that, that command. And so I will have to put like place and I'll have to teach them how to place it. Then what they will have to do is look at the paper and like it has a 1, 2, 3 minus and a Z and X and in Y. And they're gonna have to look at things that they will have to do to put it where, to want to place it. And for scale what they use -- in Logowriter to scale it, and like it is difficult to scale in Logowriter.*

Jenni's explanations of the system did for others what I had tried to do for her. I used concepts that she knew and was familiar with, from Logo, to explain something new to her. I also helped her learn her way around the computer and I was there to provide explanations for her when she requested help. We see these same kinds of concerns in Jenni's proposal of how she would teach others about the system. We also see that Jenni did understand J3D in a way that was different from David's more formal and explicit understanding.

Jenni and David represent two different ways of interacting with J3D and with me. Each child wanted something different from me and each child required different kinds of assistance. Jenni wanted a connection, a dialogue, a relationship, while and David used me as a source of information. These two children typify a number of style issue which I see as coalescing. There is the soft/hard mastery of Turkle (1984) and Turkle and Papert (1990), the field dependent/independent of Witkin (1981, 1978), and the analytic/relational learner of Hale-Benson (1986).

David demonstrated his hard mastery in his planning and his scientific approach. He would come into the sessions on the computer with all the coordinates for a new object (e.g. his sword) already written down on a piece of paper. In his first project he came with a drawing on graph paper of a person with each part labeled for the geometric primitive that he would use. Once he understood the system better, he did not even bother writing these kinds of things down. He could tell me all the commands that he would need and reconstruct whatever else was necessary. We saw this handling of the coordinations of views task (11.2.2) when he was asked where he was seeing something from. He tried to give me a possible coordinate point for the eye. The focus of David's animation (see Appendix D) was on the details of the commands for the movement for his sword. This was evident in his animation, because it was not that interesting to watch, but it was interesting for the complexity of the transformations he was trying to control. In the post-interview, his entire focus for his animation was on the complexity of rotating his sword in two different axes and moving the eye simultaneously.

David could also be described as a field-independent child. He related to space formally and often chose the most invariant and global referent for solving spatial tasks. In his description of the spatial layout he was the only child who used a formal description of the setting: "First get a grid 3 X 3." David was able to restructure and organize his space to solve the spatial tests and tasks with a high degree of consistency. If we look at David's performance on his Euclidean and Projective spatial tasks (Figure 18), we can see that he was able to measure in 2-D, and stated that he just "measured from the side and the top." His explanations to other children were tied clearly and

articulately to the formal representation of the Cartesian coordinate system. This is consistent with David's independence as a learner and especially his resistance to my interventions and my strategies. In other words, David could also be considered an analytic learner.

David is field-independent and has an analytic learning style, but this does not preclude his intense engagement in what he was doing with J3D. A look at his notebook in Appendix D, reveals that most of his comments were related to how much he like doing his projects and/or a status report of his current project. He focussed both his intellect and his aesthetics on the more technical aspects of the system.

Jenni is a child who approached J3D as a soft master. Much like David, she was very engaged in her work and in her learning process. However, her engagement was in her *relation* to things, people, ideas, as well as in the aesthetic and emotional content of her work and environment. When asked to "plan" a project she would draw lots of pictures, however, the drawings often did not have anything to do with her projects (Ackermann, 1990). We saw a similar "artistic engagement" in her pre-task in section 11.1.2, where she talked about drawing the sky. On the same problem on the post-task (section 11.1.2., in her letter to a friend), she shows us through her gluing instruction that she is a builder and a concrete thinker. This type of thinking was also revealed when she created her projects. She had an initial idea, and then she would start in a *bricoleur* fashion to add parts or new objects, look at them, think about them, then make more changes and adjustments. While David entered several commands with changes in more than one axis at a time, Jenni seldom did. She understood that the J3D allowed this, however, she wanted to see the resulting picture after almost every change she made. All of these incidents point to a negotiating way of interacting with J3D. Another aspect of her soft, relational style we have seen in her naming of objects. Everything was named after something or someone connected to her personally. She was also the only child who even noticed that the name of the hard disk on the system was called "kimba." Most of the time when she referred to the computer she called it kimba. Jenni anthropomorphized, not only the computer, but she even made jokes about the "little mouse" attached to

"kimba."

Jenni is also a child who could be described as more field-dependent. We saw her tendency to be swayed by the visual image on the screen. In sections 11.2.2. I discuss how she seemed to switch back and forth between a rotation and change of view strategy. Each time she would focus on a different part of the display she would switch strategies. She was also swayed if I questioned her on her choice. This last example of behavior is both related to her field-dependence and her relational learning style. In addition, in almost every entry in her notebook (see Appendix E), she mentions me either as a helper or someone she likes. For example, on 4/25/88 she wrote *"I was confused at the first time. But then Judy helped me a lot. I think I am finished with my animation. I like Judy a-lot. Judy is a good helper. She is nice and sweet. I thank her a-lot."* If we compare this to David's comments in his notebook, we can really see the difference between them. When he finished his animation he said "I finished my sword animation."

**Summary.** Jenni and David provided fairly clear examples of an emerging "clustering of styles" (e.g. Weir, 1986). The tendency for this emerging clustering was found in many of the other children. This clustering that I am referring to can be captured in relation to Jenni and David as a continuum with hard mastery, field-independent, analytic learner on one side and a soft mastery, field-dependent, relational learner on the other end of the continuum. What becomes especially interesting in the analysis of these children, is that this clustering also corresponds with how they related to space (especially the coordinate system) and how they related to me. In other words, their particular style was evident across the board.† For example, David understood and related to space in a very formal and logical way. He related to J3D, the computer, and to me in a very similar manner. Jenni on the other hand, was

† The children in the matching process (that was done for the purpose of population selection for the J3D group) were not categorized or tested for styles, but these categories or clusterings emerged through their learning processes, cognitive processes, problem solving, pre and post tests and tasks, through their language about space, as well as in their relation to space, to others, to the environment, and to me. This clustering came about as an emergent phenomena, it increasingly took shape and was revealed through the analysis of data.

relational in her understanding and use of space and in her interactions with J3D, with the computer, and with me. As mentioned in chapter 4, Witkin finds that the ways in which people solve spatial tasks (e.g. water level task, hidden figures, and perspective) is related to field dependence-independence. He also claims that the differentiation of articulated or global (reference frame) and degree of autonomous functioning (self-object differentiation) could be determined from his spatial tests. David and Jenni are good examples for a strong correspondence in how they relate to the reference frame, to J3D, to their knowledge, and to me as a coherent clustering.

## CHAPTER 12

# CONCLUSIONS AND DISCUSSION

### 12. Principle Findings

In this last chapter I give an overview of what the dissertation is about, and what has been achieved by it. Part of what I shall say here takes the form of conclusions: propositions which follow from the work. However, some findings cannot be summarized as a set of concise statements. I created a unique learning environment and observed children at work in it. Both the idea of 3-D computer graphics as a learning environment and the body of data are contributions to science. I hope that the corpus of data about these children will be of value to people with interests in diverse aspects of spatial thinking and learning -- value that goes beyond any summary statement.

The principle propositions that come out of my work are the following:

(1) I have demonstrated that Euclidean and Projective spatial concepts are related to each other and that the reference frame underlies both. The reference frame, more importantly, is a crucial element in spatial transformations.

(2) I have offered evidence to show that learning, even quite small amounts of learning, can drastically change performance that is sometimes taken as an index of "spatial ability." This proposition has considerable practical importance.

(3) I have shown that there are great diversities in the ways in which children think spatially and that these styles are strongly related to other aspects of their personalities.

**From this study all three categories of spatial ability (i.e. mental rotation, spatial visualization and spatial perception) significantly improved for all the children."** Other studies investigating these abilities have trained the subjects in solving one or possibly two types of spatial problems at a time (per study), however, in my study, there was no specific training for each type of spatial problem; yet, I found improvement in these types of tasks. The children's improvement was due to transference from their experience with 3-D spatial problems and the explicitness of the reference frame in J3D.

**The main findings from the children's "spatial framework" were the correspondence in performance on Euclidean space, Projective space, and FRCP.** This indicates the importance of the coordination of Euclidean and Projective concepts in spatial "ability" measures. Euclidean and Projective space both form comprehensive systems and both systems have the same underlying stable and external reference frame. The symmetry between Euclidean and Projective concepts reflects the child's construction of the reference frame. J3D's integration of Euclidean and Projective space had a clear impact on all of the children who took part.

**Usually, the coordination of perspectives task has been treated academically and abstractly. In J3D this changes: Children were able to play with rotation and perspective-change concepts, and were even able to choose which they prefer to use.** For coordinating the two, they had to understand the isomorphism between them as well as build invariants. After the intervention, the children were able to imagine more transformations (both object and view) than before working in J3D. All the children were able to solve all of the J3D tasks and many of them were able to clearly understand the multiple solutions and coordinate them.

There was a clear difference in how different children thought about space that emerged through their descriptions of the spatial relations. From various tasks it was apparent that some children thought about the image as a 2-D drawing, while others thought about a 2-D representation of a scene in 3-D space. **Many of the children, in the post-tasks shifted from the 2-D drawing strategy to the 3-D strategy. In other words, through exploring space in J3D, their thinking about space and their representation of space changed.**

This ability to think about a 3-D object (and imagine it moving, rotating, and stretching in space) coincides with the underlying ability to think of these objects in 3-D. This increased facility in representing objects, coincides with the increasing facility to imagining object transformations and how objects appear from different views. J3D provided children with the opportunity to coordinate these transformations, and therefore they were more able to build invariants of both representations and reciprocal transformations. One last aspect of this 2-D to 3-D shift can also be thought of as a shift in the "field:" from field-dependent and perceptual strategy, to a field-independent or focus on what is known (representation of a 3-D scene). This shift occurred most often among the field-dependent children. For example, Jenni, a field-dependent child, was looking at the screen she thought about the image as 2-D, however, if she thought about how she would create the scene, she shifted to a 3-D strategy. This subtle difference can reveal the style of a child as well as influence the performance on spatial tasks.

**A deeper understanding of the representation of 3-D objects was also exhibited in children's drawings (indirect learning): Their drawings changed to show a greater understanding of perspective, hidden surfaces, foreshortening, etc.** I was not teaching these children how to draw, yet their drawings improved. I saw as much improvement in the representation as I used to see when I taught drawing in high school.

**One of the contributions of this study is its very method of analysis which provided a collection of methods ranging from a grosser- to an increasingly finer-grained analysis (see Figure 9). I moved from a quantitative analysis between groups of children (control and experimental), to qualitative analysis within the experimental group, to a qualitative microanalysis between individuals, and finally to a qualitative microanalysis within individual children.** Each type of data and analysis revealed more about the spatial thinking of children. Through the integrations of the quantitative and qualitative analysis a portrait of the individual child was slowly constructed.

Figure 9 provides an overview of the analysis when looked at simultaneously from the outside and working towards the center, from the group to the individual, from quantitative to qualitative. The outer-most ring of analysis is

the most global and encompasses general trends. As we move from the outside to the inside the data becomes individualized in that it is both more personal for the children as well as more situated and fine grained.

The purpose of the data analysis was to integrate and compare various types of data at various levels of approximation. The first method was a statistical analysis of the quantitative data used to compare the the performance on standard spatial tests by different groups of children (control and experimental, boys and girls). The second method uses more qualitative data and reduces the results to a quantity for comparing the differences among the experimental children. The third method is a qualitative microanalysis of records of all the commands that the children entered into the computer, transcripts of all teaching sessions, notes from the projects, and transcripts from the post-interviews with the children.

## 12.1. Discussion

In this section I discuss five topics in more detail. **1)** Why is 3-D computer graphics a rich learning environment? **2)** Is there really a difference in between spatial cognition and mental imagery? **3)** Can we distinguish between learning and development? **4)** How can we account for individual differences in spatial strategies and representations? **5)** What are the relations between gender, style, and spatial thought? **6)** What are some educational implications of J3D. **7)** And finally, I will consider issues related to duration of learning.

**12.1.1. 3-D Computer Graphics as a Learning Environment.** J3D is an integrated learning environment which offers multiple accesses to spatial understanding. A combination of visual, verbal, formal, and informal modes encouraged, children to reflect upon and integrate different spatial representations. It is accessible for children of all styles. J3D especially provides children with the opportunity to investigate, learn, and coordinate Projective and Euclidean space, while they create images and animation. Children actively used these concepts making them explicit through perceptual and cognitive explorations in spatial problem solving.

Children were not just *learning about* spatial concepts they were *actively and explicitly using* them to construct images with J3D. Transformations of objects and views are specified through the command language which ties the name of the transformation, the metrics of the Cartesian coordinate system, and the object to be transformed with the "concrete" perspective image. All of the operations and their anchoring to the Cartesian coordinate system helped in the construction of the reference frame.

*J3D helps children explore and use the Cartesian coordinate system even if they had not yet attained coordinated measurement in 2-D. Therefore, it is a viable environment to explore these concept prior to acquiring them.*

The children improved their spatial skills quantitatively and qualitatively regardless of their styles and spatial ability. By integrating different ways of dealing with space, J3D enhanced children's understanding of space, as well as their facility to solve spatial problems both on and off the computer. Children corrected their own behavior through feedback from J3D and explored space in ways that are not available in any other medium. Changes in the children's representations became evident in many ways. They were more clear and explicit in communicating spatial information. The language they used to describe space and transformations of objects in space improved, as did their drawings representing 3-D space, and their understanding and use of the Cartesian coordinate system. There was some progress in the mathematics necessary to work in this system, and in the naming of geometric forms. In short, J3D, from a learning perspective, allows children to work with transformations, measurement, visual representations of perspective, and to manipulate these in ways that children do not normally have access.

Many children in the study gained a mastery of spatial concepts by coordinating both rotation and perspective. J3D offers both a choice and dynamic control over these operations. Thus, most of the children began to understand the relationship between mental rotation and perspective change through exploring the differences and similarities in object and view transformation.

The people I know in computer graphics (artists, researchers, programmers, and animators) are truly amazed when I tell them that I am *doing* 3-D computer graphics and animation with ten year old children. When I was writing my masters thesis at OSU, I often talked about how difficult 3-D computer graphics is for the artist, especially compared to a 2-D paint system. It is well known that the mental overhead in 3-D computer graphics is great, especially in a command based system like J3D. Often when 3-D computer graphics is taught in art and art education programs in college, it is taught in advanced or graduate courses. I find it a thrill to know that the children I worked with, although finding this system difficult still enjoyed it and created wonderful work with it, and as a result benefited from it.

Presently 3-D computer graphics is being used a great deal to study various scientific phenomena through simulations, scientific visualization, dynamic systems, and in the design process in many fields (e.g. architecture, engineering, automobile design, etc). I believe that all of these fields that currently use the medium could offer a great deal to the educational field if children had access to it to play with science and to design artifacts. It is in the power of computer simulations that this benefit becomes magnified. Not only is 3-D computer graphics a place where children can explore space, but it could also be a virtual world, where one can go from the microcosm to the macrocosm, from science to art, and back again. 3-D computer graphics has always been an interdisciplinary field of computer science, and it has moved into the world as an environment that both artists and scientist have appropriated as their medium of choice. I believe that this kind of environment also provides an opportunity even for very children to be artist and scientist without the separation we currently see in society.

I believe my *contribution to the field of computer graphics* is the task analysis (e.g. see section 10.1) of Euclidean and Projective tasks and the mental imagery and 3-D computer graphics transformational tasks. Many people are studying and building systems in this field with a focus on the ergonomics, the display algorithms, the hardware necessary for both display and computation, data structure, and of course user interface design, etc. What I offer is an analysis of the underlying spatial components that exist in this domain,

especially for the user, artist, and educator. I also believe that this can be useful for people who are interested in developing the technology, because this theoretical framework can contribute to interface design for 3-D systems. Understanding how people think about space and how different people relate to space, is essential in providing access to powerful computer graphics systems.

Finally, most research in computer graphics is in mathematical modeling, rendering techniques, user interfaces, animation, simulation, scientific visualization, etc. There is very little research on *learning about* 3-D computer graphics, and *what does it mean to know* 3-D graphics. While a great deal of teaching of 3-D graphics goes on in universities, I am not aware of studies which focus on the cognitive processes involved in learning 3-D.

**12.1.2. Spatial Cognition and Mental Imagery.** This study is a contribution to the fields of spatial cognition and mental imagery because of the unique combination and integration made possible through J3D's use of 3-D computer graphics. The main focus is on Euclidean and Projective spatial concepts, which play a crucial role in a 3-D computer graphics environment. The major area of interest and intervention were related to the explication and use of the reference frame, of the Cartesian coordinate system, and on the coordination of measurement in 2-D and 3-D -- needed to plot, place, scale, and rotate objects relative to this frame of reference.

I believe that the battery of tasks developed for this study and the analysis of the tasks can be useful to other researchers in the fields of both psychology and computer graphics because they clarify the spatial components of these tasks. Many spatial concepts are involved in mathematics (e.g. snake, temperature, and fractions), in art (e.g. drawing 3-D forms with perspective, drawing different geometric forms, and drawing and choosing multiple views), and in mental imagery (e.g. representing the movement of object through drawing and through measurement, discussing possible changes of objects and views of objects). But neither mathematics, nor art, nor mental imagery are normally employed to assess children's understanding of space. Integrating these tasks helped me gain deeper insights into the make up of the children's

so called "spatial ability" (especially Euclidean and Projective) and anticipatory imagery (through talking about and identifying transformations and the change of transformations). These tasks combined with traditional spatial tests clarified the role of Euclidean and Projective spatial concepts in mental imagery and transformations in J3D.

When imagining something moving or seen from somewhere else, mental imagery is intricately linked with such aspects of spatial cognition, such as reference frame, coordinate system, and specifically transformations in space. The culmination of anticipatory imagery is not just possible through operations of change of views (perspective) *or* objects transformations (anticipatory imagery), but through the *coordination* of these operations.

*To my surprise much of the literature separates spatial cognition and mental imagery. I see this as an artificial separation.* Most tests on spatial abilities, such as the Cubes comparison, Rotation, Flags, and even the perspective change and reference frame tests and tasks require mental imagery to be solved. In other words, Linn's three areas of spatial ability are determined by the performance on spatial tests using anticipatory imagery. Like the children in my study who reached a deeper understanding of space through coordinating several spatial concepts, I came to understand more deeply the intricate relationship between spatial cognition and mental imagery. I now realize that both are actually intimately linked through the use of a common reference frame.

Some researchers have partially related mental imagery and spatial cognition (e.g. Dean (1972, 1076), De Lisi (1976, 1981), Piaget and Inhelder (Piaget, 1960, 1967, 1971), and Olson and Bialystok (Olson, 1983)). Dean (1976) compared children's drawings of transformations and Euclidean spatial relations and found them related. De Lisi (1976) found that anticipatory imagery and spatial operations (e.g. children's performance on Piagetian water level and perspective tasks) were correlated to performance on rotation tasks. De Lisi shows that when using a discrete medium, children's understanding of the reference frame, distance relations, and measurement correspond to their understanding of geometric transformations (De Lisi, 1981).

Olson and Bialystok (Olson, 1983) unify these previous findings in their theory of spatial representation. Yet, this theory is essentially based on the idea that spatial thought consists of propositions which relate an argument to a referent through a spatial predicate. To them the most important predicates relate to 3-D Euclidean space. The construction of spatial understanding thus requires the explication and elaboration of spatial predicates and relatum. I believe that the reason for the improvement in children's ability to solve spatial tasks is due to the explication of the reference frame. Through naming transformations (syntax of J3D) and through specifying the object and view transformation which are tied to the coordinate system, the reference frame is always explicit. As mentioned in chapter 2 and 10, the reference frame underlies spatial cognition and transformational imagery. The choice of the reference frame determines both the performance of a transformation and the nature of that transformation. When the children's understanding of the reference frame improved so did their understanding of both Euclidean and Projective spatial concepts, as well as their ability to imagine transformations of objects and views.

**12.1.3. Learning and Development.** Investigating individual children's J3D learning sessions, was useful to better understand the *kinds of changes* that occurred as a result of the J3D intervention. The results in Table 9 and Figures 17 and 18 show that the children learned many things from J3D. There was improvement on J3D tasks. There were also changes in what would generally be considered developmental tasks, especially the Piagetian tasks, the perspective change task, fishing lines, and water level task. Children also improved on the many Euclidean and Projective-Space tasks that were derived from Piagetian ideas and tasks about the development of spatial cognition. This brings me to ask: Does the progress observed come as a result of learning or does it indicate development or conceptual restructuring?

Remember that Piaget separated these two processes when talking about mental growth; the first, development, results from genuine learning, and the second is learning in the narrow sense is less robust and is imposed from outside. Hart and Moore (Hart, 1973) think that learning refers to situations in which some information is presented to individuals, and change is

described as incorporating the information, and correcting erroneous responses. In contrast, they view development as changes in organization, or cognitive restructuring, resulting from the interaction between current organization and discrepancy with the environment. In short, *learning implies quantitative changes* and *development implies qualitative changes.* According to this definition, we can say that we found both quantitative as well as qualitative changes in the children participating in this experiment. The children did improve in their performance on most of the tests and task. Yet more important, the children also changed their understanding of reciprocal transformations, which suggests a change in organization.

My claim is that it may not be that easy to clearly separate learning and development. I see three main reasons for interpreting results in terms of both types of mental processes, learning and development. A first, reason is that Piaget's tasks might be more context-bound than previously thought. Piaget's focus was on differences in the quality of thinking between different age groups of children, and not on the performance of a particular child on a specific task, or Piaget did not study how the design of a task could affect children's thinking in different ways. As we saw in chapter 3, section 3.4, the particular design of Piaget's three mountain tasks (coordination of perspective) made this task very difficult. A second, reason is that Piaget's spatial tasks might not measure general levels of operativity. Maybe these tasks capture more the mobility in how children choose different reference frames and points of view, rather than in determining the stability of thinking for any developmental stage. A third, reason is that the conceptual environment of J3D might provide a framework where Euclidean, Projective, and Transformational concepts intertwine. This would enable the children to use this framework to *organize their spatial concepts.* Remember that both Werner and Piaget thought of development as degrees of organization. Maybe we are seeing changes that appear to be "developmental," because J3D offers an organization from which to solve many spatial problems. The environment provides what Papert calls "powerful ideas." Through the organization and integration of spatial concepts, children can think about spatial concepts and problems in a new way. And what we observe are global changes as well as a local

changes in thinking.

At the inception of this research I did not expect major quantitative changes. I though that I would only find very subtle qualitative changes. What I did find, however where both significant quantitative changes and qualitative changes. There is no doubt that learning and development occurred in this experiment.

**12.1.4. Individual Differences.** Through analyzing the transcripts and my interaction with the children, I realized that I had consistently used a relational style in working with children. As we saw in chapter 11, my interactions were different with each of the children. I worked with each child in different ways, sensitive to the children's styles of work and abilities. The pre-tests and pre-tasks helped me gain knowledge about each child that I could later use in helping them reach their "zone of proximal development". I used different methods and language with each child, and I was reflective about their personal development and knowledge in my questioning and hints. By analyzing the dynamics of the interaction within the learning environment, I was able to identify different learning styles, and to propose different kinds of support.

My style of teaching also allowed for multiple styles to emerge and function in this environment. Initially I did not design the system for different styles of learning. In fact, I designed it for me (my style). Yet children could none the less come in with their own emotional, cognitive, social, and spatial functioning. And I tried to provide each of them with an adequate kind of scaffolding (e.g. emotional for Melvi, social for Pauld, and cognitive for Tiffa and Jenni). Each child constructed his/her own knowledge about space and appropriated the language and concepts from J3D and from me in his/her own way. Only the finer grain analysis reveals the individual differences in what children appropriate and how they provide supports for themselves in the construction of their own knowledge. Both *how* and *what* children appropriate -- *make it their own* -- also reflects their style.

Jenni and David provided fairly clear examples of an emerging *clustering of styles,* that I found in many other children. For example, I would describe

describe David, on the other end of the continuum, as a hard master, a field-independent, and analytic learner. What I found especially interesting, is that this clustering actually corresponds with how they related to space (especially to the coordinate system) and how they related to me. In other words, their particular style was evident across the board. For example, David understood and related to space in a very formal and logical way. He related to J3D, to the computer, and to me in a very similar manner. Jenni on the other hand, was relational in her understanding and use of space, as well as in her interactions with J3D, with the computer, and with me. David and Jenni demonstrate the strong coherence in how children of different styles relate to the reference frame, to J3D, to their knowledge, and to me.

**12.1.5. Style and Gender.** I found differences in performance between boys and girls on the standard spatial tests and on the Euclidean and Projective tasks in the pre-tests and pre-task. These findings correspond to previous research in the literature. However, in the post-tests and tasks there was no significant differences between boys and girls. If there were, I would not like to characterize them as gender differences, but rather individual differences in general. Some girls exhibit strategies like the boys and vise versa. The micro analysis of individual cases did reveal differences among individuals but they cannot clearly be related to gender.

I do not believe that the gender issue is as crucial as previously thought. Even Linn and Hyde (Linn, 1989) in a recent article mentioned that gender differences in spatial ability are declining. One reason for this decline might be that education is moving more towards accommodating more styles of learners. I did not see a particular pattern of strategies, or style, that were particular to just boys or just girls. Rather, I saw individual differences that were fairly coherent within each child. And at the end, both boys and girls improved in their spatial abilities. The ANOVA showed no gender interactions, the only main effect was improvement for each individual in spatial performance. This demonstrates that different kinds of improvement or strategies do not relate to gender, but relate to preferential strategies in choosing the reference frame.

**12.1.6. Educational Implications.** J3D may be viewed as a narrow setting, with children isolated one-to-one in a room working on a computer. However, the conclusion we can draw from the examples of the children's learning and developmental changes are important. It is essential to bring spatial concepts into the life of children not only because many of these are related to mathematics and various fields of science, but because spatial ability is also related to a number of professions that actually require this particular knowledge. For instance, admittance to dental or architecture school is dependent on spatial ability. As a matter of fact many of the spatial tests I used were developed for vocational and guidance "screening." However, we should not always be concerned with what children will do in the future, but provide them with meaningful experiences here and now, helping them develop general thinking skills, flexibility, grouping, coordination, planning, and solving complex problems. Complex systems and virtual environments, such as J3D, provide an opportunity for exploration which stretches the children's thinking here and now in ways that are meaningful to them. In math, children have to memorize a definition for a point and a polygon, but it means nothing to them. When they were creating data in J3D, they needed these concepts to create images. The children were not *learning about* these concepts but they were *using* them. In the virtual world of J3D they were manipulating objects in space through doing art, doing animation, doing Euclidean mathematics, doing geometry, doing decimal fractions, doing space, doing mental imagery, and doing 3-D computer graphics.

The children also benefited from working in both Logo and J3D. This is an advantage because it is often easier to learn a second, third or even forth software environment. In introducing children to 3-D I could use what they knew about 2-D computer graphics through Logo. This made it possible to address the differences between 2-D vs. 3-D space in computer graphics and in spatial content. For example, children more often explore Topological spatial concepts in Logo, dealing with space in a more relational manner; and in J3D, they explored the Euclidean and Projective spatial concepts. This made the exploration richer and provided a flexibility not available with only on type of experience. The children brought with them what they knew from Logo to

J3D and returned to Logo with what they learned of J3D. One child, David, mentioned that he finally really understood SETPOSition (a command that takes [x y] as an input to position the Turtle on the the screen) in Logo after having worked in J3D . Another advantage of J3D is that it makes it possible to think about numbers and space in a way that was different from Logo. Children could learn about the difference of what numbers mean in Logo and J3D. Within J3D (though a change of the eye), numbers take on a relative proportion, where units reflect different distances and the number of units represents a different dimension in these different settings. These types of experiences help children think of numbers, measurement, and space in new and hopefully more cohesive ways.

If 6 hours of intervention had a significant effect on the children who participated, and if we believe that this knowledge is important for children to master, it is important to incorporate spatial concepts in educational activities in the schools. For example, in regular school work Projective and Euclidean concepts are seldom addressed. When they are, for example in graphing charts, the spatial components are normally not explicitly addressed. Currently, many researchers study children's understanding of graphs, but they do not address the underlying spatial concepts involved in graphing per se. The difficulty may not lie in the coordination of the two variables, but in the coordination of the space in which the concepts are being represented. The spatial components of graphing needs to be analyzed, as do spatial concepts in other educational activities in the schools.

Let's look again at David and Jenni in relation to this issue of educational implications. Children like David are often ignored in the existing educational system, which is designed for the "average student." But he needs challenges too -- which J3D provided for him. When I asked him if he liked using J3D, he said "I loved it!" When I asked him if other children should learn how to use J3D, he said "Yeh, it's neat and it's a lot of fun. Plus I probably, I want to be a computer engineer when I grow up." J3D was an exciting place for David to play and to learn. He benefited from it now and it may also benefit him in the future. On the other hand, children like Jenni (who started with very low spatial ability/knowledge) do not get this

knowledge in the existing educational system and will not have equal opportunity. J3D was challenging for all the children, no matter what their level of math or spatial ability, yet at the same time it provided an environment where all of these children could explore and learn.

**12.1.7 Duration Considerations for Learning.** It is important that spatial cognition be considered as a topic in the school curriculum. Usually it is not viewed as something that can be taught, however as we have seen in this study a great deal of learning went on. The duration of this study was fairly short, and yet improvement was obvious. I believe that if it had gone on longer the children's mastery of J3D and the concepts involved would have continued to improve. However, if we compare the time frame of this study with the duration of an average school project, this was a long period of time. Ideally I would have preferred to let the children work with J3D for a longer period of time, in a more integrated way (with other class activities) † but I could not due to the school's scheduling constraints the study was shorter.

## 12.2. For the Future

**J3D &Logo as Environments for Spatial Explorations.** Several claims were made here in relation to 1) the advantages of J3D in encouraging and facilitating spatial thought in children; and 2) the connections that were made, during the process of learning with J3D to their knowledge of Logo. One aspect that requires further investigation is related to the differences between J3D (3-D) and the way the Logo (2-D turtle graphics) deals with spatial concepts. One way of exploring this issue further could be by conducting a comparative study with children who are only using Logo, identifying the ways in which they understand and use space in their programming. The microanalysis and observations methods which were implemented in this study could be used in the same manner for this purpose as well.

**Further Investigation of Children's Rational-Number Knowledge.** Another aspect to be investigated is related to Harel's (1988; 1990) studies about

† In the way that Logo is used at Headlight.

children's learning fractions through instructional software design. I claimed earlier, that children needed to use a great deal of rational-number knowledge during their work with J3D. I also found that children's fractions (and decimals) knowledge improved after my experiment. In fact, Harel and I worked with the same children. Since, I conducted my study a year after Harel, it would be interesting to use my data and compare it to Harel's data. Adding a longitudinal aspect to our investigations of children's understanding of fractions and their representations through their use of different media or systems.

**Further Analysis of Children's Geometrical Knowledge.** Many geometrical concepts were explored by children through their work with J3D. Geometry was also being taught in the traditional math groups. A further analysis could focus on all the children's work in their regular math class, compared with their results on the geometry-related items in the pre- and post-tests, and also compared with their results from the pre-post spatial tests. In addition, many geometrical concepts are involved in the use of Logo. It will be interesting to investigate qualitative difference among how geometrical concept are being presented, learned, and used in these three environments (i.e. in J3D, Logo, and traditional school curriculum). Beyond the geometrical concepts themselves, it will be interesting to investigate the spatial understanding that grows out of each particular approach.

**Refining the Analysis of Children's Knowledge of Euclidean & Projective Space and its Relation to Cognitive Styles.** Originally, I planned to conduct an in-depth comparative microanalysis of 20 children (10 experimental, 10 control). Specifically, I wanted to compare the results from the Euclidean and Projective spatial tasks and to use this data to look more closely into issues related to these children's cognitive styles. However, only preliminary investigations of this kind were actually implemented by me, and for reasons related to dissertation-length constraints, I decided to exclude it from the final written thesis. I do feel, however, that such an analysis, if done in the future, could reveal interesting information, and could be a contribution to the field.

**A Finer Analysis of J3D Online Protocols.** During my study, I collected a great deal of online data about the children's use of J3D commands. I used some of this data for my discussion of individual's knowledge in Chapter 10 and 11. The children's code is a representation of their level of understanding and a representation of their process of thinking about spatial concepts (such as transformations). Therefore, an analysis of their code could assist us in investigating their trial and error techniques, and locating the moments and places where they had difficulties, problems, and insights. Tracing their process of thinking through their code could allow us to understand the particular nature of their problems, and determine how children decide between local and global changes (such as an object rotation vs. a view change).

**The need for Improvement in Experimental Design: Exploring Issues Related to Study's Length and Population.** One of my main recommendations for a future study of this kind would be to conduct it over a longer period of time. For example, I felt that by the time the study ended the children were beginning to reach a point where they could explore more deeply the dynamics and complexity of spatial concepts and arrangements. At the time the study ended, the children were still at the low end of the slope of their learning curve. Allowing more time will provide us with more information about how children think about space, rather than how well they use J3D. Moreover, for reasons related to investigating cognitive development, we might want to replicate such study with various older populations. This will reveal insightful information about developmental aspects in spatial cognition.

**Design Considerations for Future System Development.** There are several design issues that need to be discussed in relation to building a future 3-D computer graphics system for children's learning. It is beyond the scope of this section to identify and analyze these design considerations in depth. In general, my interest is in developing such system for the purpose of learning and developing spatial knowledge, and expressing ideas visually and spatially. This consideration will influence the design of future systems in what is made implicit, explicit, or accessible within a system. While J3D was more of script type environment, I recommend to expand it into a full programming

language. In addition, I recommend a hierarchical user interface. For example, a command type interface can be useful in learning about spatial concepts; but addition interactive tools (mice, slides, dials, knobs, etc.) can be useful in learning about the dynamics used for creating animations and learning about motion. In other words, I envision for the future a system that is malleable enough to have accessibility, flexibility, and functionality in allowing several kinds of interactions with the system. Both professional and young learners require different modes, appropriate for the concept they are exploring, the purpose of their explorations, and they ways in which they prefer to interact with the world.

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# APPENDIX A

# J3D REFERENCES

# J3D - Command Guide

```
=====================<OBJECT COMMANDS>==================
call <data> <object>                          [read data and give instance name]
onloff <objectlall>                                          [on/off for display]
bkface <obj> <0l1>                                   [1 turns off backface cull]
reset <objectlall>                                    [resets to default values]
===================<OBJECT MANIPULATIONS>================
scale <object> <x y z>                          [scale object along each axes]
place <object> <x y z>                           [translation to point in space]
rotate <object> <num> <axis> <angle> [<axis><angle>] [<axis><angle>]
                  [rot object num,1-3, about specified axis, and concat in order entered]
=====================<OBJECT HEIRARCHY>===================
attach <obj> <to-obj>                           [attaches one object to another ]
group <name> <num> <objects>                   [group name to attach objects to]
detach <object> <from obj> [<0l1>]              [1 (default) leaves parent xform]
===================<OBJECT INFORMATION>==================
show                                                        [lists all objects called ]
show [<objectlallIviewlhier>]                   [shows info of named parameters]
=============================================================
=====================<GLOBAL COMMANDS >===================
display [<quad> <per=0lorth=1>]                 [ fullscreen =0,  quad > 1 I 2 I 3 I 4]
bluprint [x y z]                              [display oblique, front, side, & top views]
eye <x y z>                                                              [places camera]
coi <x y z>                                   [center of interest, where  point camera]
view <angle>                                  [angle of view of camera from eyepoint]
reset <view>                                                   [resets to default values]
delete [1]                                    [1= forces delete all object, else asks]
clear <0l1>                                          [1= clears screen    0= noclear]
test [1]                                      [disp obj 0 left, obj 1right;1=timed test]
up <x y z>                                          [determines orientation of camera]
vdist <0-2>                                         [distance from eye to view window]
center <u v>                                                   [center of view window ]
===================<SCENE INFORMATION>===================
save                                                          [saves environment to a file]
history                                                  [keeps a record of all commands]
nohistory                                                [stops record of all commands]
print [quad]                                                   [prints screen or quadrant]
pause                                                          [allows  pause from script]
suhistory                                                [turns off automatic history file]
;                                                          [; is a comment and is ignored]
!                                                   [used instead of name to disable history]
edit                                              [causes user to leave J3D & enter HEDIT]
=============================================================
=====================< DEFAULT VALUES>=====================
  eye=0 0 10    coi=0 0 0    vie=45    vdi=1    up=0 1 0   cen=0 0    cle=1
    sca=1 1 1    pla=0 0 0  rot=0    bkf=0    dis=0 0    blu=3 3 10  on
 cube   cone   cylnd  hut  pyr  sq   soccer tri   horse  dog  gator lion   rhino
```

# J3D MANUAL

J3D is a view and transformation program that allows you to display one or more polyhedral objects in perspective or orthographic projection. J3D permits the scaling, translation and rotation of objects interactively by entering keyboard commands. These capabilities combine to make a flexible viewing and transforming environment: you can view objects and rotate them; position objects on the screen to create a more complicated object; or set up a scene and view it in perspective from arbitrary eyepoints. All commands can be abbreviated to the initial three letters. Brackets ('[','] ') denote optional entries; angle brackets ('<','>') denote required entries.

## OBJECT MANIPULATION COMMANDS

**bkface <object> <0|1>**

If value is '0', does not display the backfaces for making the named object. The backface refers to the back of a polygon from the observers position. When an object's backface = '0' they will not be displayed. This will give the appearance of the object being opaque. If the backface is '1' then the object will appear transparent. Default: '0'. [ie. bkf box 1].

**call <data> <object>**

Reads a model file into the current workspace and is given an object name. If the ".asc" extension of the data description is left off, the program will put it on. The same data file can be called, or instanced, many times and given different names. The maximum number of objects is fifteen. [ie. cal cube box].

**delete [1]**

Tells J3D to delete all current objects. If argument <1> left out the system prompts the user to confirm deletion.

**on <object|all>**

If an object is turned on it will be display, if it is turned off it will not be displayed. This allows you to work with one object without having to wait for other objects to be displayed. Off objects are not deleted they are just inactive and can easily be activated by the "on" command. The "on all" command turns on all objects. Default: on. [ie. on box].

**off <object|all>**

Does not display the named object the next time a "display" command is given. The "off all" command turns off all objects. [ie. off box, off all].

**place <object> <x y z>**

Translate the named object in space by the given value. Values can be either positive or negative (+ or -). Default: 0 0 0. [ie. pla box 2 .5 -1].

**reset <object|all>**

Sets all object parameters to their default values, e.g., scale of (1 1 1), translations 0 0 0, and no rotation. "Resets all" resets all objects parpameters back to the the default parameters. [ie. res box].

**rotate <object><num><axis><ang>[<axis><ang>][<axis><ang>]**

Rotate the named object by the given angle in degrees[+ or -]) about the given coordinate axis ('x', 'y' or 'z'). The 'num' tells J3D how many axis will be rotated, and allows you to rotate an object by more than one axis at a time. The rotations are concatenated in order entered. The rotate command resets all previous rotation commands in all axes. [ie. rot box 1 x 45, will tilt the box forward 45 degrees so that you see more of the top of the box; rot box 2 x 45 y -30, tilts the box forward 45 degrees also, then turns it again vertically clockwise 30 degrees so that you see the top and the right side].

**scale <object> <x y z>**

Scale object independently along any axis, or use the same value for each axis to scale uniformly. Scaling is the multiplication of original coordinates of the object by a factor (negative numbers will turn the object inside out). Scale takes one value for uniform scaling (there is a

bug in this command a blank space must be left after the one value). Default: 1 1 1. [ie. sca box 1 2 .5, the box is the same horizontally, twice as large vertically, and 1/2 as big in depth; scale box .5, will scale the box by 1/2 in all dimensions].

## OBJECT HIERARCHY

**attach <object> <to object>**

Sets up object hierarchy. This allows whatever manipulations are also done to an object are done to the objects attached to it. This can be thought of as a type of glue, although objects that are attached can still be manipulated independently in relation to the object they are attached to. [ie. att box cone].

