In-Class Problems — Week 12, Wed

Problem 1. What are each of the following quantities when n independent Bernoulli trials are carried out with probability of success p_s? (Note: at most you are allowed n trials, not an infinite number of trials)

(a) The probability of no failures.

(b) The probability of at least one failure.

(c) The probability of at most one failure.

(d) The expected number of failures in n trials.

(e) The expected number of trials for the first failure.

Problem 2. Each bag of Doritos contains a cool sticker. There are n different kinds of sticker, and I want to collect at least one sticker of each kind. (Assume that sticker kinds in Dorito bags are uniformly random and mutually independent.)

(a) Suppose that I have already collected k kinds of sticker. What is the expected number of additional bags of Doritos that I must eat to collect one additional kind of sticker?

(b) What is the expected number of bags of Doritos that I must eat to collect at least one sticker of each kind?

Problem 3. **(a)** Suppose that I roll a 4-sided die, a 6-sided die, an 8-sided die, a 10-sided die, a 12-sided die, and a 20-sided die. What is the expected number of 6’s that come up? (Assume that all of the dice are fair.)

(b) Suppose that I roll n dice that are 6-sided, fair, and mutually independent. What is the expected value of the largest number that comes up?

Hint: You may want to use the “alternative” formula for expectation: $\mathrm{E}[M] = \sum_{i=0}^{\infty} \Pr\{M > i\}$. (This formula is valid since the random variables involved are non-negative.)

A Appendix

Definition 3.1. The *expectation* of random variable, R, is:

$$\mathrm{E}[R] ::= \sum_{r \in \text{range}(R)} r \cdot \Pr\{R = r\} \tag{1}$$

If R has range $\mathbb{N}$, then this definition can also be written as

$$\mathrm{E}[R] = \sum_{r \in \mathbb{N}} \Pr\{R > r\}$$

Series:

$$\begin{aligned}
\sum_{i=0}^{\infty} x^i &= \frac{1}{1-x}, \text{ if } |x| < 1 \\
\sum_{i=0}^{n} x^i &= \frac{1-x^{n+1}}{1-x} \\
\sum_{i=1}^{\infty} i x^i &= \frac{x}{(1-x)^2} \\
\sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^i &= np \\
H_n &= \sum_{j=1}^{n} \frac{1}{j} \sim \ln n
\end{aligned}$$