**detach <object> <from object> [<0|1>]**

Detaches one object from another after they have been attached. ´1´ leaves the parent transformation, '0' deletes parent transformation from detaching object. Default: 1 [ie. detach box cone].

**group <name> <num> <objects>**

Creates a name that is used to refer to several objects simultaneously. 'Num' tells J3D how many objects will be in the group. [ie. group new 2 box cone].

## VIEWING COMMANDS

**blueprint [<x y z>]**

Displays four views of a scene. In the upper right quadrant '2' an oblique perspective view is displayed with the default eyepoint (3 3 10), although this quadrand may also be assigned any eyepoint. In the upper left quadrant '1', an orthographic view of the top is displayed. Eyepoint at (0 10 0). In the lower left quadrant '3', an orthographic view of the front is displayed with its eyepoint at (0 0 10). In the lower right quadrant `4', an orthographic view of the side is displayed with its eyepoint at (10 0 0). [ie. blu, blu 2 2 -15].

**center <u v>**

Set the center of the projection plane view window in the projection plane coordinate system. Default: (0 0) positions the window so that the view normal passes through its center. [ie. cen .2 .5]

**display [<quad> <per=0|orth=1>]**

Displays all active (on) objects. Quadrants are '0' for full screen display, lower right. Default is full screen '0' and perspective transformation. [ie. dis, dis 1, dis 2 1].

**clear <0|1>**

Clears the screen before displaying objects if it is set to '1'. If clear is set to '0', screen wil not be cleared prio to display. This can be used to build a trace of motion effect. Default: 1. [ie. cle 1]

**coi <x y z>**

Position the center of interest in world coordinates. This defines the direction of view or where the camera is pointing. Coi is always at the center of the display screen, no matter where it is in world coordinates. Default: (0 0 0). [ie. coi 0 1 0, would tilt the camera up].

**eye <x y z>**

Position the eyepoint or camera in world coordinates. Default: (0 0 10). [ie. 2 2 15]

**reset <view>**

"Reset view" set all viewing parameters to their default values; eye, coi, view, center, up, and vdist. [ie. reset view]

**test [1]**

Is to test pairs of two objects (used for Shepard-Metzler forms). It assumes there are only two objects current in the working environment and will display the first object on the left, and the second object on the right , both from a (2 4 12) eyepoint. It will do this by changing the center of the screen so that the perspective for both objects is exactly

the same. This also prints a prompt requesting (s)same or (d)different. If [1] option is chosen and a history file has been started, it will save the answer and print the length of time if took to answer the question in minutes:seconds:100th of seconds in the history following a ";" as a comment.

**up <x y z>**

This sets "up" vector about the view normal which determines the orientation of camera. Default: 0 1 0. [ie. up 0 -1 0, the picture would be upside down].

**vdist <num>**

Distance from eye to view window. Default: 1.0. [ie. vdi 3, makes the objects in the image appear larger].

**view <angle>**

Set the viewing angle in degrees. This corresponds to the field of view given by various focal length lenses. Default: 45 degrees, the field of view of a "normal" lens [ie vie 10, makes the objects appear larger, vie 90, makes the objects appear smaller].

## MISCELLANEOUS COMMANDS

**append**

Will append the current object parameters to a file. J3D will prompt for the file name. This command is similar to save.

**debug**

This causes the printing out reports on values as they are executed and prints the matrices for "show". Debug is a toggle command. To turn it on, type debug and to turn it off type debug. For "show" if debug is turned on, there is much more information about objects available, however this makes the system is much slower.

**help**

Print the command menu with a short description of the command.

**history** This file will gather all the commands entered through the keyboard and save them in a file. J3D will prompt for the file name. The file name, current time, and date are also saved as a comment.

**nohistory**

Turns off history, puts in the current time and date, and closes the file.

**pause**

Pause stops the flow of execution of commands and requires the used to enter a carriage return.

**print [<quad>]**

Prints the image screen on an attached printer. You can specify which quadrant you want printed. The default is '0', which prints the entire screen. [ie. pri 3, prints the lower left quadrant.]

**quit or q**

Leave the program. Quit asks for confirmation, but 'q' leaves immediately.

**save <file>**

Write a script which records all J3D viewing and object parameters. This script will be output to the named file current time and date. The file may be read in later to reconstruct the session.

**suhistory**

Turns off "super" history that is started up when using H3D. It records the current time and date, then closes the file.

**@file**

Begin taking input from the named file as if it were typed in from the keyboard. Files that it takes in may also have files with @ up to four levels deep. [ie. @scene1.ani]

;

All lines beginning with ';' in a script file will be ignored. It can be used for comments inside the files so that they can still be run through J3D and not be executed.

?

Like help, this prints the command menu, but if brief form.

## INFORMATION

**show**

Display the names of all current objects, their data file, and if they are 'on' or 'off'.

**show** [**object**|**all**|**hie**|**view**] Show displays the names of all current objects, their data file, and if they are 'on' or 'off'. "Object" displays all parameters of the named object. "All "displays all parameters of all the objects with pauses to slow down the output. "Hie" displays the current hierarchy of objects. "View" displays the current viewing parameters.

# APPENDIX B

## PRE-TESTS & TASKS

Johnson and Meade Instructions

Johnson and Meade Batteries (A, B, C)

Euclidean, Projective, and Mathematics Battery

J3D Related Battery

THE JM BATTERY OF SPATIAL TESTS

INSTRUCTIONS FOR ADMINISTRATION

(Middle Level)

The testing program that you are being asked to assist in is entitled "The Relation of Spatial Ability to Mathematics Aptitude." It is being funded jointly by the Virginia Department of Education and the Danville City Schools and involves the assessment of spatial ability of approximately 100 students at each of the 12 grade levels.

The reason for this program of testing is to investigate (1) the development of spatial ability in children, and (2) the relationship between spatial ability and mathematical ability. We suspect that children who are deficient in spatial ability may need specialized instructional methods in mathematics. This study is a first step toward determining if this is so.

PLEASE NOTE: Do not mention to your class any possibility of a connection between spatial and mathematical ability. Do not even bring up the topic. We do not want any students to take a "defeatist" attitude toward the spatial tests because they think the tests may require math ability.

The JM Battery is an experimental set of tests being developed at the University of North Carolina at Chapel Hill by Edward Johnson and Ann Meade of the Psychology Department. This project is the first attempt to use adult-level tests on children as young as 7 years of age. The results of pretesting show that even young children can understand these tests.

Spatial ability involves forming mental images of concrete objects and imagining how they look when twisted and turned. The seven tests in this battery represent different ways of looking at spatial ability.

There is reason to believe that people differ widely in levels of spatial ability. You might reflect on your own "directional" sense, ability to follow maps, ability to assemble toys from so-called "easy-to-follow instructions," ability to understand how machinery works, etc. Whether or not you have good spatial ability, you will be able to do a good job of administering these tests if you familiarize yourself with these instructions and the tests themselves.

In giving these tests to your students, please maintain a positive, supportive attitude. If any students seem to be bewildered by one of the tests, tell them that they may do well on the next one and to keep trying to do their best. Please do <u>your</u> best to maintain a quiet and serious atmosphere conducive to good performance. In order to include as many students as possible, try to give the tests on days when attendance is high (i.e., at least 90%).

We would like to have your written comments on the tests, the instructions, and the time limits. Please feel free to write them directly on these pages. We very much appreciate your willingness to share your opinions with us in order to improve the quality of the tests.

The JM Middle Battery consists of seven tests plus three pages of miscellaneous items. The tests are divided into three batteries each requiring less that half an hour. We suggest that you give each battery on a separate day.

Most of the tests are self-contained; instructions are written on the first page of each test. To begin a new test, simply direct the class to read the instructions and do the practice items. Allow roughly the instruction time indicated in the section below, but use your own judgment. Our main concern is to have each student understand completely what is to be done on the test. Answer any questions and emphasize the points that we have indicated below.

Three of the tests have visual aids to be used by you during the instruction period before the test. The idea is to insure that everyone understands the nature of the test items by looking at a model. Please familiarize yourself with the instructions about their use.

TIMING

Most of the tests are timed. You will need a digital watch, stop watch, or a regular sweep-second-hand watch. Please be very attentive to the count of elapsed minutes on the longer tests. Students should be encouraged to work quickly but accurately. Items that seem especially difficult should be skipped.

Be sure that everyone has a regular pencil and a red pencil. Before beginning a test have students put their red pencil on the floor beside the desk.

Start everyone together. When the time limit (specified below) is up, call a halt. At this point almost no one should have finished the test. Now have everyone put the regular pencil on the floor, pick up the red pencil, and complete the test, working quickly and accurately. Students may go back to answer skipped items but they may not change any answer they made with the lead pencil. Don't allow any erasures during red-pencil time.

Allow time for all but the most laggardly to finish before going on to the next test. To set up for the next one, have the students again place the red pencils on the floor and retrieve the regular lead pencils. At the end of the entire battery, allow the slow ones time to go back and finish (with red pencils) any tests they haven't completed. We would like to have everyone answer every item on every test.

## BATTERY A

[Time required: about 28 minutes]

Procedure

A. Make sure that everyone has a lead pencil with an eraser and one of our red pencils.

B. Hand out the tests and have everyone fill in the front page. No one is to peek ahead in the booklet. "Class" on the front page means Math, Algebra, English, or whatevery the subject of your class is. Under "Name" have students put the first and last names.

C. Emphasize the following points:

1. Spatial ability is the ability to imagine how objects look especially when they are turned and rotated. It is an important ability for architects, pilots, mechanical workers, and many others. The purpose of these tests is to see how important this ability is in school work.

2. These tests will not affect school grades in any way.

3. The tests will be timed, since we want to know how fast a student can work, as well as how accurately. After the time limit, everyone will be allowed to finish the test with red pencils. The tests are not races to see who can finish first. Work rapidly, but accurately. Do not make haphazard guesses. Do skip items that you get stuck on. After the time limit is up, you can finish the entire test, including skipped items, with the red pencil. But do not use the red pencil to change any answer marked with the lead pencil.

4. Never peek to see what's ahead in the booklet.

D. Listed below is the relevant information for each test. At the end of the instruction reading period emphasize orally the points listed and try to answer any questions. Then administer the test using a watch to time it carefully.

BATTERY A Tests

STICKS, JARS, POLES, PAGE

| | |
|---|---|
| Instruction time: | none |
| Time limit | none |
| Expected time to finish: | about 1 minute |
| Emphasize: | (toward end of time) "Please trace down one of the 3 lines for each fishing pole." |

HANDS

| | |
|---|---|
| Instruction time: | about 2 minutes |
| Emphasize: | "You can imagine that the hands are your own hands or someone else's. It doesn't matter. Keep the booklet in place. Do not turn it around." |
| Time limit: | 2:00 exactly |
| Additional red pencil time: | about 3 minutes |
| Emphasize: | "Please use the red pencil to answer all the items you've skipped." NOTE TO TEACHER: The 3 minutes is approximate. Allow time for most to finish. |

SPATIAL RELATIONS

| | |
|---|---|
| Instruction time: | about 3 minutes |
| Emphasize: | "You do these problems by imagining that the drawings can slide around on the paper." |
| Time limit: | 4:00 exactly |
| Additional red pencil time: | about 3 minutes |

HIDDEN FIGURES

| | |
|---|---|
| Instruction time: | about 2 minutes |
| Emphasize: | "The hidden figure can be in any number of the drawings (at least one of them)." |
| Time limit: | 3:00 exactly |
| Additional red pencil time: | about 3 minutes. Be alert for students who fail to realize that there are test items on the back page. |

TO FINISH

Collect all booklets and red pencils after having students double-check that they have filled out the front page correctly.

## BATTERY B

[Time required: about 25 minutes]

Procedure

As before, hand out red pencils and booklets. Have everyone fill in page 1. Be sure to cover the Virginia region of all maps.

Remind everyone about the following:

1. Try your best. Work quickly but accurately. But don't race.

2. These tests will not affect school grades.

3. The lead-pencil part of the test will be timed. The red-pencil part is not timed. During red pencil time you can go back to finish skipped items. But don't change items marked with the lead pencil.

4. Never peek ahead in the booklet.

### BATTERY B Tests

BLOCK COUNTING

| | |
|---|---|
| Instruction time: | about 2 minutes |
| Emphasize: | "All of the blocks stacked together in an item are the same size. Count all the blocks, including the ones that are hidden from view." |
| Time limit: | 4:00 exactly |
| Additional red pencil time: | about 4 minutes |

MAZE-MAP PAGE

| | |
|---|---|
| Instruction time: | none |
| Emphasize: | "Do not turn the page around to do number 1." Also emphasize that it is the mouse's right or left as he is facing each time. |
| Time limit: | none |
| Expected time to finish: | about 2 minutes |

CUBES

| | |
|---|---|
| Instruction time: | about 4 minutes |
| Visual aid: | Place the flat piece of wood on your desk so that the printing is right side up from your position behind the desk. Place the two example 1 blocks in the squares using the colored dots to position them. The class' view of the blocks should look exactly like example 1 on the test. You may have to rotate the entire display so that everyone can get a view of the block's top surface and two forward-facing surfaces. You can give them a moment to figure out the answer and then show them that the answer is "S" (same) by tipping the right hand block so that the "T" is on top.<br><br>Then set up the second example, let the class figure out the answer, and then show that the answer is "D" (different) by tipping the right block to put the circle on top. Now the blocks match on the "square" side and on the "circle" side, but the other sides do not match. This is the time to emphasize: "The cubes have a different design on each side." |
| Time limit: | 3:00 exactly |
| Additional red pencil time: | about 4 minutes |

DIRECTIONS

| | |
|---|---|
| Instruction time: | none |
| Emphasize: | "Don't draw a diagram for number 6. We want to see if you can do this in your head." |
| Time limit: | none |
| Expected time to finish: | 2 minutes. Please see that no one omits the items on the back page. |

TO FINISH

Collect all test booklets and red pencils after having students double-check that they have completed the front page correctly.

## BATTERY C

[Time required: about 22 minutes]

### Procedure

Hand out red pencils and booklets. Have everyone fill in page 1. There is no need to cover maps for this battery.

Remind everyone about the following:

1. Try your best. Work quickly but accurately. Don't race.

2. The lead-pencil part will be timed. The red pencil part won't be.

3. Never peek ahead in the booklet.

### FLAGS

Instruction time: about 2 minutes

Visual aid: Show the class the two flags. One is the mirror image of the other so that no amount of turning can make them look exactly the same.

Put the flags into the positions shown in the second example on the instruction page. Show the class that turning cannot make the two flags look alike. Therefore the answer is "D."

Emphasize to the class that they may do the test by imagining that the flags can slide around in the plane of the paper but cannot be turned over. Drive this point home by showing that the backs of the real flags are blank.

Time limit: 2:00 exactly

Additional red pencil time: about 3 minutes

Emphasize: "Those who have finished can check over the red-marked items, but don't peek ahead."

### ROTATIONS

Instruction time: about 6 minutes

Visual aid: Show the model labeled "figure A" so that the class will get the idea of what the drawings represent. Then position the model so that, from the students' view, it looks like figure A. A small rotation will then make it look like figure B. Hence the answer is "S."

Next, show figure C so that it looks, from the students' view, like figure C in the instructions. Twist and turn the figure until they are satisfied that there is no way to make it look like figure D. The answer to this example is "D."

Time limit: 4:00 exactly

Additional red pencil time: about 4 minutes. Please be alert to students who don't realize that there are test items on the back page.

TO FINISH

Collect all test booklets and red pencils after having students double-check to see that they have completed the front page correctly.

SPATIAL BATTERY
MIDDLE LEVEL A

Name_____

Today's Date_____ Teacher_____

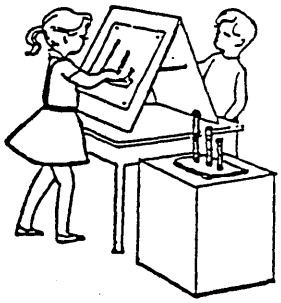
Age_____ Date of birth_____

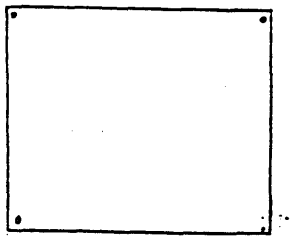
Boy____ Girl ____ Right handed_____ Left handed ____

This booklet contains several short tests. Although they are called "tests" they may seem more like games. Your teacher will tell you when to start and stop each one. Try your best to work quickly without making errors. We hope you will have fun taking these tests!

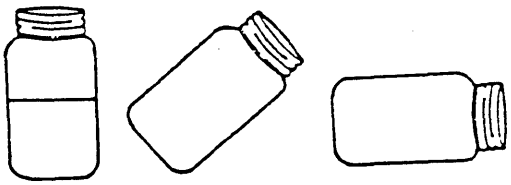
EDWARD JOHNSON & ANN MEADE
UNIVERSITY OF NORTH CAROLINA

These children are drawing three sticks of different lengths just as they see them. Pretend you are the boy. Make his drawing on this paper.



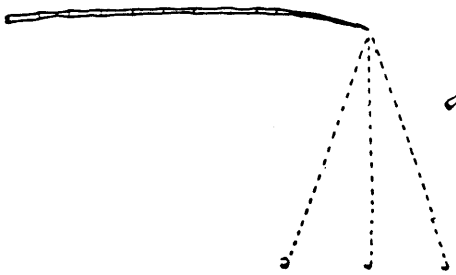


Draw the water line as it would be if the jars were half full of water. The first one has been done for you. (The cap is on tightly—it will not leak.)

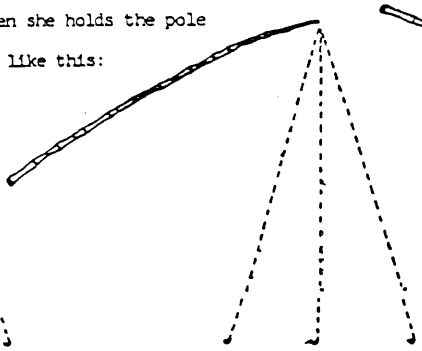



This girl is pretending she is fishing. She stands on her front porch and drops her line so that the hook almost touches the ground. Trace how the line will look when she holds it in these 3 ways:

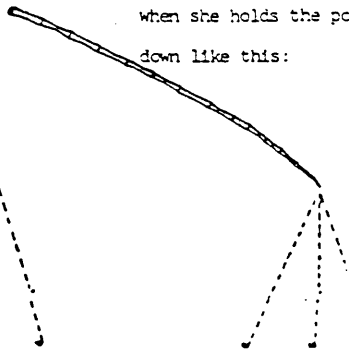
when she holds the pole straight like this:



when she holds the pole up like this:



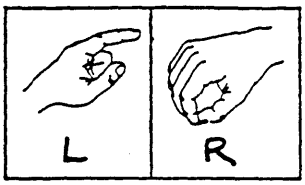
when she holds the pc down like this:



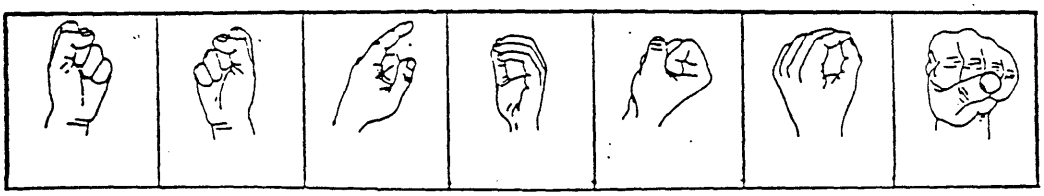
(Trace down one line for each of the 3 poles)

# HANDS

Look at the two pictures below. The first picture shows a left hand. The second picture shows a right hand.




Now look at the examples below. Write "L" below the hand if it is a left hand. Write "R" if it is a right hand.

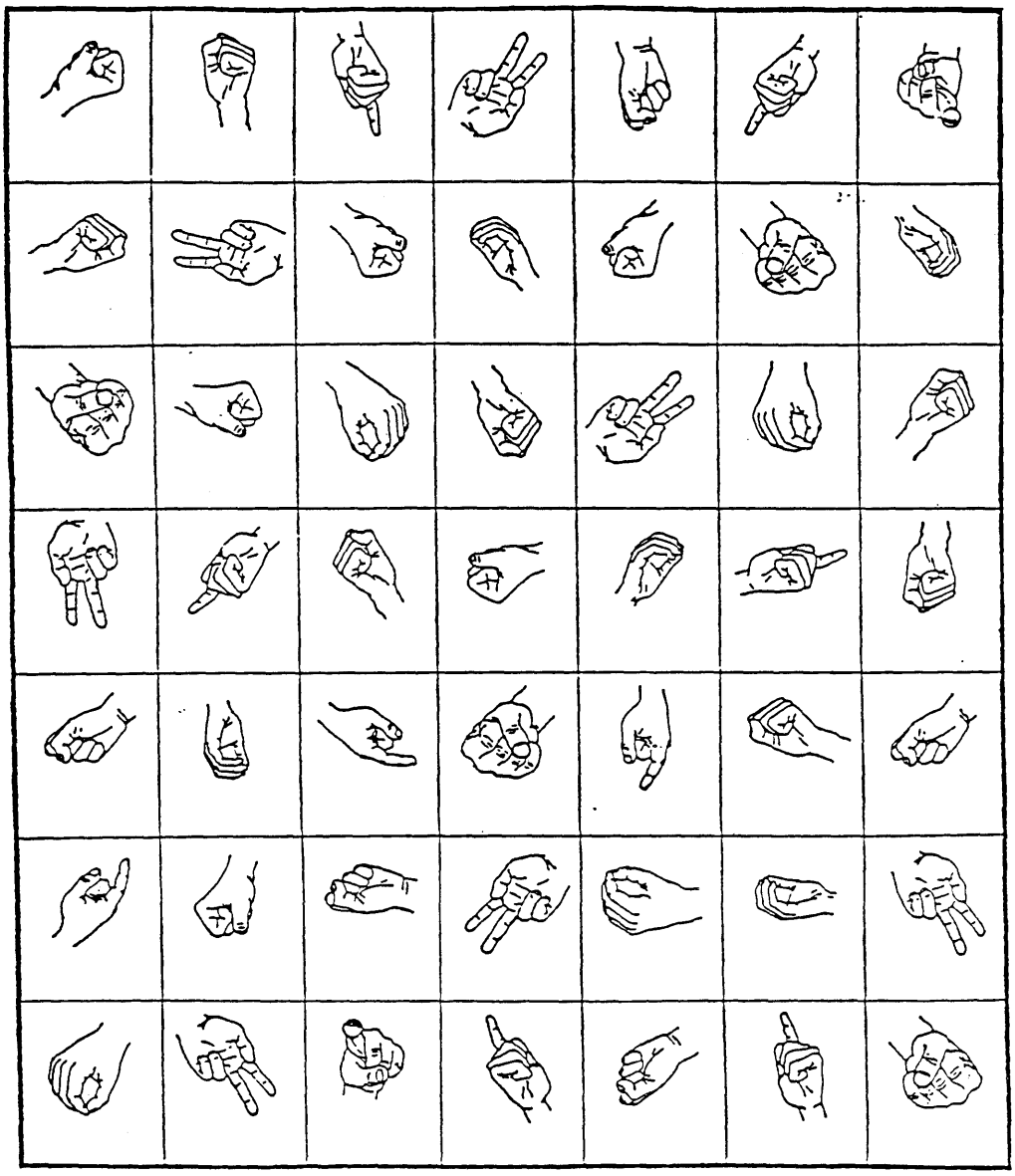


BELOW ARE ANSWERS TO THE SAMPLE PROBLEMS:


Adapted from a test
by L. L. Thurstone

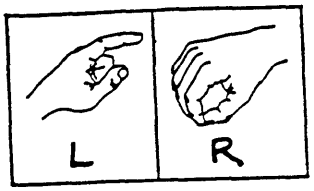
STOP. DO NOT TURN THE PAGE. WAIT FOR THE STARTING SIGNAL

IF THE PICTURE SHOWS A RIGHT HAND PUT "R" IN THE BOX. IF IT SHOWS A LEFT HAND, PUT "L" IN THE BOX.



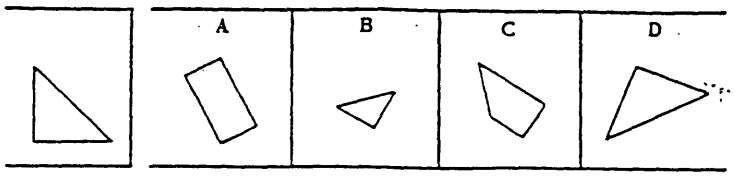
STOP HERE. WAIT FOR INSTRUCTIONS.

How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you tell how you did it, so that the younger child can do it right.



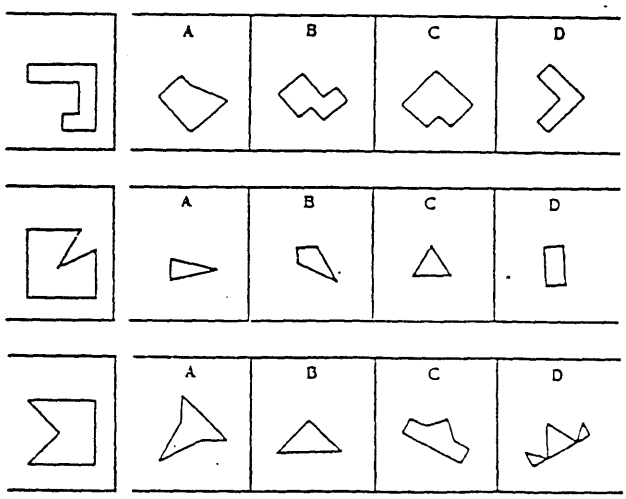
# SPATIAL RELATIONS

Look at the row of drawings below. The first drawing is part of a square. Look at the other drawings in the row and find the other part of the square.

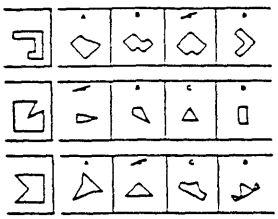



If you picked the one with the letter D above it you are right. Draw a line through the D.

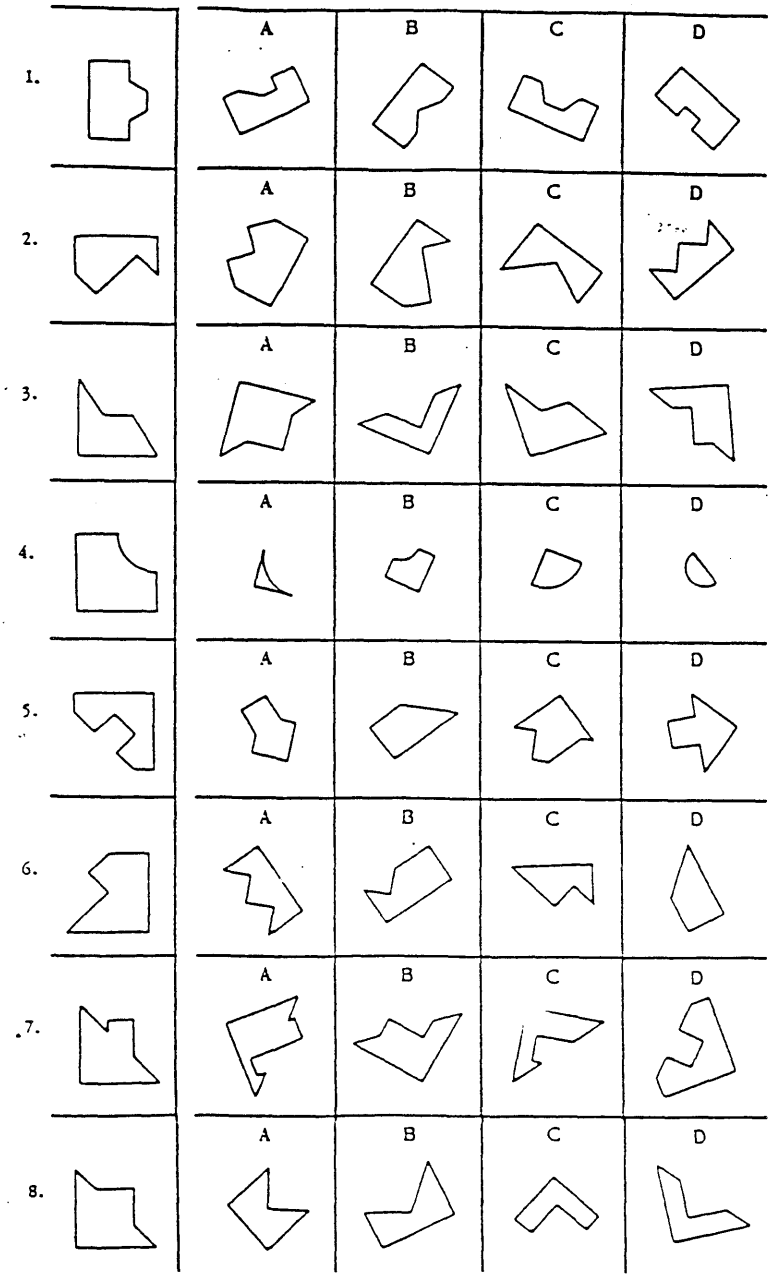
Now try these problems. The first drawing in each row is part of a square. You are to find the other part of the square. Show your answer by drawing a line through the correct answer.




Here are the answers:



Adapted from the SRA Test of Primary Mental Abilities with permission of Thelma Thurstone


1.
A
B
C
D
2.
A
B
C
D
3.
A
B
C
D
4.
A
B
C
D
5.
A
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C
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6.
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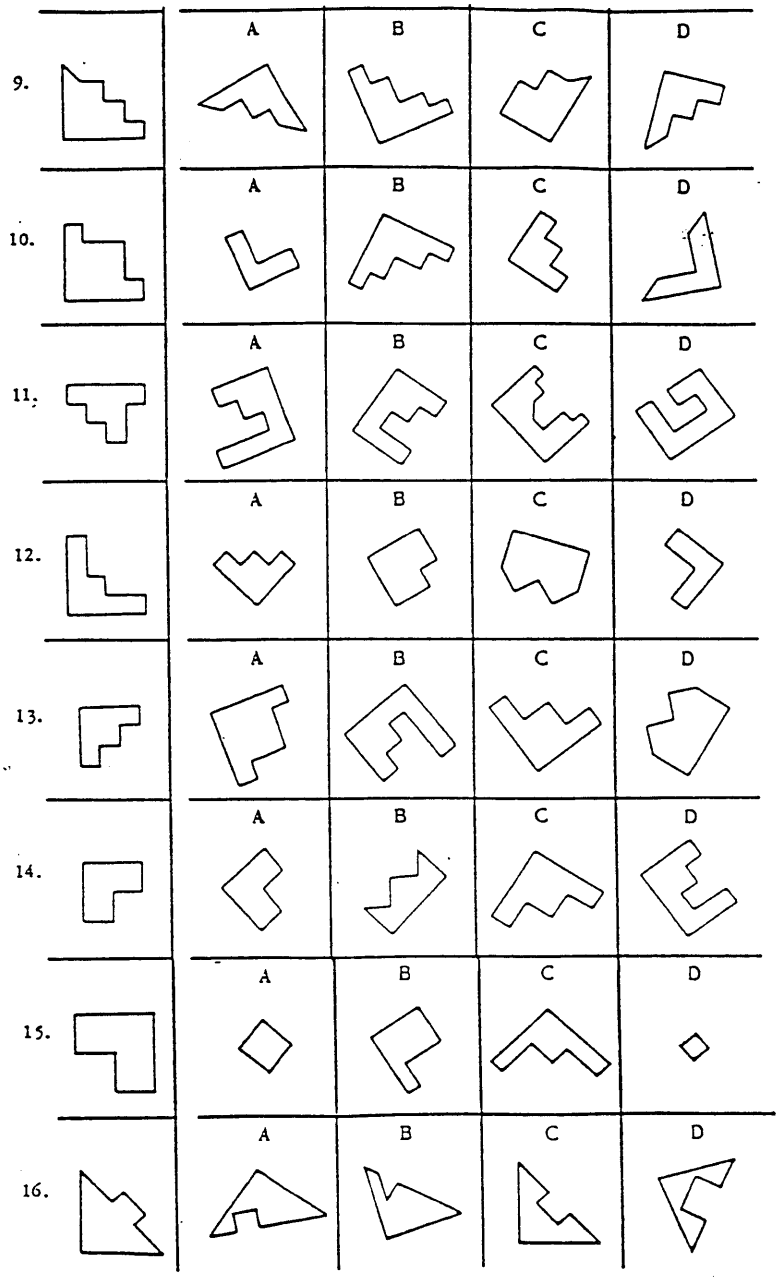
7.
A
B
C
D
8.
A
B
C
D


9.
A
B
C
D
10.
A
B
C
D
11.
A
B
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D
12.
A
B
C
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13.
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16.
A
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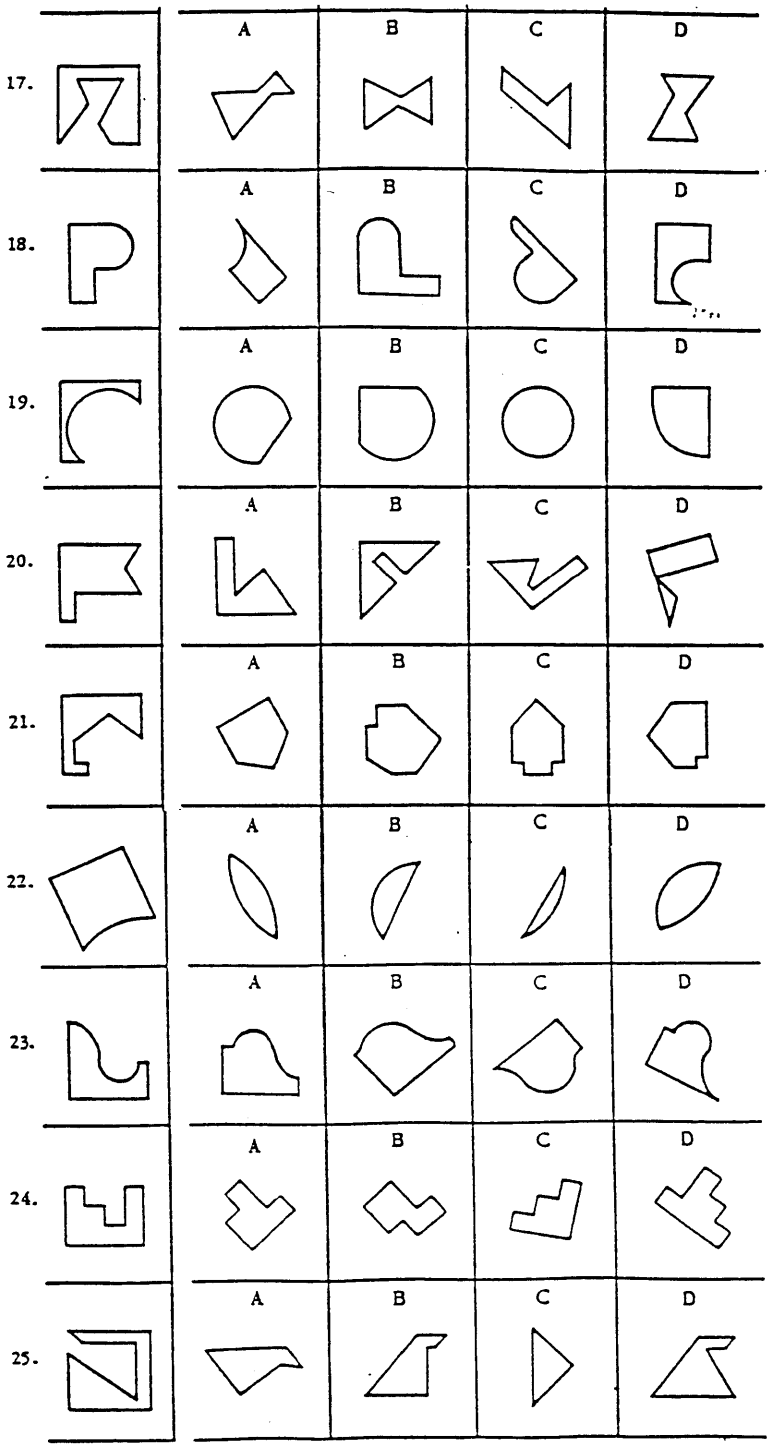
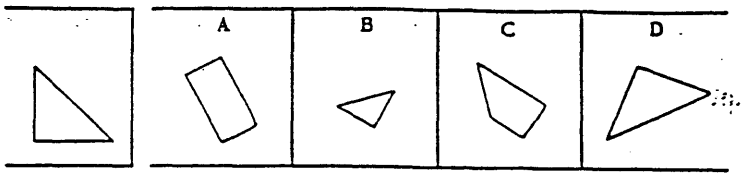
C
D

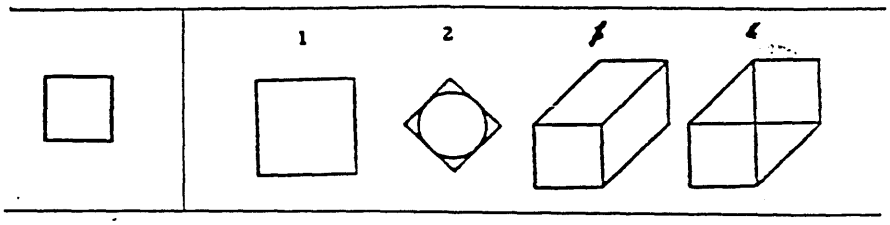

17.
18.
19.
20.
21.
22.
23.
24.
25.
A
B
C
D

How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you explain how you did it, so that the younger child does it right.

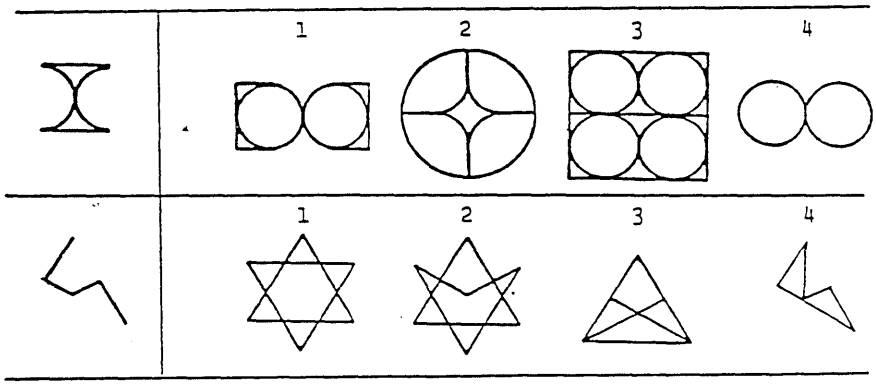


# HIDDEN FIGURES

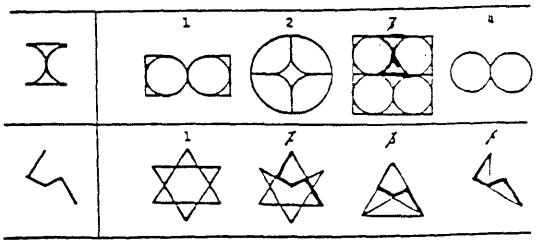
Below is a row of designs. The figure on the left is hidden in at least one of the drawings on the right. The hidden figure must be the same size and it must not be turned.




Here are two examples for you to try:

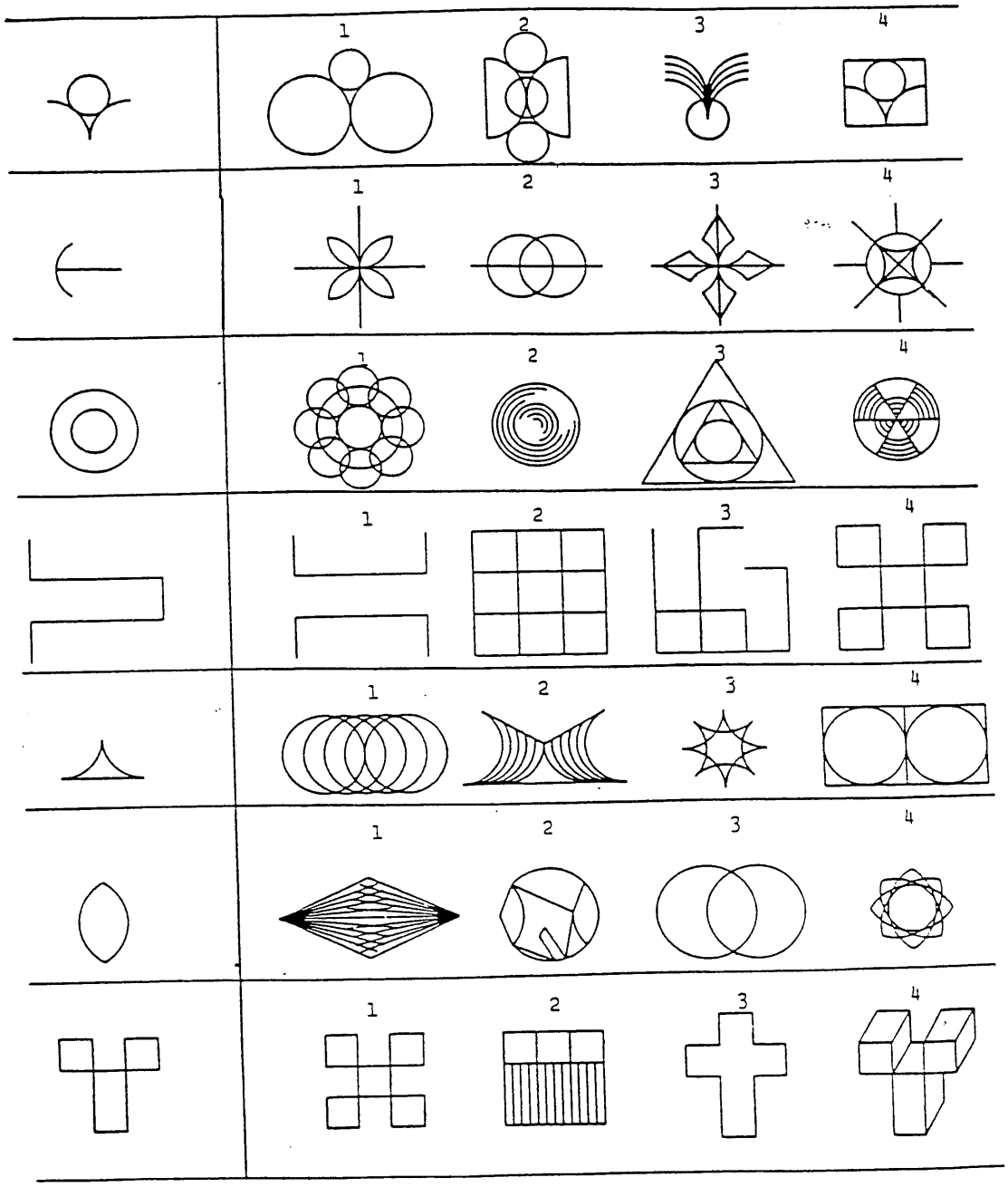



Here are the answers. The hidden part has been drawn with dark lines to help you see where it is.

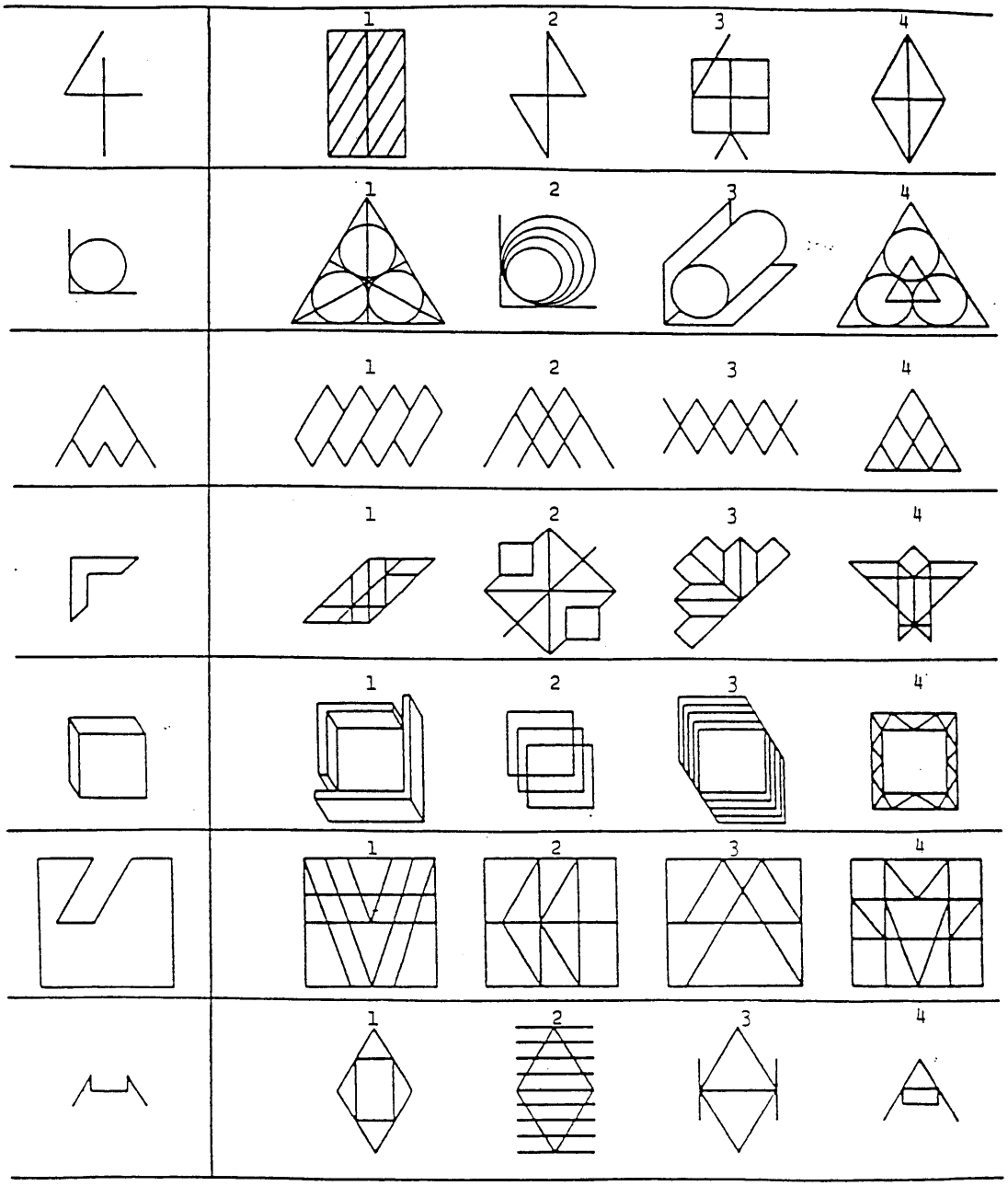



Adapted from a test by L. L. Thurstone

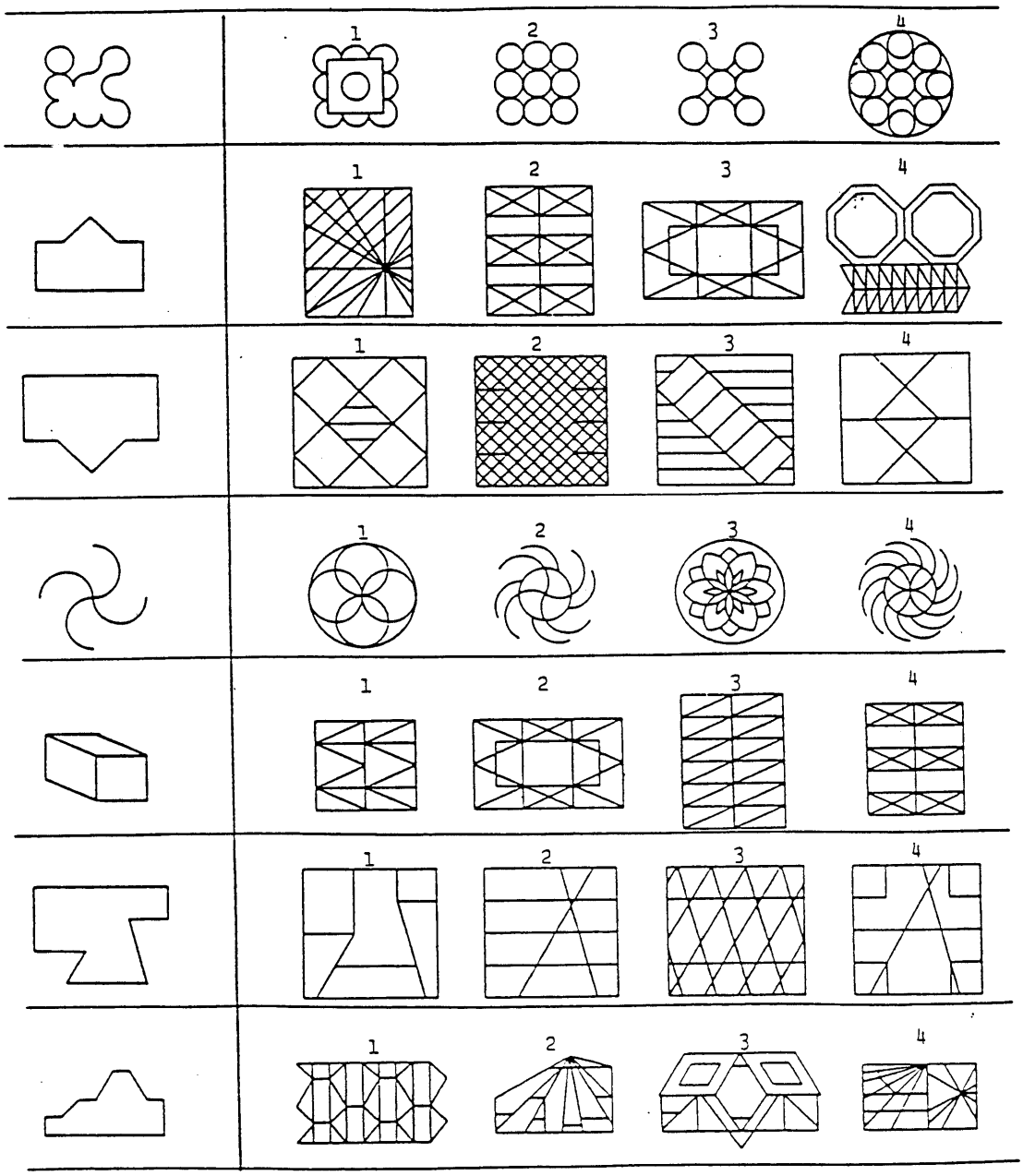
STOP. DO NOT TURN THE PAGE. WAIT FOR THE STARTING SIGNAL




GO ON TO THE NEXT PAGE.

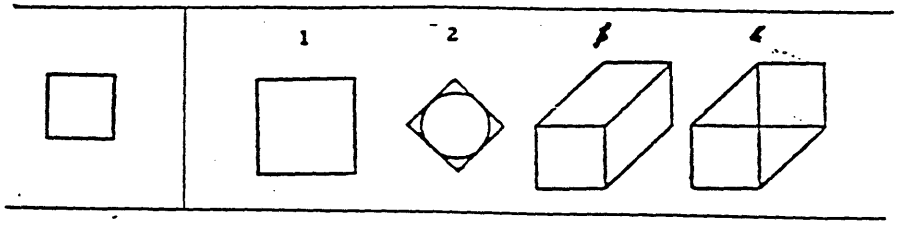



GO ON TO THE NEXT PAGE.




STOP HERE -- WAIT FOR FURTHER INSTRUCTIONS.

How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you explain how you did it, so that the younger child does it right.



1. Which of these different tests did you think was the easiest? Why?

2. Which of these different tests did you think was the hardest? Why?

# SPATIAL BATTERY

## MIDDLE LEVEL B

Danville City Schools

Name _____ School _____ Sex _____

Teacher _____ Class _____ Grade _____

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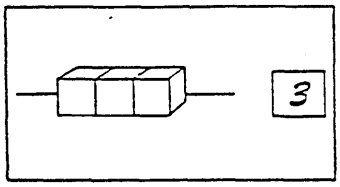
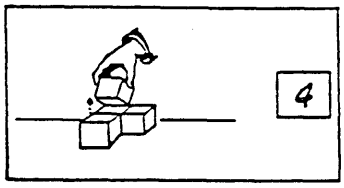
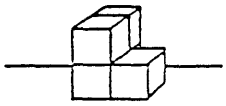
## BLOCK COUNTING

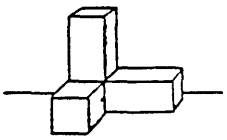
Look at the piles of blocks below. How many blocks are in each pile?

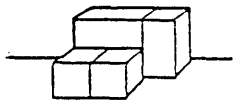


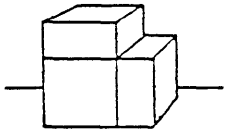




All the blocks in a particular pile are the same size and shape. Some of the blocks will be hidden because they are behind other blocks. But you can tell they are there because you can see the blocks which are on top of them. Try the practice problems below.

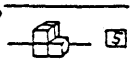
1. 

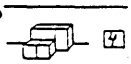
2. 

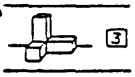
3. 

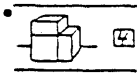
4. 

Correct Answers

- 

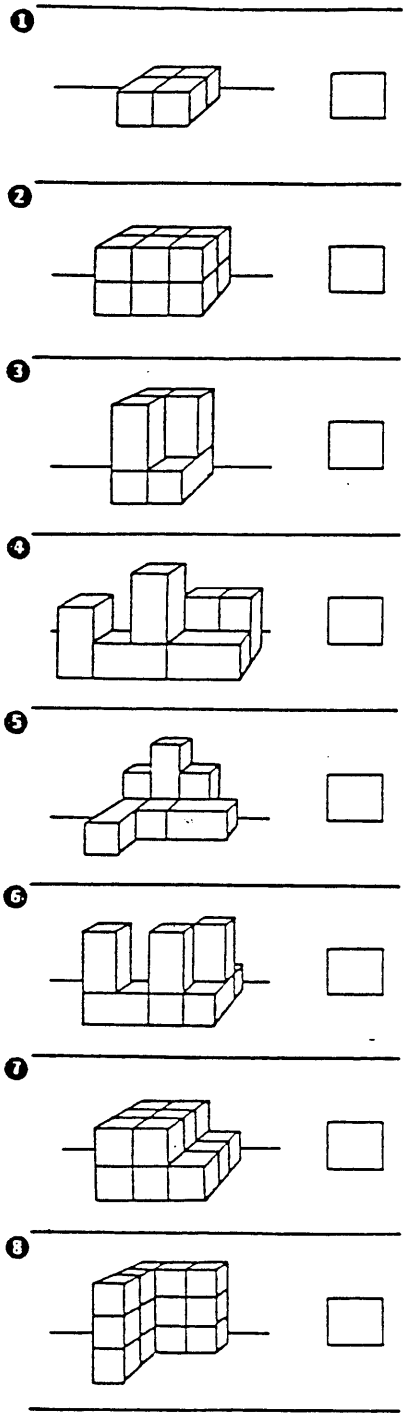
- 

- 

- 


While you are doing these problems work quickly but try to be accurate.

START HERE **1**

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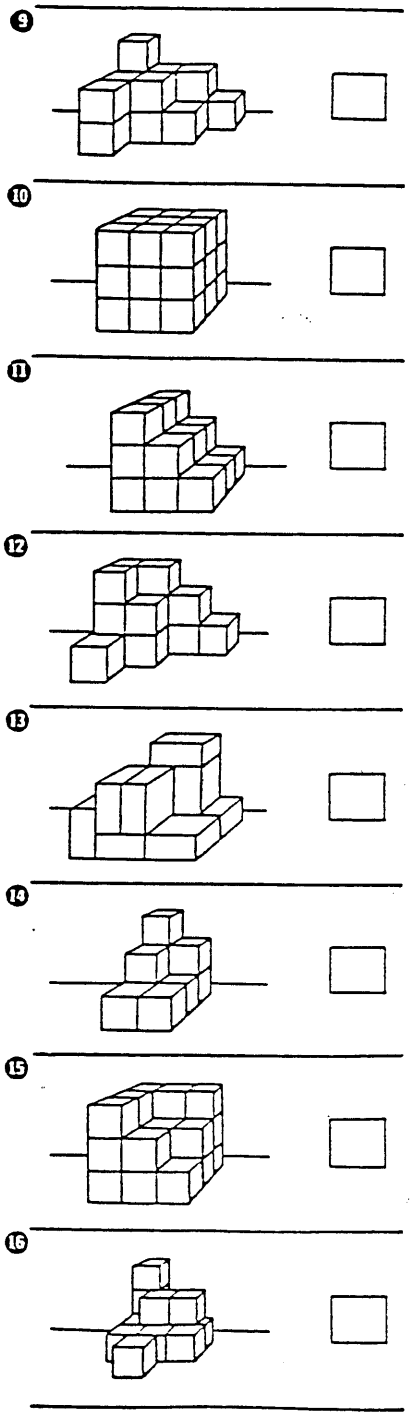


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## 3

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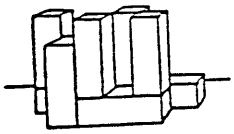

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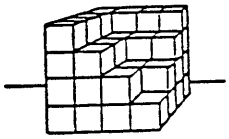

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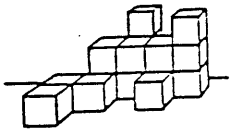

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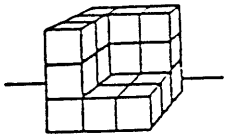

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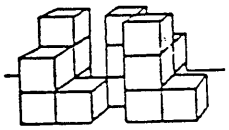

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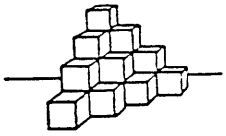

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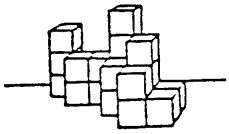

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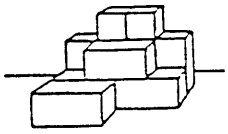

## 4

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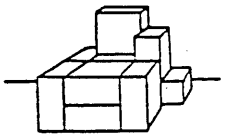

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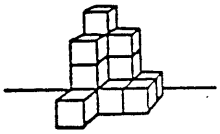

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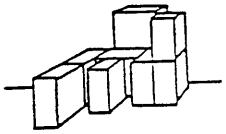

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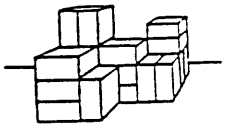

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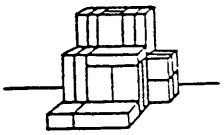

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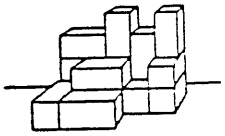

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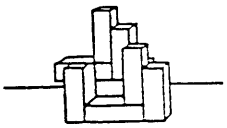

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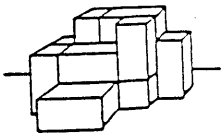
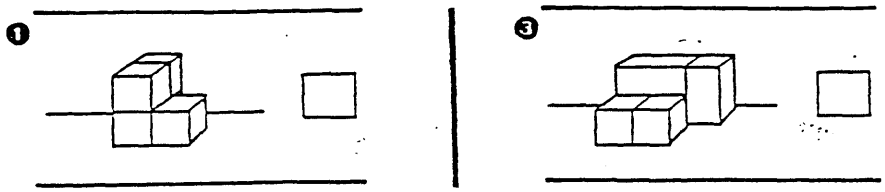
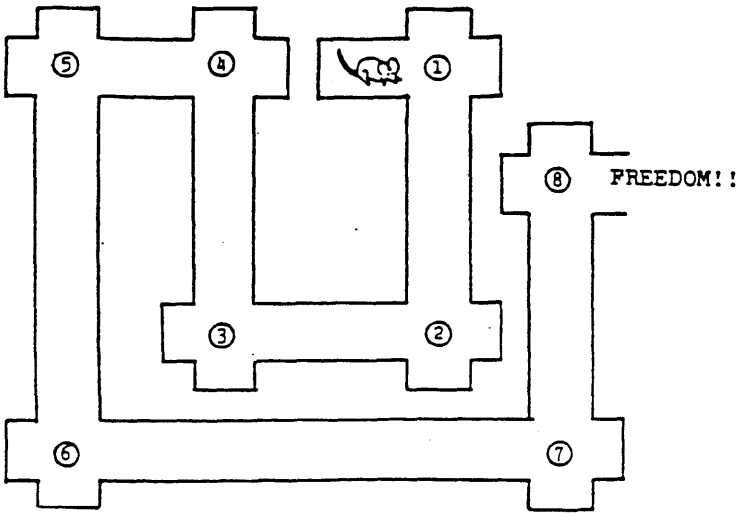


How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you tell how you did it, so that the younger child can do it right.



Help the mouse find his way to freedom. At each of the numbered places tell him whether to turn to his right or to his left.




Circle either right or left at each of the turns below.

At 1 turn right left
At 2 turn right left
At 3 turn right left
At 4 turn right left
At 5 turn right left
At 6 turn right left
At 7 turn right left
At 8 turn right left

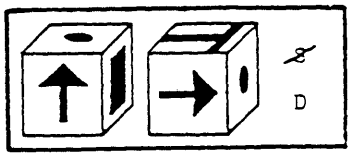
---

Make an outline drawing of the state of Massachusetts in the space below.

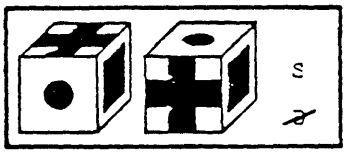
# CUBES

This is a test about cubes. The one rule you must remember is that THE SAME DESIGN IS NEVER USED TWICE ON THE SAME CUBE.

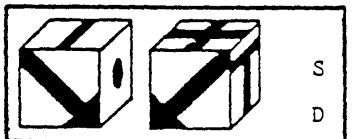
Look at the two cubes below. If you tipped the first cube over to the right it could look like the second one. The S has been marked since the cubes may be the same.

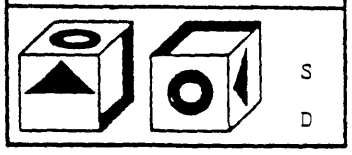


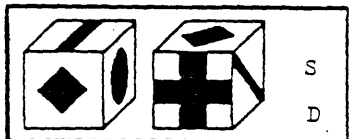

The two cubes below are DIFFERENT. If you tip the first cube toward you so that the cross is on the front, the dot will be on the bottom. There cannot be another dot on the top because the same design can't be used twice on the same cube.

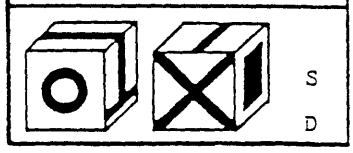



In the practice problems below, mark S if the two cubes could be the SAME. Mark D for DIFFERENT if they cannot be the same cube.

1



2



3



4


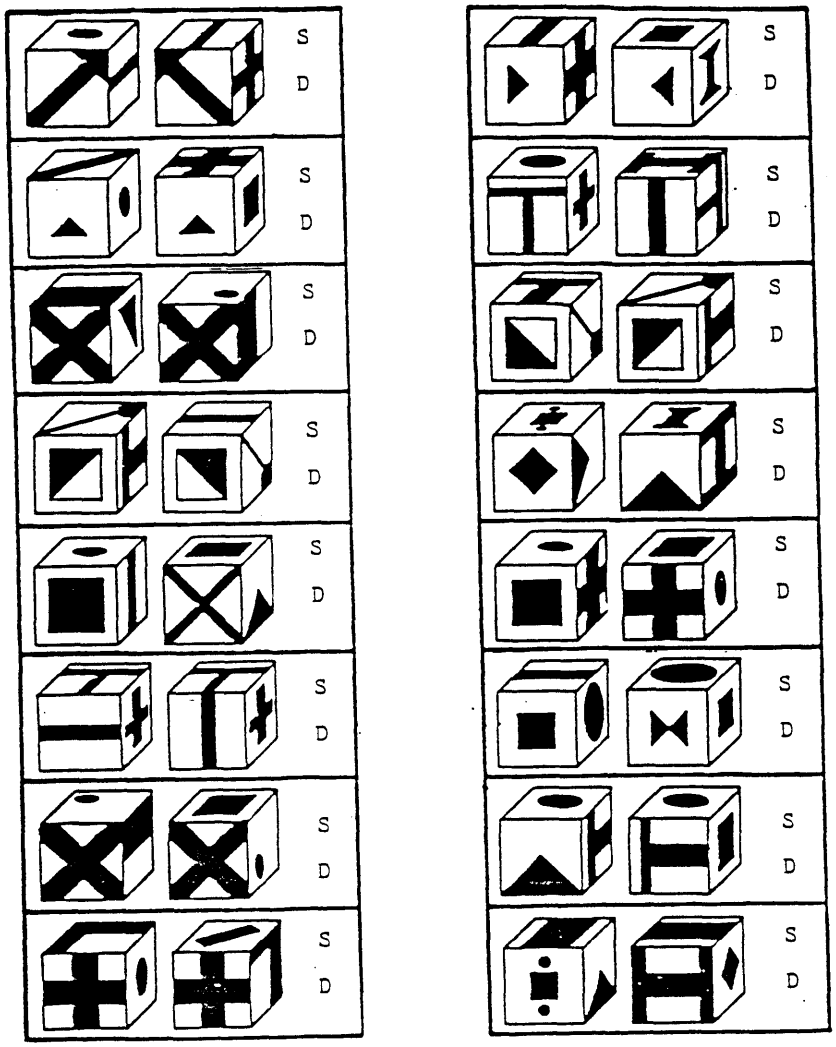

The answers to these problems are 1: S 3: D

2: D 4: S

Adapted from a test by L. L. Thurstone

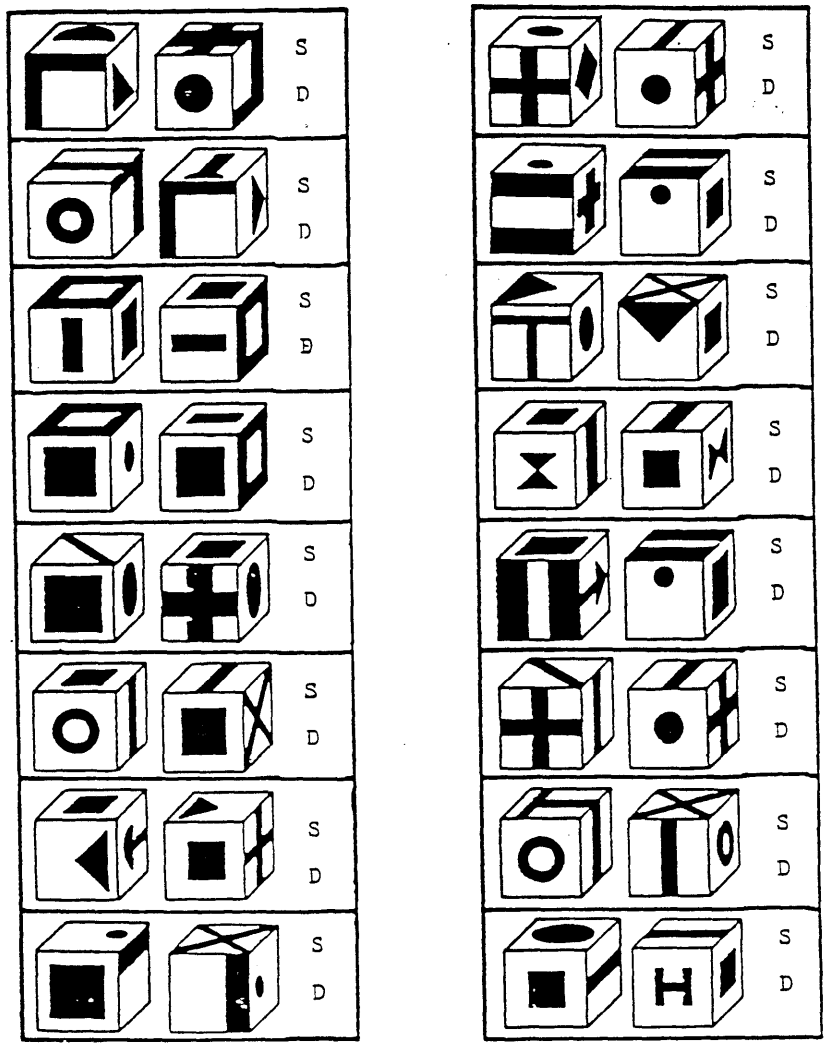
STOP. DO NOT TURN THE PAGE. WAIT FOR STARTING SIGNAL

Mark S if the cubes may be the same. Mark D if the cubes must be different.



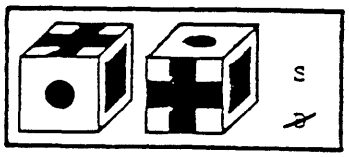

GO ON TO THE NEXT PAGE.

Mark S if the cubes may be the same. Mark D if the cubes must be different.




Stop here. Wait for further instructions.

How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you tell how you did it, so that the younger child can do it right.



# DIRECTIONS

1. In which general direction does the Charles River flow?
   _____ North _____ South

   _____ East _____ West

2. In which general direction would you travel to get to the nearest ocean?
   _____ North _____ South

   _____ East _____ West

3. If you left the Boston Area and traveled north, which state would you come to first?
   _____ New York _____ Vermont

   _____ Maine _____ New Hampshire

4. Put a check by all the states which touch Massachusettes.
   _____ New York _____ Vermont

   _____ Pennsylvania _____ Rhode Island

   _____ Connecticut _____ New Jersey

   _____ Maine _____ New Hampshire

5. Which way would you go to get to Salem?
   _____ North _____ South

   _____ East _____ West

6. (Do this one in your head. Do not make a diagram.)

   Mary stands by the flagpole at school.

   She takes 2 steps north;
   She turns right and takes 3 steps;
   She turns and takes 4 step south;
   She turns right and takes 3 steps.

   How many steps is she away from the flagpole? _____

1. Which of the tests did you think was the easiest? Why?

2. Which of these different tests did you think was the hardest? Why?

# SPATIAL BATTERY

## MIDDLE LEVEL C

Danville City Schools

Name _____ School _____ Sex _____

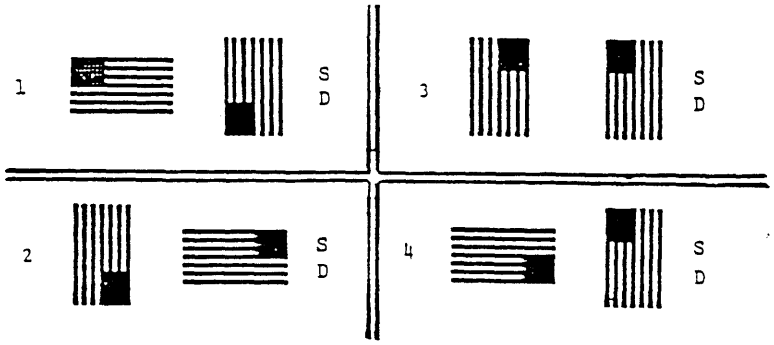
Teacher _____ Class _____ Grade _____

# FLAGS

These two pictures of a flag are the same. You can slide one picture around to fit exactly on the other picture. S is marked to show that the pictures are the SAME.


The next two pictures are different. You cannot slide the pictures around to make them fit exactly. D is marked to show that the pictures are DIFFERENT.


Here are some problems for you to try.



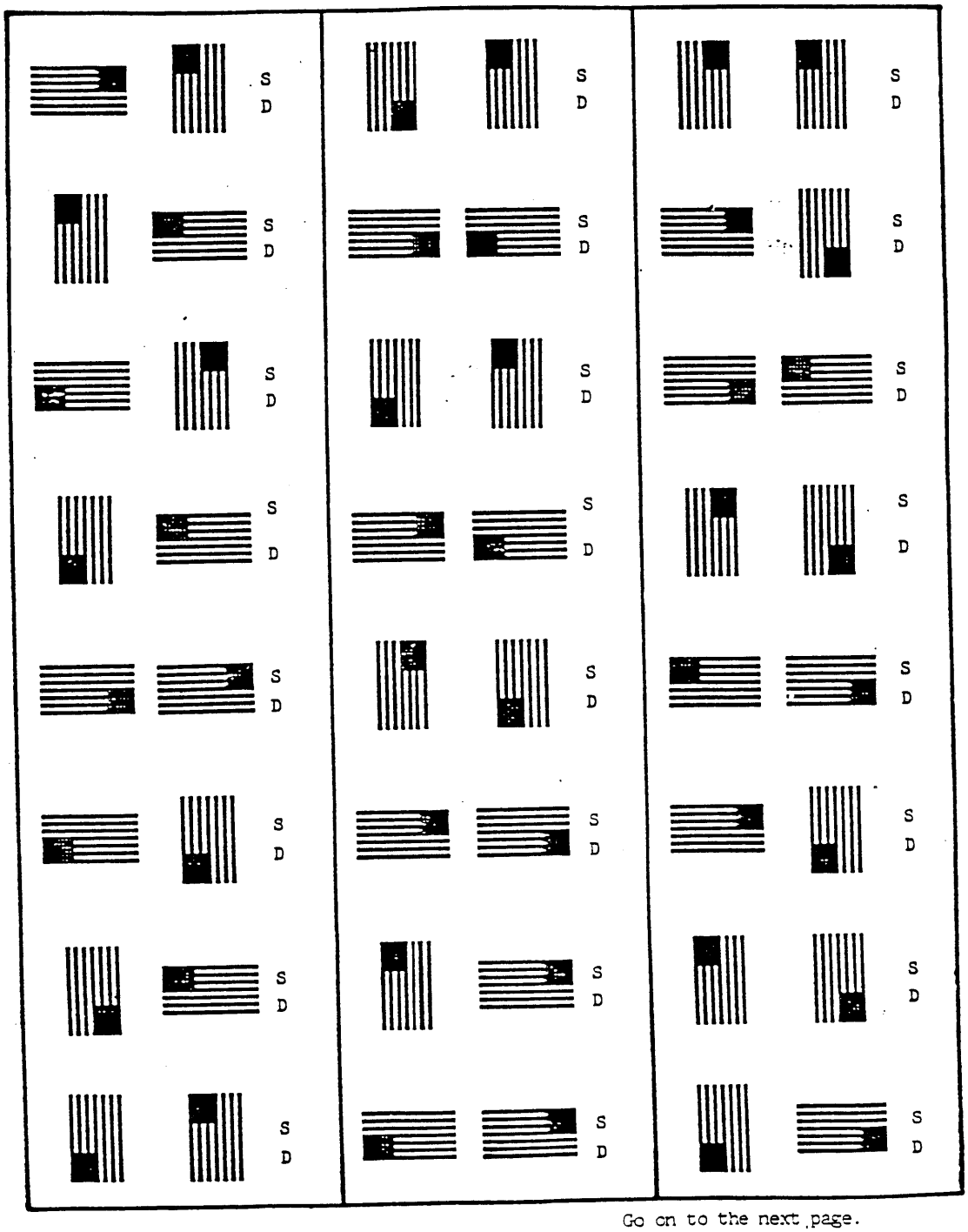

The answers to these problems are 1: S D 3: S D

2: S D 4: S D

Adapted from a test by L. L. Thurstone

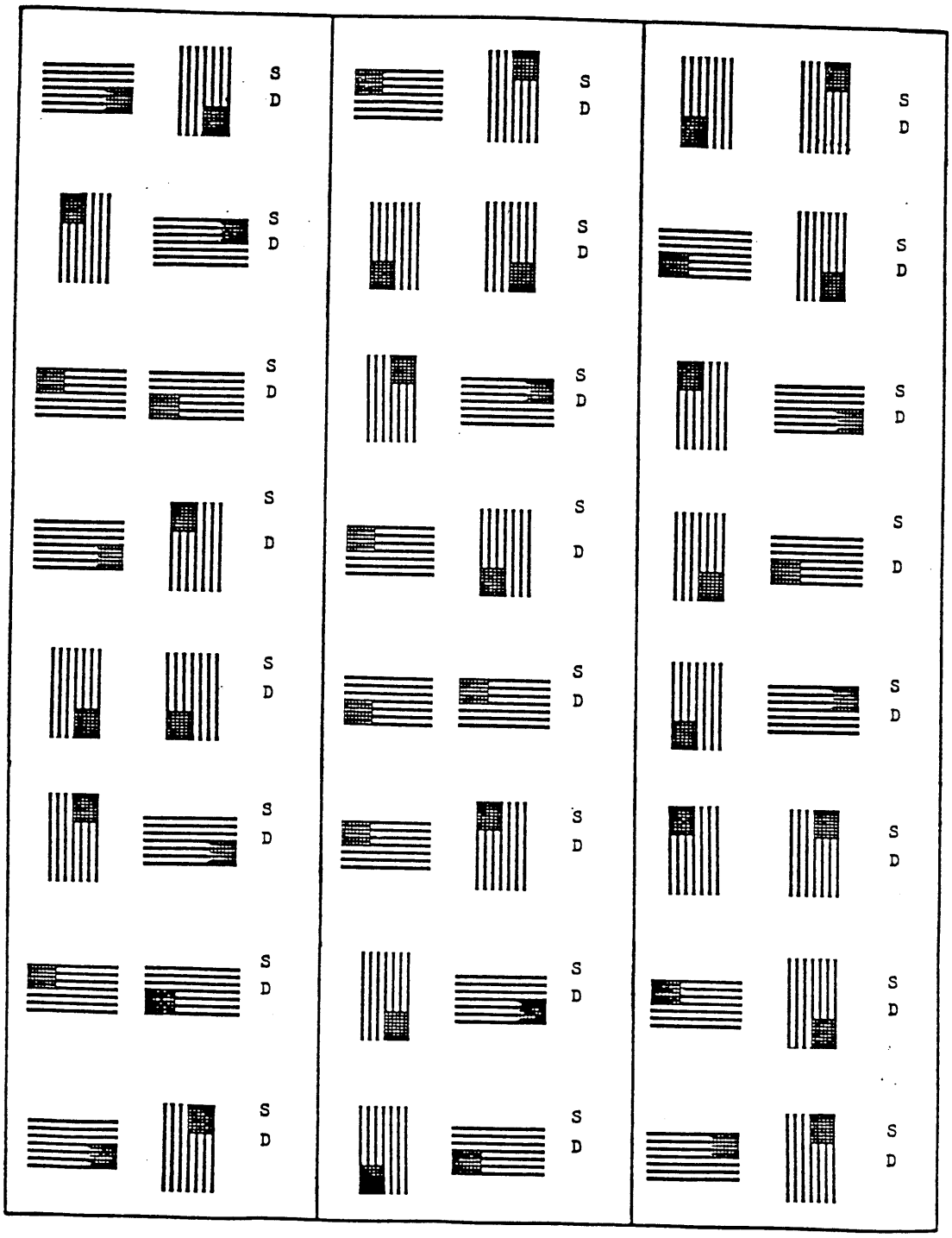
STOP. DO NOT TURN THE PAGE. WAIT FOR THE STARTING SIGNAL.

Mark S if the flags are the same; mark D if the flags are different.

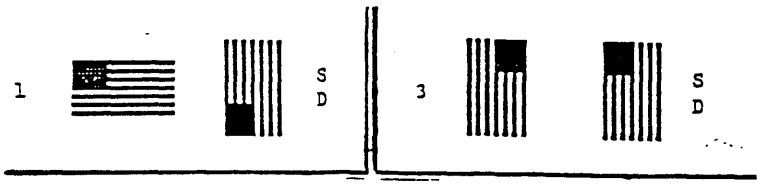



Go on to the next page.

Mark S if the flags are the same; mark D if the flags are different.



How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you tell how you did it, so that the younger child can do it right.



Adapted from a test
by Earl Hunt

# ROTATIONS

Instructions

Figures A and B are two pictures of the same object seen from different angles. Observe that you could rotate the object in Figure B so that it would be exactly the same as the object in Figure A.

Figure A

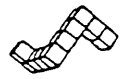


Figure B

S D

Figures C and D are pictures of two different objects. No matter how you rotated the object in Figure D, it would never match the object in Figure C.

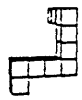


Figure C

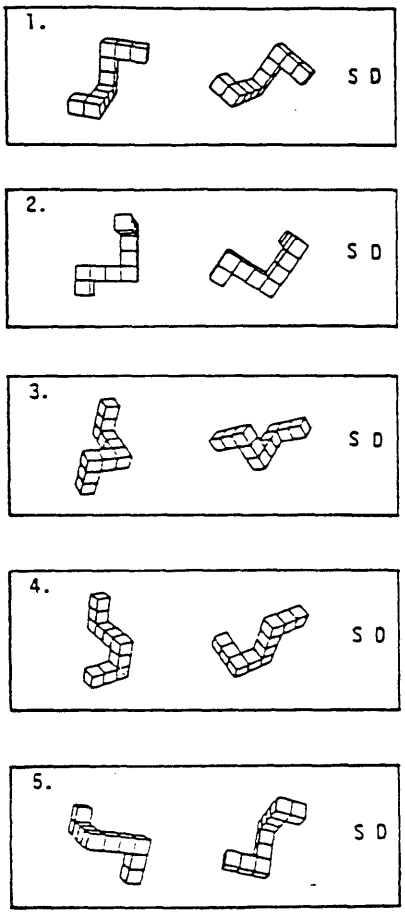
Figure D

S D

This test is made up of pairs of figures similar to those above. For each pair you must decide if the two pictures are both of the same object, or if they are pictures of different objects. If the pictures are of the same object, mark the S next to the pair. If they are of different objects, mark the D.

Now try the practice problems on the next page

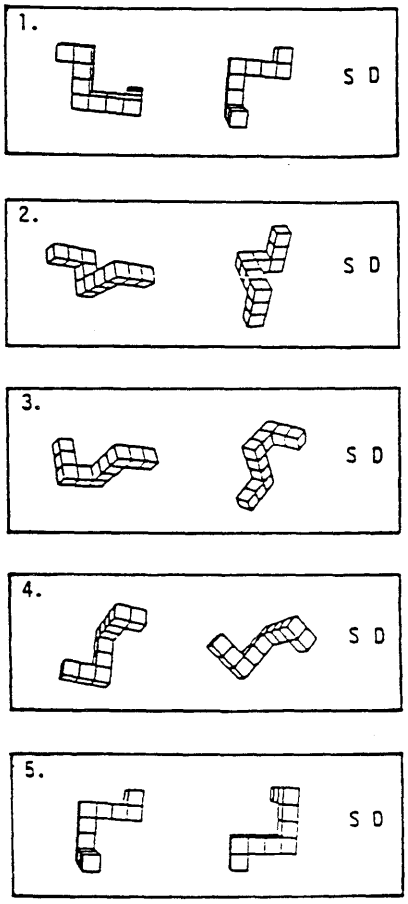
First, try the problems in this column and check your answers.




Answers:

| | | |
|---|---|---|
| 1 | ~~S~~ | D |
| 2 | S | ~~D~~ |
| 3 | ~~S~~ | D |
| 4 | S | ~~D~~ |
| 5 | S | ~~D~~ |

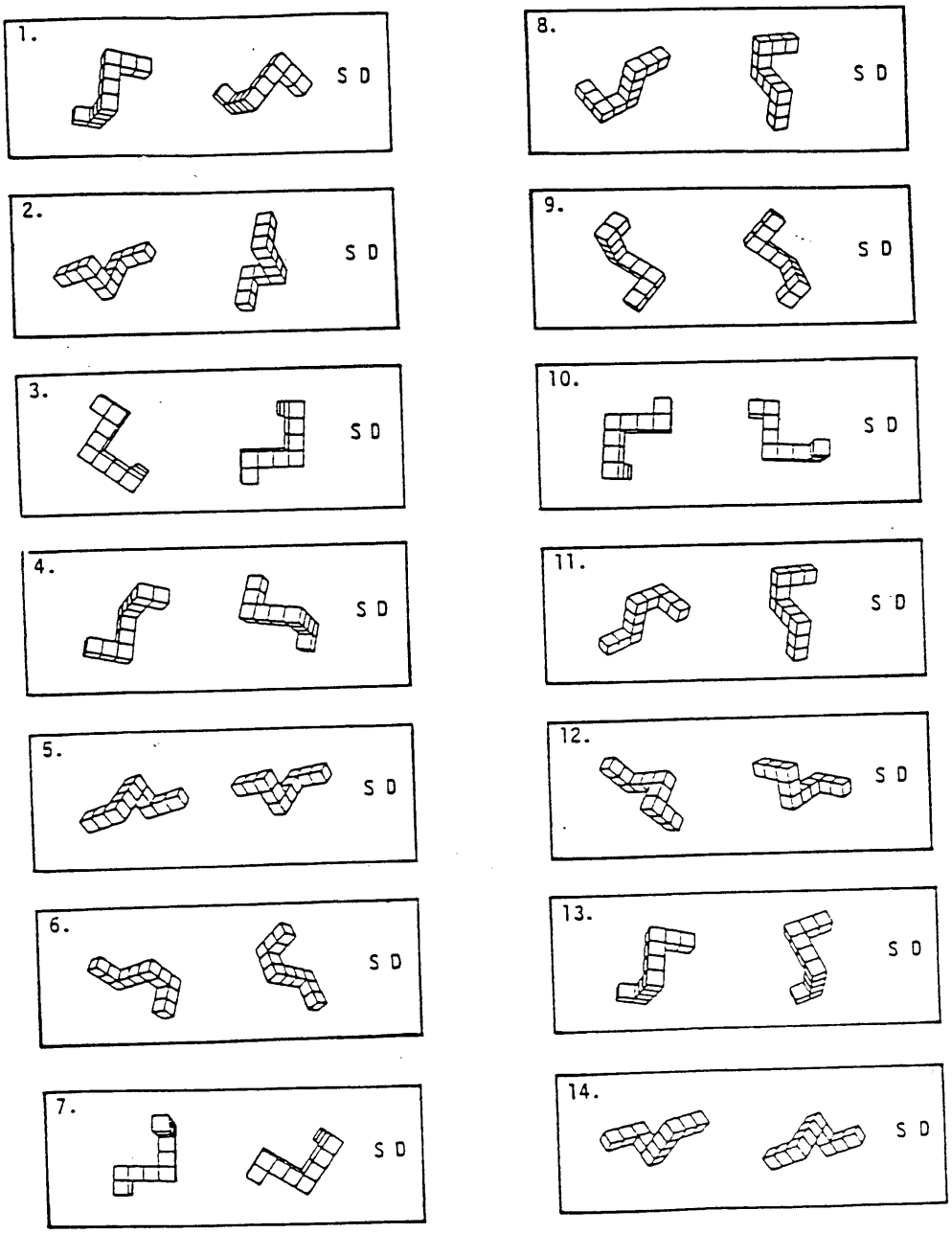
Next, try the ones below.



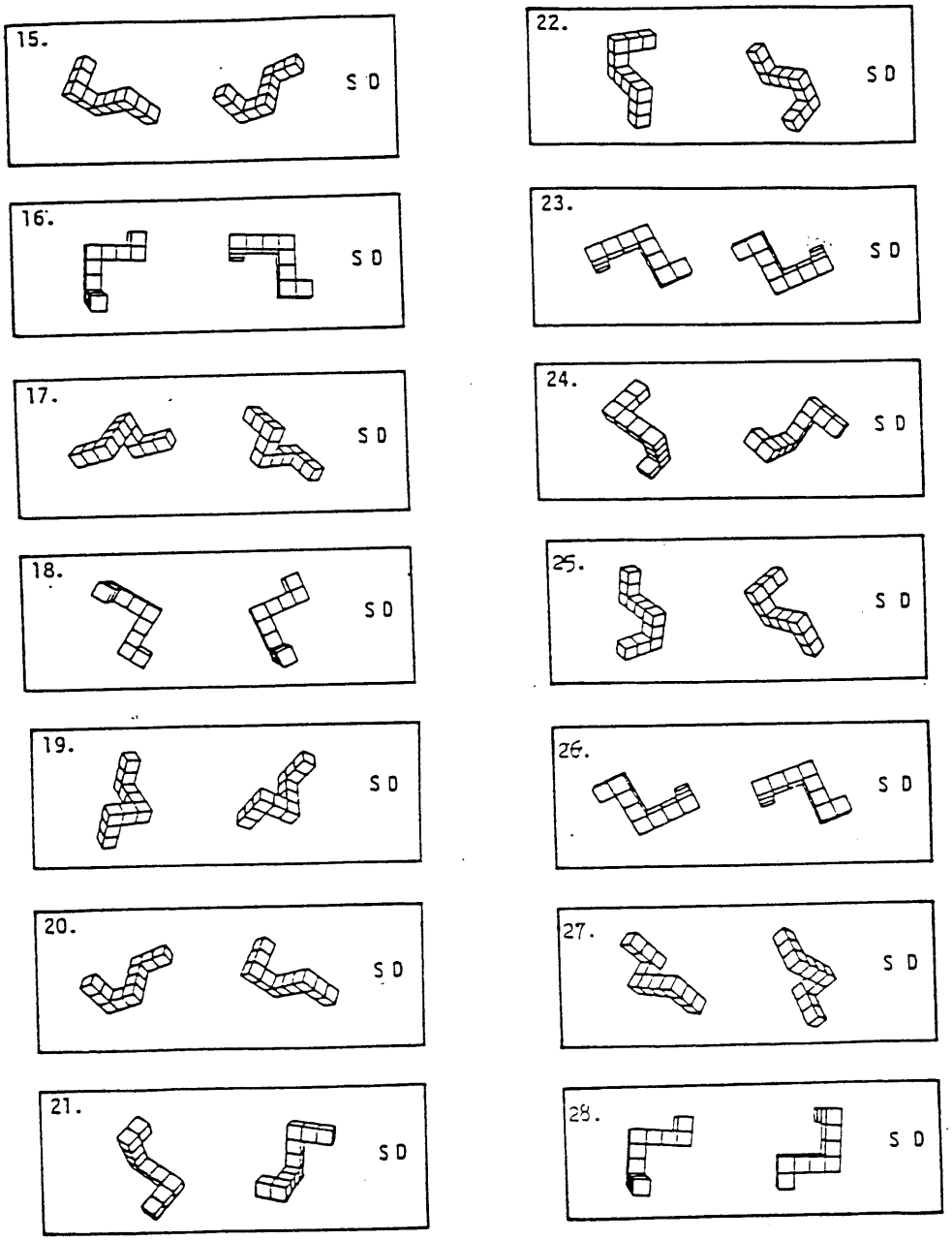

Answers:

| | | |
|---|---|---|
| 1 | S | ~~D~~ |
| 2 | ~~S~~ | D |
| 3 | S | ~~D~~ |
| 4 | ~~S~~ | D |
| 5 | S | ~~D~~ |

STOP HERE AND WAIT FOR THE STARTING SIGNAL.




Go on to the next page.




Go on to the next page.

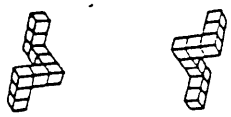
29.

S D

30.

S D

31.

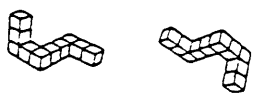


S D

32.

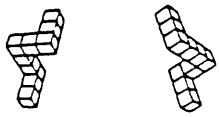
S D

33.



S D

34.



S D

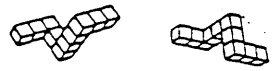
35.

S D

36.

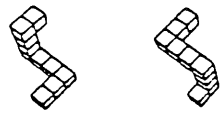
S D

37.



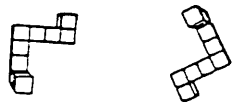
S D

38.



S D

39.

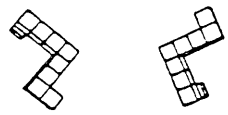


S D

40.

S D

41.



S D

42.

S D

How would you explain to a younger child what you did on the last test to figure out the answers? Use the example below to help you tell how you did it in such a way that the younger child can do it right.

Figure C

Figure D

1. Which of the tests did you think was the easiest? Why?

2. Which of these different tests did you think was the hardest? Why?

Name_____

Today's Date_____ Teacher_____

| | |
|---|---|
| a. | b. |
| c. | d. |

page 1

1. What is the difference between a square and a cube?

2. Please draw a picture of a square and a cube to show how they are different.

| square | cube |
| --- | --- |
| | |

page 2

3. This little space cadet is going to take some photographs with his camera. I want you to draw what is on the photographs.

a.

top view

b.

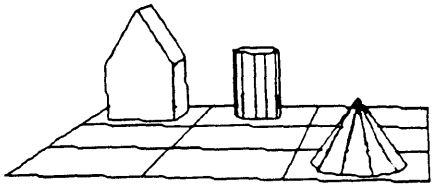
side view

c.

front view

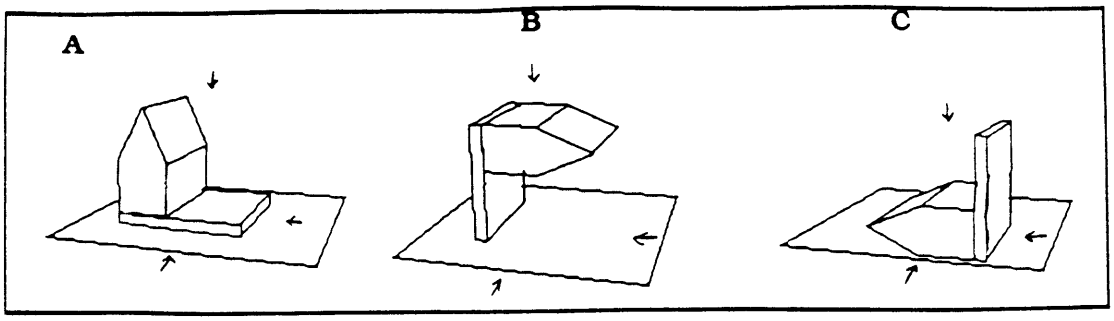
page 3

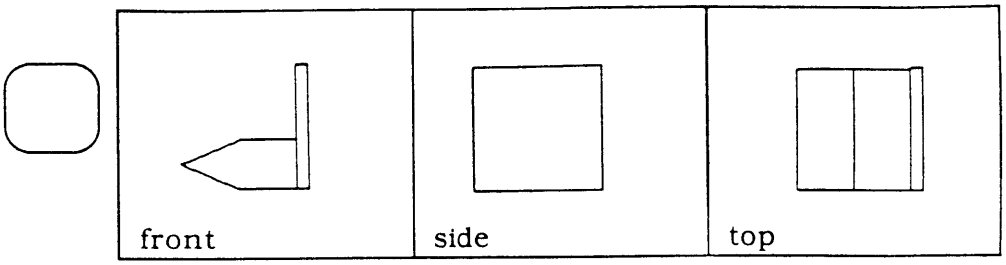
4. Your mission is to help you friend duplicate the scene below. In a short letter to your friend, tell them about the little scene well enough for them to recreate it when they get your letter. You can only help your friend by using words and not drawings.

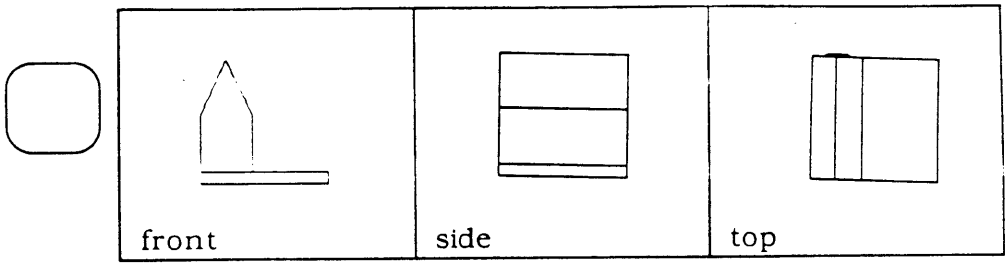


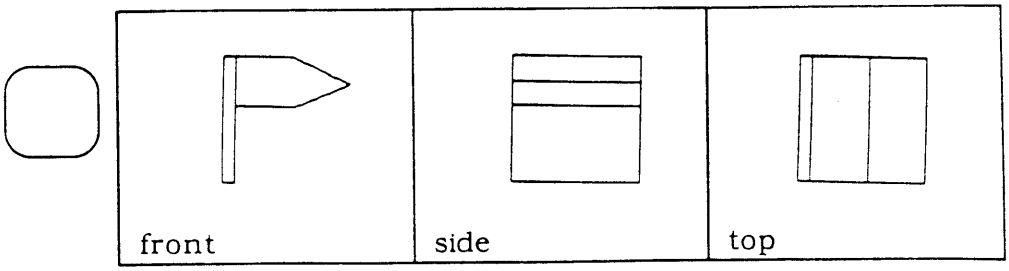
page 4

5. The little space cadet took a set of three photographs (top, front, side) of the object below when it was in three different positions. He can't remember which set of photographs went with which position. Can you help him ? Place the letter of the correct position in the box to the left of each set of photographs.

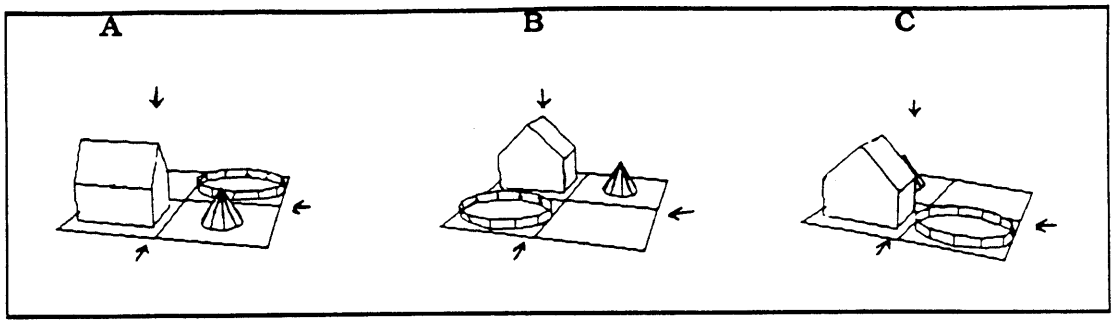


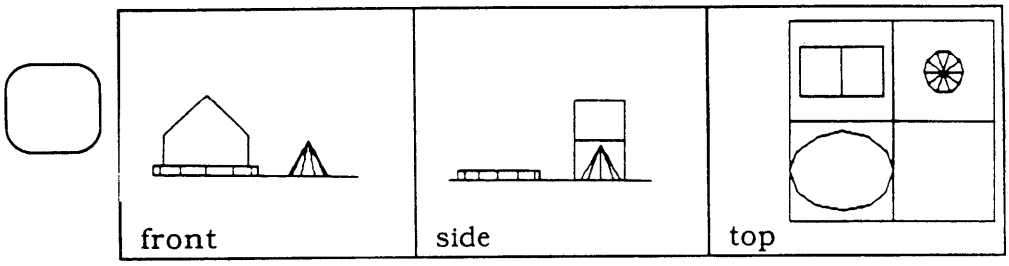



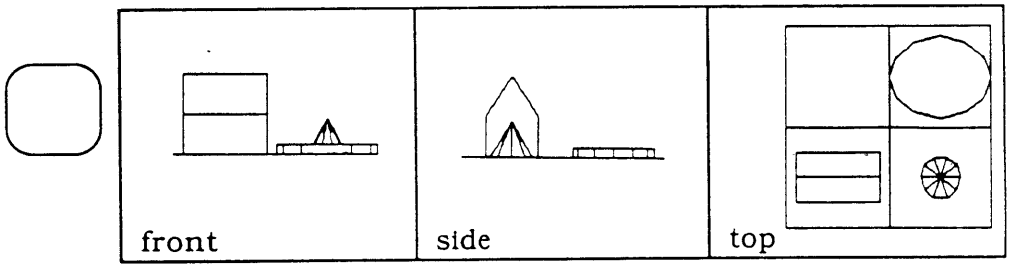



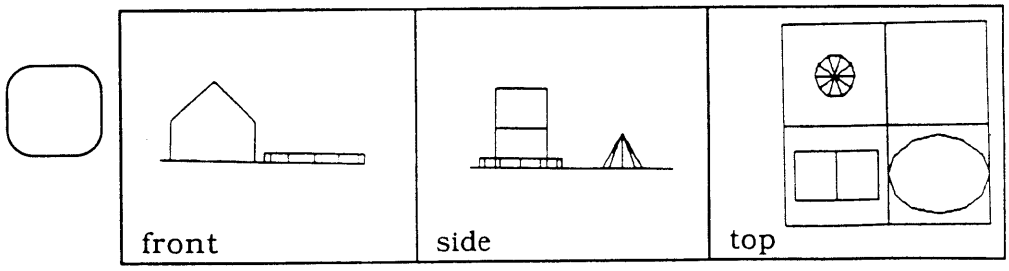



6. The little space cadet took a set of three photographs (top, front, side) of the three different scenes below. He can't remember which set of photographs went with whichscene. Can you help him ? Place the letter of the correct scene in the box to the left of each set of photographs.



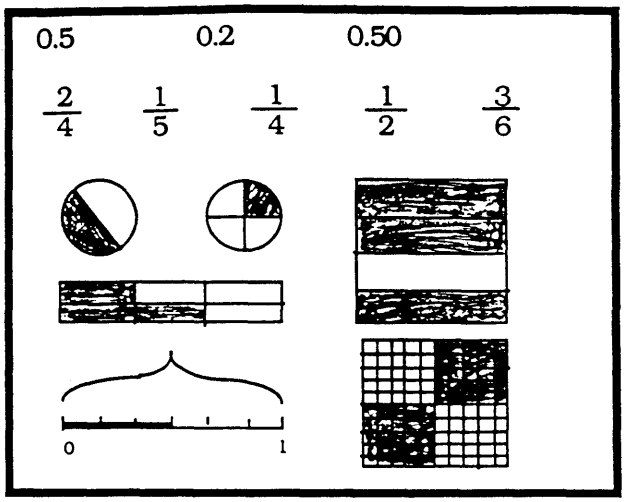
7. Draw a circle above a square.

8. Draw a circle and a triangle. You can see that the circle is behind the triangle but you can't see through the triangle.

9. Write an "R" to the left of an "E". Put an "S" under the "R". Now put a "T" above and to the right of the "R".

10 Please circle ALL the correct answers.

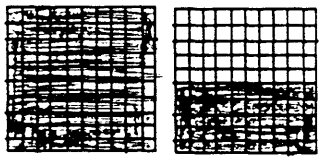
one half =




11. Please circle the correct answer.

0.1 = a. 1/2 b. 1/4 c. 1/10 d. 1/100

12. Which decimal is shown?



a. 5.1 b. 1.5

c. 10.5 c. not given

13. Can you guess what the answer to the following problems are?

a. -3 + 1 = b. -2 x 5 = c. .5 x 20 =

14. What comes next?

a. 3.4, 3.5, 3.6, 3.7, 3.8,

b. 2.8, 3.0, 3.2, 3.4, 3.6

c. 0.25, 0.50, 0.75, 1.00,

d. 1.5, 1.0, 0.5, 0.0, -0.5 ,

page 8

15. Can you guess what the answers are to the following problems?

a. $\begin{array}{r} 2 \\ -\ 3 \\ \hline \end{array}$ ( )

b. $\begin{array}{r} -2 \\ \times\ 3 \\ \hline \end{array}$ ( )

c. $\begin{array}{r} 10.0 \\ \times\ \ 0.2 \\ \hline \end{array}$ ( )

16.

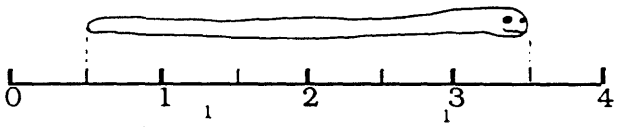
a. $\begin{array}{r} 8.40 \\ -\ 2.11 \\ \hline \end{array}$

b. $\begin{array}{r} 7.3 \\ -\ 3.8 \\ \hline \end{array}$

c. $\begin{array}{r} 1.5 \\ -\ 2.7 \\ \hline \end{array}$

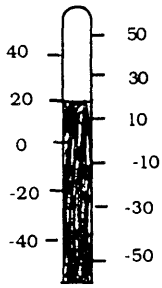
d. $\begin{array}{r} 2.60 \\ +\ 1.75 \\ \hline \end{array}$

17. How long is the snake?



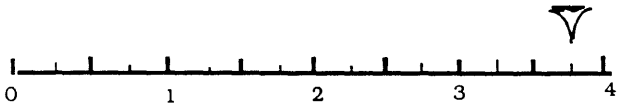

a. 2 b. $2\frac{1}{2}$ c. 3 d. $3\frac{1}{2}$ e. not given

18. If the temperature drops 30 degrees, what will the temperature be?



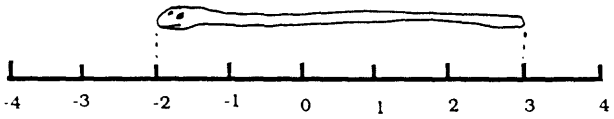

a. -30 b. 10 c. 0 d. -10 e. not given

19. Which number goes with the point?



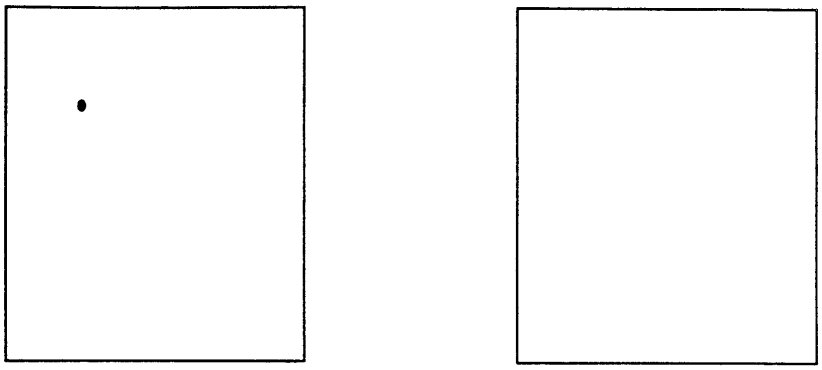

a. $1\frac{4}{4}$ b. $3\frac{4}{5}$ c. $1\frac{5}{5}$ d. $3\frac{3}{4}$ e. not given

20. How long is the snake?




a. 3 b. 4 c. 5 d. -2 e. not given

21. Please put the dot that is on the left rectangle in exactly the the same position on the rectangle on the right. You can use a ruler if you want.



21a. Please tell you had to do to solve the above problem .

22. What do you think was the easiest exercise in this booklet? Why do you think it was easy?

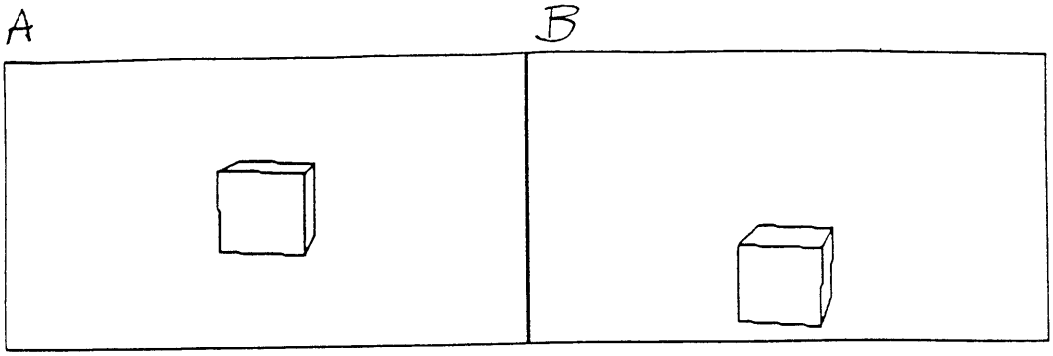
23. What do you think was the hardest exercise in this booklet? Why do you think it was hard?

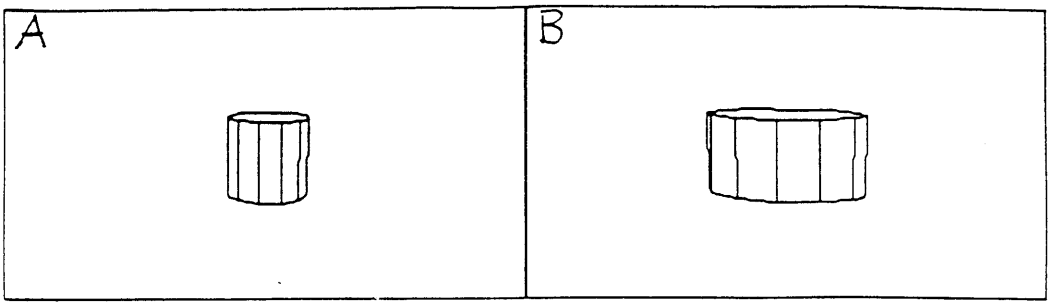
24. Of ALL the test I have given you this week one was the hardest for you? Please tell me why it was so hard.

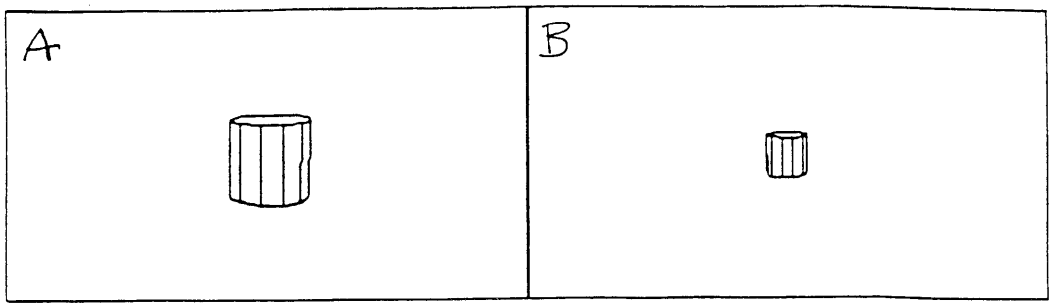
25. Of ALL the test I have given you this week one was the easiest for you? Please tell me why it was so easy.

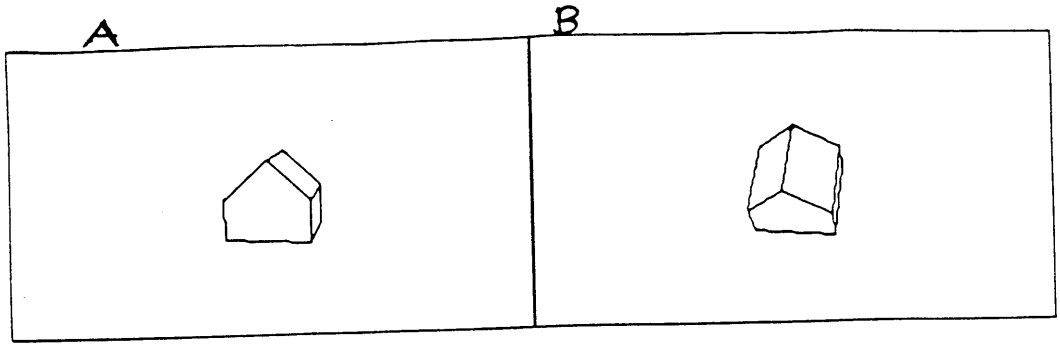
Name_____

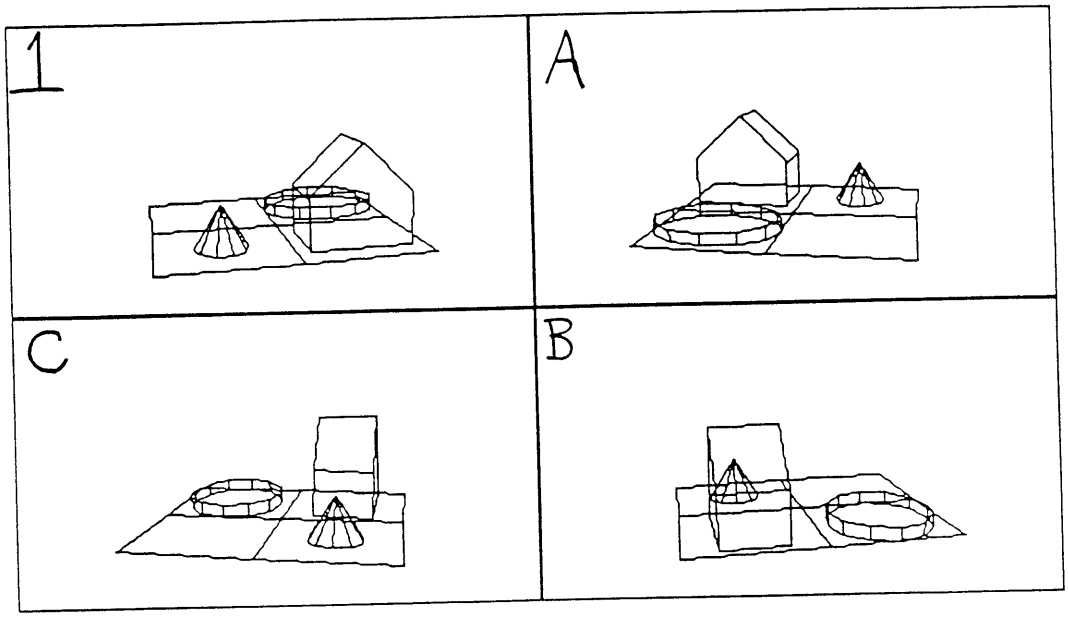
Date_____ Teacher_____

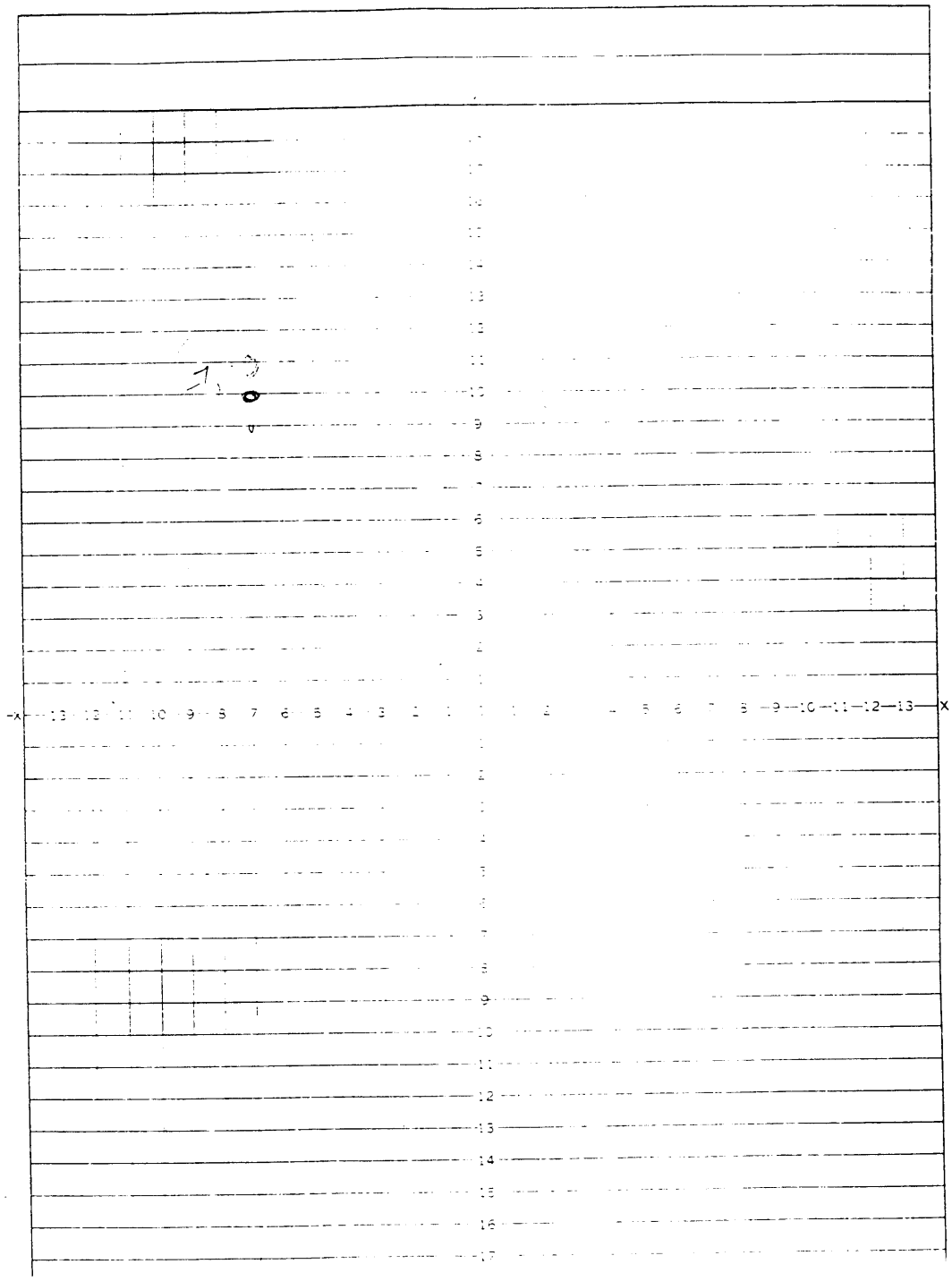

A
B


A
B


A
B


A
B


1
A
C
B


-X
X

# APPENDIX C

# POST-TESTS & TASKS

Selected Test from Johnson and Meade Batteries

Euclidean, Projective, and Mathematics Battery

J3D Related Battery

Name_____

Today's Date_____ Teacher_____

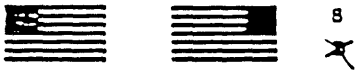
Age_____ Date of birth_____

Boy____ Girl ____ Right handed____ Left handed ____

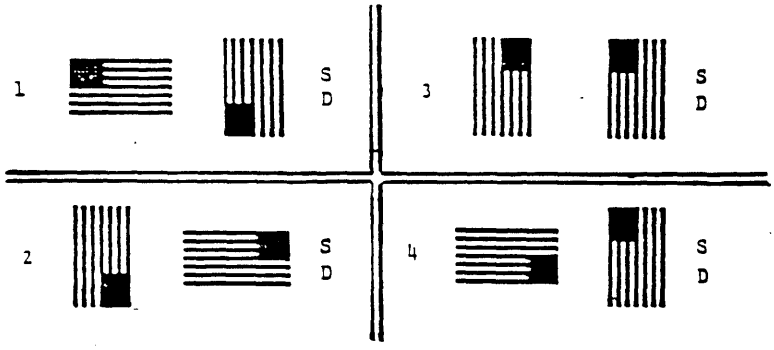
# FLAGS

These two pictures of a flag are the same. You can slide one picture around to fit exactly on the other picture. S is marked to show that the pictures are the SAME.


The next two pictures are different. You cannot slide the pictures around to make them fit exactly. D is marked to show that the pictures are DIFFERENT.




Here are some problems for you to try.

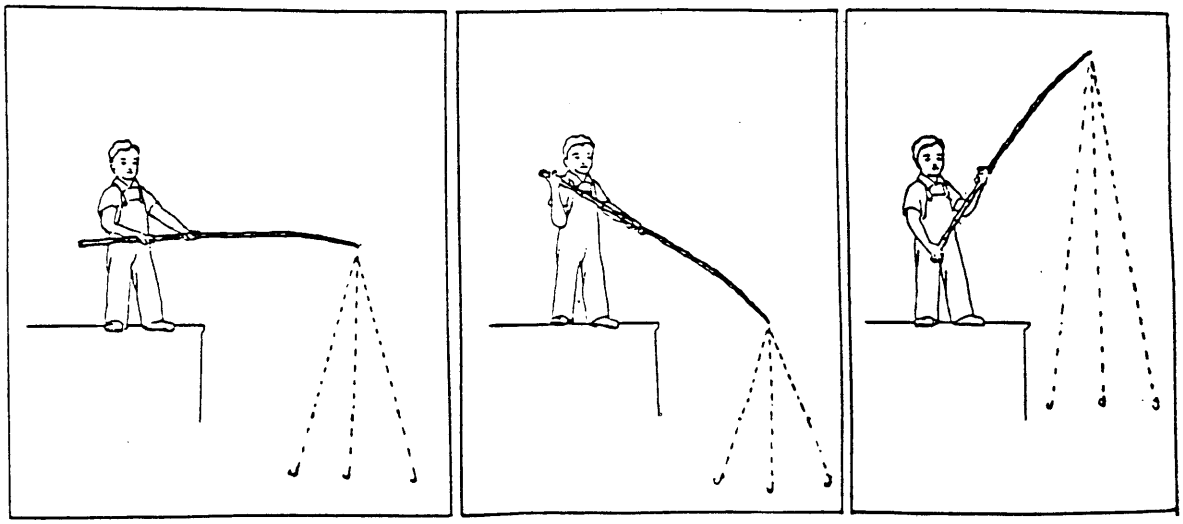



The answers to these problems are 1: D 3: S

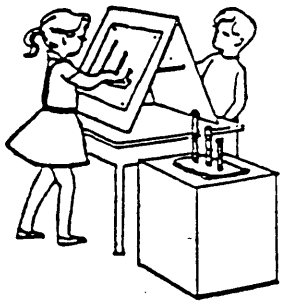
2: D 4: S

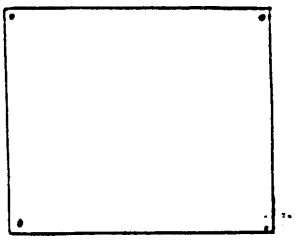
5

This boy is pretending he is fishing. He stands on his front porch and drops his line so that the hook almost touches the ground. How will the line look when he holds it in the three ways below. Trace one line for each of the pictures.



These children are drawing three sticks of different lengths just as they see them. Pretend you are the boy. Make his drawing on this paper.



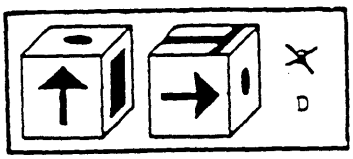


GO ON TO THE NEXT PAGE.

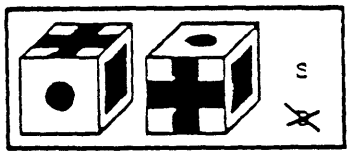
# CUBES

This is a test about cubes. The one rule you must remember is that THE SAME DESIGN IS NEVER USED TWICE ON THE SAME CUBE.

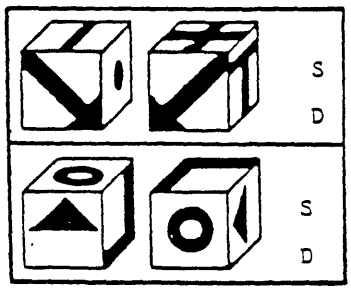
Look at the two cubes below. If you tipped the first cube over to the right it could look like the second one. The S has been marked since the cubes may be the same.

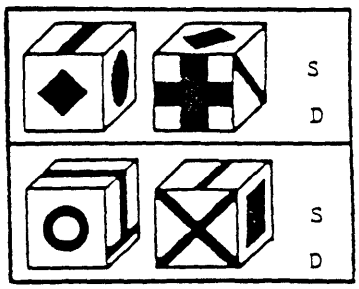



The two cubes below are DIFFERENT. If you tip the first cube toward you so that the cross is on the front, the dot will be on the bottom. There cannot be another dot on the top because the same design can't be used twice on the same cube.



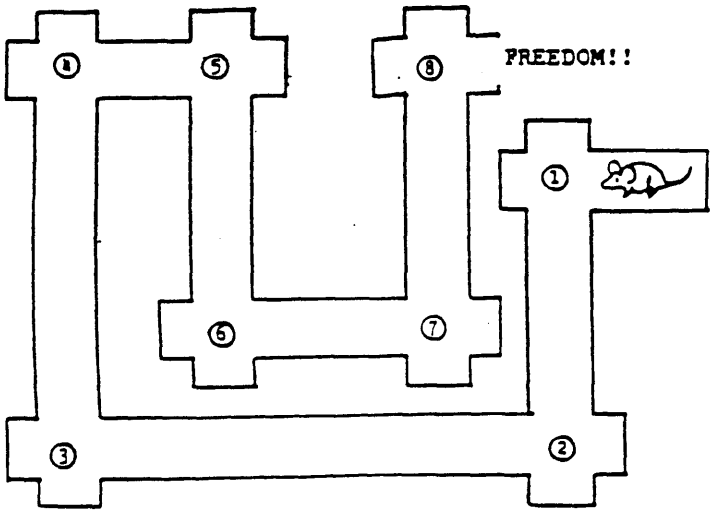

In the practice problems below, mark S if the two cubes could be the SAME. Mark D for DIFFERENT if they cannot be the same cube.

1 


3 


The answers to these problems are 1: S 3: D
2: D 4: S

Help the mouse find his way to freedom. At each of the numbered places tell him whether to turn to his right or to his left.

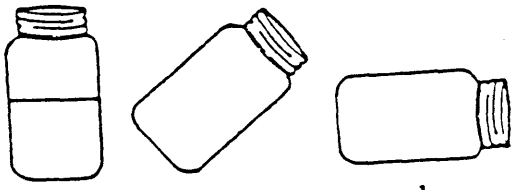



Circle either right or left at each of the turns below.

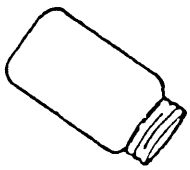
At 1 turn right left
At 2 turn right left
At 3 turn right left
At 4 turn right left
At 5 turn right left
At 6 turn right left
At 7 turn right left
At 8 turn right left

---

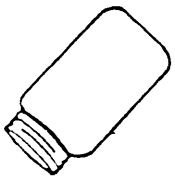
Draw the water line as it would be if the jars were half full of water. The first one has been done for you. (The cap is on tightly—it will not leak.)



1 2 3

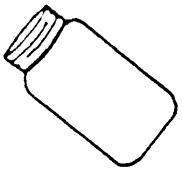


4



5

6



7

GO ON TO THE NEXT PAGE.

Adapted from a test
by Earl Hunt

# ROTATIONS

Instructions

Figures A and B are two pictures of the same object seen from different angles. Observe that you could rotate the object in Figure B so that it would be exactly the same as the object in Figure A.

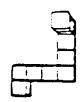
Figure A

D

Figure B

Figures C and D are pictures of two different objects. No matter how you rotated the object in Figure D, it would never match the object in Figure C.

Figure C



S

Figure D

This test is made up of pairs of figures similar to those above. For each pair you must decide if the two pictures are both of the same object, or if they are pictures of different objects. If the pictures are of the same object, mark the S next to the pair. If they are of different objects, mark the D.

STOP HERE AND WAIT FOR THE STARTING SIGNAL.

1. Which of the tests did you think was the easiest? Why?

2. Which of these different tests did you think was the hardest? Why?

Name_____

Today's Date_____ Teacher_____

| | |
|---|---|
| a. | b. |
| c. | d. |

page 1

1. What is the difference between a square and a cube?

2. Please draw a picture of a square and a cube to show how they are different.

| square | cube |
| --- | --- |
| | |

page 2

3. This little space cadet is going to take some photographs with his camera. I want you to draw what is on the photographs.

a.

top view

b.

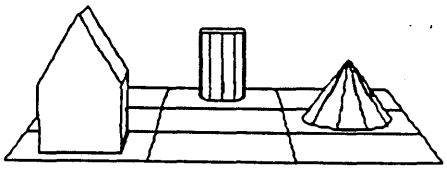
side view

c.

front view

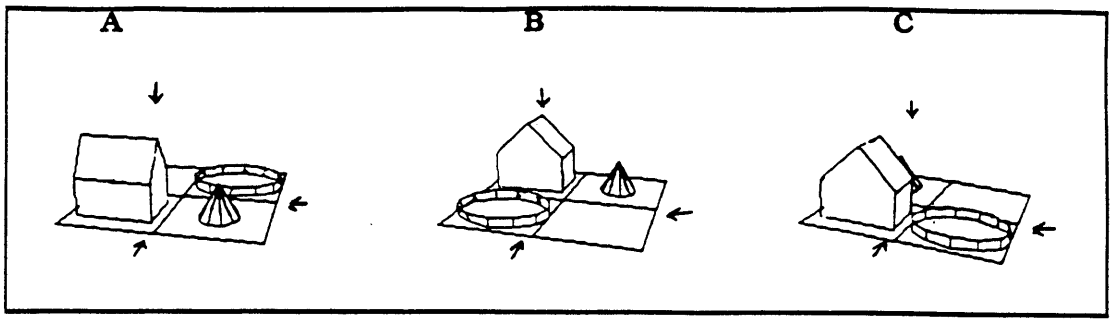
page 3

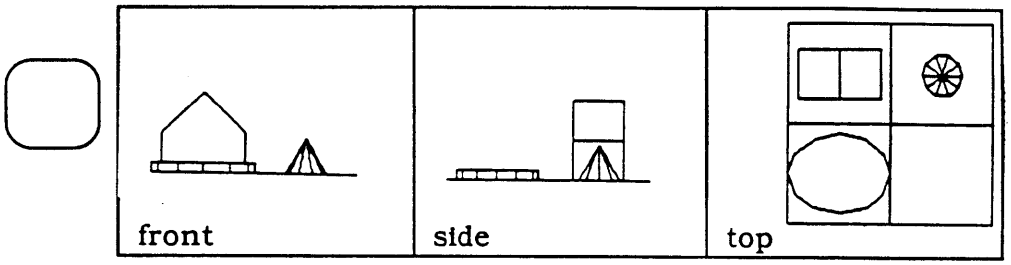
4. Your mission is to help you friend duplicate the scene below. In a short letter to your friend, tell them about the little scene well enough for them to recreate it when they get your letter. You can only help your friend by using words and not drawings.

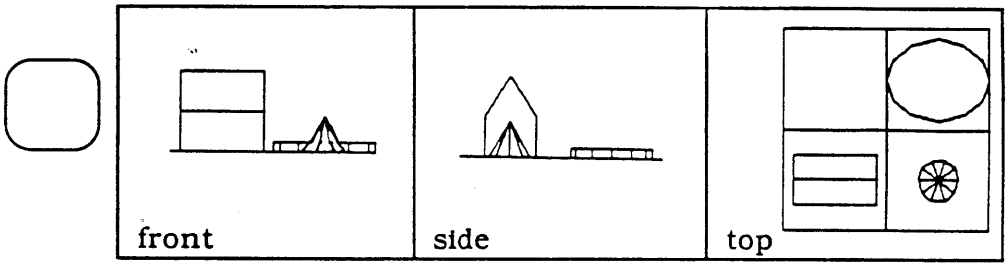


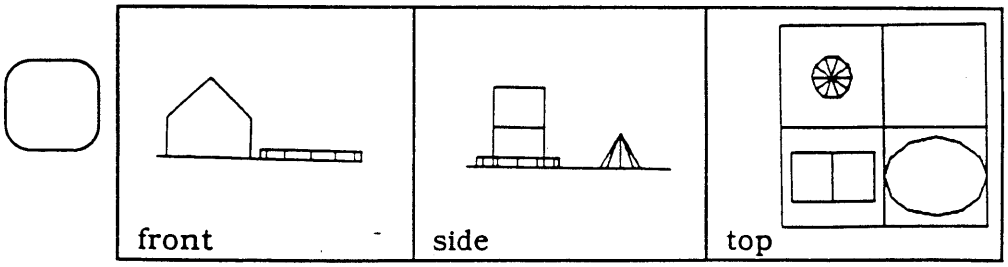
page 4

6. The little space cadet took a set of three photographs (top, front, side) of the three different scenes below. He can't remember which set of photographs went with whichscene. Can you help him ? Place the letter of the correct scene in the box to the left of each set of photographs.



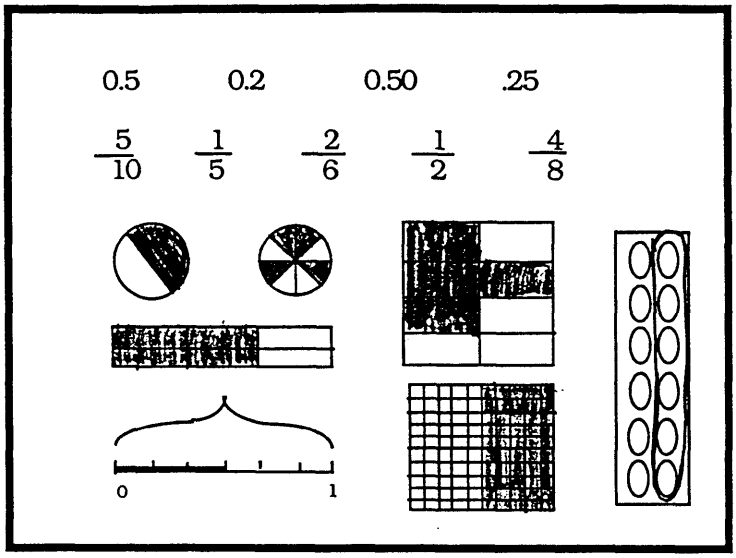



7. Draw a triangle above a rectangle.

8. Draw a circle and a triangle. You can see that the triangle is behind the circle but you can not see through the circle.

9. Write a "W" to the left of an "L". Put an "S" above the "L". Now put an "R" below and to the right of the "L".

10 Circle **ALL** of the representations in the box that mean **one half**




11. Can you think of any other ways to show one half ?
Put your answers in the box below.

12. Please circle the correct answer.

0.2 = a. 1/2 b. 2/10 c. 1/5 d. 2/100

page 8

13. Can you guess what the answer to the following problems are?

a. -3 + 2 = ☐ b. -2 x 3 = ☐ c. .2 x 10 = ☐

14. What comes next?

a. 2.4, 2.5, 2.6, 2.7, 2.8, ☐

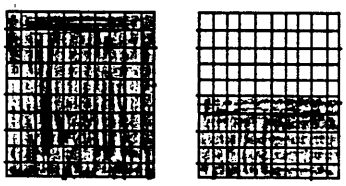
b. 1.8, 2.0, 2.2, 2.4, 2.6 ☐

c. 0.25, 0.50, 0.75, 1.00, ☐

d. 1.5, 1.0, 0.5, 0.0, -0.5, ☐

e. -1.4, -1.3, -1.2, -1.1, ☐

15. Which decimal is shown?



a. 5.1 b. 1.5

c. 10.5 c. not given

16. Can you guess what the answers are to the following problems?

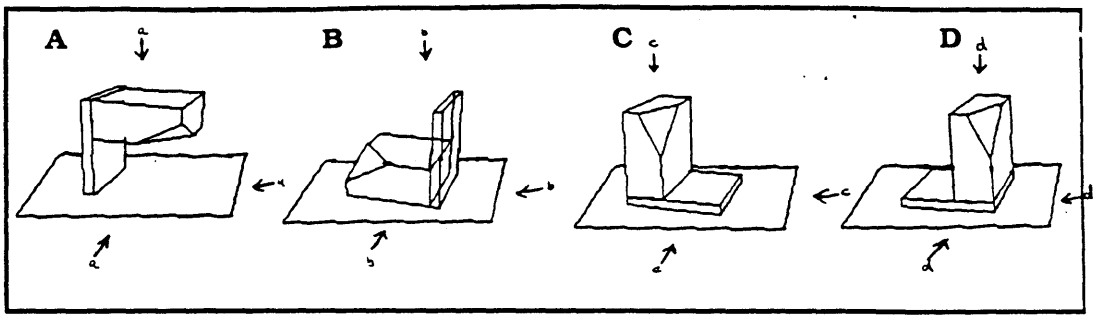
a. 1
- 3
___
( )

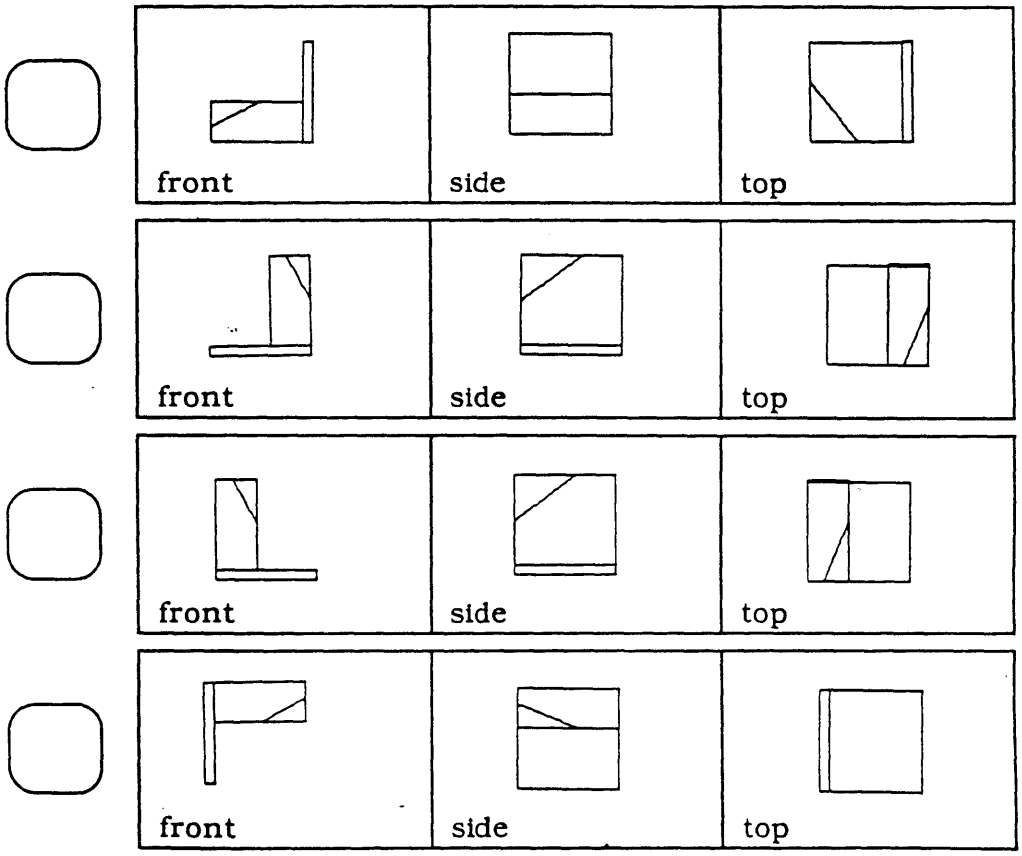
b. -2
x 5
___
( )

c. 20.0
x 0.5
___
( )

page 9

5. The little space cadet took a set of three photographs (top, front, side) of the object below when it was in three different positions. He can't remember which set of photographs went with which position. Can you help him ? Place the letter of the correct position in the box to the left of each set of photographs.



17.

a. 6.30
- 2.35

b. 8.5
- 3.2

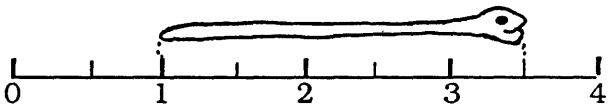
c. 1.5
- 3.7

d. 1.75
+ .25

e. -2.11
+ 8.40

f. -2.7
+ 1.5

18. How long is the snake?

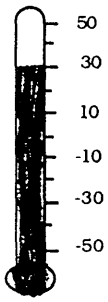



a. 2 b. $2\frac{1}{2}$ c.3 d. $3\frac{1}{2}$ e. not given

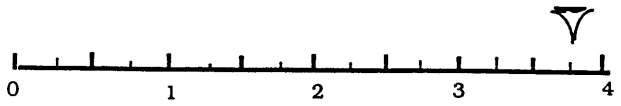
19. If the temperature drops 40 degrees, what will the temperature be?

a. -30 b. 10 c. 0

d. -10 e. not given

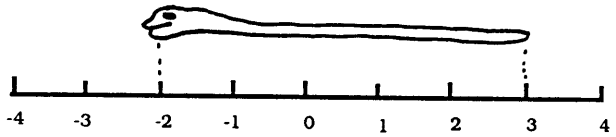


20. Which number goes with the point?




a. $\frac{1^{4}}{4}$ b. $3\frac{4}{5}$ c. $\frac{1^{5}}{5}$ d. $3\frac{3}{4}$ e. not given

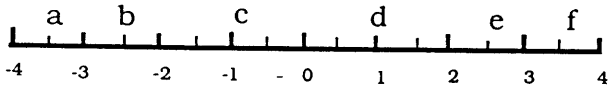
21. How long is the snake?




a. 3 b. 4 c. 5 d. -2 e. not given

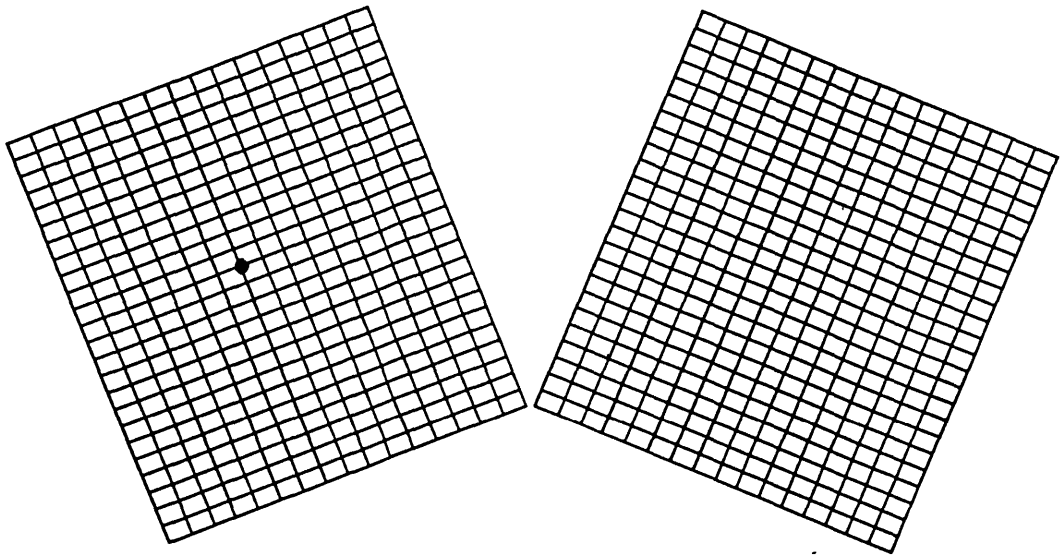
22. Which letter shows the point 2.5 ? _____

Which letter shows the point -1? _____




page 11

23. Please put the dot that is on the rectangle A in exactly the same position on rectangle B.



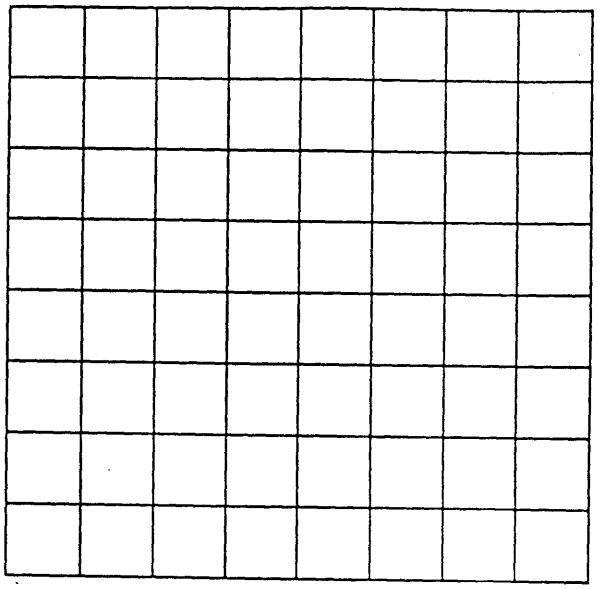
24. Explain how you solved problem 23.

25. What do you think was the easiest exercise in this booklet?
Why do you think it was easy?

26. What do you think was the hardest exercise in this booklet?
Why do you think it was hard?

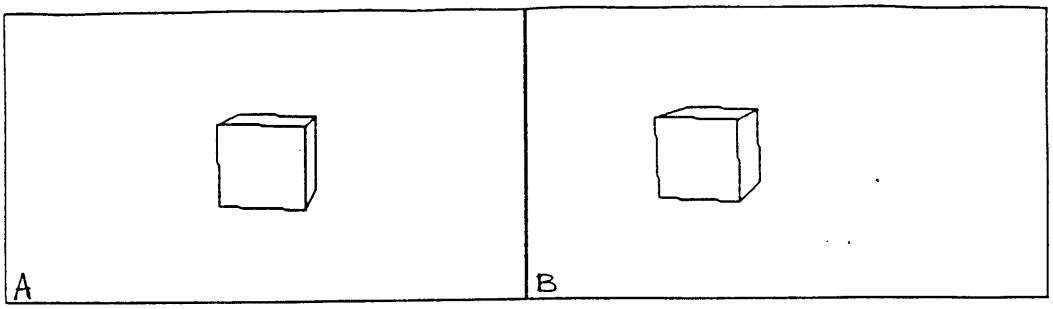
Name_____

Date_____ Teacher_____



Please put in the reference numbers in the grid and plot the following 2 points:

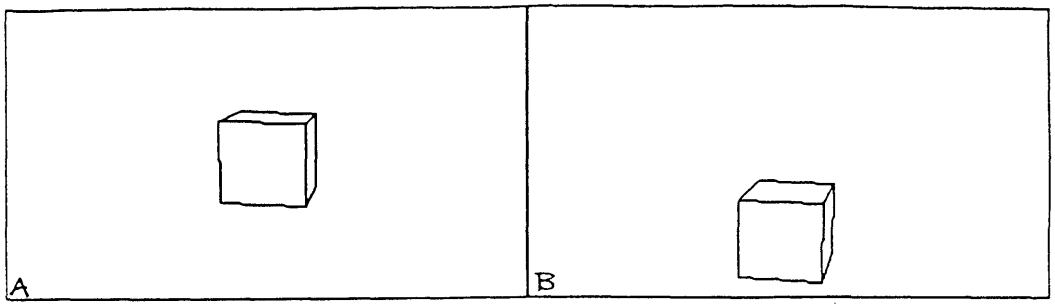
| | | |
|---|---|---|
| -1 | 2 | 0 |
| 3 | -.5 | 0 |




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

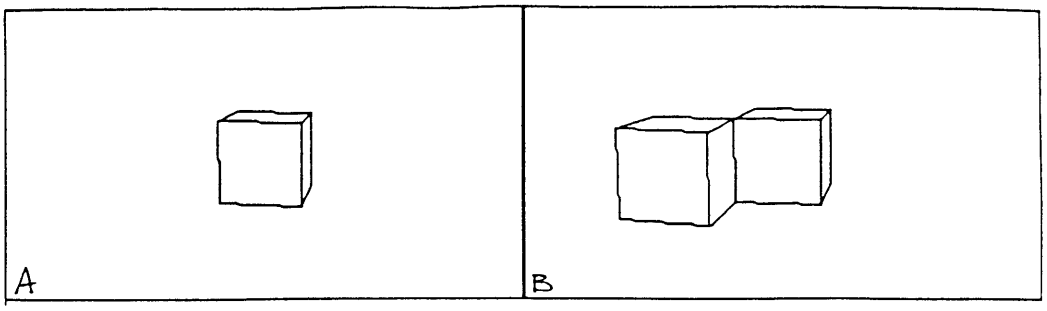
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

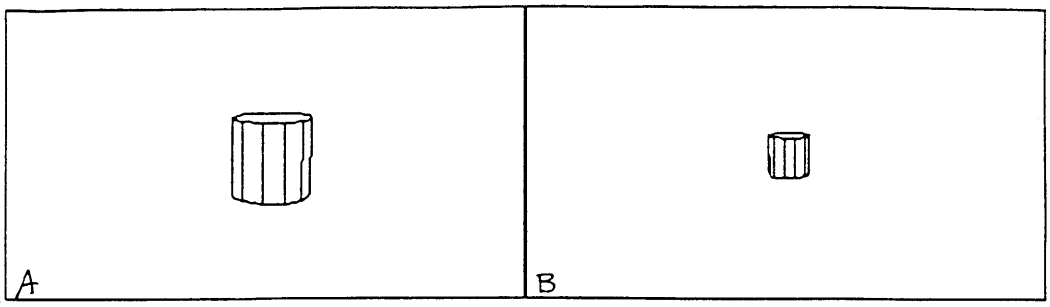
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

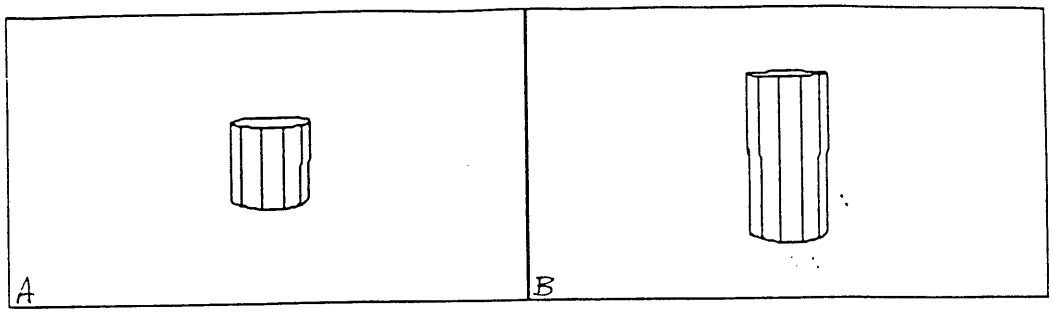
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

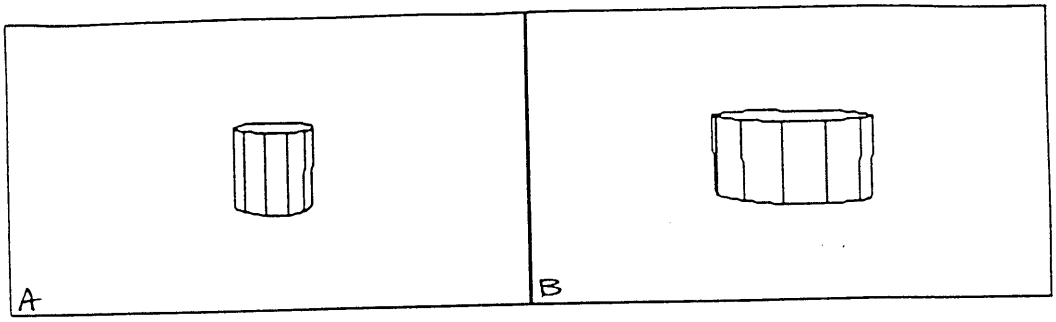
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

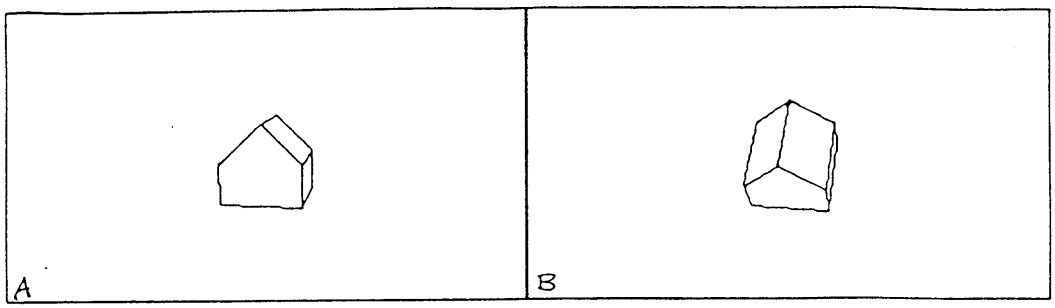
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

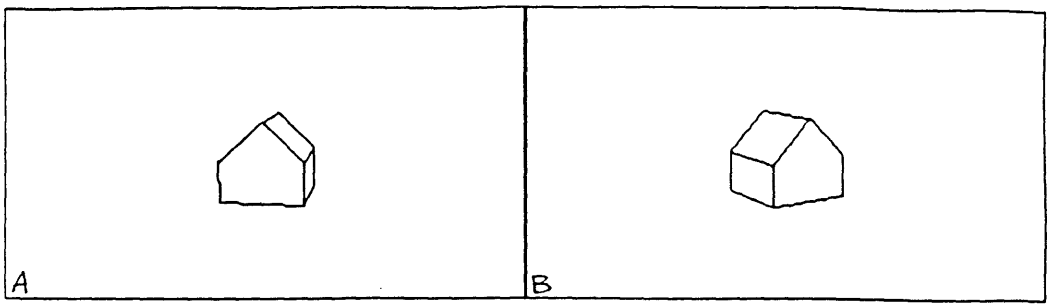
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

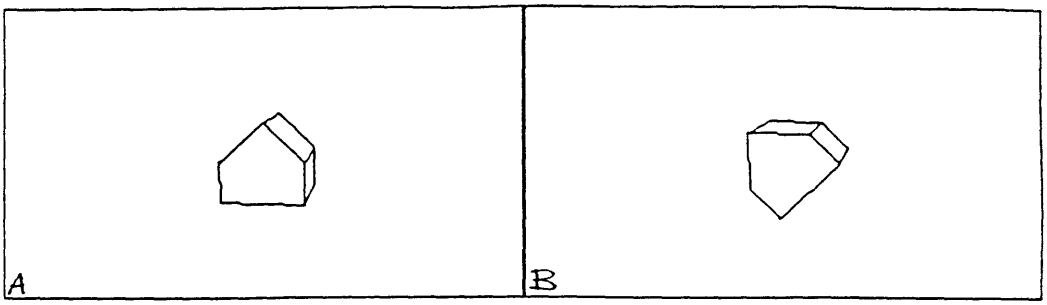
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

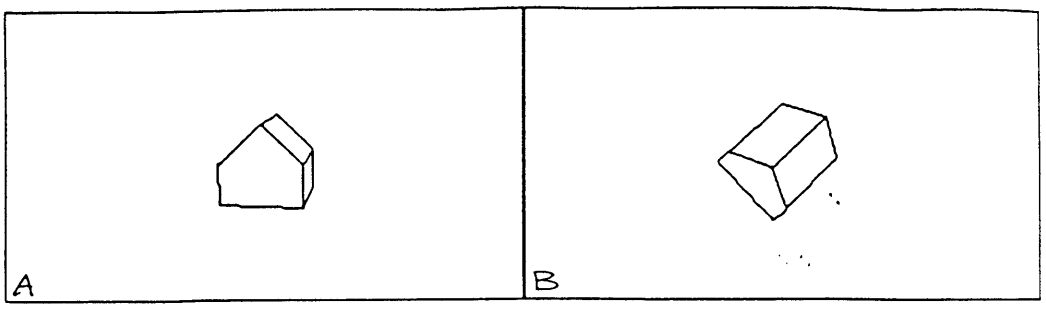
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

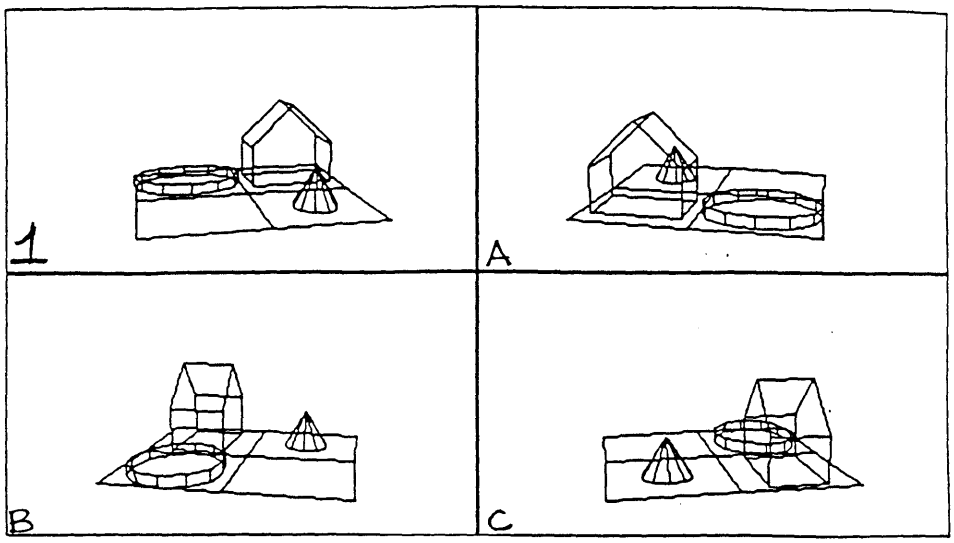
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture A to get picture B?

Can you think of other ways to explain this change?

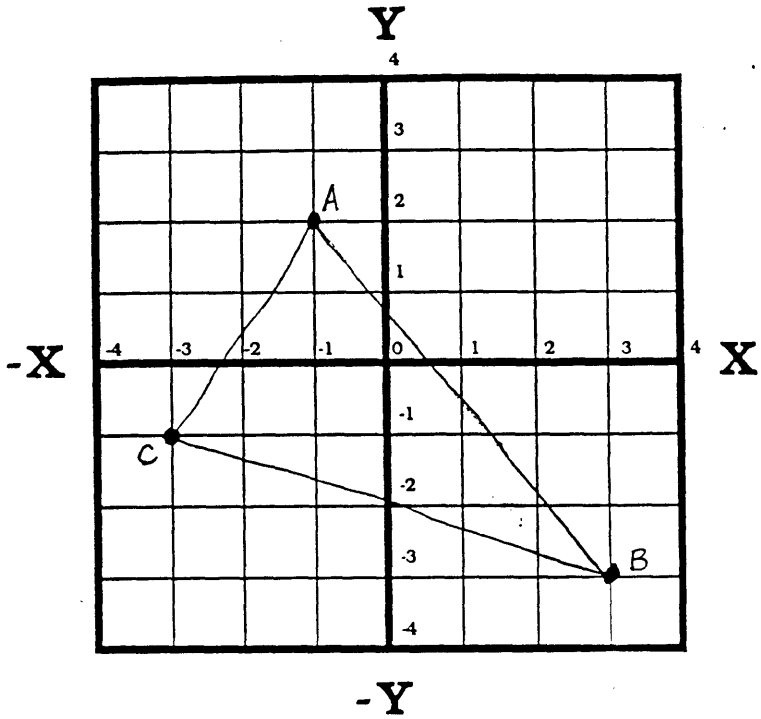
Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.




What has changed from picture 1 to get picture A, B, and C ?

Can you think of other ways to explain the changes?

Can you give the commands you would give the computer using H3D to get picture B? Guess at what numbers you would use.

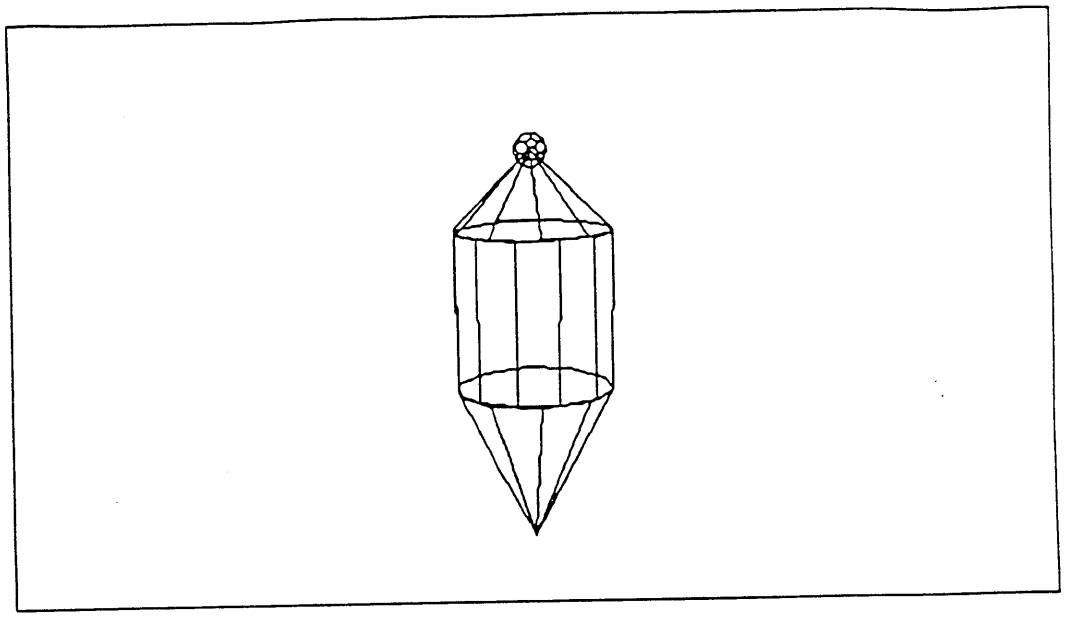



Please write down the coordinates below for points A, B, and C in the drawing above.

A.

B.

C.



# APPENDIX D

# DAVID'S WORK AND NOTES

B Name David _____ Age 10 Birthdate 12-25-77 Hnd R Math Grp H

| | HANDS (49) | | SPATIAL REL (25) | | HIDDEN FIG(84) | | BLOCK CNT (32) | | MAP |
|---|---|---|---|---|---|---|---|---|---|
| correct timed | 5/9 | .56/.18 | 15/19 | .79/.76 | 30/40 | .75/.83 | 13/17 | .76/.53 | |
| total correct | 40 | .82 | 20 | .80 | 60 | .71 | 23 | .72 | |

| | FLAGS | | | | CUBES | | | | ROTATION | | | | PERSPECTIVE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | PRE (48) | | POST (48) | | PRE (32) | | POST (48) | | PRE (42) | | POST (42) | | pre |
| correct timed | 23/23 | 1.00/.48 | 33/33 | 1.00/.69 | 15/16 | .94/.5 | 17/18 | .94/.36 | 17/22 | .77/.52 | 21/28 | .75/.66 | |
| total | 48 | 100 | 48 | 100 | 29 | .91 | 29/40 | .91/.83 | 29 | .69 | 31 | .74 | |
| rate | 11.5 | | 16.5 | | 5 | | 5.5 | | 5.5 | | 7 | | |
| | 1 9 17 25 33 41<br>2 10 18 26 34 42<br>3 11 19 27 35 43<br>4 12 20 28 36 44<br>5 13 21 29 37 45<br>6 14 22 30 38 46<br>7 15 23 31 39 47<br>8 16 24 32 40 48 | | | | 1 9 17 25 33 41<br>2 10 18 26 34 42<br>3 11 19 27 35 43<br>4 12 20 28 36 44<br>5 13 21 29 37 45<br>6 14 22 30 38 46<br>7 15 23 31 39 47<br>8 16 24 32 40 48 | | | | 1 8 15 22 29 36<br>2 9 16 23 30 37<br>3 10 17 24 31 38<br>4 11 18 25 32 39<br>5 12 19 26 33 40<br>6 13 20 27 34 41<br>7 14 21 28 35 42 | | | | post |

| | cylinder | pyramid | hex prism | cube | pre orth draw | post orth draw | VERTICAL PRE | VERTICAL POST |
|---|---|---|---|---|---|---|---|---|
| pre draw | | | | | top / front / side | top / side / front | | |
| post draw 4 | | | | | | | | |

pre post

| | FRACTIONS | | | MEASURE | | MATH + - . ops | | ORTHOGRAPHIC VIEWS | | | MOUSE | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | pre | post | gen | pre | post | pre | post | | pre | post | pre | post |
| dec | 4/4 | 5/5 | - 1 | 4/4 | 6/6 | 8/10 | 10/12 | obj | 3/3 | 4/4 | 7/8 | 8/8 |
| reg | 6/6 | 6/6 | | in 2d | in 2d | | | scene | 3/3 | 3/3 | | |
| graph | 6/6 | 7/7 | 2 | 4 | 4 | Word -> Draw pre | post | pre INC | post INC | | | |
| total | 16/16 | 18/18 | 3 | | | 3/3 | 3/3 | 5/5 | 7/7 | | | |

David

==========

==========< **OBJECT COMMANDS** >==========

**call** <data> <object> [read data and give instance name]
**on|off** <object|all> [on/off for display]
**bkface** <obj> <0|1> [1 turns off backface cull]
**reset** <object|all> [resets to default values]

==========< **OBJECT MANIPULATIONS** >==========

**scale** <object> <x y z> [scale object along each axes]
**place** <object> <x y z> [translation to point in space]
**rotate** <object> <num> <axis> <angle> [<axis><angle>] [<axis><angle>]
[rot object num,1-3, about specified axis, and concat in order entered]

==========< **OBJECT HEIRARCHY** >==========

**attach** <obj> <to-obj> [attaches one object to another ]
**group** <name> <num> <objects> [ group name to attach objects to]
**detach** <object> <from obj> [<0|1>] [1 (default) leaves parent xform]

==========< **OBJECT INFORMATION** >==========

**show** [<object|all|view|hier>] [shows info of named parameters]
**show** [lists all objects called ]

==========

==========< **GLOBAL COMMANDS** >==========

**eye** <x y z> [places camera]
**coi** <x y z> [center of interest, where point camera ]
**up** <x y z> [orientation of camera, up 01 0]
**view** <angle> [angle of view of camera from eyepoint]
**reset** <view> [resets to default values]
**display** [<quad> <per=0|orth=1>] [ fullscreen =0, quad > 1 | 2 | 3 | 4]
**bluprint** [x y z] [display oblique, front, side, & top views]
**clear** <0|1> [1= clears screen 0= noclear]
**delete** [1] [1= forces delete all object, else asks]

==========< **SCENE INFORMATION** >==========

;<any thing you want> [computer ignors lines that start with ;]
@<filename> [reads in a file]
**save** [saves environment to a file]
**history** [keeps a record of all commands]
**nohistory** [stops record of all commands]
**print** [quad] [prints screen or quadrant]
**pause** [allows pause from script]

==========

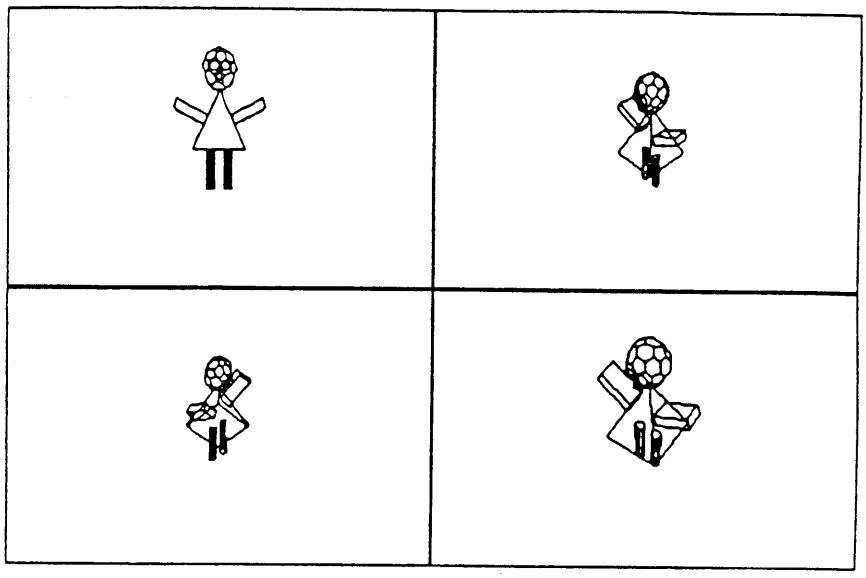
--- ----------< **DEFAULT VALUES** >----------

| | | | | | |
|---|---|---|---|---|---|
| **eye**=0 0 10 | **coi**=0 0 0 | **vie**=45 | **up**=0 1 0 | **cen**=0 0 | **cle**=1 on |
| **sca**=1 1 1 | **pla**=0 0 0 | **rot**=0 | **bkf**=0 | **dis**=0 0 | **blu**=3 3 10 |

**Objects:**

| | | | | | |
|---|---|---|---|---|---|
| cylnd | bcylnd | cube | bcube | cut | |
| circle | soccer | cone | hut | sq | nsq |

```
;dav.eyes
;Mon. 4/11/1988, 10:31:11
call pyr.asc  b
scale b 1.5 3 1.5
call soccer.asc  h
rotate h 1 z 28
call cylnd.asc  ll
scale ll 0.25 1 0.25
place ll -0.5 -1 0
call ncube.asc  la
scale la 1 0.5 0.5
place la -1.55 2.1 0
rotate la 1 z 155
call cylnd.asc  rl
scale rl 0.25 1 0.25
place rl 0.5 -1 0
call ncube.asc  ra
scale ra 1 0.5 0.5
place ra 1.55 2.1 0
rotate ra 1 z -155
call circle.asc  eye
scale eye 0.5 0.5 0.5
place eye -0.35 0 1
rotate eye 1 x 90
call circle.asc  eye1
scale eye1 0.5 0.5 0.
place eye1 0.35 0 1
rotate eye1 1 x 90
call sq.asc  mo
scale mo 0.15 0.15 0
place mo -0.05 -0.5 1
group head 4  h  eye  eye1  mo
place head 0 4 0
rotate head 1 x 0
group body 5  b  ra  rl  la  ll
group person 2  head  body
eye  0 0 30
dis 1
eye  20 20 20
dis 2
eye  -20 -20 -20
dis 3
eye  10 20 15
dis 4
```



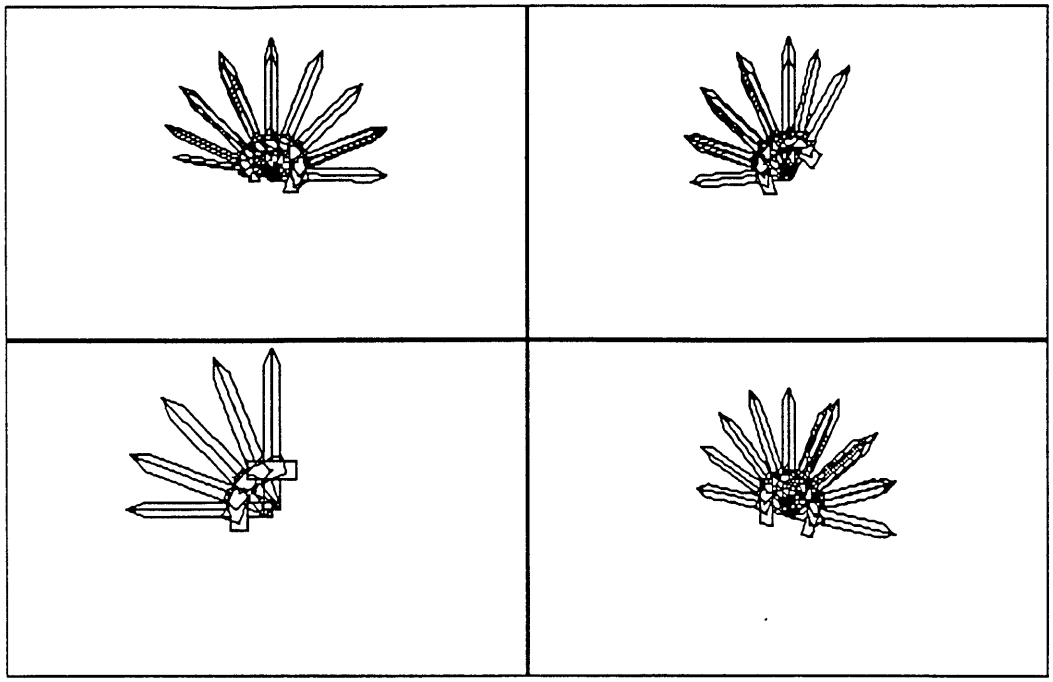
## David's Animation Script

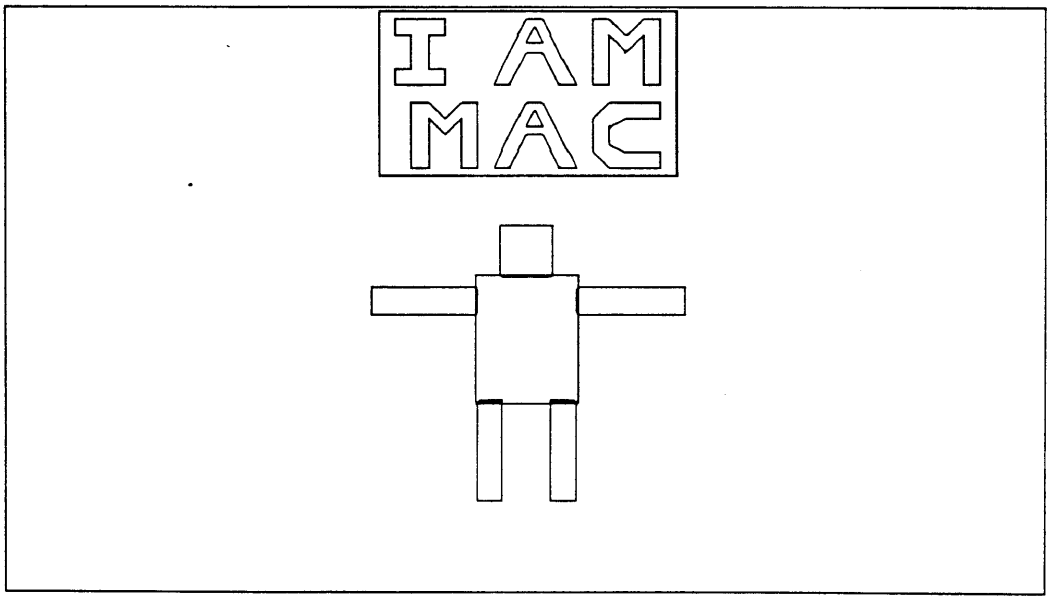
```
;dave.sw
;Wed. 4/13/1988, 12:38:29
eye  40 40 80
call sword.asc  sw
dis
rot sw 2 z 22.5 x 11.25
eye 40 37 80
dis
rot sw 2 z 45 x 22.5
eye 40 34 80
dis
rot sw 2 z 67.5 x 33.75
eye 40 31 80
dis
rot sw 2 z 90 x 45
eye 40 28 80
dis
rot sw 2 z 67.5 x 33.75
eye 40 25 80
dis
rot sw 2 z 45 x 22.5
eye 40 22 80
dis
rot sw 2 z 22.5 x 11.25
eye 40 19 80
dis
rot sw 2 z 0 x 0
eye 40 16 80
dis
rot sw 2 z -22.5 x -11.25
eye 40 13 80
dis
rot sw 2 z -45 x -22.5
eye 40 10 80
dis
rot sw 2 z -67.5 x -33.75
eye 40 7 80
dis
rot sw 2 z -90 x -45
eye 40 4 80
dis
rot sw 2 z -67.5 x -33.75
eye 40 1 80
dis
rot sw 2 z -45 x 22.5
eye 40 -38 80
dis
rot sw 2 z -22.5 x 11.25
eye 40 -34.2 80
dis
rot sw 2 z 0 x 0
eye 40 -30.4 80
dis
rot sw 2 z 22.5 x 11.25
eye 40 -26.2 80
dis
rot sw 2 z 45 x 22.5
eye 40 -22.4 80
dis
rot sw 2 z 67.5 x 33.75
eye 40 -18.6 80
dis
rot sw 2 z 90 x 45
eye 40 -14.8 80
dis
rot sw 2 z 67.5 x 33.75
eye 40 -11 80
dis
rot sw 2 z 45 x 22.5
eye 40 -7.3 80
dis
rot sw 2 z 22.5 x 11.25
dis 40 -3.5
rot sw 2 z 0 x 0
eye 40 0 80
dis

;dave.sw
;Wed. 4/13/1988, 12:38:29
eye  0 0 75
```

```
call sword.asc  sw
dis
rot sw 2 z 22.5 x 11.25
dis
rot sw 2 z 45 x 22.5
dis
rot sw 2 z 67.5 x 33.75
dis
rot sw 2 z 90 x 45
dis
rot sw 2 z 67.5 x 33.75
dis
rot sw 2 z 45 x 22.5
dis
rot sw 2 z 22.5 x 11.25
dis
rot sw 2 z 0 x 0
eye 40 40 80
dis
rot sw 2 z -22.5 x -11.25
dis
rot sw 2 z -45 x -22.5
dis
rot sw 2 z -67.5 x -33.75
dis
rot sw 2 z -90 x -45
dis
rot sw 1 z -67.5
dis
rot sw 1 z -45
dis
rot sw 1 z -22.5
dis
rot sw 1 z 0
dis
rot sw 1 z 22.5
dis
rot sw 1 z 45
dis
rot sw 1 z 67.5
dis
rot sw 1 z 90
dis
rot sw 1 z 67.5
dis
rot sw 1 z 22.5
dis
rot sw 1 z 0
dis
```

```
david/sword.asc
18 2
title sword by david
0 0 0
-1 0 0
-1 3 0
-3 3 0
-3 5 0
-1 5 0
-1 10 0
-1 15 0
0 17 0
1 15 0
1 10 0
1 5 0
0 5 0
0 10 0
3 5 0
3 3 0
1 3 0
1 0 0
26 1 2 3 4 5 6 7 8 9 10 11 12 13 6 13 14 9 14 13 12 15 16 17 3 17 18
26 18 17 3 17 16 15 12 13 14 9 14 13 6 13 12 11 10 9 8 7 6 5 4 3 2 1
```




I AM
MAC

```
;dav.robo3
;Tue. 5/10/1988, 13:13:36
eye  0 0 25
call id.asc  i
place i -10 10 0
call ad.asc  aa
place aa -4 10 0
call md.asc  ma
place ma 2 10 0
call md.asc  mb
place mb -9 5 0
call ad.asc  ab
place ab -4 5 0
call cd.asc  c
place c 2 5 0
call sq.asc  bk
scale bk 9 5 0
place bk -2 9.5 0
group sign 7  i  aa  ma  mb  ab  c  bk
place sign 2 3 -50
call cube.asc  b
scale b 1 1.25 1
place b 0 0.25 0
call cube.asc  h
scale h 0.5 0.5 0.5
place h 0 2 0
call cube.asc  ra
scale ra 0.25 1 0.25
place ra 2 1 1
rotate ra 1 z 90
call cube.asc  rl
scale rl 0.25 1 0.25
place rl 0.75 -2 0
call cube.asc  la
scale la 0.25 1 0.25
place la -2 1 1
rotate la 1 z 90
call cube.asc  ll
scale ll 0.25 1 0.25
place ll -0.75 -2 0
group robo 6  h  b  ra  rl  la  ll
place robo 0 -1 0
```

david/ad.asc

```
13 4
title a
0 0 0
1 2 0
2 4 0
3 4 0
4 2 0
5 0 0
4 0 0
3 2 0
2 2 0
1 0 0
2.5 3.5 0
2 2.5 0
3 2.5 0
10 1 2 3 4 5 6 7 8 9 10
10 1 10 9 8 7 6 5 4 3 2
3 11 13 12
3 12 13 11
```

david/cd.asc

```
12 2
title c
1 0 0
0 1 0
0 3 0
1 4 0
4 4 0
4 3 0
2 3 0
1 2.5 0
1 1.5 0
2 1 0
4 1 0
4 0 0
12 1 2 3 4 5 6 7 8 9 10 11 12
```

12 12 11 10 9 8 7 6 5 4 3 2 1

david/id.asc

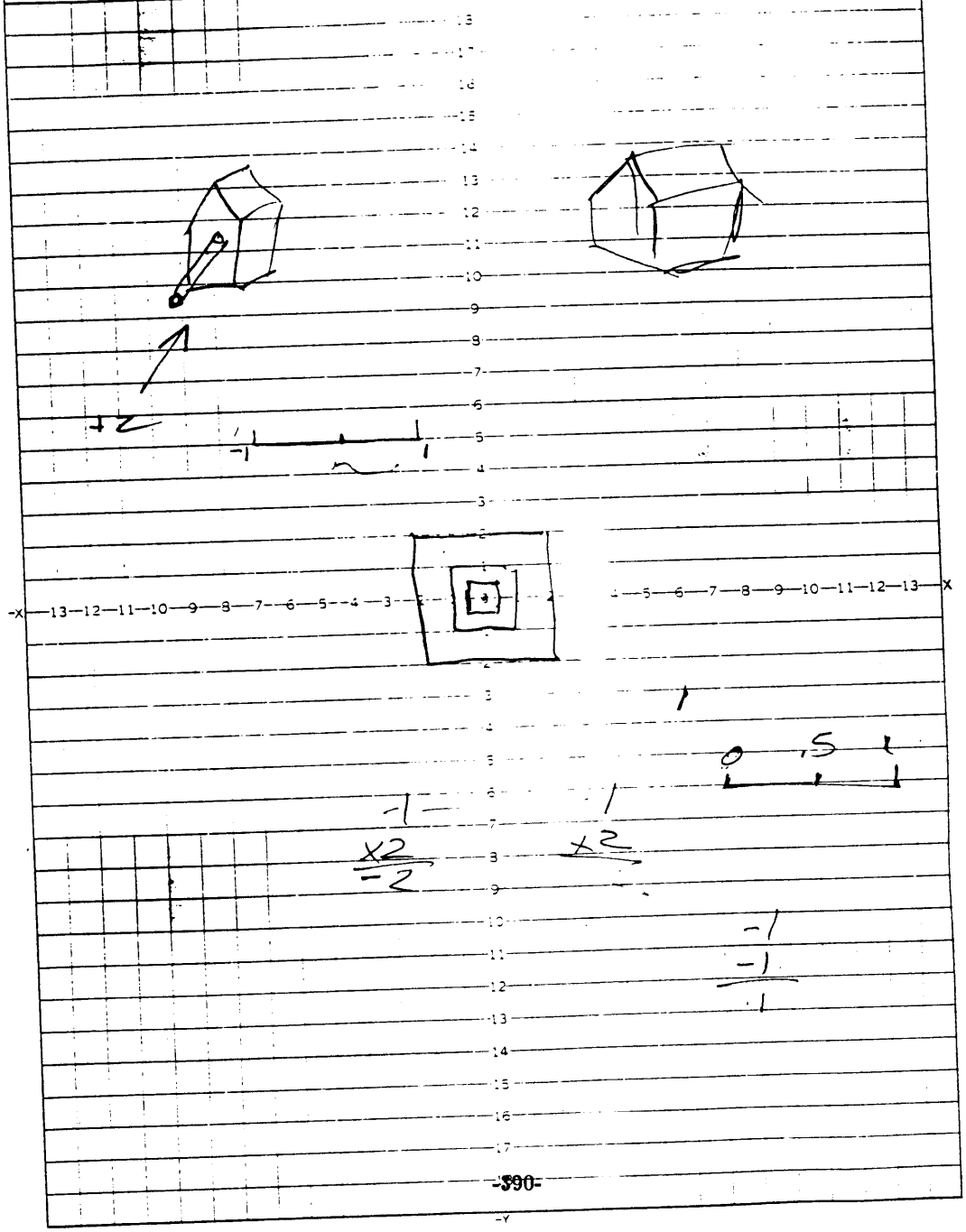
```
12 2
title i
0 0 0
0 1 0
1 1 0
1 3 0
0 3 0
0 4 0
3 4 0
3 3 0
2 3 0
2 1 0
3 1 0
3 0 0
12 1 2 3 4 5 6 7 8 9 10 11 12
12 12 11 10 9 8 7 6 5 4 3 2 1
```

Name Randi

Today's Date 7/24/02

I think this is fun! I want to make animation

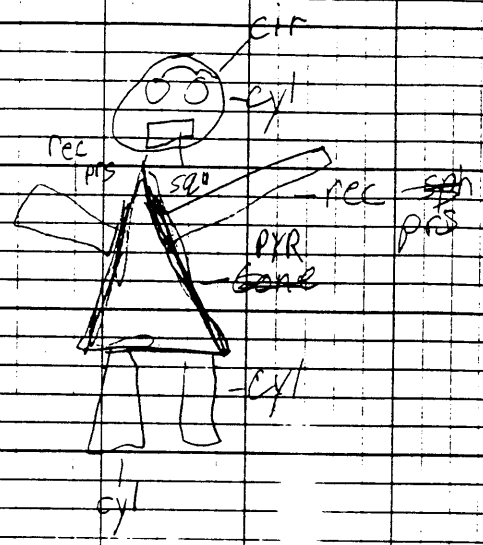
David 3-24



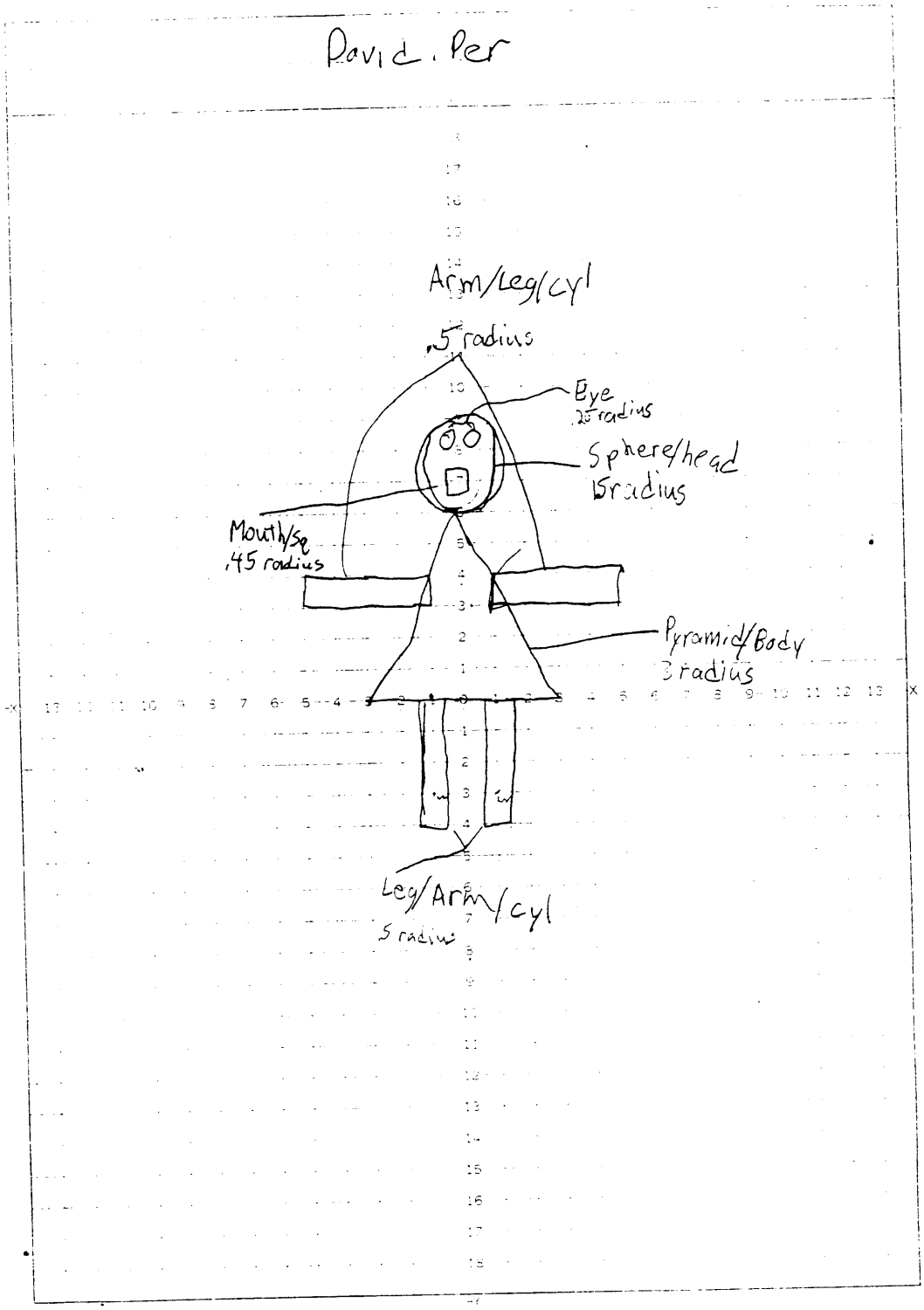



Quadratic Coordinates

David



David. Per




Quadratic Coordinates

Name David

Today's Date 4/11/88

Done!

Eye 0 0 30 saved as ~~dav~~ ~~dav.pen~~ dav.pen1

Eye 20 20 20 saved as dav.pen2

Eye -20 -20 -20 saved as dav.pen3

Dav.eyes

Name David Barner

Today's Date 4/6/88

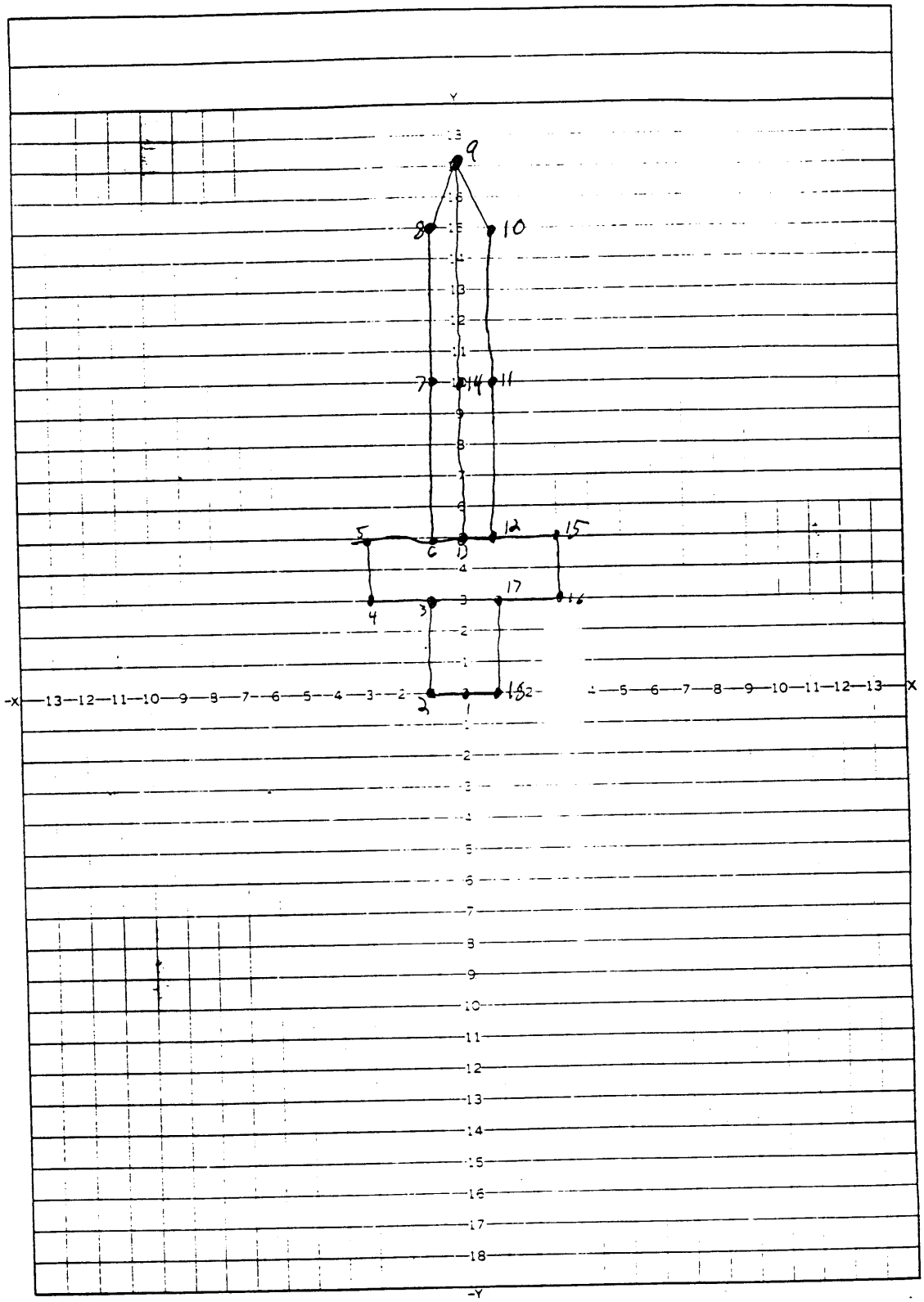
Almost done! Just need to put the head + body together

Name_____

Today's Date_____

Did my animation of sword!
Person done!
This is fun!

page 1




Quadratic Coordinates

18 2

title sword

Point Placement (x Y Z)

1. 0 00
2. -1 00
3. -1 30
4. -3 3 0
5. -3 5 0
6. -1 5 0
7. -1 10 0
8. -1 15 0
9. 0 17 0
10. 1 15 0
11. 1 10 0
12. 1 5 0
13. 0 5 0
14. 0 10 0
15. 3 5 0
16. 3 3 0
17. 1 3 0
18. 1 0 0

Sword Design connections

1-2 3 4 5 6 7 8 9 10 11 12
13 6 13 14 9 14 13 12 15 16
17 3 17 18

28

18 17 3 17 16 15 12 13 14
9 14 13 6 13 12 11 10 9
8 7 6 5 4 3 2 1

24 1 2 3 4 5 6 7 8 9 10 11 12 13 6 13 14 9 13
15 16 17 3 17 18

24

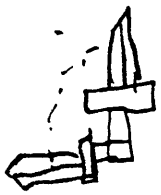
18-17-3-17-16-15-13-9-14-13-6-13-12-11-10-9-
8-7-6-5-4-3-2-1

Page _____ of _____ Pages

# Script Sheet

Date _____ 19 ____

Title: _____ Draft Number: 1 • 2 • Final



## Script

Interactive Training Systems, Inc.

Name David

Today's Date

This is fun!

Name

Today's Date 5/3/88 5/4/88 5/5/88

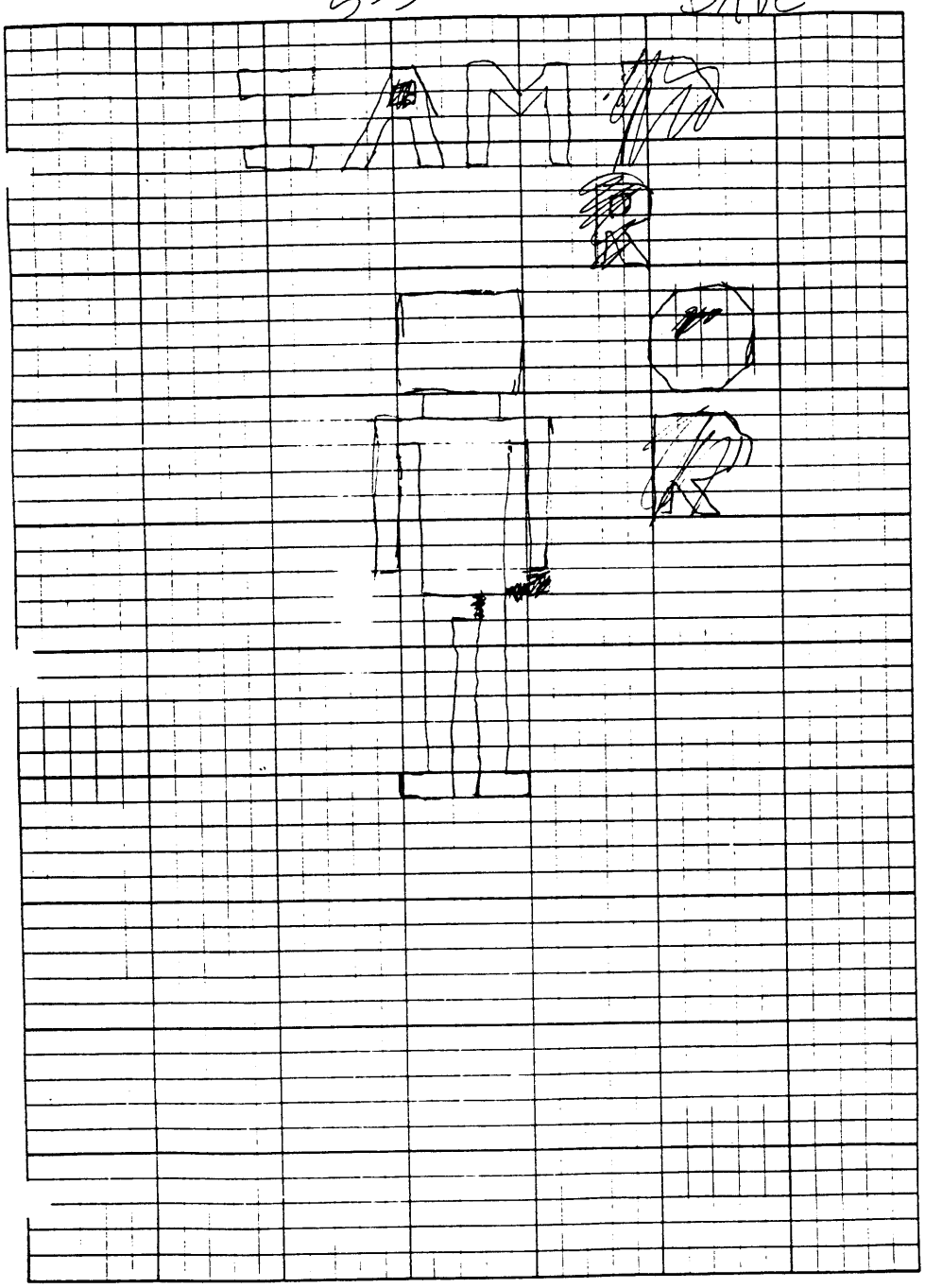
Just have to finish moving the eyes!!!

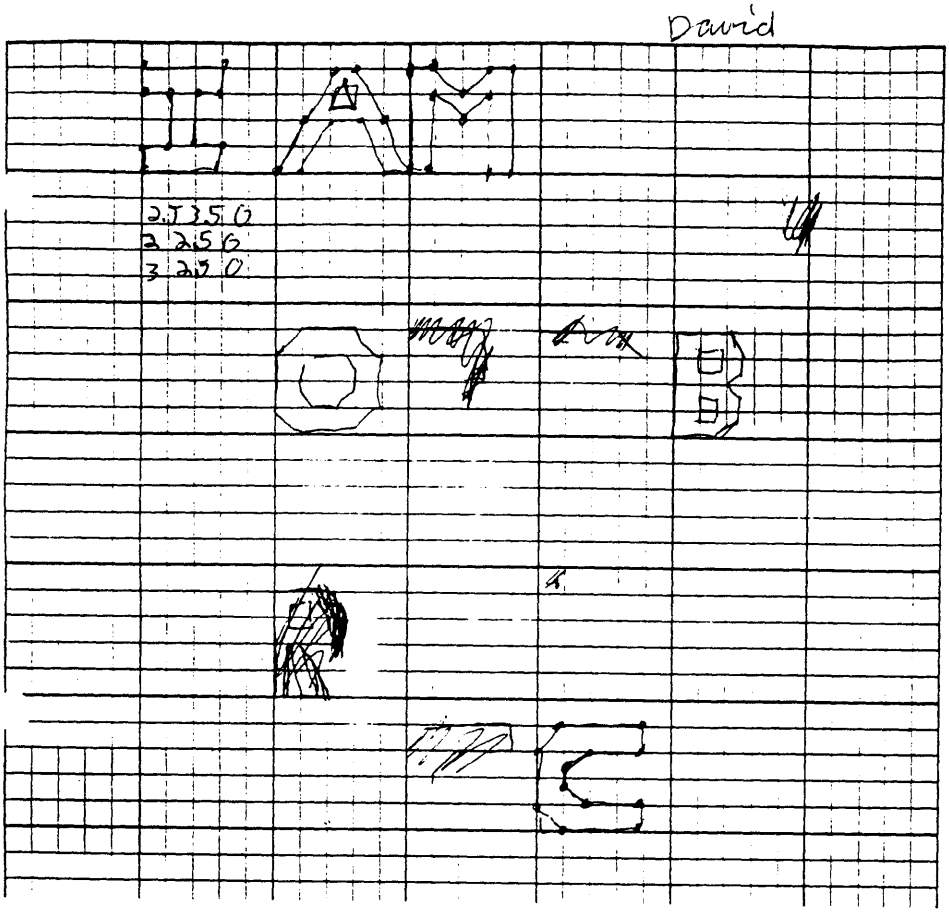
I finished my sword animation

I'm making a robot saying "I AM MAC"

I made the words

5-5 DAVE






Name David

Today's Date 5-10-86

I am almost done with robo.

# APPENDIX E

# JENNI'S WORK AND NOTES

G Name Jennifer _____ Age 10 Birthdate 11/20/77 Hnd R Math Grp M

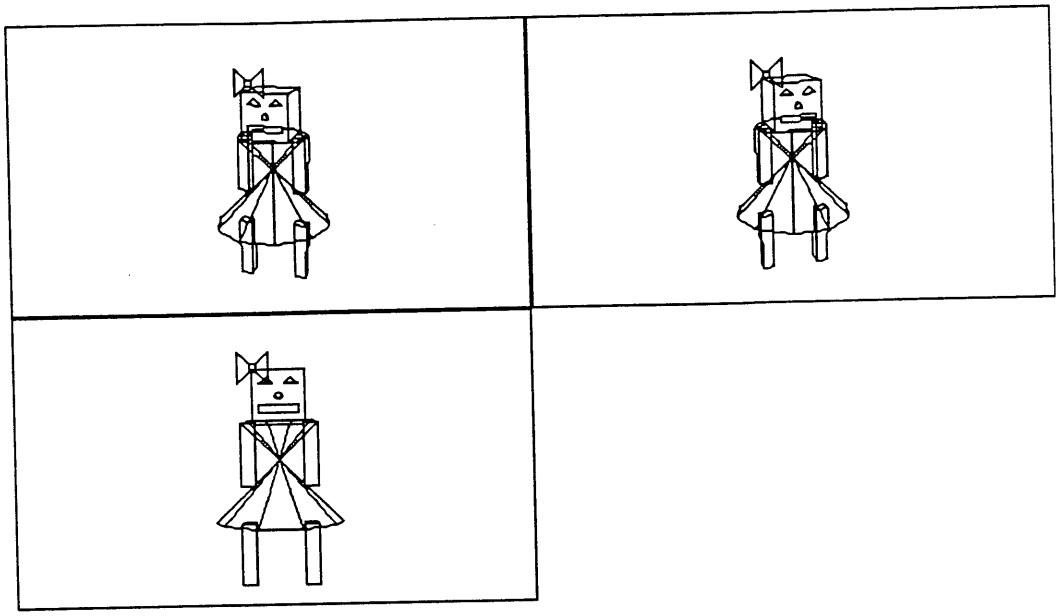
| | HANDS (49) | | SPATIAL REL (25) | | HIDDEN FIG(84) | | BLOCK CNT (32) | | MAP |
|---|---|---|---|---|---|---|---|---|---|
| correct / timed | 2 / 5 | .40 / .10 | 10 / 12 | .83 / .48 | 28 / 40 | .70 / .48 | 7 / 10 | .70 / .31 | |
| total correct | 24 | .49 | 20 | .80 | 55 | .66 | 13 | .41 | |

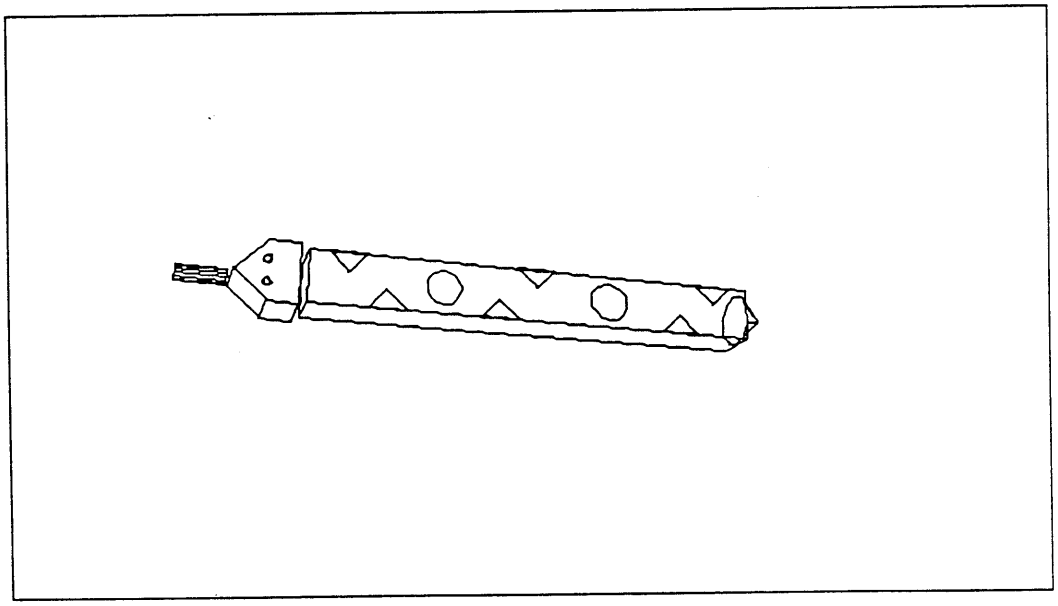
| | FLAGS | | | | CUBES | | | | ROTATION | | | | PERSPECTIVE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | PRE (48) | | POST (48) | | PRE (32) | | POST (48) | | PRE (42) | | POST (42) | | pre |
| correct / timed | 7 / 16 | .44 / .33 | 13 / 18 | .72 / .38 | 5 / 16 | .31 / .5 | 15 / 27 | .56 / .56 | 26 / 40 | .65 / .95 | 22 / 42 | .52 | |
| total | 21 | .44 | 33 | .69 | 10 | .31 | 17 / 27 | .53 / .56 | 27 | .64 | 42 | .52 | |
| rate | 8 | | 9 | | 5.3 | | 9 | | 10 | | 10.5 | | |
| | 1 - 9 1 7 - 25 - 33 - 41 - 2 1 - 10 - 18 1 - 26 1 - 34 1 - 42 1 - 3 - 11 1 - 19 1 - 27 - 35 - 43 1 - 4 - 12 1 - 20 1 - 28 1 - 36 - 44 1 - 5 1 - 13 1 - 21 1 - 29 - 37 - 45 - 6 - 14 1 - 22 - 30 - 38 - 46 1 - 7 1 - 15 - 23 - 31 1 - 39 1 - 47 1 - 8 - 16 1 - 24 1 - 32 1 - 40 1 - 48 1 - | | | | 1 1 9 1 17 1 25 1 33 - 41 - 2 - 10 1 - 18 - 26 1 - 34 - 42 - 3 1 - 11 - 19 1 - 27 - 35 - 43 - 4 - 12 1 - 20 1 - 28 1 - 36 - 44 - 5 1 - 13 - 21 1 - 29 - 37 - 45 - 6 - 14 1 - 22 - 30 1 - 38 - 46 - 7 1 - 15 1 - 23 1 - 31 - 39 - 47 - 8 1 - 16 1 - 24 1 - 32 1 - 40 - 48 - | | | | 1 - 8 1 15 1 - 22 - 29 - 36 1 - 2 - 9 - 16 - 23 1 - 30 1 - 37 - 3 1 - 10 1 - 17 1 - 24 1 - 31 - 38 1 - 4 - 11 - 18 - 25 - 32 1 - 39 - 5 1 - 12 - 19 - 26 1 - 33 1 - 40 - 6 1 - 13 - 20 - 27 - 34 1 - 41 - 7 - 14 - 21 1 - 28 - 35 - 42 - | | post |

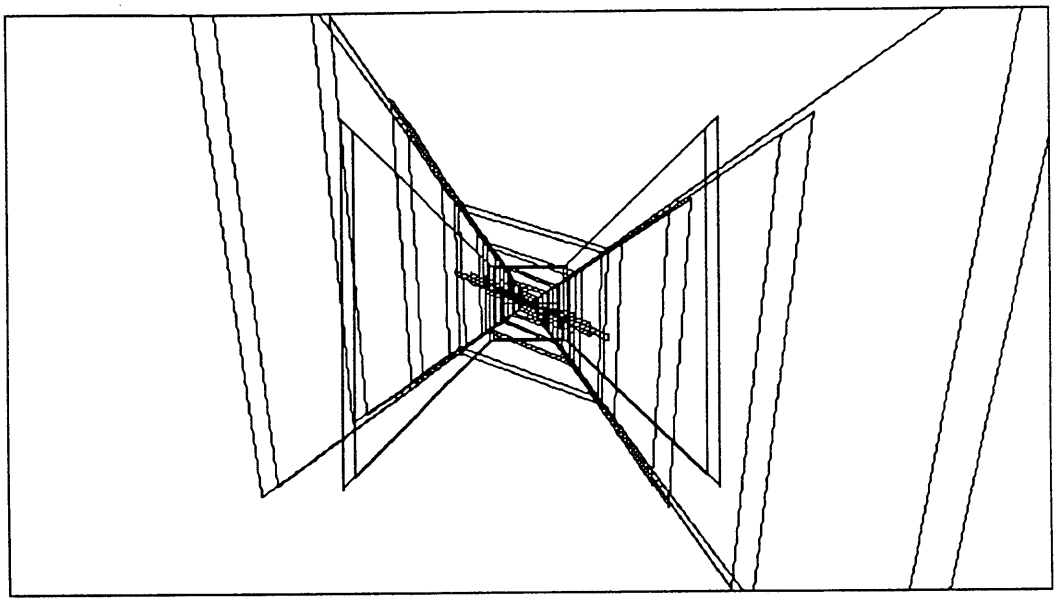
| | cylinder | pyramid | hex prism | cube | pre orth draw | post orth draw | VERTICAL PRE | VERTICAL POST |
|---|---|---|---|---|---|---|---|---|
| pre draw | | | | | top, front, side | top, side, front | | |
| post draw | | | | | | | | |

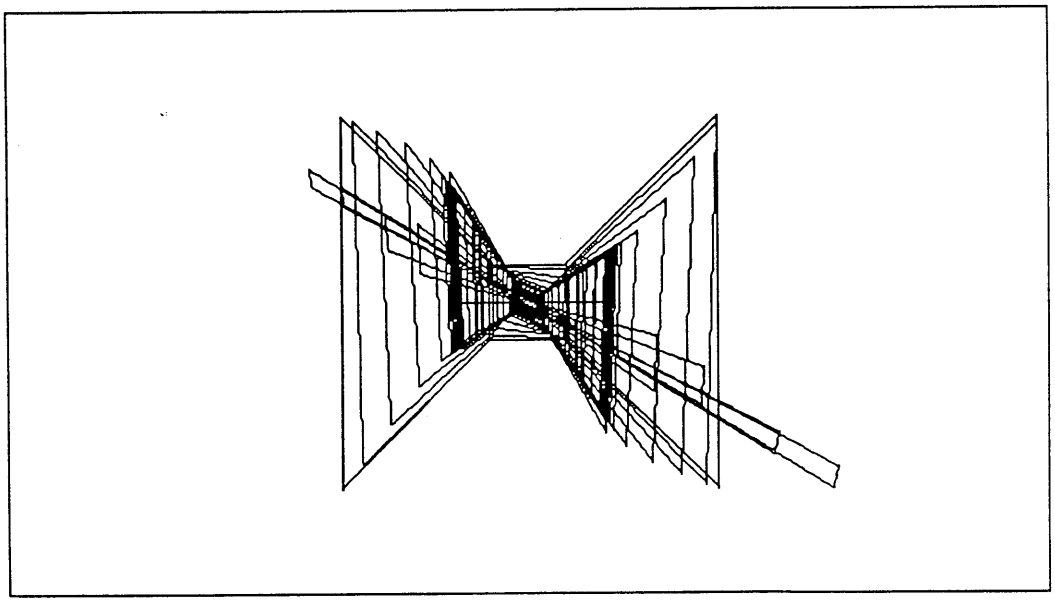
pre post

| | FRACTIONS | | | MEASURE | | MATH + - . ops | | ORTHOGRAPHIC VIEWS | | | MOUSE | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | pre | post | gen | pre | post | pre | post | | pre | post | pre | post |
| dec | 0/4 | 2/5 | - | 1/4 | 5/6 | 4.5/10 | 6/12 | obj | 3/3 | 4/4 | 5/8 | 8/8 |
| reg | 4/6 | 6/6 | 14 | 1.an | 2 d | | | scene | 3/3 | 3/3 | | |
| graph | 1/6 | 7/7 | | 2 | 4 | Word -> Draw pre | post | INC PRE | POST | | | |
| total | 5/16 | 15/18 | 14 | | | 25/3 | 2/2 | 2/5 | 5/6 | | | |









# JENNI'S PROJECT SCRIPTS

## jen.gfinal

```
;jen.gfinal final girl and eyes
;Mon. 4/11/1988, 13:44:04
call bow.asc  b
scale b 0.5 0.5 0.5
place b -0.4 1.5 0
call cone.asc  skirt
place skirt 0 -1 0
call cone.asc  top
scale top 0.6 0.6 0.6
place top 0 0.6 0
rotate top 1 x 180
call cube.asc  head
scale head 0.4 0.4 0.4
call tri.asc  le
scale le 0.1 0.1 0.1
place le -0.2 0.2 0.4
call tri.asc  re
scale re 0.1 0.1 0.1
place re 0.2 0.2 0.4
call sq.asc  mo
scale mo 0.3 0.05 0.1
place mo 0 -0.2 0.4
call circle.asc  nose
scale nose 0.1 0.1 0.1
place nose 0 0 0.4
rotate nose 1 x 90
group face 5  le  re  head  nose  mo
place face 0 1 0
call cube.asc  la
scale la 0.1 0.5 0.1
place la -0.5 0.1 0
call cube.asc  ra
scale ra 0.1 0.5 0.1
place ra 0.5 0.1 0
call cube.asc  lleg
scale lleg 0.1 0.5 0.1
place lleg -0.5 -1.5 0
call cube.asc  rleg
scale rleg 0.1 0.5 0.1
place rleg 0.5 -1.5 0
eye 3 3 10
dis 1
eye -3 3 10
dis 2
eye 0 0 10
dis 3
```

## jen.ani1ey

```
cal bow b
sca b 3 3 3
eye 0 0 10
dis
sca b 2.8 2.8 2.8
dis
sca b 2.6 2.6 2.6
eye 1.8 1.8 2.6
dis
sca b 2.4 2.4 2.4
dis
sca b 2.2 2.2 2.2
eye 2.6 2.6 3.4
dis
sca b 2 2 2
dis
sca b 1.8 1.8 1.8
eye 3.4 3.4 4.2
dis
sca b 1.6 1.6 1.6
dis
sca b 1.4 1.4 1.4
eye 4.2 4.2 6
dis
sca b 1.2 1.2 1.2
dis
sca b 1 1 1
eye 5.2 5.2 6.1
dis
```

```
sca b .9 .9 .9
dis
sca b .8 .8 .8
eye 6.3 6.3 7.4
dis
sca b .7 .7 .7
dis
sca b .6 .6 .6
eye 7.5 7.5 8.9
dis
sca b .5 .5 .5
dis
sca b 1 .4 1
eye 8.8 8.8 10.6
dis
sca b 1.5 .3 1.5
dis
sca b 2 .2 2
dis
sca b 2.5 .1 2.5
eye 10.2 10.2 12.5
dis
sca b 3 .1 3
dis
```

## jen.ani2e

```
eye 3 3 3
cal bow b
sca b 3 .1 3
dis
sca b 2.5 .1 2.5
dis
sca b 2 .2 2
eye 3.3 3.3 3.7
dis
sca b 1.5 .3 1.5
dis
sca b 1 .4 1
eye 3.6 3.6 4.4
dis
sca b .5 .5 .5
dis
sca b .6 .6 .6
eye 3.9 3.9 5.1
dis
sca b .7 .7 .7
dis
sca b .8 .8 .8
eye 4.2 4.2 5.8
dis
sca b .9 .9 .9
dis
sca b 1 1 1
eye 4.9 4.9 6.5
dis
sca b 1.2 1.2 1.2
dis
sca b 1.4 1.4 1.4
eye 5.7 5.7 7.3
dis
sca b 1.6 1.6 1.6
dis
sca b 1.8 1.8 1.8
eye 6.5 6.5 8.1
dis
sca b 2 2 2
dis
sca b 2.2 2.2 2.2
eye 7.4 7.4 9
dis
sca b 2.4 2.4 2.4
dis
sca b 2.4 2.4 2.4
eye 8.3 8.3 10.1
dis
sca b 2.6 2.6 2.6
dis
sca b 2.8 2.8 2.8
eye 10.3 10.3 12.1
dis
```

```
sca b 3 3 3
dis
eye 10 10 10
dis
eye 8 8 10
dis
eye 6 6 10
dis
eye 4 4 10
dis
eye 2 2 10
dis
eye 0 0 10
dis
```

## jen.sfin

```
;jen.sfin  snake
;Wed. 5/11/1988, 14:16:26
eye  -10 -10 30
call hut.asc  h
scale h 0.8 0.8 0.8
place h -7 0 0
rotate h 1 z 90
call cube.asc  y
scale y 0.7 6 0.7
rotate y 1 z 90
call cone.asc  c
scale c 0.7 0.7 0.7
place c 6 0 0
rotate c 1 z -90
call circle.asc  cc
scale cc 0.2 0.2 0.2
place cc -7 0.3 0.8
rotate cc 1 x 90
call circle.asc  cl
scale cl 0.2 0.2 0.2
place cl -7 -0.3 0.8
rotate cl 1 x 90
call cylnd.asc  l
scale l 0.2 0.6 0
place l -8.5 0 0
rotate l 1 z 90
call tri.asc  tr
scale tr 0.5 0.5 0.5
place tr -5 0.7 0.7
rotate tr 1 z 180
call tri.asc  tr1
scale tr1 0.5 0.5 0.5
place tr1 0 0.7 0.7
rotate tr1 1 z 180
call tri.asc  tr2
scale tr2 0.5 0.5 0.5
place tr2 5 0.7 0.7
rotate tr2 1 z 180
call circle.asc  e
scale e 0.9 0.9 0.9
place e -2.5 0 0.7
rotate e 1 x 90
call circle.asc  1
place 1 2 0 0.7
rotate 1 1 x 90
call tri.asc  3
scale 3 0.5 0.5 0.5
place 3 -1 -0.7 0.7
call tri.asc  4
scale 4 0.5 0.5 0.5
place 4 -4 -0.7 0.7
call tri.asc  5
scale 5 0.5 0.5 0.5
place 5 4 -0.7 0.7


8 2
title bow jen
-.5 .5 0
-.1 .1 0
-.1 -.1 0
-.5 -.5 0
12 8 1 2 3 4 5 6 3 6 7 2 7
12 1 8 7 2 7 6 3 6 5 4 3 2
```

```
==========================================================================
=====================< OBJECT COMMANDS >====================
call <data> <object>                          [read data and give instance name]
on|off <object|all>                           [on/off for display]
bkface <obj> <0|1>                            [1 turns off backface cull]
reset <object|all>                            [resets to default values]
=====================< OBJECT MANIPULATIONS >===================
scale <object> <x y z>                        [scale object along each axes]
place <object> <x y z>                        [translation to point in space]
rotate <object> <num> <axis> <angle> [<axis><angle>] [<axis><angle>]
     [rot object num,1-3, about specified axis, and concat in order entered]
=====================< OBJECT HEIRARCHY >===================
attach <obj> <to-obj>                         [attaches one object to another ]
group <name> <num> <objects>                  [ group name to attach objects to]
detach <object> <from obj> [<0|1>]            [1 (default) leaves parent xform]
=====================< OBJECT INFORMATION >===================
show [<object|all|view|hier>]                 [shows info of named parameters]
show                                          [lists all objects called ]
==========================================================================
=====================< GLOBAL COMMANDS >====================
eye <x y z>                                   [places camera]
coi <x y z>                                   [center of interest, where  point camera ]
up <x y z>                                    [orientation of camera, up 01 0]
view <angle>                                  [angle of view of camera from eyepoint]
reset <view>                                  [resets to default values]
display [<quad> <per=0|orth=1>]               [ fullscreen =0,  quad > 1 | 2 | 3 | 4]
bluprint [x y z]                              [display oblique, front, side, & top views]
clear <0|1>                                   [1= clears screen    0= noclear]
delete [1]                                    [1= forces delete all object, else asks]
=====================< SCENE INFORMATION >===================
;<any thing you want>                         [computer ignors lines that start with ;]
@<filename>                                   [reads in a file]
save                                          [saves environment to a file]
history                                       [keeps a record of all commands]
nohistory                                     [stops record of all commands]
print [quad]                                  [prints screen or quadrant]
pause                                         [allows  pause from script]
==========================================================================
-----------------------< DEFAULT  VALUES >-----------------------
eye=0 0 10   coi=0 0 0   vie=45   up=0 1 0   cen=0 0   cle=1   on
sca=1 1 1    pla=0 0 0   rot=0    bkf=0      dis=0 0   blu=3 3 10
```

**Objects:**

| | | | | | |
|---|---|---|---|---|---|
| cylnd | bcylnd | cube | bcube | cut | pyr |
| circle | soccer | cone | hut | sq | nsq |

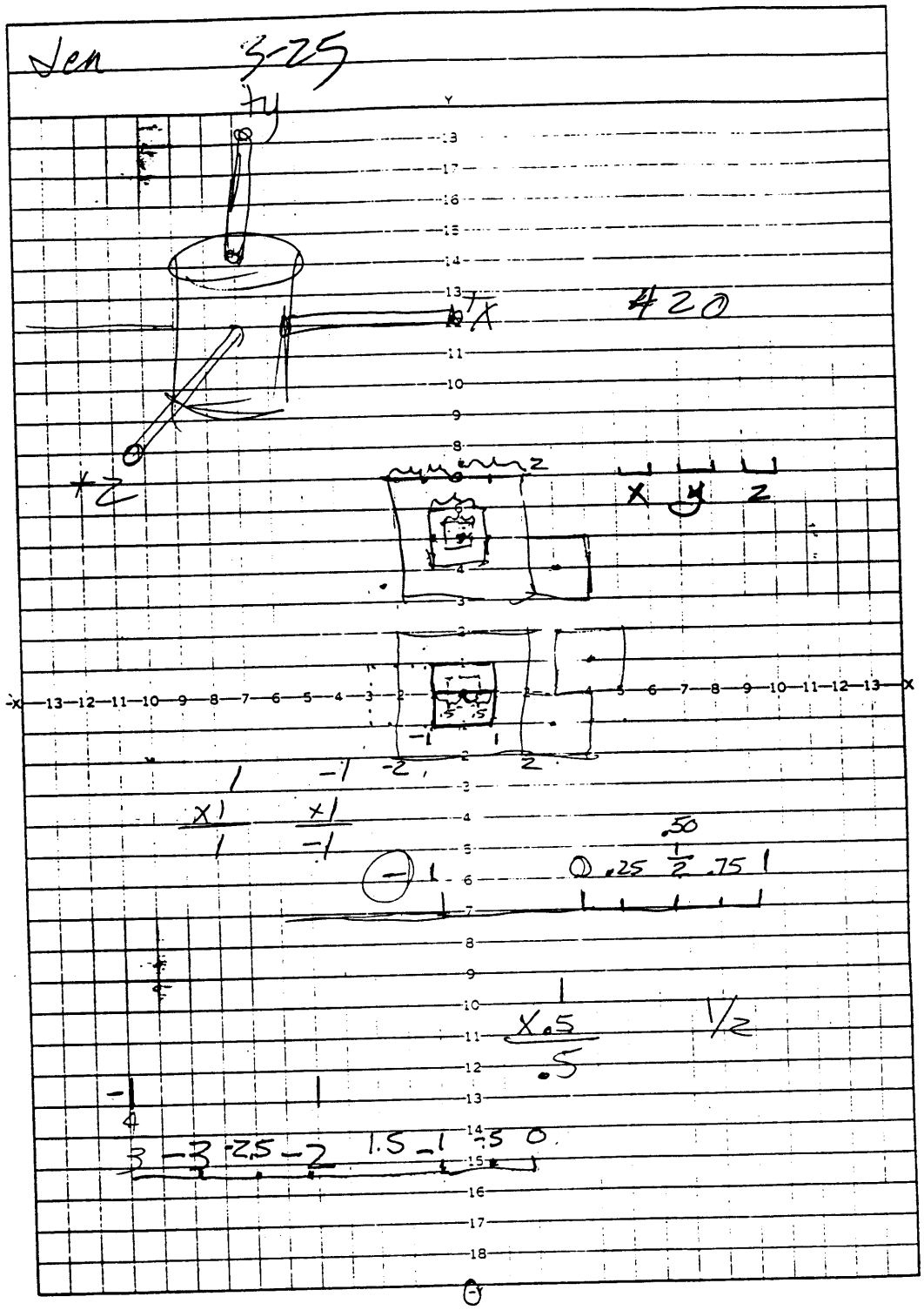
Name Jenni

Today's Date 3/28/88

I learn a lot of things. How to rotate cylinderss and oct. I like having Judy for my 2 computer teacher. She is fun.

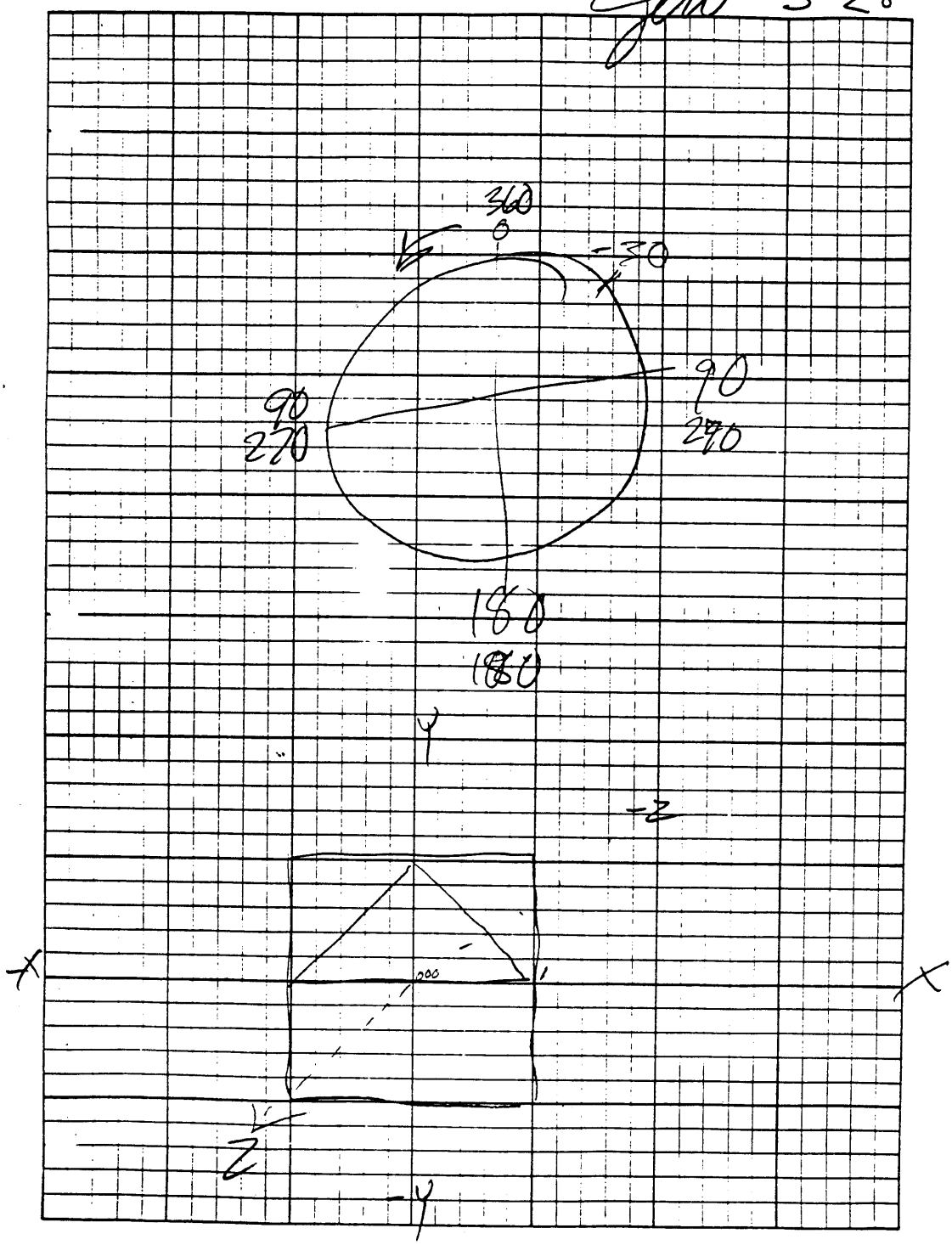
3-30 I like this a lot of making any design.

page 1




Quadratic Coordinates

Jan 3-28

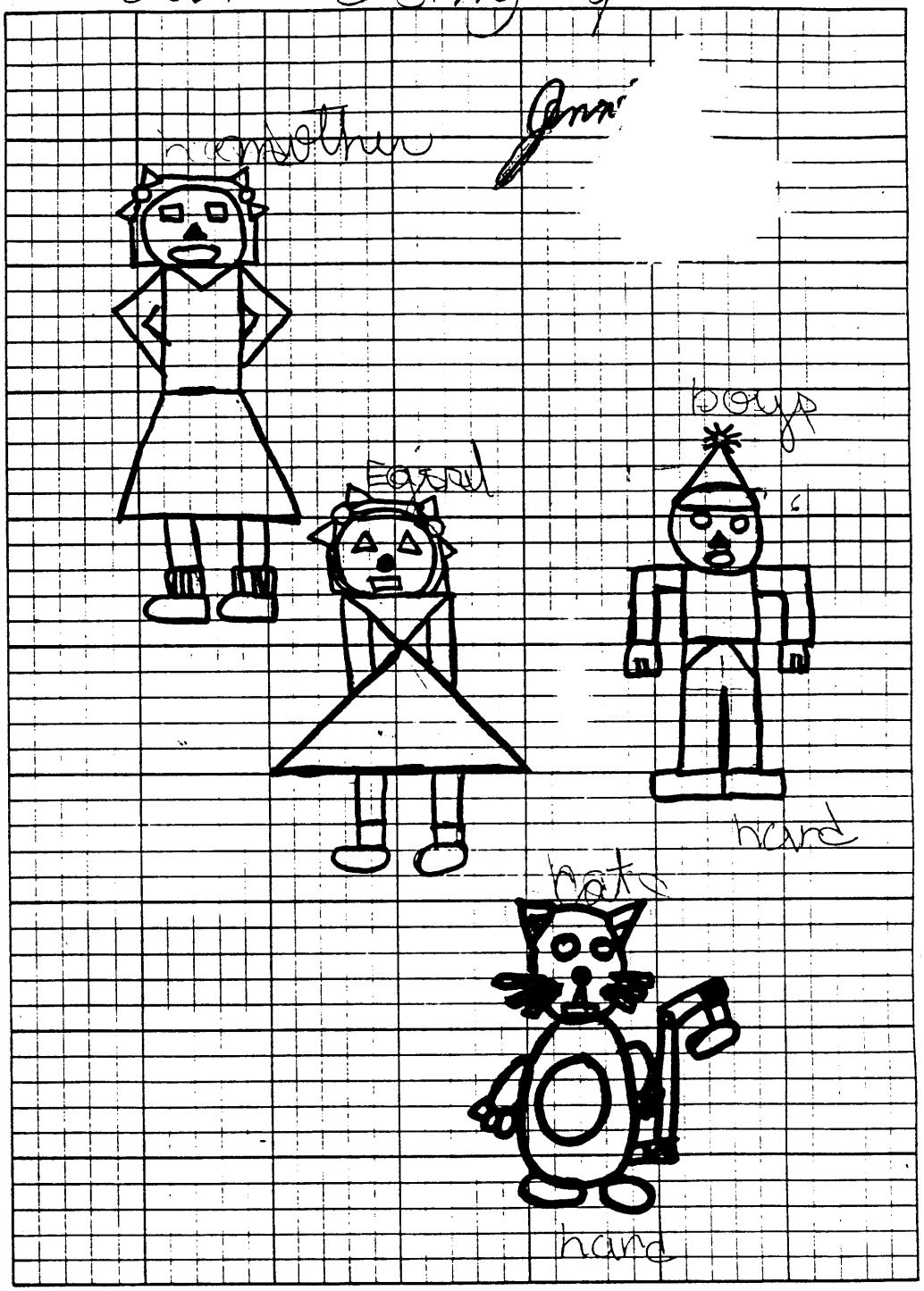


Name [illegible]

Today's Date 4-4-88

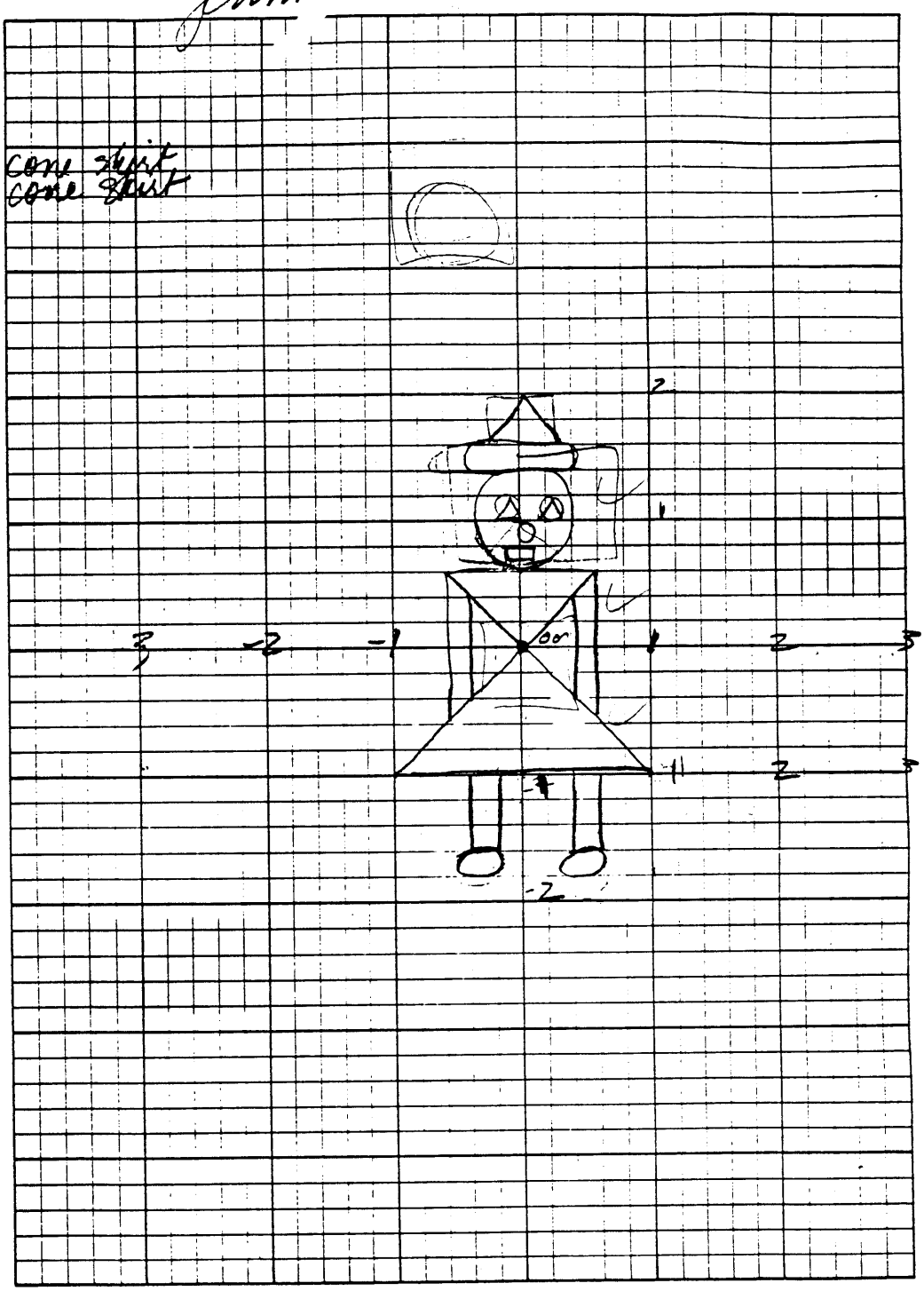
What's about my design. It's outragous.

~~disine~~ Dising by



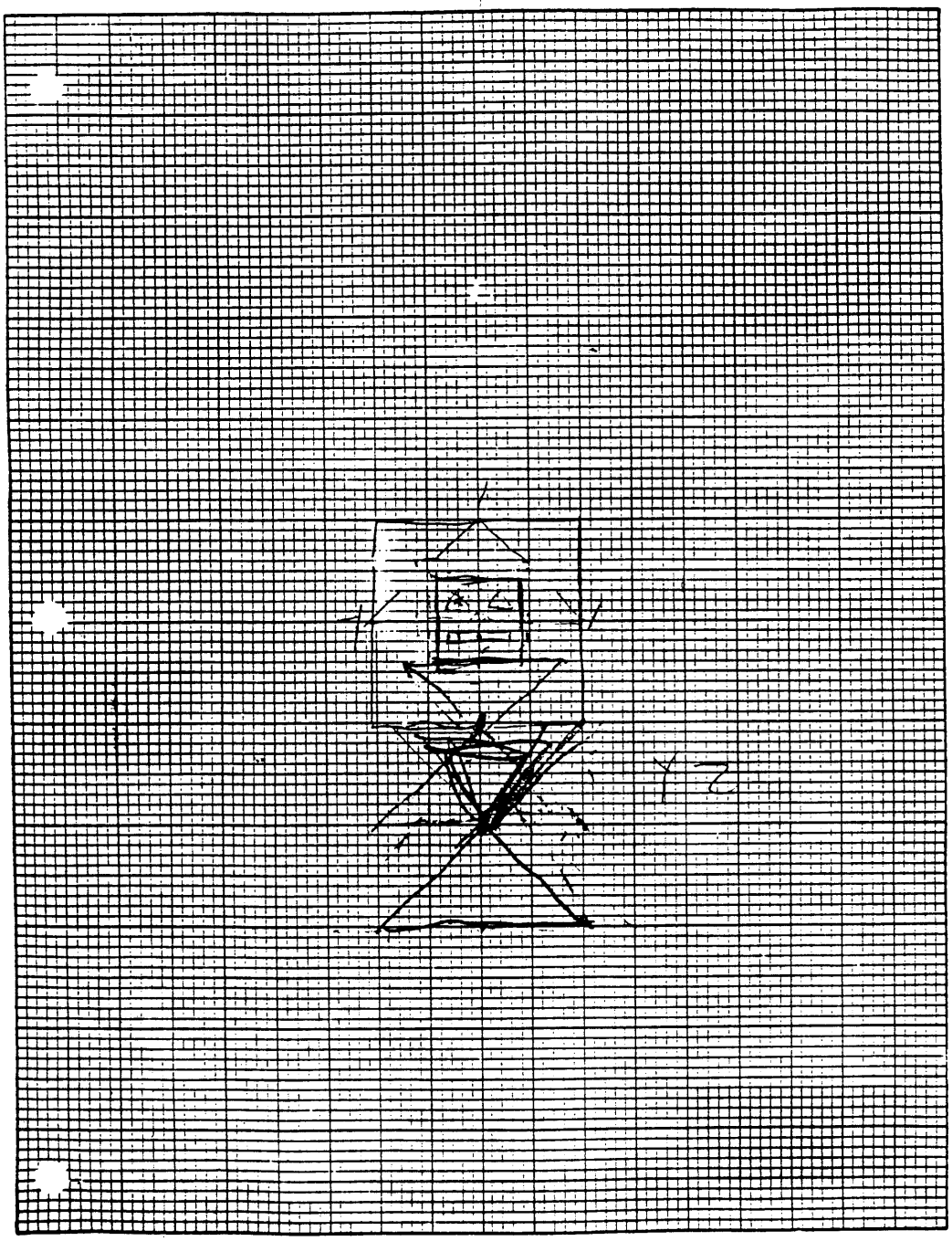
Jim

cone shirt
cone shirt



Sen 4-4-88

Form 680—Rev. 7/67
Gen. Stm. 253640



Name Jenni

Today's Date 4-7-88

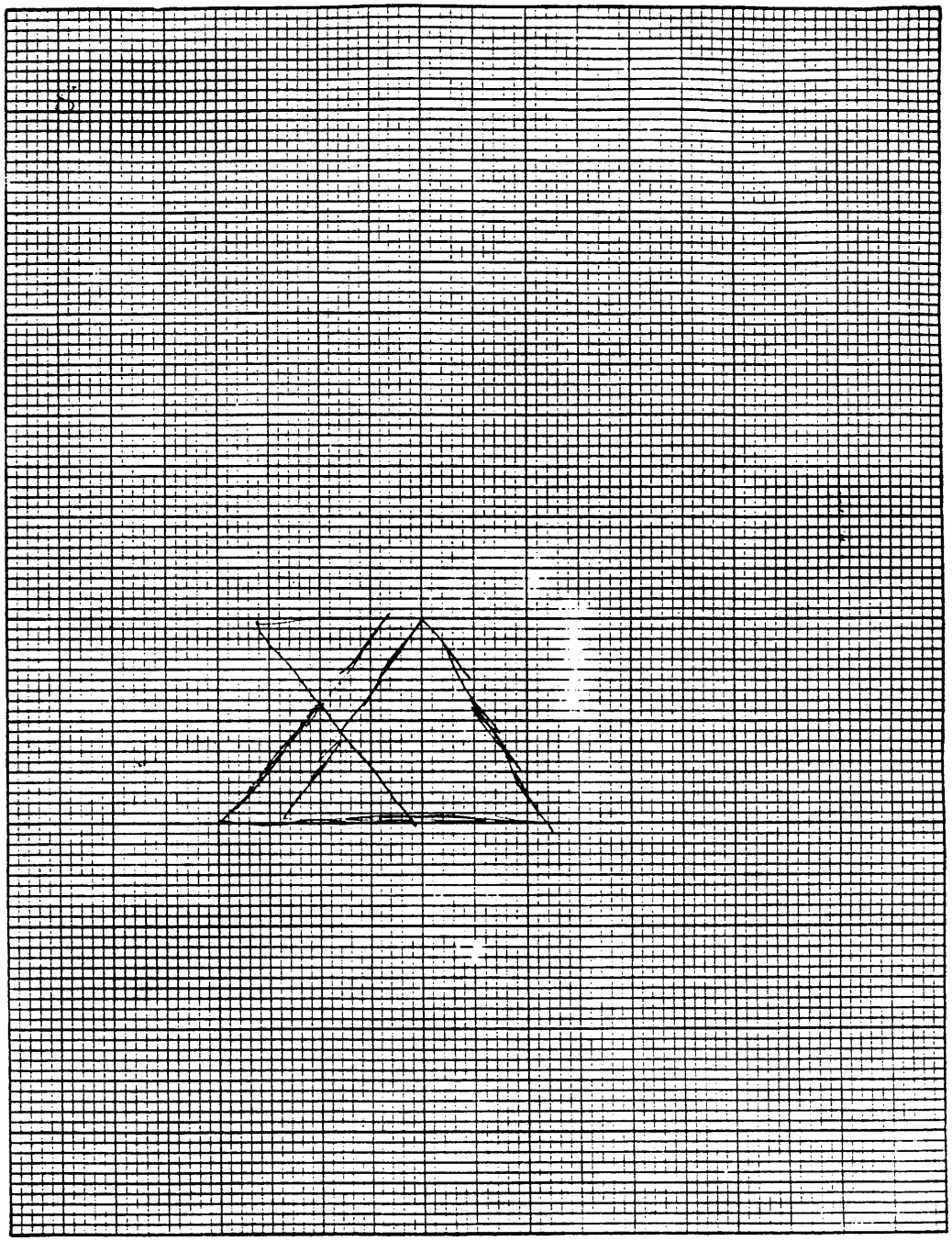
I'm confused in the bow and the dot-to-do. I need more help from Judy. This is getting hard.

7-10

-.4 15 0

1 3310
2 -3310
3 0010

4to-88 sca- 0.1, 0.5, 0.1 Legs pla- 0.5, 0.1, 0

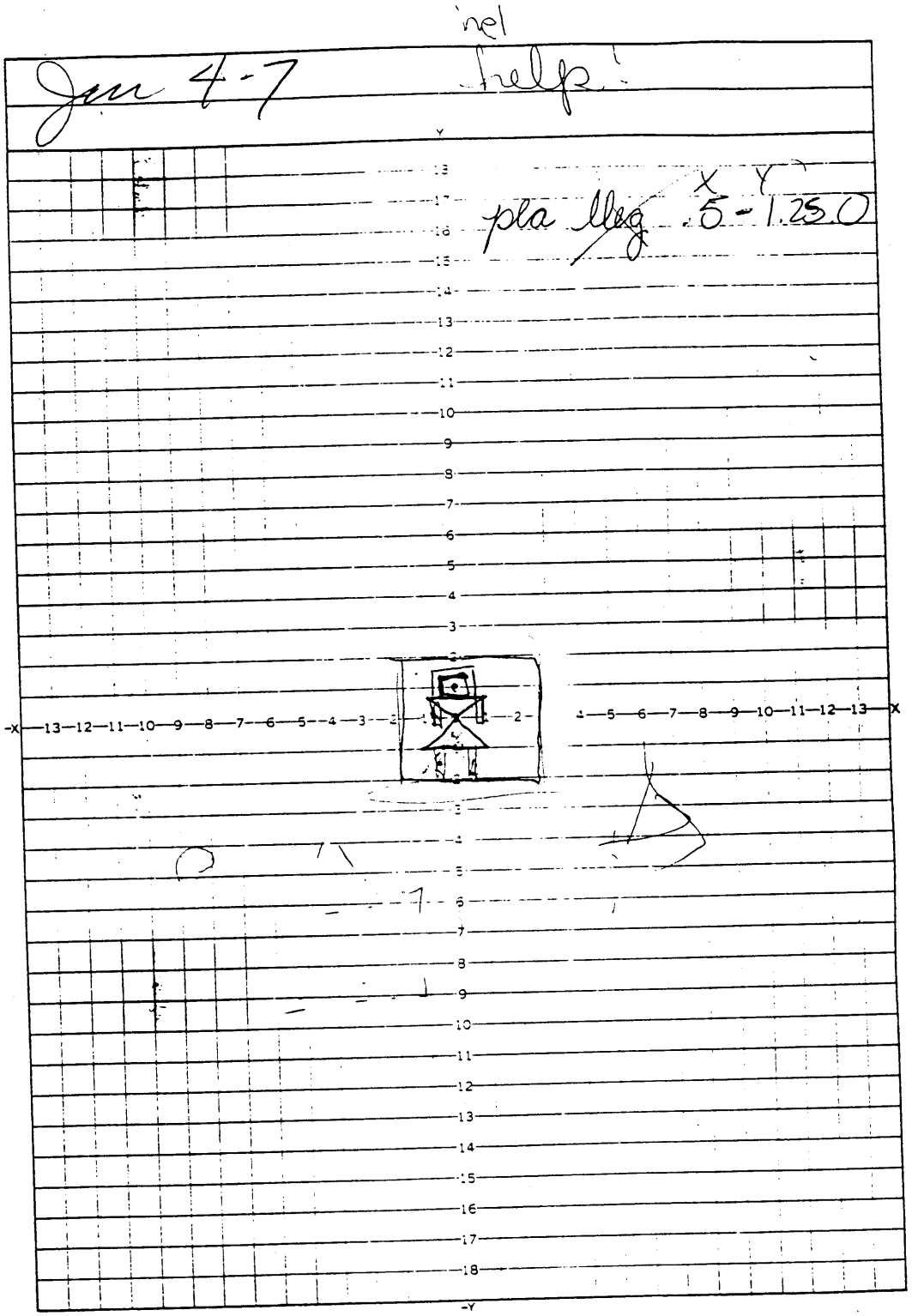


Form 680—Rev. 7-67
Gen. Strs. #53540

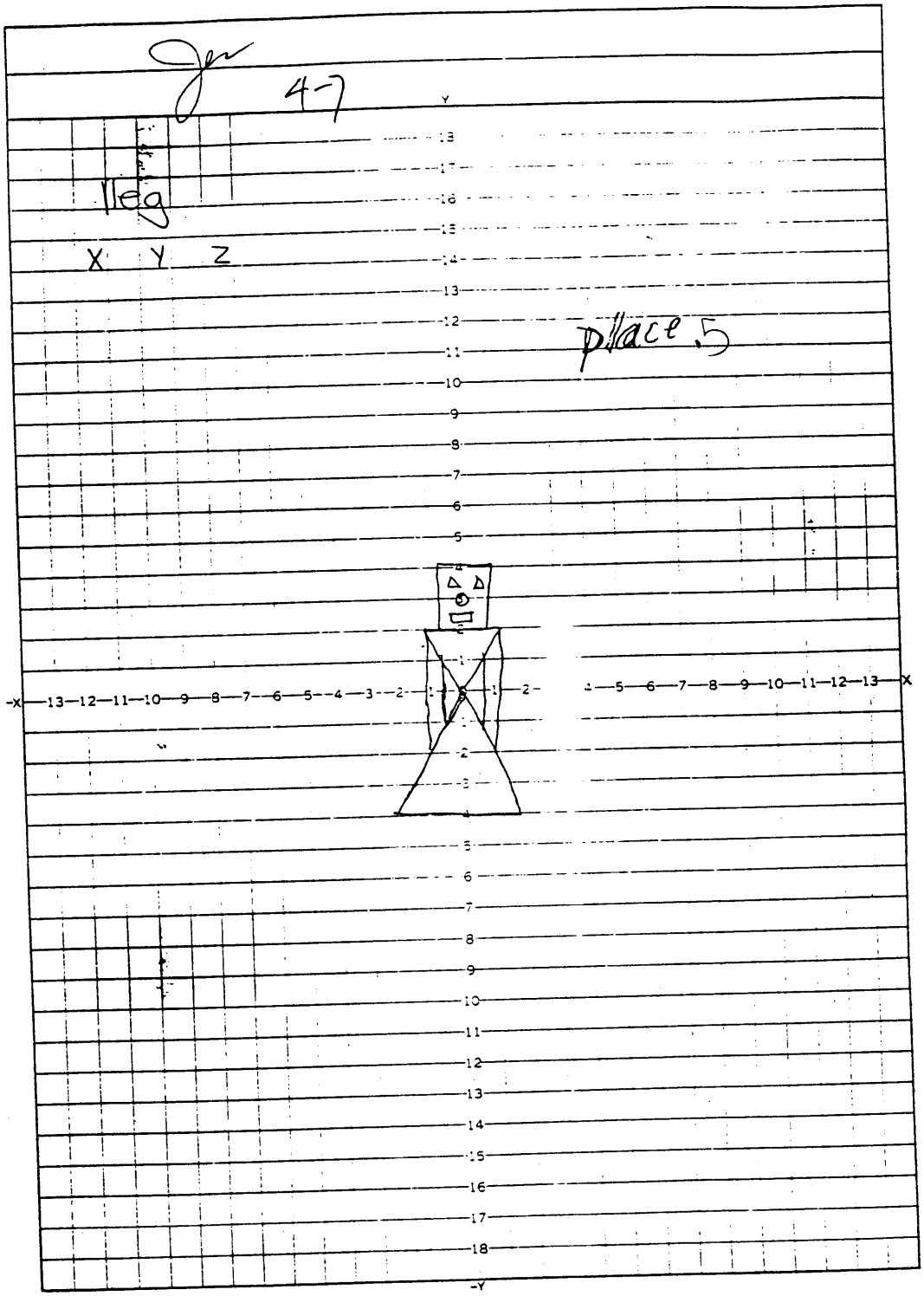
Name Jenni

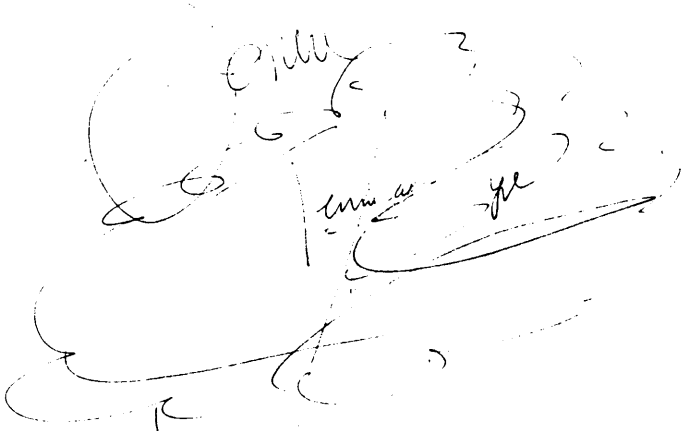
Today's Date 4-6-88

I learn how to do the arms and legs. Judy is nice. This is no very complecated.




Quadratic Coordinates

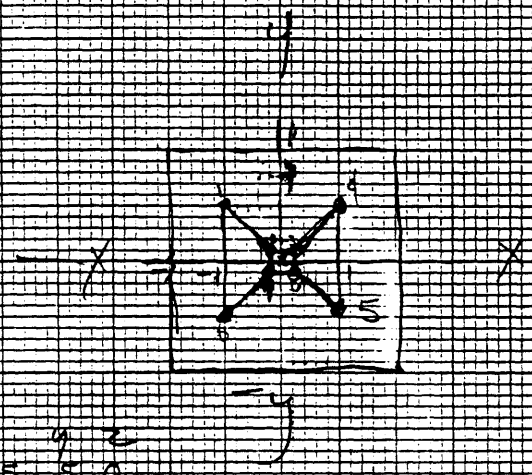

Jr
47
leg
X Y Z
place 5
Y
-Y
-X
X



help,
Judy, help
me.

Jenni

4-7-88




| | x | y | z |
|---|---|---|---|
| 1a | -.5 | .5 | 0 |
| 2a | -.1 | .1 | 0 |
| 3a | .1 | .1 | 0 |
| 4a | -.5 | .5 | 0 |
| 5a | .5 | -.5 | 0 |
| 6a | .1 | -.1 | 0 |
| 7a | .1 | .1 | 0 |
| 8a | -.5 | -.5 | 0 |

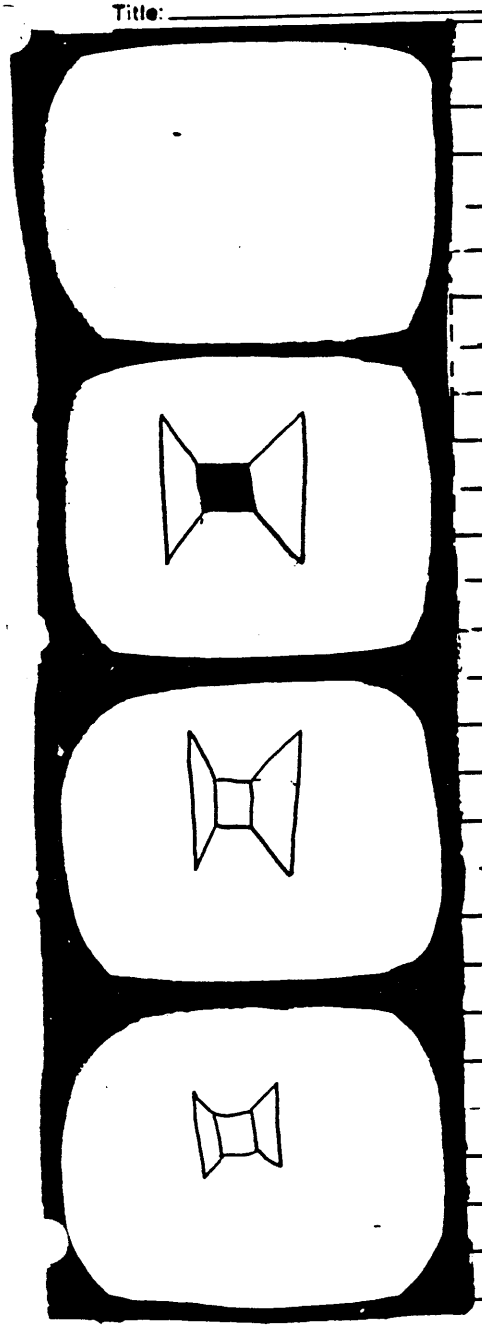
Form 680—Rev. 7/67
Gen. Stra. #53640

Page _____ of _____ Pages

# Script Sheet

Date _____ 19 ____

Title: _____ Draft Number: 1 • 2 • Final



## Script

Interactive Training Systems, Inc.

Jan 4-13

Page ____ of ____ Pages **Script Sheet** Date ____ 19__

Title: ____ Draft Number: 1 • 2 • Final

**Script**

big bow=scale:

| X | Y | Z |
|---|---|---|
| 3 | 3 | 3 |

medium bow scale:

| X | Y | Z |
|---|---|---|
| 2 | 2 | 2 |

small bow scale:

| X | Y | Z |
|---|---|---|
| 1 | 1 | 1 |

tiny bow scale:

| X | Y | Z |
|---|---|---|
| .5 | .5 | .5 |

medium squash bow scale:

| X | Y | Z |
|---|---|---|
| 2 | -.4 | 2 |

squash bow scale:

| X | Y | Z |
|---|---|---|
| ~~3~~ | ~~.1~~ | ~~3~~ |
| 3 | -.1 | 3 |

Interactive Training Systems, Inc.

Page 1 of bow Pages

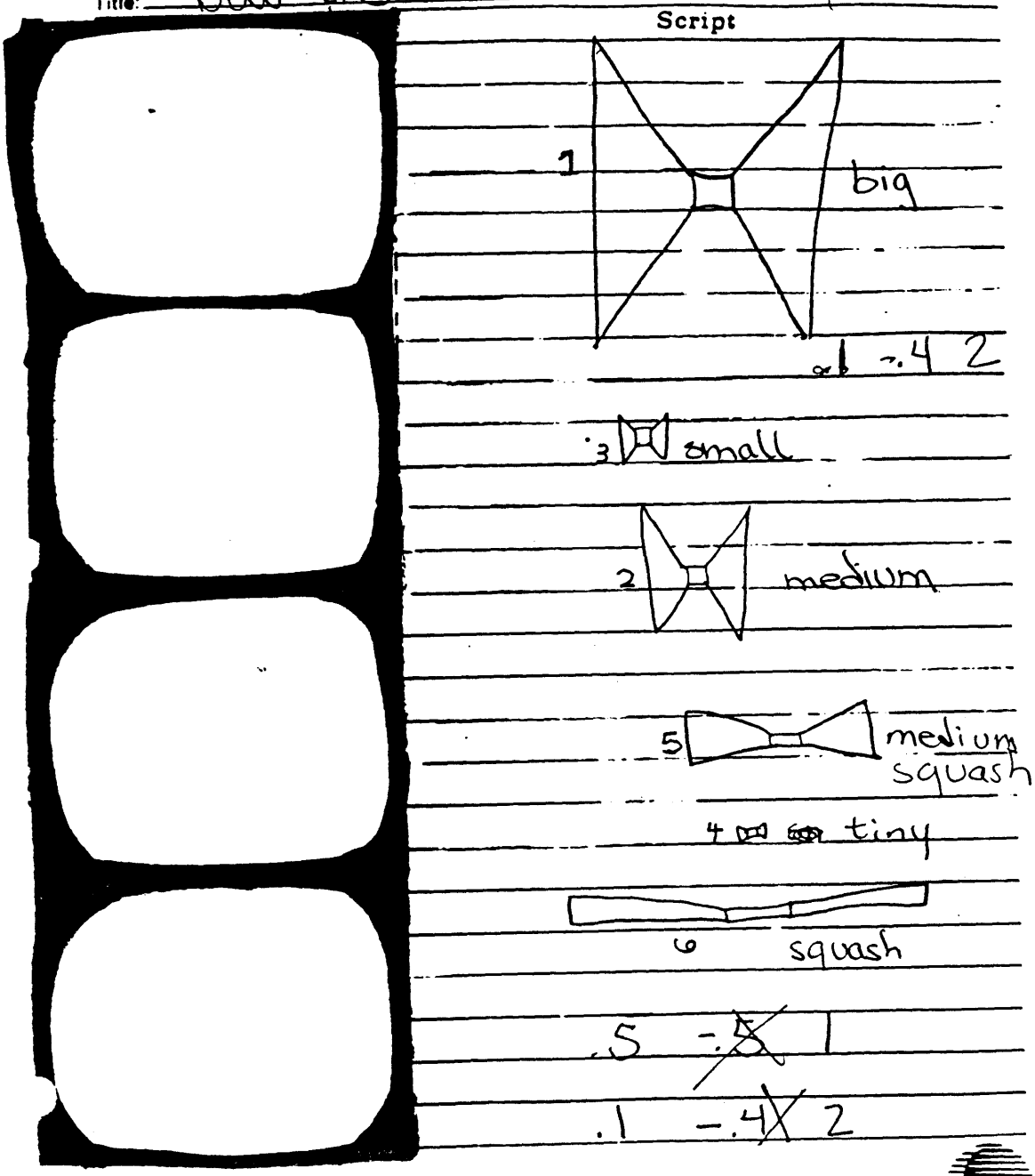
**Script Sheet**

Date 4/13/88 19

Title: Bow bRB

Draft Number: 1 • 2 • Final

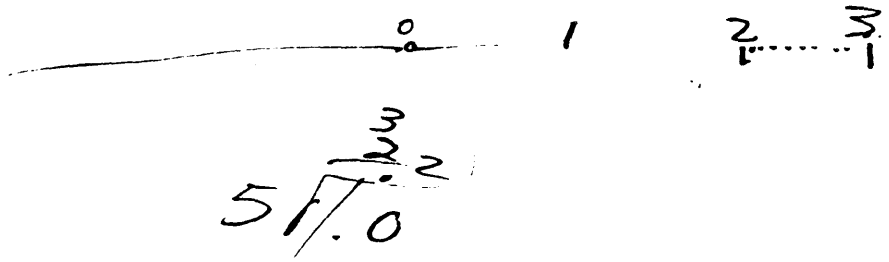
**Script**




Interactive Training Systems, Inc.

Name_____

Today's Date_____

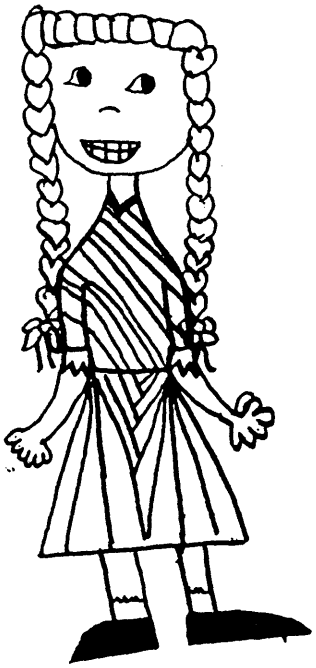


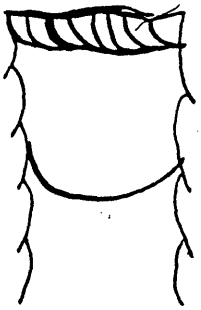

3
2.8
2.6
2.4
2.2
2.0

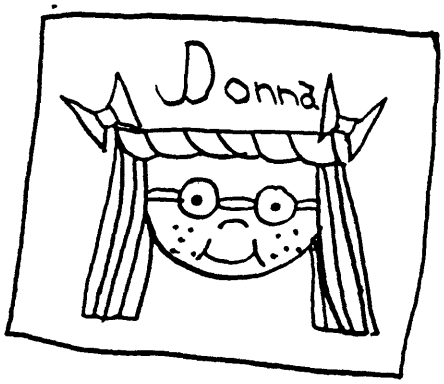
page 1

Name_____

Today's Date_____



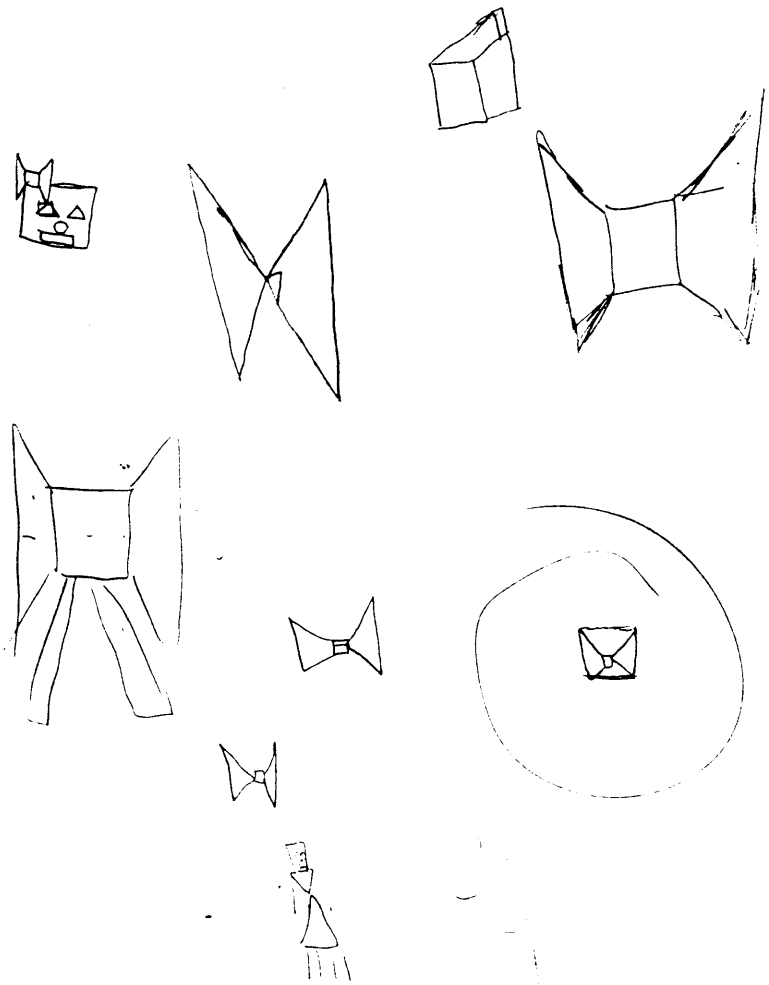





page 1

Name_____

Today's Date_____



page 1

4-25-88 Jen

~~Start~~ end 0-0-10

eye 333

bow - stitched
eye 333

0 0 10

eye 2-1 3

gen. an 1

EZ 11

3 3 3

0 0 10 | 33.7

10 | 3.0 .3 10 | 7.0 .7

3 3 3
3.3 3.3 3.7
3.6 3.6 4.4

3.7
.7
4.4

place

place

Name _____

Today's Date _____

| | |
|---|---|
| 1.0 | 1.8 |
| +.8 | +.8 |
| 1.8 | 2.6 |
| +.8 | +.8 |
| 2.6 | 3.4 |
| +.8 | +.8 |
| 3.4 | 4.2 |
| +.8 | +.8 |
| 4.2 | 5.0 |

| | |
|---|---|
| 4.2 | 5.0 |
| +.9 | +1.0 |
| 5.1 | 6.0 |
| +.9 | +.9 |
| 6.0 | 6.8 |
| +1.0 | +1.0 |
| 7.0 | 7.8 |
| +1.0 | +1.0 |
| 8.0 | 8.8 |

page 1

Name Jenn

Today's Date 4/25/88

~~edit~~
~~eye points~~

I was confused at the first time. But then Judy helped me a lot. I think I am finished with my animation. I like Judy a-lot. Judy is a good helper. She is nice and sweet. I thank her a-lot.

Name Jenni

Today's Date 4-27-88

I have finished
my animation. It's finished.
Now I'm doing my own
thing. I think this is
going to be fun and
easy. Judy is a good-
helper. I have learn to
do many things and to
run the H3D. She is
very fun. She helps me
with my work and
she takes every thing
very sereously. I understood
every thing.

page 1

Name_____

Today's Date_____

to del. design
off ____
{ sint
pier
letter }

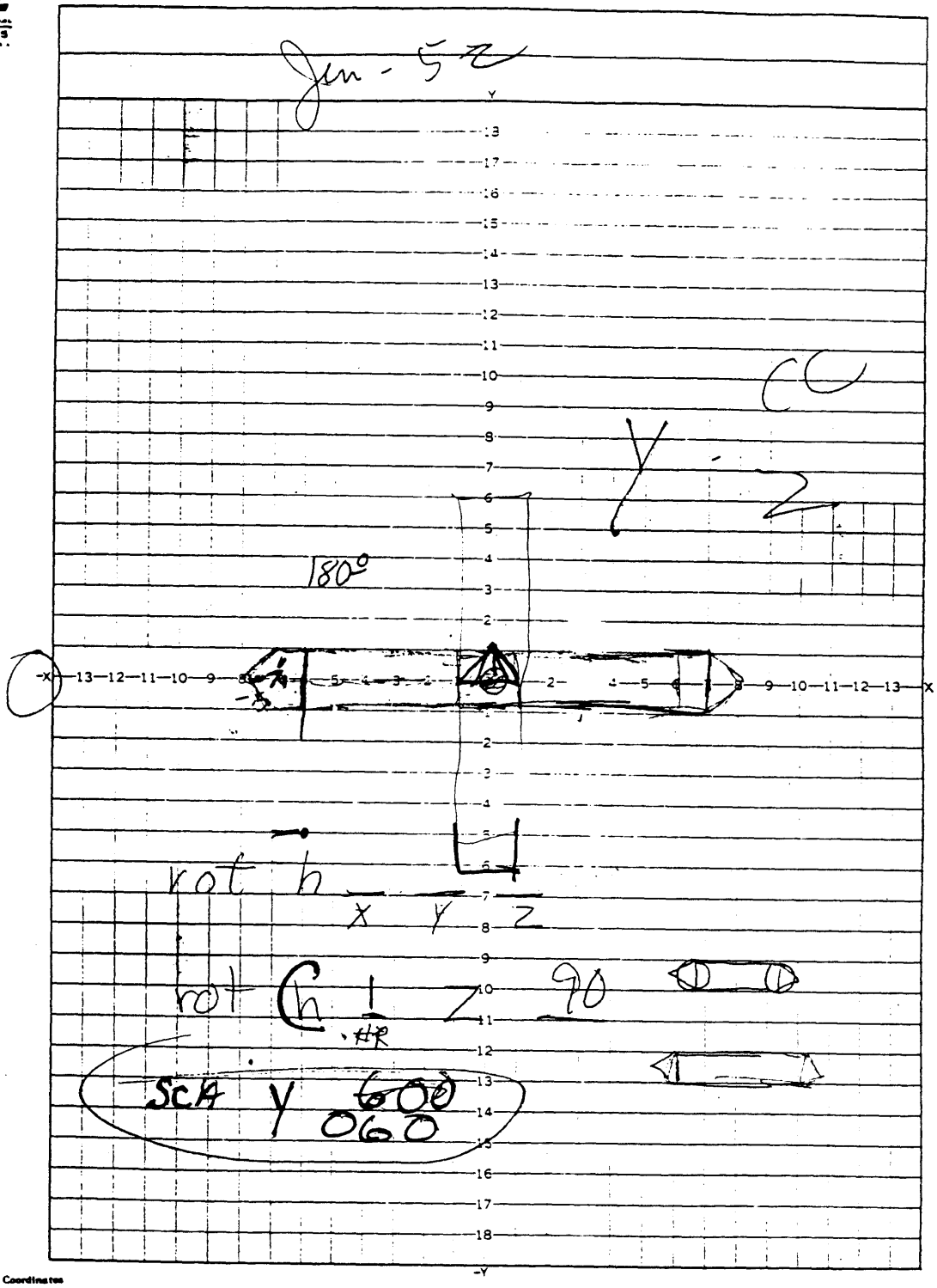
cat dog d
cat horse h
cat rhino r
cat gator g
cat gull l
cat arrow a
cat cut c
cat lion n
cat nsq q

Name Jenni

Today's Date 4-27-88

| | |
|---|---|
| 4.2 | |
| +1.0 | |
| 5.2 | 6.0 |
| +1.1 | +1.1 |
| 6.3 | 6.1 |
| +1.2 | +1.3 |
| 7.5 | 7.4 |
| +1.3 | +1.5 |
| 8.8 | 8.9 |
| +1.4 | +1.7 |
| 10.2 | 10.6 |
| | +1.9 |
| | 12.5 |

page 1

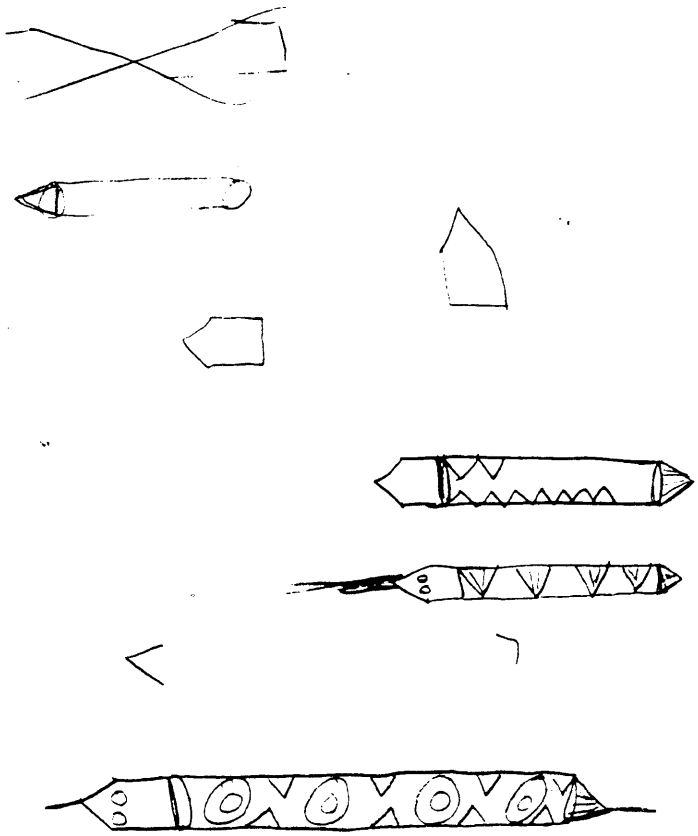



Quadratic Coordinates

Name Sam

Today's Date 5-2-88

Snake



page 1

Name Jenni

Today's Date 5-2-88

Today I'm doing my
own animation.
It's a snake.
It's fresh.
Bye-Bye'
I hafta
Leave.

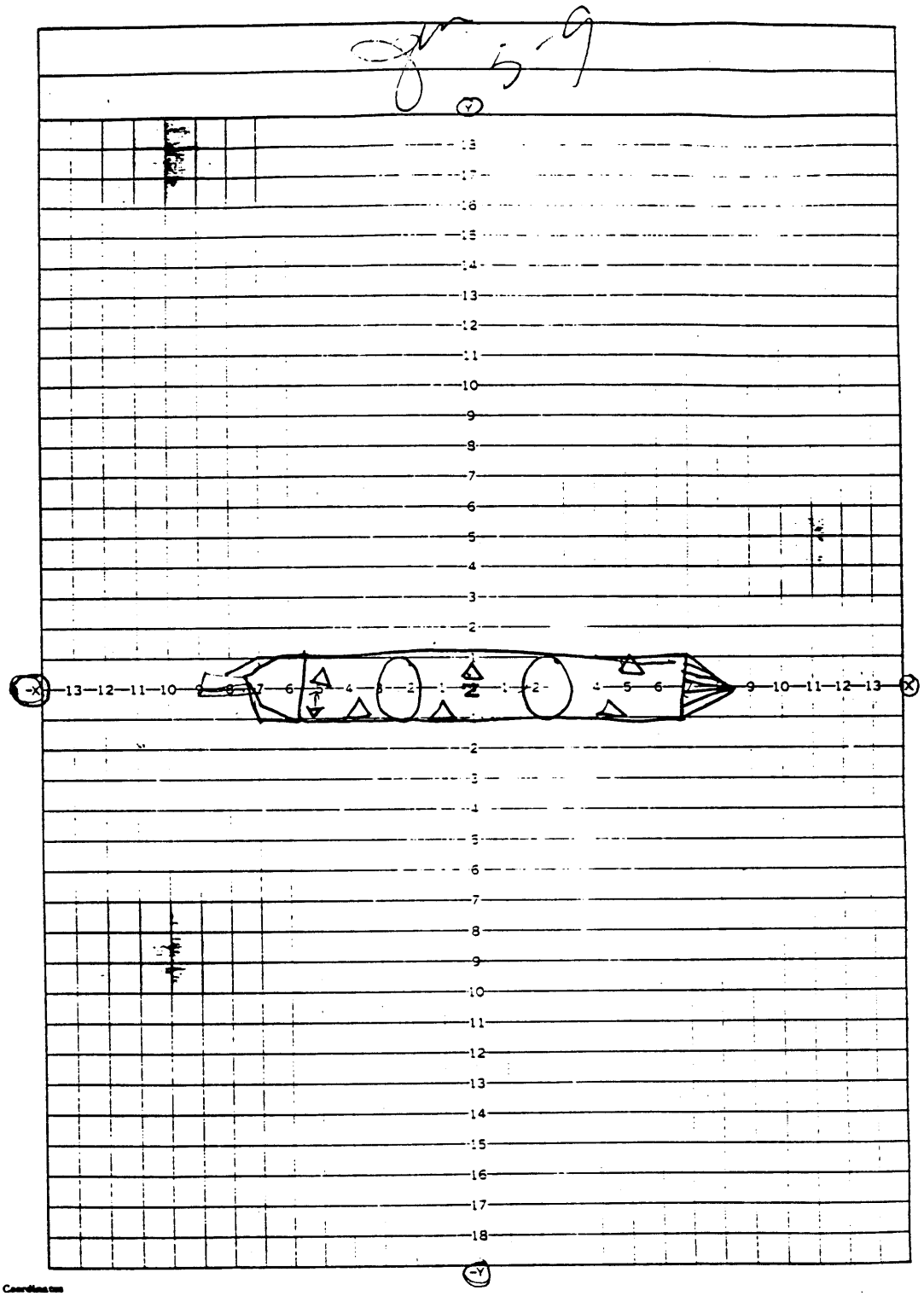
page 1

Name_____

Today's Date 5-4-88

I finished
my snake.
It's fresh.
Have ~~to~~
run.
See you!

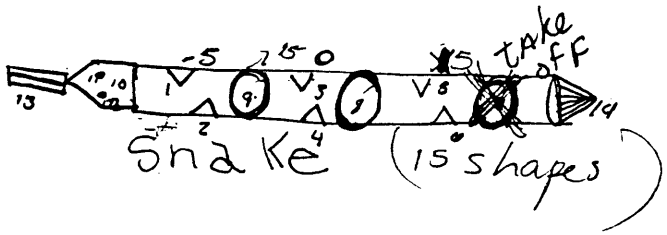
page 1




Quadratic Coordinates

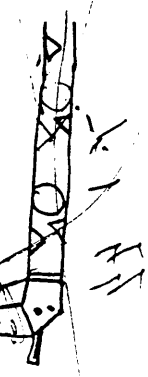
Name Jen.

Today's Date 5-9-88




<rot tr Z 180>

(rot tr 1 Z 180)



rot 4 1 Z 90
rot 5 1 Z 90

-5 5 5 5 5

-5 7 1

page 1

Name dennis

Today's Date 5-11-88

Today I finished my animation of the snake. It was great. Judy helps me alot. She is a good helper

Bye.

Love Judy

page 1